NOVEL ASPECTS OF VERTICAL GROUND MOTION MODELLING IN EARTHQUAKE ENGINEERING

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Abstract

The analysis of the vertical component of site ground motion is one of the main challenges in Geotechnical and Structural Engineering due to recent observations of failure and damage of buildings and structures under seismic action. The most common approach employed in current engineering practice for saturated soils is the simplified u-p formulation of the Biot's equations describing the coupled hydro-mechanical behaviour, although more refined formulations are available (e.g. the u-U formulation), which include all the inertial terms. The aim of this study is to perform a novel theoretical validation of the u-p formulation as compared with the u-U formulation for different levels of permeability and dynamic actions that are representative of a wide scenario of site ground properties and seismic hazard in the vertical direction. A detailed analysis of the response in term of acceleration and pore pressure time history, frequency content, acceleration response spectrum, and amplification rate of acceleration. This allows to extend the discussion of the limits of applicability of the u-p formulation to the context of a complex dynamic regime provided by the vertical components of real earthquake records.

1. Introduction

It is well known that the detrimental effects of earthquakes in terms of damage to buildings, structures and bridges may arise from both the horizontal and the vertical components of site ground motion. However, attention is usually given to the horizontal component, as it is considered to be more relevant for different constructions, although this consideration is not fully general. Therefore, seismic protection systems are investigated mainly for the horizontal component of seismic actions (Larkin, 2008, Carta et al., 2016), despite constructions may experience relevant damages due to the vertical component, especially when the constructions lie in the near-field domain (Housner & Trifunac, 1967). Although different formulations for the Biot's equations are available in literature (e.g. the u-U formulation, as well as the u-w, u-w-p, and u-U-p formulations) to describe the saturated soil response when pore fluid accelerations are not negligible with respect to those of the solid phase, the simplified u-p formulation is usually employed in the current practice.It should be remarked that the simplifications included in the u-p formulation limit its range of validity in terms of maximum frequency content of input motion, as well as thickness and permeability of soil layers (Zienkiewicz et al. 1980).
In this work, a novel theoretical validation of the u-p formulation as compared to the u-U formulation of the Biot's equations for the analysis of the vertical component of site ground motion taking into account of the complex dynamic regime of real earthquakes (Argani & Gajo, 2021). This novel validation provides an extension of the validation proposed by Zienkiewicz et al. (1980): the analysis

is limited to the case of elastic response of the soil and only the longitudinal waves are considered (therefore a one-dimensional model is considered), however, differently from Zienkiewicz et al. (1980), the general case of seismic ground motions encompassing an interval of frequencies and applied at a specific depth is investigated, instead of a single frequency soil motion applied to the top surface of a soil layer, as proposed by Zienkiewicz et al. (1980).
It is shown that the validity ranges proposed by Zienkiewicz et al. (1980) in the case of a complex dynamic regime should be accompanied with a thorough analysis of the errors in the acceleration and pore pressure time history, the frequency content, the acceleration response spectrum and amplification, in order to define the appropriate limits of the applicability of the u-p formulation, thus paving the way for further investigations.

2. Methods

1.1 Governing equations

The well-known u-p formulation for the dynamic behaviour of saturate porous media can be expressed by the following set of equations (Zienkiewicz & al., 1980) for a linear-elastic soil response:

$$\mathrm{d}\sigma_{ij} = \mathrm{d}\sigma''_{ij} - \alpha\,\delta_{ij}\,\mathrm{d}p,$$
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),$$
$$\mathrm{d}\sigma''_{ij} = \mathbb{D}_{ijkl}\left(\mathrm{d}\varepsilon_{kl} - \mathrm{d}\varepsilon^0_{kl}\right),$$
$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = \rho\frac{\partial^2 u_i}{\partial t^2},$$
$$\alpha\frac{\partial \varepsilon_{ii}}{\partial t} + \frac{K_\mathrm{D}}{g}\frac{\partial}{\partial x_i}\left(-\frac{1}{\rho_\mathrm{f}}\frac{\partial p}{\partial x_i} + g_i - \frac{\partial^2 u_i}{\partial t^2}\right) + \frac{1}{Q}\frac{\partial p}{\partial t} = 0,$$

