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Abstract

In the framework of microwave imaging applications, an innovative strategy aimed at addressing the multi-objective inverse scattering problem is proposed. Starting from the spatial-domain integral formulation, the original multi-objective problem is recast into a single-objective one by defining a suitable cost function as a linear combination of the data and state terms according to variable weighting parameters. By iteratively tuning these parameters, the optimization procedure is forced to solve an "almost" multi-objective problem avoiding the use of *ad-hoc* multiple-objective optimization methods and satisfying different objectives in a balanced way. Selected numerical results indicate that the use of such a strategy yields to accurate reconstructions, with noise-corrupted data as well, by improving the performances of the adopted optimization procedure.

Index Terms - Microwave imaging, inverse scattering, adaptive weighting, genetic algorithms

1 Problem Overview

In the framework of inverse scattering techniques for microwave imaging, let us consider the 2-D TM-polarization case. An unknown object, belonging to an investigation domain D, is successively illuminated by a set of known incident electric fields f_v^{inc}, $v = 1, ..., V$. For each incident field f_v^{inc}, the scattered field f_v^{scatt} is measured on a surface S outside D and satisfies the following integral equation (*data equation*)

$$f_v^{scatt} = G_{ext}\tau\, f_v \tag{1}$$

where f_v denotes the total electric field related to f_v^{inc}, τ is the material contrast, and G_{ext} is the external Green operator [1]. Moreover, the total field f_v inside the investigation domain is known to satisfy the so called *state equation*

$$f_v^{inc} = f_v - G_{int}\tau\, f_v \tag{2}$$

G_{int} being the internal Green operator [1].

The profile reconstruction problem consists of determining the material contrast τ and the field distribution f_v inside D which satisfy the *data* and the *state* equations starting from the knowledge of the incident fields f_v^{inc}, $v = 1, ..., V$ on D and the scattered fields f_v^{scatt}, $v = 1, ..., V$ on S. Clearly, this is a *multi-objective* problem (MOP) since the solution is required to satisfy two goals simultaneously. Mathematically, it can be described as follows: "*to minimize* $\underline{\Phi} = \underline{G}(\tau,\, f_v) = \{g_{data}(\tau,\, f_v),\, g_{state}(\tau,\, f_v)\}$ *subject to the constraint* $\underline{\Omega} = \underline{H}(\tau,\, f_v) = \{h_i(\tau,\, f_v),\, i = 1, ..., I\} \leq 0$" where $\underline{\Phi}$ and $\underline{\Omega}$ are the objective-array and the constraint-array, respectively. More in detail,

$$g_{data}(\tau,\, f_v) = \frac{\sum_{v=1}^{V}\left\| f_v^{scatt} - G_{ext}\tau\, f_v \right\|^2}{\sum_{v=1}^{V}\| f_v^{scatt}\|^2} \qquad g_{state}(\tau,\, f_v) = \frac{\sum_{v=1}^{V}\left\| f_v^{inc} - \{f_v - G_{int}\tau\, f_v\} \right\|^2}{\sum_{v=1}^{V}\| f_v^{inc}\|^2} \tag{3}$$

and the constraint-array is defined according to the available *a-priori* information on the problem's unknowns $(\tau,\ f_v)$ (e.g., $h_1(\tau) = -Re\,(\tau)$, $h_2(\tau) = Re\,(\tau) - \tau_{max}^{re}$, $h_3(\tau) = Im\,(\tau)$, $h_2(\tau) = \tau_{max}^{im} - Im\,(\tau)$). The MOP generally presents more than one acceptable solution or there may not exist one solution that is the best with respect to all objectives. As far as the MOP's solution is concerned, such a problem can be directly addressed by considering a customized optimization procedure. Many real-world electromagnetic problems involve the simultaneous optimization of multiple objectives that often are competing. Usually, customized methodologies for MOP's are aimed at determining a trade-off surface (i.e., a set of non-dominated solutions [2]) known as *Pareto front* (see [3] and the references cited therein).

