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RESPONSE SPECTRUM ANALYSIS OF LIGHT TIMBER-FRAME BUILDINGS BY MEANS OF AN ITERATIVE APPROACH

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ABSTRACT: Light timber-frame buildings are nowadays widely spread even in seismic prone regions tanks to several researches undertaken in the last decade, which highlighted their potentialities in terms of seismic response. Anyway, a definitive method to assess their dynamic properties as well as an analytical method to apply the Response Spectrum Analysis (RSA) are missing both in standards and literature. The present paper deals with the seismic analysis of this type of building. Specifically, through the use of a numerical simplified model accounting for the main deformation contributions, three analytical-iterative method to apply the RSA are presented. The methods are iterative because both of the hold-downs behaviour and the vertical load actions. In fact, the hold-downs display an on-off behaviour depending on their state which can be active (hold-down in tension) or not active (hold-down not in tension), whereas the vertical load modifies the seismic force distribution within the wall. These methods can be used to assess the dynamic properties of a building and to design non-regular buildings. In order to allow readers to better understand the procedures a numerical example is included.

KEYWORDS: Light timber-frame buildings, stiffness matrix, response spectrum analysis, UniTn-Model, seismic

1 INTRODUCTION

The Response Spectrum Analysis (RSA) has been applied to buildings which cannot be defined regular in elevation and it should be considered the reference method for the seismic design because it can be applied to any type of building without any geometric limitation. Moreover, its results could be considered more reliable compared to *Lateral Force Method* (LFM) because the analysis takes into account all the significant modes of vibration participating to the seismic response of the structure. Anyway, despite the construction of timber building is more and more increasing, the use of the RSA has not been established yet, because a definitive procedure is still missing.
The main objective of the present work is to develop a suitable analytical procedure to allow researches and engineer to apply the RSA to light timber-frame buildings. In fact, nowadays the LFM is mainly used by designers (even if the RSA is mandatory for non-regular buildings, see [2]) because a definitive method to apply the RSA which correctly accounts for its recursive nature is missing.

The seismic behaviour of timber buildings is not fully-developed as they are more widespread in countries of North Europe (e.g. Sweden, Norway, Finland, etc.) as well as in Germany and Austria, where the seismic activity is not particularly intense. Nowadays, thanks to the potentialities in terms of seismic response, the timber buildings are becoming to spread to countries where the earthquake are more frequent and damaging as in Southern Europe and Central America. It is therefore necessary to have standard requirements and the methods of analysis to use for timber buildings in the design phase in seismic areas, but for now they are still poorly developed.
To better understand the behaviour of the timber building subjected to horizontal load (i.e. seismic event) analytical model is needed. For this purpose, an analytical iterative model suitable to predict the mechanical behavior of timber walls subjected to horizontal load is presented.
This paper is based on a previous work () focused on the elastic analysis of one-storey timber shear-wall, both Light-Timber frame wall (TF) and Cross Laminated Timber walls (CLT). In that work an analytical procedure and a simplified numerical model (called UNITN model) were proposed to assess the elastic-horizontal behaviour of single-storey buildings.

In Europe, timber buildings are traditionally more widespread in northern countries (e.g. Sweden, Norway, Finland etc.) as well as in Germany and Austria. The fact that the seismic activity in these areas is not particularly intense has led to a not fully-developed awareness about the potentialities of timber wall buildings in terms of seismic response. Research undertaken in last decades have highlighted these potentialities, so that timber building are becoming widespread even in south Europe and in Central America, where earthquakes are more frequent and damaging. For this reason it is necessary to deepen the knowledge of the dynamic behaviour of timber buildings; in fact, both standard requirements and method of analysis are still poorly developed. Some useful

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researches have been done, but they are mainly focused on case-study buildings and based on the use of finite-element analysis, therefore analytical models as well as numerical procedures and practical tools for designer are needed.

This paper is based on a previous work [1] focused on the elastic analysis of one-storey timber shear-walls, both Light-Timber frame walls (TF) and Cross Laminated Timber walls (CLT). In that work an analytical procedure and a simplified numerical model (called UNITN model) were proposed to asses the elastic-horizontal behaviour of single-storey buildings.

The objective of the paper is to develop a correct procedure to apply the *linear lateral force method* (LFM) and a suitable procedure in order to give an answer to the open technical problem of the application of the *response spectrum analysis* (RSA) for multi-storey light-timber frame buildings. In fact, nowadays, LFM is mainly used by designers because a definitive procedure to extend the analytical modal analysis to timber wall building is still missing. The implementation of the common LFM could lead to analysis errors because it is suitable for buildings which behaviour can be mainly attributed to the first modal shape (i.e. regular buildings in elevation,

In the first part of the paper, a procedure to evaluate the horizontal deformation of one timber shear-wall composed by m storeys (system of m x 1 walls) is presented. The analysis is then extended to a more complex system composed of n timber shear-walls of m storeys (system of m x n walls), which reproduces a full-scale building. Both the procedures are iterative due to the presence of the hold-downs (i.e. the connection devices which prevent the uplift of the wall), which is responsible of a geometrical and mechanical non-linear behaviour. The hold-down action in fact, should be taken into account only when the overturning moment produced by a horizontal force exceeds the stabilizing moment due to the vertical load.

In the second part of the paper, the system of m x n walls is further developed; a procedure to asses the dynamic properties of a timber building (e.g. natural frequencies, mode-shapes, participating masses) is presented. Three methods to apply the RSA are also proposed. The procedures, better explained in Section 6.2, differ in the way they consider the vertical load effects; in fact, the vertical load affects both the stiffness-matrix of the building and the shear-force distribution between the walls.

2 SINGLE WALL MODEL

2.1 Analytical formula

The elastic displacement Δ_C of Light-Timber frame (TF) shear-wall subjected to a horizontal external force *F,* see *Figure 1*(a), can be evaluated using Eq. (1), which considers the four main deformation contributions, namely the sheeting to framing connection, the rigid-body rotation, the rigid-body translation and the sheeting-boards respectively.

$$\Delta_C = \begin{cases} \frac{\lambda\cdot F\cdot s_c}{l\cdot n_{bs}\cdot k_c}+\frac{F\cdot i_a}{k_a\cdot l}+\frac{F\cdot h}{l\cdot G_p\cdot n_{bs}\cdot t_p} & F\cdot h\leq\frac{q\cdot l^2}{2} \\ \frac{\lambda\cdot F\cdot s_c}{l\cdot n_{bs}\cdot k_c}+\left[\frac{h}{\tau\cdot l\cdot k_h}\cdot\left(\frac{F\cdot h}{\tau\cdot l}-\frac{q\cdot l}{2}\right)\right]+\frac{F\cdot i_a}{k_a\cdot l}+\frac{F\cdot h}{l\cdot G_p\cdot n_{bs}\cdot t_p} & F\cdot h>\frac{q\cdot l^2}{2} \end{cases} \quad (1)$$

where:

- F: is the applied horizontal force;
- s_c: is the fasteners spacing;
- l: is the wall length:
- b: is the breadth of the sheathing-panel
- λ : is a parameter related to the sheeting-panel dimensions;
- k_h: is the hold-down stiffness;
- q: is the vertical distributed load;
- k_a: is the angle-brackets (or screws) stiffness;
- i_a: is the angle-brackets (or screws) spacing;
- G_p:is the shear modulus of sheathing panels;
- t_p: is the sheathing panel thickness;
- h: is the height of the panel;
- k_c: is the fasteners stiffness;
- n_{bs}:is the number of braced sides of the wall;
- τ: is a number accounting for the distance between the hold-downs, typically $\in$[0.9;1];

Other contributions (as frame deformation, bending deflection, etc.) could be taken into account, but for the wall typologies most used in Europe, these contributions are negligible compared to the other. For more details see [1],[4] and [5].