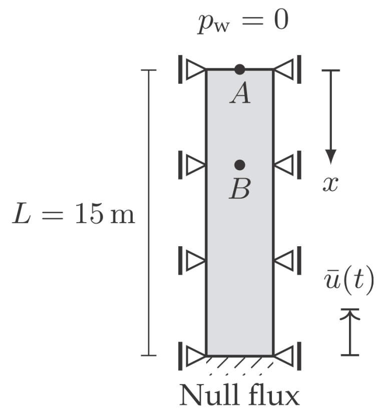
where u_i is the displacement of the solid skeleton, σ_{ij} is the total stress tensor, σ''_{ij} is the Nur and Byerlee effective stress tensor, ε_{ij} is the strain tensor, ε^0_{kl} is the initial (creep or thermal) strain tensor, $\mathbb{D}_{ijkl}$ is the elastic stiffness tensor, K_D is the Darcy permeability coefficient, ρ is the density of the whole porous medium, ρ_f is the pore fluid density, g_i is i-th component of the gravity acceleration, (having modulus g), α is the Biot's coefficient, Q is the bulk modulus of the mixture. The u-p formulation is implemented as a user defined 2D finite element (through a UEL subroutine) in the commercial finite element code Abaqus Unified FEA®.
The governing equations for the u-U formulation within the small strain framework are given by (Gajo et al., 1994; Gajo, 1995):

$$\mathbb{D}_{ijkl}\varepsilon_{kl} + (\alpha - n)^2 Q\frac{\partial \varepsilon_{jj}}{\partial x_i} + n(\alpha - n)Q\frac{\partial^2 U_j}{\partial x_j \partial x_i} + (1-n)\rho_\mathrm{S} g_i - (1-n)\rho_\mathrm{S}\frac{\partial^2 u_i}{\partial t^2} - \rho_\mathrm{a}\left(\frac{\partial^2 u_i}{\partial t^2} - \frac{\partial^2 U_i}{\partial t^2}\right) - \frac{n^2}{k}\left(\frac{\partial u_i}{\partial t} - \frac{\partial U_i}{\partial t}\right) = 0,$$
$$n(\alpha - n)Q\frac{\partial \varepsilon_{jj}}{\partial x_i} + n^2 Q\frac{\partial^2 U_j}{\partial x_j \partial x_i} + n\rho_\mathrm{f} g_i - n\rho_\mathrm{f}\frac{\partial^2 U_i}{\partial t^2} - \rho_\mathrm{a}\left(\frac{\partial^2 U_i}{\partial t^2} - \frac{\partial^2 u_i}{\partial t^2}\right) - \frac{n^2}{k}\left(\frac{\partial U_i}{\partial t} - \frac{\partial u_i}{\partial t}\right) = 0,$$

where U_i is the absolute displacement of the pore fluid, ρ_a is the added mass of pore fluid (which is neglected here for the sake of consistency with u-p formulation), and ρ_s is the density of the solid constituent. The u-U formulation is implemented in an in-house 1D FEM code (Gajo et al., 1994), in which both the solid and the pore fluid displacements are approximated with quadratic elements.

1.2 Numerical simulations

The transient response of a finite length, saturated soil column subjected to longitudinal dynamic excitation is investigated using both u-p and u-U formulations and assuming linear elastic isotropic material properties (figure 1). The soil column (figure 1) has a length of 15 m ad is discretised with 30 elements and is laterally constrained, so that lateral displacements and horizontal strains are equal to zero. Since the response of the system is thought as an incremental response, no gravity, null initial stress state, and null pore pressure are assumed. A prescribed longitudinal displacement is applied at the bottom surface, which represents the vertical component of the selected real earthquakes: Christchurch (2011, New Zealand, see figure 2), L'Aquila (2009, Italy), Emilia (2012, Italy), and Norcia (2016, Italy) (Bhanu et al., 2018).



Parameter	Symbol	Value
Density	ρ	2020 kg/m^3
Porosity	n	0.4
Young modulus	E	1200 MPa
Poisson's ratio	ν	0.3
Bulk modulus	K_s	$+\infty$
Solid phase density	ρ_s	2700 kg/m^3
Fluid bulk modulus	K_f	2.1771 GPa
Fluid density	ρ_f	1000 kg/m^3

Fig 1. Geometry reference for the soil column and material properties of the components of the two-phase medium. Properties referred to the solid phase and to the fluid phase are denoted with subscripts "s" and "f" respectively.

