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Applications of angular power spectra to problems in Astroparticle and Cosmological Physics

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*To my grandparents and my mother:
you went through much harder roads.
To Mimma: we wrote this together.*

Abstract

The overarching topic of this thesis is the use of angular power spectra formalisms in the spherical harmonic space, aimed at inferring information on observables relevant to astroparticle physics and cosmology applications.

Part I describes the work I developed at the University of Turin under the supervision of Prof. Nicolao Fornengo and Prof. Stefano Camera, collected in (Rubiola et al., 2024, 2025). It focuses on the astroparticle-cosmological topic of indirect detection techniques for Dark Matter particle candidates belonging to the Weakly Interacting Massive Particle (WIMP) class, endowed with masses spanning the GeV-TeV energy scale. The subject is dealt with in terms of a statistical cross-correlation formalism between the 2MRS galaxy clustering spectroscopic sample and the unresolved gamma-ray background (UGRB), which was modelled including observational features based on the Fermi-LAT satellite measured performances. The nature of the UGRB sources has been under scrutiny long since (Camera et al., 2013, 2015; Fornengo & Regis, 2014), for it possibly conceals, next to the emission coming from astrophysical processes such as different classes of Active Galactic Nuclei (AGNi) and of Star Forming Galaxies (SFGs), signatures of Beyond the Standard Model (BSM) particle signals – in particular, those coming from the potential annihilation of WIMP particles. This general physical endeavour is complemented with a number of innovations at the crossroad between the statistical and the physical levels – namely, the application of the multi-tracer formalism and of a signal filtering analogous to the one known as Wiener filter. The use of the Wiener filter can be understood as the introduction of a theoretical Ansatz concerning the connection between the halos hosting the galaxies and the halos hosting the astrophysical and dark matter UGRB sources. This first idea is combined with the use of a multi-tracer analysis aimed at simultaneously exploiting the constraining power coming from the galaxy autocorrelation signal, the galaxy-UGRB cross-correlation signal and the UGRB autocorrelation signal. Multiple strategies are described, eventually translated in a discussion of the smallest measurable annihilation cross-section for WIMP candidates at different masses comprised in the interval from 10 GeV to 1 TeV, capable of ensuring the detection of an excess dark matter signal against the irreducible astrophysical background at the 2σ level in the explored statistical configurations. We find that the multi-tracer innovation is particularly effective for light WIMP candidates ($10 \leq m_\chi \leq 100$ GeV): the sensitivities to the bounds on the annihilation cross-section are forecast to attain values more than an order of magnitude lower than the thermal WIMP cross-section rate and up to about a factor of five below the results obtained with standard cross-correlation approaches. On the other hand, the features of the Wiener filter chiefly improve the detectability of signals at the high-mass end of the discussed WIMP mass interval.

The second part of this work (Part II) is instead a fundamental cosmology study based on

the research work developed under the supervision of Prof. David Alonso, Dr Carlos García García and Dr Matteo Zennaro, during my visits at the Department of Physics of the University of Oxford (May–September 2024 and June–August 2025), whose outcome is published in (Rubiola et al., 2025). The project concentrated on measuring the growth factor of cosmic structures and on constraining the two parameters of the Λ CDM Ω_m and σ_8 as well as their combination, the parameter S_8 . By doing so, we were able to elaborate on the so-called “ σ_8 tension” (also known as “ S_8 tension”, the precise distinction will be exposed in the text), that is, the $2\text{--}3\sigma$ mismatch between the variance of the density perturbations with respect to the average cosmic matter density as measured in the low-redshift Universe, chiefly reported by some main observational campaigns focusing on measuring cosmic shear signals (Asgari et al., 2021; Doux et al., 2022; Dalal et al., 2023), once the results are compared with the value obtained from the Planck mission observations of the Cosmic Microwave Background. The situation is made more unclear because other probes of the Large Scale Structure provide mixed evidence on potential divergences between high-redshift and low-redshift measurements on those parameters, spanning from a $2\text{--}3\sigma$ tension to full agreement with Planck (Di Valentino et al., 2025). The community is thus pondering different alternative answers to this potential inconsistency, spanning from statistical fluctuations, modelling and instrumental systematics affecting the power spectrum at the smallest scales and particularly severe in the case of cosmic shear statistics, up to interpreting such divergence as a signature of a falsification of the Λ CDM requiring the introduction of new physics in terms of additional fields or modifications of the gravitational interaction.

Taking a rather agnostic stance, oriented in the first place to properly measuring the relevant quantities, we build upon a number of previous works, in particular, on (García-García et al., 2021; Sailer et al., 2025a), that laid out tomographic and multi-tracer analyses combining CMB lensing, galaxy clustering and weak lensing / cosmic shear observables from several surveys. These and other works indeed identified the shear signal as the main source of the tension, driving the discrepancy to a value as high as $\sim 3.5\sigma$, a value which gets typically reduced when cosmic shear samples are excluded from the same analysis configurations and only galaxy clustering signals are retained. Altogether, the onset of the divergence between the predictions based on the Λ CDM model having the cosmological parameters set at the values provided by Planck (Aghanim et al., 2020) and the predictions calculated starting from the measurement of the same parameters from Large Scale Structure (LSS) statistics was identified in the redshift interval $0.25 \leq z \leq 0.5$, while, at higher redshifts, the measurements based on the LSS appear more consistent with the CMB-based prediction. Building on these conclusions, constraints are presented in my work, focusing on measuring the amplitude of matter perturbations using statistics on the clustering of galaxies and in cross-correlation with the gravitational lensing signal of the cosmic microwave background (CMB): we developed a low-redshift analysis ($z \lesssim 0.3$), in order to complement the results in the extant literature we mentioned above. This allowed us to complement the redshift intervals explored by extant tomographic analyses by covering almost all of the remaining epochs: we worked indeed with three redshift bins derived from the data of the 2MASS photometric survey (2MPZ) and the WISE $\times$ SuperCOSMOS galaxy survey having as average redshifts $z_{m,1} \sim 0.07$, $z_{m,2} \sim 0.18$, $z_{m,3} \sim 0.32$, combined with the CMB lensing maps from Planck: next to completing extant available results, we could thus concentrate on a regime particularly sensitive to Dark Energy models different from the baseline cosmological constant assumed by the Λ CDM model. The combined study of low redshifts and small scales allowed us to achieve sufficient constraining power on the parameters of interest just by using the well-understood galaxy clustering data

without introducing cosmic shear observable that, as mentioned, might be more problematic to handle: we strived to have all potential systematics under control, in order to introduce cosmic shear in future analysis having already predisposed the clearest and cleanest analysis conditions. After having set all the other cosmological parameters at mutually compatible LSS-based and CMB-based values – our data would not be able to constrain the whole set of Λ CDM parameters independently –, we considered two main strategies: by imposing a prior on Ω_m derived from the DESI measurements of the baryon acoustic oscillation (BAO), we measure $S_8 = 0.79 \pm 0.06$, sufficiently in agreement with Planck constraints and recover the Ω_m value corresponding to the prior. When instead this prior is not included, we find a value of $S_8 = 0.82 \pm 0.07$ – compatible with Planck – and $\Omega_m = 0.245 \pm 0.024$, 2.8σ lower than the value reported by Planck. We interpret the somehow different results on Ω_m (although of course no incompatibility or tension can be claimed) in terms of the fact that, in absence of the BAO prior, the measurement of this quantity is driven by the general functional form of the galaxy auto-correlation power spectrum (its “broadband” shape), more sensitive to theoretical uncertainties in the power spectrum models and with a relatively loose constraining power, whilst, in presence of a prior on Ω_m , the value is tightly controlled by the very specific sensitivity of this quantity to the BAO, from whose measurement the prior was derived. Also in analogy with (García-García et al., 2021; Sailer et al., 2025a), the low-redshift growth history is reconstructed: we recover substantial agreement with the Planck predictions, as well as with other reported results using the lensing tomography technique.

Both research topics reserve specific attention to intermediate and small scales of the Large Scale Structure of the Universe, whose appropriate modelling calls for going beyond the linear perturbation theory: analytic tools spanning from the halo model to the use of emulators and of the Hybrid Effective Field Theory Lagrangian Perturbation Theory (HEFT LPT) calculations, devised to more accurately describe the non-linear matter field as well as that of biased tracers such as galaxy clustering signals are discussed. In particular, the 2MPZ-CMB lensing data are also used in order to test the phenomenology of non-linear galaxy bias terms and their coevolution relations, that is, the possibility of expressing the higher-order bias terms as functions of the well-understood and directly measurable linear galaxy bias term. By comparing our measurements of the bias parameters for our galaxy samples with analogous measurements available in the literature, we find a qualitative agreement among them and consistency with the available empirical coevolution scaling laws. Eventually, computational techniques concerning more efficient marginalisation over nuisance parameters in the likelihood MCMC sampling process, as well as the development of extrapolation schemes aimed at extending the dynamic ranges of the **baccoemu** emulator, were also implemented, thus complementing my research with specific technical and computational aspects.

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Dark substances and observational tensions: a “big picture” for this thesis

Our current physical description of the Universe relies on the Λ CDM cosmological model (Efstathiou et al., 1990; Carroll et al., 1992; Ostriker & Steinhardt, 1995). As such, this intellectual building can be considered an application of General Relativity and its description of Gravitation^{*}, condensed in the two Friedmann equations (Friedmann, 1922), regulating the dynamics of the cosmic expansion, combined with a perturbative theory of structure formation (Lifshitz, 1946; Sachs & Wolfe, 1967; Peebles, 1980), that includes a handful of non-gravitational physics arguments required to account for additional effects which have been relevant in different phases of the cosmic history, especially in the Early Universe, between the Big Bang epoch and the time at which the Cosmic Microwave Background (CMB) formed. Stemming from more than a century of astrophysical observations, inaugurated by the pioneering result achieved by Georges Lemaître in 1926-1927 (Lemaître, 1926; Lemaître, 1927) and by Edwin Hubble in 1929 (Hubble, 1929) concerning the recession of distant galaxies, suggestive of an expanding Universe, and having, as another fundamental pillar, the body of scientific evidence derived from galactic and cluster dynamics suggestive of the existence of non-baryonic forms of matter regulating most part of the gravitational potentials at the cosmological scales (Zwicky, 1933; Rubin & Ford, 1970; Rubin, 1983), the Λ CDM model aims to be a fundamental Standard Model for Cosmology in a vein quite similar to the Standard Model of Particle Physics (White, 2007): a consistent physical description of the Universe with few free parameters to be fit against the data – as we will see, six in its most simple formulation – possibly to be interpreted as fundamental constants of Nature or as quantities that a more fundamental theory could be able to predict in the future.

The effort behind this intellectual construction started with the Einsteinian formulation of General Relativity (Einstein, 1915) and progressively led to the formulation of a Hot Big Bang model, accounting for light element nucleosynthesis, by Alpher, Bethe, Herman and Gamow in the 1940s (Alpher & Herman, 1948; Alpher et al., 1948; Gamow, 1948) and predicting the properties of the Cosmic Microwave Background (CMB), later observed by Penzias and Wilson in 1965 (Penzias & Wilson, 1965). Next to the earliest phases of the cosmic evolution, estimated to have occurred about 13.7 billions of years ago, the model is also capable of describing the formation of galaxies and clusters and their observed organisation in a sort of spider-net of matter articulated in nodes, filament, voids and walls extending for hundreds of Megaparsecs in space and having been evolving for about 10^{10} years, the Large Scale Structure of the Universe (LSS) (Peebles, 1980, 1982; Cooray & Sheth, 2002). Further endowed with an inflationary paradigm (Linde, 1974; Guth, 1981) capable of setting the initial conditions and introduced in order to solve some critical aspects associated

^{*}The discussion of the standard cosmological theoretical “toolkit”, from the definition of redshift up to cosmological summary statistics, will be presented in chapter 1.

to the early stages and to the geometry of the Universe, the Λ CDM model is often referred to as a “concordance” physical framework precisely because of its capability of reconciling most of the astrophysical and cosmological evidence across so many spatial, temporal and energy scales in a unique and consistent picture.

Dark Matter and Dark Energy as fields beyond the Standard Model of Particle Physics

Undoubtedly an almost majestic scientific and cultural result – a quantitative theory of Cosmology might be interpreted as a scientific, and successful, counterpart to the creation myths so ubiquitous in human cultures (Luminet, 2016; Bertolami, 2010) – our cosmological model can be equally seen as a rather challenging and radical theory from its very name. The Λ CDM acronym enshrines indeed an almost disturbing assumption on the fundamental matter and energy composition of the Universe: it indeed implies the existence of two “dark substances” representing altogether about 95.4 % of the total matter-energy composition of the Universe, completely different from those we experience both at Earthly or astronomical scales below the size of galaxies. The dominant elementary constituents of the Universe seem therefore to be particles and fields beyond the summary of fundamental particles, interactions and fields and provided by the Standard Model of Particle Physics, whose particle inventory is partially improperly referred to as “baryonic” matter and radiation in the cosmological parlance: the phenomenology of matter we experience in colliders would therefore just represent the physics of about 4.6% of the total building blocks of the Universe.

Concerning the Λ symbol, it stands for the requirement of including, in the cosmological inventory of fields, matter and energy, a **Cosmological Constant** described as a negative-pressure field, corresponding to about 63 % of the matter-energy composition of the Universe. It thus appears that the dominant fluid permeating the Universe is endowed with properties completely different from any other fluid (radiation, baryons and dark matter) affecting the cosmic evolution. A positive cosmological constant – whose name recalls Einstein’s original cosmological constant, although the modern formulation reverses its effect, determining an acceleration of the cosmic expansion instead of keeping the size of the Universe stationary in time – was already being suggested before and at the turn of the 1990s (Turner et al., 1984; Efstathiou et al., 1990; Carroll et al., 1992; Ostriker & Steinhardt, 1995), based on structure formation simulations and measurements and on the estimation of the age of the oldest astrophysical objects in the Universe. The prediction was fully incorporated in the standard model once the measurement of distances based on SNIa in 1998 (Perlmutter et al., 1999; Riess et al., 1998) led to the conclusion that not only is our Universe expanding, but also that such expansion is *accelerating* with time. The acceleration is indeed suggestive of the action of a Cosmological Constant in that a negative pressure fluid with constant density is the simplest ingredient to be inserted in the Friedmann equations capable of inducing such a phenomenology and correctly fitting the data. More generally, the observed acceleration can be interpreted as a hint for the existence of a “Dark Energy” component in the Universe, a broader term addressing a fluid whose pressure is negative but whose equation of state might evolve in time. Since the density and pressure of the different cosmic fluids decrease as the the expansion of the Universe proceeds, and are therefore smaller today than they were in the past, the effect of the Cosmological Constant (whose density is by definition time-independent, instead) becomes more relevant only in the Late Universe, when all other contributions from the other fluids become

smaller than its energy density value. The “Dark Energy domination” epoch, that started in the past, at a time associated to the redshift of $z \sim 2$ (see chapter 1.1), when its density becomes larger than the one of the other cosmic fluids. At earlier epochs, and especially at the time of CMB formation, cosmological probes show less sensitivity to the effects of Dark Energy compared to that of fluids behaving like ordinary matter, i.e., exerting a positive or null pressure, yet the CMB observations of WMAP and Planck were able to independently confirm a similar abundance and to eventually provide the reference values for the parameters characterising the cosmological constant sector of the Λ CDM model (Jarosik et al., 2011; Komatsu et al., 2011; Clements, 2017; de Zotti et al., 1999; Planck Collaboration et al., 2011, 2020a).

The acronym CDM stands instead for **Cold Dark Matter**: compared to the topic of Dark Energy existence and characterisation, this is an even more seasoned open problem in our understanding of the Universe and, possibly, of gravitation. It dates indeed back to the observations of the kinematics of galaxies in galaxy clusters performed by Fritz Zwicky in the 1930s (Zwicky, 1933, 1937), when he found that the total amount of matter sourcing the gravitational potential responsible of the observed velocity dispersions of galaxies in clusters was not compatible with the mass value recovered by independently estimating the mass of the light-emitting, baryonic matter component residing in the same astrophysical objects. By summing the mass of the galaxies and of the gas in the intracluster medium, one typically recovers a mass value for a cluster that is about ten times lower than the mass found by the application of the virial theorem to the gravitational dynamics of the system. This result was further reinforced by the observation of the flatness of galactic rotation curves (Rubin & Ford, 1970; Rubin, 1983), which were found to stay constant at large radii away from the galactic centre, contradicting expectations based on the assumption of an almost Keplerian decaying trend at large radii if the total matter constituting the galaxies were the one in the form of stars, gas, dust and other baryonic astrophysical bodies. The observation of such unexpected shape of the rotation curves could be explained again if the “baryonic galaxy” were embedded in a more extended halo of additional invisible matter not accounted for – hence also its description in terms of “missing matter” – such that the Keplerian decaying trend would show only at larger radii and be unobservable if no luminous tracer is present at such larger distance. Throughout the years, the development of Physical Cosmology on one hand incorporated dark matter in its scientific paradigm and on the other hand refined the definition of its properties by relying on the growing body of observational information about the features of the LSS. The term itself thus shifted from the generic adjective “dark” (an adjective written without capital letters), to be intended as “non light-emitting matter”, although of possible baryonic origin, to a novel kind of matter (Bertone & Hooper, 2018) (hence capitalised as “Dark Matter”), whose fundamental feature is that of being much more abundant than baryon matter, organised in almost spherical halos, and constituting therefore the gravitational backbone of the LSS, hosting the light-emitting objects, whose structure and dynamic is determined by the host dark matter halos.

In the contemporary understanding and from an exquisitely cosmological and astrophysical point of view, the inclusion of Dark Matter (intended as a “missing, invisible mass”) is required by the many measurements of galactic and cluster dynamics and “weighting” of astrophysical objects by means of strong and weak gravitational lensing effects (Massey et al., 2010; Clowe et al., 2006), all confirming that the mass driving the observed phenomena significantly exceeds the luminous content of the studied objects and consistently setting the abundance of the invisible source of additional gravitational attraction between 20% and 30% of the total mass-energy density in the Universe. At the same time, the description of the matter content in the Universe as a

combination of a subdominant baryon matter component with a dominant Dark Matter one proved to be predictive, allowing gravitational potentials to shape the cosmic microwave background (CMB) anisotropies in the way we observe them, determining the properties of the Baryon Acoustic Oscillation feature in both the CMB and in the LSS clustering power spectra (Eisenstein et al., 2005), up to the comparison of the results of numerical simulation of the formation of cosmological objects at several scales, from the LSS as a whole to individual galaxies, with observational evidence (see (Cirelli et al., 2024) for a wider review). On the other hand, the abundance of baryonic matter can be independently predicted starting from Big Bang Nucleosynthesis (BBN), which accurately calculates the primordial abundances of Hydrogen, Deuterium, Helium, and, with some open problems, of Lithium, and then compared with the census of light elements on astrophysical objects and the gas abundance in the cosmic web, from the BAO and the CMB (Cyburt et al., 2016; Grohs & Fuller, 2022; Pospelov & Pradler, 2010; Schöneberg, 2024; Fields, 2011; Spite et al., 2012; Crespi et al., 2025; Pettini & Cooke, 2025). All these measurement repeatedly set the amount of Standard Model matter once also electromagnetic radiation and neutrinos are included, to be no larger, than 4-5% of the total composition of the Universe.

Next to the above-mentioned successful results suggestive of or relying on the existence of Cold Dark Matter, the associated conceptual paradigm is also related to a number of open problems and controversial points, since the Λ CDM model struggles to systematically justify a number of properties of cosmic structures, especially at the smallest scale (Weinberg et al., 2015; Kroupa et al., 2023). Among them, we can quote the disputed shape of the dark matter halos density profiles at small radii, whether saturating to a constant density value or instead growing (known as core-cusp problem (de Blok, 2010; Del Popolo & Le Delliou, 2021)), the reduced number of observed dwarf galaxies – objects that should be associated to the smallest dark matter halos – lower than what is usually recovered in numerical simulations (Kase et al., 2007; Bullock, 2010); the absence of stars in high-mass subhalos, again contrary to predictions based on numerical simulations (the “too big to fail” problem (Boylan-Kolchin et al., 2011; Jiang & van den Bosch, 2015; Kaplinghat et al., 2019)), the discrepancy between the observed manifold dynamics of galactic rotation curves opposed to the rather uniform behaviour we should find according to what is predicted by the cosmological model (Brook, 2015); or the fact that not all astrophysical objects show forms of dynamical friction (Chiba & Kataria, 2024), i.e., to be slowed down by gravitational interaction with the gas of Dark Matter particles permeating the halos where objects reside.

Calculating and testing these predictions in the Λ CDM model requires to manage a delicate interplay between non-linear phenomena in the complex environment of the smallest cosmological scales. Overall, they are difficult to systematically test or reproduce in both observations and simulations and require to take into account additional baryon plasma processes next to the pure gravitational dynamics. The presence of so many concurring physical effects makes it extremely difficult to develop a unique consistent theory: one could well relax some of the hypotheses concerning the nature of Dark Matter, but should then take care of conserving the excellent agreement between the predictions of the theory and the observations at the larger scales and more linear regimes, where phenomena are simpler to describe. The gravitational anomalies explained in terms of both Dark Matter and Dark Energy can therefore be seen as addressing the status of General Relativity as the reference physical theory of gravitation at our disposal, suggesting the possibility of extending it with alternative, more powerful theories capable of reconciling the regimes of low acceleration, the one experienced by objects at the outskirts of astrophysical and cosmological-sized gravitational potentials, without resorting to dark/missing/non-baryonic matter,

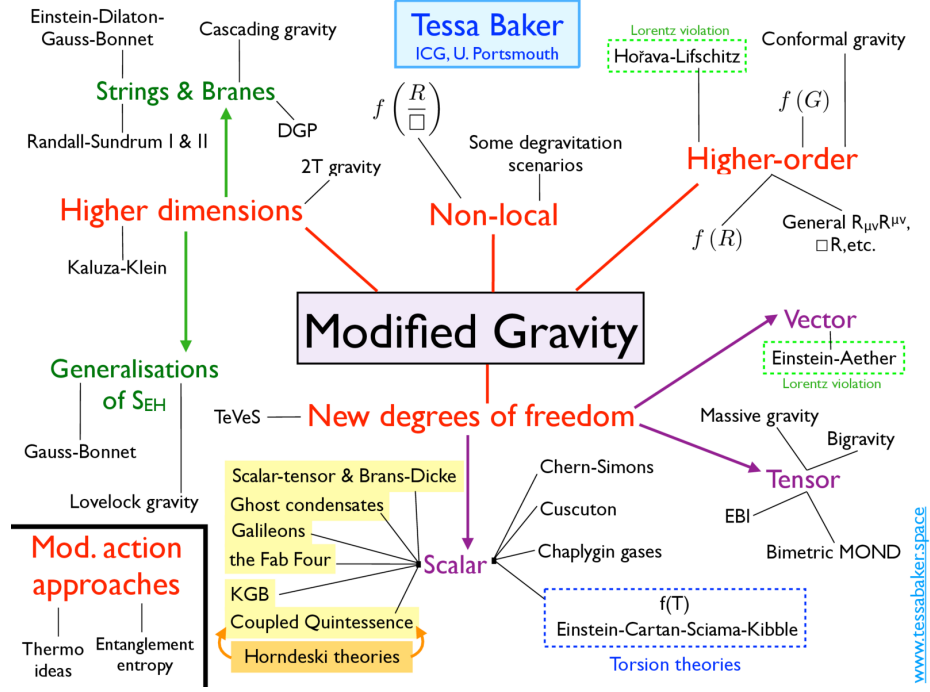


Figure 1: The landscape of Modified Gravity theories explored by cosmologists, displaying their relations and the main assumptions determining their features, from Tessa Baker’s personal page (Baker, 2019) .

or by invoking a smaller amount of it. These theories may handle the problem either from first principles –i.e., the full landscape of Modified Gravity (MG) theories (Baker, 2019; Saridakis et al., 2021; Bajardi & Capozziello, 2023)– or starting from phenomenological considerations (such as MODified Newtonian Dynamics, MOND (Milgrom, 2015, 2001; Famaey & Durakovic, 2026; Desmond, 2025), which in some cases have also been systematised in first-principle fundamental theories of gravity). These theories (many of them and their relationships are displayed in fig. 1) can be still considered to belong to the “astrophysical”, dynamic description of Dark Matter and Dark Energy, trying to reframe the observed anomalous celestial dynamics at the largest scales exclusively in terms of baryonic effects, or to combine the existence of novel kinds of matter with more refined descriptions of the gravitational interaction; in this second case, however, nothing can be said about the nature of Dark Matter as an actual substance – nor does General Relativity: this is instead the approach of major relevance for this thesis.

The problem of the elementary nature of Dark Matter sharply shifts the perspective from astrophysics to fundamental physics: if Dark Matter is to be intended as an additional typology of fundamental particle to be added to the Standard Model of Particle Physics, it should then be consistently described in terms of Quantum Field Theories, the physical paradigm currently most effective at describing the fundamental features of the elementary constituents of our world (we note in passing that this work will not be dealing with the hypotheses, also currently being discussed in the field, of Dark Matter being constituted by Primordial Black Holes, PBH, or anyway by models not considering Dark Matter as an elementary particle (Arbey, 2024; Escrivà et al., 2024; Kaplan et al., 2010)). This is the point of view taken by theoretical physicists and astroparticle physicists that will represent the results described in the first part of this work: it can be summarised in the collection of strategies and considerations aimed at establishing *how to test the particle, microphysical properties of Dark Matter* and how to eventually detect it

by means of particle physics techniques. The particle physics characterisation of Dark Matter can be better understood by attentively looking again at its name:

- “**Coldness**” means that CDM is a form of matter having a non-relativistic behaviour. More precisely, the properties and interactions of dark matter with Standard Model particles are determined by specific conditions imposed on it during the earliest phases of the Universe, before the CMB formation. In particular, we concentrate on two distinct stages. The dark matter fluid should first have *chemically and thermally* decoupled from the rest of the primordial plasma (freeze-out time): before that instant, the abundance of dark matter could be modified by interactions creating or destroying new particles, while, after that instant, dark matter abundance was fixed to the value we measure today in absence of relevant subsequent decay or self-annihilation processes of the dark matter particles themselves. Subsequently to chemical and thermal decoupling, Dark Matter is still allowed to interact kinetically, i.e., to scatter, with itself and with the other particle species in the Universe by channels *other than gravitation*. Yet, eventually, if the Universe expands, the distance between two particles becomes larger than the one they can travel given their velocity. When dark matter enters the regime where also scattering processes cease, this determines its *kinetic* decoupling and the onset of the Dark Matter free streaming phase. Now, a general result of the cosmological perturbation theory is that particles can escape gravitational fields of a given spatial scale λ if they have a sufficiently high escape (“free-streaming”, see e.g. (Gondolo et al., 2012)) velocity. When particles are not trapped inside a gravitational potential of some scale, the related perturbation gets disrupted because escaping particles cause the loss of coherence of the perturbation which is said to be “smoothed”, driven towards the average matter density $\bar{\rho}(z)$. Perturbation erasure occurs at scales corresponding to distances equal or smaller than the distance $\lambda \sim v_s(t_{\text{today}} - t_{\text{kinetic decoupling}})$, called the free-streaming length, determined by the particle streaming velocity v_s and by the time interval $(t_{\text{today}} - t_{\text{kinetic decoupling}})$ corresponding to the stage of cosmic history during which Dark Matter particles did not experience any form of coupling with the rest of matter, neither chemical nor kinetic in nature, other than gravitation. For many dark matter models, especially those related to heavy mass dark matter particles (see below and sec. 2.3.1), $t_{\text{kinetic decoupling}}$ occurs rather soon after the Big Bang, so that the free streaming time interval is almost equal to the age of the Universe. Now, the typical observational result, further supported by insights from numerical simulations, is that very small scales in the Universe are suppressed, yet we do observe that structures form at relatively small scales, meaning that, overall, small-size perturbations are for the most part conserved (Kolb & Turner, 1990; Dodelson & Schmidt, 2020; Peacock, 1999). The conclusion is that the maximum free streaming length λ is rather small: since the free streaming time, if corresponding to the age of the Universe, is fixed and set to about 13.7 billion years, it follows that v_s for Dark Matter cannot be too large, otherwise we would expect the minimum size of early structure to correspond to a much larger λ . In particular, v_s is in the order of about 200 km/s, associated to a non-relativistic particle. If dark matter had *a)* an already non-relativistic velocity at kinetic decoupling and *b)* satisfies the condition on small-scale structure preservation described above, it is said to be “cold”. The combination of these conditions as formulated above is the most typical framework for dark matter characterisation, although other potential configurations in which Dark Matter particles could well have had a relativistic speed at the chemical decoupling /

freeze-out time –yet granting consistent predictions with the current cosmological data– can still be considered among the viable dark matter models.

Given the limited rate of interaction for Dark Matter by channels other than gravitation, after the chemical decoupling its role is chiefly that of “structure builder”. By effect of their own gravity, Dark Matter particles began to cluster, while baryon matter was still coupled to photons in the Early Universe plasma, and, instead, was not clustering. The clustering process of Dark Matter continued successfully above the free streaming scales and started the formation of those Dark Matter halos, whose gravitational attraction also began to attract baryon matter, once, after the CMB formation epoch and the decoupling of baryons and photons species and the dissolution of the primordial plasma, no other interaction other than gravity was controlling its dynamics. That was the beginning of a process still ongoing today, ultimately determining the formation of galaxies and the observed structures (Dodelson & Schmidt, 2020; Kolb & Turner, 1990; Peacock, 1999).

- “**Darkness**” can be more precisely expressed in terms of a very weak coupling of Dark Matter particles via electromagnetic interactions or a lack thereof, meaning that Dark Matter neither absorbs and emits light (or this process occurs very rarely) nor, more generally, interacts strongly with Standard Model particles through forces different from the gravitational one. Since we are able to probe the Universe exclusively through telescopes collecting photons or by cosmic ray detectors sensitive to ordinary matter and Standard Model interactions, Dark Matter remains invisible to conventional instruments.

Dark Matter is therefore expected to be radically different from non-relativistic or relativistic electromagnetically interacting matter (ordinary baryons in the Universe and photons, which, due to their low abundance, have nevertheless been able to erase small-scale perturbations) and from neutrinos, which are substantially non-interacting particles, hence “dark” in nature, but at least for the lighter species, still relativistic at the decoupling time and thus at least partially “hot” forms of Dark Matter.

The alternatives opening to the researcher get immediately extremely large (see (Cirelli et al., 2024)), because one is in principle left with the freedom of specifying:

- **The number of particles constituting Dark Matter:** it could be assumed that there is one single particle species representing the whole of Dark Matter, or, alternatively, many particles with different properties, just like the several ones of the Standard Model, thus postulating the existence of a whole “dark sector”.
- **The particle mass(es), charge(s) and other quantum numbers.**
- **The nature of its (their) coupling(s),** either within the dark sector or with Standard Model particles. This point is relevant because, if there is no mutual dialogue between the two sectors other than the gravitational interaction, Dark Matter by definition would not be detectable by other channels.

Of course, observational and theoretical constraints allow us to prune at least partly the forest of alternatives, yet the landscape of models available in the literature is quite broad, especially in terms of the many orders of magnitude spanned by the possible mass values of the dark matter candidates, as displayed in fig. 2. The work outlined in this thesis will partially consider different particle Dark Matter models, but will soon concentrate on one specific class, among the simplest,

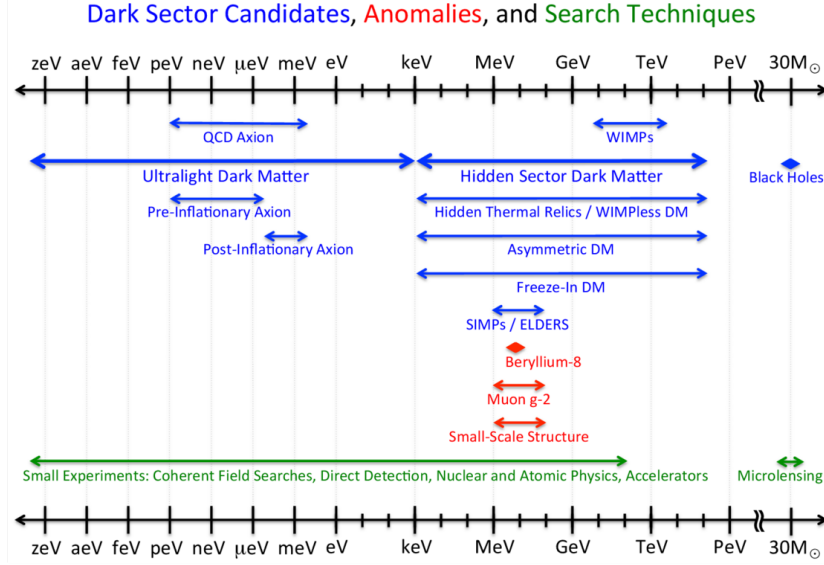


Figure 2: Dark Matter particle candidates (including non-particle candidates such as Primordial Black Holes) as a function of their mass ranges and detection techniques. The picture is taken from (Battaglieri et al., 2017)

oldest and most explored Dark Matter paradigms, that of Weakly Interactive Massive Particles (WIMP, see e.g. (Feng, 2023)). WIMP particles satisfy the coldness condition because of their heavy mass, in the GeV-TeV regime – if DM particles are rather massive, they have entered rather soon a non-relativistic regime in the Early Universe –, while it is sufficient to impose that their abundance after production during the Big Bang matches the observed Dark Matter abundance to find that they are endowed with weak-scale interaction features, justifying therefore the observed scarce degree of interaction with baryon matter (weak being used here with the precise meaning of “having a coupling similar to that of the weak interaction in the Standard Model”) (Lee & Weinberg, 1977).

Their particle features can be connected to a wide array of astrophysical-cosmological observables and techniques, as we shall outline in chapter 2, with a specific focus on the unresolved electromagnetic backgrounds populating the Universe. These backgrounds can be interpreted as “fogs” of photons not originating by identified sources: rather, they should be the cumulative outcome of many processes too faint or too distant to be individually resolved. As displayed in fig. 3, apart from the well-known, cosmological-only, Cosmic Microwave Background (CMB), the Universe is permeated, e.g., by a Cosmic Infrared Background (CIB, dominated by dust emission in Star Forming Galaxies), a Cosmic Optical Background (COB, mainly sourced by unresolved stars and dust) and, at the extreme high-energy end of the spectrum, by the Unresolved Gamma Ray Background (UGRB) (Hill et al., 2018; Biteau, 2025). The latter is defined as the total measured cosmic γ -ray emission after the subtraction of the resolved sources (belonging to both the Milky Way and to the extragalactic Universe, and constituted by active galaxies belonging to the AGN class and Star Forming Galaxies, together with compact objects such as pulsars, supernova remnants, binary stars...) and of the diffuse Galactic foreground (the latter, which

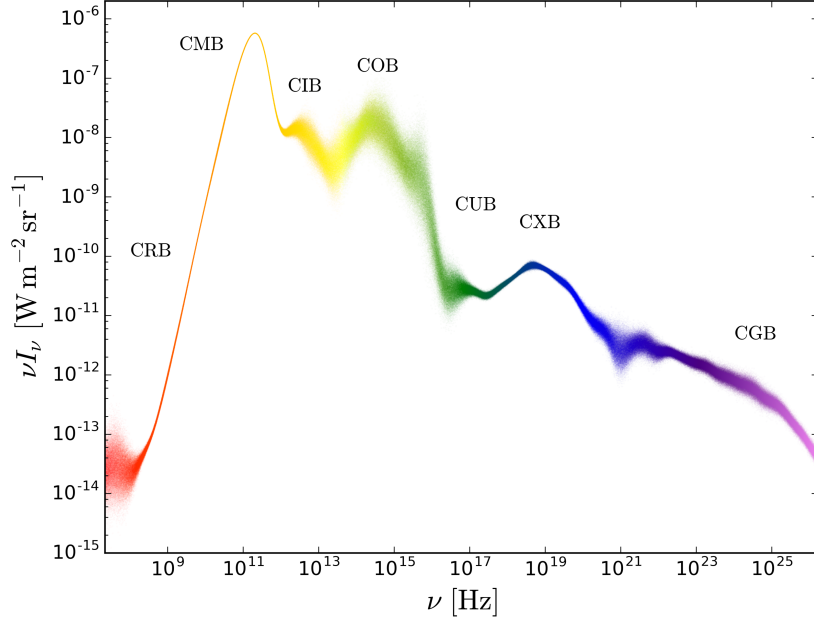


Figure 3: The full spectrum of the Cosmic Electromagnetic Backgrounds, as a function of the photon frequency, from (Hill et al., 2018). The UGRB is here called the Cosmic Gamma Background (CGB).

in parts may still leak in the ultimate UGRB contaminating it especially at the largest angular scales, sums up to about 80% of all photons detected by the Fermi-LAT instrument and is chiefly contributed by highly energetic scattering processes of cosmic rays) (Fornasa & Sánchez-Conde, 2015). The general criterium to discriminate between resolved and unresolved sources is based on the number of photons measured by the detector and exceeding the mean counts expected if no source were present. Since the instrument acquires an increasing amount of data as time goes by, hence achieving longer and longer exposures, every source detection grows in significance, and more and more sources overcome the minimum detection threshold, thus being classified as “resolved”. The finite duty cycle of the instrument implies however that not all gamma sources will be eventually resolved: we will gain more and more information about the overall inventory of sources emitting γ -rays if we are able to assign to every detected source a specific typology of object, but the remaining signal could always host either faint sources exclusively belonging to the same category of objects constituting the resolved source subset or there could be some additional typology of source that for some reason has no representative in the resolved class.

Revealed for the first time already in 1972 by the OSO-3 satellite (Kraushaar et al., 1972), the current most detailed mapping of the UGRB, performed by the Fermi-LAT satellite starting from the year 2008 (Ackermann et al., 2012a) and capable of covering energies from 100 MeV to 820 GeV, has better defined its most relevant features (Ackermann et al., 2015, 2018). In particular, it should be accommodating a number of astrophysical extragalactic sources: the main contributions are represented by some classes of Active Galactic Nuclei (AGN), namely Blazars – further subdivided into BL Lacs objects and Flat Spectrum Radio Quasars (FSRQ) according to their respective spectral properties – and misaligned AGNs (mAGN); the last relevant contribution is represented by γ emission sourced by Star Forming Galaxies (SFGs) (Fornasa & Sánchez-Conde, 2015; Ackermann et al., 2015; Camera et al., 2015; Cuoco et al., 2017; Thakore et al., 2025, 2026). Some residual

astrophysical processes might be contributing to the total intensity, but they are expected to be many orders of magnitude below the above-mentioned ones and, most important, much below potential Beyond the Standard Model (BSM) phenomena. Signatures of BSM Physics are still to be observed: yet, if dark matter is constituted by WIMP particles and if they are decaying or annihilating, then we should expect that part of their decay or annihilation products should be high-energy photons associated to WIMP large masses, belonging to the gamma wavelength interval (Camera et al., 2013; Fornengo & Regis, 2014). Synergies between astroparticle physics and cosmology can thus start from the consideration that dark matter is expected to cluster into halos representing the scaffolding of the LSS: ignoring galaxies hosted in voids (but see (Arcari et al., 2022) for a work including them in this same line of research), also astrophysical objects and baryonic gas all reside in such halos and behave therefore as light-emitting tracers of the underlying dark matter distribution. The observed property of the original dark matter field and of its tracers of being organised in the LSS web-like structure translates into a degree of anisotropy and inhomogeneity in their distributions across the Universe that can be quantified in terms of statistical quantities such as the angular and 3-D power spectra (see chapter 1, especially secs. 1.2.1 and 1.4), respectively calculated in the spherical harmonics and Fourier spaces. The UGRB is found to be substantially isotropic, yet, if at least part of this background is indeed sourced by objects clustered in halos, we therefore need to recover an anisotropic component, which might be made more visible by statistically cross-correlating it with some LSS tracer (Camera et al., 2013; Fornengo & Regis, 2014): both astrophysical objects and dark matter particles would display this anisotropic component, yet the signal from annihilating or decaying dark matter would show a stronger degree of correlation with the cosmic web and, comparatively, better defined signatures of anisotropy with respect to pure astrophysical contributions (see fig. 3.2 and the associated discussion). In the last decade, the existence of the global UGRB-LSS cross-correlation has indeed been demonstrated on data, although the exact composition, either purely astrophysical or combining astrophysical (BL Lac, mAGN, FSRQ and SFG being found the likely dominant sources) and dark matter components remains undecided, especially at the lowest redshifts, where a pure annihilating dark matter contribution might be a competing explanation to astrophysics only or astrophysics and dark matter combined explanations. On the contrary, higher redshifts should be chiefly dominated by astrophysical contributions (Ackermann et al., 2015, 2018; Xia et al., 2015; Cuoco et al., 2015; Regis et al., 2015; Ammazzalorso et al., 2018, 2020; Thakore et al., 2025; Ando et al., 2014). The discussion of the use of high-energy photons in combination with Large Scale Structure statistics leaves therefore open many investigation pathways, possibilities to improve our power spectra estimators and attempts to infer in greater detail the composition of the UGRB, especially in terms of potential contributions from dark matter particles (Feng, 2023; Fornengo & Regis, 2014). This conceptual framework, being developed with particular attention to the low-redshift regime probed by the 2MRS galaxy survey (Huchra et al., 2012; Ando et al., 2018), specifically apt to probe the potential dark matter signal (see fig. 3.3) will thus represent the main theoretical foundation of my research as exposed in chapter 3.

