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BORESIGHT SLOPE OPTIMIZATION OF SUB-ARRAYED LINEAR ARRAYS THROUGH THE CONTIGUOUS PARTITION METHOD

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Boresight Slope Optimization of Sub-arrayed Linear Arrays through the Contiguous Partition Method

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Abstract

The optimization of the normalized boresight slope of the difference pattern in sub-arrayed linear monopulse antennas is presented. The knowledge of the independently optimum difference excitations, which provide the maximum normalized boresight slope, is exploited with an efficient excitation matching technique based on the contiguous partition method. A set of numerical experiments are provided to assess the effectiveness of the proposed method in reaching the best achievable performances even though with a small number of sub-arrays.

Index Terms

Monopulse array antennas, Sum and difference patterns, Boresight slope.

I. Introduction

In the framework of radar applications, a key feature of the antenna systems is the ability to afford a difference pattern with a null as deep as possible in the boresight direction [1]. As a matter of fact, such a characteristic determines the sensitivity of the radar in term of angle resolution. In [2], it has been shown how to obtain the maximum angular sensitivity (i.e., the deepest slope on boresight) in the case of a linear odd amplitude distribution. As regards to monopulse radar [3], Bayliss distributions are usually used since they allow the synthesis of patterns with a good trade-off between low-sidelobe and narrow beamwidth. Unfortunately, the synthesized patterns do not present the maximum normalized slope for a given array geometry. Moreover, a complete and dedicated feed network would be required to generate such a difference mode [3][4]. The use of two independent feed networks for the sum and difference patterns is often unacceptable, because of the complexity of the HW realization and the arising costs. In order to overcome these drawbacks, several techniques, which share parts of the feed network to generate the sum and the difference patterns, have been presented in the literature [5]-[10]. More in detail, one set of excitations (either the sum or the difference coefficients) is *a-priori* fixed to afford an optimum pattern. The other pattern is obtained by properly grouping the array elements into sub-arrays and assigning to each sub-array a suitable gain to match some constraints on the generated beam.

As far as the literature on such a topic is concerned, the approximation of a reference pattern has been considered in [5][10], wherein the "best compromise" has been computed by means of excitation matching procedures. On the other hand, in [6]-[9] the optimization of the sidelobe level (SLL) of the difference pattern, for a pre-fixed sum mode, has been considered.

The optimization of other pattern features has been faced in [11] and in [9] where the directivity and the slope on the boresight, together with a proper control of the SLL, of the difference pattern have been optimized through a differential evolution (DE) method and a hybrid approach, respectively. In this letter, the contiguous partition method (CPM) [10] is applied to the optimization of the boresight slope of the difference pattern. In particular, since the CPM has shown its effectiveness not only in synthesizing a difference pattern close as much as possible to the optimum one in the Dolph-Chebyshev sense [10], but also in minimizing the SLL [12] of difference beams, this work is aimed at showing its potentialities and limitations as well as its flexibility also in this context. Moreover, a comparison with the results in [9] is also reported to shown how the proposed approach compares with others in the literature.

The paper is organized as follows. In Sect. 2, the problem is mathematically formulated by detailing the synthesis procedure. In Sect. 3, selected results are reported to assess the validity and versatility of the CPM-based technique. Finally, some conclusions are drawn (Sect. 4).

II. MATHEMATICAL FORMULATION

Let us consider a linear array of $M = 2 \times N$ elements uniformly-spaced of d. Following the monopulse principle, the sum pattern is given by the set α_n, $n = \pm1, ..., \pm N$ of symmetric excitations ($\alpha_n = \alpha_{-n}$), while anti-symmetric coefficients ($\beta_n = -\beta_{-n}$) generate on receive the difference beam. Accordingly, sum and difference patterns are obtained by adding and subtracting the two halves of the antenna aperture [13].

When a sub-arraying technique is adopted to generate the difference mode from the sum one [5], the synthesis problem is recast as the definition of a suitable grouping, described through the integer indexes $c_n \in [1 : Q]$, $n = 1, ..., N$, and the sub-array gains w_q, $q = 1, ..., Q$, to fit some user-defined requirements. In particular, the compromise difference pattern is obtained from the coefficient set

$$\underline{B} = \{b_n = -b_{-n} = \alpha_n \delta_{nq} w_q;\, n \in [1 : N]\,;\, q \in [1 : Q]\}\,. \tag{1}$$

where δ_{nq} is the Kronecker delta equal to $\delta_{nq} = 1$ if $c_n = q$ and $\delta_{nq} = 0$ otherwise.