Unlike these approaches, generally inverse scattering techniques do not treat the multi-objectives as they are. The original problem is transformed into a single-objective (SOP) one by defining an artificial scalar objective function generally obtained through some linear combination of the weighted objectives [4][1]:

$$
\begin{aligned}
\phi(\tau,\ f_v) &= \phi_{data}(\tau,\ f_v) + \phi_{state}(\tau,\ f_v) \\
\phi_{data}(\tau,\ f_v) &= \alpha_{data}\left\{g_{data}(\tau,\ f_v)\right\} \\
\phi_{state}(\tau,\ f_v) &= \alpha_{state}\left\{g_{state}(\tau,\ f_v)\right\}
\end{aligned}
\tag{4}
$$

α_{data} and α_{state} being two positive weighting parameters[1]. Such a formulation allows the use of any single-objective optimization algorithm [5]-[9], but it needs to define the weights associated to each objective. This is one of the main drawbacks of the approach since the behavior of the optimization algorithm is very sensitive and is biased by the values of these parameters [10]. Although some very interesting attempts for selecting optimum values of the weighting factors have been carried out [11] (even though the search of optimum regularization parameters is conceptually different from the determination of the weighting factors, some interesting analogies and strategies for the unsupervised evolution

[1]The term "weighting" parameters has been used since the term "regularization" has been assumed to be mainly associated with the use of *a-priori*, "external" information, which imposes additional constraints to be satisfied besides state and data terms (e.g., Tikhonov, total variation, edge preserving, etc.).

can be found in [11]), it is recognized as a very difficult problem from a mathematical point of view. Generally, the choice of the values of the weighting parameters is performed through a computationally expensive numerical experimentation or by considering a supervised tuning process. Moreover, such a calibration presents some arbitrariness due to the adopted model and is largely dependent on the scatterer involved, so that a general theory providing the optimum values of α_{data} and α_{state} cannot be easily developed and this certainly represents an obstacle preventing widespread industrial applications.

The unsupervised adaptive setting of these parameters may be a solution. As an analogy to the unsupervised evolution of weighting parameters, but within the framework of the adaptive choice of the regularization factors, a significant contribution has been proposed by Abubakar *et al.* [12]. By fully exploiting some *a-priori* information[2] on the scatterer under test, a suitable adaptive multiplicative parameter is defined. Such a parameter is a function of the total variation regularization factor and it is determined by the iterative inversion procedure itself. This eliminates the choice of the values of the parameters completely.

In this letter, an alternative adaptive weighting strategy is proposed. The approach defines an adaptive and unsupervised procedure for tuning the weighting parameters without any information on the class of the profiles to be reconstructed. After a mathematical description of the iterative process (Sect. 2), selected numerical results will be presented in order to preliminary illustrate the effectiveness of the proposed approach.

2 The Adaptive Weighting Strategy

Let us consider a multiple-agent strategy (e.g., the real-coded genetic algorithm proposed in [6]) where a set of L trial solutions $\left\{(\tau^{(l)},\, f_v^{(l)}),\, l = 1, ..., L\right\}$ is defined and iteratively (k being the iteration number) updated in order to define the optimal solution minimizing the cost function $(\tau^{(opt)},\, f_v^{(opt)}) = arg\left\{min_k\left\{min_l\left[\phi(\tau_{(k)}^{(l)},\, f_{v(k)}^{(l)})\right]\right\}\right\}$. Generally speaking,

[2] The TV-factor takes into account for the nature of the scatterer under test. The norm used in defining the TV-factor favors a 'blocky' contrast (L^1-norm) or a smooth profile (L^2-norm) [13].

the underlying idea of the adaptive weighting strategy (AWS) is to reduce the original multiple-objective problem in a single-objective one. In particular, the proposed approach forces the decreasing not-only of the greatest term but it also favors the minimization of the other which is already better satisfied. Mathematically, such a strategy is detailed in the following.