Tensions on fundamental parameters

Next to the foundational problems outlined above, related to the inventory and properties of cosmic fluids, measurements of fundamental parameters of the Λ CDM model are also puzzling cosmologists. In natural sciences two quantities can be considered “equal” only in a statistical

sense: once some statistical criterion is established, compatibility between the outcomes of distinct experiments supposed to be measuring the same quantity is usually associated to a distance between the measured values typically less than $3\text{--}5\sigma$ apart from each other, assuming that the measured parameters can be described as Gaussian random variables and their uncertainties be associated to standard deviations for a normal probability distribution. Criteria establishing compatibility between measurements, such as the z -test for Gaussianly distributed variables or the Student’s t -test, depend at the same time on precision and accuracy, so that the value of the measurement is as important as the associated uncertainty (Vidgen & Yasseri, 2016; Chén et al., 2020; Cohen, 1994). In spite of the conceptual similarities, one of the main differences between Particle Physics and Cosmology is that performing measurements in Cosmology usually demands to deal with larger potential systematics and complex degeneracies between them: whilst Particle Physics experiments can be performed on Earth and in “real time”, Cosmology probes phenomena extremely distant in space and time, requiring mutual calibration between observables that further propagate uncertainties and with limited possibility of empirical reproducibility of the observed phenomena on Earth, numerical simulations apart. In spite of the large impact of uncertainties on cosmological measurements, years of scientific and technological advancements have led to an often exquisite control of many systematics and to a deep understanding of spurious effects biasing our measurements, that, combined with the devising of clever theoretical strategies in the development of the estimators, has been progressively reducing uncertainties on measurements to the order of percent or below. The achievement is so important that the current stage of Physical Cosmology is also known, as already mentioned, as the age of “Precision Cosmology” (Turner, 2022).

Overall, most of measurements of cosmological quantities, and in particular of fundamental parameters, can be subdivided according to two main criteria: indirect measurements opposed to direct measurements, and Early Universe probes contrasted to Late Universe (or Local Universe) probes. Broadly speaking, indirect measurements are those involving methodologies based on fitting the templates of “summary statistics” (Slepian et al., 2025) of cosmological observables such as the CMB and the LSS. The leading direct methods are instead those based on standard candle methodologies relying on the delicate calibration of a “distance ladder” starting from stellar parallaxes and going as far as Cepheids and Supernovae Ia, thus requiring sound models for stellar chemical and physical evolution and a careful control on how instrumental systematics may propagate from “rung” to “rung” in the distance ladder (Freedman & Madore, 2023; Moresco et al., 2022). Next to these two main strategies, other methodologies (Moresco et al., 2022), either already applied or expected to be technologically attainable in the coming years, are available: from time-delay measurements in strong gravitational lensing (Birrer et al., 2024; Treu & Shajib, 2024), to very-large scale standard rulers such as BAO (Heavens et al., 2014; Cuceu et al., 2019) and the equality scale at the power spectrum turnover (Alonso et al., 2025; Cunningham, 2022; Bahr-Kalus et al., 2023; Dube et al., 2025), up to the Sandage test (Sandage, 1962; Loeb, 1998) or the use of gravitational waves as “standard sirens” (Soares-Santos et al., 2017). Further methodologies, combining traditional astrophysical approaches with cosmological theory exist, in a wider gradient between actually model-independent direct measurements and strongly model-dependent techniques.

Ordered by decreasing chronological proximity to the Big Bang, the Early Probe *par excellence* is the CMB, which provides access to the Universe up to recombination ($z \simeq 1100$), about 300,000 years after the Big Bang, corresponding to the most remote stage of the cosmic history we are able to probe. In order to perform measurements based on the observation of the LSS or of astrophysical

objects, we need to wait until structures form: the definition of Late Universe thus encompasses epochs from around $z \simeq 10 - 15$ down to the very local Universe at $z \simeq 0$; as already stated, most of the measurements performed in the past decades concentrated on redshifts smaller than 2, but the situation is changing and will further change in the coming years. The general ambition is to uniformly probe the Universe at many redshifts and epochs [†], in order to observe the full evolution of the Universe throughout its history, to test for the possible “activation” of fields and physical processes which were not exerting any visible effect at higher redshift and could be hints of novel physics.

Although overall the Λ CDM and the consensus it attracts strongly rely on the concordance among measurements performed through the diverse methodologies described above, relevant or at least mild but concerning statistical inconsistencies between the evaluation of cosmological parameters from the CMB and the same quantities inferred from direct measurements or through LSS summary statistics in the Late Universe have emerged, especially in the last decade, determining the rise of two infamous “tensions”:

- the tension on the Hubble parameter H_0 , determining the expansion rate of the Universe today,
- the tension on the σ_8 parameter (or, almost equivalently, on the S_8 parameter, see sec. 1.2.1 for more precise distinctions) – describing the amplitude of matter perturbations, and thus indirectly the amount of structure with respect to a uniform distribution of matter in the Universe, as measured within radii of $8 \text{ Mpc } h^{-1}$.

The tension on H_0 is the most problematic one (Di Valentino et al., 2021a, 2025): rigorously defined, the quantity H_0 is the free integration constant of the Fridman equations and, as such, it must be the same for every cosmic epoch [‡]. We can on one hand infer its value from the fit of the CMB angular power spectrum of temperature and polarisation fluctuations. These observables yield, as its most updated value, $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Aghanim et al., 2020), in agreement with other CMB experiments such as ACT (Louis et al., 2025). On the other hand, its most competitive direct measurements in the local Universe, obtained from the distance ladder method and published by the SH₀ES collaboration (using a distance ladder relying on Cepheids to calibrate intermediate distance “rungs” between the close and the more distant Universe probed by supernovae) yield a value of $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al., 2022), a result generally compatible with other local Universe probes and direct measurements, but at variance with the Planck outcome as it can be seen from fig. 4. Depending on the dataset and on its related error bar size, the statistical discrepancy attains different levels of significance: generally speaking, most of the low-redshift direct measurements or the results derived from relatively Λ CDM-independent approaches tend to prefer higher H_0 or to lay midway between the CMB value and the SH₀ES one, and often the larger error bars ensure statistical agreement with both low and high-redshift probes. On the other hand, Λ CDM model-dependent probes, or probes depending on Early Universe not through the CMB but by Big Bang Nucleosynthesis (BBN) tend to attain values closer to the CMB value. The result of SH₀ES, whose fits are characterised by very small uncertainties, attains the level of a 5σ discrepancy from Planck results on the CMB, a result which has understandably elicited a lively

[†]The James Webb Space Telescope has already been able to construct samples of galaxies at redshift as high as $z=17-25$ (Pérez-González et al., 2025).

[‡]Note that this is opposed to the expansion rate itself $H(z)$, which can be obtained by the solution of the Fridman equations and that evolves in time.

debate about the possibility that either some aspects of the measurements and physical processes studied by Cosmology are still poorly understood, more than what we believe, although without the necessity of greatly extending Physics beyond what we already know, or, more radically, suggesting that New Physics beyond the standard model is at play. The literature on the topic is immense (Knox & Millea, 2020; Di Valentino et al., 2021a, 2025; Valentino et al., 2025; Binney et al., 2025), and possible solutions include revisiting our description of Early Universe physics, modifying the Λ CDM model, introducing new fluids in the equations, that could affect the expansion history and that may be related to the nature of Dark Matter, or leveraging on Modified Gravity theories. Among the most recent and significant results suggesting instead reconciliation between the Early and the Late Universe fully within the Λ CDM model, we mention the outcome of the study on a supernovae datasets provided by the Chicago-Carnegie Hubble Program (CCHP) (Freedman et al., 2024), combining many available supernovae catalogues and notably adding those observed more recently by the James Webb Space Telescope. The measurements published by the CCHP research programme are found to agree with the Planck values not only because of large error bars, but also because the obtained best-fit values are particularly lower than any other measurement performed in the local Universe.

This result was achieved through the use of two alternative distance calibrators: namely, a first distance ladder based on Tip of the Giant Branch (TRGB) stars and a second, independent one, based on J-Region Asymptotic Giant Branch (JAGB) stars. Cepheid-calibrated distances are reported, but the work leading to a sound result for a Cepheid-based measurement is declared to be still ongoing. The most precise result quoted by the paper is the following: $H_0 = 70.39 \pm 1.22$ (stat) ± 1.33 (sys) $\pm 0.70 \sigma_{\text{SN}}$ km s⁻¹ Mpc⁻¹ (σ_{SN} stands for an additional uncertainty accounting for the proper motions of galaxies hosting the observed supernovae and photometric uncertainties). The paper is actually an extensive discussion on a number of relevant methodological subtleties, its goals being far beyond that of being a report of a new distance measurement, in fact appearing as a detailed review of observational techniques, calibration procedures, systematics affecting the measurements. Among its messages, there is the conclusion that, regardless of the final measurement outcome, the claim of a *very high* tension might be excessive. Further Late Universe probes of H_0 in agreement with the CMB are in particular estimates of the Hubble constant obtained by combining the observations of the BAO radial and transverse scales with a calibration of the sound horizon based on the properties of baryons and relativistic degrees of freedom derived by the BBN and thus equally related to the Early Universe but not depending on the CMB (Schöneberg et al., 2019), and from LSS summary statistics observations again typically using a range of Early Universe priors, either from the BBN or from the CMB, in order to break degeneracies among the parameters. Such results are therefore at the same time important as independent tests of the value of the H_0 with respect to direct measurements in the Local Universe but, importantly, not completely informed exclusively by low-redshift only data. Similarly, a discussion of the tension reframing it in terms of a possible opposition between the results obtained by means of the Cepheid-based distance ladder methodology and other measurement strategies, both in the Early and in the Late Universe, is presented in (Pantos & Perivolaropoulos, 2026).

The tension on the S_8 parameter (Di Valentino et al., 2025; Abdalla et al., 2022; Douspis et al., 2019), which will be the subject of Part II of this thesis, is formulated in a way similar to the H_0 tension, but, due to its lower significance, it is less controversial. Here we briefly sketch its nature,

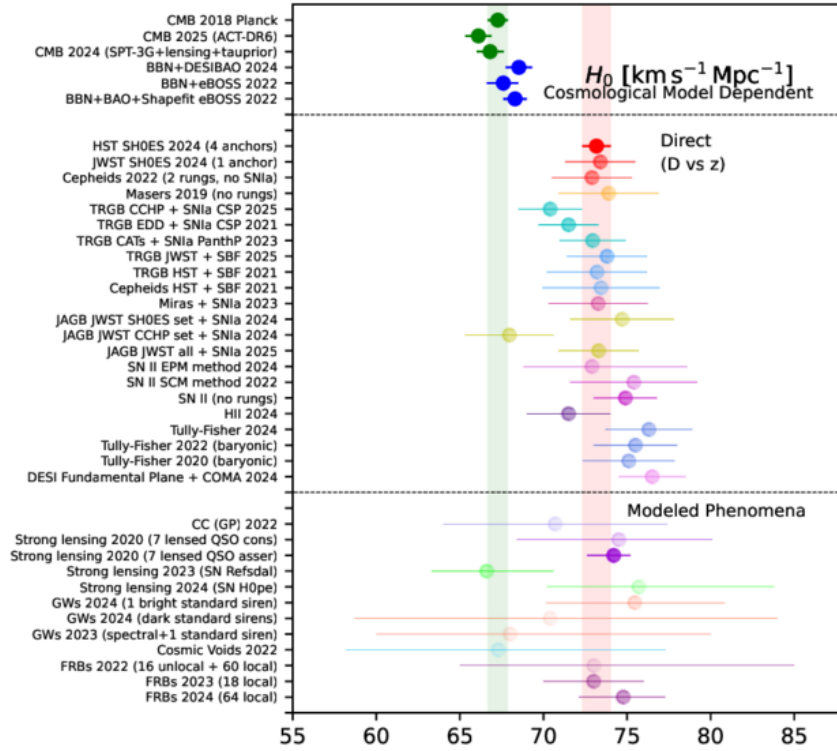


Figure 4: A summary of the H_0 measurements based on distinct probes and methodologies as of March 2025, displaying the tension between the CMB/Early Universe-based measurements and the direct measurements. The picture is taken from (Di Valentino et al., 2025).

while a more in-depth discussion will be presented in chapters 4 and 5[§]. Generally speaking, this tension behaves in the opposite way compared to the tension on H_0 : Early-Universe measurements on the CMB provide a low value for its quantity, while Late-Universe observations suggest higher values. More precisely, the tension is defined in terms of the parameter

$$S_8 = \sigma_8 \left(\frac{\Omega_m}{0.3} \right)^\alpha$$

where Ω_m is density parameter of all matter in the Universe (dark matter+baryon matter) and α is usually set to 0.5. This reformulation allows to break most of the degeneracy between σ_8 and Ω_m one usually finds when measuring the actual fundamental quantities. We shall hereafter present the topic in terms of “ S_8 tension” as usual in the literature and quote the values of this parameter rather than the ones of σ_8 .

Now, measurements on the CMB tell us that $S_8 \sim 0.83$ -0.85 (the reference value being set by the Planck satellite observations at 0.832 ± 0.013 in the TT, TE, EE+lowE+lensing analysis configuration, combining all primordial data, temperature maps T, polarisation maps E and their combinations, together with the CMB lensing signal determined by the gravitational interaction of the CMB photons with the LSS forming in the Late Universe (Aghanim et al., 2020)), whereas

[§]The detailed definition of the σ_8 parameter and its connection to the theory of cosmological perturbation is presented in sec. 1.2.1 of chapter 1.

local Universe measurements, especially when relying on cosmic shear as a probe, at the same time the most sensitive observable to the parameter and a particularly difficult to handle cosmological tracer, are associated to values as low as $S_8 \sim 0.76$ (Heymans et al., 2013; Doux et al., 2022; Amon et al., 2022; Loureiro et al., 2022; Asgari et al., 2021; Miyatake et al., 2023; Li et al., 2023; Dalal et al., 2023; García-García et al., 2021; Di Valentino et al., 2025). The discrepancies between the better-constrained datasets and Planck this time do not exceed the threshold of $\sim 3.5\sigma$ significance, hence the inconsistency might not be a deep fundamental problem and could just be a matter of better understanding systematics and of more accurately modelling the growth of perturbation at small, non-linear scales, where complex interactions involving gravity and the physics of baryons might be acting. Solutions based on the fact that this tension is actually present and fundamental in nature might try to reconcile the Late Universe value by finding a way to enlarge its value, making it compatible with the one measured on the CMB or, conversely, to predict the CMB power spectrum but allowing for a lower variance of perturbations in the Early Universe, which can thus be reconciled with the reference value measured on the LSS at low redshifts. It is finally worth mentioning that the two tensions might be at least partially related, since both H_0 and σ_8 show degeneracies with the matter density parameter Ω_m : thus, a model able to solve one of the tensions might be automatically fix the other one or, conversely, a model capable of solving one tension should at least not exacerbate the other one – a delicate technical issue, part of the current open debates in Cosmology.

The attitudes of the cosmological community towards the foundational problems and tensions affecting the field are manifold, as it can be observed by the daily stream of publications, or in the context of conferences, seminars and research projects (López-Corredoira, 2017, 2022; Turner, 2022; Valentino et al., 2025). The spectrum of suggested answers is wide. Leveraging also on other inconsistencies, such as the problem posed by the nature of dark matter and dark energy or concerns about the actual level and soundness of the assumption of homogeneity and isotropy properties of the Universe (Kumar Aluri et al., 2023; Peebles, 2022), considerations concerning the crisis and falsification of the Λ CDM model and that cosmology necessitates a profound rethinking, possibly accepting to relax some achieved precision in measurements, could be raised (Kroupa et al., 2023). As well, positions can be mixed, without a clear opposition between researchers who are totally dismayed and those who are totally confident in the Λ CDM model (Blanchard, 2025; Perivolaropoulos & Skara, 2022; Turner, 2025a,b). Alternative stances may acknowledge the existence of tensions but argue that these can be addressed by extending the existing model, complementing it with additional hypotheses, or suggest the need of incorporating new physics, although keeping the general conceptual edifice, looking at the Universe as the largest available fundamental physics experiment, allowing to test physics at different time and distances. Moderate or radical opinions would often agree on the wish to extend the amount and diversity of data, devising novel observables, developing more advanced instruments and/or theoretical approaches either in the hope that many problems might be solved soon, fixing systematics and shedding light on current observational limitations, or in order to achieve actual novel discoveries (Schmitz, 2022; Binney et al., 2025; Madhavacheril, 2025; Peebles, 2022).

Awareness on the problems concerning the “big picture” of the discipline comes in parallel, and can also be superseded, by the everyday necessities of research activity, a job dealing with very specific and technical issues, characterised by the typical routine of “normal science” times, in the famous words of Thomas Kuhn (Kuhn, 1996). This can be seen as the framework in which my research activity has been developed: more details on the tension topic will be presented in chapter

4, complemented with a broader discussion of the measurements of the growth of structures and properties of the cosmic web by means of multiple strategies, not necessarily having the topic of tensions in mind, just focusing on probing specific chronological and spatial regimes of the LSS. This is the spirit of the study I shall discuss in chapter 5, where I will expose tomographic measurements of galaxy clustering-based datasets – the 2MPZ photometric sample (Skrutskie et al., 2006) and the samples derived from the combination of the WISE and SuperCosmos (Bilicki et al., 2016) catalogues– without the need of modelling the complex cosmic shear signals, cross-correlated with the CMB lensing signal and selected in order to access very low redshifts, between $z = 0$ and $z = 0.6$ (centred around the redshift values of 0.07, 0.18, 0.32), and complementing extant works such as (Sailer et al., 2025a; Bianchini & Reichardt, 2018; Peacock & Bilicki, 2018; Sailer et al., 2025b; García-García et al., 2021).

An outline of this thesis

Combining thus “big picture” inspirations and more “normal science” activities, encompassing the implementation of a number of specific technical methodologies, the exposition of my activity in this thesis can be outlined as follows:

We start from chapter 1, intended to serve as a “toolkit” section, presenting most of the theoretical material required for the topics developed in the two subsequent parts of this work. Subsequently, Part I discusses the synergies between astroparticle physics and cosmology. In chapter 2, an outline of the current status of the research in the field, with a specific focus on Dark Matter indirect detection methodologies and the investigation of the Unresolved Gamma Ray Background, is presented; chapter 3 will collect instead my work on the application of multi-tracer and weighting techniques on LSS-UGRB cross-correlations, corresponding to the preliminary results published in (Rubiola et al., 2024) and to the subsequent consolidated conclusions collected in the paper (Rubiola et al., 2025).

Part II will instead be dedicated to the research work I have been developing on the topic of the “ S_8 tension” and the growth of the Large Scale Structure investigated at low redshift and non-linear scales. Similarly to the organisation of Part 1, chapter 4 will be a general review of the topic, focusing on the most important observational results and approaches to the “tension”, whether of fundamental nature or just requiring more refined modelling of the processing shaping the LSS, its growth and evolution, whilst chapter 5 will present my research work, derived from the results published in (Rubiola et al., 2025). We will eventually present the **general conclusions**, recapitulating the achieved results and presenting some perspectives concerning their future extension.

Chapter 1

From the Fridman equations to angular power spectra: a theoretical toolkit for this work

This chapter will provide an overview of the fundamental analytic tools required to describe the buildup and temporal evolution of the Large Scale Structure (LSS) of the Universe, from the description of an expanding homogeneous and isotropic background, modelling the Universe as a whole, to the cosmological perturbation theory required to predict and describe the formation of cosmic structures and of the CMB. This framework can be seen as made of two main ingredients: a “physical” part of the theory, rooted in Einstein’s General Relativity, Gravitation being the dominant force determining the phenomenology of the Universe, through which we can measure distances and time intervals, and a “statistical” part: the physical properties of the fields can be combined and related to a number of statistical considerations, thus summarising and compressing the complexity of the information we obtain from LSS observations into simpler and more readily interpretable quantities. These latter quantities are indeed often referred to as “summary statistics”, that can be intuitively connected to estimates of the degree of correlation among the fields associated to the studied LSS tracers, such as galaxies, cosmic shear, voids, HI Intensity Mapping, CMB, and other electromagnetic backgrounds (Slepian et al., 2025). When working in the real space, the most customary summary statistics is known as *two-point correlation function* (2PCF), measuring the excess probability of finding two points separated by a distance $\mathbf{r}$ with respect to a uniform distribution. The same quantity, when expressed in the Fourier space, is known as *power spectrum* and it is directly related to the variance of the fields under scrutiny (Peacock, 1999; Dodelson & Schmidt, 2020). Most of the work presented in this manuscript will imply discussion of quantities expressed in the Fourier space, so that the focus will chiefly be on different derivations, formulations and applications of the power spectrum.

1.1 Describing the expanding background: From Einstein’s field equations to cosmological perturbation theory

Modern physical cosmology results can be considered as solutions and applications of General Relativity, condensed in Einstein’s field equations (EFE)* (Einstein, 1915)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (1.1)$$

The discussions on the application of General Relativity to the Universe as a whole and the derivation of the first models of dynamic Universe by Fridman are actual prior (1922) (Friedmann, 1922) to George Lemaître and Edwin Hubble’s observational results on galaxy recession (1926-1927 and 1929, respectively) (Lemaître, 1926; Lemaître, 1927; Hubble, 1929). At the same time, it is useful to introduce first some observational quantities and subsequently embed them in the formal framework provided by General Relativity. At the turn of the 1920s it was discovered that the spectra of distant galaxies – whose velocities could be better measured under the assumption of negligible proper motions induced by the gravitational attraction of the Local Group – displayed a systematic shift of their spectroscopic lines to larger (hence “redder”, the red colour being associated to the larger wavelengths of the visible electromagnetic spectrum) wavelengths λ_1 compared to the rest-frame value λ_0 . We can call this displacement redshift z and define it as

$$\frac{\lambda_1 - \lambda_0}{\lambda_0} = z. \quad (1.2)$$

The phenomenon was interpreted kinematically and combined with the determination of the distance of the studied galaxies from the observer: it was thus found that, the further away the observed galaxy, the greater the redshift its spectral lines displayed, whence the conclusion that galaxies in the Universe were moving away from the observer with a velocity proportional to their distance R and, for sufficiently large distances, this velocity was the only relevant one, much larger than peculiar motions induced by local gravitational potentials. The observed velocity can be linked to the redshift as $v_{\text{obs}} \equiv cz$ and, by Hubble’s result, we have

$$v_{\text{obs}} = HR \quad (1.3)$$

with H a redshift-dependent quantity. Such coherent motion by galaxies and clusters (also known as “Hubble flow”) is therefore suggestive of the expansion of the Universe as a whole: the cosmos can be interpreted as a “grid” on which astrophysical objects are placed. As such, two objects have a fixed relative comoving distance χ . The comoving distance can of course change in time because of peculiar motions of the objects “within” the grid, but, even in absence of peculiar motions, the observed distance R is observed to change in time because it is a function of the expansion of the Universe itself. The evolution law for R in time corresponds to a re-scaling of the comoving distance by means of a time-dependent coefficient, the scale factor $a(t)$:

$$R(t) = a(t)\chi.$$

From the position above, we can recover Hubble’s law by deriving R with respect to time[†]:

$$\dot{R} = \dot{a}\chi + a\dot{\chi}. \quad (1.4)$$

*The expression already includes the effect of the cosmological constant in its modern formulation of a negative energy pressure determining an accelerated expansion of the Universe, rather than Einstein’s original, *positive* cosmological constant introduced to keep the Universe size stationary in time.

[†]The number of dots denotes the order of the derivative we apply to quantities with respect to time.

The quantity $\dot{R}$ is the observed velocity v_{obs} , while on the right-hand side the second addendum represents changes in the comoving distance, hence displacements between galaxies related to relative peculiar motions, that, we said, at sufficiently large distances become negligible. Under this approximation, we are left with

$$v_{\text{obs}} \approx \dot{a}\chi. \quad (1.5)$$

If we multiply and divide the right-hand side by the scale factor a , we have:

$$v_{\text{obs}} \approx \frac{\dot{a}}{a}a\chi \approx HR \quad (1.6)$$

which is indeed Hubble's law. We stress that we defined $\frac{\dot{a}}{a} \equiv H$, the Hubble factor at a given redshift or time, as opposed to its value today H_0 . The last preliminary and semi-formal derivation we need is the connection between time, redshift and scale factor: we stress that what we *observe* from astrophysical data is the spectroscopic redshift, so we want to ultimately express all of our results in terms of the observed quantity. For a signal propagating at the speed of light c in an interval dt in an expanding background, we have that the travelled distance is $dR = a d\chi = c dt$.

We can integrate the two sides and focus on the right-hand side, considering the following physical process: an astrophysical object emits an electromagnetic wave at instant t_0 for a short time interval δt_0 , so that the emission ends at $t_0 + \delta t_0$. At the receiver, the signal is detected at the time instant t_1 until $t_1 + \delta t_1$. If the position of the receiver can be considered fixed, the comoving distance $d\chi = c dt/a$ photons travel between t_0 and t_1 is the same they travel between $t_0 + \delta t_0$ and $t_1 + \delta t_1$. We thus have:

$$\int_{t_1}^{t_0} \frac{c dt}{a(t)} = \int_{t_1 + \delta t_1}^{t_0 + \delta t_0} \frac{c dt}{a(t)}. \quad (1.7)$$

By recombining the integration extremes, the relation can be rearranged (simplifying c) as

$$\int_{t_1}^{t_1 + \delta t_1} \frac{dt}{a(t)} = \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)} \quad (1.8)$$

and, for small δt , the two integrals can be approximated with

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}. \quad (1.9)$$

The intervals δt_0 and δt_1 correspond to the respective time intervals at the emitter (for δt_0) and receiver (for δt_1) between two subsequent peaks or troughs of the same electromagnetic wave, whose wavelength in the emitter's rest frame is measured having value λ_0 and in the observer's one is measured to be equal to λ_1 . By recalling the redshift definition of eq. 1.2 we have:

$$1 + z = \frac{\lambda_1}{\lambda_0} = \frac{a(t_1)}{a(t_0)}, \quad (1.10)$$

and, setting the scale factor today $a(t_1) = 1$, for every time instant in the past we eventually recover the connection between redshift and scale factor given by

$$a(t_0) = a(t) = \frac{1}{1 + z}.$$

With these preliminary results in hand, the first step to derive a physical description of the Universe starting from the EFE is to solve the equations for the unknown components of the metric tensor $g_{\mu\nu}$, entering both in the Ricci scalar R and the Riemann tensor $R_{\mu\nu}$, as a function of a number of energy and matter components, usually defined “cosmic fluids” or “cosmic species”, whose energy

and momentum densities contribute the stress-energy tensor $T_{\mu\nu}$. This latter term requires the specification of all the types of matter we observe or we assume according to some model to populate the Universe, and a relation between their pressure p and density ρ – an equation of state – by means of the barotropic index w :

$$p = w\rho/c^2.$$

Given our standard Λ CDM cosmological model, we have therefore to specify contributions from: radiation, neutrinos, baryon matter, Dark Matter and the Cosmological Constant as a specific Dark Energy model. Thermodynamics and statistical Physics considerations (see chapter 2) allow us to state that, for most part of the cosmic history and in particular after the formation of the CMB, when the LSS formation process starts to proceed more steadily, baryon and cold dark matter components are non-relativistic fluids and have $w = 0$, relativistic fluids such as photons and (at least in the past) neutrinos have $w = 1/3$, while the Cosmological Constant has $w = -1$ (Kolb & Turner, 1990; Dodelson & Schmidt, 2020; Peacock, 1999). We can put further constraints on the EFE by making additional assumptions on the properties of our manifold: combined with the observational evidence based on Hubble’s law, we indeed require our Universe to follow the Cosmological Principle. That is, we describe our Universe to be, on sufficiently large scales, statistically homogeneous and isotropic. These two requirements imply that, on average, all physical quantities can depend on the cosmic epoch by means of the redshift z but not on the direction (the isotropy condition) and that they take everywhere the same average values, regardless of the position (the homogeneity condition); Hubble’s law does not show position dependence and can thus be interpreted as consistent with the Cosmological Principle.

These requirements can be summarised by endowing our manifold with a background metric known as Fridman-Lemaître-Robertson-Walker (FLRW), reading:

$$ds^2 = -c^2 dt^2 + a(t)^2 [(1 \pm kr^2)^{-1} dr^2 + r^2(\sin^2 \theta d\theta^2 + d\varphi^2)]. \quad (1.11)$$

Once a specification for the constant curvature k – for which a number of measurement techniques exist, both statistical and astrophysical in nature, and which is measurable as a function of the cosmic fluids we introduce in the stress-energy tensor (Coley & Ellis, 2020; Dinda et al., 2023; Moresco et al., 2016; Di Valentino et al., 2021b)– is provided, the only unknown we are left with and for which we have to solve the EFE is the scale factor a as a function of time t or, given the relationship between scale factor and redshift, $a = \frac{1}{1+z}$, as a function of the redshift z .

The final requirement is a law for the scaling of density and pressure of the involved fields as a function of the scale factor, which can be derived by imposing the conservation of the time-like component of the stress-energy tensor $\nabla_\mu T_0^\mu = 0$, which imposes a general evolution for the density of a fluid:

$$\rho = \rho_0 a^{-3(1+w)} \quad (1.12)$$

with ρ_0 its observed density today. The ultimate outcome of this setup is the derivation of two paired ordinary differential equations having a as unknowns, called Fridman equations (Friedmann, 1922; Peacock, 1999; Dodelson & Schmidt, 2020; Davis, 2026; Ferreira & Roskill, 2025).

The second equation will no longer be required, we display it here for completeness and limited to a single fluid with density ρ and pressure P , observing that it is connecting the second derivative of the scale factor and the scale factor itself to the pressure and density of a given fluid

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho c^2 + 3P). \quad (1.13)$$

The first Fridman equation (we shall hereafter work by setting $c = 1$) instead reads:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i(a) + \frac{\Lambda}{3} - \frac{k^2}{a^2} \quad (1.14)$$

with i running on all the cosmic fluids required in our model. The terms entering in the right-hand side can be further rearranged:

1. We define a critical density value at present time as $\rho_c(z=0) \equiv \frac{3H_0^2}{8\pi G}$.
2. We connect all densities to their associated density parameter $\Omega_i(z) \equiv \frac{\rho_i(z)}{\rho_c(0)}$.
3. Factoring out the redshift dependence in the density of every cosmic fluid according to $\rho = \rho_0 a^{-3(1+w)}$, we can express the present-day density parameter as $\Omega_{0,i} = \frac{\rho_i(0)}{\rho_c(0)}$. Such quantities are the parameters usually fitted in cosmological models and are quoted as density parameters *tout-court*, without mentioning that they are evaluated at $z = 0$.

The standard expression of the first Fridman equation is thus:

$$\frac{\dot{a}}{a} = H(z) = H_0 \sqrt{\Omega_{0,m}(1+z)^3 + \Omega_{0,r}(1+z)^4 + \Omega_\Lambda + \Omega_{0,K}(1+z)^2}, \quad (1.15)$$

that, after integration, allows us to retrieve the time or redshift evolution of the scale factor, from which we can determine all the relevant properties of the metric (especially, distance and time interval measurements) of a homogeneous and isotropic Universe and their evolution dynamics. To characterise the metric and the scale factor means also to completely characterise the properties of the fluids populating the Universe, thanks to the relationship between pressure, density and scale factor of eq. 1.12, thus providing us with a full description of the so-called “background” properties of the Universe. Furthermore, it is possible to find that the evolution of the scale factor depends on the equation of state of the fluid whose density is the largest at a given stage of the cosmic history and that, therefore, is dominating the expansion dynamics. In particular, we have that:

1. The earliest phases of the cosmic history are characterised by “radiation domination”, when the density of relativistic fluids was larger than the one of any other fluid. At those times, $a \propto t^{1/2}$.
2. Since the density of radiation scales as $\rho_{\text{rad}} \propto a^{-4}$ while that of non-relativistic matter scales as $\rho_{\text{matter}} \propto a^{-3}$, at around $z \sim 3600$ the density of matter became larger than the density of radiation (the “equivalence” epoch) determining the onset of the matter-dominated phase, with $a \propto t^{2/3}$.
3. Eventually, when the matter density became smaller than the Cosmological Constant density, around $z \sim 2$, we entered in the Dark Energy-dominated phase of the cosmic history, characterised by $a \sim \exp(H_0 t)$.

The formulation of the first Fridman equation as stated in eq. 1.15 exposes most of the parameters of the Λ CDM model in their most compact and standardised form: these quantities can be interpreted as summarising the cosmic properties at a fundamental level, not differently from the way the Standard Model of Particle Physics summarises the fundamental particles properties in a simple and consistent framework. Cosmology from the point of view of fundamental physics can indeed be interpreted as a discipline aiming at measuring with the greatest possible precision and

accuracy the value of such fundamental quantities and connecting astrophysical phenomenology to microphysical, fundamental descriptions.

We can stay a bit more general and just classify the cosmological parameters based on the equation of state of the associated fluids: we might always insert more fluids in the right-hand side of the Fridman equation than the ones of the Λ CDM model, just requiring them to obey some equation of state. If we limit the cosmic inventory to baryons, Cold Dark Matter, electromagnetic radiation and neutrinos, we fall back to the Λ CDM case. In particular, $\Omega_{0,r}$ is the value of today's density parameter for all fluids with a relativistic equation of state, such as electromagnetic radiation and neutrinos, $\Omega_{0,m}$ is the value of today's density parameter for all fluids with a non-relativistic equation of state, such as baryons and Dark Matter, $\Omega_{0,\Lambda}$ describes the contribution from the cosmological constant (the simplest form of Dark Energy currently best describing our observations) while $\Omega_{0,K}$ contains the information on the curvature of our metric and can be reformulated as a function of the other density parameters by means of the relation

$$\Omega_{0,K} = 1 - \sum_{i \in \{r, m, \Lambda\}} \Omega_{0,i}.$$

From now on, we will no longer display the 0 subscript in the density parameters, that should nevertheless always be intended as measured at present time and opposed to the redshift-dependent quantities noted with $\Omega_i(z)$. At the same time, our inventory of cosmic species, including a Cosmological Constant Λ as specific Dark Energy contribution, will always be associated to the relativistic and non-relativistic species contemplated by the Λ CDM cosmological model. Altogether, the density of the cosmic species and the Hubble constant, under the assumption of a null curvature define three out of the six parameters (H_0 , $\Omega_c + \Omega_b = \Omega_m$ and Ω_Λ) of the Λ CDM cosmological model, the ones which can be measured by studying the properties of the cosmic background. Their reference values as established by the CMB observations performed by Planck ([Planck Collaboration et al., 2020b](#)) will be exposed in table 1.1 of sec. 1.2.1, after the discussion in the next section of the concepts specific of the perturbative theory, related to the remaining three fundamental parameters of the theory.

1.2 Perturbation theory: from to the perturbations of the metric and structure formation

If we want to understand the formation, growth and evolution of the Large Scale Structure of the Universe (LSS) or predict the temperature anisotropies of the Cosmic Microwave Background (CMB), we have to take a further step beyond the analysis of the equations dictating the homogeneous and isotropic background evolution. The general idea behind the theory of structure formation in the Universe is that the LSS is the ultimate outcome of the growth of slight deviations from an homogeneous distribution of matter, seeded at the quantum scale shortly after the beginning of the expansion (the “Big Bang”), inflated to macroscopic scales by the cosmic expansion during the inflation process ([Linde, 1974](#); [Guth, 1981](#)), whose evolution is, generally speaking, driven by gravity and, more specifically, by Dark Matter, since this appears to be the dominant matter component of the Universe and thus the ultimate responsible of the shape and intensity of the gravitational potentials in the Universe.

By similar arguments it is possible to calculate the anisotropy perturbations of the photon fluid from the beginning of the cosmic history up the recombination epoch, allowing us to also derive

the theory underlying the perturbative treatment of the Cosmic Microwave Background. Overall, General Relativity implies the existence of scalar, vector and tensor observables (also known as “degrees of freedom” of the metric) in the Universe: a typical example of scalar observables are density perturbations in the matter field, typical vector perturbations are transverse velocities (vorticity) of the matter field, whilst tensor perturbations can be understood as gravitational waves. A perturbative treatment for all of these quantities can be consistently derived; this thesis will focus however on observations and features of scalar perturbations, and we shall concentrate on them only. The second important point to bear in mind is the order of our perturbative theory: if we introduce the density contrast of the local density field $\rho(\mathbf{x}, t)$ with respect to the background density of the Universe $\bar{\rho}(t)$ and defined as

$$\delta(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}, \quad (1.16)$$

the first approximation for δ is to assume it to be much smaller than 1, allowing a linear perturbative expansion of the fields. This assumption has historically laid the foundations for perturbation theory, from the pioneering works of (Lifshitz, 1946; Lifshitz & Khalatnikov, 1963) up to the more modern and standard formulations applied to the CMB (Sachs & Wolfe, 1967) and to the Large Scale Structure of (Peebles, 1980), (Bardeen, 1980) (providing the rigorous gauge-invariant linear theory). Linear theory is also the standard discussion presented in textbooks, either in a simplified Newtonian gravity framework (e.g., (Peacock, 1999)) or in a full general relativistic formulation encompassing also vector and tensor degrees of freedom (Dodelson & Schmidt, 2020). It is also implemented in a number of computational tools of everyday use in Cosmology, such as the Boltzmann solvers CAMB (Lewis & Challinor, 2011) and CLASS (Blas et al., 2011): indeed, in spite of its simplicity, linear theory is as much fundamental as of practical interest for scientific research, since the linear regime is associated to either high-redshifts, such as the one of the CMB formation, thus allowing the extremely accurate prediction of the CMB angular power spectrum, or to large and very large scales of the LSS. These scales have been repeatedly observed and studied in the years, showing indeed agreement with the predictions of linear theory, including the observation of distinctive effects, such as Baryon Acoustic Oscillations, that the linear theory is capable of deriving (Eisenstein et al., 2005). However, at very low redshifts and smaller scales, characterised by $O(1)$ or larger density contrasts, linear theory may no longer be enough, and further theoretical tools are required. These will be outlined later in the text but can be better understood as corrections and extensions of the standard linear perturbation theory and its formulation; at the same time, all these higher-order perturbative formulations must be consistent with the linear theory when the large scale or the small density contrast limits are taken.