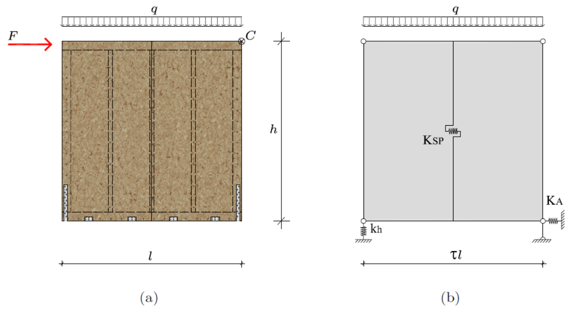


Figure 1: *(a) Light timber-frame wall; (b) UNITN simplified model.*

2.2 Simplified numerical model

The behaviour of a timber wall can be faithfully reproduced by means of simplified numerical model, called UniTN-Model (see *Figure 1*(b)). It is called simplified because it reproduces the behaviour of a timber wall by means of three linear-elastic springs placed in series; each spring has a stiffness related to the corresponding deformation contribution, namely the hold-down, the angle brackets and the coupled action of the sheathing-panels with the fasteners:

$$\frac{1}{K_{SP}}=\frac{1}{K_P}+\frac{1}{K_{SH}}=\frac{h}{G_p\cdot n_{bs}\cdot t_p\cdot l}+\frac{s_c\cdot\lambda}{n_{bs}\cdot k_c\cdot l} \quad (2)$$

$$\frac{1}{K_A}=\frac{i_a}{k_a\cdot l} \quad (3)$$

$$\frac{1}{K_H} = \frac{h^2}{k_h \cdot (\tau \cdot l)^2} \quad (4)$$

Considering Eq. 4 it is evident that when the hold-down is in tension the behaviour of the wall is not linearly-proportional to the wall length, indeed the wall stiffness depends on the squared length. Furthermore, it is important to remark that when the hold-down is in tension, the horizontal displacement of a timber wall is reduced by the vertical load:

$$\Delta = \frac{F}{K_{tot}} - \Delta_N \quad (5)$$

where K_{tot} is the global stiffness of TF wall:

$$K_{tot} = \left(\frac{1}{K_{SP}} + \frac{1}{K_A} + \frac{1}{K_H}\right)^{-1} \quad (6)$$

And Δ_N is the equivalent horizontal force produced by the vertical load (which counteracts the horizontal external force):

$$\Delta_N = \frac{q \cdot l/2 \cdot h}{\tau \cdot l \cdot k_h} = \frac{N \cdot h}{\tau \cdot l \cdot k_h} \quad (7)$$

3 MODEL OF HORIZONTALLY ALIGNED WALLS

A system of several horizontally-aligned walls, i.e. a system of *1xn* walls modelling a single-storey building (see *Figure 2*) can be obtained by connecting each wall-model to the next one by means of an infinite rigid pinned-beam simulating the effects of the upper floor (diaphragms are assumed to be rigid, namely they do not undergo any deformation during earthquake), which imposes to the walls the same horizontal displacement. Therefore, the horizontal force acting on the building is supported by all the walls, which can be regarded as a system of springs in parallel (see *Figure 3*)

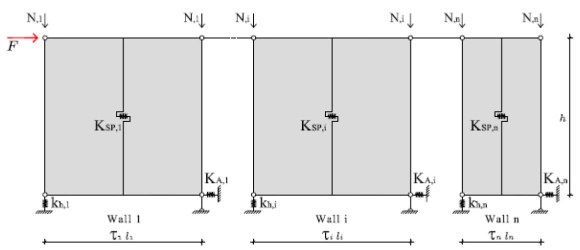


Figure 2: *System of 1 x n walls.*

The horizontal displacement Δ of the system of *1 x n*, as well as the horizontal force carried by each wall F_i, can be assessed by the two following equations:

$$\Delta = \frac{F - \sum_{i=1}^{n}(K_{tot,i} \cdot \Delta_{N,i})}{\sum_{i=1}^{n} K_{tot,i}} \quad (8)$$

$$F_i = V_i = \frac{K_{tot,i}}{\sum_{j=1}^{n} K_{tot,j}} \cdot \left[F - \sum_{j}^{n}\left[K_{tot,j} \cdot (\Delta_{N,j} - \Delta_{N,i})\right]\right] \quad (9)$$

These equations were determined using the compatibility as well as the constitutive and equilibrium laws.
It is important to note that for one-storey building, the horizontal force carried-out by each wall is equal to the shear-force acting on it.

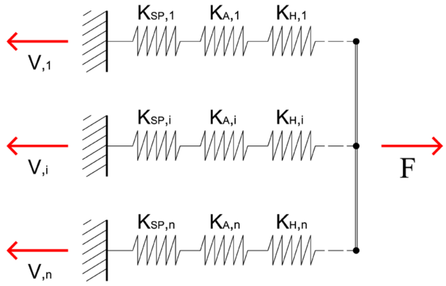
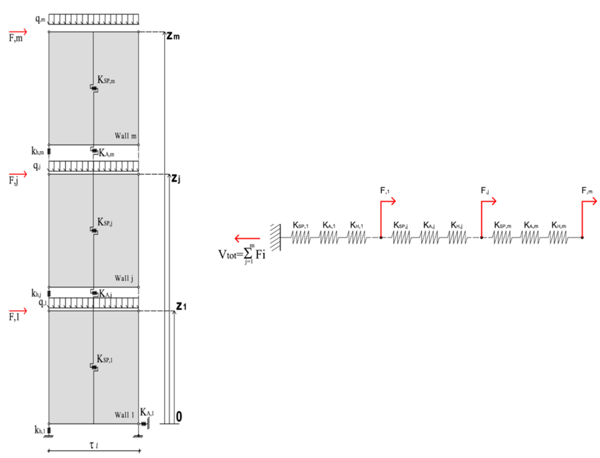


Figure 3: *Rheological model of a one-storey building.*

Eqs. (8) and (9) demonstrate that both the displacement and the shear-force distribution are influenced by the presence and the magnitude of the vertical load. Moreover, the presence of the vertical load leads to the need of adopting an iterative procedure. In fact, the stiffness of a wall depends on the hold-down state (hold-down in tension = active or hold-down in compression = not active) which, in turn, depends on the shear-force distribution.

4 MODEL OF VERTICALLY-ALLIGNE WALLS

A system of vertically-aligned walls, modelling a timber shear-wall of *m*-storey, can be obtained by means the superimposition of *m* single walls (see *Figure 4*(a)). Each wall model is characterised by three springs which stiffness can be determinate through Eqs. (2)-(4). The wall uplifting corner is vertically connected to the lower wall by means of a spring with stiffness equal to $k_{h,j}$ which models the *j-th* hold-down behaviour. In the other corner, instead, a pinned-axially-rigid beam is placed. This pinned beam is used to avoid the corner lowering but, at the same time, to allow a mutual translation between the walls connected by it. Moreover, a horizontal spring with a stiffness equal to $K_{A,j}$ reproduces the behaviour of the shear connection to the lower support (i.e. angle brackets or screws, as well as steel plates).



