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Wave propagation control in active acoustic metamaterials

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Abstract. Focus is on the design of an innovative class of tunable periodic metamaterials, conceived for the realization of high performance acoustic metafilters with settable real-time capabilities. In this framework the tunability is due to the presence of a piezoelectric phase shunted by a suitable electrical circuit with adjustable impedance/admittance. It follows that the acoustic properties of the metamaterial can be properly modified in an adaptive way, opening up new possibilities for the control of pass- and stop-bands.

1. Introduction

The study of metamaterials is increasingly emerging as an innovative interdisciplinary area, including physics, material science and engineering. Metamaterials are architected composites, specifically conceived to exhibit outstanding constitutive properties, well above those achievable with classical materials. In this context, the present paper proposes the design of high-performance tunable acoustic filters for the passive wave propagation control. More specifically a special class of periodic metamaterials with a phase shunted by either a dissipative or non dissipative electrical circuit is proposed.

2. Active metamaterial with a piezoelectric shunting phase

Consider an infinite metamaterial with periodic microstructure characterized by three phases, as shown in the Figure 1(a). This metamaterial is made by in-plane tessellation of the centrosymmetric periodic cell \mathcal{A} , schematically depicted in the Figure 1(b), along two orthogonal periodicity vectors $\mathbf{v}_1 = d\mathbf{e}_1$, $\mathbf{v}_2 = d\mathbf{e}_2$, being \mathbf{e}_1 and \mathbf{e}_2 unit vectors of a given orthogonal base. The microstructure of the cell \mathcal{A} , with out-of-plane thickness w , is composed by a stiff and massive frame of outer edge b , embedding both an outer ring made of a soft piezoelectric material of thickness h and a inner disk of radius r , made of the same material as the frame. The geometry is completed by four flexible and light ligaments of length $l = (d - b)/2$ and width a . Each piezoelectric ring is connected in parallel to an external electrical circuit with tunable equivalent impedance/admittance, see Figure 1(c). It follows that the frequency band structure of the acoustic metamaterial can be properly controlled by modifying the equivalent impedance/admittance together with the constitutive properties of the shunted piezoelectric material. The stop- and pass-band frequency amplitudes as well as their central frequencies can be thus manipulated to exploit the performance of the periodic tunable metamaterial as acoustic filter. In the following, first the dynamic governing equations of the three phase active metamaterial, together with the constitutive relations of the shunted piezoelectric phase, are introduced. Then the capabilities of the



free wave propagation control in such acoustic metamaterial are investigated and a relevant example is provided.

3. Governing equations of the three-phase active metamaterial

The in-plane governing equations of the infinite metamaterial are expressed, accordingly with [1], in the transformed Laplace space, as follows

$$\left(C_{ijhl}^{\diamond}(s) \widehat{u}_{h,l} \right)_{,j} + \widehat{b}_i = \rho s^2 \widehat{u}_i, \quad (1)$$

where $C_{ijhl}^{\diamond}(s)$ are the components of the constitutive tensor, s is the complex Laplace variable, \widehat{u}_i are the in-plane transformed displacement components, \widehat{b}_i the transformed source term components, ρ is the mass density and the comma denotes the partial derivative with respect to x_j , i.e. the components of the in-plane position vector $\mathbf{x} = x_j \mathbf{e}_j \in \mathbb{R}^2$, with $i, j, h, l = 1, 2$. Due to the medium periodicity, both the constitutive tensor and the mass density are \mathcal{A} periodic, so that

$$C_{ijhl}^{\diamond}(\mathbf{x} + n\mathbf{v}_{\beta}, s) = C_{ijhl}^{\diamond}(\mathbf{x}, s), \quad \rho(\mathbf{x} + n\mathbf{v}_{\beta}) = \rho(\mathbf{x}), \quad \forall \mathbf{x} \in \mathcal{A}, \quad (2)$$