The first-order perturbation theory of the metric starts from the following modification of the FLRW line element:

$$ds^2 = -(1 + 2\psi)dt^2 + (1 - 2\phi)a^2 [(1 \pm kr^2)^{-1}dr^2 + r^2(\sin^2\theta d\theta^2 + d\varphi^2)]. \quad (1.17)$$

The function ψ can be related to the Newtonian gravitational potential of the non-relativistic theory, whereas ϕ can be related to the curvature of the space. Overall, a full-fledged, rigorous cosmological perturbation theory would further redefine such quantities in terms of the gauge-invariant variables known as Bardeen’s potentials (Bardeen, 1980) Φ and Ψ (notice the use of the capital equivalent of the previously employed Greek letters ϕ and ψ), and develop the calculations consequently. The stress-energy tensor on the right-hand side of eq. 1.1 can be similarly perturbed, assuming

small local deviations from spatially homogeneous and isotropic density, pressure and velocity fields, identified with the '0' subscript (notice that $\mathbf{v}_0 = 0$, meaning that, on average, no peculiar velocities are present in the fields: every point, if unperturbed, would be just comoving with the expanding background):

$$\rho = \rho_0 + \delta\rho \quad (1.18)$$

$$\mathbf{v} = \delta\mathbf{v} \quad (1.19)$$

$$p = p_0 + \delta p \quad (1.20)$$

The rigorous treatment of the problem would require to perturbatively expand Einstein's Field Equations under the described assumptions. Nevertheless, most of the fundamental results can be obtained also in a classical Newtonian framework, given the weakness of the gravitational fields at these cosmic scales and typically slow motions of the involved fluids, the main correction being associated to the behaviour of electromagnetic radiation (inherently relativistic) and for matter fields in the Early Universe, when their relativistic behaviour cannot be neglected. We can thus sketch the main arguments and results of cosmological linear perturbation theory also in the classic Euler's description (Peacock, 1999), defining first the continuity equation

$$\dot{\rho} + \nabla \cdot (\rho\mathbf{v}) = 0, \quad (1.21)$$

then the dynamic equation

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0 \quad (1.22)$$

and, eventually, by coupling the two equations above with Poisson's equation for the gravitational potentials:

$$\nabla^2\varphi = 4\pi G\rho. \quad (1.23)$$

In order to close the system, one has to recall the equations of state linking pressure and density for each one of the fluids taken into consideration

$$p = p(\rho). \quad (1.24)$$

Starting from these considerations, the standard line of reasoning goes as follows:

1. The perturbed quantities are inserted in the equations above.
2. The calculations are developed bearing in mind that we are set in an expanding background.
3. The equations are expressed in the Fourier space instead of the real one, so that we study $\delta(\mathbf{k}, t)$ instead of $\delta(\mathbf{x}, t)$.

The latter position is in the first place a mathematical device ensuring a more straightforward solution of the equations themselves. Furthermore, from a physical point of view, it will make the results more readily interpretable. Indeed, by Fourier-transforming the fields, instead of focusing on their value as a function of the *position* $\mathbf{x}$, we are decoupling them in terms of superposed modes of different wavelengths associated to different *scales* $\mathbf{k}$. Reasoning in terms of scales is rather natural in Cosmology, since we can separate the largest-scale effects, referred to the Large Scale Structure as a whole (superclusters, walls, the cosmic web “at large”), for which the density

fluctuation with respect to the average density is low (in the order of $10^{-2}-1$), fully justifying a linear perturbation approach, from the smallest scales (galaxy clusters, galaxies and astrophysical, “virialised” objects in general), forming in regions where the density contrast is much larger than 1, ultimately breaking the linear perturbation theory assumptions, whose correct description calls for alternative, more complex and non-linear approaches. In addition to this, whilst the dynamics of the LSS in the linear regime can be assumed to be chiefly driven by dark matter through gravitational interactions (allowing to properly study the Universe in cosmological N -Body simulations, where all interactions are gravitational in nature), at the smallest scales the contribution of plasma and baryons starts to be significant and we are thus urged to scale up and work in the much more complex context of hydrodynamical simulations (Dolag et al., 2008; Angulo & Hahn, 2022; Schaye et al., 2023). Furthermore, given the causal nature of General Relativity and the fact that gravity can at most propagate at the speed of light c , the concept of “scale” and the definition of an horizon corresponding to the largest causally connected scale at every given time – thus distinguishing between super-horizon and sub-horizon perturbations –, comes in quite handy in order to suitably define what cosmological scales are coupled and interacting in every phase of the cosmic history.

Eventually, the equations for the scalar matter density contrasts $\delta(k, t)$ (where we treat the Fourier modes k as scalars under the assumption of isotropy and homogeneity of our observables, hence a function of the modulus of the scale but not of a specific direction) read:

$$\frac{d^2\delta(k, t)}{dt^2} + 2H(t)\frac{d\delta(k, t)}{dt} + \left[\frac{w}{a^2(t)}k^2 - 4\pi G\bar{\rho}(t) \right] \delta(k, t) = 0. \quad (1.25)$$

This is the typical form of a damped harmonic oscillator equation of motion, where the dissipative term is represented by the so-called “Hubble friction” contribution $2H(t)\frac{d\delta(k, t)}{dt}$, meaning that the background expansion opposes the clustering effect of gravity by damping the density fluctuations, and inducing a reversion of their value towards the average density value $\bar{\rho}(t)$. The last term in eq. 1.25 is instead describing the equilibrium between the thermal pressure term $\frac{w}{a^2(t)}k^2$ induced by the fluids, identified by some equation of state with barotropic index w , and the gravitational attraction codified in the term proportional to $4\pi G\bar{\rho}(t)$. Observe that, in a non-expanding background, where $H(t) = 0$, eq. 1.25 describes the standard theory of the Jeans collapse for astrophysical object formation (Binney & Merrifield, 1998; Binney & Tremaine, 1987).

Always at the linear theory level, the solution to eq. 1.25 can be expressed as a summation of a growing mode (identified with the $+$ subscript hereafter), whose amplitude grows with time and a decreasing mode, which instead is progressively being damped and progressively vanishes:

$$\delta(k, t) = \delta_+(k, t) + \delta_-(k, t).$$

We are also allowed to rewrite each term of the solution as the product of a purely time-dependent function $D(t)$ (called “growth factor”, since we implicitly discard the contribution of the decaying mode of the perturbation, which becomes less and less observable the further away in time we are from the onset of the perturbations) and a purely mode-dependent function $\delta(k, t = t_c)$, being evaluated at a suitable time t_c . We can for example set $t_c = t_{z=0}$, i.e., today’s time, since it is the most natural epoch at which we have direct access to the density contrast, and set $\delta(k, t) = D(t)\delta_{\text{today}}(k)$, usually by also rescaling $D(t)$ so that today its value is $D(t_{\text{today}}) = 1$. A final observation is that the whole formalism can be expressed, as we shall do hereafter, in terms of the observed redshift z , instead of the time t , whose value can be inferred from the redshift in a model-dependent way, whence $\delta(k, z) = D(z)\delta_{z=0}(k)$.

We observe in passing that non-linear, more complex models for the calculation of the value and evolution of perturbations no longer allow for such a simple separation between time-dependent and scale-dependent contributions, thus inducing what we generally call scale-dependent effects, since $D(z)$ gets promoted to a function of scale as well: $D(z) \rightarrow D(k, z)$. $D(z)$ can be explicitly calculated in a FLRW metric reading:

$$D(z) = \frac{5}{2} \Omega_m E(z) \int_z^\infty \frac{1+z'}{E(z')^3} dz', \quad (1.26)$$

where we define the shorthand $E(z) = \sqrt{\Omega_{0,m}(1+z)^3 + \Omega_{0,r}(1+z)^4 + \Omega_\Lambda + \Omega_{0,K}(1+z)^2}$. We thus find that the growth factor, entering in the formalism of linear perturbative regimes, is a function of background quantities only.

1.2.1 Introducing the power spectrum

Our discussion has thus far addressed the *physics* of perturbations: from this point onwards, the physical treating of the LSS is accompanied by a *statistical* description of it. The introduction of statistical concepts is due to a number of reasons:

- Even in the luckiest case of being able to track the time and space evolution of a given mode $\delta(k, z)$, non-linear and local effects would most likely disrupt the linear evolution of our perturbations. One perturbation mode observed in the Universe would most likely show a very low degree of coherence with the same mode sampled in another point of the cosmos, local interactions having washed out most of the primordial information. The picture described by the theory of perturbation appears rather as an *average* property of the LSS, so that the predicted matter field properties are obtained by sampling every scale at different locations and then performing an average of such measurements. All the relevant quantities, we will see, will ultimately be expressed as ensemble averages (between angled brackets $\langle \cdot \rangle$) assumed to be obtained by sampling many realisations of a given observable of interest.
- Linked to the above concept, there is an intrinsic need to do some form of “data compression” of the LSS complexity, and to summarise it in simpler, more interpretable quantities. Usual standard summary statistics can be the average density value of the field, for which we have the physical constraint that it must be equal to $\bar{\rho}(z)$, its variance (related to the power spectrum we shall discuss later), and higher-order moments (such as the skewness, linked to the bispectrum), an approach introducing an interesting connection between the study of the LSS and widely used signal processing and image processing techniques.

We might reframe the use of statistical approaches also in terms of our ignorance on the exact value of the initial conditions: since we do not have direct observational access to the Big Bang epoch, statistical reasoning in absence of exact measurements could be more suitable. At an even more fundamental level, if the initial conditions were actually set at the Big Bang epoch, possibly by an inflation process, they were determined by quantum physics phenomena, inherently non-deterministic and statistical in nature. This also allows to see the LSS as a “magnifying glass” of pristine quantum effects whose signature can then be detected in heavens.

Given the above points, we can endow the initial density contrast $\delta_{in}(k, z)$ with some statistical properties: first, we can assume that the initial value of the density contrast (we recall, defined as $\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$) was actually a random variable sampled from a Gaussian distribution. It thus defines

a stochastic field with zero average (this condition corresponding to the fact that we want the average value of the density field to be equal to $\bar{\rho}(z)$) and a scale-dependent variance. Starting from the primordial fluctuations, the scale dependent variance is measured by the correlator

$$\langle \delta_{prim}^*(\mathbf{k}', z) \delta_{prim}(\mathbf{k}, z) \rangle = (2\pi)^3 \delta_D^3(\mathbf{k} - \mathbf{k}') P_{prim}(k, z) \quad (1.27)$$

where the Dirac delta $\delta_D^3(\mathbf{k} - \mathbf{k}')$ imposes the absence of correlation between distinct modes $\mathbf{k}$ and $\mathbf{k}'$ (in other words, the covariance between the modes is diagonal). With a slight abuse in notation and terminology, given the effect of the Dirac delta, we can directly call “power spectrum” the quantity

$$\langle \delta_{prim}^*(\mathbf{k}, z) \delta_{prim}(\mathbf{k}, z) \rangle = P_{prim}(\mathbf{k}, z), \quad (1.28)$$

mixing a bit the correlator and the actual power spectrum. The isotropy and homogeneity in the Fourier modes allows us to discard the vector notation for the Fourier modes and refer to them with the scalar notation k . The values of the amplitude of the primordial density fluctuation value, subdivided scale by scale, can then be considered as realisations of a Gaussian field: in these terms, every fluctuation amplitude can be treated as a normally distributed random variable – in statistical notation: $\delta_{prim}(k) \sim \mathcal{N}(0, P_{prim}(k))$. The value of the initial conditions can vary with the scale: the typical assumption, related to some fundamental considerations on the properties of the matter field and on the observed structure formation process led to the definition of the pristine value of the density contrasts in terms of the Harrison-Zel’dovich power spectrum (Peebles & Yu, 1970; Harrison, 1970; Zeldovich, 1972):

$$P_{prim}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1}, \quad (1.29)$$

where the amplitude A_s , the reference scale k_* and the spectral index n_s can be measured, for instance, with the CMB. Concerning the value of n_s , it is also possible to provide a prediction based on the inflationary theory (Linde, 1974; Guth, 1981), according to which $n_s \sim 0.96$, a value indeed closely matching the one measured with the CMB. The primordial power spectrum is said to be “processed” by the gravitational interaction throughout the LSS evolution, both in time and across spatial scales: the time evolution is encoded in the growth factor $D(z)$ we defined above, whilst specific scale-dependent deviations from the primordial spectrum are a function of whether a given scale was within the causal horizon or outside of it at the time of the transition between a radiation-dominated Early Universe, when the dominant cosmic component (that is, the one having the largest density at the time, thus driving expansion) was radiation, to a matter-dominated Universe, where the largest density value is the one of the CDM fluid (the equality epoch). Generally speaking, scales which were super-horizon at the equality epoch get preserved in their original spectral form, whereas sub-horizon scales get “processed” and modified. A further physical phenomenon adding a distinctive signature to sub-horizon modes and instead due to the coupling of baryon matter and radiation until the CMB formation epoch at $z \sim 1100$ are Baryon Acoustic Oscillations (BAO) (Bassett & Hlozek, 2010; Dodelson & Schmidt, 2020), associated to the linear regime at scales $k \sim 150 \text{ Mpc}^{-1}$. All these effects can be condensed in terms of a “transfer function” $T(k)$ acting differently according to the scale, such that $\delta_{z=0}(k) = T(k) \delta_{prim}(k)$. In conclusion, the linear power spectrum observed at an arbitrary redshift can be written in terms of the primordial power spectrum according to the equation

$$\langle \delta^*(\mathbf{k}, z) \delta(\mathbf{k}, z) \rangle = P(k, z) = D^2(z) T(k)^2 A_s \left(\frac{k}{k_*} \right)^{n_s-1}. \quad (1.30)$$

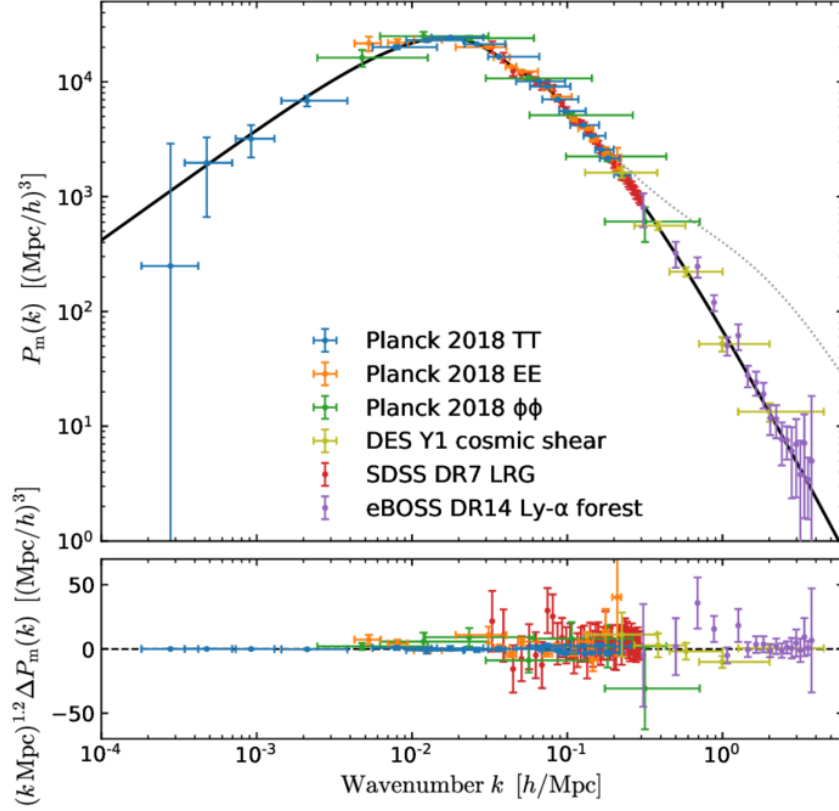


Figure 1.1: The LSS power spectrum prediction with superposed the main observational data capable of probing its various scales, from (Chabanier et al., 2019a)

The resulting distinctive shape of the power spectrum function, over which a number of observations capable of probing its several scale regimes are superposed, is displayed in fig. 1.1. In general, the expression reported in eq. 1.30 encloses a number of dependencies from fundamental parameters both in term of background quantities and of perturbative quantities, making the LSS summary statistics, and in particular its simplest and lower-order moment, the power spectrum, a fundamental and handy tool to measure cosmological parameters, complementary to direct measurements performed by means of standard candles or standard sirens, which instead address the evolution of the cosmic expansion at the background level.

A last important remark is related to the way we actually measure $P(k, z)$: strictly speaking, the quantity expressed in eq. 1.30 is connected to the variance of the dark matter field. This quantity can be directly measured only by means of gravitational effects, so that the sole suitable observables directly tracing Dark Matter are the strong and weak lensing effects and their statistics. Otherwise, the Dark Matter field is not directly measurable by definition, since it does not generate detectable phenomena other than gravitational interaction. We are thus obliged to access information on it by means of light-emitting tracers (see again fig. 1.1 for further examples): the most immediate tracer for this aim is represented by galaxies, that are abundant, relatively easy to detect even at moderately high redshifts and whose properties are well known. As already mentioned in the [Introduction](#), most of the great expectations in the field of Cosmology for the coming years reside indeed in the opportunity we will have to gain access to more tracers (X-ray surveys, Intensity mapping of gas in galaxies and in the LSS, voids...), all detectable with a greater observational quality, probing different LSS scales and regimes as well as different redshift intervals, allowing us

to study in detail a greater number of the Universe features and history. In addition, this means that we can extend our statistical tools: we are indeed not limited to measuring the variance of a single field, a procedure providing the “autocorrelation” power spectrum for the tracer under investigation, but also the covariance between two distinct tracers i and j (e.g., galaxies and weak lensing), in which case we are measuring the cross-correlation power spectrum, formally defined as the following correlator:

$$\langle \delta_i^*(\mathbf{k}, z) \delta_j(\mathbf{k}, z) \rangle = P_{ij}(k, z). \quad (1.31)$$

The cross-correlation is a powerful statistical method, allowing the simultaneous study of different combinations of variables. Indeed, next to the benefit of augmenting the data at our disposal, we usually assume that most of the observational systematics are specific of every field and independent among each other. Consequently, whilst the autocorrelation is always burdened by such systematics, when we cross-correlate two fields their systematic errors either enter with just half the power of the autocorrelation case or are completely eliminated because they do not correlate with the other observed field (Alimi et al., 1988; Blanchard, 2022). Also in this work cross-correlations will be a recurring topic: they will be explored at a conceptual level in chapters 2 and 4 for what concerns their use respectively in astroparticle physics and in cosmology, and, most important, in terms of direct applications in the discussion of the results exposed in chapters 3 and 5.

Bias factors

Although fundamental observables for probing the matter field, all cosmological tracers but weak lensing, and galaxies in particular, are known to form in regions that are typically much more dense compared to the matter density contrasts at a given scale (Kaiser, 1984), as if they were “overdensities forming on overdensities”. The quantity acting as a rescaling between the underlying matter density field and the observed tracer field is known as “bias factor”: in the simplest, linear framework, this quantity can be assumed to evolve in redshift but not to be a function of position in space neither to depend on other variables. We shall hereafter define it the “linear bias” term, b_1 , since, in the most general, non-linear theory, this can always be mapped to the first term of a perturbative expansion. Limiting the discussion to the number density field of galaxies, $n_g(\mathbf{x}, z)$, the link between the galaxy density contrast and the matter density contrast can be stated, at the linear level, in this form:

$$\delta_g(\mathbf{x}, z) \equiv \frac{n_g(\mathbf{x}, z)}{\bar{n}_g(z)} - 1 = b_1 \delta(\mathbf{x}, z) = b_1 \left(\frac{\rho_m(\mathbf{x}, z)}{\bar{\rho}(z)} - 1 \right). \quad (1.32)$$

In linear theory, the biased power spectrum between two tracers i, j , requires that eq. 1.30 gets thus modified:

$$P_{ij}(k, z) = b_i(z) b_j(z) D^2(z) T(k)^2 A_s \left(\frac{k}{k_*} \right)^{n_s-1}. \quad (1.33)$$

This of course translates into an effort towards appropriate modeling of the bias for each cosmological tracer, either at the linear level or at higher orders – as we shall discuss in sec. 1.3.2.

The fundamental parameters of the Λ CDM Cosmological Model

The theory outlined above allows us also to present and discuss the complete set of free cosmological parameters of the Λ CDM model and the main strategies aimed at their measurement. The Fridman equations, and hence any measurement at the level of cosmological background, such as space

Parameter	<i>Planck</i> TT,TE,EE+lowE+lensing	+BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.1193 ± 0.0009
$100, \theta_{\text{MC}}$	1.0409 ± 0.0003	1.0410 ± 0.0003
n_s	0.965 ± 0.004	0.966 ± 0.004
τ	0.054 ± 0.007	0.056 ± 0.007
$\ln(10^{10} \Delta_{\mathcal{R}}^2)$	3.044 ± 0.014	3.047 ± 0.014
h	0.674 ± 0.005	0.677 ± 0.004
σ_8	0.811 ± 0.006	0.810 ± 0.006
Ω_m	0.315 ± 0.007	0.311 ± 0.006
Ω_Λ	0.685 ± 0.007	0.689 ± 0.006

Table 1.1: The fundamental parameters of the Standard Cosmological Model Λ CDM as presented in (Lahav & Liddle, 2024). The central column displays their best-fit value when exclusively CMB data are used – namely, temperature maps (T), E-modes polarisation maps (E), and CMB lensing. The right column shows the best-fit results once also Baryon Acoustic Oscillation (BAO) data from Large Scale Structure observations are included. The first six quantities are the actual parameters fitted on CMB data, the last four below the black line are the more immediate fundamental parameters that can be recovered by rearranging the previous six above.

and time intervals, display sensitivity exclusively to the Hubble constant H_0 and to the density parameters, while the perturbative sector of the theory introduces three additional free parameters. Overall, the reference cosmological observable sensitive to the full set of Λ CDM parameters (or to combinations thereof) is the CMB: fitting the angular power spectrum of its temperature fluctuations is equivalent to putting a constraint on the whole Λ CDM model. The reference values are thus those obtained by the Planck satellite measuring the Cosmic Microwave Background and whose last and definitive data release published in 2018 (Aghanim et al., 2020), is summarised in table 1.1. The table allows us to recognise four sub-categories for the parameters:

- **Parameters measuring the matter-energy content of the Universe:** the already mentioned density parameters Ω_i , distinguished in Cold Dark Matter density Ω_c and baryon matter density Ω_b or combined in the total (cold, in that both act as a pressureless field) matter density Ω_m , degenerate with the dimensionless expansion rate h , i.e., the Hubble constant H_0 divided by $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- **Parameters determining the features of the spectrum of primordial perturbations and related to the inflationary paradigm:** namely, the spectral index n_s , setting the slope of the primordial power spectrum of matter density perturbation and $\ln(10^{10} \Delta_{\mathcal{R}}^2) = A_s$, setting instead the amplitude normalisation of the primordial power spectrum.
- **The parameter τ , setting the optical thickness of the Universe at Cosmic Reionisation:** Cosmic Reionisation is the only event subsequent to the CMB formation in the cosmic history during which CMB photons could have been absorbed at a high efficiency, determining a significant damping of the CMB signal we observe with respect to the one emitted at the source.

- **The parameter $100\theta_{\text{MC}}$ setting the angular size of the acoustic horizon at CMB formation:** this parameter determines the angular size of the BAO and indirectly probes the Hubble constant H_0 or its rescaled version h .

Given these quantities and under the assumptions of a flat Universe ($\Omega_K = 0$), of massless neutrinos ($\Sigma m_\nu = 0$) and setting (without fitting) a small, although non-zero, electromagnetic radiation density $\Omega_r \sim 10^{-4}$, one can rearrange the degenerate or less immediate parameters as measured from the CMB in terms of the more directly interpretable quantities displayed in the lower half of table 1.1, completing the full set of the free cosmological parameters $\{\Omega_b, \Omega_c, n_s, \sigma_8, H_0, \tau\}$ thus recovering the remaining ones:

- The rescaled Hubble parameter h .
- The variance of the cosmic perturbations σ_8 .
- The total matter density parameter (baryons+Dark Matter) Ω_m .
- The dark energy density parameter Ω_Λ .

Introducing the σ_8 and S_8 parameters and how they are measured The review of the Λ CDM parameters exposed above allows us to provide more detail concerning the quantities subject of the research work discussed in Part II, the parameter σ_8 and its closely related kin S_8 . The parameter can be interpreted as a reformulation of the power spectrum amplitude normalisation. We have seen that the power spectrum summarises in practice the value of the variance of a given cosmological field broken down scale-by-scale: it could be useful to further compress the variance of the matter field in a single number with immediate physical understanding, such as the variance of the field at a specific, typical size for a structure in the Universe. Following the exact normalisation definition by the Planck collaboration (Planck Collaboration et al., 2014a), one considers the **linear** power spectrum computed for the total matter field at $z = 0$ including baryons + neutrinos + dark matter, $P_{L,M}$ (other works might apply slightly different definitions (Madhavacheril, 2025)) and evaluates its variance filtered on a reference size corresponding to a sphere with radius $R = 8h^{-1}$ Mpc, about or slightly above the typical radius of galaxy clusters:

$$\sigma_8^2 \equiv \frac{1}{2\pi^2} \int_0^\infty k^2 P_{L,M}(k) W^2(kR) dk, \quad (1.34)$$

where $W(kR)$ represents the Fourier transform of a spherical top-hat window function of radius R

$$W(\mathbf{x}, R) = \frac{3}{4\pi R^3} \begin{cases} 1 & \text{for } |\mathbf{x}| \leq R \\ 0 & \text{for } |\mathbf{x}| > R \end{cases} \quad (1.35)$$

such that

$$W(kR) = \frac{3}{(kR)^3} [\sin(kR) - (kR) \cos(kR)], \quad (1.36)$$

a quantity also known as spherical Bessel function of order 1 (Planck Collaboration et al., 2014a). Furthermore, this estimate of the level of LSS “clumpiness” enforces a normalisation for the matter power spectrum. We can thus fit our preferred model of the power spectrum as measured through a certain tracer and then calculate the integral of eq. 1.34, retrieving an estimate for this parameter. Given the expression of $P(k)$ as a function of the growth factor $D(z)$ (as we can see from eq. 1.26,

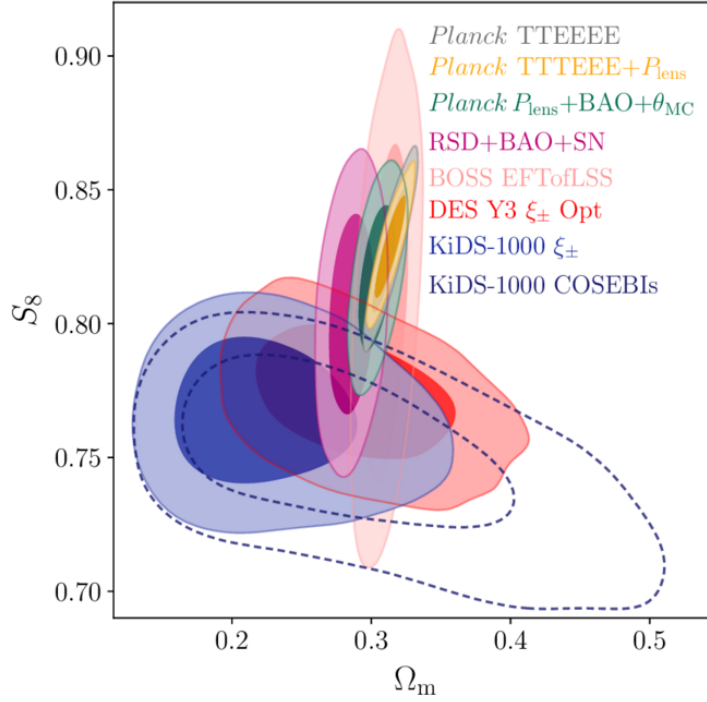


Figure 1.2: $S_8 - \Omega_m$ joint posterior profiles as probed by different surveys and techniques, from (Amon & Efstathiou, 2022)

where Ω_m enters at least twice, in the pre-factor and in $E(z)$), σ_8 is strongly correlated with Ω_m : it is thus customary to define a partially decoupled parameter S_8 , defined as follows:

$$S_8 \equiv \sigma_8 \left(\frac{\Omega_m}{0.3} \right)^\alpha, \quad (1.37)$$

α being a free index typically set to 0.5, a position that substantially removes the dependence from the $D(z)$ prefactor (fig. 1.2). Given the small scale of the filtering radius, in order to properly measure S_8 some requirements to perform accurate measurements have to be met: typically, better constraints on the parameter can be put by looking at small, mildly or fully non-linear scales; at the same time, some tracers are more or less statistically constraining according to the redshift we consider, depending on their evolutionary stages and where their abundances or kernels peak – some tracers might be even be absent at some redshifts. Therefore, also in an optic of tomographic analyses, where the dataset is partitioned in a number of bins referred to different redshifts, in order to potentially constrain the time evolution of some parameters (see sec. 1.4 and chapter 5), we might contemplate distinct tracers with specific requirements. The main ones we can quote (Madhavacheril, 2025; Hanson et al., 2010; Lewis & Challinor, 2006) (well summarised in fig. 1.3, where they are plotted as a function of the scale along the x -axis and of the redshift where they can be detected or are more effective on the y -axis), whose observational evidence will be reviewed in chapter 4 and, most important, in chapter 5, are thus:

- **The Cosmic Microwave Background**, whose temperature fluctuation and polarisation power spectra can be fitted up to very high multipoles ℓ , up to $\ell_{max} \sim 1500 - 2500$ depending on whether we only use Planck, attaining $\ell = 1500$ (Aghanim et al., 2020), or if we also include ground-based experiments like ACT, capable of measuring the smallest scales (Louis et al., 2025). The primary, *unlensed* signal is chiefly probing the very early stages and largest

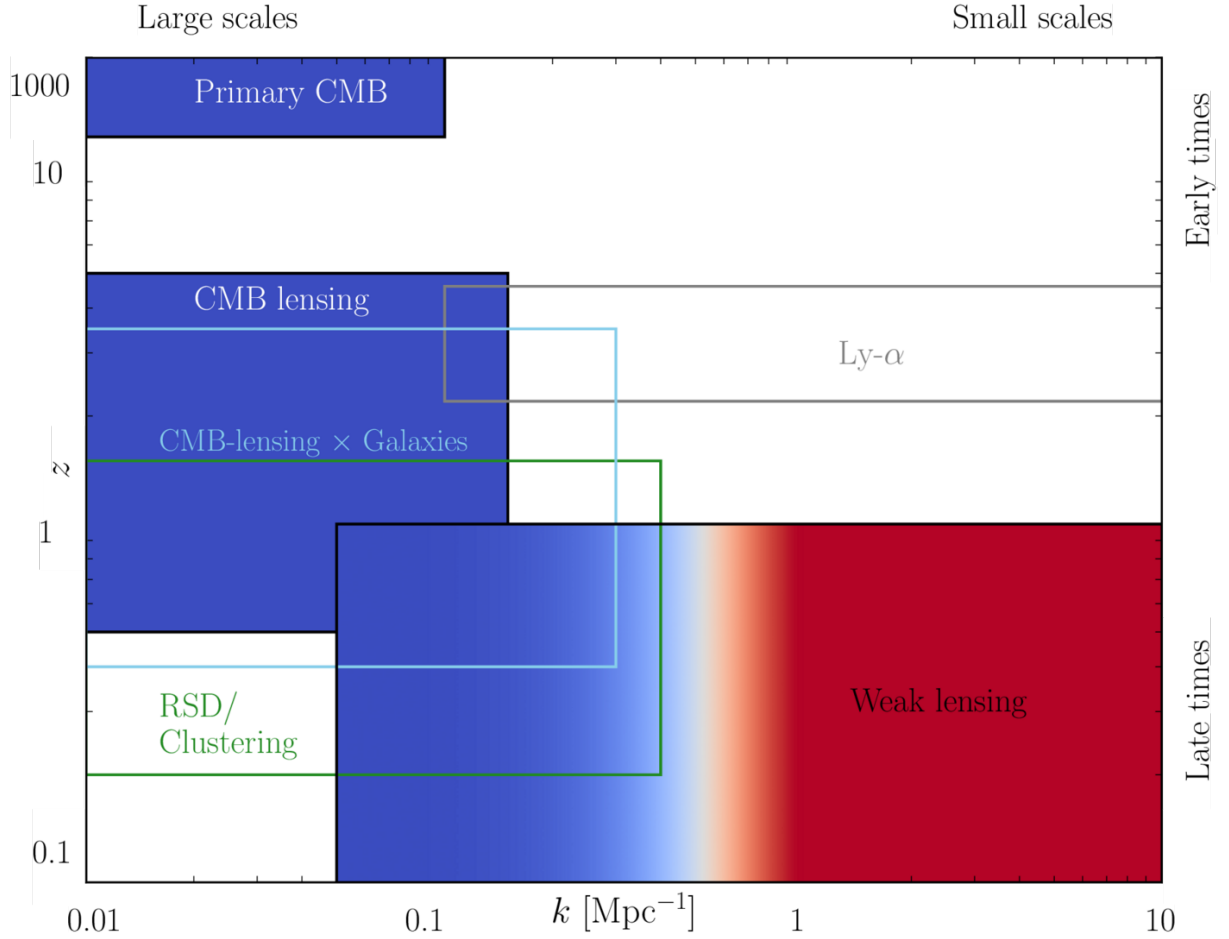


Figure 1.3: Breakdown of the sensitivity of cosmological probes to σ_8 or S_8 in terms of scales and redshifts from (Preston et al., 2023)

scales of linear perturbation growth, whereas the *lensed* CMB, due to its interaction with the LSS evolution through many cosmic epochs and scales is among the most relevant tracers for the task.

- **Strong lensing and cosmic shear-weak lensing** measurements, directly accessing the dark matter field. They are a well-established techniques as long as they involved the measurement of image distortions by means of photon messengers. An innovative pathway which is starting to receive attention is the development of analogous techniques based instead on gravitational waves as messengers. Several methodologies have been proposed, all relying on the fact that certain physical phenomena, e.g. mergers of compact objects like black holes and neutron stars, are characterised by standardised emission intensity of the gravitational signal (“standard sirens”) in a way similar to SN Ia acting as standard candles. The gravitational emission, especially in combination with an electromagnetic signal, can thus be used in order to infer the distance and the redshifts of the emitters and leveraged as an LSS tracer (Congedo & Taylor, 2019; Vaskonen, 2026; Palmese & Kim, 2021).
- **Galaxy clustering signals**, especially at the smallest scales, provided we are able to suitably model this regime through one of the techniques we will be outlining in sec. 1.3.
- **Higher-order perturbative effects, such as Redshift Space Distortions (RSD)** (Kaiser, 1987; Tsagas et al., 2025). They are probes targeting peculiar velocities of galaxies

in the LSS gravitational potentials. They represent the first intuitive example of non-linear effect, rather easy to measure and to model and, given the current precision standards of instruments, far from being a subdominant contribution. Thus, due to a combination of necessity and utility, the measurement of the velocity field is a rather customary result, always included in cosmological surveys next to the density field estimates and the associated RSD non-linear correction included in the power spectra also when the studies are limited to the linear regime. The RSD contribution can be effectively thought of as a correction to the linear bias term:

$$b_{\text{RSD}}(z) = b_{\text{lin}}(z) \left(1 + \frac{f(z)}{b_{\text{lin}}(z)} \cos^2 \theta \right), \quad (1.38)$$

where $f(z)$ is the growth function, that is, the logarithmic derivative of the growth factor with respect to the scale factor

$$f(z) = \frac{d \ln D(z)}{d \ln(a)}. \quad (1.39)$$

Since the usual normalisation condition for $D(z = 0) = 1$ corresponds to the rescaling $\sigma_8(z) = \sigma_8 D(z)/D(0)$, the dependence of $f(z)$ on $D(z)$ also translates into a dependence on σ_8 : if we measure the usual power spectrum of density fluctuations, corrected by the RSD, we have a more precise summary statistics of the matter field, but σ_8 is still degenerate with the bias factors. If, instead of measuring the density fluctuation power spectrum, one just measures the *velocity* field power spectrum, we have that $\delta_v \propto b(z) \frac{f(z)}{b(z)} \cos^2 \theta = f(z) \cos^2(\theta)$ (Tsagas et al., 2025). In this expression, at the first order, σ_8 is just degenerate with $f(z)$, not with the bias factors –the quantity is often made explicit and written as $\sigma_8 f(z)$ – and we can thus decouple the measurement of the parameter from the bias terms, allowing for a more direct measurement of σ_8 .

- **Baryonic probes:** the thermal Sunyaev-Zel’dovich effect measured on the CMB (Louis et al., 2025) caused by the scattering of CMB photons on relativistic electrons located in the LSS, in particular in galaxy clusters, is sensitive to σ_8 because cluster abundance is highly sensitive to the amplitude of matter fluctuations. The kinetic Sunyaev-Zel’dovich effect, probing peculiar velocities of the ionised electrons can be used as well (Calafut et al., 2021). Lyman- α measurements, probing several properties of the interstellar medium of objects along the line of sight and at different redshifts by means of its capability of absorbing photons emitted by quasars at high redshift (Rogers & Poulin, 2025), are a peculiar probe, because they access at the same time high redshifts (above $z \sim 3$) and small scales along the line of sight, displaying sensitivity to σ_8 and requiring a simpler modelling of a less non-linear LSS regime, given the early stage of its development they probe. Measurements of σ_8 derived from this probe provided mixed results, spanning from full agreement with Planck to tension and preference for alternative models (Rogers & Poulin, 2025; Chabanier et al., 2019b; Palanque-Delabrouille et al., 2020). More recently, σ_8 measurements based on X-Ray detection of clusters performed by state-of-the-art instruments such as eROSITA have been published (Ghirardini et al., 2024). Eventually, the use of radio observations with HI Intensity Mapping techniques have also been suggested and forecast based on the future performances of the SKAO radiotelescope have been made available in the literature (Bull, 2016; Dlamini et al., 2024; Berti et al., 2023; Cunningham et al., 2025).

1.3 Dealing with non-linearities in the perturbative regime

Another important theoretical element we require in this work is a discussion of the non-linear regime of the matter field and of the non-linear description of its biased tracers. The matter field and, generically, the LSS, can be assumed to enter the non-linear regime around scales of order $k \geq 0.1 \text{ Mpc}^{-1}$. At those scales, the dynamics of the LSS becomes increasingly more complex: on one hand, by the sheer effect of structure growth and gravitational interactions, the density contrast of the underlying matter field starts to become comparable if not much larger than 1, while different spatial scales start to be coupled. These phenomena invalidate the simplest assumptions underlying the linear perturbative theory and its predictions about the growth and value of the LSS summary statistics. On the other hand, the evolution of the LSS tracers themselves gets more complicated, and their degree of biasing with respect to the underlying (already inherently non-linear) matter field density contrast also evolves in a non-linear fashion. We therefore no longer expect the biasing to be properly captured by the linear bias term b_1 .

In the two research topics we will discuss, two distinct methodologies will be utilised in order to deal with non-linearity. For the astroparticle topics discussed in chapter 3, the properties of the window functions of the tracers under scrutiny, peaking at low redshift, and the interest in probing intermediate-to-small scale Dark Matter halos require the use of a non-linear clustering formalism known as the halo model. This formalism is better suited to describe very small scales and strongly non-linear regimes, with the additional benefit of exactly recovering by construction the linear regime at the largest scales, while showing instead some accuracy limitations in the transition between the two regimes (Cooray & Sheth, 2002; Asgari et al., 2023). For the sake of the S_8 and structure growth measurement, instead, we need again to probe small scales because they are the most sensitive to the parameters of interest, as discussed in section 1.2.1, combined with the appropriate description of non-linear contributions to the biased clustering of the low-redshift galaxy samples we will use. This time, the Lagrangian Perturbation Theory (LPT) formalism, associated to a series expansion of the density contrast up to the second order and a related non-linear galaxy bias description will be leveraged, capable of consistently covering scales as small as $k \sim 0.42 \text{ Mpc}^{-1}$ (Desjacques et al., 2018; Matsubara, 2008; Zennaro et al., 2023).