(a) Numerical m x 1 model *(b) Rheological model*

Figure 4: *System of m x 1 walls modelling a single-shear wall of m-storey.*

The rigid-rotation deformation becomes extremely relevant for a multi-storey shear-wall; in fact, the elongation of a hold-down produces an increasing horizontal displacement at each upper floor, because a multi-storey shear-wall can be considered as a system of several springs in series, see *Figure 4*(b). Moreover, the non-linear relationship between the wall length and its stiffness, as reported in Eq. (4), is directly related to the rigid-body rotation, which increases significantly for multi-storey walls.
For a shear-wall of m-storey the displacement $\Delta_{j,\xi}$ at the *j-th* storey provoked by a horizontal force placed at the *ξ-th* storey is equal to:

$$\Delta_{j,\xi} = \sum_{r=1}^{\min(j,\xi)} F_\xi \cdot \left[\frac{1}{K_{SP,r}} + \frac{1}{K_{A,r}}\right] + \sum_{r=1}^{\min(j,\xi)} \left[\left(F_\xi \cdot \frac{z_\xi - z_{r-1}}{\tau \cdot l} - \frac{M_r}{|M_r|}\sum_{y=r}^{m} N_y\right) \cdot \frac{z_j - z_{r-1}}{k_{h,r} \cdot \tau \cdot l}\right] \tag{10}$$

where N_y is the vertical concentrated load at the *y-th* floor:

$$N_Y = \frac{q_y \cdot l}{2} \tag{11}$$

and where M_r is the bending moment acting at the *r-th* storey and the quantity $M_r/|M_r|$ is needed to give the correct sign to the equivalent displacement $\Delta_{N,j}$.
It is important to remark that when the vertical load at a certain storey exceeds the tensile force produced by the horizontal loads, the hold-down contribution at that level has to be removed from Eq. (10); in fact, when a hold-down is in compression, it does not undergo any deformation and it does not produce any horizontal displacement. In order to assess the magnitude of the tensile force in the hold-down the following equation can be used:

$$T_{HD,j} = \left|\sum_{r=j}^{m}\left[F_\xi \cdot \frac{z_\xi - z_{r-1}}{\tau \cdot l}\right]\right| - N_j \tag{12}$$

where $T_{HD,j}$ is the tensile force in the hold-down of the *j-th* storey. A positive value means tension whereas a negative value means compression. This allows to define two states: hold-down active when it is in tension, hold-down not active when it is in compression. Because a not-in-tension hold-down does not produce any deformation, its stiffness can be considered infinite.
The number of degree of freedom increases with the building storey and so a matrix formulation is needed. Eq. (10) can be written in the following matrix form:

$$\boldsymbol{\Delta} = \boldsymbol{\tilde{U}F} - \boldsymbol{\Delta_N} \tag{13}$$

where $\boldsymbol{\tilde{U}}$ is the flexibility matrix and $\boldsymbol{\Delta_N}$ is the displacement array due to the vertical load.
The *j,ξ*-element of the flexibility matrix can be obtain from:

$$\tilde{U}_{j,\xi} = \sum_{r=1}^{\min(j,\xi)} \frac{1}{K_{SP,r}} + \frac{1}{K_{A,r}} + \frac{(z_\xi - z_{r-1}) \cdot (z_j - z_{r-1})}{k_{h,r} \cdot (\tau \cdot l)^2} \tag{14}$$

The *j*-element of the displacement array Δ_N can be determined from:

$$\Delta_{N,j} = \sum_{r=1}^{j} \frac{M_r}{|M_r|}\left(\sum_{y=r}^{m} N_y\right) \cdot \frac{z_j - z_{r-1}}{k_{h,r} \cdot \tau \cdot l} \tag{15}$$

The bending moment M_r can be evaluated by means of the following equation:

$$M_r = \sum_{p=r}^{m} F_P \cdot (z_p - z_{r-1}) \tag{16}$$

Eq. (13) can be rearranged as:

$$\boldsymbol{F} = \boldsymbol{\tilde{U}}^{-1}(\boldsymbol{\Delta} + \boldsymbol{\Delta_N}) = \boldsymbol{K\,\Delta} + \boldsymbol{K\,\Delta_N} = \boldsymbol{K\,\Delta} + \mathbf{F_N} \tag{17}$$

Where the inverse of the flexibility matrix represents the stiffness matrix of one timber shear-wall of m-storey:

$$\boldsymbol{K} = \boldsymbol{\tilde{U}}^{-1} \tag{18}$$

And $\mathbf{F}_N$ is the array of the equivalent force due to the vertical load.

5 MODELLING OF A FULL-SCALE BUILDING

A full scale building can be modelled connecting each-other several multi-storey walls, see *Figure 5*. According to the UNITN model, each wall is represented by means of three springs, $K_{SP,ji}$, $K_{A,ji}$ and $k_{h,ji}$, and each wall is properly connected to the upper, lower and side wall. Moreover, the load pattern is composed by the vertical loads $q_{j,i}$ and the horizontal force distribution F_j.

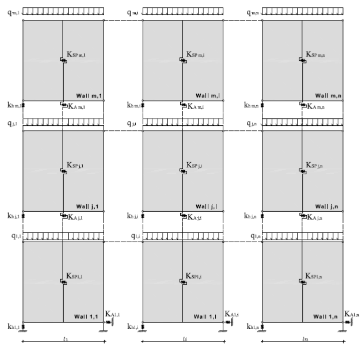


Figure 5: *System of m x n walls modelling a full-scale building.*

The constitutive law of a system of *m x n* walls, can be derived from (17) ad it is:

$$\boldsymbol{F} = \boldsymbol{K_{sys}\,\Delta} + \boldsymbol{F_{N,sys}} \tag{19}$$

where K_{sys} is the stiffness matrix of the model of *m x n*, and it is given by the sum of stiffness matrices K_i (see Eq. (18)) of each multi-storey wall *m x 1*:

$$\boldsymbol{K_{sys}} = \sum_{i=1}^{n} \boldsymbol{K_i} \tag{20}$$

and the array of the equivalent forces $F_{N,sys}$ due to the vertical load is equal to:

$$F_{N,sys} = \sum_{i=1}^{n} K_i \Delta_{N,i} \tag{21}$$

Generally, know the applied external force distribution F (e.g. wind or seismic load), the horizontal displacement vector Δ is given by:

$$\boldsymbol{\Delta} = \boldsymbol{K_{sys}}^{-1}\left(\boldsymbol{F} - \boldsymbol{F_{N,sys}}\right) \tag{22}$$

The external force array on the acting on the a multi-storey shear-wall *m x 1* can be determined by the following equation:

$$\mathbf{F_i} = \boldsymbol{K_i}\left(\boldsymbol{\Delta} - \boldsymbol{\Delta_{N,i}}\right) \tag{23}$$

the shear-force distribution, the bending-moments as well as the hold-down force of the *j,i-th* shear-wall can be determined as:

$$\mathrm{V}_{j,i} = \sum_{r=j}^{m} F_{r,i} \tag{24}$$

$$\mathrm{M}_{j,i} = \sum_{r=j}^{m} F_{r,i} \cdot \left(z_r - z_{j-1}\right) \tag{25}$$

$$\mathrm{T}_{HD,j,i} = \frac{\left|M_{j,i}\right|}{\tau \cdot l_i} - N_{j,i} \tag{26}$$

Anyway, the complexity of the analysis is not seated in the matrix formulation, but it is due to the non-linearity introduced by the binary behaviour of the hold-downs. In fact, the state of the hold-downs (i.e. activation or not) depends on the shear distribution; but this depends on the stiffness of the walls which depends in turn on the state of the hold-downs and so on. Moreover, for the evaluation of the equivalent horizontal displacement of each wall (see Eq. (15)) the bending moment sign should be predetermined but this depends in turn on the force distribution.
Therefore, the problem shows a recursive nature and it requires an iterative procedure of solution. In detail, as shown in the flow-chart of *Figure 6*, it is necessary to assume the model initial condition (i.e. state of the hold-downs), which allows to evaluate a first-attempt solution in terms of displacement, shear-distribution, bending moment sign and hold-downs stress. This solution is correct if the state of the hold-downs is consistent with the initial condition as well as with the calculation of equivalent horizontal displacement due to the vertical load; otherwise, it has to be rejected, the model has to be updated and the procedure has to be iterated up-to the achievement of the correct solution.