Since the problem at hand is concerned with the maximization of the boresight slope of the difference pattern and the CPM is an excitation matching approach aimed at fitting a reference pattern, it is needed to determine the optimal pattern in terms of slope. Concerning the metric to be used to quantify the boresight slope of an array of discrete elements, the *difference slope ratio* is considered [14]. It is defined as $K_r = \frac{K}{K_0}$, K and K_0 being the normalized boresight slope of the actual difference beam and the maximum value that would be achieved with a line source distribution on the antenna aperture of size L/λ, respectively. In the linear case, it has been shown in [2] that the distribution providing the maximum value of K_0 is a linear odd (with respect to the center of the antenna aperture) distribution. Accordingly, since K_0 is known once the array geometry is given, the synthesis procedure for a discrete element array is aimed at maximizing the value of the normalized boresight slope K. Such a value for an anti-symmetric set of excitations is given by [14]

$$K = \frac{\sum_{n=1}^{N} \{k_n \beta_n\}}{\sqrt{2 \sum_{n=1}^{N} \sum_{m=1}^{N} \{\beta_n G_{nm} \beta_m\}}} \tag{2}$$

where $k_n = \frac{2n-1}{2N-1}$ and $G_{nm} = \frac{sin[(m-n)kd]}{(m-n)kd} - \frac{sin[(m+n-1)kd]}{(m+n-1)kd}$. Accordingly, the first step of the compromise synthesis procedure is aimed at computing the excitation coefficients $\underline{B}^{opt} = \{b_n^{opt};\, n = \pm1, \ldots, \pm N\}$ that afford a pattern with the maximum normalized boresight slope K_{max} in the case of discrete element arrays. Towards this end, the functional (2) is maximized by means of a standard steepest-descent method according to the procedure described in [14]. Afterward, the CPM is exploited to find the "best compromise" between sum and difference patterns such that the excitations $\underline{B}$ be close as much as possible to the reference ones $\underline{B}^{opt}$. In particular, once the sum mode coefficients α_n, $n = 1, ..., N$ are fixed to provide an optimum sum pattern (e.g., a Taylor pattern [15]), the following cost function

$$\Psi\left(c_n, w_q\right) = \frac{1}{N}\left\{\sum_{n=1}^{N} \left|g_{nq}\right|^2\right\} \tag{3}$$

where $g_{nq} = \alpha_n \left[\gamma_n - \sum_{q=1}^{Q} \delta_{nq} w_q\left(c_n\right)\right]$ and $\gamma_n = \frac{b_n^{opt}}{\alpha_n}$, is minimized with respect to the unknowns (c_n, w_q), $n = 1, ..., N$; $q = 1, ..., Q$.

It is worth to notice that, equation (3) mathematically formalizes a minimum variance problem, where each term is related to a different sub-array. Since the value minimizing the sum of the square distances, for a given set of real values, is the weighted arithmetic mean, the sub-array weights turn out to be

$$w_q\left(c_n\right) = \frac{\sum_{n=1}^{N} \left(\alpha_n\right)^2 \delta_{nq} \gamma_n}{\sum_{n=1}^{N} \left(\alpha_n\right)^2 \delta_{nq}}, \quad q = 1, \ldots, Q. \tag{4}$$

As a consequence, the problem solution recast as the definition of only the sub-array aggregations c_n, $n = 1, ..., N$. With reference to (3), let us observe that such a solution is a least square partition and Fisher in [16] proved that it is a *contiguous*

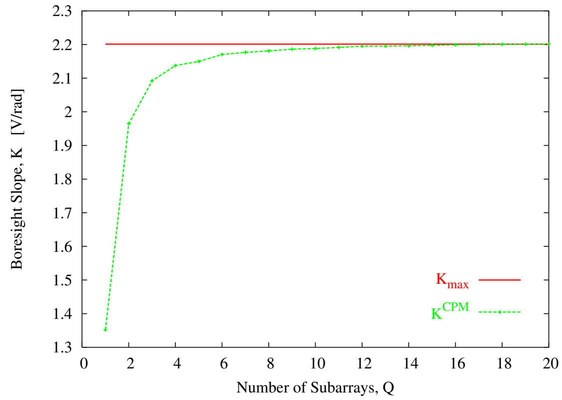


Fig. 1. *Test Case 1* ($N = 20$, $d = 0.7\lambda$, *Taylor sum pattern* [15] - $SLL = -30\,dB$, $\overline{n} = 6$) - Value of the normalized boresight slope versus the number of sub-arrays Q.