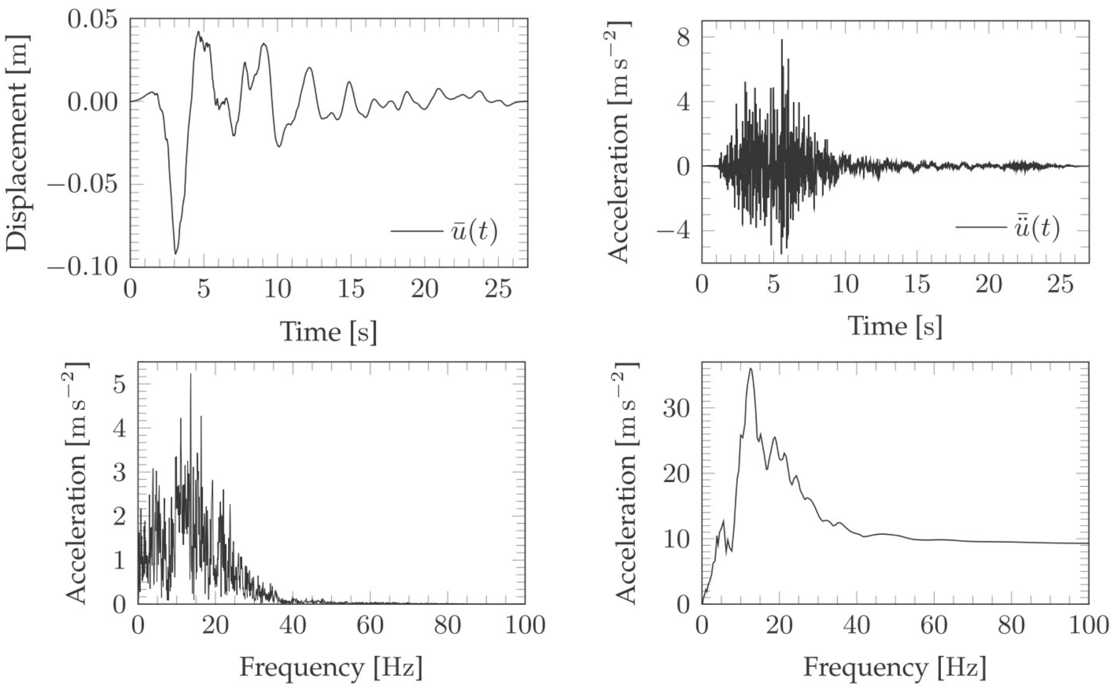
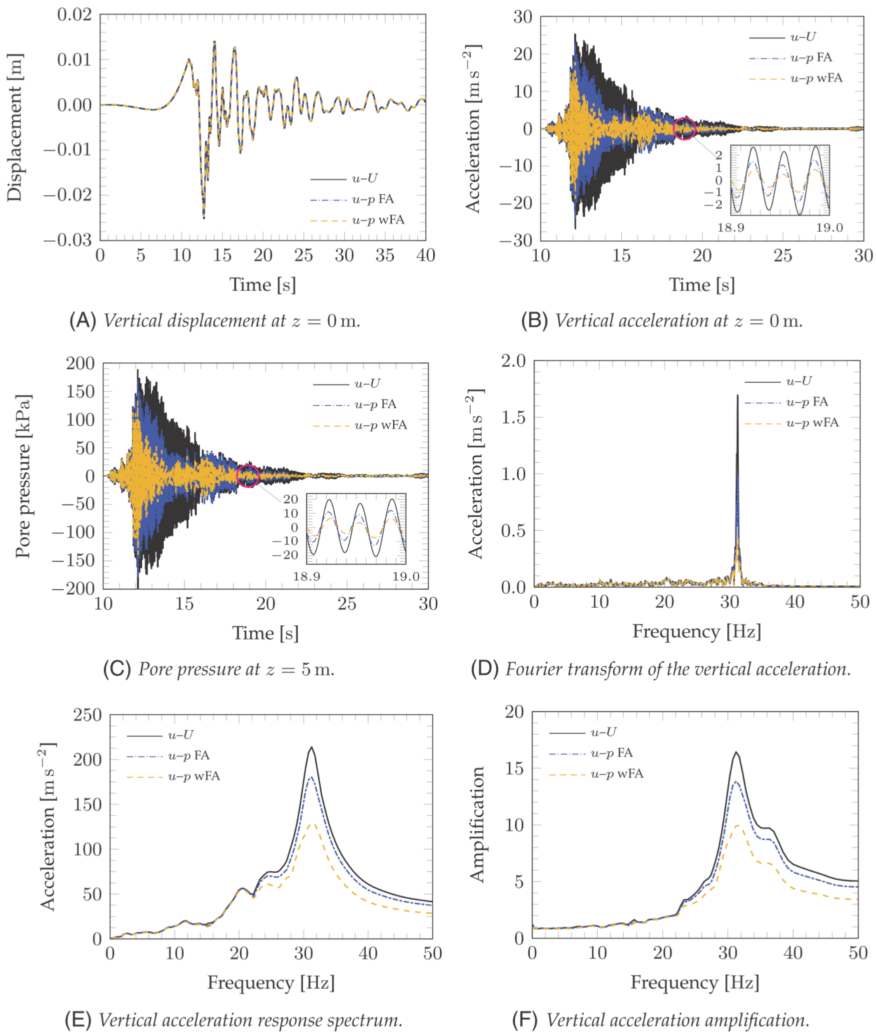