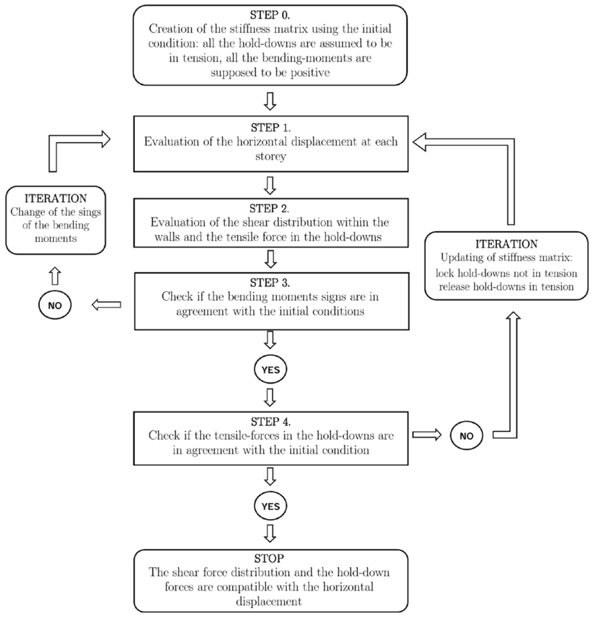


Figure 6: *Flow-chart for the iteration.*

5.1 PRACTICAL EXAMPLE OF A BUILDING COMPOSED BY 3X2 WALL

In order to explain the iteration procedure presented in Section 5, it is applied here to a system of 3 x 2 walls, which mechanical and geometrical properties as well as the vertical loads are shown in *Table 1*. Despite the procedure can be used to analyse real three-dimensional buildings, a simplified bi-dimensional building is analysed. In fact, the study of a 3D building would not have added any benefit; on the contrary, the seismic force distribution within the walls would be more complex due to the eccentricity between stiffness-center and center of mass.
The system is assumed to be loaded by the following set of external horizontal forces:

$$\mathbf{F} = \begin{bmatrix} 10 \\ 20 \\ -5 \end{bmatrix} \text{ [kN]} \tag{27}$$

Table 1: *Example 3 x 2. Geometrical and mechanical properties of the walls.*

Wall index	1,1	2,1	3,1	1,2	2,2	3,2
Length: l [mm]	2500	2500	2500	1250	1250	1250
Height: h [mm]	2500	2500	2500	2500	2500	2500
Vertical distributed load: q [kN/m]	5	5	5	0	0	0
Braced sides: n_{bs}	2	2	2	2	2	2
Shear Modulus: G_p [N/mm]	1000	1000	1000	1000	1000	1000
Sheathing pan. thickness: t_p [mm]	15	15	15	15	15	15
Sheathing pan. breadth: b [mm]	1250	1250	1250	1250	1250	1250
Fasteners stiffness: k_c [N/mm]	500	500	500	500	500	500
Fasteners spacing: s_c [mm]	100	100	100	100	100	100
HD stiffness: k_h [N/mm]	5000	2500	2500	5000	2500	2500
Angle-brackets stiffness: k_a [N/mm]	3000	2000	2000	3000	2000	2000
Angle-brackets number: n_a	4	4	4	2	2	2

Assuming all the hold-downs active as initial conditions, see *Figure 6* (a), the flexibility matrices of the two 3-storey can be evaluated by means of Eq. (14) (the values are given in mm/kN):

$$\tilde{\boldsymbol{U}}_1 = \begin{bmatrix} 0.4976 & 0.6976 & 0.8976 \\ & 1.8369 & 2.6369 \\ Sym & & 4.7761 \end{bmatrix};$$

$$\tilde{\boldsymbol{U}}_2 = \begin{bmatrix} 1.3949 & 2.1949 & 2.9949 \\ & 6.0732 & 9.2732 \\ Sym & & 17.1515 \end{bmatrix} \quad (28)$$

known the flexibility matrices, using Eq. (18), the initial stiffness matrices become (the values are given in kN/mm):

$$\boldsymbol{K}_1 = \begin{bmatrix} 4.51 & -2.39 & 0.47 \\ & 3.89 & -1.70 \\ Sym & & 1.06 \end{bmatrix};$$

$$\boldsymbol{K}_2 = \begin{bmatrix} 1.79 & -0.97 & 0.21 \\ & 1.47 & -0.63 \\ Sym & & 0.36 \end{bmatrix} \quad (29)$$

the global stiffness matrix of the system of 3 x 2 walls K_{sys}, can be determined through Eq. (20):

$$\boldsymbol{K}_{sys} = \begin{bmatrix} 6.30 & -3.36 & 0.68 \\ & 5.36 & -2.32 \\ Sym & & 1.42 \end{bmatrix}; \quad (30)$$

the arrays of the equivalent horizontal displacement produced by the vertical load $\Delta_{N,i}$ are calculated by means of Eq. (15) assuming all the bending-moments positive, whereas the array of the equivalent force $F_{N,sys}$ produced by the vertical load is determined by means of Eq. (21) (values are given in kN):

$$\boldsymbol{\Delta}_{N,1} = \begin{bmatrix} 3.75 \\ 12.50 \\ 23.75 \end{bmatrix}; \boldsymbol{\Delta}_{N,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \boldsymbol{\Delta}_{N,3} = \begin{bmatrix} -1.75 \\ -0.68 \\ 5.68 \end{bmatrix}; \quad (31)$$

the total displacement at each storey (at the end of the first iteration) can be determined by means of Eq. (22) (values are given in mm):

$$\boldsymbol{\Delta}^{1\text{st}} = \begin{bmatrix} 7.86 \\ 13.38 \\ 10.62 \end{bmatrix} \quad (32)$$

The arrays of external force carried out by each multi-storey wall can be determined by the Eq. (23) (values are shown in kN):

$$\mathbf{F}_1 = \begin{bmatrix} 6.70 \\ 14.58 \\ -2.11 \end{bmatrix}; \mathbf{F}_2 = \begin{bmatrix} 3.30 \\ 5.42 \\ -2.89 \end{bmatrix} \quad (33)$$

the bending moment are determined from Eq. (25):

$$\mathbf{M}_1 = \begin{bmatrix} 73.8 \\ 25.9 \\ -5.28 \end{bmatrix}; \mathbf{M}_2 = \begin{bmatrix} 13.7 \\ -0.89 \\ -7.22 \end{bmatrix} \quad (34)$$