At each iteration and for each trial solution l, the AWS requires the tuning of $\{\alpha_{data}\}_k^{(l)}$ and $\{\alpha_{state}\}_k^{(l)}$ according to the values of $s_k^{(l)} = \frac{g_{data}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)}{g_{state}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)}$ and $d_k^{(l)} = \frac{g_{state}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)}{g_{data}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)}$, by comparing the data term $g_{data}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)$ and the state term $g_{state}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)$. More in detail, if $g_{data}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right) \leq g_{state}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)$ then

$$\{\alpha_{state}\}_k^{(l)} = \begin{cases} s_k^{(l)} & if\; s_k^{(l)} < \gamma_k \\ \gamma_k & otherwise \end{cases} \qquad \{\alpha_{data}\}_k^{(l)} = 1.0 \tag{5}$$

where γ_k is a threshold adaptively tuned iteration-by-iteration. Its value is decreased ($\gamma_{k+1} \leftarrow \frac{\gamma_k}{2}$) when k_{data} or k_{state} is equal to a fixed percentage η of the maximum number of iterations K. Moreover, two counters are updated: $k_{state}^{(l)} \leftarrow k_{state}^{(l)} + 1$ and $k_{data}^{(l)} \leftarrow 0$. Otherwise $\left(g_{data}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right) > g_{state}\left(\tau_{(k)}^{(l)}, f_{v(k)}^{(l)}\right)\right)$, the following complementary rule is adopted

$$\{\alpha_{data}\}_k^{(l)} = \begin{cases} d_k^{(l)} & if\; d_k^{(l)} < \gamma_k \\ \gamma_k & otherwise \end{cases} \qquad \{\alpha_{state}\}_k^{(l)} = 1.0 \tag{6}$$

and $k_{state}^{(l)} \leftarrow 0$, $k_{data}^{(l)} \leftarrow k_{data}^{(l)} + 1$.

Successively, the genetic operators are applied by taking into account as fitness measure of the lth solution its corresponding weighted cost function $\phi\left(\tau_k^{(l)}, f_{v(k)}^{(l)}\right)$.

3 Numerical Validation

In this section, selected numerical results are provided in order to show the effectiveness of the proposed methodology. The assumed imaging configuration is shown in Fig. $1(a)$

(please note that the black pixel near the low right corner is used for reference). The scattering object is an off-centered square homogeneous cylinder of side $d = \frac{9}{8}\lambda$ (λ being the free-space wavelength) illuminated by a set of $V = 8$ unit TM-polarized plane waves. The coordinates of the center of the cylinder are $x_0 = y_0 = -\frac{27}{80}\lambda$ and $\tau = 0.5$. The input scattering data are analytically computed and a gaussian noise, characterized by a signal-to-noise ratio equal to $20\,dB$, has been added.

The initial set of L trial solutions has been generated by considering complete random values in the following ranges: $0.0 \leq \tau_0^{(l)} \leq 1.0$, $-1.5 \leq Re\left\{f_{v(0)}^{(l)}\right\} \leq 1.5$, and $-1.5 \leq Im\left\{f_{v(0)}^{(l)}\right\} \leq 1.5$. In particular, Fig. 1$(b)$ shows the dielectric distribution related to $\tau_{(0)}^{(opt)}$ satisfying $\left(\tau_0^{(opt)}, f_{v(0)}^{(opt)}\right) = arg\left\{min_l\left[\Im\left(\tau_0^{(l)}, f_{v(0)}^{(l)}\right)\right]\right\}$ $\left(\tau_{(0)}^{(opt)}, f_{v(0)}^{(opt)}\right) = arg\left\{min_l\left[\phi\left(\tau_{(0)}^{(l)}, f_{v(0)}^{(l)}\right)\right]\right\}$.