Fig 2. Example of loading time history for the numerical simulation: vertical components of the displacement (top left) and of the acceleration (top right) for the Christchurch earthquake (2011, NZ); Fourier transform of the vertical acceleration (bottom left) and vertical acceleration response spectrum (bottom right).

The time step of the simulation employed in the simulations is equal to 10^{-4} s in order to obtain the best accuracy level in the results. In fact, such time step is smaller than the time needed for the longitudinal wave to travel the distance between two adjacent nodes (1.35×10^{-4} s) and are much smaller than the time step that could be deduced from the highest frequency of the input signal (40 Hz from figure 2) according to Nyquist theorem (80 sampling points are much less than the 250 points that are employed in the simulation performed for this study).



(A) *Vertical displacement at $z = 0$ m.* (B) *Vertical acceleration at $z = 0$ m.*

(C) *Pore pressure at $z = 5$ m.* (D) *Fourier transform of the vertical acceleration.*

(E) *Vertical acceleration response spectrum.* (F) *Vertical acceleration amplification.*

Fig 3. Comparison between u-p and u-U formulations for the L'Aquila earthquake assuming permeability K_D = 10^{-3} m/s, porosity n = 0.4, Young's modulus E = 1200 MPa, and soil layer thickness L = 15 m.

3. Results

The water pore pressure is evaluated at the reference point B (figure 1), which is 5 m below the ground level, whereas the vertical displacement and acceleration are evaluated at the top of the soil column (point A in figure 1). The system response is evaluated in the time domain as well as in the frequency domain (Fourier transform and response spectrum of the vertical acceleration, and vertical acceleration amplification) in order to provide a detailed and comprehensive analysis of the effects of the frequency content of a real earthquake. The study includes a parametric analysis on the effects of different values of Young's modulus, of the porosity, and of the permeability for the same soil column length (15 m). The investigations include also the analysis of the case of a 100 m soil column length, not reported here for conciseness (see Argani & Gajo, 2021). The results of the u-p formulation are provided both for the cases in which the pore fluid inertial force in the mass balance equation is neglected and is taken into account (label "wFA" and "FA" respectively in figure 3) and are compared with the results obtained with the u-U formulation.

The results can be summarised in a compact form by employing the dimensionless chart provided by Zienkiewicz et al. (1980) (see their figure 3), as illustrated in figures 4which highlights the limits of validity of u-p formulation proposed by these authors in terms of frequency response. The results are plotted in terms of two non-dimensional quantities Π_1 and Π_2 defined as follows:

$$\Pi_1 = \frac{K_{\mathrm{D}} V_{\mathrm{c}}^2}{g \beta \omega L^2}, \qquad \Pi_2 = \frac{\omega^2 L^2}{V_{\mathrm{c}}^2},$$

where V_c is the compression wave velocity (assumed equal to 1869.26 m/s) and β is the ratio between the fluid density and the total density. Differently from Zienkiewicz et al. (1980), in this work (due to the complex dynamic regime embracing a wide range of frequencies) the value of ω is selected as the angular frequency associated with the largest acceleration amplitude.