The signs of the bending-moments are not in agreement with the initial condition, hence the $\Delta_{N,j}$ have to be recalculated using the values of Eq. (34). The updated arrays are:

$$\boldsymbol{\Delta}_{N,1} = \begin{bmatrix} 3.75 \\ 12.50 \\ 18.75 \end{bmatrix}; \boldsymbol{\Delta}_{N,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \boldsymbol{\Delta}_{N,3} = \begin{bmatrix} -4.12 \\ 7.82 \\ 0.38 \end{bmatrix}; \quad (35)$$

the new displacement array becomes:

$$\boldsymbol{\Delta}^{2\text{nd}} = \begin{bmatrix} 7.88 \\ 13.54 \\ 14.59 \end{bmatrix} \quad (36)$$

the updated values of bending moments are:

$$\mathbf{M}_1 = \begin{bmatrix} 73.3 \\ 24.9 \\ -8.6 \end{bmatrix}; \mathbf{M}_2 = \begin{bmatrix} 14.2 \\ 0.1 \\ -3.9 \end{bmatrix} \quad (37)$$

Comparing the signs of the bending moments of Eqs. (37) and (34) it can be noted that the value of the $wall_{2,2}$ is not consistent, hence, the procedure has to be iterated once-again. The necessity to iterate the procedure depends on the fact that the array of the equivalent force F_{Nsys} as well as the displacement arrays due to the vertical load Δ_N have been determined with a not correct signs-distribution of moments. $\Delta_{N,j}$ have to be recalculated. The updated values are:

$$\boldsymbol{\Delta}_{N,1} = \begin{bmatrix} 3.75 \\ 12.50 \\ 18.75 \end{bmatrix}; \boldsymbol{\Delta}_{N,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \boldsymbol{\Delta}_{N,3} = \begin{bmatrix} -4.12 \\ 7.82 \\ 0.38 \end{bmatrix}; \quad (38)$$

the new displacement array becomes:

$$\boldsymbol{\Delta}^{3\text{rd}} = \begin{bmatrix} 7.88 \\ 13.54 \\ 14.59 \end{bmatrix} \quad (39)$$

the updated values of bending moments are:

$$\mathbf{M}_1 = \begin{bmatrix} 73.3 \\ 24.9 \\ -8.6 \end{bmatrix}; \mathbf{M}_2 = \begin{bmatrix} 14.2 \\ 0.1 \\ -3.9 \end{bmatrix} \quad (40)$$

It is important to note that in this case the new iteration does not produce any change because the equivalent horizontal displacement $\Delta_{N,2}$ is zero since no vertical load is applied, hence it is independent by the bending-moments sings.

The sings of the bending-moments of Eq. (40) are in agreement with the values of Eq. (37) therefore according to STEP 3 of the flow-chart of *Figure 6*, the force in the hold-downs can be assessed by means of Eq. (12):

$$\mathbf{T}_{HD,1} = \begin{bmatrix} 10.59 \\ -2.53 \\ -2.80 \end{bmatrix}; \mathbf{T}_{HD,2} = \begin{bmatrix} 11.33 \\ 0.07 \\ 3.10 \end{bmatrix} \quad (41)$$

According to STEP 4 of the flow-chart of *Figure 6*, the force in the hold-downs is not in agreement with the initial condition of STEP 0 (all hold-down were supposed to be in tension), therefore the procedure goes back to STEP 1 after updating the stiffness matrices, namely all the hold-down in compression has a infinite stiffness. The new stiffness matrices are:

$$\boldsymbol{K}_1 = \begin{bmatrix} 6.30 & -2.94 & -0.08 \\ & 5.89 & -2.95 \\ Sym & & 2.34 \end{bmatrix};$$

$$\boldsymbol{K}_2 = \begin{bmatrix} 1.79 & -0.97 & 0.21 \\ & 1.47 & -0.63 \\ Sym & & 0.36 \end{bmatrix}; \quad (42)$$

$$\boldsymbol{K}_{sys} = \begin{bmatrix} 8.08 & -3.91 & 0.13 \\ & 7.37 & -3.57 \\ Sym & & 2.70 \end{bmatrix};$$

Using the values of the new stiffness matrix of Eq. (42) and the sing of the bending-moments of Eq. (40), the $\Delta_{N,j}$ and the $F_{N,sys}$ becomes:

$$\mathbf{\Delta}_{N,1} = \begin{bmatrix} 3.75 \\ 7.50 \\ 11.25 \end{bmatrix}; \ \mathbf{\Delta}_{N,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \ \mathbf{\Delta}_{N,3} = \begin{bmatrix} 0.54 \\ 0 \\ 3.87 \end{bmatrix}; \qquad (43)$$

the new displacement array becomes:

$$\mathbf{\Delta}^{4\text{th}} = \begin{bmatrix} 7.90 \\ 14.38 \\ 15.38 \end{bmatrix} \qquad (44)$$

The force distribution within the wall is:

$$\mathbf{F_1} = \begin{bmatrix} 6.57 \\ 16.14 \\ -3.21 \end{bmatrix}; \ \mathbf{F_2} = \begin{bmatrix} 3.43 \\ 3.86 \\ -1.79 \end{bmatrix} \qquad (45)$$

the updated values of bending moments are:

$$\mathbf{M_1} = \begin{bmatrix} 73.0 \\ 24.3 \\ -8.0 \end{bmatrix}; \ \mathbf{M_2} = \begin{bmatrix} 14.4 \\ 0.7 \\ -4.5 \end{bmatrix} \qquad (46)$$

The sings of the bending-moments of Eq. (46) are in agreement with the sings adopted (see Eq. (40)) therefore no iteration is needed and according to STEP 3 of the flow-chart of *Figure 6*, the force in the hold-downs can be assessed by means of Eq. (12):

$$\mathbf{T}_{HD,1} = \begin{bmatrix} 10.47 \\ -2.78 \\ -3.04 \end{bmatrix}; \ \mathbf{T}_{HD,2} = \begin{bmatrix} 11.56 \\ 0.56 \\ 3.57 \end{bmatrix} \qquad (47)$$

The hold-down forces obtained at the end of the second iteration are in agreement with the boundary condition, the procedure therefore can be considered completed. The obtained results comply the equilibrium, the compatibility and the constitutive law.

6 LINEAR SEISMIC ANALYSIS OF TIMBER SHEAR-WALL BUILDINGS

Two types of linear analysis are suggested by [2]: the Lateral Force Method (LFM) and the Response Spectrum Analysis (RSA). The former should be used only if the structure can be considered regular in elevation, the latter is applicable to all types of building.

6.1 Lateral Force Method – LFM

LFM can be considered as a particular case of the method introduced in Section 5. The LFM, indeed, assumes the seismic action as an equivalent static horizontal forces distribution. The use of the LFM has been already presented in the previous section, and the applied horizontal forces can be determined according to simplified expressions reported in several national standards, which assumes that the building response is not significantly affected by the contribution of the higher modes of vibration. For these reason, the LFM of analysis shall be applied if the fundamental period of structure is smaller than a given value and the building meets the criteria for regularity in elevation.
Differently from the common practice, the method presented (see Section 5) considers the influence of the hold-downs even in the elastic response. Usually, designers prefer to neglect the hold-down presence because it introduces a non-linear behaviour since from the elastic-analysis, which significantly increases the difficulty and the time of analysis becoming iterative. The need to compute the hold-down contribution is however clear, both the shear-distribution and the tensile force in the hold-downs change magnitude compared with the actual-common approach. Therefore, in order to correctly asses the seismic action in a timber shear-walls building, the iterative approach is required.