partition [1] (CP) of the ordered list of the *optimal gains* γ_n. Since the number of CPs is equal to $U = \begin{pmatrix} N-1 \\ Q-1 \end{pmatrix}$, the dimension of the solution space of the CPM considerably reduces compared to that of classical optimization-based approaches [6]-[9]. In order to sample such a space, the *Border Element Method* (BEM) [10] is used. Starting from a randomly chosen contiguous partition $\underline{C}^{(0)} = \left\{c_n^{(0)};\, n = 1, ..., N\right\}$, the trial solution is updated, $\underline{C}^{(i)} \leftarrow \underline{C}^{(i+1)}$, taking into account that the *border elements* (i.e., those elements whose adjacent values γ_{n-1} or/and γ_{n+1} are assigned to a different sub-array) can change the sub-array membership without violating the condition of contiguous partition. The process is iterated until the termination criterion, based on the maximum number of iterations I (i.e., $i > I$) or on the stationariness of the cost function value (i.e., $\frac{\left|K_\Psi \Psi^{(i-1)} - \sum_{j=1}^{K_\Psi} \Psi^{(j)}\right|}{\Psi^{(i)}} \leq \eta_\Psi$, being K_Ψ and η_Ψ two user-defined control parameters), is verified.

III. Numerical Results

In the first example, an array of $M = 40$ elements spaced by $d = 0.7\,\lambda$ is considered. The sum pattern has been fixed to a Taylor pattern with $SLL = -30\,dB$ and $\overline{n} = 6$. The reference difference pattern $\underline{B}^{opt}$, which guarantees the maximum boresight slope ($K_{max} = 2.2013$) has been computed [14]. Concerning the compromise solution, the number of sub-arrays used in the non-complete feed network has been varied in the range $Q \in [1, 20]$. As far as the initialization of the BEM is concerned, the initial aggregation $\underline{C}^{(0)}$ has been chosen with the array elements uniformly distributed among the Q sub-arrays. The values of K in correspondence with the solutions obtained by the CPM are shown in Fig. 1. By quantifying the closeness of the synthesized normalized difference slope on boresight to the optimal value $K_{max} = 2.2013$ with the index $\xi_K \triangleq \frac{K_{max} - K^{CPM}}{K_{max}} \times 100$, it turns out that $\xi_K \leq 3$ when $Q \geq 4$ and $\xi_K \leq 1$ for $Q \geq 8$. On the other hand, the simplification of the network architecture when $\left.\frac{N}{Q}\right\rfloor_{Q=3} \simeq 7$ and $\left.\frac{N}{Q}\right\rfloor_{Q=2} = 10$ causes a strong reduction of the performance (i.e., $\left.\xi_K\right\rfloor_{Q=3} = 4.95$ and $\left.\xi_K\right\rfloor_{Q=2} = 10.74$).

The effectiveness of the CPM in sampling the solution space is pointed out by the values in Tab. I, I_{end} and T being the number of cost function evaluations to get the final solution and the total CPU-time (on a $3.4\,GHz$ PC with $2\,GB$ of RAM), respectively. As a matter of fact, starting from an uniform clustering ($i = 0$), the trial solution is closer to the reference one just increasing the number of sub-arrays ($\left.\Psi^{(i)}\right\rfloor_{Q=3} \simeq 2.9 \times 10^{-2}$, $\left.\Psi^{(i)}\right\rfloor_{Q=5} \simeq 1.2 \times 10^{-2}$, $\left.\Psi^{(i)}\right\rfloor_{Q=10} \simeq 2.8 \times 10^{-3}$, and $\left.\Psi^{(i)}\right\rfloor_{Q=15} \simeq 8.2 \times 10^{-4}$). Moreover, it should be observed that at most 40 iterations are enough to reach the convergence solutions whose excitations and corresponding patterns are shown in Fig. 2.