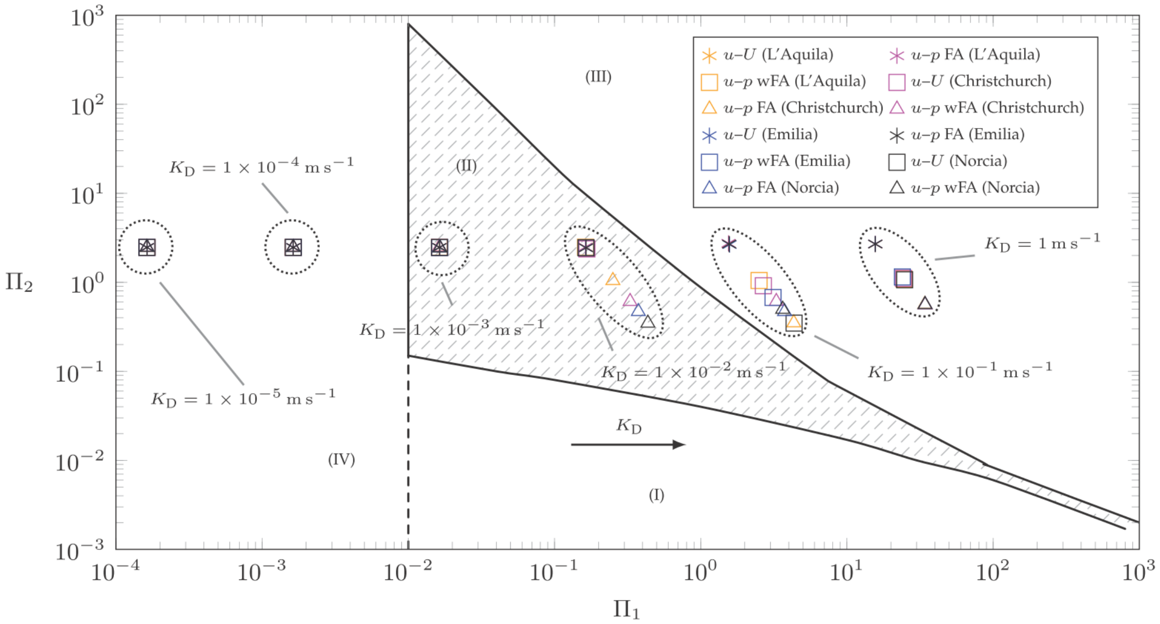


Fig 4. Comparison between u-p and u-U formulations in terms of the zones of applicability following Zienkiewicz et al. (1980) for the four real earthquakes in the case of E = 1200 MPa, n = 0.4, and soil layer thickness equal to 15 m. According to their work, zone (I) denotes the zone of slow phenomena; zone (II) and (III) denote, respectively, the zone of moderate speed and the zone of fast phenomena; zone (IV) corresponds to the zone of undrained behaviour.

The results show that the limits of applicability shown in figure 4 that were suggested by Zienkiewicz et al. (1980) for the case of a single frequency loading may no longer hold true when dealing with an input ground motion that includes a wide range of frequencies. The boundaries of this diagram should be modified by observing the response in terms of acceleration history, pore pressure, acceleration response spectrum, frequency content, and acceleration amplification, in order to verify the overall agreement between the u-p formulation and the more refined u-U formulation.

For instance, according to Zienkiewicz et al. (1980) the u-p formulation is expected to be within a reliability zone for permeability $K_D \leq 10^{-2}$ m/s for a single frequency input, whereas, according to the present study, in the case of complex dynamic regimes the errors (both in the frequency domain and in the time domain) can be about 15% (in terms of pore pressure and acceleration peaks) and become negligible only for lower permeabilities, such as for $K_D \leq 10^{-4}$ m/s. For lower permeabilities, for instance $K_D \leq 10^{-5}$ m/s, the results of u-p formulation and those of u-U formulation are practically superposed. Therefore, it is clear that a more thorough analysis of the results of the simulations is needed in order to assess whether the u-p formulation can be considered reliable or more refined formulations are needed to model the soil column behaviour.

4. Conclusions

The transient response of a finite length, saturated soil column subjected to a longitudinal dynamic input provided by the vertical component of real seismic ground motion (embracing a large number of frequencies associated with different amplitudes) is investigated in order to assess the validity limits of u-p formulation as compared to u-U formulation.
It is shown that in such a complex dynamic regime and assuming a linear elastic response of the saturated soil, the results can lead to validity ranges that are slightly different from those identified by Zienkiewicz et al. (1980) (which is valid for a single frequency ground motion), so that their dimensionless Π_1-Π_2 domain needs to be updated based on a thorough analysis in terms of acceleration and pore pressure time history, acceleration response spectrum, frequency content and acceleration amplification.

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