6.2 Response spectrum analysis

RSA is usually applied to buildings which cannot be defined regular in elevation and it should be considered the reference method for determining the seismic effects because it can be applied to any type of building without any geometric limitation. Moreover, its results could be considered more reliable (compared to LFM) because the analysis takes into account all the significant modes of vibration participating to the seismic response of the structure. The effects of the analysis E^{ζ} are then combined to asses the design actions; the effects can be combined using a modal superimposition techniques such as the *Complete Quadratic Combination* [*CQC*] or the *Square Root Sum of Square* [*SRSS*], see [6].
Two key aspects have to be investigated to apply the RSA to timber wall buildings: the hold-downs non-linear behaviour (hold-downs can have two states) and the presence of the vertical load.

6.2.1 Modal analysis for a single-story timber shear-walls building

In order to apply the RSA to a building, the dynamic properties of the building have to be assessed through a modal analysis. Namely, the natural periods, the mode shapes and the participating masses have to be determined using a modal analysis.
In order to perform the modal analysis of a timber shear-wall (see *Figure 7*) a concentrated-mass *m* is added on the top-plate of the wall.
The equation of motion can be written regarding to the equilibrium of the concentrated mass subjected to its inertial force F_{in} and the wall elastic force F_{el}:

$$\mathrm{F_{in}} + \mathrm{F_{el}} = 0 \qquad (48)$$

The inertial force can be expressed as:

$$\mathrm{F_{in}} = \mathrm{m} \cdot \ddot{\Delta} \qquad (49)$$

where $\ddot{\Delta}$ is the mass acceleration. The wall elastic force F_{el}, according to Eq. (5), is obtained by:

$$\mathrm{F_{el}} = \mathrm{K}_{tot} \cdot (\Delta + \Delta_N) \qquad (50)$$

where K_{tot} is the wall stiffness accounting for all the deformation contributions. Therefore, Δ can be regarded as the horizontal displacement of the concentrated-mass and Δ_N is the wall horizontal displacement due to the vertical load. Eq. (48) can be rewritten as:

$$\mathrm{m} \cdot \ddot{\Delta} + \mathrm{K}_{tot} \cdot (\Delta + \Delta_N) = 0 \qquad (51)$$

Eq. (51) can be rearranged as follows:

$$\mathrm{m} \cdot \ddot{\Delta} + \mathrm{K}_{tot} \cdot \Delta = -\mathrm{K}_{tot} \cdot \Delta_N \qquad (52)$$

Eq. (52)} is a second order linear differential equation; each term can be divided by the mass *m* to get:

$$\ddot{\Delta} + \omega^2 \cdot \Delta = -\omega^2 \cdot \Delta_N \qquad (53)$$

where $\omega = \sqrt{\frac{K_{tot}}{m}}$ is the circular frequency.

The solution of Eq. (53) can be obtained easily considering the homogeneous and particular terms as:

$$\Delta(t) = A \cdot e^{i\omega t+\theta} + \Delta_N \quad (54)$$

where: A and ϑ are the amplitude end the phase of the motion respectively; they are constant and can be calculated using the initial conditions.

If the rigid body rotation contribution is not considered the wall stiffness can be expressed by $K_{tot,nt}$ and the term Δ_N becomes zero.

The period and the natural frequency of the wall can be obtained directly from the circular frequency as:

$$T = \frac{\omega}{2\pi}; \qquad f = \frac{2\pi}{\omega} \quad (55)$$

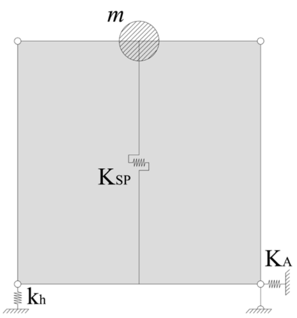


Figure 7: *UNITN model with a concentrated mass.*

6.2.2 *Modal analysis for a system of m x n walls modelling a multi-storey timber shear-wall building*

The modal analysis of a system of *m x n* walls can be performed not in a much different way from the previous case. In order to consider the mass-distribution along the height, an equivalent concentrated-mass is added at each storey and not to each wall; this assumption can be considered valid until the floor of the building can be regarded as a rigid-diaphragm. The mass-distribution hence becomes a diagonal matrix and it is defined as follows:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_j & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_m \end{bmatrix} \quad (56)$$

For a system of *m x n* walls , the modal analysis allows to evaluate the ζ *Natural Periods* $\mathbf{T}^\zeta$ and the relative *mode-shapes* φ^ζ, where ζ is equal to the degrees of freedom, i.e. the number of storeys. Due to the fact that the mode-shapes can be determined using the following equation:

$$\left(\boldsymbol{K} - \omega_\zeta^2 \cdot \boldsymbol{M}\right) \cdot \varphi^\zeta = 0 \quad (57)$$

it is clear that both the *Natural Periods* and the relative *mode-shapes* strictly depend on the stiffness **K** of the structure, hence, they strongly depend on the hold-downs state. It is therefore clear once again that an iterative procedure is needed to solve the problem.

7 *RSA* BY MEANS OF AN ITERATIVE PROCEDURE

Three different methods for applying the RSA are presented. All the procedures are iterative in order to determine a shear-distribution and a hold-downs force consistent with the model of the building.

7.1 Method 1: VTM

The first method is called VTM (*Vertical load To Main mode*) and it is suggested to be used when a prevailing mode-shape exists. The method consists of five steps and it performs a static analyses for any shape-mode of the model; for the analysis associated to the main shape-mode the simultaneous presence of the vertical and horizontal load is taken into account.

The first method presented is called VTM (*Vertical load To Main mode*) and it is suggested to be used when a prevailing mode-shape exists. The method consists of five steps as shown in *Figure 8*.

In the first phase the modal analysis is performed in order to determine the dynamic properties of the model, namely the natural periods T^ζ, the mode-shapes φ^ζ and the modal participation factor Γ^ζ, which is used to identify the most important mode-shape and determined as follow:

$$\Gamma^\zeta = \frac{\left(\varphi^\zeta\right)^T \cdot \mathbf{M} \cdot \mathbf{R}}{(\varphi^\zeta)^T \cdot \mathbf{M} \cdot \varphi^\zeta} \quad (58)$$

where **R** is a ones-array.

In the second step of the procedure, ζ-static analyses are performed; for any shape-mode the model of the structure is loaded by an equivalent static horizontal force distribution related to the shape-mode itself. For the analysis associated to the main shape-mode (i.e. the mode with the higher participation factor), the simultaneous presence of the vertical load is taken into account in order to consider its influence on the shear-distribution.

The static force distribution related to the *ζ-th* mode-shape can be evaluated as:

$$\mathbf{F}^\zeta = S_d(T^\zeta) \cdot \Gamma^\zeta \cdot \mathbf{M} \cdot \Phi^\zeta \quad (59)$$

The values of shear-force and bending-moment (given by the ζ-static analyses) acting on each wall are combined by the SRSS procedure in the fourth step. This modal superimposition technique allows to estimate the actual response of the structure. The net force acting in the hold-downs is determined from the bending moment and the vertical load:

$$T_{HD-j,i} = \frac{M_{j,i}^{SRSS}}{\tau_i \cdot l_i} - N_{j,i} \quad (60)$$

The last step requires to verify that the values of hold-downs forces are in agreement with the state set in the initial conditions (i.e. STEP 0, see *Figure 8*). If the forces are consistent, the procedure can be stopped; otherwise, the model and its stiffness matrix have to be updated changing the state of the hold-downs not compatible and the procedure has to be iterated.