1.3.1 The halo model formalism

The halo model formalism (Scherrer & Bertschinger, 1991; Seljak, 2000; Ma & Fry, 2000; Peacock & Smith, 2000; Cooray & Sheth, 2002; Asgari et al., 2023) tackles the limitations of linear perturbation theory and allows to describe density contrasts up to $\mathcal{O}(100)$: conceptually, it is an almost “modular” framework relying on a number of building blocks, also associated to many numerical implementations such as `halomod` and its online applet counterpart `TheHaloMod`[‡] (Murray et al., 2013; Murray, 2014; Murray et al., 2021), and the suites `Colossus` (Diemer, 2018) and `CCL` (Chisari et al., 2019a), the latter having been the fundamental computational tool for this thesis. At the same time, its predictions can be suitably tested both on small-scale tracers of the observed cosmic web as well as on numerical simulations, that have been long since capable to probe highly non-linear clustering regimes of the LSS.

The basic idea behind the halo model description is that all matter in the Universe can be assumed to exclusively reside in Dark Matter halos of different sizes and mass, excluding voids

[‡]Available at [this link](#).

and other observed structures. We will show that the calculation of halo model power spectra may be loosely traced back to the calculation of mass integrals (or integrals of some quantity, such as luminosities, to be associated to the mass of the host dark matter halos by means of suitable scaling laws): the choice of the integration extremes, especially the lower one, can therefore be considered a free parameter of the problem or a decision motivated on theoretical grounds. It is important to remember that not all matter in the Universe is actually hosted in halos, although the statement is sufficiently valid for most current applications: an effort towards greater completeness is being nevertheless performed by the community and voids, although hosting very little matter, can be included in the description (Voivodic et al., 2020; Arcari et al., 2022), as well as structures far from the dynamic equilibrium of virialised halos, such as filaments, sheets and walls (Brieden et al., 2025). The description of a tracer can thus be connected back to the properties and distribution of the hosting halos, and will require three main ingredients:

- A functional form for the number density per unit mass and redshift of the dark matter halos – their halo mass function (HMF).
- A functional form for the dark matter halo bias (notice that *halos are themselves biased* against the background matter field, they need to be more strongly clustered than matter, to form), again as a function of mass and redshift,
- A functional form for their density profile (usually assumed to be the standard Navarro-Frenk-White, NFW density profile (Navarro et al., 1996; Navarro et al., 1997)) as a function of redshift, mass and spatial scale and usually under the assumption of a spherical symmetry for the object.

The power spectra can be then derived under the assumption that they are the summation of two terms, respectively known as 1-halo (P_{1h}) term and 2-halo (P_{2h}) term:

$$P(k, z) = P_{1h}(k, z) + P_{2h}(k, z). \quad (1.40)$$

The two contributions can be better understood by recalling the definition of the power spectrum as the Fourier transform of the two-point correlation function of two fields evaluated at positions $\mathbf{x}$ and $\mathbf{x} + \mathbf{r}$.

The 1-halo term represents the contribution to the field auto-correlation (or fields cross-correlation, if we are calculating a cross-correlation power spectrum) coming from objects (either of the same type of tracer if we are calculating an autocorrelation power spectrum, or of two distinct tracer categories if we are evaluating a cross-correlation power spectrum) residing in the same halo, i.e., at a separation $\mathbf{r}$ smaller than the size of the host halo. The 2-halo term represents instead contributions coming from objects residing in two distinct halos, hence at a separation $\mathbf{r}$ larger than the size of the host halos (fig. 1.4). The 1-halo contribution is expected to be dominant at the smallest scales and represents the true non-linear correction to the linear power spectrum, in that it enhances local correlations of objects residing in the same environment, not captured by the linear theory. At the largest scales, this contribution translates in a subdominant shot-noise contribution, representing the fact that objects residing in the same very large halo but at great distance are substantially randomly scattered: although residing in the same host halo, if they are very far apart, local effects no longer induce correlations. At the largest scales we are instead left with the dominant contribution of the 2-halo term, matching the linear power spectrum prediction: this second term provides a measure of the degree of correlation between distinct halos induced by

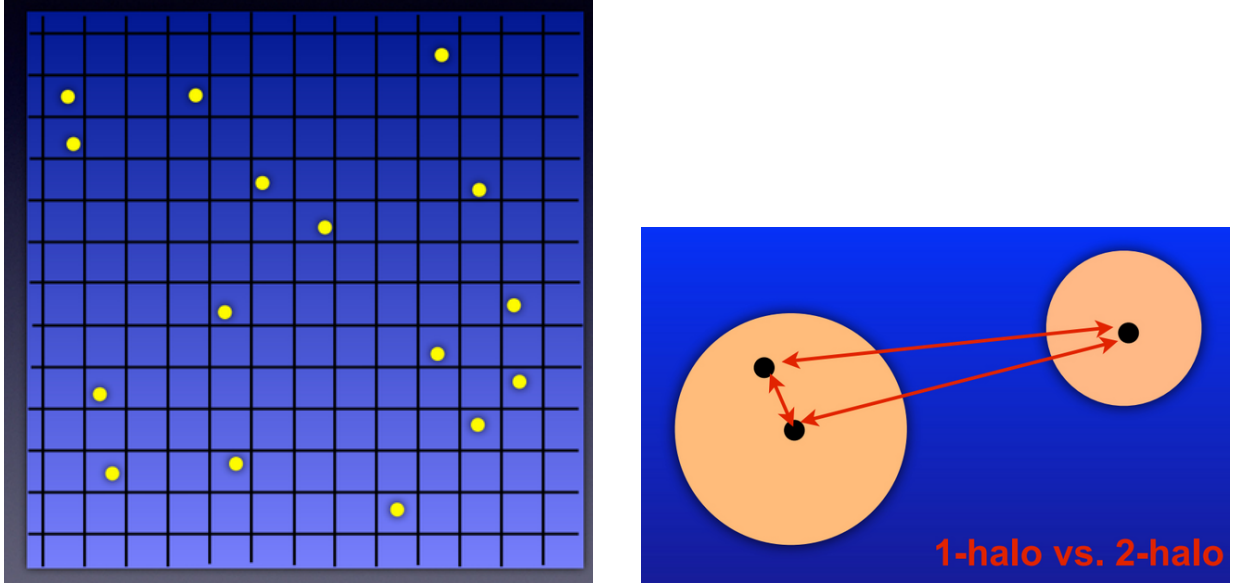


Figure 1.4: The geometric intuition behind the halo model (Neyman et al., 1953; Mo et al., 2010): the matter field is partitioned in a grid (left), and each cell in the grid can host one or none Dark Matter halos center (yellow dots). Halos extend on one or more cells depending on their radius size and each halo is subsequently populated of objects: if objects reside in the same halo, with a separation smaller then the halo radius, they will contribute to the one-halo term. If instead we correlate two objects at a separation larger then the size of a given halo, we are computing a two-halo contribution to the power spectrum. The pictures are taken from the lecture notes of Frank van den Bosch (Yale University) available [at this link](#) .

the fact that they are probing different peaks or troughs of the same long-wavelength perturbation mode.

We can derive the expression of eq. 1.40 for the dark matter field by describing each halo as an independent, discrete mass seed, as we are about to outline (Fornengo & Regis, 2014): the calculations can be developed starting from the ratio $f(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\bar{\rho}}$ between the density profile of the dark matter halos ($\rho(\mathbf{x})$) normalised to the $\bar{\rho}(z=0)$ value defined as the density today of Dark Matter *and* baryons together.

Starting from the density contrast field and by associating each a -th seed with a different mass m_a and position $\mathbf{x}_a$, we can state that the total density contrast (leaving redshift or time dependence implicit) reads:

$$f(\mathbf{x}) = \sum_a f(m_a, \mathbf{x} - \mathbf{x}_a) = \int dm \int d^3\mathbf{x}' \sum_a \delta_D^3(\mathbf{x}' - \mathbf{x}_a) \delta_D(m - m_a) f(m_a, \mathbf{x}_a - \mathbf{x}') \quad (1.41)$$

(notice that the seeds are discrete in number, so we use the symbol of summation over a , but the mass and position meshes are continuous, hence the position and mass integrals and the use of Dirac delta functions). We introduce the statistical distribution of the seeds in terms of their number density per unit mass, the halo mass function $dn/dm = \langle \sum_a \delta_D^3(\mathbf{x} - \mathbf{x}_a) \delta_D(m - m_a) \rangle$, and compute the real-space 2-point correlation function of the field as

$$\xi^{(2)}(\mathbf{x}, \mathbf{y}) = \int dm_1 dm_2 d^3 \mathbf{x}_1 d^3 \mathbf{x}_2 \left\langle \sum_a \delta^3(\mathbf{x}_1 - \mathbf{x}_a) \delta(m_1 - m_a) \sum_b \delta^3(\mathbf{x}_2 - \mathbf{x}_b) \delta(m_2 - m_b) \right\rangle \times f_1(m_1, \mathbf{x} - \mathbf{x}_1) f_2(m_2, \mathbf{y} - \mathbf{x}_2). \quad (1.42)$$

If we now plug the halo mass functions of the two fields (ξ_s is the two-point correlation function between two field seeds, opposed to ξ being the two-point correlation function of the two fields), and using the fact that

$$\left\langle \sum_a \delta_D^3(\mathbf{x}_1 - \mathbf{x}_a) \delta_D(m_1 - m_a) \sum_b \delta^3(\mathbf{x}_2 - \mathbf{x}_b) \delta_D(m_2 - m_b) \right\rangle = \frac{dn}{dm_1} \frac{dn}{dm_2} \left[1 + \xi_s^{(2)}(m_1, m_2, \mathbf{x}_1, \mathbf{x}_2) \right] + \frac{dn}{dm_1} \delta_D^3(\mathbf{x}_1 - \mathbf{x}_2) \delta(m_1 - m_2), \quad (1.43)$$

we can introduce eq. 1.43 into eq. 1.42, finding

$$\xi^{(2)}(\mathbf{x}, \mathbf{y}) = \int dm d^3 \mathbf{x}_1 \frac{dn}{dm} f_1(\mathbf{x} - \mathbf{x}_1, m) f_2(\mathbf{y} - \mathbf{x}_1, m) + \int dm_1 dm_2 d^3 \mathbf{x}_1 d^3 \mathbf{x}_2 \frac{dn}{dm_1} \frac{dn}{dm_2} f_1(\mathbf{x} - \mathbf{x}_1, m_1) f_2(\mathbf{y} - \mathbf{x}_2, m_2) \xi_s^{(2)}(m_1, m_2, \mathbf{x}_1, \mathbf{x}_2). \quad (1.44)$$

We can now take the Fourier transform of the correlation function in order to recover the three dimensional power spectrum P_{ij} :

$$P_{ij}(k) = \int dm \frac{dn}{dm} f_i^*(k|m) f_j(k|m) + \int dm_1 dm_2 \frac{dn}{dm_1} \frac{dn}{dm_2} f_i^*(k|m_1) f_j(k|m_2) P^{(s)}(k, m_1, m_2), \quad (1.45)$$

with $P^{(s)}$ being defined as the Fourier transform of ξ_s . We can now recognise the 1-halo and 2-halo contributions to the halo model power spectrum of eq. 1.40 in the two terms. Indeed, the first addendum is the 1-halo term:

$$P_{1h} = \int dm \frac{dn}{dm} f_i^*(k|m) f_j(k|m) \quad (1.46)$$

since, due to the above reasonings, the two fields in the integral were forced to be evaluated in the same position $\mathbf{x}$, that is, within the same halo, whereas the second term corresponds to the 2-halo term,

$$P_{2h} = \int dm_1 dm_2 \frac{dn}{dm_1} \frac{dn}{dm_2} f_i^*(k|m_1) f_j(k|m_2) P^{(s)}(k, m_1, m_2) \quad (1.47)$$

since we had instead to evaluate the position integrals in two distinct positions, i.e., in two distinct halos. We have now to treat the $P^{(s)}(k, m_1, m_2)$ term: if we assume that its mass dependence can be factored out in a product of bias terms, we are left with $P^{(s)}(k, m_1, m_2) = b_i(m_1) b_j(m_2) P^{\text{lin}}(k)$, where $P^{\text{lin}}(k)$ is the standard linear matter power spectrum predicted by linear perturbation theory as per eq. 1.30. Under the assumption of independence between terms being a function of m_1 and terms being a function of m_2 , we can rewrite the double integral of the 2-halo term as a product of independent integrals:

$$P_{2h}^{ij}(k) = \left[\int dm_1 \frac{dn}{dm_1} b_i(m_1) f_i(k|m_1) \right] \left[\int dm_2 \frac{dn}{dm_2} b_j(m_2) f_j(k|m_2) \right] P^{\text{lin}}(k) \quad (1.48)$$

Notice that the the following relations must hold:

$$\bar{\rho}(z=0) = \int dm \frac{dn(m, z)}{dm} \int d^3 \mathbf{x} \rho(\mathbf{x}, z, m) \quad (1.49)$$

where we reiterate that $\bar{\rho}(z=0)$ is the density today of Dark Matter *and* baryons together, and also

$$\bar{\rho} = \int_0^\infty m b(m) \frac{dn}{dm} dM; \quad (1.50)$$

this latter equality ensures that at large scales we can recover the linear matter power spectrum and, although analytically exact, it can be hard to recover computationally and many codes impose it by hand as a forced normalisation (Asgari et al., 2023). Once the non-linear halo model description of the matter field has been laid out, it is possible to implement the treatment of tracers in the same formalism as long as we are able to connect the features of the chosen tracers to the features of the host halos. For galaxies, the requirement is to define how galaxies occupy halos, including some corrections due to the discrete nature of the galaxy field opposed to the continuous features of the other tracers. This is usually obtained by specifying a halo occupation distribution function (HOD) that treats galaxies by dividing them into central and satellite galaxies and can be calibrated on simulations and observations; we observe in passing that galaxy clustering in the halo model completely avoids, or at least reformulates, the problem of describing the galaxy bias, only requiring the bias of Dark Matter halos, as it will become evident below. Other tracers require different treatments, but the framework allows to proceed in a systematic way that allows to include many more tracers, provided one is capable of capturing the often complex relationships connecting astrophysical and cosmological features of the observables under scrutiny.

Of particular relevance for this work, it is possible to describe in a similar way astrophysical sources contributing to the UGRB in terms of their luminosity functions instead of their masses (Camera et al., 2015; Tröster et al., 2017; Camera et al., 2015), but halo-model-based descriptions are also available for cosmic shear (Heymans et al., 2013; Joudaki et al., 2017), SZ effect (Komatsu & Seljak, 2001), X-Rays (Hurier et al., 2014; Ferreira et al., 2024), 21 cm and Lyman- α lines signals (Feng et al., 2017), or HI IM Intensity Mapping (Wolz et al., 2019), with different degrees of advancement; theoretical effort is however ongoing in order to refine their predictions.

Building a halo model formalism to describe the Unresolved Gamma Ray Background-galaxy clustering cross-correlations

We can now put the halo model formalism at work: we will here outline how it can be used in order to construct the three-dimensional power spectrum estimators that will be employed in chapter 3. The essential aspects of the physical and observational properties of the Unresolved Gamma Ray Background have been presented in the Introduction and further considerations will be exposed in chapter 2, especially 2.3.2, and in chapter 3. Overall, the ingredients required to set up the full galaxy-UGRB cross-correlation formalism are the following:

- **The characterisation of UGRB tracers**, in turn subdivided in two main source categories:
 - *Annihilating dark matter signal*: that can be directly treated in a “standard” halo model formalism, building up mass integrals starting from the features of the dark matter halos.
 - *Astrophysical sources*: misaligned AGN (mAGN), Blazars (BL Lac), Flat Spectrum Radio Quasars (FSRQ) and Star Forming Galaxies (SFG), whose power spectra can be constructed in a modified halo model framework requiring the conversion between mass and luminosity of the sources (Camera et al., 2015; Tröster et al., 2017; Fornengo & Regis, 2014).

- **The characterisation of galaxies:** whose discrete nature will be summarised by the definition of a specific HOD model tuned to the selected galaxy survey, 2MRS (Huchra et al., 2012; Ando et al., 2018).

Because of the intervening halo model building blocks, our theoretical formalism requires the analytic form of :

- Autocorrelation power spectra between each tracer (hence, e.g., galaxy-galaxy, annihilating DM-annihilating DM, SFG-SFG power spectra...).
- Cross-correlation power spectra between galaxy and UGRB tracers, such as galaxy-annihilating DM power spectra.
- Cross-correlation power spectra between the several DM and astrophysical sources of the UGRB, such as DM-SFG, SFG-mAGN power spectra, etc.

All the related formulas will be stated hereafter, together with considerations dealing with the building blocks of the tracers.

Shared model choices among all tracers

The following halo model building blocks (all also having an implicit dependence on redshift) will be employed consistently among all tracers when required (Cooray & Sheth, 2002; Chisari et al., 2019a; Pinetti, 2022):

Halo Mass Function:

introduced above at the “mass seed” level as $dn/dm = \langle \sum_a \delta_D^3(\mathbf{x} - \mathbf{x}_a) \delta_D(m - m_a) \rangle$, the general form for this quantity, expressing the number of halos per unit volume and mass at a given redshift, is:

$$\frac{dn}{dM}(M, z) = \frac{\bar{\rho}(z=0)}{M} f(\nu) \frac{d\nu}{dM}. \quad (1.51)$$

with $\bar{\rho}(z=0)$ being the total matter (baryons and Dark Matter) average background density today while ν represents the quantity $\nu = \frac{\delta_{sc}(z)^2}{\sigma(M, z)^2}$. This latter quantity estimates the ratio between the critical density contrast $\delta_{sc} = 1.686(1+z)$ at which structure formation onsets in the spherical collapse theory (Cooray & Sheth, 2002; Mo et al., 2010) and the variance of the matter field, usually estimated at $R \approx 8 \text{ Mpc } h^{-1}$ and thus related to σ_8 as defined in eq. 1.34. Given the general form of the HMF, specific prescriptions in the literature change the analytic expression of the so-called multiplicity function $f(\nu)$. This quantity defines the fraction of mass within halos in units of $d\ln(\nu)$, either based on theoretical considerations or calibrated on observations and simulations, with the normalisation prescription $\int d\nu f(\nu) = 1$. The selected model for our HMF functional form, and hence of the multiplicity function, is the one usually known as **Sheth-Tormen 1999** (Sheth & Tormen, 1999).

Halo bias model: regardless of the biasing of the observable tracers, dark matter halos are collapsed, non-linear structures: they are themselves biased tracers of the uniform underlying dark matter background. Their biasing is determined by the quantity $b_h(M)$ and the general theory treats the quantity as a functional correction to the assumption of unbiaseding, corresponding to $b_h = 1$, in terms of simple polynomial contributions expressed as a function of the powers of the above-defined ν quantity. In our specific case, the selected model is the one known as **Sheth-Tormen 2001** (Sheth et al., 2001).

Halo concentration model $c(M)$. The role of concentration is better understood after the full description of the dark matter halo density profiles is provided, so its meaning will be better outlined below (eq. 1.53). The purpose of these paragraphs is to provide a handy list of “ingredients” and we report we are using the one known as [Correa 2015](#) ([Correa et al., 2015](#)).

Halo density profile $\rho(\mathbf{x}, M)$: we choose the standard reference profile known as Navarro-Frenk-White (NFW) profile ([Navarro et al., 1996](#); [Navarro et al., 1997](#)), characterised by a simple model and a good degree of universality, defined as the property for the functional form of density profile to be consistently recovered when measuring the properties of dark matter halos at different redshifts, size and masses.

Importantly, the mass integrals are assumed to be *virial* mass integrals, that is, the mass we quote corresponds to the one enclosed in the structure region within the virial radius r_{vir} at a given redshift. This corresponds to the halo region for which we may apply the virial theorem to describe its dynamics. This is a standard result in astrophysics derived from the Newtonian description of gravity, descending from the assumption of hydrostatic equilibrium in a spherical gas distribution. Under these conditions, virial equilibrium implies that the following relation between the gas thermal energy K and its gravitational potential energy U holds: $2K + U = 0$ ([Binney & Tremaine, 1987, 2008](#)). Concerning the extremes of the mass integrals of eqs. 1.46 and 1.47, they will always be extended from $10^{-6}M_{\odot}$ to $10^{18}M_{\odot}$, in order to cover the mass range permitted by WIMP Dark Matter models from the smallest allowed mass for dark matter halos under free-streaming conditions, ([Fornengo & Regis, 2014](#); [Moliné et al., 2017](#); [Hofmann et al., 2001](#); [Berezinsky et al., 2003](#); [Green et al., 2004, 2005](#); [Bringmann & Hofmann, 2007](#); [Bringmann, 2009](#); [Berezinsky et al., 2014](#)), up to the mass of the most massive structures observed the Universe, between $10^{15}M_{\odot}$ and $10^{17}M_{\odot}$ ([Reisenegger et al., 2000](#); [Böhlinger et al., 2025](#)). Due the typical shapes of the Halo Mass Function, regardless of the specific model, the lower-mass cutoff for the halo model integral is a delicate topic: observed structures associated to halos, hence from galaxies upwards, do not attain masses below the value of about $10^6 M_{\odot}$, typically associated to dwarf spheroidal galaxies ([Collins et al., 2014](#)), so that our minimum mass might elicit realism concerns. In terms of Dark Matter halo modelling, free streaming arguments (see the [Introduction](#), and sec. 2.2.3) ensure us that the choice of $M_{\text{min}} = 10^{-6}M_{\odot}$ is motivated. When it comes to modelling galaxies and astrophysical sources, the chosen HOD formulations as well as the luminosity-mass conversions ensure that the integrals, as well as the cross-correlations with annihilating dark matter signals, do actually start at observationally motivated minimum mass values of about $10^6 M_{\odot}$. Concerning the maximum mass in the integrals, next to the observational upper limits discussed above, we remark that the halo model integrals do not show strong dependence from the maximum mass cut off due to the sharp decrease of the HMF at those values. Our choices are therefore such that our theoretical model is encompassing all available dark matter structures in the Universe.

Finally, the linear power spectrum entering the 2-halo terms is, in theoretical terms, the one derived by standard linear perturbation theory and corresponding to the one calculated by CCL through its CAMB wrapper function.

Annihilating Dark Matter

Dark Matter halos are defined as spherical dark matter overdensities having mass M and size defined by a virial radius $r_{\text{vir}} = [3M/4\pi\rho_c(z)\delta_c(z)]^{\frac{1}{3}}$, with $\rho_c(z)$ the Universe critical density at redshift z . The term $\delta_c(z)$ corresponds to the density contrast at which we can assume that a stable

halo forms: its analytic expression may vary based on different structure formation formalisms: here we adopt the **Bryan-Norman** formulation, $\delta_c(z) = 18\pi^2 - 82\Omega_\Lambda(z) - 39\Omega_\Lambda^2(z)$ (**Bryan & Norman, 1998**), where $\Omega_\Lambda(z)$ is the dark energy density parameter.

The halo density profile $\rho(r)$ is modeled as an NFW profile (**Navarro et al., 1996; Navarro et al., 1997**), normalized to the halo mass M inside its virial radius, reading:

$$\rho(r, M, z) = \rho_s \left(\frac{r_s}{r} \right) \left(1 + \frac{r}{r_s} \right)^{-2}, \quad (1.52)$$

depending on two free parameters, r_s and ρ_s . The parameter r_s known as the scale radius, defined as the halo region where its density starts to scale with a logarithmic slope of ~ -2 , is related to the universality of the NFW profile and can be obtained from the virial radius of the halo through the halo-mass-dependent concentration parameter. The concentration is a quantity calibrated on simulations or observations (**Diemer & Kravtsov, 2015; Coe, 2010**) through the expression

$$c_{\text{vir}} = r_{\text{vir}}/r_s. \quad (1.53)$$

The selected concentration model is the one of (**Correa et al., 2015**), that we adjust: since the original work defined halos by means of an overdensity of 200 with respect to the critical density instead of virial terms, we connect c_{200} to c_{vir} as: $c_{\text{vir}} = b + a c_{200}$, with $a = -1.119 \log_{10}(\delta_c) + 3.537$, $b = -0.967 \log_{10}(\delta_c) + 2.181$ (**Coe, 2010**). The normalisation ρ_s can be actually removed from the NFW formula by imposing that the mass of the halo is the mass enclosed between $r = 0$ and r_{vir} , namely: $M = \int_0^{r_{\text{vir}}} \rho_{\text{NFW}}(\mathbf{r}) d^3\mathbf{r}$, which leads to

$$\rho_s = \frac{M}{4\pi r_s^3} \left[\ln(1 + c_{\text{vir}}) - \frac{c_{\text{vir}}}{1 + c_{\text{vir}}} \right]^{-1}. \quad (1.54)$$

A further requirement specific to the description of annihilating dark matter halos is the clumping factor. This is introduced because, in this case, the gamma ray flux depends on the squared dark matter density. Given the general definition for the variance of a random variable x , $\text{Var}[x] = \langle x^2 \rangle - \langle x \rangle^2$ (angled brackets representing the averaging operation) the clumping factor measures how different the average density of the dark matter density field is from the average *density squared* of the same field:

$$\Delta^2(z) \equiv \frac{\langle \rho^2(z) \rangle}{\langle \rho(z) \rangle^2}, \quad (1.55)$$

where

$$\langle \rho^2(z) \rangle = \int dM \frac{dn}{dM}(z) \int d^3\mathbf{r} \rho^2(\mathbf{r}, M, z), \quad (1.56)$$

and $\langle \rho(z) \rangle$ is the same integral, this time calculated for the dark matter density. A remark on notation: $\langle \rho(z) \rangle$, in *angled brackets* exclusively refers to the average **dark matter** density and thus differs from $\bar{\rho}(z)$, with the *bar* superscript, that instead measures the average density of Dark Matter and baryons together. In the case of annihilating Dark Matter the contribution to the signal coming from substructures is relevant: they can be interpreted as a boost in the squared density profile obtained by setting the following modification: $\rho^2(\mathbf{r}, z, M) \rightarrow \rho^2(\mathbf{r}, z, M)[1 + B(M, z)]$; out of the many available models, we adopt the boost function $B(M, z)$ of Ref. (**Moliné et al., 2017**):

$$\log_{10} B(M, z) = \sum_{i=0}^5 b_i \left[\log_{10} \left(\frac{M(z)}{M_\odot} \right) \right], \quad (1.57)$$

which is scale-independent and thus not a function of the dark matter halo size, with $b_i = (-0.186, 0.144, -8.8 \cdot 10^{-3}, 1.13 \cdot 10^{-3}, -3.7 \cdot 10^{-5}, -2.0 \cdot 10^{-7})$, where the values in parenthesis run over $i = 0, \dots, 5$.

Galaxies

Galaxies, being discrete tracers instead of a continuous field, can be included in the halo model formalism in terms of a description based on their halo occupation distribution: that is, given their number density distribution as a function of their redshift $\bar{n}(z)$ and their number N_{gal} , we can associate them to the underlying matter field through the quantity $g(\mathbf{x}, M, z) = \rho(\mathbf{x}, M, z) \langle N_{\text{gal}}(M, z) \rangle / \bar{n}_{\text{gal}}(z)$, with the condition $\bar{n}_{\text{gal}} = \int dM dn/dM \langle N_{\text{gal}} \rangle$, ρ again being the host DM halo density profile.

N_{gal} is a model-dependent quantity: it is usually calibrated on simulations or on observations performed by specific surveys and distinguished according to the targeted galaxy populations (so that each major galaxy survey programme usually calibrates and publishes its own HOD (Yuan et al., 2022; Avila et al., 2020; Ando et al., 2018)) and usually it is formulated by separately modelling the central and the satellite galaxies contributions in each halo (Berlind & Weinberg, 2002; Zheng et al., 2005).

In full generality, HOD theory defines the average number density of galaxies $\langle n_g(r|M, z) \rangle$ at radial position r , in a halo of mass M and at redshift z , as follows:

$$\langle n_g(r|M, z) \rangle = \langle N_{\text{cen}}(M, z) \rangle f_{\text{cen}}(z) \delta_D(r) + \langle N_{\text{sat}}(M, z) \rangle u_{\text{sat}}(r|M, z). \quad (1.58)$$

where $\delta_D(r)$ is a Dirac delta function. A survey-specific model capable of predicting the average number of central galaxies $\langle N_{\text{cen}}(M, z) \rangle$ is required: it becomes thus a function of the fraction of central galaxies $f_{\text{cen}}(z)$, of the average number of satellite galaxies $\langle N_{\text{sat}}(M, z) \rangle$ and of the chosen model for the density profile for the halos hosting the satellite galaxies. $u_{\text{sat}}(r|M, z)$. Our analysis considers a low-redshift galaxy catalogue modeled on the characteristics of 2MRS (Huchra et al., 2012; Ando et al., 2014, 2018), whose parameterisation assumes $f_{\text{cen}}(z) = 1$, meaning that each halo hosts on average at most one central galaxy, independently of the redshift. The quantity $u_{\text{sat}}(r|M, z)$ follows an NFW density profile, as done for the halo dark matter profile discussed above in sec. 1.3.1, but with a concentration parameter c_{sat} which is rescaled from the halo one as: $c_{\text{sat}}(M, z) = b_m/b_g \cdot c_{\text{vir}}(M, z)$, with $b_m = 6.7$ e $b_g = 0.66$ (Ando et al., 2018). The specific chosen functional form for the 2MRS HOD derives from (Ando et al., 2018):

$$\langle N_{\text{cen}}(M, z) \rangle = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\log M(z) - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right], \quad (1.59)$$

$$\langle N_{\text{sat}}(M, z) \rangle = \left(\frac{M(z) - M_0}{M_1} \right)^\alpha \Theta(M(z) - M_0), \quad (1.60)$$

where masses are intended in units of solar masses and the free parameters read $\log_{10} M_{\text{min}} = 11.85$, $\log_{10} M_1 = 11.97$, $\sigma_{\log M} = 0.15$, $\alpha = 0.846$ and we have set $M_0 = M_{\text{min}}$.

We further have to specify a number of features of the 2MRS galaxy catalogue: its redshift interval, which we take as $z_{\text{min}} = 0.0012$ and $z_{\text{max}} = 0.1$, and the average number density of galaxies per unit volume $\bar{n}_g(z)$, related to the average number density of galaxies per unit redshift dN_g/dz as:

$$\bar{n}_g(z) = \frac{dN_g}{dV} = \frac{dN_g}{dz} \left[\frac{4\pi c}{H(z)} \frac{D_A^2(z)}{(1+z)^2} \right]^{-1} \quad (1.61)$$

where $D_A(z)$ is the angular diameter distance and $H(z)$ the Hubble rate. For $\bar{n}_g(z)$, we derive the parametrization from the $\frac{dN_g}{dz}$ parametrisation of (Ando et al., 2018):

$$\frac{dN_g}{dz} = \frac{N_g \beta}{z_0 \Gamma[(m+1)/\beta]} \left(\frac{z}{z_0} \right)^m \exp \left[- \left(\frac{z}{z_0} \right)^\beta \right] \quad (1.62)$$

Source class	Γ	$\mathcal{L}_{\min} [\text{erg s}^{-1}]$	$\mathcal{L}_{\max} [\text{erg s}^{-1}]$
BL Lacs	2.11	7×10^{43}	10^{52}
mAGN	2.37	10^{40}	10^{50}
FSRQ	2.44	10^{44}	10^{52}
SFG	2.7	10^{37}	10^{42}

Table 1.2: Table reporting the values for the spectral index Γ , for the minimum luminosity $\mathcal{L}_{\min}$ corresponding to the lower integration bounds in the halo model calculations, alongside with the maximal luminosity $\mathcal{L}_{\max}$, determining the upper bound of the halo model integrals, for the four astrophysical contributors to the unresolved gamma ray background under consideration.

with

- $N_g = 43182$, equal to the galaxies belongin to the 2MRS catalogue in the unmasked pixel.
- The remaining shape parameters read : $\beta = 1.64$, $z_0 = 0.0266$ and $m = 1.31$. The Γ symbol stands for the Gamma function.

Astrophysical objects

Based on extant literature (see for reference (Fornasa & Sánchez-Conde, 2015)), the main *astrophysical* contributions to the UGRB can be associated to four classes of objects: blazars (BLA), misaligned active galactic nuclei (mAGN), flat spectrum radio quasars (FSRQ) and star forming galaxies (SFG), which have been extensively recognised as guaranteed components of the UGRB since we know from observations of individually resolved objects that they are among the most abundant gamma-sources in the Universe, an insight then confirmed by fitting the total UGRB intensity spectrum as the outcome of the summation of the emission coming from unresolved objects belonging to those four classes of sources (Ando et al., 2007; Stecker & Venters, 2011; Fornasa & Sánchez-Conde, 2015; Negro, 2018; Xia et al., 2015).

It is more suitable to define their clustering in a halo model framework starting from luminosity integrals instead of mass integrals, in terms of the source luminosity $\mathcal{L}$, the gamma-ray luminosity function (GLF) $\phi(\mathcal{L}, z)$ and the spectral energy distribution (SED) dF/dE .

For UGRB sources, the luminosity $\mathcal{L}$ of each element is conventionally defined ad integrated over the energy interval (0.1, 100) GeV:

$$\mathcal{L} = \int_{0.1 \text{ GeV}}^{100 \text{ GeV}} dE_r \frac{dF}{dE_r} \quad (1.63)$$

where E_r is the photon energy in the rest frame of the source, that we can connect to the observed energy by $E_r = (1+z)E$, and dF/dE is the spectral energy distribution (SED) of the sources. Each SED is modelled as a power law with index Γ :

$$\frac{dF}{dE} = A(\mathcal{L}, z) E^{-\Gamma} \quad (1.64)$$

We report the values of Γ and E_{cut} for the four classes of astrophysical sources entering our model in table 1.2, alongside with the respective minimal and maximal luminosities $\mathcal{L}_{\min}$ and $\mathcal{L}_{\max}$. The amplitude $A(\mathcal{L}, z)$, instead, can be recovered by observing that (Hogg, 1999; Pinetti et al., 2025):

$$\mathcal{L} = \frac{4\pi d_L^2(z)}{(1+z)} \int_{0.1 \text{ GeV}}^{100 \text{ GeV}} dE_r E_r \frac{dF}{dE_r} = 4\pi d_L^2(z) \int_{0.1 \text{ GeV}/(1+z)}^{100 \text{ GeV}/(1+z)} dE E \frac{dF}{dE} \quad (1.65)$$

from which we obtain:

$$A(\mathcal{L}, z) = \frac{\mathcal{L}}{4\pi d_L(z)^2} \left[\int_{0.1 \text{ GeV}/(1+z)}^{100 \text{ GeV}/(1+z)} dE E^{(-\Gamma+1)} \right]^{-1} \quad (1.66)$$

Combining all the results, we obtain the following expression for the SED:

$$\frac{dF}{dE}(E, \mathcal{L}, z) = \frac{\mathcal{L}}{4\pi d_L^2(z)} \left\{ \frac{1}{2-\Gamma} \left[\left(\frac{100}{1+z} \right)^{2-\Gamma} - \left(\frac{0.1}{1+z} \right)^{2-\Gamma} \right] \right\}^{-1} \left(\frac{E}{\text{GeV}} \right)^{-\Gamma}. \quad (1.67)$$

Mass-to-luminosity relations The formulation of the halo model as a function of luminosity integrals requires to connect the luminosity of the source $\mathcal{L}$ and to the mass M of the halo that hosts the same source. The models we have followed are reported (Camera et al., 2015; Pinetti, 2022). The details of the calibration are extensively dealt with at particularly interesting detail level in (Camera et al., 2015), where further references on the literature and astrophysics of the modelled objects can also be suitably found.

BL Lac and Flat Spectrum Radio Quasars. BL Lacs and FSRQ have the same expression for the mass-to-luminosity ratio (Pinetti, 2022):

$$M(\mathcal{L}) = 10^{13} M_{\odot} \left(\frac{M_{\star}}{10^{8.8}(1+z)^{1.4}} \right)^{0.645} \quad (1.68)$$

with

$$M_{\star} = 10^9 \left(\frac{\mathcal{L}}{10^{48} \text{ erg/s}} \right)^{0.36} \quad (1.69)$$

Misaligned active galactic nuclei. For mAGNS, we adopt the following expression (Pinetti, 2022):

$$M(\mathcal{L}) = 10^{13} M_{\odot} \left(\frac{M_{\star}}{10^{8.8}(1+z)^{1.4}} \right)^{0.645} \quad (1.70)$$

with

$$M_{\star} = 4.6 \cdot 10^9 \left(\frac{\mathcal{L}}{10^{48} \text{ erg/s}} \right)^{0.16} \quad (1.71)$$

Star forming galaxies. For SFG we adopt the following relation (Pinetti, 2022):

$$M(\mathcal{L}) = \frac{10^{12} M_{\odot}}{(1+z)^{1.61}} \left(\frac{\mathcal{L}}{6.8 \cdot 10^{39} \text{ erg/s}} \right)^{0.92} \quad (1.72)$$

Gamma-ray Luminosity Functions The GLF $\phi(\mathcal{L}, z)$ defines the number density of astrophysical objects per unit luminosity, unit volume and unit spectral index:

$$\phi(\mathcal{L}, z, \Gamma) = \frac{dN}{d\mathcal{L} dV d\Gamma} \quad (1.73)$$

Again following (Pinetti, 2022), we list below the GLF of each source class we adopt in our analysis.

	A	$\mathcal{L}_\star$ [erg s ⁻¹]	γ_1	γ_2	p_1	p_2	$z_\star$	β_1	β_2	σ
BL Lacs	$9.20 \cdot 10^{-11}$	$2.43 \cdot 10^{48}$	1.12	3.71	4.50	-12.88	1.67	0.0604	0.0446	0.26
FSRQ	$3.06 \cdot 10^{-9}$	$0.84 \cdot 10^{48}$	0.21	1.58	7.35	-6.51	1.47	0.	0.21	0.18

Table 1.3: Summary table for the chosen values of the model parameters entering the gamma-ray luminosity functions for BL Lacertae and Flat-Spectrum Radio Quasars sources. Notice that the parameter A is expressed in Mpc⁻³dex⁻¹, while $\mathcal{L}_\star$ is in units of erg s⁻¹.

BL Lac and Flat Spectrum Radio Quasars. We adopt the luminosity-dependent density evolution (LDDE) model of (Ajello et al., 2014) for BL Lacs and (Ajello et al., 2012) for FSRQ, that further factors the function in a redshift-dependent term and a quantity calculated at redshift 0:

$$\phi_\gamma(\mathcal{L}, z, \Gamma) = \phi(\mathcal{L}, \Gamma) \times e(z, \mathcal{L}) \quad (1.74)$$

The redshift-independent term (Ajello et al., 2012, 2014) encompasses most of the physics of the astrophysical objects under scrutiny and is in turn parametrised as a broken power law in luminosity. Importantly at a conceptual level, we allow for a degree of variability in the photon spectral index, implying that the sources might not be exactly associated to a universal spectral index. The variability of spectral indexes within the classes of models is summarised by a Gaussian function, representing the probability distribution of the photon spectral index values, multiplying the GLF:

$$\phi(\mathcal{L}, \Gamma) = \frac{A}{\log(10)} \left(\frac{\mathcal{L}}{\text{erg s}^{-1}} \right)^{-1} \left[\left(\frac{\mathcal{L}}{\mathcal{L}_\star} \right)^{\gamma_1} + \left(\frac{\mathcal{L}}{\mathcal{L}_\star} \right)^{\gamma_2} \right]^{-1} \exp \left[-\frac{(\Gamma - \mu(\mathcal{L}))^2}{2\sigma^2} \right]. \quad (1.75)$$

Notice that we allow for the mean of the Gaussian distribution to be a function of physical parameters as well, effectively providing a theoretical model for the mean spectral index μ as a function of $\mathcal{L}$:

$$\mu(\mathcal{L}) = \mu^\star + \beta_1 \left[\log_{10} \left(\frac{\mathcal{L}}{\text{erg s}^{-1}} \right) - 46 \right] \quad (1.76)$$

The redshift-dependent term in the GLF entails the astrophysically interesting topic of redshift evolution of BL Lacertae and Flat-Spectrum Radio Quasars, exposed with greater detail in (Dunlop & Peacock, 1990; Ajello et al., 2012, 2014). The cosmic history of those objects can be encoded in the function $e(z, \mathcal{L})$ (Pinetti, 2022; Ajello et al., 2014, 2012):

$$e(z, \mathcal{L}) = \left[\left(\frac{1+z}{1+z_c(\mathcal{L})} \right)^{-p_1} + \left(\frac{1+z}{1+z_c(\mathcal{L})} \right)^{-p_2} \right]^{-1} \quad (1.77)$$

where $z_c = z_\star (\mathcal{L}/10^{48} \text{ erg/s})^{\beta_2}$.