As far as the regularization strategy is concerned, the proposed technique is independent from the type of the multi-agent minimization algorithm. However, because of the stochastic nature of the considered multi-agent procedure, the AWS-based minimization process has been executed 30 times for each numerical experiment with the same randomly generated initial population. The average results of a complete set of executions are then presented.

The set of genetic parameters used is chosen according to the values suggested in the literature on this subject [14][15]: $L = 81$ (dimension of the set of trial solutions), $P_c = 0.6$ (crossover probability), $P_m = 5\times10^{-2}$ (mutation probability), $P_{bm} = 1\times10^{-3}$ (single-gene mutation), and $K = 2\times10^4$. Moreover, the numerical thresholds and parameters of the adaptive regularization strategy are $\eta = 1\times10^{-3}$ and $\gamma_0 = 1.0$.

In figure 2 the behaviors of the optimal value of the weighted cost function after the application of the genetic operators (i.e., $\{\phi\}_k^{(opt)} = min_{h=1,\ldots,k}\left\{min_l\left[\phi\left(\tau_{(h)}^{(l)}, f_{v(h)}^{(l)}\right)\right]\right\}$) and related data and state terms are shown. For comparison purposes, the results obtained by considering a standard procedure with constant weighting parameters ($\{\alpha_{data}\}_k^{(l)} = \{\alpha_{state}\}_k^{(l)} = 1.0$ - Fig. 2(a)) and an adaptive regularization with fixed threshold ($\gamma_k = \gamma_0$) in the updating of $\{\alpha_{data}\}_k^{(l)}$ and $\{\alpha_{state}\}_k^{(l)}$ (Fig. 2(b)), are also reported. During the iterative process, a similar behavior can be observed for the case of the reference (Fig. 2(a)) as well as for those of the adaptive strategies (Fig. 2(b)-(c)). The optimization algorithm forces $\{\phi_{data}\}_k^{(opt)}$ and $\{\phi_{state}\}_k^{(opt)}$ to assume the same order of magnitude. On the contrary, not-negligible differences turn out in the corresponding data $\{g_{data}\}_k^{(opt)}$ and state $\{g_{state}\}_k^{(opt)}$ terms given in Fig. 3. In particular, the constant-weights strategy (Fig. 3(a)) reduces the data term of about two-orders in magnitude during the iterative process. On the other hand, when the adaptive strategies are used, the same term is decreased of about three (Fig. 3(b)) and four-orders (Fig. 3(c)) in magnitude, respectively.

As a consequence, starting from the same random distribution of the initial guess population, at the end of the iterative optimization process performed with the same minimization algorithm, the profile reconstructions shown at the bottom of Fig. 4 are obtained. For completeness, the estimated distributions at intermediate iterations are also given (Fig4 - $k = 1000$ (*top*) and $k = 5000$ (*middle*)). As can be seen, the adaptive strategies allow an improvement of the reconstruction accuracy as confirmed by the values of the error figures (Tab. I.) computed as follows

$$\Xi_j = \left\{ \int_{D_j} \frac{\tau^{(opt)}(x,y) - \tau(x,y)}{\tau(x,y)} \, dx\, dy \right\} \times 100 \tag{7}$$

where $\tau^{(opt)}$ is the reconstructed object function and D_j indicates the whole domain ($j \Rightarrow tot$), or the area where the actual scatterer is located ($j \Rightarrow int$), or the background belonging to the investigation domain ($j \Rightarrow ext$).

4 Conclusions

In this paper, a new adaptive regularization strategy for microwave imaging purposes has been presented. The proposed approach considers only a very limited *a-priori* information (e.g., some physical constraints on the values of the material contrast τ, $\tau_{min} \leq \tau(x,y) \leq \tau_{max}$) about the scenario under test and does not require neither some knowledge on the shape of the scatterer under test nor a supervised tuning procedure. The updating of the regularization parameters is adaptively determined by means of an on-line analysis of the values of the cost function terms. This allows a more efficient management of the original multi-objective inverse scattering problem without recurring to an *ad-hoc* optimization strategy. Numerical experiments have been carried out and the achieved results are very promising.