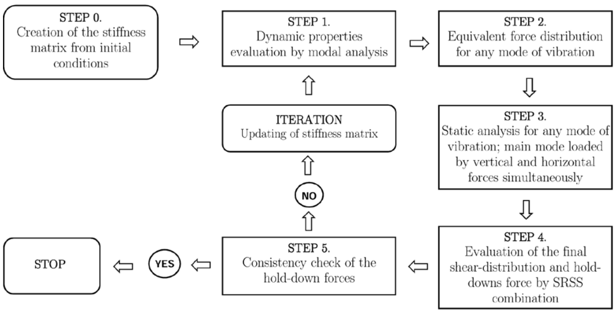


Figure 8: *Flow chart of the VTM method.*

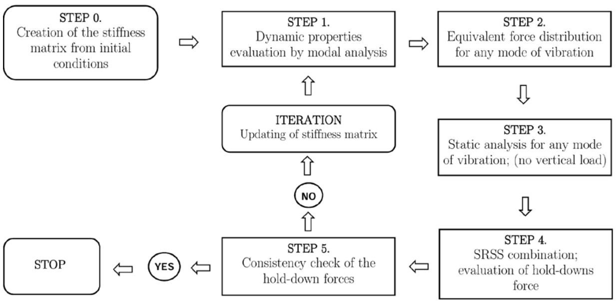


Figure 9: *Flow chart of the VNA method.*

7.2 Method 2: VNA

The method called VNA (*Vertical load Not Applied*) is similar to the previous one; it differs from the VTM method for the fact that the vertical load is taken into account only at the end of the analysis for the evaluation of the tensile force in the hold-downs.

The method called VNA (*Vertical load Not Applied*) is similar to the previous one; it differs from the VTM method by the fact that the vertical load is not considered during the analysis phase (STEP 3), but it is taken into account only at the end of the analysis for the evaluation of the tensile force in the hold-downs (in the same way as Eq. (60)).

Synthetically, the VNA method consists of the following steps (see *Figure 9*): in the STEP 0, the structure is modelled and the stiffness matrix is evaluated; in the STEP 1 the modal analysis is performed to determine the dynamic properties and the modal participation factor: T^{ζ}, φ^{ζ}, Γ^{ζ}; then in the STEP 2, ζ-static equivalent force distributions are determined using Eq. (59), which are used in the STEP 3 to statically analyse the structure ζ-times, applying each time one of the static equivalent force distributions, the vertical load is not considered. In the STEP 4, the shear-forces and the bending moments values are combined by the SRSS and the net forces in the hold-downs are assessed by Eq. (60); in the last STEP the consistency of the result is checked in terms of compatibility between hold-downs forces and hold-downs state; if the compatibility is not satisfied the procedure has to be iterated.

It is important to remark that the vertical load is considered by the method only as a reduction of the tensile forces of the hold-downs and therefore its influence on the shear-force distribution is not taken into account. This approach leads to more approximated results but, on the other hand, it is faster and easier compared to the other ones.

7.3 Method 3: CNA

The method of analysis called CNA (*Complete Numerical Analytical*) was developed with the aim to estimate as correctly as possible the influence of the vertical load on the shear-distribution. The key point of the method is the evaluation of the horizontal load and of the vertical load effects separately.

According to the flow-chart of *Figure 10*, both the modal analysis and the RSA (without the vertical load) are firstly performed in order to obtain the dynamic properties and modal shear-force distribution respectively. Then, the model of the structure is analysed only applying the vertical load, which causes an auto-balanced shear-distribution.

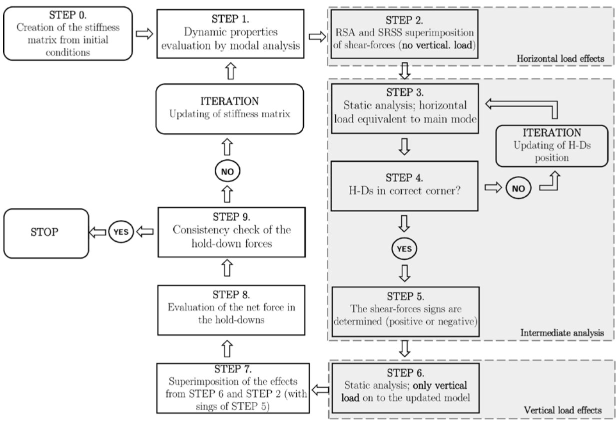


Figure 10: *Flow chart of the CNA method.*

The RSA allows to evaluate only the modulus of the shear-forces but not their sign, moreover it is not influenced by the hold-downs position (the result of the RSA is not depended on the corner where the hold-downs are placed, left-right). On the contrary, the shear-forces produced by the vertical-load can change magnitude and orientation with the hold-down position. Hence, the superimposition of the shear-forces due to the RSA and the vertical-load static analysis can not be automatically performed since only the modulus of the shear-forces is determined by means of RSA .

In other words, both the RSA shear-forces sign and the orientation of shear-forces produced by vertical-load have to be previously determined. Therefore, an intermediate analysis is needed for the evaluation of the correct position of the hold-downs (left or right) and for the evaluation of a sign-pattern, which has to be assigned to the RSA shear-distribution. This intermediate analysis is carried-out loading the structure only with a horizontal force distribution related to the main shape mode (which allows to evaluate the correct position of the hold-down). After evaluating the final shear-force distribution by the superimposition of the RSA effects with the vertical-load effects, the net forces in the hold-downs as well as the consistency as to be determined. In the case that the results are not in accordance with the initial conditions, the procedure are to be iterated.

8 NUMERICAL EXAMPLE FOR THE APPLICATION OF THE RSA PROPOSED METHODS

With the aim to make readers understand correctly the system of *3 x 2* walls of Section 5.1 is analysed.
The mass matrix adopted is (the masses are given in tons):

$$\mathbf{M} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (61)$$

For the initial condition all the hold-downs are considered active.

8.1 Modal analysis

All the three proposed methods require, as STEP 1, the modal analysis of the structure-model in order to assess its dynamic properties as well as the mass participating factor.
From the mass matrix of Eq. (61) and the global stiffness matrix of Eq. (30), according to Eq. (57), the periods result:

$$T^1 = 0.63 \text{ sec};\ T^2 = 0.16 \text{ sec};\ T^3 = 0.09 \text{ sec} \quad (62)$$

And the related shape-modes are:

$$\Phi^1 = \begin{bmatrix} 0.21 \\ 0.59 \\ 1.00 \end{bmatrix};\ \Phi^2 = \begin{bmatrix} 1.00 \\ 0.80 \\ -0.68 \end{bmatrix};\ \Phi^3 = \begin{bmatrix} -1.00 \\ 0.95 \\ -0.35 \end{bmatrix} \quad (63)$$

The participation factors, evaluated using the Eq. (58), are the following:

$$\Gamma^1 = 1.29;\ \Gamma^2 = 0.53;\ \Gamma^3 = -0.19 \quad (64)$$

It is clear that the main mode of vibration is the first one, This can be more emphasized evaluating the participation masses:

$$\widetilde{\mathrm{M}}^1 = 4.66 \text{ ton};\ \widetilde{\mathrm{M}}^2 = 1.19 \text{ ton};\ \widetilde{\mathrm{M}}^3 = 0.15 \text{ ton} \quad (65)$$

In order to evaluate the equivalent force distributions related to the mode-shapes, the reduced response spectrum (called design spectrum) of *Figure 11* is considered. For each period, the spectral values are:

$$S_d(T_1) = 0.42\text{g};\ S_d(T_2) = 0.56\text{g};\ S_d(T_3) = 0.64\text{g} \quad (66)$$