The two classes of objects BL Lacs and FSRQs exhibit thus the same functional form for the GLF – they are indeed defined “sibling” astrophysical sources in (Ajello et al., 2014) – but the different values in the parameters they adopt, displayed in table 1.3, encompass their differences, translating also in different redshift distributions (see fig. 3.3 and sec. 3.1); in general, it is not clear whether the two classes represent two distinct classes of objects, or represent separate evolutionary stages of the same active galaxy typology, observed at different epochs of their “life” (Cavaliere & D’Elia, 2002; Ajello et al., 2014).

Our model treats the possible non-universality of the spectral indexes of BL Lacs and FSRQs without accounting for the full Gaussian probability distribution of eq. 1.75; rather, we set the

spectral index of each source class fixed at its best fit value reported in table 1.2; in other words, we assume that the probability distribution has a very narrow dispersion, approximated by a Dirac delta, allowing us to write the following substitution in eq. 1.75:

$$\exp \left[-\frac{(\Gamma' - \mu(\mathcal{L}))^2}{2\sigma^2} \right] \rightarrow \sqrt{2\pi}\sigma \delta_D(\Gamma' - \Gamma) \quad (1.78)$$

where Γ is reported in table 1.3 while $\sqrt{2\pi}\sigma$ is a prefactor defined to keep the normalization. An additional modification to the baseline models available in the literature derives from the slight rescaling in the normalizations of BL Lac and FSRQ we introduce in order to fit the auto-correlation of UGRB as measured by Fermi (Ackermann et al., 2018). The intensity rescaling, since the GLF enters the source window function as we will describe in eq. 3.11 and the window function is in turn connected to the source average intensity (see eq. 3.5), translates into a modification of the amplitudes of the GLF by two rescaling factors of 0.494 for BL Lacs and 0.714 for FSRQ. This rescaling can always be understood in the framework of the general non-universality of the two astrophysical sources, not fully captured by the available models, since it is equivalent to setting to 1 the exponential function of eq. 1.78 and rescale down the GLF of both BL Lacs and FSRQ by a common factor of 0.322; of course, this variability is well within the uncertainty range on the measurements of both the specific source and the total UGRB intensity and we do not exceed the upper limit represented by the value of the total cumulative intensity (Ackermann et al., 2015; Ammazzalorso et al., 2018).

Misaligned active galactic nuclei We do not use a direct modelling of the mAGN GLF: they represent a strongly subdominant component in the total intensity flux of the UGRB compared to BL Lac and FSRQ, hence calibration and direct insights are more difficult; yet, they are non-negligible sources especially due to the rather high abundance we observe in resolved sources, partially compensating for the limited power in their emission (Fornasa & Sánchez-Conde, 2015; Negro, 2018). A possible approach is thus that of estimating the GLF starting from the better understood radio luminosity functions (RLF) of the same class of sources. mAGN are indeed stronger radio than gamma emitters, so that their luminosity in that other electromagnetic band is more easily measurable, and we can eventually recover an expression from the GLF by imposing, since they are the same galaxies, that the number of objects predicted by the RLF matches that of the GLF and constraining consequently a number of conversion coefficients. This endeavour has been pursued by a number of works (Willott et al., 2001; Inoue, 2011; Di Mauro et al., 2014; Lara et al., 2004) leading to the relation:

$$\phi_\gamma(\mathcal{L}, z) = \frac{k}{\log(10)\mathcal{L}} \rho_r(\mathcal{L}_r, z) \frac{d \log_{10} \mathcal{L}_r}{d \log_{10} \mathcal{L}}, \quad (1.79)$$

where $\mathcal{L}_r$ is the radio luminosity and we allow for an additional constant k to be tuned in order to provide the numbers of mAGN the γ -ray detector has observed. The quantity $\rho_r(\mathcal{L}_r, z)$ is known as RLF, corresponding to the number of radio sources per unit of co-moving volume and per base-10 logarithmic unit of luminosity (dex), and expressed as (Willott et al., 2001; Pinetti, 2022):

$$\rho_r(\mathcal{L}_r, z) = \rho_l(\mathcal{L}_r, z) + \rho_h(\mathcal{L}_r, z), \quad (1.80)$$

where

$$\rho_l = \rho_{l\star} \left(\frac{\mathcal{L}_r}{\mathcal{L}_{l\star}} \right)^{-\beta_l} \exp \left(-\frac{\mathcal{L}_r}{\mathcal{L}_{l\star}} \right) (1+z)^{k_l} \quad \text{for } z < z_{l\star} \quad (1.81)$$

$$\rho_l = \rho_{l\star} \left(\frac{\mathcal{L}_r}{\mathcal{L}_{l\star}} \right)^{-\beta_l} \exp \left(-\frac{\mathcal{L}_r}{\mathcal{L}_{l\star}} \right) (1+z_{l\star})^{k_l} \quad \text{for } z \geq z_{l\star} \quad (1.82)$$

and

$$\rho_h = \rho_{h\star} \left(\frac{\mathcal{L}_r}{\mathcal{L}_{h\star}} \right)^{-\beta_h} \exp \left(-\frac{\mathcal{L}_{h\star}}{\mathcal{L}} \right) f_h(z) \quad (1.83)$$

The function f_h encompasses further redshift-dependent variability, reading:

$$f_h(z) = \exp \left\{ -\frac{1}{2} \left(\frac{z - z_{h\star}}{z_{h0}} \right)^2 \right\} \quad (1.84)$$

The parameters of the previous equations are: $\rho_{l\star} = 10^{-7.523} \text{ Mpc}^{-3}$, $\beta_l = 0.586$, $\mathcal{L}_{l\star} = 10^{26.48} \text{ W}/(\text{Hz sr})$, $k_l = 3.48$, $z_{l\star} = 0.710$, $\rho_{h\star} = 10^{-6.757} \text{ Mpc}^{-3}$, $\beta_h = 2.42$, $z_{h\star} = 2.03$ and $\mathcal{L}_{h\star} = 10^{27.39} \text{ W}/(\text{Hz sr})$.

Depending on whether $z < z_{h\star}$ or $z \geq z_{h\star}$, we use $z_{h0} = 0.568$ for the former and $z_{h0} = 0.956$ for the latter case. The formulation requires some care in the units of measurement and associated conversions: importantly, given the units of $\mathcal{L}_{l\star}$ and of $\mathcal{L}_{h\star}$, the luminosities in $\rho_{r,\text{tot}}$ must be expressed in units of $\text{W Hz}^{-1} \text{ sr}^{-1}$ for consistency. We thus have to convert the luminosities in erg s^{-1} first into units of Watt (W) and then to rescale them at the reference frequency value, 151 MHz, divided by 4π .

Since mAGN are typically distant objects, set at cosmologically relevant distances, some of their observed properties have to be measured using an explicit cosmological model: compared to when the first studies of (Willott et al., 2001) were published, the availability of the updated constraints on the Λ CDM model provided by the Planck satellite required the implementation of corrections on the redshift evolution laws for the mAGN. These calculations are published in (Inoue, 2011) and (Di Mauro et al., 2014), where the correlation between the core radio and the γ -ray luminosities are derived again within the more recent cosmological information. Furthermore, the study of mAGN astrophysics is usually based on connecting observations of the emission coming from its more luminous core emission to the total radio luminosity, including fainter emission from the galactic outskirts. The calibration among core and total radio luminosity is provided by (Lara et al., 2004)

$$\log_{10} \mathcal{L} = 2 + 1.008 \log_{10} \mathcal{L}_{r,\text{core}}^{5\text{GHz}} \quad (1.85)$$

$$\log_{10} \mathcal{L}_{r,\text{core}}^{5\text{GHz}} = 4.2 + 0.77 \log_{10} \mathcal{L}_{r,\text{tot}}^{1.4\text{GHz}}, \quad (1.86)$$

that in turn requires a scaling between the core and total RLF. The two quantities are related by:

$$\rho_{r,\text{core}}(\mathcal{L}_{r,\text{core}}, z) = \rho_{r,\text{tot}}(\mathcal{L}_{r,\text{tot}}, z) \frac{d \log_{10} \mathcal{L}_{r,\text{tot}}}{d \log_{10} \mathcal{L}_{r,\text{core}}}. \quad (1.87)$$

Again, a word of caution concerning the unities: in eq. 1.85, the luminosities are expressed in units of erg s^{-1} , whilst in eq. 1.86 they are expressed in units of $\text{W}/(\text{Hz})$. All the derived laws are calibrated from observations at a reference frequency, in this case 151 MHz: hence after having solved eqs. 1.85 and 1.86 for $\mathcal{L}_{r,\text{tot}}^{1.4\text{GHz}}$ as a function of $\mathcal{L}_{r,\text{core}}^{5\text{GHz}}$ (that is, scaling laws in turn expressed at reference frequencies) and of the free variable $\mathcal{L}$ (representing the integration variable of the halo model integrals for the astrophysical sources), such value has to be expressed in terms of the luminosity at 151 MHz as (Inoue, 2011):

$$\mathcal{L}_r = \mathcal{L}_r^{1.4\text{GHz}} \left(\frac{\nu}{1.4\text{GHz}} \right)^{-\alpha_r}, \quad (1.88)$$

where $\alpha_r = 0.80$ for the total radio emission. The scaling law from the original results of (Willott et al., 2001) and the ones obtained using a Planck 2015 cosmology can be obtained by relating the comoving volume element adopted in (Willott et al., 2001):

$$\frac{d^2 V_W}{dz \, d\Omega} = \frac{c^3 z^2 (2+z)^2}{4H_{0,W}^3 (1+z)^3} \quad (1.89)$$

where $H_{0,W} = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the comoving volume in the standard Λ CDM cosmology:

$$\frac{d^2 V}{dz \, d\Omega} = \frac{cd_L^2(z)}{H_0(1+z)^2 \sqrt{(1-\Omega_\Lambda - \Omega_m)(1+z)^2 + (1+z)^3 \Omega_m + \Omega_\Lambda}} \quad (1.90)$$

If we condensate the result in a conversion factor:

$$\eta = \frac{d^2 V_W / dz \, d\Omega}{d^2 V / dz \, d\Omega}, \quad (1.91)$$

the GLF can be therefore be expressed as:

$$\phi_\gamma(\mathcal{L}, z) = \frac{k \eta}{(1+z)^{2-\Gamma} \ln(10)} \frac{1}{\mathcal{L}_{\text{tot}}^{151\text{MHz}}} \frac{d\mathcal{L}_{\text{tot}}^{151\text{MHz}}}{dL} \rho_{r,\text{tot}}(\mathcal{L}_{\text{tot}}^{151\text{MHz}}(\mathcal{L})), \quad (1.92)$$

where $k = 3.05$. The spectral index is set to $\Gamma = 2.37$, while the factor $(1+z)^{2-\Gamma}$ is known as K-correction, a typical astrophysical effect that takes into account the additional variation between the emitted energy and the observed one due to the cosmic expansion and therefore the possible degeneracy between intrinsic variability in power of the sources and their different distances, and the effects it has on the correct calibration of the scaling laws (Hogg et al., 2002).

Star forming galaxies. Considerations analogous to the ones we exposed for the mAGN sources also hold for the SFG source class: they are expected to be overall subdominant sources, although, especially at the lowest energies, their contribution to the total intensity of the UGRB can be competitive with that of the BL Lac and of the FSRQ contribution, if not the dominant one (see fig. 3.1). Furthermore, their number is again expected to be rather high: although their total intensity quickly decays in redshift and therefore represent overall a secondary contribution, the exposed considerations require however to properly include them in the correct modelling of the UGRB sources inventory (Fornasa & Sánchez-Conde, 2015; Ando & Pavlidou, 2009).

Again, SFG are stronger emitters in a band different from the infrared one, and, instead of directly deriving their GLF, we present a model based on matching their gamma emission with that of their properties in the infrared band, where they are better understood and more easily detectable objects. Our reference model is that of (Gruppioni et al., 2013): the study recognises three separate and independently evolving sub-classes of SFG: quiescent spiral galaxies, starburst galaxies and SFG hosting a concealed or low-luminosity AGN (SF-AGN); their total IR luminosity function is obtained by summing the three contributions:

$$\hat{\phi}_{\text{IR}} = \hat{\phi}_{\text{spiral}} + \hat{\phi}_{\text{starburst}} + \hat{\phi}_{\text{SF-AGN}}. \quad (1.93)$$

Splitted by sub-class, the total model is built as:

$$\hat{\phi}_i = \hat{\phi}_{0,i} \left(\frac{\mathcal{L}_{\text{IR}}}{\mathcal{L}_{0,i}} \right)^{1-\gamma_i} \exp \left(-\frac{1}{2\sigma_i^2} \right) \log_{10}^2 \left(1 + \frac{\mathcal{L}_{\text{IR}}}{\mathcal{L}_{0,i}} \right) \quad (1.94)$$

where $i = \text{spiral, starburst and SF-AGN}$. we distinguish between $\mathcal{L}_{0,i}$ for spirals, defined in the following way:

$$\mathcal{L}_{0,\text{spiral}} = \begin{cases} \mathcal{L}_{\star,sp} \left(\frac{1+z}{1.15} \right)^{k_{Lsp}} & \text{for } z \leq 1.1 \\ \mathcal{L}_{\star,sp} \left(\frac{2.1}{1.15} \right)^{k_{Lsp}} & \text{for } z \geq 1.1 \end{cases} \quad (1.95)$$

	γ	σ	$\log_{10}(\mathcal{L}_*/\mathcal{L}_\odot)$	$\log_{10}(\phi_*/(\text{Mpc}^{-3}\text{dex}^{-1}))$	k_L	k_{R1}	k_{R2}
spiral	1.0	0.50	9.78	-2.12	4.49	-0.54	-7.13
starburst	1.0	0.35	11.17	-4.46	1.96	3.79	-1.06
SF-AGN	1.2	0.40	10.80	-3.20	3.17	0.67	-3.17

Table 1.4: Parameters used in our models for the infrared luminosity function of the gamma-ray source represented by SFG, in turn subdivided in three distinct galaxy populations: spiral galaxies, starburst galaxies and star-forming galaxies hosting an AGN.

and the one for starburst and SF-AGN, where we have:

$$\mathcal{L}_{0,i} = \mathcal{L}_{*,i} (1+z)^{k_{Li}}. \quad (1.96)$$

The quantity $\hat{\phi}_{0,i}$ for spirals reads:

$$\hat{\phi}_{0,\text{spiral}} = \begin{cases} \phi_{*,\text{sp}} \left(\frac{1+z}{1.15} \right)^{k_{R1,\text{sp}}} & \text{for } z \leq 0.53 \\ \phi_{*,\text{sp}} \left(\frac{1.53}{1.15} \right)^{k_{R1,\text{sp}}} \left(\frac{1+z}{1.53} \right)^{k_{R2,\text{sp}}} & \text{for } 0.53 < z \leq 1.7 \\ 0 & \text{for } z > 1.7 \end{cases} \quad (1.97)$$

while for starburst and SF-AGN we have:

$$\hat{\phi}_{0,j} = \begin{cases} \phi_{*,j} \left(\frac{1+z}{1.15} \right)^{k_{R1,j}} & \text{for } z \leq 1.1 \\ \phi_{*,j} \left(\frac{2.1}{1.15} \right)^{k_{R1,j}} \left(\frac{1+z}{2.1} \right)^{k_{R2,j}} & \text{for } z > 1.1 \end{cases} \quad (1.98)$$

In the cross-correlation study exposed in chapter 3, combining the candidate UGRB sources with the very-low redshift galaxy survey 2MRS, limited at $z \sim 0.1$, we use the parameters for the model discussed in Ref. (Gruppioni et al., 2013) and limited to their first redshift bin ($0 \leq z \leq 0.3$), as listed in table 1.4.

We again have to state a scaling relation between the GLF for energies between 0.1 GeV and 100 GeV, and the IR luminosity for wavelengths between $8\mu\text{m}$ and $1000\mu\text{m}$, and we use the one given in Ref. (Ackermann et al., 2012a):

$$\log_{10} \left(\frac{\mathcal{L}_{0.1-100\text{GeV}}}{\text{ergs}^{-1}} \right) = \alpha_{\text{IR}} \log_{10} \left(\frac{\mathcal{L}_{8-1000\mu\text{m}}}{10^{10} \mathcal{L}_\odot} \right) + \beta_{\text{IR}} \quad (1.99)$$

where $\alpha_{\text{IR}} = 1.09$ and $\beta_{\text{IR}} = 39.19$. This leads eventually to a functional form for the GLF that can thus be derived using eqs. 1.93 and 1.99:

$$\phi_\gamma(\mathcal{L}_\gamma, z) = \frac{\hat{\phi}_{\text{IR}}}{\mathcal{L} \log(10)} \frac{d \log_{10}(\mathcal{L}_{\text{IR}})}{d \log_{10}(\mathcal{L}_\gamma)} \quad (1.100)$$

For practical purposes we stress again that both eqs. 1.92 and 1.100 are expressed as an explicit function of the radio and IR luminosities but, when they are used in window function and in the halo model integrals, the free integration variable is the *gamma* luminosity; this means that some of the provided expressions, such as eq. 1.85 and eq. 1.99 are actually formulated in an *implicit* way; that is, given the gamma luminosity, they must be inverted in order to recover the desired radio and IR luminosities that represent the independent variables entering in the exposed GLFs.

Full list of 3D power spectra

Collecting all of the previous elements and by recalling the theory on the halo model and the quantities common to all tracers combinations (namely, bias factors, HMF, concentration relations, HOD, mass definition, minimum and maximum mass and luminosity values) exposed in sec. 1.3.1, we can now present the expressions for all required power spectra:

Galaxies auto-correlation For the galaxies auto-correlation, we have (Fornengo & Regis, 2014):

$$P_{1h}^{\text{gg}}(k, z) = \int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} \frac{\langle N_g (N_g - 1) \rangle}{\bar{n}_g^2} \tilde{v}(k|M)^2 \quad (1.101)$$

$$P_{2h}^{\text{gg}}(k, z) = \left[\int_{m_{\min}}^{m_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\langle N_g \rangle}{\bar{n}_g} \tilde{v}(k|M) \right]^2 P^{\text{lin}}(k, z) \quad (1.102)$$

$\tilde{v}(k|M)$ being the Fourier transform of $\rho(\mathbf{r}|M)/\bar{\rho}$, $\bar{\rho}$ the present-day mean value of the matter density of the Universe, considering both baryons and CDM, $b_h(M, z)$ is the halo bias, modeled as in Ref. (Sheth et al., 2001), N_g the number of galaxies of the HOD, $\bar{n}_g = \int dM \, dn/dM \, \langle N_g \rangle$ and $P^{\text{lin}}(k)$ is the matter linear power spectrum, which we have calculated by using the package `pycc1` (Chisari et al., 2019a), based on `CAMB` (Lewis & Challinor, 2011).

Galaxies-DM annihilation cross-correlation power spectra When we cross-correlate galaxies with the gamma-ray emission from DM annihilation (which depends on the square of the dark matter density), the power spectra are (Fornengo & Regis, 2014):

$$\begin{aligned} P_{1h}^{\text{g}\delta^2}(k, z) &= \int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} \frac{\langle N_g \rangle}{\bar{n}_g} \tilde{v}(k|M) \frac{\tilde{u}(k|M)}{\Delta^2} \\ P_{2h}^{\text{g}\delta^2}(k, z) &= \left[\int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\langle N_g \rangle}{\bar{n}_g} \tilde{v}(k|M) \right] \times \\ &\quad \times \left[\int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\tilde{u}(k|M)}{\Delta^2} \right] P^{\text{lin}}(k, z) \end{aligned}$$

where $\tilde{u}(k|M)$ is the Fourier transform of $\rho^2(\mathbf{r}|M)/\langle \rho(z) \rangle^2$ and $\Delta(z)$ the clumping factor introduced in 1.55. Notice that, although $\rho^2(\mathbf{x})$ is the square of the NFW profile, the squaring operator does not commute with the Fourier transform, hence $\rho^2(\mathbf{k}) \neq (\rho(\mathbf{k}))^2$.

Annihilating DM autocorrelation power spectrum The autocorrelation of gamma-ray emission from DM annihilation reads (Fornengo & Regis, 2014):

$$\begin{aligned} P_{1h}^{\delta^2\delta^2}(k, z) &= \int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} \left(\frac{\tilde{u}(k|M)}{\Delta^2} \right)^2 \\ P_{2h}^{\delta^2\delta^2}(k, z) &= \left[\int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\tilde{u}(k|M)}{\Delta^2} \right]^2 P_{\text{lin}}(k, z) \end{aligned}$$

Galaxies-astrophysical objects cross-correlation power spectrum Focusing instead on the gamma-ray emission with any generic astrophysical source S , multiple terms have to be considered.

The cross-correlation with galaxies reads (Fornengo & Regis, 2014):

$$\begin{aligned}
P_{1h}^{Sg}(k, z) &= \int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_S \rangle} \frac{\langle N_g \rangle}{\bar{n}_g} \tilde{v}(k|M) \\
P_{2h}^{Sg}(k, z) &= \left[\int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) b_S(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_S \rangle} \right] \times \\
&\quad \times \left[\int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\langle N_g \rangle}{\bar{n}_g} \tilde{v}(k|M) \right] P_{\text{lin}}(k, z),
\end{aligned}$$

the astrophysical sources having been labelled with S . Consistently with the previous sections, $\mathcal{L}$ denotes the gamma luminosity of the source, $\phi(\mathcal{L}, z)$ is the gamma-ray luminosity function, $\langle g_S \rangle = \int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) \mathcal{L}$ and finally $b_S(M, z)$ the source bias. This quantity is derived from $b_h(M, z)$ (Sheth et al., 2001), having defined the conversions from luminosity to mass $M(\mathcal{L})$ as discussed in sec. 1.3.1. i.e. $b_S(M, z) = b_h(M(\mathcal{L}), z)$. Notice that the one-halo integral runs on luminosities, whereas the 2-halo integrals run on masses for the galaxy-dependent factor and on luminosities for the astrophysical sources-dependent factor. The maximal luminosity $\mathcal{L}_{\max}(z)$ in this and in the other halo model luminosity integrals is related to the detector and the detection threshold for resolved sources, by:

$$\mathcal{L}_{\max}(z) = \min(\mathcal{L}_{\max}, \mathcal{L}_{\text{sens}}(z)) \quad (1.103)$$

where $\mathcal{L}_{\max}$ is given in table 1.2 for each source class and $\mathcal{L}_{\text{sens}}(z)$ is obtained as:

$$\mathcal{L}_{\text{sens}}(z) = 4\pi d_L^2(z) F_{\text{sens}}^{\text{LAT}} \frac{\int_{0.1 \text{ GeV}/(1+z)}^{100 \text{ GeV}/(1+z)} dE E^{-\Gamma+1}}{\int_{1 \text{ GeV}}^{100 \text{ GeV}} dE E^{-\Gamma} \exp[-\tau(E, z)]} \quad (1.104)$$

where $F_{\text{sens}}^{\text{LAT}}$ is the Fermi-LAT flux sensitivity calculated in (1,100) GeV. We assume $F_{\text{sens}}^{\text{LAT}} = 10^{-10} \text{ cm}^{-2} \text{ s}^{-1}$: our forecast is therefore based on assuming a slight improvement on the current sensitivity (Manconi et al., 2020). Another important assumption we make is that the flux sensitivity, instead of having a dependence from the source spectral index, takes a flat value $F_{\text{sens}}^{\text{LAT}}$. See e.g. (Manconi et al., 2020) for alternative formalisms.

Annihilating Dark Matter-astrophysical sources cross-correlation power spectra The cross-correlation with annihilating Dark Matter is developed analogously to the cross-correlation between galaxies and astrophysical sources, involving again mass-to-luminosity conversions and that the one-halo integral runs on luminosities, whereas the 2-halo integrals run on masses for the galaxy-dependent factor and on luminosities for the astrophysical sources-dependent factor (Fornengo & Regis, 2014):

$$\begin{aligned}
P_{1h}^{S\delta^2}(k, z) &= \int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_S \rangle} \frac{\tilde{u}(k|M)}{\Delta^2} \\
P_{2h}^{S\delta^2}(k, z) &= \left[\int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) b_S(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_S \rangle} \right] \times \\
&\quad \times \left[\int_{M_{\min}}^{M_{\max}} dM \frac{dn}{dM} b_h(M, z) \frac{\tilde{u}(k|M)}{\Delta^2} \right] P_{\text{lin}}(k, z).
\end{aligned}$$

Autocorrelation between two astrophysical sources i and j Finally, the autocorrelation between two gamma-ray emission i, j of astrophysical sources is (Fornengo & Regis, 2014):

$$P_{1h}^{S_i S_j}(k, z) = \int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \phi(\mathcal{L}, z) \left(\frac{\mathcal{L}}{\langle g_S \rangle} \right)^2 \delta_{ij} \quad (1.105)$$

$$P_{2h}^{S_i S_j}(k, z) = \left[\int_{\mathcal{L}_{i,\min}}^{\mathcal{L}_{i,\max}(z)} d\mathcal{L} \phi_i(\mathcal{L}, z) b_{S_i}(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_{S_i} \rangle} \right] \left[\int_{\mathcal{L}_{j,\min}}^{\mathcal{L}_{j,\max}(z)} d\mathcal{L} \phi_j(\mathcal{L}, z) b_{S_j}(\mathcal{L}, z) \frac{\mathcal{L}}{\langle g_{S_j} \rangle} \right] P_{\text{lin}}(k, z). \quad (1.106)$$

Importantly, we make the approximation that the astrophysical objects are point-like and that there must be **at most one** single object per halo, two astrophysical gamma emitters of different classes are not allowed; the one-halo term is thus null for $i \neq j$, as enforced by the Kroenecker delta.

1.3.2 Non-linearly biased tracers and Lagrangian Perturbation Theory

Alternatively to the halo model, which shall be chiefly used in chapter 3, another well-developed formalism aimed at describing the non-linear regime of the LSS is represented by the vast variety of higher-order perturbation theories. These will be the main instrument we will use in order to model non-linear power spectra in chapter 5 and, generally speaking, instead of basing their description of non-linearity in the LSS on properties related to the number counts and abundance statistics of the objects, this second class of approaches tries to extend the linear perturbation theory by introducing higher order powers of the density contrast field as non-linear corrections to the basic linear perturbative theory. We shall start by introducing the theory of non-linear bias (Desjacques et al., 2018): the general idea remains that of relating δ_g and the matter density contrast by means of multiplicative factors; this time, however, δ_g will be the outcome of a series expansion of the product of a number of bias factors, b_n , times the n -th component of some series expansion of the matter field density contrast. We anticipate that the expansion can be interpreted, depending on the chosen non-linear perturbation theory formalism, either as a true power series expansion of the matter fields (that, we remind, are stochastic in nature and associated to some assumptions concerning their statistical distribution and moments) up to some order n , or in terms of a summation over a “phenomenological” collection of fields O_n fields associated to operators encoding features and physical processes of the LSS, such as contributions from the largest over- and underdensities, or tidal effects, selected based on the impact these variables seem to exert on the biasing of tracers observed in simulations and observations. This latter approach, combined with the more general Lagrangian Perturbation Theory (LPT) approach, is known as Hybrid Effective Field Theory (HEFT) (Zennaro et al., 2023, 2022; Lazeyras et al., 2016; Lazeyras & Schmidt, 2018; Barreira et al., 2021; Modi et al., 2020). We remark that, in this general definition, the concept of operator O_n indicates either a proper *calculus* operator acting on a field (e.g., a derivative applied to some power of the density contrast) or a field –in other words, both δ and $\nabla^2 \delta$ are “operators” in this formulation. The generalised non-linear bias formalism can be expressed as

$$\delta_g(\mathbf{x}, z) = \sum_n b_n(z) O_n(\mathbf{x}, z). \quad (1.107)$$

Notice that, in this formulation, the generalised bias factors $b_n(z)$ still do not exhibit position dependence, but exclusively redshift dependence. The O_n terms can therefore include, depending on the the specific field, constant coefficients (numbers), first-order powers of the density contrast

(therefore linear functions of the “operator” δ , endowed with specific physical meanings and *correcting* the basic δ of the linear theory) or higher order powers of δ that might be reformulated in terms of lower order powers and consequently truncated at some perturbative order. This connects the specific HEFT perturbative theory formulation with the formulations based on the direct expansion in power series of the density contrast: (see below sec. 1.3.2). For example, for commuting operators, $\delta = \delta^{(1)} + \delta^{(2)} + \dots$ is the second-order expansion of the linear density contrast operator, while $\delta^2 = (\delta)(\delta) = (\delta^{(1)} + \delta^{(2)}) (\delta^{(1)} + \delta^{(2)}) = (\delta^{(1)})^2 + 2\delta^{(1)}\delta^{(2)}$ is the second order expansion of the squared density contrast operator. This gives us a handle towards the description of the broader landscape of the different theoretical frameworks that have been elaborated in the last forty years, often originating from fluid mechanics and particle physics, capable of predicting with different accuracy and simplicity the features of the non-linear power spectrum. Concerning the range of application of these different theoretical pictures, similarly to the different choices in the description of the bias factors, the use of one formalism against the other might also depend on the specific problem, contemplating, among the requisites, the computational feasibility of integrating the equations when the modes k get coupled and higher powers of the density contrast are involved (Carlson et al., 2009; Ivanov, 2022).

Non-linear expansion techniques based on power series

The most straightforward approach to a non-linear theory of the matter power spectrum, expressing it as a function of the power series expansion of the matter density contrast up to some power order, is known as Standard Perturbation Theory (SPT) (Peebles, 1980; Juszkiewicz, 1981; Vishniac, 1983; Goroff et al., 1986). Its formulation is rather intuitive, but its accuracy can be limited even when rather high-order corrections are introduced (Carlson et al., 2009). The inclusion of an higher-order series expansion of the density contrast would be thus required to recover appropriate predictions but, since, however, the ultimate quantities we want to calculate, such as summary statistics, are typically a product of density contrasts (as in eq. 1.120), this quickly leads to a heavy computational burden and to substantial difficulties in finding tractable expression. Starting from SPT, theories have stemmed aiming to achieve greater tractability by trying to make order among the different products of terms, individuating the dominant ones order by order and focusing on strategies to explicitly calculate their value. This usually allows explicit power spectra calculations only at very large or very small k , where simplifications can be more effectively performed, but suitable interpolation techniques can also be devised in order to calculate the intermediate regimes. Theories belonging to this class are the Renormalized Perturbation Theory (RPT) (Crocce & Scoccimarro, 2006a,b, 2008), the Time-Renormalisation Group (Time-RG) theory (Pietroni, 2008) and Renormalization Group Perturbation Theory (RGPT) of (McDonald, 2007). Approaches instead starting from the Vlasov equation, working on the distribution function in the position and momentum space $f(\mathbf{x}, \mathbf{p}, t)$, have been explored in the works of (Valageas, 2001, 2002, 2004, 2007).

The combined HEFT and Lagrangian Perturbation Theory approach

The Lagrangian Perturbation Theory (LPT) formulation adopted in chapter 5 can instead be traced back to the works of (Juszkiewicz, 1981; Vishniac, 1983; Fry, 1984; Goroff et al., 1986) for the “pioneering” phase, further developed in a cosmological context by (Buchert, 1992; Ehlers & Buchert, 1997; Buchert, 1994) and formulated in the most contemporary form by (Matsubara, 2008,a). It takes a different perspective on the problem of modelling non-linearities in the matter

field. The final goal of this formulation is that we want to find an expression for the matter field or of some biased tracer power spectrum as a Fourier transform of quantities calculated in the Eulerian space of fluid dynamics. The Eulerian description of a fluid corresponds to evaluating the density contrast and thus the dynamics of a fluid element at fixed positions in the real space (subsequently converted to the Fourier space) while it evolves in time or redshift. The Eulerian picture is the ultimate description we want to use because it corresponds to the one of the observables used in cosmology (we have a fixed “image” of the sky out of which we sample different time-redshift slices). Instead of working from the beginning in the Eulerian space to derive the quantities of interest, however, LPT theory first expresses perturbations in the Lagrangian space. This second approach corresponds to a formulation of fluid mechanics in which the kinematics of fluid elements is studied, instead, by tracking the motion in space of fluid elements, starting from some initial position $\mathbf{q}$ at some initial time t_0 . The mapping of a field from its Lagrangian to its Eulerian description in fluid mechanics is known in jargon as “advection”, and consists into calculating the value of a field at a given time t and Eulerian position $\mathbf{x}$, considering all those fluid elements that in the Lagrangian framework started at different positions $\mathbf{q}$ at t_0 and are ending up in $\mathbf{x}$ at t . The formalism to perform advection and the mapping between the coordinates $\mathbf{q}$ and $\mathbf{x}$ lies at the heart of the specific LPT point of view and will be discussed in the next paragraphs. The subsequent step is to state the advection law between Eulerian and Lagrangian positions: this expression in cosmology takes the typical form of the Zel’dovich approximation (Zel’dovich, 1970):

$$\mathbf{x} = \mathbf{q} + \boldsymbol{\Psi}(\mathbf{q}, t) \quad (1.108)$$

where $\boldsymbol{\Psi}$ is known as the “displacement field”. In other words, the remapping between Lagrangian and Eulerian positions is assumed to be exclusively a function of positions, and of no other dynamics variables of the studied fields other than time. The features of the displacement field can be determined from simulations and observations (Eisenstein et al., 2007), or modelled through different theoretical approaches aimed at defining suitable estimators (Padmanabhan et al., 2009; Noh et al., 2009; White, 2015; Schmittfull et al., 2015; Ota et al., 2021): the Zel’dovich approximation has historically proven to be extremely powerful and effective, since it always provides finite velocities and positions up until when two particle trajectories intersect (an event known as shell crossing, corresponding to the onset of gravitational collapse of objects) but, in principle, can be used even for very high density contrasts (Zel’dovich, 1970). Such combination of simplicity and efficiency is in general the motivation which has led to the most contemporary formulations of LPT; furthermore, some results are relevant also in regimes that are usually considered linear in nature, but for which, due to the current accuracy standards of cosmology, the simplest treatments require improvements in order to ensure the correct measurement of BAO (Eisenstein et al., 2005) or the inclusion of RSD in the matter field (Matsubara, 2008b). Equipped with the above considerations and passing from a single particle to a continuous distribution of particles and Lagrangian positions, the mapping for a general Lagrangian field $F(\mathbf{q})$ to the Eulerian field $\delta_F(\mathbf{x})$ reads

$$1 + \delta_F(\mathbf{x}) = \int d^3\mathbf{q} F(\mathbf{q}) \delta_D(\mathbf{x} - \mathbf{q} - \boldsymbol{\Psi}(\mathbf{q})), \quad (1.109)$$

where δ_D is a Dirac delta. In other words, the field F is considered as a collection of localised Lagrangian elements, the position of each one of them corresponding to the Dirac delta, embedded in a continuous position mesh, over which we integrate. The integral of eq. 1.109 can be developed starting from the assumption that the displacement field can be in turn expanded in powers up to

some order (in our case, we truncate the expansion to the second order): $\Psi \simeq \Psi^{(1)} + \Psi^{(2)} + \dots$. Subsequently, we prepare ourselves to calculate the ultimate quantity of interest, the power spectrum in the Fourier space, by performing a Fourier transform of the expression in eq. 1.109:

$$\begin{aligned}\delta(\mathbf{k}) &= \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \delta_F(\mathbf{x}) \\ &= \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \left[\int d^3\mathbf{q} F(\mathbf{q}) \delta_D(\mathbf{x} - \mathbf{q} - \Psi(\mathbf{q})) - 1 \right] \\ &= \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \int d^3\mathbf{q} F(\mathbf{q}) \delta_D(\mathbf{x} - \mathbf{q} - \Psi(\mathbf{q})) - \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} 1.\end{aligned}\quad (1.110)$$

We focus on the first term of the difference above: we first invert the integration order between the integrals in $\mathbf{x}$ and $\mathbf{q}$ in order to let the Dirac delta act, eliminating one of the two integrals and replacing every instance of $\mathbf{x}$ with $\mathbf{q} + \Psi(\mathbf{q})$ by effect of the Zel'dovich approximation, obtaining

$$\delta_F(\mathbf{k}) = \int d^3\mathbf{q} e^{-i\mathbf{k}\cdot(\mathbf{q}+\Psi(\mathbf{q}))} F(\mathbf{q}). \quad (1.111)$$

The second addendum of eq. 1.110 can be discarded at every k , since the Fourier transform of 1 only adds a Dirac delta function when $k = 0$ (a scale we cannot measure, since it is associated to infinite distances) and 0 otherwise. The next passage is to Taylor-expand the complex exponential $e^{-i\mathbf{k}\cdot\Psi(\mathbf{q})}$ combining the series expansion with the assumption of truncating Ψ at the second order but keeping only terms that are quadratic at the *first* order in Ψ . That is, we keep at most terms of the form $(\Psi^{(1)})^2$:

$$\begin{aligned}\delta_F(\mathbf{k}) &= \int d^3\mathbf{q} e^{-i\mathbf{k}\cdot\mathbf{q}} F(\mathbf{q}) \left\{ 1 - i\mathbf{k} \cdot \Psi^{(1)}(\mathbf{q}) - \frac{1}{2} [\mathbf{k} \cdot \Psi^{(1)}(\mathbf{q})]^2 \right. \\ &\quad \left. - i\mathbf{k} \cdot \Psi^{(2)} \right\}.\end{aligned}\quad (1.112)$$

Provided with these preliminary reasonings, the final calculation of the LPT non-linear power spectrum starts from the expression of eq. 1.109 for two fields 1 and 2 in the real space, parametrised by the respective Lagrangian coordinates $\mathbf{q}_1$ and $\mathbf{q}_2$. Of these two fields, we take the Fourier transform and combine them constructing the general definition of the power spectrum as correlation between the density contrasts $\langle |\delta_F(\mathbf{k}') \delta_F^*(\mathbf{k})| \rangle$, evaluated at different modes $\mathbf{k}'$ and $\mathbf{k}$. Combined with the assumption that the density contrasts are Gaussian random variables, we have:

$$P(\mathbf{k}) = \int d^3\mathbf{q} e^{-i\mathbf{k}\cdot\mathbf{q}} \left\langle F(\mathbf{q}_1) F(\mathbf{q}_2) e^{-i\mathbf{k}\cdot[\Psi(\mathbf{q}_1) - \Psi(\mathbf{q}_2)]} \right\rangle \quad (1.113)$$

where we have redefined $\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$, and used one of the two Dirac deltas in the definition of $\delta_F(\mathbf{x})$ to eliminate one of the two integrals in $\mathbf{q}_1$ and $\mathbf{q}_2$. We can discard the terms integrating the 1 term or depending on integrals of $\langle F \rangle$ (whose value is 1, since, by applying the ensemble average to eq. 1.109, $\delta_F(\mathbf{x})$ is equal to 0 as every density contrast and therefore we are left with the condition that the right-hand side expression must be equal to the $1 + 0$ term on the right hand side), because they are all contributing again only with a term proportional to the Dirac delta centered in $k = 0$.