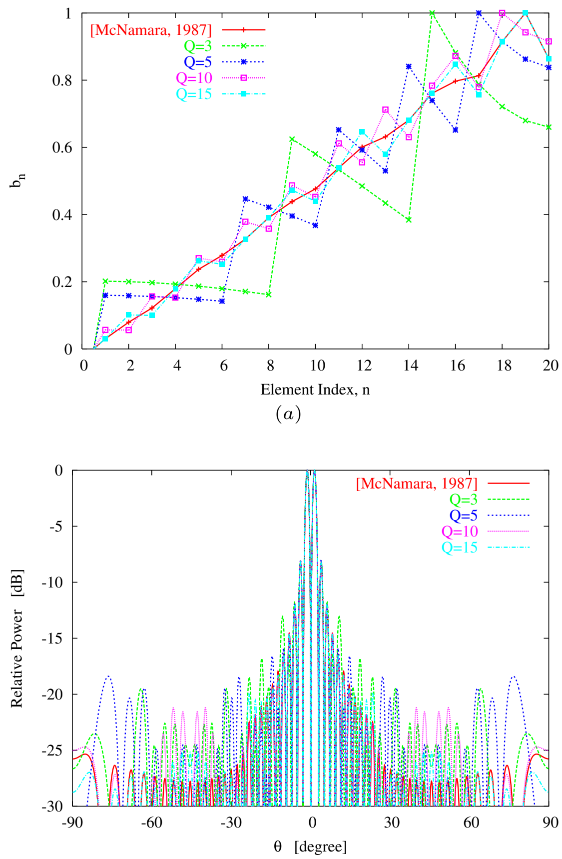
The second example deals with a linear array of $N = 20$ equally-spaced ($d = 0.5\lambda$) elements. The sum excitations have been set to those of the Dolph-Chebyshev pattern with $SLL = -20\,dB$ [17]. Regarding the definition of the reference set of excitations $\underline{B}^{opt}$, it has been observed [14] that the element coefficients, for the half-wavelength spacing case, are simply computed by sampling the continuous line-source distribution in [2] (Fig. 3 - McNamara, 1987). Thus, the maximum value

[1] A grouping of array elements is a *contiguous partition* when given two elements γ_i and γ_n which belong to the q-th sub-array, if another element exists such that the condition $\gamma_i < \gamma_j < \gamma_n$ holds true, hence γ_j has to be assigned to the same sub-array.

TABLE I

Test Case 1 ($N = 20$, $d = 0.7\lambda$, *Taylor sum pattern* [15] - $SLL = -30\,dB$, $\overline{n} = 6$) - COMPUTATIONAL INDEXES.

	U	I_{end}	T [sec]
$Q = 3$	741	3	3.0×10^{-8}
$Q = 5$	82251	31	3.1×10^{-7}
$Q = 10$	211×10^{6}	33	3.3×10^{-7}
$Q = 15$	1.50×10^{10}	9	9.0×10^{-8}



(a)

(b)

Fig. 2. *Test Case 1* ($N = 20$, $d = 0.7\lambda$, *Taylor sum pattern* [15] - $SLL = -30\,dB$, $\overline{n} = 6$) - Plots of the (a) excitation coefficients and of the (b) corresponding relative power pattern for various values of Q.

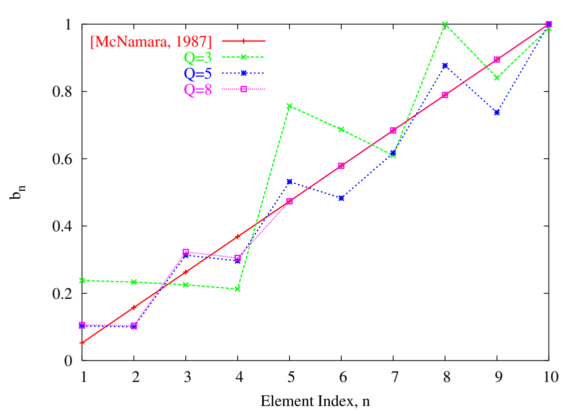


Fig. 3. *Test Case 2* ($N = 10$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - Plot of the values of the excitation coefficients for various values of Q.