The equivalent static force distribution are determinate by-means of Eq. (59) (the forces are given in kN):

$$F^1 = \begin{bmatrix} 2.26 \\ 6.27 \\ 10.67 \end{bmatrix};\ F^2 = \begin{bmatrix} 5.84 \\ 4.67 \\ -3.98 \end{bmatrix};\ F^3 = \begin{bmatrix} 2.45 \\ -2.33 \\ 0.85 \end{bmatrix} \quad (67)$$

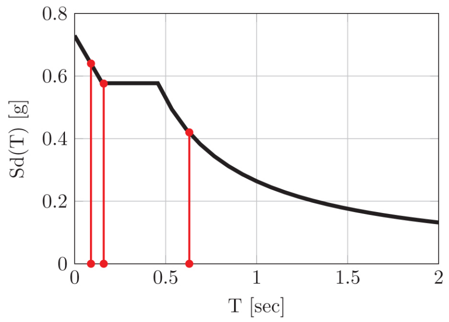


Figure 11: *Design spectrum adopted.*

8.1.1 VTM method

According to the Step 3 of the VTM method, three static analyses of the model are performed, namely one for each static force distributions. It has to be reminded that the analysis related to the main mode-shape considers the simultaneous presence of the equivalent horizontal and the vertical loads. Conversely, the analyses related to the other mode-shapes consider only the equivalent horizontal loads.
The shear-force distributions and the bending-moments for each mode-shape are shown in *Table 2*; for each equivalent static analysis, the initial-condition assumed considers all the bending-moments to be positive. According to the procedure of Section 5, the states of the hold-down were checked at every step; for the first mode-shape no iteration has been required; conversely, the other two methods needed two iterations each.
he Step 4 of the VTM method consists in the modal superimposition of the shear-forces and the bending moments, the SRSS procedure gives the values shown in *Table 3*.
The net vertical-force acting in the hold-downs and the related compatibility verification are shown in *Table 4*.
The final result of the first iteration are in agreement with the initial condition adopted, therefore the results are consistent with the boundary condition used and no more iteration are needed. It is important to remark that if the hold-down forces would have not been compatible with the hypothesized hold-down state, the procedure would have been iterated after the updating of the boundary condition, hence the updating of the stiffness matrix.

Table 2: *VTM shear-force distribution and bending moments for the 1st interaction.*

Shape-mode 1							
$V^1_{11} =$	15.07	$V^1_{12} =$	4.13	$M^1_{11} =$	97.93	$M^1_{12} =$	19.09
$V^1_{21} =$	14.23	$V^1_{22} =$	2.71	$M^1_{21} =$	60.26	$M^1_{12} =$	8.76
$V^1_{31} =$	9.89	$V^1_{32} =$	0.79	$M^1_{31} =$	24.69	$M^1_{12} =$	1.98
Shape-mode 2							
$V^2_{11} =$	4.73	$V^2_{12} =$	1.79	$M^2_{11} =$	5.63	$M^2_{12} =$	2.47
$V^2_{21} =$	0.50	$V^2_{22} =$	0.18	$M^2_{21} =$	−6.21	$M^2_{12} =$	−2.01
$V^2_{31} =$	−2.99	$V^2_{32} =$	−0.99	$M^2_{31} =$	−7.47	$M^2_{12} =$	−2.47
Shape-mode 3							
$V^3_{11} =$	0.68	$V^3_{12} =$	0.28	$M^3_{11} =$	0.59	$M^3_{12} =$	0.26
$V^3_{21} =$	−1.06	$V^3_{22} =$	−0.41	$M^3_{21} =$	−1.12	$M^3_{12} =$	−0.45
$V^3_{31} =$	0.62	$V^3_{32} =$	0.23	$M^3_{31} =$	1.54	$M^3_{12} =$	0.59

Table 3: *SRSS values of shear-force and bending moments for the 1st iteraction VTM.*

Shear-forces V_{ij} [kN]		Bending-moments M_{ij} [kN m]	
$V_{11} = 15.80$	$V_{12} = 4.51$	$M_{11} = 98.1$	$M_{12} = 19.2$
$V_{21} = 14.27$	$V_{22} = 2.75$	$M_{21} = 60.6$	$M_{12} = 9.00$
$V_{31} = 10.33$	$V_{32} = 1.29$	$M_{31} = 25.8$	$M_{12} = 3.22$

Table 4: *Hold-down force and compatibility verification, 1st iteraction VTM*

Tensile force T_{ij} [kN]	
$T_{HD,11} = 20.47$	$T_{HD,12} = 7.7$
$T_{HD,21} = 11.73$	$T_{HD,22} = 3.6$
$T_{HD,31} = 4.1$	$T_{HD,32} = 1.3$
Compatibility verification	
Verified	Verified
Verified	Verified
Verified	Verified

9 CONCLUSION

In this paper three iterative method to apply the *Response Spectrum Analysis* to light timber frame buildings were presented. In fact, the RSA cannot be directly applied to this buildings because their dynamic features (like periods, mode of vibration etc.) are dependent on the stiffness of the building, which depends on the magnitude of the seismic force which in turns depends on the stiffness itself. The methods differ in the way in which the hold-down they consider the effect of the vertical load on the hold-down forces and how it modifies the shear-distribution within the walls. The CNA method is the most correct one from the mathematical point of view, but it requires a high computational effort. The VNA method is faster than the other but, at the same time is the less accurate. The VTM method represents a compromise between computational expensiveness and accuracy; for this reason, the VTM should be the reference method.

This paper has dealt with the static and dynamic seismic elastic analysis of light timber-frame multi-storey buildings by-meas of the UNITN model (presented in a previews work), assuming a cantilever-behaviour for the shear-walls.
In the first part of the paper an iterative procedure for the static analysis has been developed. This procedure is necessary to determine the horizontal-forces distribution proportionally to the walls stiffness and it can be conveniently used for applying the seismic lateral force method. The procedure involves two level of iteration; the first iteration-level is required to evaluate which of the two hold-downs of each wall is working and the auto-balanced shear forces due to the vertical load. The second level of iteration is necessary to update the stiffness matrix of the system in the case that some hold-downs are in compression.
The second part of the paper is focused on the modal analysis of the timber light-frame buildings; in particular three approaches for the application of the RSA are proposed. The modal analysis can not be directly applied to these type of timber building due to the non-linearity introduced by the hold-down and the vertical load. In fact, the seismic force depends on the stiffness of the structure which depends on the state of the hold-downs; but the hold-downs state is in turn influenced by the magnitude of the seismic force. Therefore three iterative methods with varying levels of accuracy have been developed. The CNA method is the most correct from the mathematical point of view, but the its complexity makes it the most expensive in terms of time of analysis. The VNA method is less accurate because it accounts for the vertical load effect only in the post-process phase, but at the same time it results faster than the other ones. The VTM method represents a compromise between computational expensiveness and accuracy; for this reason the VTM should be the reference method. \par

The three procedures may be expensive from the computational point of view because, generally, they require several steps of iteration to achieve the solution. However, they allow to get a balanced and compatible solution (allowing their use even in the S.L.S). In particular, through the iteration process, a stiffness matrix modelling properly the building, can be determined which enables to get reliable values of the periods of vibration. Other methods, on the contrary, solve directly the problem without any iteration, neglecting the behaviour of the hold-downs (active: tension; not active: compression) and assuming them active. This leads to high values of period of vibration as well as to an underestimation of the seismic force (which produces a dangerous under-design of the building).

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