where $n \in \mathbb{Z}$ and the index $\beta = 1, 2$. Note that the components $C_{ijhl}^{\diamond}(s)$ characterize the constitutive properties of each material phase pertaining to the microstructure that in Figure 1 are distinguished by color. In particular, the gray phase, referred to ligaments, is assumed as a linear elastic isotropic material. The same constitutive behavior is considered for the blue phase, pertaining both to the stiff frame and to the stiff disk. On the other hand, the red phase, i.e. the internal ring, is made by a shunted piezoelectric material. It stands to reason that the components C_{ijhl}^{\diamond} of the linear elastic materials are s -independent, while those of the shunting piezoelectric phase, denoted with the apex EL , are in general s -dependent and take the following form

$$C_{ijhl}^{EL}(\lambda(s)) = C_{ijhl} + \frac{e_{ij3} \widetilde{e}_{3hl}}{\beta_{33}^{EL}(\lambda(s))} - \left(C_{ij33} + \frac{e_{ij3} \widetilde{e}_{333}}{\beta_{33}^{EL}(\lambda(s))} \right) \begin{pmatrix} C_{33hk} + \frac{e_{333} \widetilde{e}_{3hl}}{\beta_{33}^{EL}(\lambda(s))} \\ C_{3333} + \frac{e_{333} \widetilde{e}_{333}}{\beta_{33}^{EL}(\lambda(s))} \end{pmatrix}, \quad (3)$$

where C_{ijhl} are the components of the fourth order elasticity tensor, e_{ij3} are the components of the third order stress-charge coupling tensor of the piezoelectric material and $\widetilde{e}_{pqs} = e_{qsp}$ its transpose. Moreover,

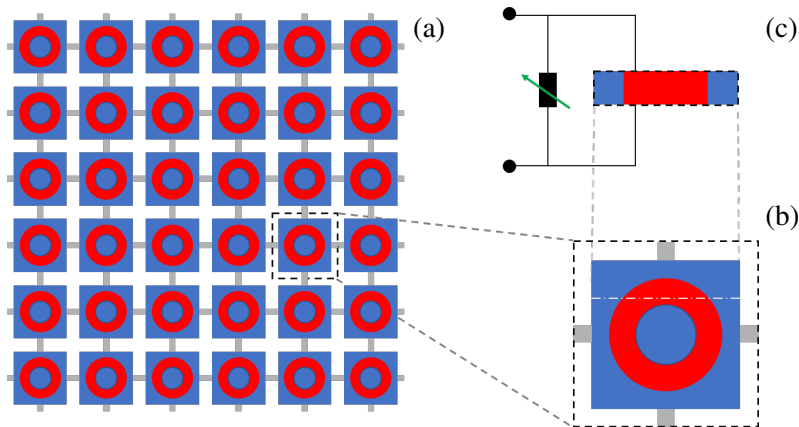


Figure 1. (a) A generic portion of the heterogeneous material; (b) scheme of the periodic cell containing the shunted piezoelectric material; (c) scheme of the tunable electric circuit.

the auxiliary s -dependent function $\beta_{33}^{EL}(s) = \beta_{33}(1 + \lambda(s))$ is introduced, with β_{33} the component of the second order permittivity tensor and the so-called *tuning function* $\lambda(s) = L^{(P)}Y_{33}^{SU}(s)/(s\beta_{33}A^{(P)})$ linearly depending on the generic equivalent shunting admittance $Y_{33}^{(SU)}(s)$ that, in turn, may depend on one or more tuning parameters characterizing the specific electrical circuit. Note that $A^{(P)}$ is the area of the piezoelectric phase (red annulus in Figure 1(b)), and $L^{(P)} = w$.