In the authors' opinion, although further analysis should be performed for a complete assessment of the method, the obtained results are indicative of the potentialities, robustness and flexibility of the proposed approach. Future works will be devoted to further refine the adaptive tuning of the regularization parameters in order to improve the convergence rate of the minimization process.

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Figure Caption

- Figure 1. Dielectric profile reconstruction. Actual dielectric distribution (a). Initial trial dielectric distribution (b).

- Figure 2. Behavior of the cost function versus the iteration number k: (a) Constant-weights approach, (b) Adaptive-weights approach - Constant-threshold strategy ($\gamma_k = \gamma_0$), and (c) Adaptive-weights approach - Variable-threshold strategy.

- Figure 3. Behavior of the terms (g_{data} and g_{state}) of the cost function versus the iteration number k: (a) Constant-weights approach, (b) Adaptive-weights approach - Constant-threshold strategy ($\gamma_k = \gamma_0$), and (c) Adaptive-weights approach - Variable-threshold strategy.

- Figure 4. Dielectric profile reconstruction. Dielectric distribution estimated at $k = 1000$ (*top*), $k = 5000$ (*middle*), and at the convergence $k = K$ (*bottom*) with the Constant-weights approach (*left*), with the Adaptive-weights approach - Constant-threshold strategy (*center*), and with the Adaptive-weights approach - Variable-threshold strategy (*right*).

Table Caption

- Table I. Dielectric profile reconstruction. Error figures.