We recall that $F(\mathbf{q})$ is itself a series expansion, so that the explicit expression of the power spectrum will actually be the summation of different orders in the field operator expansion basis, combined with the expansion of the complex exponential in $\Psi(\mathbf{q}_1) - \Psi(\mathbf{q}_2)$ following the guidelines of eq. 1.110. Because of the assumption of homogeneity underlying the density contrast field properties (i.e., that the density contrast at some scale is the same at every position we are

evaluating it), the ensemble average in the integrand of the right-hand side (RHS) will only depend on the separation vector $\mathbf{q}$ and not on the specific positions $\mathbf{q}_1$ and $\mathbf{q}_2$. Combined with the hypothesis that $\langle \Psi \rangle = 0$ and following (Zennaro et al., 2023; Matsubara, 2008), it turns out that, given the perturbative truncation exposed above, only terms of order (11) and (22) in the powers of Ψ are retained (i.e., we perform an “one-loop” truncation of the non-linear expansion, higher truncations would have led to “two-loop”, “three-loop” etc. corrections), while mixed terms in the form (12) are not present under the assumption of Gaussian fluctuations. A second order perturbation term can be indeed written as the product of two first-order perturbation terms (i.e., their square), leading to expressions of the form: $\langle \delta_F^{(1)} \delta_F^{(2)} \rangle = \langle \delta_F^{(1)} \delta_F^{(1)} \delta_F^{(1)} \rangle$. Combined with the fact that $\langle \delta_F^1 \rangle = 0$, we can use the Wick-Isseerlis’ theorem (Wick, 1950; Isserlis, 1916, 1918) (see also sec. 3.3 and eq. 3.33). In general, the theorem allows to simplify complex correlations among fields and ensures that correlators featuring an odd number of fields always provide a null expectation value. The final outcome for the non-linear matter power spectrum is an expression in the form

$$P_{\text{matter, nl}}(k) = P(k) + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} P(\mathbf{p}) P(|\mathbf{k} - \mathbf{p}|) C(\mathbf{p}, \mathbf{k}) \quad (1.114)$$

with $P(k)$ (without subscripts) being the linear power spectrum: the linear formulation appears thus corrected by an additional term within the integral, which also encapsulates the computational complexity of the theory, especially the integral on the kernel C , capturing the desired non-linear scale dependencies and scale couplings represented by being simultaneously a function of two scales $\mathbf{p}$ and $\mathbf{k}$. Although mathematically difficult and intimidating, LPT is not necessarily more involved than SPT and related theories at comparable perturbative orders. On the contrary, the relative simplicity of the Zel’dovich approximation from which it is stemming has made it of great interest, alongside with the possibility of linking some of the kernel integrals to a physical interpretation, although only at the lowest orders. Furthermore, it displays a better convergence behaviour at high k compared to power-series based theories, while formal equivalence between it and SPT-related theories has also been discussed and proven in the literature (Matsubara, 2008). As an additional benefit, LPT at the level of the matter power spectrum also features the possibility of readily including RSD and allows for a more immediate calculation of Fourier transforms and thus a better capability of providing both n -point correlation functions in the real space and in higher-order spectra in the Fourier space (Matsubara, 2008,a). But, most important for this work, it can be especially be gauged to a HEFT approach for biased tracers of the matter field. The extension to biased tracers starts from the observation that, if for the matter field it holds that the weighting function $F(\mathbf{q}) = 1$, this, in the Lagrangian fluid formulation, can be interpreted as the expression of an unbiased field. Therefore, as mentioned in sec. 1.2.1, in the operator/HEFT framework, non-linear bias terms and non-linear bias fields can be expressed as a function of a number of physical properties which are found to dominate the clustering dynamics. Following the HEFT approach and requiring for the total biased Lagrangian density contrast F to be a function of at most quadratic powers of the density contrast, we said that the additional bias terms and fields can be connected to operators, in turn active on different powers of the density contrasts, so that LPTs can be in turn expressed at different orders and ultimately connected to the power series expansions; a number of classes of operators can be further recognised (Zennaro et al., 2023):

- *Local bias operators*: including those operators that involve two spatial derivatives of the gravitational potential Φ , as well as powers of the tidal and density fields, such as 1 (“0-th”

power of δ), δ , δ^2 ... They are known as “local” in that they are connected to local observables of long-wavelength perturbations.

- *Higher-derivative bias operators*: including operators with three or more spatial derivatives, still associated to local observables but typically acting as sub-leading terms in a series expansions for long-wavelength density perturbations.
- *Non-local bias operators*: they are operators involving a number of derivatives less than two and possibly of fractional order. They cannot be associated to local physical processes because the equivalence principle of General Relativity requires that we cannot directly measure the gravitational potential Φ and its first derivatives $\partial_i\Phi$, but can for example capture primordial features, such as primordial non-Gaussianity or scale dependence in the bias terms, so that they are usually referred in the literature as “scale-dependent” bias terms (Sullivan & Chen, 2025).

One of the possible selections of fields, the one indeed associated to the HEFT-LPT formulation, can be interpreted as an expansion of the weighting function (Zennaro et al., 2023) including a number of first-order terms, not simply understood as first order density contrast corrections to the matter weighting function, rather, as physically interpretable fields, to which some terms that “formally” appear as second order powers of the density contrast with an intrinsic physical meaning are added:

$$F(\mathbf{q}) = 1 + b_1\delta(\mathbf{q}) + \frac{b_2}{2} [\delta^2(\mathbf{q}) - \langle\delta^2\rangle] + \frac{b_{s^2}}{2} [s^2(\mathbf{q}) - \langle s^2\rangle] + b_{\nabla^2} [\nabla^2\delta(\mathbf{q}) - \langle\nabla^2\delta(\mathbf{q})\rangle] . \quad (1.115)$$

The physical interpretation of the bias terms and the associated terms is:

- 1 is the homogeneous field associated to the matter field *in the Lagrangian framework*.
- δ is a linearly biased field (with zero average) in the Lagrangian framework for density perturbations. It is associated to the linear bias but notice that, to recover the usual *Eulerian* linear bias of eq. 1.33, we have to set $b_{\text{Eul}} = 1 + b_1$ (this is why the unbiased matter field is associated to the “1” field/operator, since its $b_{\text{Eul}} = 1$ requires that $b_1 = 0$). From a phenomenological point of view, the term expresses the sensitivity of the field to collapsed regions such as nodes of the cosmic web, most intuitively associated to galaxies and clusters.
- δ^2 can be interpreted as a quadratic term in the density contrast power series expansion, enhancing the locations where it takes the largest values. Being quadratic, it acts on both collapsed regions and voids. In other words, it expresses specific sensitivity of the clustering process to both the largest over- and underdensities.
- $s^2(\mathbf{q}) = s_{ij}(\mathbf{q})s_{ij}(\mathbf{q})$ is the shear field, defined by the tidal tensor $s_{ij}(\mathbf{q}) = \partial_i\partial_j\Phi(\mathbf{q}) - \delta_{ij}\delta(\mathbf{q})$, with $\Phi(\mathbf{q})$ the local matter gravitational potential and δ_{ij} is Kroenecker’s delta. It therefore expresses dependence on the tidal fields between different objects in the same environment and their influence on the clustering and formation of galaxies.
- ∇^2 can be understood as a term derived from limited resolution in observations and simulations, setting the minimum scale of observable halos, combined with possible smoothings and loss of coherence in the field induced by gas pressure and other baryon effects at small scales.

Given the results of eq. 1.107 and of eq. 1.114, we can put everything together and recover the expression of the non-linear autocorrelation power spectrum for galaxies as a function of non-linear bias terms:

$$P_{\text{gg}} = \sum_i \sum_j b_i b_j P_{ij} . \quad (1.116)$$

where the terms P_{ij} can be recovered by repeating the calculations that led to eq. 1.114 but considering the new expression for $F(\mathbf{q})$. Always keeping at most second order terms in the displacement and in the density contrast fields, each P_{ij} term reads

$$P_{ij}(k) = A_{ij}(k)P(k) + B_{ij}(k) \int \frac{d^3\mathbf{p}}{(2\pi)^3} P(\mathbf{p})P(|\mathbf{k} - \mathbf{p}|)C_{ij}(\mathbf{p}, \mathbf{k}), \quad (1.117)$$

where $P(k)$ (without subscripts) is again the linear power spectrum. Compared to eq. 1.114, we have a further complication due to the introduction of the terms $A_{ij}(k)$, $B_{ij}(k)$ (equal to one when $i = j = 1$, corresponding to the HEFT *matter* power spectrum of eq. 1.114), and in the multiple kernels $C_{ij}(\mathbf{p}, \mathbf{k})$ instead of the single C term. The explicit expressions of these quantities are collected in the Appendix A of (Zennaro et al., 2023). We can similarly quote the matter-galaxy cross-correlation power spectrum, reading

$$P_{\text{gm}} = \sum_i b_1 b_i P_{1i} \quad (1.118)$$

and recall that the matter power spectrum, from the definition of $F(\mathbf{q})$ for unbiased fields reads $P_{\text{mm}} = P_{11}$.

In the above expressions, then, indexes i and j run on the subset of bias operators we considered and P_{ij} is the corresponding matter power spectrum term. In the HEFT formulation, the considered Lagrangian bias parameters b_1, b_2, b_{s^2} , and b_{∇^2} can be understood as free coefficients. We might fit them against the data assuming that they are scale-independent. We emphasize that the bias factors are multiplying a *summation* of power spectrum-like terms (15 in total), each one satisfying the relation

$$P_{ij} \equiv \langle |\delta_i(\mathbf{k})\delta_j^*(\mathbf{k}')| \rangle (2\pi)^2 \delta_D^3(\mathbf{k} - \mathbf{k}') \quad (1.119)$$

or, for greater simplicity hereafter and leaving implicit the effect of the Dirac delta in selecting only the same mode for every scale:

$$P_{ij} \equiv \langle |\delta_i(\mathbf{k})\delta_j^*(\mathbf{k})| \rangle, \quad (1.120)$$

while each correlator must be separately calculated starting from the treatment of the non-linear field $\delta_i(\mathbf{k})$ associated to the i -th operator. Expectations, confirmed on simulations (Zennaro et al., 2022), are that out of the 15 terms determining the total power spectrum of eq. 1.117, the dominant ones at large scales are closely associated to linear quantities – $\langle 11 \rangle$, $\langle 1\delta \rangle$, and $\langle \delta\delta \rangle$ – with an agreement of about 1% with respect to theoretical predictions, whilst on smaller scales the linear power spectrum is modified by the onset of dominant effects by fields involving δ^2 – the combinations $1\delta^2$, $\delta\delta^2$, and $\delta^2\delta^2$. On the other hand, the fields involving the tidal field – $1s^2$, δs^2 , s^2s^2 – are expected to be subdominant at all scales and indeed found to be as such.

Bias coevolution relations The physical meaning attributed to the bias operators in the HEFT-LPT framework has stimulated attempts to directly measure them on observational data (Philcox & Ivanov, 2022): at the same time, the complex formulation of the power spectra and the interactions between the high-order fields induces relevant degeneracies among them even when restricted to the four terms described above or at relatively low power orders. Partial breaking of such degeneracies can be achieved by using statistics of order higher than the power spectrum, in particular the bispectrum, together with careful use of the priors imposed on such parameters. By balancing between non-informative values in order not to force convergence of the analyses and sensible limitations to the acceptable values, it is possible to achieve up to $\sim 30\%$ (Philcox & Ivanov, 2022; Ivanov et al., 2022) reduction on the uncertainties on some of them. It would nevertheless be desirable to further simplify our modelling by combining empirical evidence with theoretical arguments, aiming at writing higher order parameters as a function of the well-understood and directly measurable linear bias parameter b_1 . Quantitative relationships of this kind are known as *coevolution relations* and can be derived by the combined fitting of data from observations (Philcox & Ivanov, 2022), simulations (Zennaro et al., 2022) and from further theoretical reasonings. These latter have stemmed from original efforts aimed at modelling *dark matter halo* non-linear biasing with respect to the underlying dark matter field as an alternative to the peak-background approach from which the halo bias functional forms used in the halo model are derived (Sheth & Tormen, 1999; Desjacques et al., 2018). Using the notation of eq. 1.51, Lagrangian *halo* bias terms (Lazeyras et al., 2016) can be consistently derived from higher order derivatives of the halo mass function $n(M)$:

$$b_n^L(\nu_B) = \frac{1}{n(\nu_B)} \left. \frac{\partial^n ([B(\sigma_0) - \delta_0]/\sigma)}{\partial \delta_0^n} \right|_{\delta_0=0}. \quad (1.121)$$

The quantity can be defined starting from the barrier-crossing argument (Kaiser, 1984; Mo & White, 1996; Cole & Kaiser, 1989): instead of focusing on the specific value of the density contrast (usually defined δ , without further subscripts) associated to the threshold required by the spherical collapse theory (Coe, 2010; Cooray & Sheth, 2002; Mo et al., 2010), one extends the argument considering the halos whose overdensity is associated to a more general “barrier” value B , specified together with a reference background density contrast δ_0 corresponding to a perturbation whose wavelength is much larger than the local scale at which the halo form. If, like in eq. 1.121, we set $\delta_0 = 0$, the long-wavelength background perturbation has a density corresponding to the average density of the Universe. We can then calculate the corresponding variance σ_0 (again, not necessarily corresponding to the σ_8 we described in eq. 1.34), define $\nu_B \equiv \frac{B}{\sigma_0}$ and then derive the higher-order Lagrangian bias terms accordingly, leveraging on the greater flexibility allowed by the “barrier-picture” with respect to the “spherical-collapse picture” to better model local effects. The subsequent step is to adapt the same approach to the context of biased tracers of the LSS residing within halos and, in particular, to consider the relations between non-linear galaxy bias terms.

The theoretical arguments based on the recursive nature in the definition of higher-order bias terms obtained by applying multiple times the derivative operator on the mass function (eq. 1.121) can be used in order to derive an analytic coevolution relation of higher-order bias terms as a function of b_1 . Alternatively, or in combination, these derivations can be associated to a phenomenological polynomial fitting strategy: relations between the linear bias and its higher order counterparts get calibrated on N -body simulations, in the case of dark matter halos, or through galaxy clustering simulations (either hydrodynamic simulations or N -body simulations

over which the galaxy field is “painted” through suitable techniques such as HODs) for galaxies (Lazeyras et al., 2016; Lazeyras & Schmidt, 2018; Barreira et al., 2021; Zennaro et al., 2022). In this work, and in particular in chapter 5, we shall concentrate on testing the “empirical” form of coevolution relations, by comparing the recovered values and trends of b_2, b_s, b_{∇^2} against b_1 . The theory is known not to be mature enough to provide tight and exact fitting result. Rather, general phenomenological trends and the definition of the maximum order of the polynomial chosen for the coevolution relation fitting will be presented, together with the test of possible differences between the coevolution laws for halos and for galaxies (Lazeyras et al., 2016).

Emulators and the use of `baccoemu`

In contemporary cosmology, implementation of analytic results in computer code is often pursued for a number of scopes. Linear results can be obtained with codes such as `CAMB` or `CLASS`, and it is clear that possessing computational tools capable of reproducing the more complex results of higher-order perturbative theories and easily providing non-linear power spectra template, both for the matter field and, even more so, for biased tracers, is extremely interesting to the community for several goals (Dvorkin et al., 2022; Alvarez et al., 2022; Anninos, 2001; Lahav, 2023; Ting, 2025).

Concerning the *matter* power spectrum, a fundamental tool is represented by the `HALOFIT` (Takahashi et al., 2012) power spectrum templates: available since the beginning of the Millennium, and among the first frameworks capable of overcoming simplified analytic results and concurring with the development of the halo model formalism, it is a relatively cosmology-independent and robust interpolation of power spectra extracted from different simulations. It is a simple and useful computational prescription accounting for non-linear corrections to the matter power spectrum, which still leaves open the problem of treating biased fields, which can be generated only by including multiplicative factors correcting the underlying `HALOFIT` matter power spectrum (Smith et al., 2003) in the same spirit of the linear formulation of eq. 1.33.

In general, closed-form analytic expressions for kernels and non-linear power spectrum terms formulated in higher-order perturbative expansions are not granted to be found. We might thus rely on computers also in order to perform the required integrals, such as `CLEFT` and `CLEFT-GSM` (Vlah et al., 2016; Modi et al., 2017), `velocileptors` (Chen et al., 2020, 2021) or `FastPT` (McEwen et al., 2016; Fang et al., 2017; Blazek et al., 2019), but this task can be computationally heavy, with typical calculation times for every integral in the order of 0.2 seconds (Zennaro et al., 2023). Blending the idea at the base of the `HALOFIT` approach of extracting non-linear templates from simulations with theoretical results, methodologies based on advanced interpolation strategies relying on Machine Learning and Neural Networks have thus been progressively suggested, opening the field of research on cosmological emulators. Emulators in Cosmology represent a wider class of computational tools: the general idea is that of using available simulation datasets and snapshots (Aricò et al., 2021b; Jense et al., 2025) calculated at different redshifts, scales and with different fiducial cosmologies, either derived from a FLRW metric but with the parameters set at values different from the Λ CDM ones (“different cosmologies”), as well as relying on other metrics and theories of gravity different from General Relativity. These snapshots represent multidimensional points in the space of the variables of interest that can be interpolated by means of suitable strategies, spanning from Gaussian Processes to Neural Networks. The interpolated mesh can then be used to generate customised cosmologies within the dynamic range of the original datasets and summary statistics spanning from power spectra to halo mass functions can be generated in a

similar way, by interpolating the summary statistics, and it has become customary for many large collaborations to release emulators of their reference cosmological simulations (Euclid Collaboration et al., 2019, 2021; Zhang et al., 2025). Even emulators of Boltzmann codes such as CAMB or CLASS have been made available to the cosmological community, thus skipping the calculation overhead required even in linear theories (Bolliet et al., 2024; Jense et al., 2025).

The emulator used in this work, **baccoemu**, is capable of generating power spectra templates at any given redshift, scales and reference cosmologies within a range based on a suite of 8 N -body simulations obtained within the BACCO simulation project (Angulo et al., 2021), conceived in order to cover an adequate range of cosmological parameters. The extensive dynamic range is obtained by means of a computational technique known as cosmology-rescaling (Angulo & White, 2010): the methodology starts from the idea that the non-linear structure in a given cosmology can be mimicked by rescaling the outputs of a simulation carried out in a nearby cosmology, once the length units are changed and the redshift of the original snapshot is remapped into an epoch in the final cosmology having at each scale the same field variance of the original simulation output. A comparison among different strategies to calculate the linear and non-linear matter and galaxy three dimensional power spectra, comprising linear theory, **halofit** and HEFT-LPT results based on the **baccoemu** emulator is displayed in fig. 1.5.

In the specific case, the “parent” cosmological simulations are generated from N -body codes calculated on boxes of side length $L = 1440 h^{-1}\text{Mpc}$ populated with 4320^3 particles. Four distinct cosmologies are then evolved, the parameters being chosen so that they can cover a 10σ dispersion around the best-fit values of the ΛCDM model obtained from Planck observations, calculating the gravitational evolution with an updated version of **L-Gadget3** (Springel, 2005; Angulo et al., 2021) and adapted to non-linear scales with a convergence better than 2% at $k \sim 10 h \text{Mpc}^{-1}$, further compared with other available codes. Once the original dataset is generated, further snapshots at desired cosmological parameter sets, redshift and scales are generated by means of the rescaling strategy. In order to improve the result, large-scale modes are further modified by additionally displacing the positions assigned to the particles in a simulation by calculating, and adding, the difference, calculated with a two-loop LPT approach, in the displacement fields between the original and target cosmologies; similar treatments are applied to velocities as well.

Given the suite of simulations, many fields, of either actual cosmological observables or of theoretically useful quantities, as well as different summary statistics, can be extracted based on the final goals: for the LPT emulator generator, the researchers started from a non-linear bias operator formulation where the following five fields $\{1, \delta, \delta^2, s^2, \nabla^2 \delta\}$ are employed. The fields are extracted from the N -body simulation suite in Lagrangian coordinates using a grid of 1080^3 points, on which a further Gaussian smoothing with a scale of size $k_d = 0.75 h^{-1}\text{Mpc}$, associated to the typical size of Dark Matter halos in the Lagrangian space, is added, mimicking possible exclusion effects and other contributions deriving from the limited resolution of the simulations, which are assumed to be captured by the b_{∇^2} term. The results are then advected to the Eulerian space and stored as the datasets from which further interpolations can be performed. Eventually, the emulation phase is performed, following the recipe of (Angulo et al., 2021) and (Aricò et al., 2021a). A feed-forward Neural Network performs the interpolations between the extracted LPT power spectrum addenda one by one (in other words, the Neural Network is interpolating, for every fixed i, j pair, the different P_{ij} as recovered from the available original simulation-based fields). The architecture is based on two fully-connected hidden layers with 200 neurons each and a Rectified Linear Unit activation function and it is preferred to other methodologies because less computationally expensive in

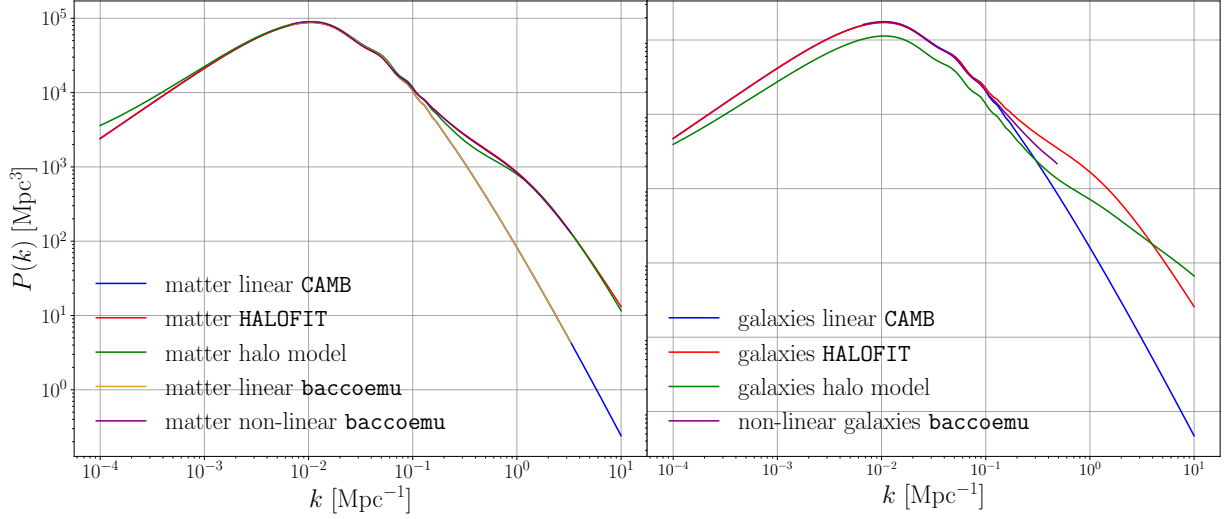


Figure 1.5: Comparison among different theoretical frameworks predicting the three-dimensional matter power spectrum (left) and the galaxy autocorrelation power spectrum (right) at $z = 0$ for a reference Planck cosmology (Planck Collaboration et al., 2020b). On the left, the linear power spectrum as can be recovered by exactly solving the linear theory equations (blue) is compared with the non-linear, simulation-calibrated template of HALOFIT (red) (Takahashi et al., 2012; Smith et al., 2003), with the linear prediction from the baccoemu emulator (gold) and the corresponding non-linear one (purple). The halo model power spectrum (green) is also displayed, showing the peculiar different trend at the transition scales between the linear and non-linear regimes ($10^{-1} \leq k \leq 1 \text{ Mpc}^{-1}$). The spectrum is calculated assuming a virial mass definition, a Sheth-Tormen 1999 (Sheth & Tormen, 1999) HMF, the Sheth-Tormen 1999 (Sheth & Tormen, 1999) bias function, a NFW (Navarro et al., 1997) density profile and a Duffy 2008 (Duffy et al., 2008) concentration. On the right, the discussion is extended to biased tracers – namely, galaxy clustering autocorrelation power spectra. The linear power spectrum (blue) and the HALOFIT-based non-linear (red) predictions are calculated assuming a linear bias factor $b_1 = 1.40$ multiplying the matter power spectrum. The non-linear HEFT power spectrum (purple) is calculated assuming the values of (0.4, -0.14, 0.01, 2.43) for the four HEFT bias factors ($b_1, b_2, b_s, b_{\nabla^2}$), defined such that $b_1^{\text{Eulerian}} = 1.4$ corresponds, in agreement with the LPT formulation, to $1 + b_1$. Due to the original baccoemu training range, the scale range for which this latter power spectrum is defined is narrower than the ones covered by the other models. The halo-model based prediction (green) uses the same recipe of the halo model matter power spectrum, adding the default HOD and two-point correlators from CCL as utilised in the tutorial at the official GitHub repository shown at this link. Since the halo model uses a different bias formulation which cannot be directly matched to the previous ones, it is displayed only for completeness.

high dimensions and for abundant training data. Every cross-spectrum term is emulated by a separate neural network which is first trained and then tested for validation. Notice that, in this framework, the power spectra returned by baccoemu § when its functions are invoked do not include a “stochastic bias” term ϵ that other LPT formulations include, accounting for the impact of small-scale density fluctuations on galaxy formation (Desjacques et al., 2018; Rubira & Schmidt, 2025). In the simplest case, the term can be added *a posteriori* by treating galaxies as a Poisson

§Details, source code and documentation about the emulator can be recovered at this link.

sampling process of an underlying smooth galaxy density field, so that the noise in auto-correlation power spectrum reads $P_{\epsilon\epsilon} = 1/\bar{n}_g$, where $\bar{n}_g$ is the comoving number density of galaxies. Notice that **baccoemu** is capable of producing LPT power spectra for both biased (especially, galaxies) fields and also for the matter field only, having all bias parameters set to 1, as well as cross-correlations between fields. In this latter case, the shot-noise term is always zero. In the end, the emulator is capable of producing the fifteen P_{ij} entering in the calculation of the full LPT power spectra as its ultimate output, letting the user the freedom to have them calculated at arbitrary redshifts, cosmologies and scales within the admitted dynamic range, effectively acting as power spectrum generators without the requirement of explicitly solving the perturbative equations, but only the interpolation of the quantities calculated at reference redshifts, scales and cosmologies.

1.4 Angular power spectra for observables projected on the celestial sphere

The discussion entertained thus far provides us with a theoretical prediction of the three dimensional power spectrum $P(k)$, enabling us to study the properties of the fields under scrutiny by sampling their modes both in the transverse direction and in depth along the line of sight. This ideal property, giving us the maximum of the information concerning two-point estimators of an almost Gaussian field, sliced in redshift with arbitrary precision and with complete information about all spatial directions, might however be difficult to exploit in practice even in absence of observational systematics. Strictly speaking, in cosmology, as well as in astrophysics, this situation can actually be rather unnatural (Bonvin & Durrer, 2011; Asorey et al., 2012; Alonso et al., 2019). Indeed, observations along the line of sight imply observations of tracers across different redshifts and thus, if the temporal evolution of the quantities is fast enough, it invalidates the assumption that we are actually observing a constant “snapshot” of the LSS status at some reference redshift z_{ref} and its related power spectrum $P(k, z_{\text{ref}})$, actually studying a superposition of different evolutionary stages. There might be additional properties of the cosmological observables making the use of $P(k, z_{\text{ref}})$ questionable or delicate:

- They might be actually lacking of redshift information, appearing as inherently projected fields on the sphere, giving us the cumulative contribution of all physical processes virtually happened from the Big Bang Epoch up to today –it is the case of electromagnetic backgrounds such as the UGRB, of gravitational wave backgrounds or of weak lensing maps both from the LSS and the CMB.
- The redshift information can be recovered but with limited precision, as it is the case for photometric galaxy clustering samples. In the limit, since also spectroscopic measurements of redshifts are affected by uncertainties, *all* cosmological observables can be considered as lacking a perfectly reliable redshift information.
- They provide us information about processes that happened at a well-defined redshift, with almost no time evolution, so that they are intrinsically two-dimensional observables. This is the case for the CMB: recombination process can be considered to have occurred almost instantaneously at $z \sim 1100$.

All these considerations lead to the consideration that the correctness in the estimation of the three-dimensional power spectrum is always limited and subject to the effect of more assumptions than

the ones we should have to make if we would limit ourselves to measuring pairs of points projected on the celestial sphere. In this second case, we could indeed discard the cosmology-dependent definition of spatial positions, distances and sizes, considering instead the *angular* separation between tracers. We could then define the two-point *angular* correlation function in the real space or a corresponding transform, constructing more intuitive, safer or more meaningful statistics, extracted from the only actual thing we are actually measuring, projected fluctuations of fields on the celestial sphere (Alonso et al., 2019). Although implying a further level of spatial compression, working therefore on a two-dimensional projection of the power spectrum on the celestial sphere might be more natural and free from biases due to possible assumptions on redshift information. Furthermore, even if our redshift information is reliable enough to measure an appropriate $P(k)$, we may nevertheless decide to work with narrow “tomographic”, slices of the observables, studied at a fixed redshift as two-dimensional projected fields, a task for which again we need to calculate the two-dimensional angular power spectrum. In cosmology, this quantity is usually noted as C_ℓ and, in analogy with the Fourier-space formulation of the $P(k)$, two-point correlations are usually dealt with in the spherical harmonics space rather than in the real space, so that, instead of angular separations we actually work with the multipoles ℓ as a measure of angular scales, bearing in mind the relation between an angular separation scale θ in radians and the associated multipole $\ell = \frac{\pi}{\theta}$, which in the literature is either more correctly reported in inverse radians units or treated as a dimensionless coordinate. The “conversion” from a three-dimensional fluctuation to a two-dimensional one and hence from the $P(k)$ to the C_ℓ is derived below. We consider a generic three dimensional density field $\rho(\mathbf{x}, z)$, that we might be able to actually measure as either both a bi-dimensional or three-dimensional quantity (as for galaxy clustering observations) or be a purely theoretical quantity that we have to model and constrain “backwards” given only its bi-dimensional realisation (as in the case of unresolved astrophysical backgrounds). $\rho(\mathbf{x}, z)$ can be reparameterised as a function of the comoving radial coordinate χ [¶] and of the direction versor $\mathbf{n}$ (with unitary module), so that the following relation holds:

$$I(\mathbf{n}) = \int d\chi \rho(\chi, \mathbf{n}) \tilde{W}(\chi) , \quad (1.122)$$

The quantity $\tilde{W}(\chi)$ will not be directly used in the following discussion: we therefore first introduce the average quantity $\langle I \rangle$ so that, applying the ensemble average operator $\langle \cdot \rangle$ on eq. 1.122 we have $\langle I \rangle = \int d\chi W(\chi)$, leading to the actual quantity of interest $W(\chi) = \bar{\rho} \tilde{W}(\chi)$, where $\bar{\rho}$ is the ensemble average of the field associated to the tracer we are considering (notice that, depending on the case, the field might be a physical quantity, such as a density or an intensity, or a “contrast” with respect to the average). $W(\chi)$ (without the tilde superscript) is known as the “window function” of the tracer and it is specific to it. It depends on the radial distance but not on the specific direction $\mathbf{n}$ and encapsulates either a theoretical assumption on the redshift distribution of the tracer (angular power spectra can thus be exploited also as a tool to reconstruct the actual redshift distribution of a tracer) or, if available as in the case of galaxy surveys, its actual redshift distribution.

[¶]Note that with this definition we are working using the comoving distance χ as integration variable, but this is just a matter of consistency with the reference derivation we follow, (Fornengo & Regis, 2014), and with the notation used in the research chapters of this thesis. The C_ℓ can be recovered also by working directly in terms of z instead of χ and the conversion between the two quantities can be performed at the very end of the derivation, without influencing the previous calculations.

We can then construct the fluctuation field $\frac{\delta I(\mathbf{n})}{\langle I \rangle} \equiv \frac{I(\mathbf{n}) - \langle I \rangle}{\langle I \rangle}$ that, if it is a scalar field like in our case, can be expanded in the scalar, spin-0 spherical harmonics base $Y_{\ell m}$, by setting

$$\delta I(\mathbf{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\mathbf{n}). \quad (1.123)$$

Given the general theory of harmonic functions, the decomposition above effectively corresponds to the projection of the original three-dimensional field ρ on the sphere by means of the spherical harmonics as basis functions where the $a_{\ell m}$ coefficients can be expressed by:

$$a_{\ell m} = \frac{1}{\langle I \rangle} \int d\mathbf{n} \delta I(\mathbf{n}) Y_{\ell m}^*(\mathbf{n}). \quad (1.124)$$

We can reformulate the quantity above in terms of the three-dimensional density contrast $\delta(\chi, \mathbf{n})$, in order to proceed more closely to the $P(k)$ formalism, thanks to the following chain of identities:

$$\begin{aligned} \frac{\delta I}{\langle I \rangle} &= \frac{I}{\langle I \rangle} - 1 = \frac{1}{\langle I \rangle} \int d\chi \rho(\chi, \mathbf{n}) \frac{W(\chi)}{\bar{\rho}(\chi)} - 1 = \frac{1}{\langle I \rangle} \int d\chi W(\chi) \left(\frac{\rho(\chi, \mathbf{n})}{\bar{\rho}(\chi)} - 1 + 1 \right) - 1 = \\ &= \frac{1}{\langle I \rangle} \int d\chi W(\chi) \delta(\chi, \mathbf{n}) + \frac{1}{\langle I \rangle} \int W(\chi) d\chi - 1 = \frac{1}{\langle I \rangle} \int d\chi W(\chi) \delta(\chi, \mathbf{n}) \end{aligned} \quad (1.125)$$

where the last step is obtained by recalling that $\langle I \rangle = \int d\chi W(\chi)$. We can thus write

$$a_{\ell m} = \frac{1}{\langle I \rangle} \int d\mathbf{n} d\chi \delta(\chi, \mathbf{n}) W(\chi) Y_{\ell m}^*(\mathbf{n}). \quad (1.126)$$

Notice that, in spite of the close analogy with the three-dimensional formulation, we cannot fully reabsorb $\frac{\delta I(\mathbf{n})}{\langle I \rangle}$ in the density contrast δ because of the intervening integrals and we are thus forced to carry with us the normalisation $\langle I \rangle$ and keep it explicit. Using the Rayleigh expansion of a plane wave into spherical harmonics and spherical Bessel functions j_ℓ we can derive the following expression:

$$\begin{aligned} a_{\ell m} &= \frac{1}{\langle I \rangle} \int d\mathbf{n} d\chi \frac{d\mathbf{k}}{(2\pi)^3} \delta(\chi, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} W(\chi) Y_{\ell m}^*(\mathbf{n}) \\ &= \frac{1}{\langle I \rangle} \int d\mathbf{n} d\chi \frac{d\mathbf{k}}{2\pi^2} \delta(\chi, \mathbf{k}) \left[\sum_{\ell' m'} i^{\ell'} j_{\ell'}(k\chi) Y_{\ell' m'}^*(\mathbf{k}) Y_{\ell' m'}(\mathbf{n}) \right] W(\chi) Y_{\ell m}^*(\mathbf{n}) \\ &= \frac{i^\ell}{\langle I \rangle} \int d\chi W(\chi) \int \frac{d\mathbf{k}}{2\pi^2} \delta(\chi, \mathbf{k}) j_\ell(k\chi) Y_{\ell m}^*(\mathbf{k}), \end{aligned} \quad (1.127)$$

with the position $\mathbf{r} = \chi \mathbf{n}$, so that the modulus of $\mathbf{r}$ is $r = \chi$, $\mathbf{n}$ being a unit vector. We can now define the correlator between two fields f^i and f^j expanded in the spherical harmonics basis in analogy with the definition of the three-dimensional power spectrum expressed in eq. 1.27

$$\langle f_{\ell m}^i f_{\ell' m'}^j \rangle = \delta_{\ell \ell'}^K \delta_{m m'}^K C_{\ell}^{ij}, \quad (1.128)$$

with the two Kroenecker deltas $\delta_{\ell \ell'}^K$ and $\delta_{m m'}^K$ enforcing the lack of correlation between distinct angular scales. The angular power spectrum is defined as

$$C_{\ell}^{(ij)} = \left\langle \sum_m a_{\ell m}^{(i)} a_{\ell m}^{(j)*} \right\rangle, \quad (1.129)$$

and, invoking also the relation $a_\ell = (-1)^m a_{\ell-m}^*$, we can take further steps:

$$\begin{aligned}
C_\ell^{(ij)} &= \frac{1}{\langle I_i \rangle \langle I_j \rangle} \int d\chi W_i(\chi) \times \\
&\quad \int d\chi' W_j(\chi') \int \frac{d\mathbf{k}}{2\pi^2} \int \frac{d\mathbf{k}'}{2\pi^2} \langle \delta_i(\chi, \mathbf{k}) \delta_j^*(\chi', \mathbf{k}') \rangle j_\ell(kr) j_{\ell'}(k'r') Y_{\ell m}(\mathbf{k}) Y_{\ell' m'}^*(\mathbf{k}') \\
&= \frac{1}{\langle I_i \rangle \langle I_j \rangle} \int d\chi W_i(\chi) \int d\chi' W_j(\chi') \int \frac{d\mathbf{k}}{2\pi^2} \int \frac{d\mathbf{k}'}{2\pi^2} (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P_{ij}(k, \chi, \chi') \times \\
&\quad \times j_\ell(kr) j_{\ell'}(k'r') Y_{\ell m}(\mathbf{k}) Y_{\ell' m'}^*(\mathbf{k}') \\
&= \frac{2}{\pi \langle I_i \rangle \langle I_j \rangle} \int d\chi W_i(\chi) \int d\chi' W_j(\chi') \int d\mathbf{k} P_{ij}(k, \chi, \chi') j_\ell(kr) j_{\ell'}(kr') Y_{\ell m}(\mathbf{k}) Y_{\ell' m'}^*(\mathbf{k}) \\
&= \frac{2}{\pi \langle I_i \rangle \langle I_j \rangle} \int d\chi W_i(\chi) \int d\chi' dk k^2 W_j(\chi') P_{ij}(k, \chi, \chi') j_\ell(kr) j_{\ell'}(kr') \\
&= \frac{1}{\langle I_i \rangle \langle I_j \rangle} \int \frac{d\chi}{\chi^2} W_i(\chi) W_j(\chi) P_{ij} \left(k = \frac{\ell + 1/2}{\chi}, \chi \right)
\end{aligned}$$

where the three-dimensional power spectrum is introduced by recalling that $\langle \delta_i(\chi, \mathbf{k}) \delta_j^*(\chi', \mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P_{ij}(k, \chi, \chi')$. The transition from the penultimate to the last line is performed by resorting to the Limber approximation (Limber, 1953; Kaiser, 1992) that allows us to eliminate the Bessel functions j_ℓ . Under the Limber approximations, both Bessel functions $j_\ell(kr)$ and $j_{\ell'}(kr')$ can be approximated by two Dirac deltas:

$$j_\ell(x) \simeq \sqrt{\frac{\pi}{2\ell+1}} \delta \left(\ell + \frac{1}{2} - x \right). \quad (1.130)$$

Combined with the fact that $r = \chi$ in module, this allows to eliminate one of the two integrals in the comoving distance and to simplify the other one. Without the Limber approximation, the C_ℓ would have explicit dependence on k whereas, with this approximation, k can be written as a function of the integration variable χ and therefore eliminated. Limber approximation holds true as long as the scales of interest are at intermediate or large ℓ . This property is typically associated to small χ and large k , since it physically means that the Bessel functions at a given angular scale are almost constant in their spatial dependence or, equivalently, that their oscillatory pattern varies more slowly than the variation of the window functions $W(\chi)$ within the corresponding spatial scale associated to the comoving distance value χ . Importantly, the relation established between ℓ and k is redshift dependent: in general, smaller k mean smaller ℓ and vice versa, but, at very low redshift, even rather small ℓ , associated to large angular separations, correspond to relatively large k , hence small scales. This on one hand means that one must be careful with understanding whether observations are accessing the linear or non-linear regime of the projected tracers, while on the other hand that even large angles convert, at low redshifts, into relatively small “fields of view”, limiting the access we have to larger scales (we show and comment this in fig. 1.6). Focusing on

$$C_\ell^{ij} = \frac{1}{\langle I_i \rangle \langle I_j \rangle} \int \frac{d\chi}{\chi^2} W_i(\chi) W_j(\chi) P_{ij} \left(k = \frac{\ell + 1/2}{\chi}, \chi \right) = \int \frac{d\chi}{\chi^2} \mathcal{I}(\chi, \ell) \quad (1.131)$$

and possibly reabsorbing the $\langle I \rangle$'s in the definitions of the window functions, as it often happens in the literature, we are thus left with a connection between the angular and the three-dimensional power spectrum. Building up an estimator for the C_ℓ^{ij} calculated between two tracers can thus be connected to the combined specification of a model for the associated window functions $W(\chi)$ and a model for the three-dimensional power spectrum $P_{ij}(k)$, either formulated in the linear theory or

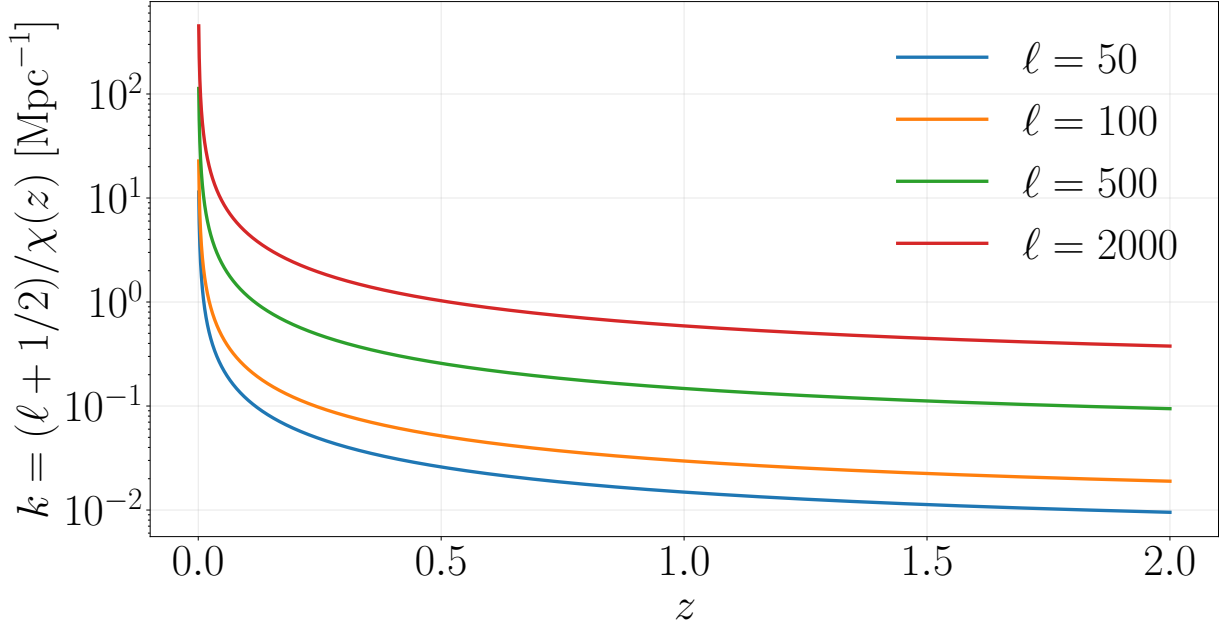


Figure 1.6: “Conversion” between the Fourier modes k and the multipoles ℓ as a function of various redshifts allowed by the Limber approximation (Limber, 1953). In general, larger ℓ are associated to larger k , so that, comparatively, a smaller multipole implies that we are probing larger scales. However, the corresponding k scales inversely with the redshift, so that at low redshifts the spatial scale associated to even relatively large multipoles correspond in fact to rather small, non-linear scales, effectively limiting the “width” of our field of view and the number of data points we may be actually analysing. Furthermore, we cannot measure an ℓ smaller than 0 (the monopole), therefore some very small k will never be accessed within a given redshift.

by means of more complex approaches, combined in the integrand $\mathcal{I}$ of eq. 1.131. Concluding this section, notice that the effect of the window function can be interpreted at least from two points of view, which will be at the basis of the weighting/Wiener filtering formalism described in chapter 2:

1. As a weight entering the sum of different power spectra being calculated at different distances (implicitly, redshifts) χ along the line of sight.
2. As an actual window or filter in a sense analogous to signal processing theory that, combined with the relation $k = (\ell + 1/2)/\chi$, allows to probe only a limited regime of scales k from the power spectrum given an angular separation ℓ (or vice-versa) so that also the redshift location of the tracers and the width of the window (see figs. 1.6 and 1.7) determines the amplitude and features of the observed signal. Furthermore, at every order for some tracers or for higher-order formulations of otherwise exclusively redshift-dependent tracers, the window function might be also an explicit function of ℓ or k , inducing additional complications, generally referred to as “mode-coupling” phenomena (Chisari et al., 2019a).