4. Wave propagation modeling

With the aim of investigating the free wave propagation in the infinite periodic tunable metamaterial, accordingly with the Floquet-Bloch theory [4], the Floquet-Bloch decomposition is introduced as

$$\hat{u}_i = \hat{U}_i e^{Ik_j x_j} \quad (4)$$

with \hat{U}_i the \mathfrak{A} -periodic Bloch amplitudes of the displacement in the transformed space, k_j the wave vector components and I the imaginary unit. By plugging Eq. (4) in the field equation (1), the following field equation in terms of the Bloch amplitudes is obtained as

$$\left(C_{ijhl}^{\diamond}(s) \hat{U}_{h,l} \right)_{,j} - \left(C_{ijhl}^{\diamond}(s) k_l k_j + s^2 \rho \delta_{ih} \right) \hat{U}_h + Ik_j \left[\left(C_{ijhl}^{\diamond}(s) + C_{ilhj}^{\diamond}(s) \right) \hat{U}_{h,l} + C_{ilhj,l}^{\diamond}(s) \hat{U}_h \right] = 0, \quad (5)$$

where δ_{ih} is the Kronecker delta. By assuming $k_i = kn_i$ with k the wave number and $\mathbf{n} = n_i e_i \in \mathbb{R}^2$ the unit vector of propagation, the previous field equation takes the form

$$\left(C_{ijhl}^{\diamond}(s) \hat{U}_{h,l} \right)_{,j} - \left(C_{ijhl}^{\diamond}(s) k^2 n_l n_j + s^2 \rho \delta_{ih} \right) \hat{U}_h + Ikn_j \left[\left(C_{ijhl}^{\diamond}(s) + C_{ilhj}^{\diamond}(s) \right) \hat{U}_{h,l} + C_{ilhj,l}^{\diamond}(s) \hat{U}_h \right] = 0. \quad (6)$$

By exploiting the medium periodicity, Eq. (6) can be solved by applying the well-known periodic boundary conditions on the cell boundaries. Moreover, with the aim of solving the field equation (6) through a standard Galerkin procedure, the weak formulation is obtained by integrating, over the periodic cell \mathfrak{A} , this equation multiplied by a properly chosen \mathfrak{A} -periodic test function $\hat{\Psi}_i$, resulting in

$$\int_{\mathfrak{A}} \left\{ \left(C_{ijhl}^{\diamond}(s) \hat{U}_{h,l} \right)_{,j} - \left(C_{ijhl}^{\diamond}(s) k^2 n_l n_j + s^2 \rho \delta_{ih} \right) \hat{U}_h + Ikn_j \left[\left(C_{ijhl}^{\diamond}(s) + C_{ilhj}^{\diamond}(s) \right) \hat{U}_{h,l} + C_{ilhj,l}^{\diamond}(s) \hat{U}_h \right] \right\} \hat{\Psi}_i d\mathbf{x} = 0. \quad (7)$$

By means of the divergence theorem and observing that the boundary integrals vanish due to the \mathfrak{A} -periodicity of the test function, the following equation is obtained

$$k^2 \int_{\mathfrak{A}} C_{ijhl}^{\diamond}(s) n_l n_j \hat{U}_h \hat{\Psi}_i d\mathbf{x} - Ik \int_{\mathfrak{A}} \left[\left(C_{ijhl}^{\diamond}(s) + C_{ilhj}^{\diamond}(s) \right) \hat{U}_{h,l} \hat{\Psi}_i + C_{ilhj,l}^{\diamond}(s) \hat{U}_h \hat{\Psi}_i \right] n_j d\mathbf{x} + \int_{\mathfrak{A}} C_{ijhl}^{\diamond}(s) \hat{U}_{h,l} \hat{\Psi}_{i,j} d\mathbf{x} - s^2 \int_{\mathfrak{A}} \rho \delta_{ih} \hat{U}_h d\mathbf{x} = 0. \quad (8)$$

In the framework of the finite elements method, the following homogeneous assembled matrix equation can be obtained by discretized the weak form (8) together with its boundary terms, in agreement with the procedure proposed by [5] and detailed in [6, 7], i.e.