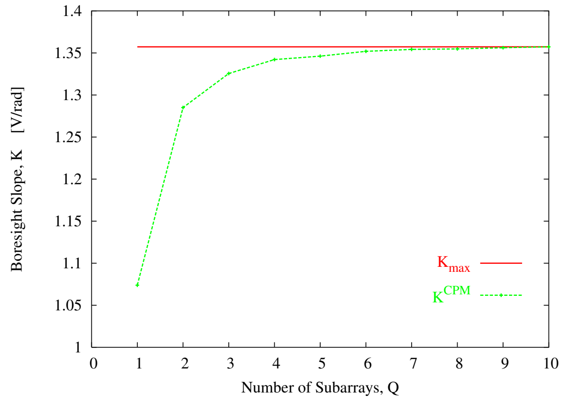


Fig. 4. *Test Case 2* ($N = 10$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - Plot of the normalized boresight slope values versus the number of sub-arrays Q.

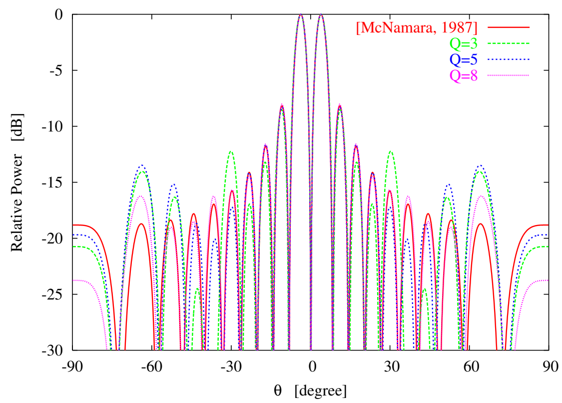


Fig. 5. *Test Case 2* ($N = 10$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - Plot of the relative power pattern for different values of Q.

of the boresight slope for an aperture of length $L/\lambda = 10$ is $K_{max} = 1.3572$ [14]. Also in this case, the estimated values of K are close ($\xi_K < 1$) to the reference one when $Q > 4$ (Fig. 4). Such a circumstance is further pointed out in Fig. 3 where the synthesized coefficients get closer and closer to the reference set $\underline{B}^{opt}$ when $Q \to N$.

For completeness, the difference patterns for the experiment considered in Fig. 3 are reported in Fig. 5. Moreover, the sub-array configurations and the corresponding gains are summarized in Tab. II. As far as the computational issues are concerned, the dimensions of the solution spaces are equal to $U\rfloor_{Q=3} = U\rfloor_{Q=8} = 36$ and $U\rfloor_{Q=5} = 126$. Furthermore, the numbers of iterations to reach the convergence solutions are $I_{end}\rfloor_{Q=3} = 4$, $I_{end}\rfloor_{Q=5} = 1$, and $I_{end}\rfloor_{Q=8} = 3$. As a result, the CPU-time for the synthesis is lower that $10^{-6}\,sec$.

Finally, let us compare with the result reported in [9] where the constrained (sidelobe-wise) optimization of the boresight slope is considered when $N = 10$ and $Q = 8$. Towards this end, the sub-array weights are now computed solving a Convex Programming (CP) problem as in [9] starting from the sub-array configuration obtained by means of the CPM. Figure 6 shows

TABLE II

Test Case 2 ($N = 10$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - SUB-ARRAY CONFIGURATIONS AND SUB-ARRAY GAINS.

c_n	w_q
1 1 1 1 2 2 2 3 3 2	0.2328, 0.8925, 1.6912
1 1 2 2 3 3 4 5 5 4	0.1145, 0.3696, 0.7152, 1.0299, 1.6912
1 1 2 2 3 4 6 7 8 5	0.1145, 0.3696, 0.6184, 0.8325, 1.0, 1.1087, 1.4783, 1.9923

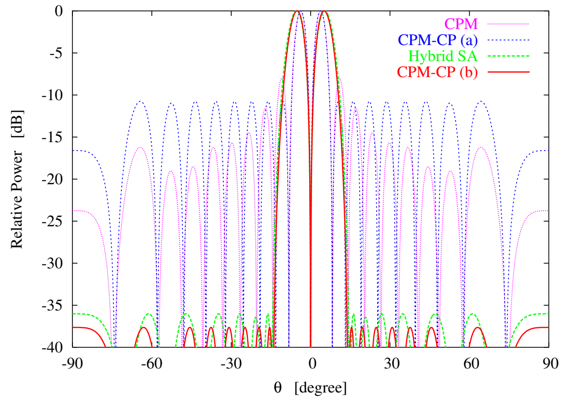


Fig. 6. *Test Case 3* ($N = 10$, $Q = 8$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - Plot of the relative power pattern obtained by the herein proposed hybrid method and that of [9].