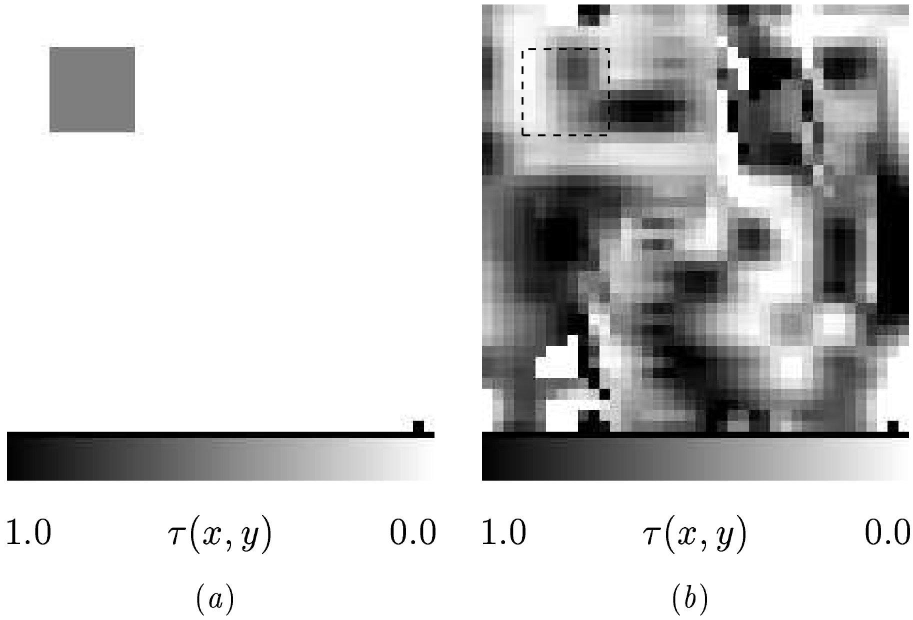
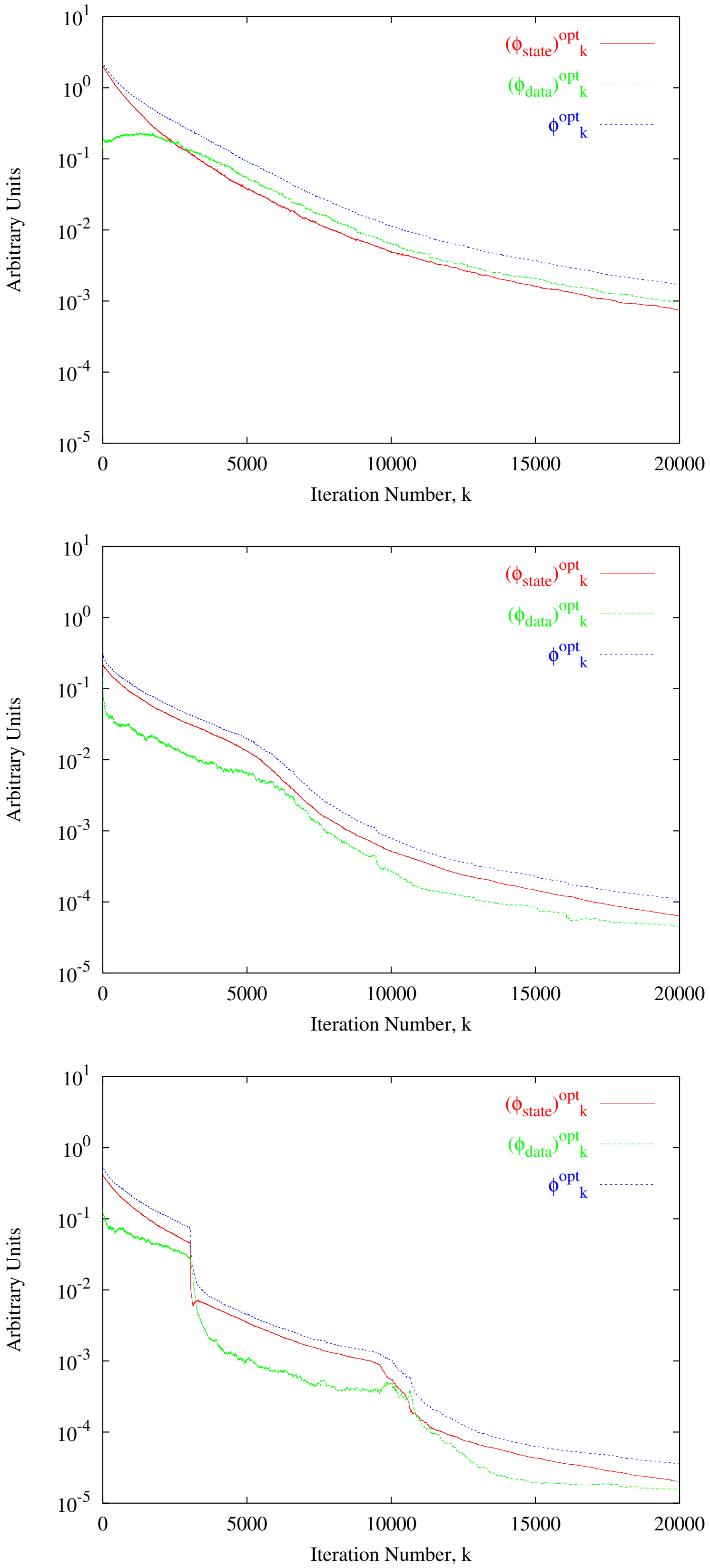


Fig. 1 - E. Bort *et al.*, "An adaptive regularization strategy ..."



(a)

(b)

(c)

Fig. 2 - E. Bort *et al.*, "An adaptive regularization strategy ..."