$$\mathbf{H}(k, s, \mathbf{n}) \hat{\mathbf{U}} = \left(k^2 \mathbf{H}^{(2,0)}(s, \mathbf{n}) + Ik \mathbf{H}^{(1,0)}(s, \mathbf{n}) + \mathbf{H}^{(0,0)}(s) + s^2 \mathbf{H}^{(0,2)}(s) \right) \hat{\mathbf{U}} = 0, \quad (9)$$

where $\hat{\mathbf{U}}$ is the vector collecting nodal Floquet-Bloch amplitudes and $\mathbf{H}^{(2,0)}$, $\mathbf{H}^{(1,0)}$, $\mathbf{H}^{(0,0)}$ and $\mathbf{H}^{(0,2)}$ are s -dependent linear operators. More specifically, the matrix equation (9) is a generalized eigenvalue

problem characterizing the free propagation of Bloch waves with angular frequency s , wave number k and polarization vector $\widehat{\mathbf{U}}$, in the infinite periodic metamaterial along the generic propagation direction \mathbf{n} . Aiming at investigating the temporal damping [8, 9, 7], the angular frequency s and the polarization vector $\widehat{\mathbf{U}}$ are, respectively, the eigenvalue and the eigenvector of the generalized eigenproblem (generally rational) depending on parameters \mathbf{n} and k . It stands to reason that the eigenproblem (9) has non trivial solution in the case the following characteristic equation is fulfilled

$$\mathfrak{D}(k, s, \mathbf{n}) = \det[\mathbf{H}(k, s, \mathbf{n})] = 0, \quad (10)$$

determining the complex frequency band structure $s(k) \in \mathbb{C}$ in terms of the wave number $k \in \mathbb{R}$ for a fixed propagation direction \mathbf{n} . In the particular case of remarkable technological interest, in which the equivalent electrical circuit is characterized by a purely capacitive equivalent admittance $Y_{33}^{SU}(s) = sC$, the tuning functions turn out to be s -independent, i.e. $\lambda = CL^{(p)}/(\beta_{33}A^{(p)})$, as well as the constitutive tensor components C_{ijkl}^{EL} . In this case, as shown in [1], the complex frequency s is purely imaginary, i.e. $s = I\omega$, being $\omega \in \mathbb{R}$ the angular frequency of Bloch waves. The eigenproblem (9) takes the form

$$\mathbf{K}(k, \omega, \mathbf{n})\widehat{\mathbf{U}} = \left(k^2 \mathbf{K}^{(2,0)}(\mathbf{n}) + Ik \mathbf{K}^{(1,0)}(\mathbf{n}) + \mathbf{K}^{(0,0)} - \omega^2 \mathbf{K}^{(0,2)} \right) \widehat{\mathbf{U}} = 0, \quad (11)$$

where the linear operators $\mathbf{K}^{(2,0)}$, $\mathbf{K}^{(1,0)}$, $\mathbf{K}^{(0,0)}$ and $\mathbf{K}^{(0,2)}$ are s -independent so that the eigenproblem turns out to be in the polynomial form. It has non trivial solution in the case the following characteristic equation is satisfied

$$\mathfrak{D}(k, \omega, \mathbf{n}) = \det[\mathbf{K}(k, \omega, \mathbf{n})] = 0, \quad (12)$$

determining the frequency band structure $\omega(k) \in \mathbb{R}$ in terms of $k \in \mathbb{R}$ for fixed \mathbf{n} .

5. Conclusions

A class of tunable mechanical metamaterials characterized by periodic piezoelectric microstructure, shunted by an electrical circuit is here designed. Numerical experiments highlight that it is possible to open low-frequency band gaps and modifying on demand their amplitude and central frequencies by adjusting the tuning parameter, also starting from dense spectra. It is noted that by decreasing the tuning parameter up to about the resonant value, a stiffening of the piezoelectric material is exhibited, together with enlargments of low-frequency band gaps. Moreover, as expected the central frequencies of such band gaps tend to slightly increase.

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