TABLE III
Test Case 3 ($N = 10$, $Q = 8$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - PERFORMANCE INDEXES.

	K $[V/rad]$	SLL $[dB]$	BW $[degree]$
CPM	1.35	-8.0	3.95
$CPM - CP\,(a)$	1.28	-10.8	3.90
$Hybrid\,SA$	0.90	-35.7	5.90
$CPM - CP\,(b)$	0.97	-37.5	5.60

the results of the hybrid approach ($CPM - CP$, [18]) as well as those synthesized by the CPM and in [9]. With reference to the configuration in Tab. II, the SLL of the solution computed through the hybrid method $[CPM - CP\,(a)]$ is almost $3\,dB$ below that with the CPM, but the slope at boresight slightly worsen. Successively, more stringent constraints on the SLL are imposed to fairly compare with the solution of the $Hybrid\,SA$ in [9]. Accordingly, a new reference pattern has been assumed (namely a Zolotarev pattern with $SLL = -39\,dB$ [4]), which presents a high value of the boresight slope for a given SLL. In this case, the synthesized aggregation is $\{c_n\} = \{1\,2\,3\,4\,4\,4\,4\,5\,6\,7\,8\}$. The corresponding solution $[CPM - CP\,(b)]$ outperforms that in [9] for both the boresight slope, the beamwidth (BW), and the SLL (Tab. III).

Similar conclusions hold true for the case also dealt with in [9] with $Q = 6$, thus confirming the effectiveness and versatility of the CPM-based approach. In particular, Figure 7 and Tab. IV report the radiation patterns obtained with the bare CPM as well as the hybrid approaches (i.e., $CPM - CP$ and $Hybrid\,SA$ [9]) and their performance, respectively.

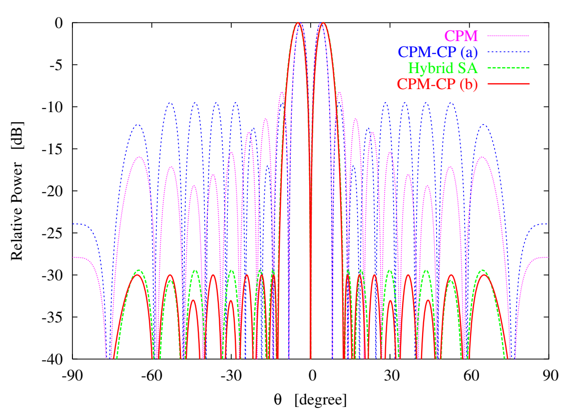


Fig. 7. *Test Case 4* ($N = 10$, $Q = 6$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - Plot of the relative power pattern obtained by the herein proposed hybrid method and that of [9].

TABLE IV

Test Case 4 ($N = 10$, $Q = 6$, $d = \lambda/2$, *Dolph-Chebyshev sum pattern* [17] - $SLL = -20\,dB$) - PERFORMANCE INDEXES.

	$K\ [V/rad]$	$SLL\ [dB]$	$BW\ [degree]$
CPM	1.35	-8.2	3.94
$CPM - CP\,(a)$	1.25	-9.5	3.92
$Hybrid\,SA$	1.05	-29.5	5.26
$CPM - CP\,(b)$	1.06	-30.0	5.21

IV. CONCLUSIONS

In this paper, the optimization of the normalized boresight slope of the difference pattern of monopulse array antennas has been carried out by means of the CPM. In particular, the sub-arraying configuration has been taken into account in order to reduce the complexity of the synthesized antennas and the knowledge of the independently optimum difference excitations, which provide the maximum normalized boresight slope has been exploited. The numerical experiments have pointed out that a proper definition of the sub-array configurations and the corresponding gains allows one to obtain good boresight slope values even though with a limited number of sub-arrays. Constraints on the SLL have been also taken into account through a hybrid $CPM - CP$ approach in order to compare with other state-of-the-art methods dealing with slope maximization.

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