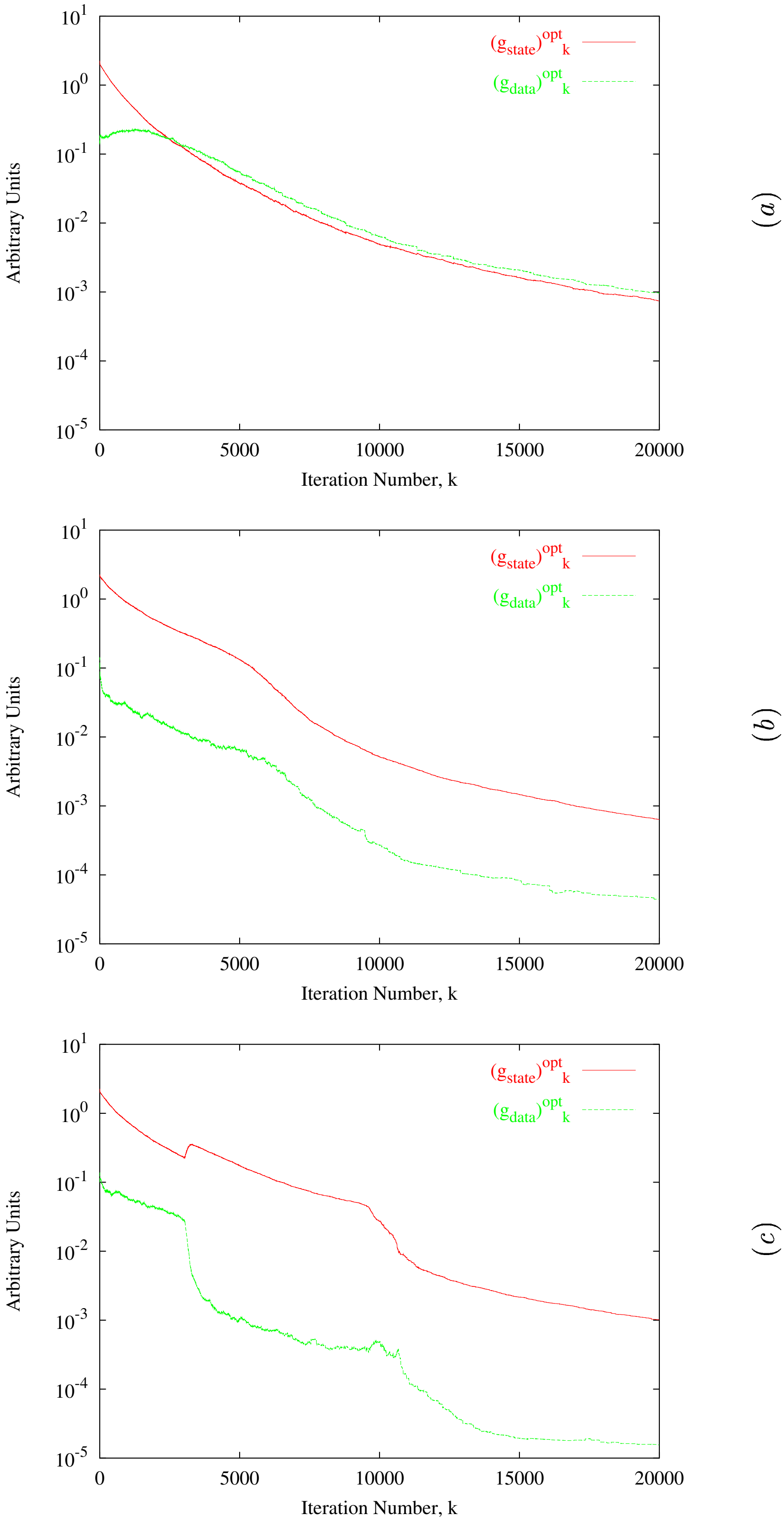


Fig. 3 - E. Bort *et al.*, "An adaptive regularization strategy ..."