Convolution with instrumental systematics: including the Fermi beam model in the angular power spectrum Working with the power spectra, either in the spherical harmonics or in the Fourier space, provides the additional benefit of treating some systematics in a rather simple way, as long as their convolution with the cosmological signal in the real space can be transformed into a product of terms in the harmonic space. A typical effect of this kind is the limited resolution

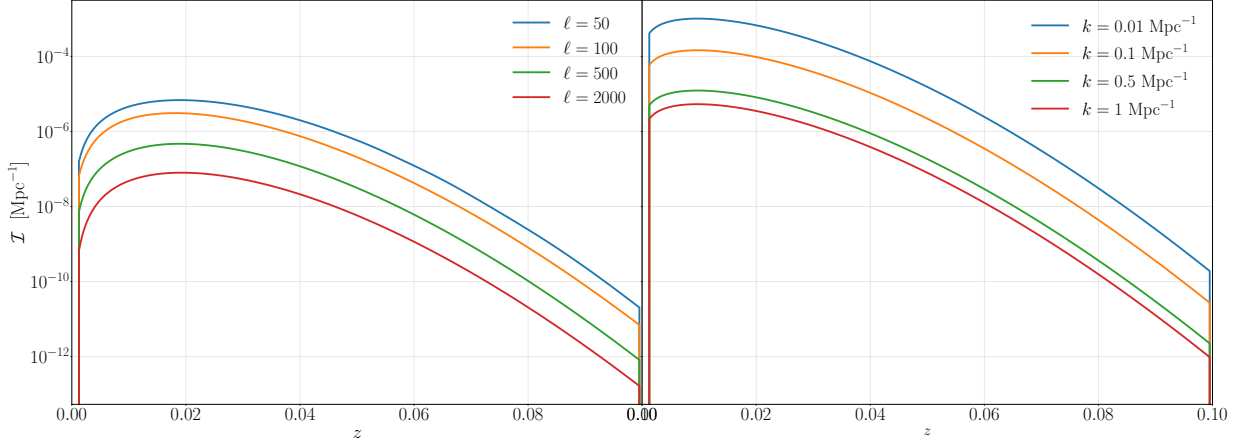


Figure 1.7: The integrand $\mathcal{I}$ of eq. 1.131. The displayed quantities are based on the window function (combining eqs. 1.62 and 3.12), the halo-model based power spectrum (defined in eq. 1.102) and the HOD (define in eqs. 1.59 and eqs. 1.60) of 2MRS, therefore truncated at the survey maximum redshift $z = 0.1$. $\mathcal{I}$ is displayed as a function of the redshift, both at fixed multipole ℓ (left) and Fourier mode k (right). We see that we can interpret the argument of the integral leading to the definition of C_ℓ as a weighted sum of different power spectra and that smaller multipoles are associated to larger power. In the left picture, we see that the amplitude of $\mathcal{I}$ is larger for smaller ℓ at every scale, although decreasing with z . At the same time, the contribution from larger scales (smaller k) always dominates at every redshift (right). However, because of the relation between ℓ and k of fig. 1.6, the very large scales k might not be accessed by low-redshift surveys, given the scale cuts of a real analysis, combined with the potential filtering effects of the window functions of the tracers.

of the instruments (usually denominated “beam” or Point Spread Function, PSF), which is usually included also in theoretical work as an unavoidable systematic effect downgrading the quality of the observations and related figures of merit.

In the galaxy clustering-UGRB cross-correlation formalism, theoretical terms involving γ -rays should be corrected because of the effect of the Fermi point spread function (PSF), a function of the energy and of the angular mode ℓ . The effect in angular space translates thus in multiplying correlation functions of the type $C_\ell^{g\gamma}$ by the expression of the Fermi PSF in the angular space, denoted with $B_{\ell,i}$, in a specific energy bin (i.e. $C_{\ell,i}^{g\gamma} \rightarrow C_{\ell,i}^{g\gamma} B_{\ell,i}$) while correlation functions of the type $C_{\ell,ij}^{\gamma\gamma}$ are corrected by applying the beam twice (i.e. $C_{\ell,ij}^{\gamma\gamma} \rightarrow C_{\ell,ij}^{\gamma\gamma} B_{\ell,i} B_{\ell,j}$). The noise terms derive instead from the Poisson fluctuations in photon number counts, calibrated *on the instrument* and consequently not requiring further beam corrections. In both the research chapters of chapters 3 and 5, terms involving galaxies instead do not resent of a beam function. Indeed, the angular resolution for galaxy catalogues is largely better than the one of the gamma-ray telescope. The galaxy survey limited resolution starts to become relevant only for multipole we exclude because they belong to a regime already significantly affected by the limitations of the Fermi PSF, above $\ell \sim 1000$.

The limited resolution imposed by the Fermi PSF implies that significant information from our summary statistics can be recovered only up to about $\ell \sim 1000$, associated to angular scales of about $0.1^\circ - 0.2^\circ$, much larger than the typical angular resolution of galaxy catalogues, instead in the order of the arcsec. In theoretical context, the actual complexity of the finite Fermi beam resolution in the real space can be simplified in terms of a Gaussian function model also displaying

a characteristic energy dependence (Ackermann et al., 2018), while of course actual data analyses require more realistic assumptions. The transform in the harmonic space of the Gaussian PSF is again a Gaussian term, using the parametrisation of Ref. (Pinetti et al., 2020), based on the measured Fermi PSF (Ackermann et al., 2018)

$$B_\ell(E_b) = \exp\left(-\frac{\sigma_b(\ell, E_b)^2 \ell^2}{2}\right), \quad (1.132)$$

where :

$$\sigma_b(\ell, E_b) = \sigma_0^{\text{Fermi}}(E_b) [1 + 0.25 \sigma_0^{\text{Fermi}}(E_b) \ell]^{-1}. \quad (1.133)$$

with

$$\sigma_0^{\text{Fermi}}(E_b) = (\sigma_0^{\text{Fermi}}(E_{\text{ref}}) \times (E_b/E_{\text{ref}})^{-0.95} + 0.05) \frac{\pi}{180} [\text{rad}]. \quad (1.134)$$

$E_b = \sqrt{E_{\text{min},b} E_{\text{max},b}}$ stands for the reference energy of the b^{th} energy bin, with $E_{\text{min},b}$ and $E_{\text{max},b}$ its boundary energies. All the quantities related to the energy bins, together with the values of $\sigma_0^{\text{Fermi}}(E_{\text{ref}})$, are listed in table 3.1.

Part I

Techniques for indirect detection of WIMP Dark Matter by means of LSS tracers-UGRB cross-correlations

Chapter 2

Astroparticle Physics and Dark Matter indirect detection techniques

2.1 General framework: addressing the particle nature of Dark Matter

As presented in the [Introduction](#), the Dark Matter (DM) paradigm poses on one hand problems of astrophysical-dynamical nature, related with how the formation of cosmic structures, their density profiles and the kinematics of celestial bodies undergoing gravitational effects from Dark Matter should appear, especially at the smallest scales. As well, under its understanding as an additional kind of matter to be added to the one described by the Standard Model of Particle Physics, the dark matter model asks for determining its microphysical properties and, possibly, on how to eventually detect it by means of particle physics technique. Relying by now on about fifty years of research efforts, the number of models and techniques, theoretical aspects as well as observational approaches is extremely vast, covering several mass ranges (see [fig. 2](#)), interaction channels and results, that are indeed thoroughly exposed in sometimes imposing works such as ([Cirelli et al., 2024](#)) and, more specific to the UGRB and WIMP paradigm, in ([Fornasa & Sánchez-Conde, 2015](#); [Negro, 2018](#); [Pinetti, 2022](#)), all readings which can act as a main reference and that we will use as a fundamental guideline for this exposition as well.

Related to the theoretical landscape is the quest for observational counterparts as final confirmation of the correct theory of Dark Matter. Setting about to eventually detect and demonstrate the existence of particle Dark Matter, three main strategies can be devised: production of DM through interaction of SM particles, a process perhaps observable in colliders, which nonetheless requires that potential Beyond Standard Model (BSM) particles observed on Earth are the same we are looking for in Heavens; DM interaction with SM particles (direct detection), being tested in dedicated experiments, and, finally, the possibility for DM particles to annihilate or decay in SM particles (indirect detection) ([Drees, 2018](#); [Navas et al., 2024](#)). Different DM particle models assume different interaction channels and production mechanisms: the direct detection setup substantially treats Dark Matter as a particular form of cosmic ray and aims to observe interactions between DM particles with ordinary matter by measuring the recoil energy imparted to nuclei in a detector, using a number of experimental setups, such as cryogenic detectors (examples include the CDMS ([Aralis et al., 2020](#)) and EDELWEISS ([Armengaud et al., 2013](#)) experiments), noble liquid detectors (notable experiments are XENON ([Aprile et al., 2016](#)), LUX ([Akerib et al.,](#)

2013), and PandaX (Cheng et al., 2021)), scintillators (DAMA/LIBRA (Bernabei et al., 2018) and COSINE-100 (Adhikari et al., 2022) experiments) or directional detectors, i.e. time projection chambers able to track both energy and direction of nuclear recoils (Misiaszek & Rossi, 2024).

Indirect detection methodologies (Conrad, 2014) are the most relevant for this work and focus on the hypothesis that DM particles, either self-interacting, or through interaction with Standard Model particles, produce specific final states (such as gamma rays, neutrinos, or cosmic rays) that we might observe and that may not be fully explainable in terms of Standard Model particle interactions or, in the most fortunate case, that might even be unambiguous signatures of BSM processes. Methodologies include: gamma-ray observations by means of space-based telescopes like Fermi-LAT (Ackermann et al., 2012b, 2015, 2018; Atwood et al., 2009) and ground-based Cherenkov telescopes such as CTA (Actis et al., 2011; Maier et al., 2017), HESS (Cornils et al., 2003; Bernlöhner et al., 2003; Abdalla et al., 2021) and MAGIC (Aleksić et al., 2016), able to observe the Universe in the gamma-ray band, focusing both on the cosmos as a whole and on specific regions where the density of Dark Matter is expected to be higher and thus interactions more likely, for instance the Milky Way centre or dwarf spheroidal galaxies. Similarly, neutrino telescopes like Km3Net (KM3NeT Collaboration et al., 2024; Margiotta, 2022, 2014), IceCube (Aartsen et al., 2017) and ANTARES (ANTARES Collaboration et al., 2016) can detect high-energy neutrinos that could in part result from Dark Matter annihilation in celestial bodies like the Sun or Earth, while cosmic ray detectors such as AMS-02 (Tomasetti & AMS Collaboration, 2015; Battiston, 2008) and PAMELA (Adriani et al., 2014) measure or have measured cosmic ray fluxes, searching for excesses in positrons or antiprotons that might indicate DM interactions. Eventually, collider production experiments attempt to create Dark Matter particles through high-energy collisions, observing missing energy and momentum as a signature of Dark Matter particles escaping the detector without significant traces due to their weakly interacting nature (Boveia & Doglioni, 2018).

Concerning achieved results, direct detection experiments have not been able to claim any consensus detection so far, but, from a strictly technical point of view, they are cutting-edge instruments: the XENON-1T instrument, for example, has not detected any Dark Matter-nucleon scattering, showing in any case the capability to probe physical processes having cross-sections as small as $6.3 \cdot 10^{-42} \text{ cm}^2$ in some mass regimes (Aprile et al., 2020, 2024, 2019), however useful for exploring the physics of very weak physical processes as well as stimulating important instrumental progresses. Similarly, the body of knowledge coming from collider production experiments has not claimed a consensus Dark Matter production signature as of 2025, although the absence of evidence has allowed to constrain or exclude some models (Boveia & Doglioni, 2018; De Roeck, 2024). Proposals concerning the construction of next-generation colliders are also being developed having in mind energy and luminosity levels able to probe DM production events (Naseem et al., 2019; Gonzalez Suarez, 2022; Bernardi et al., 2022; Andreev et al., 2025; Cid Vidal et al., 2018).

2.2 Indirect detection techniques

Indirect detection techniques representing the core topic of my research work, we will hereafter restrain the discussion to this sub-field, its definition and results, progressively heading towards the specific field of research about heavy-mass Dark Matter particle candidates and what results have or can be obtained in this context. The essential concept behind indirect detection techniques is the assumption that DM particles might undergo either pair-annihilation or decay processes

having as final states Standard Model particles. If this is actually the case, signals of DM processes would contribute to the overall cosmic ray fluxes measured on Earth or from space; at the same time, we would like to find processes for which an alternative astrophysical explanation for these signals is of the lowest possible likelihood. In the best-case scenario, an actual “smoking-gun” signal could be recognised as exclusively coming from some Dark Matter candidate particle, without any astrophysical alternative explanation of similar constraining power (Gaskins, 2016; Klasen et al., 2015; Pérez de los Heros, 2020; Chau & Aguilar, 2023; Fornengo & Regis, 2014; Ullio et al., 2002; Bergström et al., 2001; Stecker, 1978; Rudaz & Stecker, 1988). In terms of increasing ambiguity between an astrophysical and DM interpretation of the processes and signatures they probe, the messengers we are currently able to detect thanks to the variety of cosmic ray telescopes at our disposal span from photons, neutrinos, positrons, anti-protons, up to anti-deuterium; heavier anti-nuclei such as anti-helium are also considered. In general, we shall chiefly focus on “prompt” messengers, i.e., we look for products of processes in which DM directly converts into the particles we then detect. Researchers are nevertheless working on the topic of secondary particles as well, that is, SM particles being produced as an outcome of interactions among SM particles who were originally produced by DM annihilation or decay processes, see e.g. (Cirelli et al., 2024).

Broadly speaking, the main “competitors” to any suggested Dark Matter signature are the astrophysical processes in the same energy ranges capable of producing the same final states and, possibly, with greater experimental evidence and statistical significance, so that, in more precise terms, the idea for DM indirect detection techniques can be formulated as *looking for DM or other Beyond-Standard-Model (BSM)* processes in terms of excesses against the reference signal represented by the predicted or measured irreducible astrophysical contributions*. Sometimes, and interestingly, theory predicts that DM events should display distinctive and different features compared to astrophysical ones: for instance, in terms of their spatial location or signal typology. A typical case is represented by Dark Matter searches in the Milky Way environment: DM in our galaxy should almost isotropically occupy the Milky Way spheroidal halo, with a density increasing at decreasing distance from the center of our Galaxy, whereas astrophysical objects would typically reside in the disk of the Milky Way. In the case of extragalactic observations, some objects in the Universe are expected to be richer in Dark Matter, such as dwarf spheroidal galaxies (DSph), and therefore interesting targets to observe. Next to theoretical modelling, the quality of the observations and the conclusions we can infer from them are always conditional on the signal-to-noise ratios instruments can achieve, so that additional variability factors such as competition among foregrounds, intensity and time frequency of the sought-after signals and technological capability are also relevant factors guiding the attention of researchers towards the most promising strategies and the setting of priorities in research programmes.

*BSM processes not associated to Dark Matter span, but are not exhausted by, from unification of fundamental interactions at larger energy scales, to symmetries or novel properties of neutrinos (Baer, 2024; Lee, 2021; Peskin, 1997; Lykken, 2010)

2.2.1 The main astrophysical and cosmological candidate messengers of Dark Matter

Photons

High-energy electromagnetic radiation: γ -rays Photons belonging to the γ -ray band of the electromagnetic spectrum provide information both at the level of energy spectrum and of angular (morphological) distribution of the associated sources (Cuoco, 2020; Leane, 2020; Bergström et al., 2001; Stecker, 1978; Ullio et al., 2002; Cirelli et al., 2024). At least for what concerns the galactic environment, absorption is negligible, whilst this effect must be taken into account when considering extragalactic photons: signals get suppressed in intensity and this absorption might determine the loss of part of the direction information, determining a “blurring” of the morphological data (Franceschini et al., 2008). Typical observational targets are the Milky Way galactic centre, dwarf galaxies and galaxies belonging to the Local Group, up to scales of cosmological interest, represented by galaxy clusters and the distribution of DM in the Universe, which should determine the ultimate shape the LSS.

Low-energy electromagnetic radiation: from X-rays to the radio band “Prompt” low-energy photons can be emitted by decaying light DM particles in the keV to MeV ranges, typically associated to candidates such as sterile neutrino, axions or as evaporating primordial black holes (these latter representing alternative DM candidates not of fundamental particle nature). “Secondary” photons in this wide range of the electromagnetic spectrum can be also considered: for example, the outcome of interactions between electrons and positrons originated by DM processes, in turn, for instance, scattering with matter populating the galactic or the cosmic environment through channels such as inverse Compton scattering, synchrotron emission, or Bremsstrahlung (Todarello & Regis, 2025; Todarello et al., 2024; Regis et al., 2025; Boyarsky et al., 2019; Cirelli et al., 2024).

Charged electromagnetic messengers

Positrons, electrons, anti-protons and light anti-nuclei, being charged particles, interact with galactic and cosmic magnetic fields, that thus erase information on their original position. We hence lose morphological information and we can only measure their energy spectrum, which in turn might undergo relevant processing because of the many interactions particles experience in their travel towards the observer (Stecker et al., 1985; Rudaz & Stecker, 1988; Beischer et al., 2009; Cuoco et al., 2019; Salamon et al., 1990; Carlson et al., 2014; Donato et al., 2008; Cirelli et al., 2024). The solar wind is a specific source of disturbance below 1 GeV, while, generally speaking, the higher we go in energy, the lesser is the degree of “processing” the original signal undergoes, granting the possibility of extracting more relevant information from messengers at higher energies (Salati, 2022; Cuoco, 2020; Leane, 2020). Electrons and positrons often undergo many information-erasing scattering processes and can lose energy by means of Bremsstrahlung or synchrotron emission before reaching the observer, while typically antiprotons experience less scattering processes and are thus more likely to maintain “memory” of the energy of the original process. The lower degree of interaction associated to antiprotons is such that their total flux combines both local (events occurring in the direct surroundings of the Solar System) contributions and events originating further away in the Milky Way, thus requiring more articulate modelling of intervening astrophysical processes. (Rudaz & Stecker, 1988; Cuoco et al., 2019, 2017; Eichler et al.,

2016). Antinuclei fluxes, finally, are typically rather low because particle production probabilities are lower, although the competition from astrophysical processes is equally rather low, and peaking at different energies compared to the potential DM ones: this property makes this typology of cosmic rays particularly worth examination (Cirelli et al., 2014; Carlson et al., 2014; Donato et al., 2000, 2008; Baer & Profumo, 2005; Kadastik et al., 2010; Cirelli et al., 2024).

Neutrinos

Neutrinos are characterised by very low degrees of interaction, since their cross-sections are a function of the weak interaction: this is a blessing because this means that similarly to photons, neutrinos propagate freely in the Galaxy, undergoing even less disruptions than the former, but also a curse, since also their degree of interaction with detectors is extremely low, making them among the most elusive particles in nature. Furthermore, we are not able to directly detect them, and we are bound to measure the outcome of their scattering with charged particles we are then collecting at detectors. Energy and direction information reconstruction is thus limited, the main benefit being that the neutrino interaction cross-section increases with energy, allowing us to probe models assuming a very large DM particle mass, in a way similar to the way we use photons (Gallo Rosso et al., 2018; Herrera & Horiuchi, 2025; Meighen-Berger et al., 2025; Abazajian, 2021; Aartsen et al., 2018; Valle, 2006) .

2.2.2 Astrophysical and Cosmological techniques for indirect detection

Given the inventory of observables and candidate processes discussed in the previous section, we now concentrate our discussion on the potential body of evidence the scientific community has collected so far. Particular attention will be deserved to high-energy processes, since they are more tightly related to the WIMP dark matter particle class being the subject of this work, whose properties will be better outlined in section 2.3.1. We will distinguish between two main categories of phenomena:

- On one hand, **actual anomalies**, that is, observations qualified as puzzling or unexplained in astrophysical terms, which could be considered actual signatures of some kind of Dark Matter in the Universe, if explainable or predicted by some of our available models.
- On the other hand, we will examine and discuss the main classes and strengths of **constraints** that have been put on fundamental parameters. Especially, we shall concentrate on the upper bounds on the maximum value of Dark Matter annihilation or decay rates as a function of the candidate particle mass several observational campaigns have been able to put.

The research of Dark Matter signatures by directly looking at cosmological and astrophysical probes is also related to circumscribing the most interesting locations for its detection: some regions in the Milky Way and in the Universe are of particular interest for DM indirect detection strategies because they are expected to host high-density concentrations of dark matter or they are relatively free from astrophysical contamination. The main targets shall be outlined in the following lines (Fornasa & Sánchez-Conde, 2015; Cirelli et al., 2024) but, whether we are only able to put constraints or we will eventually associate some anomaly to actual detections of Dark Matter, we are required to define measurable quantities acting as an “identity card” for Dark Matter particles. In the field of indirect research techniques, these can be summarised in two main quantities: the candidate particle mass m_χ and its decay rate Γ or its average annihilation cross-section rate $\langle\sigma v\rangle$.

In this vein, available constraints represent the experimental results of measurement strategies having resulted in null detection of dark matter signatures, providing upper limits on the aforementioned Γ or $\langle\sigma v\rangle$ parameters as a function of the mass and mechanism through which the specific DM candidate is supposed to be observed. Results are usually displayed in terms of exclusion plots that often also include the comparison between the measured constraint and some “reference”, “typical” values for the measured parameters in reference models. Focusing, as this work will do, on annihilating dark matter particles, the reference quantity is usually represented by the annihilation rate parameter for a *thermal* Dark Matter relic, $\langle\sigma v\rangle_{\text{th}}$. This quantity can be derived under the assumption that Dark Matter particle production occurs in thermal equilibrium with the SM particles bath during the Early Universe. It is therefore among the first and simplest mechanisms having been proposed, also at the base of the WIMP Dark Matter particle paradigm (Lee & Weinberg, 1977; Baer et al., 2015; Kolb & Turner, 1990). Simplicity, an intuitive underlying physical process and the possibility of explicitly calculating the essential quantities make it a standard reference mechanism: the thermal equilibrium hypothesis allows us to develop the calculations regulating the synthesis of dark matter particles and defining its abundance and annihilation cross-section using the treatment also applied to Standard Model particles during the BBN, combining the description of a primordial plasma with that of an expanding background whose dynamics is governed by the Fridman equations discussed in chapter 1 and providing the expansion rate H (eq. 1.14).

Now, given two species a and b , their interaction rate R_a depends on the number density of the targets n_b , their mutual interaction cross-section σ_{ab} and the relative velocity between two particles v_r :

$$R_a = \sum_b n_b v_r \sigma_{ab}. \quad (2.1)$$

We start with imposing thermal, chemical and kinetic equilibrium on the reactions. The fundamental details to bear in mind are that:

1. Thermal equilibrium requires that all species and the related fluids in the bath have the same temperature, while chemical equilibrium allows particles to change their number by converting into each other. For greater precision, we remark that all equilibrium conditions are subordinated, in a relativistic framework, to causality conditions, so that it is more accurate to always speak about Local Thermal Equilibrium (LTE).
2. Kinetic equilibrium, instead, does not allow for changes in the number of particles (hence, if chemical equilibrium is broken but kinetic equilibrium is present, the abundance of a species stays fixed), but allows for reactions in which momentum is exchanged among species.

When thermal and chemical equilibrium will cease (**chemical decoupling**), the *abundance* of Dark Matter will be set, because the reactions will cease to be “two way”: the production channels for particles from initial to final states cease to be equally likely and final states become equilibrium states. The abundance of the reaction products of a species thus reaches its “freeze out” value. The term is used because, after that instant, the number of particles produced no longer changes (it “freezes”) by effect of interactions with the original bath of particles. The cosmological abundance of a cosmic fluid is thus definitively established, determining the value we should measure in the Universe in absence of processes of cosmological scale influencing their number (that for Dark Matter are no longer expected to occur), or might be modified only if a given species is expected to annihilate or decay. This also means that, if we are able to calculate the number of particles of some

species being produced during the Big Bang, we can predict the corresponding density parameter at every epoch $\Omega_i(z)$ and compare it with observed quantities, possibly falsifying our model or – conversely – to use the observed $\Omega_i(z)$ to constrain the free parameters for a given Dark Matter particle model. We remind that, in order to define the “coldness” property of the Dark Matter model, it is also important to consider a stage (typically holding also for some time after chemical decoupling) in which we allow for the **kinetic** equilibrium condition: until kinetic equilibrium holds, dark matter particles are **still exchanging momentum** by collision with other species or with themselves. After the end of kinetic equilibrium, all non-gravitational interactions between dark matter and the rest of the cosmic fluids cease and the free streaming process of the dark matter fluid can start, determining the largest suppressed scale and letting dark matter interact with the other cosmic species exclusively through its gravitational interaction (see also the [Introduction](#)). To calculate $\langle\sigma v\rangle_{\text{th}}$ we first have to concentrate on the thermal and chemical equilibrium phase and define the number density of DM particles in terms of the phase-space distribution f as a function of the bath temperature T and on the modulus of the particles 3-momentum p , from which we will derived the equilibrium number density of a species and other useful quantities. After having discussed this first stage, since we know that dark matter must have exited equilibrium conditions, we will set ourselves in a more general inequilibrium framework and use the Boltzmann equation framework in order to derived the same and other relevant quantities useful for our ultimate goals. We start with the definition of some quantities, useful both in equilibrium and non-equilibrium conditions: in natural units, and calling g_a the internal degrees of freedom of some species a (corresponding to its specific quantum numbers, e.g. 2 for the electron, connected to its permitted spin values) we have:

$$n_a(T) = \frac{g_a}{(2\pi)^3} \int d^3p f_a(p, T). \quad (2.2)$$

The energy density and the pressure can be obtained as statistical momenta of f_a : the energy density for a given energy E satisfies

$$\rho_a(T) = \frac{g_a}{(2\pi)^3} \int d^3p E f_a(p, T), \quad (2.3)$$

while the pressure reads:

$$P_a(T) = \frac{g_a}{(2\pi)^3} \int d^3p \frac{p^2}{3E} f_a(p, T). \quad (2.4)$$

We can also calculate the average energy $\langle E \rangle$ by normalising the energy density

$$\langle E \rangle = \frac{\int d^3p E f_a(p, T)}{\int d^3p f_a(p, T)} = \frac{\rho_a(T)}{n_a(T)}. \quad (2.5)$$

Furthermore, we introduce the total, temperature-weighted number of degrees of freedom for the relativistic species g_* defined as

$$g_*(T) = \sum_{b \in \text{Bosons}} g_b \left(\frac{T_b}{T} \right)^4 + \frac{7}{8} \sum_{f \in \text{Fermions}} g_f \left(\frac{T_f}{T} \right)^4, \quad (2.6)$$

which changes every time a particle becomes non-relativistic, namely, when the plasma temperature T becomes smaller than its mass m . We also introduce the entropy of the system S in a volume V for a given entropy density $s = \frac{P+\rho}{V}$:

$$S = sV. \quad (2.7)$$

The expression of the entropy density as a function of pressure and density of the relativistic fluids allows us to recover the following quantities:

$$s_i(T) = \begin{cases} \frac{2\pi^2}{45} g_i T^3 & \text{for Bosons} \\ \frac{7}{8} \frac{2\pi^2}{45} g_i T^3 & \text{for Fermions} \end{cases} \quad (2.8)$$

$$s(T) = \frac{2\pi^2}{45} g_{*s}(T) T^3 \quad (2.9)$$

$$g_{*s}(T) = \sum_{b \in \text{Bosons}} g_b \left(\frac{T_b}{T} \right)^3 + \frac{7}{8} \sum_{f \in \text{Fermions}} g_f \left(\frac{T_f}{T} \right)^3 \quad (2.10)$$

where $s(T)$ is the total entropy density given the individual species entropy density s_i and the total degrees of freedom g_s . Now we take the first step and set ourselves in *equilibrium* conditions. In this case, we can express explicitly the form of f_a :

$$f_a(T) = \frac{1}{e^{\frac{E-\mu_a}{T}} \pm 1}.$$

The expression with the $+1$ is the Fermi-Dirac distribution for Fermionic particles, whilst using the formula with the -1 we have the Bose-Einstein distribution valid for Bosonic particles. When the plasma is extremely hot, $\frac{E-\mu}{T} \sim \frac{E}{T}$, hence the distribution function does not depend on the chemical potential and, consequently, on the particle species. Under the approximation of

$$f_a(T) \sim \frac{1}{e^{\frac{E}{T}} \pm 1},$$

we can obtain

$$\frac{dn}{dp} = \frac{4\pi g_a}{(2\pi)^3} p^2 \left[e^{\frac{p}{T}} \pm 1 \right]^{-1}. \quad (2.11)$$

This allows us to recognise a relativistic regime when $p \gg m$ and $T \gg m$, and a non-relativistic regime when $T \ll m$. In the relativistic regime, recalling from Special Relativity that $E^2 = p^2 + m^2 \sim p^2$, we are thus allowed to replace E with p and to calculate the integral of eqs. 2.2, 2.3, 2.4, 2.5, finding:

$$n_a(T) = \frac{g_a}{\pi^2} T^3 \zeta(3) \begin{cases} 1 & \text{for Bosons} \\ \frac{3}{4} & \text{for Fermions} \end{cases} \quad (2.12)$$

$$\rho_a(T) = \frac{\pi^2}{30} g_a T^4 \begin{cases} 1 & \text{for Bosons} \\ \frac{7}{8} & \text{for Fermions} \end{cases} \quad (2.13)$$

$$P_a(T) = \frac{\rho_a(T)}{3} \quad (2.14)$$

$$\langle E \rangle = \begin{cases} 2.701 T & \text{for Bosons} \\ 3.151 T & \text{for Fermions} \end{cases} \quad (2.15)$$

where ζ is the Riemann Zeta function. In the non-relativistic limit we can approximate $E \sim m + \frac{p^2}{2m}$ and derive

$$n_a(T) = g_a \left(\frac{m_a T}{2\pi} \right)^{3/2} e^{-m_a/T} \quad (2.16)$$

$$\rho_a(T) = m_a n_a(T) \quad (2.17)$$

$$P_a(T) \sim 0 \quad (2.18)$$

$$\langle E \rangle = m_a \quad (2.19)$$

We observe in passing that the derived expressions for P_a in the two relativistic and non-relativistic cases justify why the barotropic index w for the equation of state $P = w\rho$ described in sec. 1.2 is $1/3$ for relativistic fluids (eq. 2.18) and zero for non-relativistic fluids (eq. 2.18).