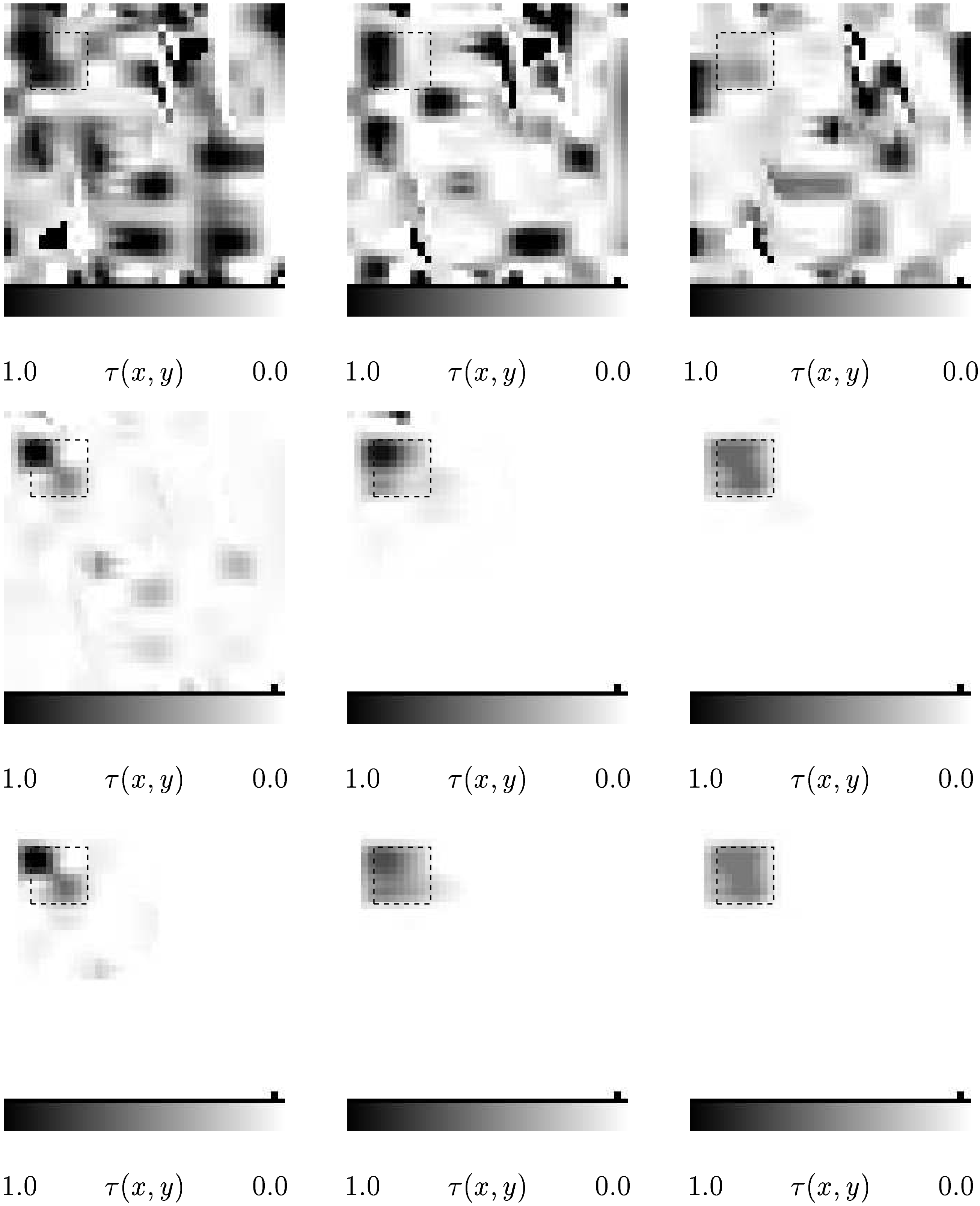


Fig. 4 - E. Bort *et al.*, "An adaptive regularization strategy ..."

Percentage Error	Ξ_{tot}	Ξ_{ext}	Ξ_{int}
Constant-weights	2.09	1.29	21.32
Adaptive-weights $(\gamma_k = \gamma_0)$	1.11	0.28	16.00
Adaptive-weights (γ_k)	0.72	0.21	10.98

Tab. I - E. Bort *et al.*, "An adaptive regularization strategy ..."