By recalling the definition of the number and entropy densities per species, we can define the equilibrium abundance of a species $Y_{\text{eq},a}$, that reads

$$Y_{\text{eq},a} \equiv \frac{n_a}{s} = \begin{cases} \frac{45\zeta(3)}{2\pi^4} \frac{g_a}{g_{*s}(T)} \left(\frac{T_a}{T} \right)^3 & \text{(in the relativistic regime),} \\ \frac{45\zeta(3)}{4\sqrt{2}\pi^{7/2}} \frac{g_a}{g_{*s}(T)} \left(\frac{m_a}{2\pi T} \right)^{3/2} e^{-m_a/T} & \text{(in the non-relativistic regime)} \end{cases} \quad (2.20)$$

(notice that the abundance is the ratio between the *species* number density n_a of eq. 2.2 and the *total* entropy s of eq. 2.10). We can thus notice that the equilibrium abundance evolves with temperature, scaling with it and, since the temperature changes with the redshift, with the expansion of the Universe. This evolution is however related to equilibrium conditions, which at some point, as we said, will be broken: the scaling will thus be modified, as we will discuss in the following lines. Before doing so, we have to formally define the condition associated to the end of equilibrium: a particle exits from some condition of equilibrium (be it kinetic or chemical) with the plasma at the temperature T_{ineq} corresponding to the instant when its equilibrium reaction rate $R_a(T_{\text{ineq}})$ equates the expansion rate $H(T_{\text{ineq}})$. We can thus individuate a freeze-out, chemical decoupling T_f temperature, associated to the end of the chemical equilibrium and thus to the final setting of the abundance of the species, followed by a kinetic decoupling temperature T_k associated to the end of kinetic equilibrium, not involving changes in the number of particles, but just the scattering among the various species. Furthermore, concerning chemical decoupling, we shall be interested in particles whose freeze-out time corresponds to a temperature $T_{f,a} \ll m_a$, when they are no longer in the relativistic regime, the simplest condition satisfying the “coldness” property of dark matter: namely, that its velocity is non-relativistic and that therefore its streaming distance from the subsequent kinetic decoupling time onwards is associated to the suppression exclusively of very small scales. After chemical decoupling occurs, the number of particles is no longer modified by reactions with the rest of the primordial bath, but will evolve (or stay constant) exclusively in terms of what the Boltzmann equation, allowing to extend the calculation for distribution functions, number densities and abundances beyond equilibrium, will predict, as we shall now describe. We start with formulating the Boltzmann equation for a species assuming that the only relevant remaining interaction for it is its self-annihilation, associated to the cross-section rate $\langle \sigma v \rangle$ (notice, without the “th” subscript associated to the thermal cross-section rate. This latter quantity, which is the typical reference parameter for annihilating WIMP dark matter models will

be obtained at the end of this discussion). The equation can thus be stated using as unknown its number density $n_a(t)$ and formulated in an expanding background, as a function of the deviation between the *actual*, unknown abundance at time t ($n_a(t)$) and the corresponding abundance we would find if equilibrium conditions would still be holding at the same instant ($n_{\text{eq},a}(t)$):

$$\frac{dn_a}{dt} = -3Hn_a - \langle \sigma v \rangle (n_a^2(t) - n_{\text{eq},a}^2(t)). \quad (2.21)$$

We define the auxiliary variable $x = m_\chi/T$ relating the mass $m_a = m_\chi$ of the species a with the temperature of the Universe at some instant t and rewrite the Boltzmann equation in terms of the abundance Y_a . Since in an expanding background governed by the Fridman equations the temperature scales with the the scale factor ($T \propto 1/a$, in all regimes, regardless of which kind of matter or energy is dominating and determining the expansion rate), and the time can be expressed as function of the scale factor too by inverting the solutions $a(t)$ of the Fridman equations, we can eliminate the time variable and use the temperature, or the auxiliary variable x , as a “clock”, obtaining ((Kolb & Turner, 1990)):

$$\frac{dY_a}{dx} = -0.264 \frac{g_{*s}}{g_*^{1/2}(T)} M_{\text{Pl}} m_\chi \frac{\langle \sigma v \rangle}{x^2} (Y_a^2(x) - Y_{\text{eq},a}^2(x)), \quad (2.22)$$

with the value of the Planck mass being set to $M_{\text{Pl}} = \sqrt{\hbar c^5/G}$. The cold relic equilibrium abundance, therefore also holding at the last instant before freeze out, can be recalled from eq. 2.20 by considering the non-relativistic regime, and reformulated in terms of the auxiliary variable x as

$$Y_{\text{eq},a}(x) \equiv \frac{n_{\text{eq},a}}{s_a} \simeq \frac{45\zeta(3)}{4\sqrt{2}\pi^{7/2}} \frac{g_a}{g_{*s}(T)} \left(\frac{x}{2\pi}\right)^{3/2} e^{-x}. \quad (2.23)$$

This expression allows to recover the density n of the species at equilibrium, to be inserted in eq. 2.1 and equated to the condition $R_a(T_{\text{ineq},a}) = H(T_{\text{ineq},a})$ in order to recover the freeze-out temperature T_f . Importantly, the solution of the Boltzmann equation can be approximated by using the observation that *at today's temperature* T_0 it must hold $Y_a(T_0) \gg Y_{\text{eq},a}(T_0)$: notice that $Y_{\text{eq},a}(T_0)$ is the abundance we would calculate at the present temperature, the condition is not true for the freeze-out temperature, where instead we are expecting that $Y_{\text{eq},a}(T_f) \gg Y_a(T_0)$. The positions above follow from the fact that, as the Universe expands and the temperature decreases, $Y_{\text{eq},a}$ would be expected to vanish, since the exponential of eq. 2.23 would become smaller and smaller, whereas we expect the generic, out-of-equilibrium value for Y_a , not to vanish since we are indeed looking for the relic, remnant non-zero abundance of the species under scrutiny (see fig. 2.1). We can thus simplify the right-hand-side of eq.2.21 that becomes a function of Y_a only, finding a simpler first-order ordinary differential equation. Concerning the left-hand side, the integration in dY_a will be extended from the freeze-out abundance (when chemical equilibrium was still holding, hence corresponding to the value for $Y_{\text{eq},a}$ at the freeze-out temperature) up to the desired today's abundance $Y_{0,a}$, while the right-hand side is reformulated again as an integral in terms of the temperature. We make the dependence on T explicit again and integrate from the freeze-out temperature (the instant in time after which no new particle is produced in the primordial bath) T_f up to today (T_0). We thus find:

$$\frac{1}{Y_{0,a}} - \frac{1}{Y_{T_f,a}} \sim \frac{1}{Y_{0,a}} = 0.264 M_{\text{Pl}} \int_{T_0}^{T_f} dT \frac{g_{*s}(T)}{\sqrt{g_*(T)}} \langle \sigma v \rangle (T). \quad (2.24)$$

where the second passage is allowed by the fact that at freeze-out we expect $Y_{T_f,a}$ to be larger than $Y_{0,a}$ and that therefore the difference between the two reciprocals can be approximated by

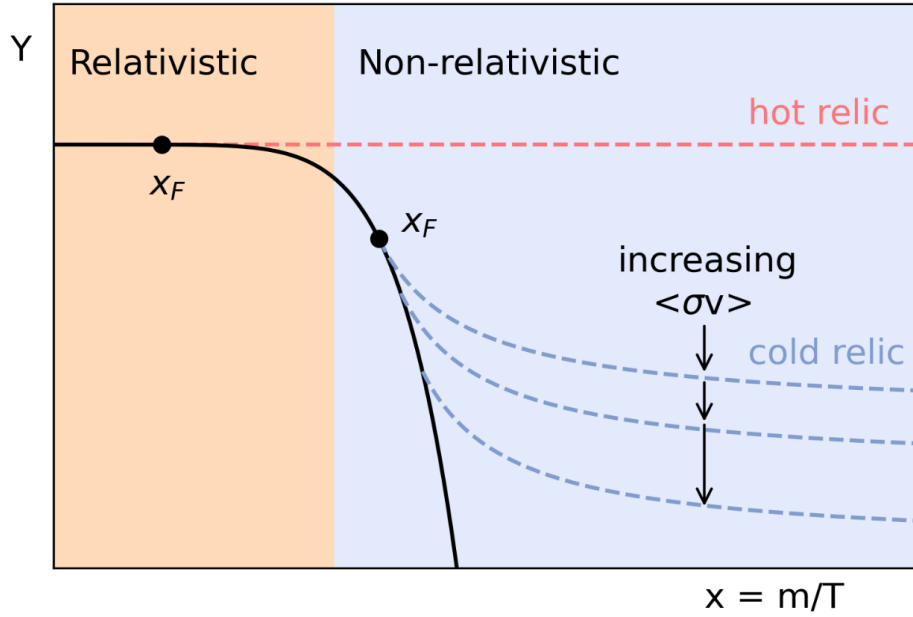


Figure 2.1: Relic abundance Y evolution in term of the variable $x = m/T$. Focusing on the “cold relic” case, we see how the relic abundance, after having transitioned from the relativistic to the non-relativistic regime when its mass becomes sensibly larger than the bath temperature, it first follows the equilibrium abundance (solid black line) of eq. 2.23. Should the particle be still in equilibrium at arbitrary x , in the end its abundance would be vanishing because all dark matter particles would annihilate (the solid black line intercepts the horizontal axis). This does not happen because at some point the particle exits from the chemical equilibrium with the rest of the bath, at the time/temperature instant x_F . At that point, the particle will follow the light-blue dashed line, corresponding to the solution $Y(x)$ we can obtain by solving the Boltzmann equation. We see that it soon reaches an almost constant value, the relic abundance, with an inversely proportional scaling to the cross-section rate magnitude $\langle\sigma v\rangle$ (the more likely are particles to self-annihilate, the lower their relic abundance, even out of equilibrium), as eventually found in eq. 2.26. This $\langle\sigma v\rangle$ corresponds to the reference $\langle\sigma v\rangle_{\text{th}}$ parameter. The picture is taken from (Pinetti, 2022).

the first term. We can define an integrated cross-section rate per unit mass as follows

$$\langle\sigma v\rangle_{\text{int}} = \frac{1}{m_\chi} \int_{T_0}^{T_f} dT \langle\sigma v\rangle \quad (2.25)$$

that allows us to condensate eq. 2.24 in

$$Y_0 \sim \frac{1}{\langle\sigma v\rangle_{\text{int}}}. \quad (2.26)$$

The result sets today’s abundance of the dark matter relics. We thus see that the relic abundance is a constant value and a function of the annihilation rate of dark matter particles: substantially, the dark matter particles are allowed to reduce their number by decay or annihilation but this process, given their typical small interaction cross-sections, is rather ineffective and the abundance can be considered substantially not changing with time. This can also be seen as the last relevant form of interaction for Dark Matter, other than in the gravitational channel, is just among Dark Matter particles themselves, self-annihilating and possibly producing Standard Model particles. Similar interactions, as well as interaction between Standard Model and Dark Matter particles, were occurring during the Early Universe, but with the relevant difference that at those early

epochs and until chemical equilibrium was holding, the reactions were “two way”, allowing to Dark Matter to be regenerated by Standard Model particles, whilst today it would exclusively be in the direction from Dark Matter particles to Standard Model final states.

Given the expression for the abundance and its relation with the number density, and combining it with the definition of the density parameter today for a species $\Omega_\chi = \frac{\rho_\chi}{\rho_c}$, we can connect the two expressions obtaining the density parameter as a function of the cross-section rate:

$$\Omega_\chi h^2 \simeq 8.5 \cdot 10^{-11} \frac{\sqrt{g_*(T_F)} \text{ GeV}^{-2}}{g_*(T_F) \langle \sigma v \rangle_{\text{int}}} \quad (2.27)$$

The cross-section rate can be expanded in polynomials that can be in turn identified with the angular momentum states of the particle wave function at annihilation:

$$\langle \sigma v \rangle = a + \frac{b}{x},$$

with a known as the s-wave annihilation (associated to the s angular momentum state) contribution to the cross-section rate and b contributions associated to the p-wave, $l = 1$ angular momentum states, component. The annihilation processes in the s-wave channels are independent on particle velocities and are usually dominant. We can then calculate the related quantity at the freeze-out (chemical decoupling) instant

$$\langle \sigma v \rangle_{\text{int}} = \frac{1}{x_F} \left(a + \frac{b}{x} \right)$$

with $x_F = m_\chi/T_F$ typically comprised between 15 and 25. $\langle \sigma v \rangle_{\text{int}}$ can be connected to the Fermi constant and to the weak interaction (see, for pedagogical introductions, (Kolb & Turner, 1990; Pinetti, 2022; Cirelli et al., 2024)), and we thus have

$$\langle \sigma v \rangle_{\text{int}} = \frac{1}{x_F} G_{\text{Fermi}} m_\chi^2.$$

Given eq. 2.27 and eq. 2.2.2, we can perform two relevant calculations: on one hand, since, for a given cold relic model, its density parameter must not be larger than the observed dark matter density parameter, we can derive a minimum mass limit for the particle (the Lee-Weinberg limit (Lee & Weinberg, 1977)) since we can solve the inequality for m_χ

$$\Omega_\chi h^2 \leq (\Omega_\chi h^2)_{\text{obs}}, \quad (2.28)$$

finding that $m_\chi \geq 0.8 \text{ GeV}$. Below this limit, Dark Matter models satisfying the observational requirements can be devised (such as axions (Kolb & Turner, 1990; Ringwald, 2024; O’Hare, 2024; Cirelli et al., 2024)), but the particle candidates no longer belong to the class of the *thermal* cold relics. Also, by solving eq. 2.27 for $\langle \sigma v \rangle$ again imposing that $\Omega_\chi h^2 = (\Omega_\chi h^2)_{\text{obs}} \sim 0.11$, we obtain that the velocity-averaged and temperature-integrated cross-section annihilation rate value (hereafter we shall note and quote it by $\langle \sigma v \rangle_{\text{th}}$) is around $3 \cdot 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. The value of $\langle \sigma v \rangle_{\text{th}}$ scales slowly with the mass value in the 10 GeV-1000 GeV range (corresponding to the typical WIMP candidate masses), the thermal dark matter model probing hence particularly handy as a reference. For a given candidate particle mass, exclusion plots imply that, if we want it to still be considered valid, its signal must be assumed to be weaker than current instrumental capabilities or theoretical strategies able to isolate it. If the bound we find falls below the predicted thermal parameter value, we have to assume that the given candidate cannot be produced in thermal equilibrium, pointing at a more complex model.

Having set the fundamental concepts, we can move on and briefly review the main observational targets for dark matter searches in astrophysical and cosmological processes. A graphic summary of their respective strong and weak points can also be found in fig. 2.2, where the various observables are located in a two-dimensional space, assessing on the y -axis the likelihood each tracers has to provide an intense DM signal and on the x -axis, instead, the typical size of the associated uncertainties, determining whether the potential detection of a strong signal can actually be claimed as statistically reliable against a background.

The Milky Way, its environment and its satellites

Our galaxy and its surroundings are a prime interesting target for DM searches (Yepes et al., 2014; Famaey, 2014): in general, galaxies and clusters are expected to form in spherical dark matter halos of increasing size. We should thus be immersed in a bath of DM particles at a relatively low distance from the Earth, a simple yet important consideration which opens the way to a number of places where to look for interesting signals.

The Milky Way The Galactic Center (GC) and its surroundings, the disk and the wider galactic halo can be inspected, the latter extending for many tens of degrees in latitude and longitude, the various regions displaying different degrees of astrophysical “contamination” due to signals emitted in the region of the Galactic plane (Famaey, 2014). Because of the typical expected Dark Matter density profiles (e.g., a Navarro-Frenk-White, NFW, profile (Navarro et al., 1997)) the maximum of the density is expected towards the centre, whilst the DM density in the halo is expected to be much lower but more free from astrophysical competitors (Manconi et al., 2024; De Laurentis et al., 2023; Hooper, 2022; Abazajian et al., 2014).

Globular clusters The Milky Way environment is also characterised by the presence of globular clusters, objects made of gravitationally bound stars. Galaxy clusters are objects with masses in the order of $10^5 M_\odot$, orbiting in the Milky Way galactic halo and typically found in the subhalos that host dwarf galaxies. Globular clusters are interesting because they are small, relatively low-mass objects, possibly originated by a DM subhalo. They should be hosting DM particles today and might even be accreting further DM by effect of their gravitational potential (Reynos-Cordova et al., 2022; Scarpa et al., 2007).

Satellite galaxies The Milky Way is surrounded by smaller galaxies, occupying in turn DM subhalos: many of these bodies belong to the dwarf spheroidal class, such as the galaxies called Sagittarius, Segue 1, Draco... (Drlica-Wagner et al., 2020). These galaxies are rather poor in star abundance and are largely occupied, according to the standard theory, indeed by DM particles (McConnachie, 2012; Chang & Necib, 2021; Drlica-Wagner et al., 2020), making them particularly interesting targets and “laboratories” for Dark Matter signal with possibly low baryon physics contamination and ambiguity.

Astrophysical and cosmological approaches: from planets to the CMB

Astrophysical objects Next to DM indirect detection by means of techniques which can be generically recognised as belonging to “cosmic rays” physics, observables more related to the fields of astrophysics and cosmology have been considered in the literature. Among them, we might

mention the proposal of stars, including the Sun, and planets capturing dark matter particles (Krauss et al., 1986; Vincent, 2020; Leane & Smirnov, 2023) and accreting them in their interior, with the possibility of subsequently producing signals related to their annihilation or decay and resulting in excesses in particle fluxes and electromagnetic frequency bands.

Galaxies, galaxy clusters and the Large Scale Structure as a whole Given the hierarchical structure formation paradigm predicted by the Λ CDM model, just like every galaxy is hosted in a halo, also galaxy clusters should be hosted in larger DM halos, with a relative ease in their observation since some of them are quite close to us (the Virgo, Coma, Fornax, and Perseus clusters). Higher up in the hierarchy, a good target could be the LSS as a whole (Fornengo & Regis, 2014): DM halos and their gravitational pull should be the “backbone” of the LSS; since gravity takes a finite time to communicate its effects, not all points in the Universe are undergoing clustering and the remotest and largest scales are more homogeneous, reproducing the initial conditions of the almost unperturbed Universe, whose fundamental property is expanding according to the Fridman equations. This process is of course a function of the Dark Matter abundance, but provides a limited number of phenomena to test its properties. The “homogeneity scale”, after which the matter distribution attains its average properties lies at around 100-200 Mpc (Scrimgeour et al., 2012; Laurent et al., 2016; Camacho-Quevedo & Gaztañaga, 2022; Nesseris & Trashorras, 2019) but, at scales smaller than this value, clustering, structure formation and interaction processes are ongoing, thus ensuring us a wide interval of scales over which we can probe the effects of Dark Matter on the Large Scale Structure and having observables alternative to the homogeneous background dynamics. Many physical processes occurring in the LSS might be instrumental to the study of the properties of Dark Matter, including its particle nature: the study of the dynamics and clustering properties of LSS tracers can be ultimately translated into indirect gravitational evidence of the Dark Matter properties according to perturbative theory, and are a function of its density parameter Ω_c . As well, Dark Matter processes might be producing photons and cosmic rays detectable on Earth and again informing us about the properties of Dark Matter precisely where it should be more abundant. Statistical techniques might thus help us to correlate emissions of light and cosmic rays detected at cosmic scales with the LSS, on one hand in terms of astrophysical sources residing within it and that we cannot detect individually but contribute to observed cumulative signals and, on the other hand, with Dark Matter itself should it be contributing to the observed particle signals, determining excesses and phenomena we cannot explain exclusively in astrophysical terms. Particularly relevant to this work is the use of this approach to the observed fluxes of γ -rays sourced by extragalactic objects residing in the LSS up to about redshift 2-3, before absorption becomes excessively strong (Franceschini et al., 2008). At the first order, this diffuse signal, known as Unresolved Gamma Ray Background (UGRB), can be assumed to be isotropic: deviations from perfect isotropy have indeed been observed and found to be statistically correlated with the LSS, regardless of its origin (Ackermann et al., 2012b, 2018; Xia et al., 2015). The subsequent step being pursued and relying in particular on the stream of data the Fermi-LAT satellite is collecting since 2008 is to obtain sound evidence on the sources and processes emitting the UGRB, both in terms of astrophysical sources and of heavy-mass WIMP-like dark matter particles undergoing annihilation or decay. The topic will be discussed in greater detail in the next sections 2.3, 2.3.2 and chapter 3, in this general list it can be interpreted as one of the most relevant examples of a strategy that can, of course, be extended to other wavelengths and other

kinds of tracers (Lauer et al., 2022; Nakayama & Yin, 2022; Kalashev et al., 2019; Cirelli et al., 2024).

The CMB The Cosmic Microwave Background might also be influenced by DM annihilation and decay processes, either in the Early Universe (from the Big Bang prior to CMB formation), or at the redshift of CMB formation, the Recombination Epoch ($z \sim 1100$) when, due to the progressive expansion and cooling of the Universe, the free electrons and nuclei in the primordial plasma definitively recombined in atoms of neutral light (chiefly Hydrogen and Helium) elements, as well as in the subsequent cosmic epochs until nowadays, by further interaction of CMB photons in the evolving Universe. In the first case, annihilating and decaying DM processes might have behaved as additional heating sources of the plasma in which nucleosynthesis and then recombination were occurring, leaving signatures both at the level of the CMB blackbody spectrum and of the angular power spectrum fluctuations. Signatures on the blackbody spectrum belong to the wider class of CMB spectral distortions, which also include standard and well-known processes both in the Early and in the Late Universe and from which Dark Matter processes should be distinguished (Chluba, 2025; Cyr, 2024; Lucca, 2023; Liu et al., 2023). The same processes might also modify the angular power spectrum of temperature fluctuations both in the primordial Universe and between the Recombination Epoch and the Reionisation Epoch (that is, between CMB formation epoch at $z=1100$ and down to $z=5-10$, especially in terms of correction to the predicted absorption rate and features of the CMB signal by the LSS, finding general upper bounds on the Dark Matter annihilation rate parameter (see sec. 2.2.3 for its meaning and calculation), required to be as low as $\langle\sigma v\rangle \leq 10^{-25} \text{ cm}^3 \text{ s}^{-1}$ for $m \approx 100 \text{ GeV}$ (Galli et al., 2009; Slatyer et al., 2009; Kawasaki et al., 2021). The CMB as a Dark Matter probe is interesting in that the observable is quite robust against systematics depending on many astrophysical uncertainties, being instead more strongly dependent on Dark Matter features in the Early Universe (see e.g. (Cirelli et al., 2024)).

Big Bang Nucleosynthesis and Early Universe bounds The Big Bang Nucleosynthesis (BBN) is another powerful instrument for constraining Dark Matter: since the abundance of baryonic matter produced during BBN can be very precisely predicted and this prediction very precisely matches the abundances we observe in the Universe (Schöneberg, 2024; Cyburt et al., 2016; Grohs & Fuller, 2022; Crespi et al., 2025), Dark Matter effects can only modify them at a very slight level, putting stringent constraints on its properties and the occurring physical processes, typically involving injection of particles in the plasma, disrupting the standard nucleosynthesis process (Pospelov & Pradler, 2010; Ripamonti et al., 2007a; Jedamzik & Pospelov, 2009; Hisano et al., 2009; Giovanetti et al., 2022). Usual expectations are that, under the assumption of DM injecting leptons and photons in the primordial plasma, light nuclei would have been undergoing more frequent photodissociation processes, determining modifications in the abundances of ^4He , deuterium and ^3He fraction (Cirelli et al., 2024). DM could act also as a catalyst (Pospelov, 2007): it has been proposed as an enhancer of BBN processes by creating intermediate bound states modifying the ultimate abundance of light elements. The interest for this scenario has been enhanced by the fact that a DM-sourced mechanism capable of enhancing the predicted ^6Li abundance could allow then for a reconciliation with the observed cosmic ^6Li fraction, since this quantity is notoriously being underestimated by the standard BBN calculation framework and we still do not dispose of a clear explanation for such discrepancy (Fields, 2011; Spite et al., 2012).

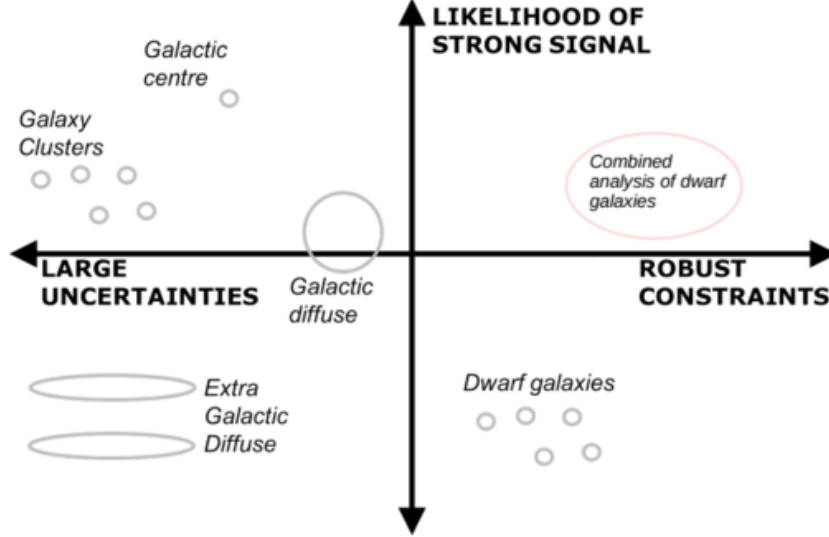


Figure 2.2: Summary of astrophysics and cosmology-based DM indirect detection strategies in terms of strategy effectiveness, from (Conrad, 2014)

Peculiar cases of not unique classification can be that of a dark sector not interacting at all with SM particles (Cirelli et al., 2024); if dark sector particles are relativistic, the case can be however tested in terms of modifications ΔN_{eff} in the number of relativistic degrees of freedom N_{eff} in the cosmic inventory, where N_{eff} is defined as the number of relativistic species, other than photons, in the primordial bath (Gerbino & Lattanzi, 2017). The Λ CDM prediction (which only assumes three neutrinos as additional relativistic particles next to photons, all of them still relativistic before the CMB formation epoch) is $N_{\text{eff}} = 3.0440$, whilst the DM-induced ΔN_{eff} scales as $\Delta N_{\text{eff}} \sim (T_{\text{DM}}/T)^4$, thus depending on the relativistic status of the candidate DM particles and their temperature T_{DM} (Kolb & Turner, 1990). Eventually, Dark Matter could be also indirectly detected as a responsible of intergalactic medium heating processes: particles emitted by decaying and annihilating DM relics would thus alter the environment in which stars form, reducing their formation rate (Ripamonti et al., 2007b; Muñoz & Loeb, 2017).

2.2.3 Reviewing the observational status: constraints and main anomalies

Once outlined the main techniques and available observables, and keeping a particular focus on DM candidates whose mass is in the GeV-TeV regime (“weak scale” masses), we can now move to the observational status of Dark Matter indirect searches as of 2024-2025. In general, observational outcomes can be broadly distinguished between “constraints” and “anomalies”.

Constraints

The discussed results involve both direct observations of cosmic rays as well as statistics-based methodologies of cosmological flavour, namely, statistical auto-correlations and cross-correlations between tracers of the LSS/DM field, which are of particular interest for this work. Focusing on the γ -ray detection channel, an exclusion plot from (Cirelli et al., 2024), thus updated to about the year 2024, is displayed in fig. 2.3 (the underlying calculations assume NFW DM halo profiles in every case), collecting the main relevant observables, from dwarf galaxies, among the most effective tracers at our disposal, to the CMB and the UGRB. The measurements based on radio

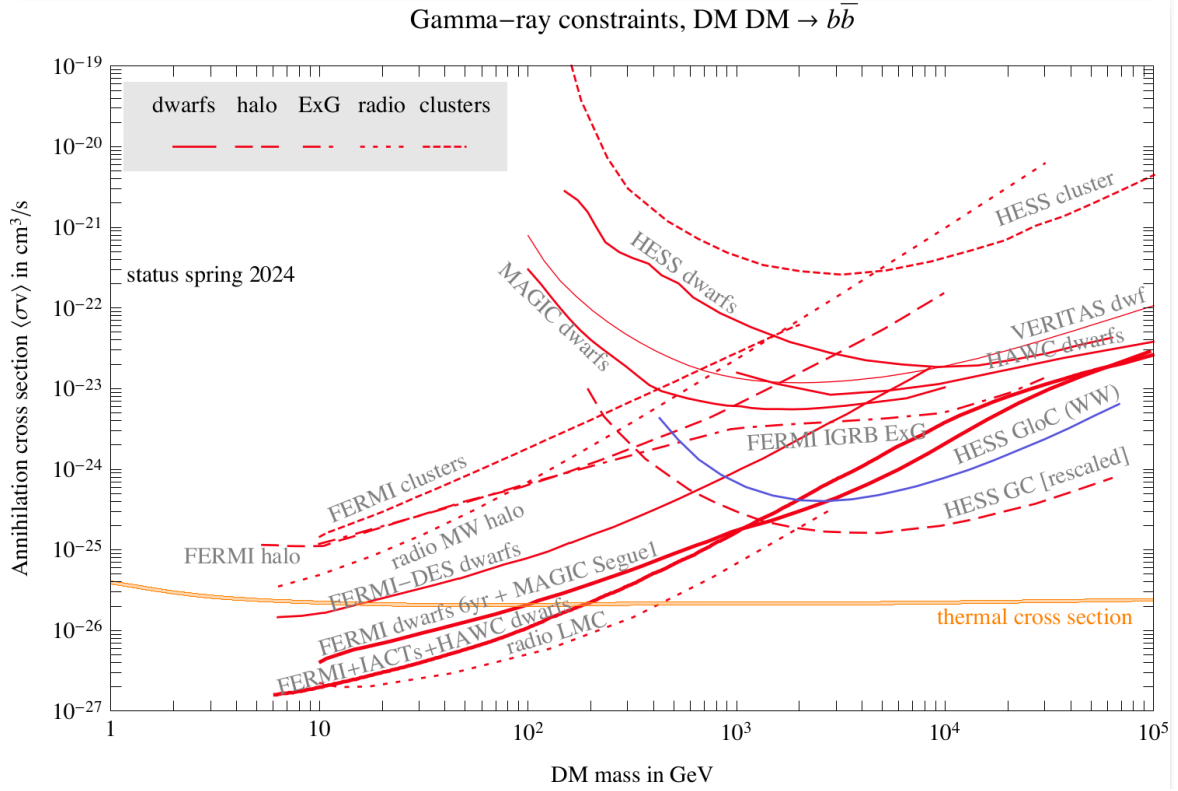


Figure 2.3: Bounds on weak-scale DM annihilation rate deriving from electromagnetic messengers, in the gamma and radio bands, as a function of the Dark Matter candidate mass. The bounds are obtained under the assumption of DM annihilating through the fairly typical $b\bar{b}$ channel. The almost horizontal orange thin band is the thermal annihilation rate. The plot is taken from (Cirelli et al., 2024).

signals, capable of extending the probed mass range for heavy DM candidate models and effective especially on objects belonging to the Milky Way and Local Group environment, are summarised in the picture as well. Weak-scale DM can be probed also through charged particles, and the main results are instead summarised in fig. 2.4: attained sensitivities are rather competitive with the ones achieved through neutral particles, in spite of the limitations burdening charged messengers we discussed in the previous sections. Particularly of interest for the topic is the detection of antideuterons as probes of WIMP-like DM particle annihilation processes (Donato et al., 2000, 2008; Baer & Profumo, 2005; Kadastik et al., 2010): astrophysical processes producing this kind of nuclei peak at an energy value very different from the one of processes producing the same nuclei but due to DM effects, so that probing the presence or absence of antideuteron signals in the energy range exclusively attainable by DM processes would ensure access to a region potentially “clean” from astrophysical disturbance. Neutrino constraints have a mixture of positive and negative aspects: they are typically less competitive than electromagnetic messengers, so that most of the regimes they are informative about are typically already ruled out by photon measurements; at the same time, at high masses, above many TeV, processes can be probed by neutrinos while they would be suppressed and hardly detectable in the electromagnetic channel (Yüksel et al., 2007; Argüelles et al., 2021).

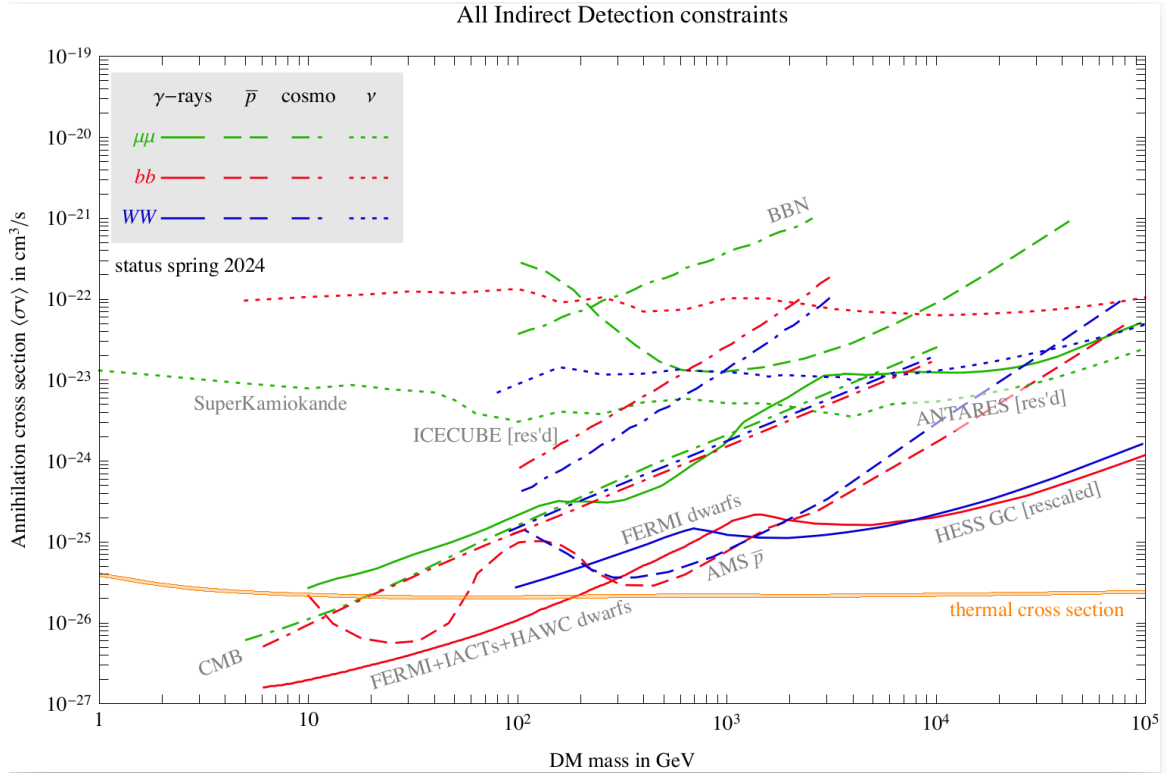


Figure 2.4: Bounds on heavy DM annihilation cross-section including both electromagnetic and non-electromagnetic messengers (photons, antiprotons, neutrinos, and the cross-correlations, denominated “cosmo”, with LSS tracers and the CMB) for the three reference intermediate annihilation states of Dark Matter particles, namely $\mu^+\mu^-$ (green), $b\bar{b}$ (red) and W^+W^- (blue). The plot is taken from (Cirelli et al., 2024).

Anomalies

Among the relevant anomalies, we might quote those eliciting the greater debate, starting from the one known as “Arcade-2 excess”. The phenomenon was detected in 2009 during a CMB observational campaign: the instrument, a balloon-borne experiment, reported a significant excess compared to the expected contributions in the 22 MHz band up to frequencies of a few GHz (Fixsen et al., 2011). Proposed astrophysical mechanisms generated in AGNs or Star Forming Galaxies are unable to predict the observed, quite high, flux intensity, 5-6 times higher than any considered “standard” hypothesis (see fig. 2.5) (Singal et al., 2010; Kogut et al., 2011; Fornengo et al., 2011). DM explanations would point at a cosmic signal from DM particles residing in the LSS and having a mass in the 10-25 GeV regime (Fornengo et al., 2011). The DM particles would be annihilating into $\mu^+\mu^-$ pairs, whose interaction would then produce the $e^\pm$ particles which in turn are responsible for the observed radio signal by synchrotron radiation. Trying to clarify the situation, researchers have also addressed the spatial morphology of the signal: if it were coming from clustered halos in the LSS, it should show some degree of anisotropy compatible with the one displayed by the cosmic web. Yet, the signal distribution through angular scales appears substantially isotropic (Hooper et al., 2012; Fang & Linden, 2015). Interestingly, this argument is also a challenge for astrophysical explanations, since, if the signal is of cosmic origin, also baryonic processes should show some degree of clustering, which instead remains undetected. The interest for the Arcade-2

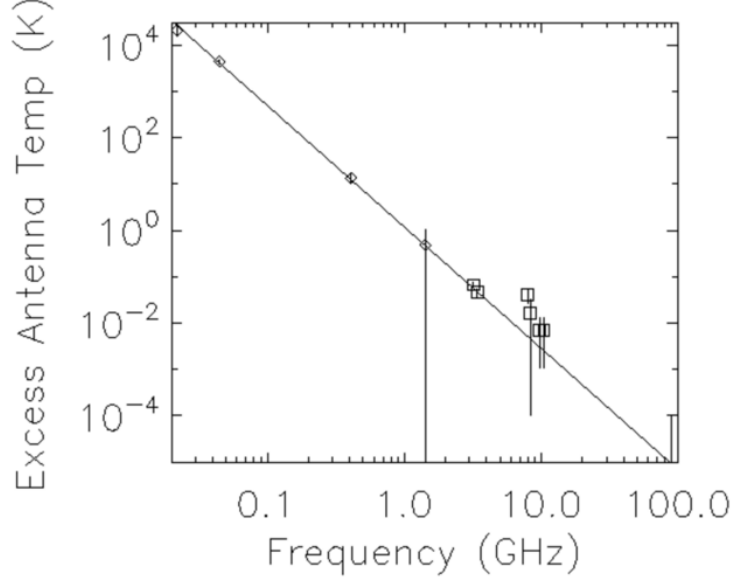


Figure 2.5: The Arcade excess antenna temperature as reported by the original paper (Fixsen et al., 2011), displaying the unexpected additional radio signal as a function of the observed frequency.

anomaly remains high and it will be a target of next-generation instruments such as the SKAO radiotelescope (Hooper et al., 2012; Fang & Linden, 2015; Caputo et al., 2023; Cirelli et al., 2024).

Always in the field of cosmic backgrounds, particularly interesting for this thesis, some excesses in the Cosmic Optical Background (COB) and in the Cosmic Infrared Background (CIB) have also been targeted. These two electromagnetic bands should experience a large contribution from the stars, dust and other rather standard astrophysical processes occurring in the Universe. Measurements of these two backgrounds are rather easy, although contaminated by many local contributions, and the COB has been subject of specific study by the LORRI instrument flying on the New Horizons spacecraft (Lauer et al., 2022). Surprisingly, the COB amplitude value one obtains by fitting the LORRI data is two times higher than the one derived from galaxy counts obtained from Hubble Space Telescope data, the discrepancy attaining a 4σ significance. A similar line of reasoning holds for the detected excess in the CIB, although at a lower significance level, related to larger uncertainties in the data (Matsuura et al., 2017). DM explanations of such excesses are related to axion dark matter candidates with a mass in the order of some eV, but morphological studies again point at very low levels of anisotropy in the detected backgrounds, conflicting with the emission we should expect from some clustered source such as DM halos (Nakayama & Yin, 2022; Bernal et al., 2022; Kalashev et al., 2019; Caputo et al., 2021).

The above-mentioned background emissions are continuous in frequency. Unexplained line-emissions have also been detected: in the X-ray band, for instance, (Bulbul et al., 2014) and (Boyarsky et al., 2014) reported a line detection at $E_\gamma \simeq 3.55$ keV from XMM-Newton and Chandra data. The signal appears to be emitted by several galaxy clusters (the one emitted by the Perseus cluster is particularly strong) and by the Andromeda galaxy (Bulbul et al., 2016; Boyarsky et al., 2014). Follow-up observations detected – sometimes in a disputed way – the same line at the centre of the Milky Way, in its halo and in other galaxy clusters (Conlon et al., 2017; Malyshev et al., 2014; Ruchayskiy et al., 2016). At the same time, nevertheless, other galaxy clusters do not display it at all, nor is it detected in wide regions of the sky, a behaviour not expected from DM

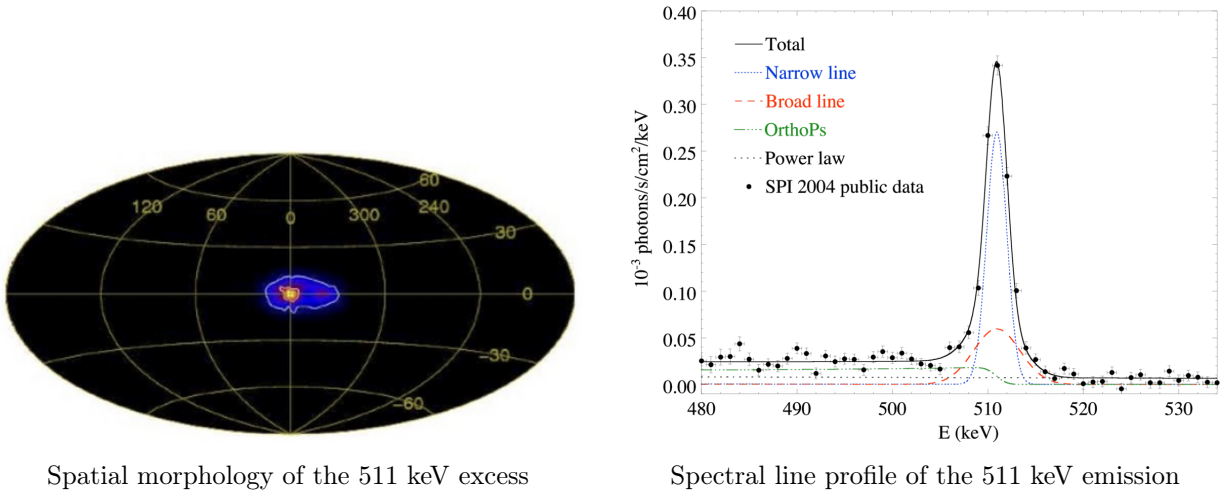


Figure 2.6: The spatial morphology of the 511 keV excess (left) and its spectral line fit (right), from (Cirelli et al., 2024).

which should be permeating the whole Universe (Jeltema & Profumo, 2016, 2015; Dessert et al., 2020) .

Subsequent literature reconsidered the robustness of the signal detection itself , whose robustness has undergone criticism and potential rejection (Ruchayskiy et al., 2016; Riemer-Sørensen, 2016; Jeltema & Profumo, 2015; Malyshev et al., 2014; Dessert et al., 2020; Conlon et al., 2017). The line emission nature of the signal has fostered a wide range of studies and debates linked to the possibility of applying spectroscopic techniques aimed at checking the explanation of the signal in terms of rare or still unobserved line emission from ordinary elements, while, on the DM interpretation side, next to devising the source processes (Shah et al., 2016), further constraints have been discussed based on finding connections between the observed properties of the line with the dynamic state of DM particles in clusters, the broadening induced by their velocity dispersions, its Doppler shifting and its presence in baryon-poor dwarf galaxies (Conlon et al., 2017; Jeltema & Profumo, 2016; Anderson et al., 2015). Similar reasonings can be applied to the 511 keV γ -ray line, which has a long history of detections dating back to the 1970s (Johnson & Haymes, 1973; Leventhal et al., 1978; Knödlseeder et al., 2005). The emission corresponds to the one expected from e^+e^- annihilations, which indeed have to peak at 510.954 ± 0.075 keV, corresponding to the electron mass, including the correction due to temporary formation of positronium before annihilation (Prantzos et al., 2011; Jean et al., 2006). The emission is particularly intense and extended, displaying a spherical symmetry around and on the galactic plane (fig. 2.6). Whilst the disk emission can fairly easily be related to astrophysical processes driven by stars, supernovae and compact, extreme objects, the bulge emissions appears even in regions where stars are not observed, thus, again, suggestive of light DM particles populating the halo and converting in positron-electron pairs (Ascasibar et al., 2006; Bartels et al., 2018; Hooper & Wang, 2004; Vincent et al., 2012). The main problem in this case is related to the properties of the original DM particles: the abundance is not compatible with that of a thermal relic and requires further theoretical specifications (Hooper & Wang, 2004; Wilkinson et al., 2016). Furthermore, a similar signal would be expected in dwarf galaxies, which the theory predicts to be particularly rich in Dark Matter and with a weak baryon background competition, but the evidence is scarce (Cordier et al., 2004; Hooper et al., 2004). The region of the Galactic Center seems to host another excess, detected starting from 2009 and ranging between 0.5 and 5 GeV (Hooper & Goodenough, 2011; De Laurentis et al., 2023) (see fig. 2.7).

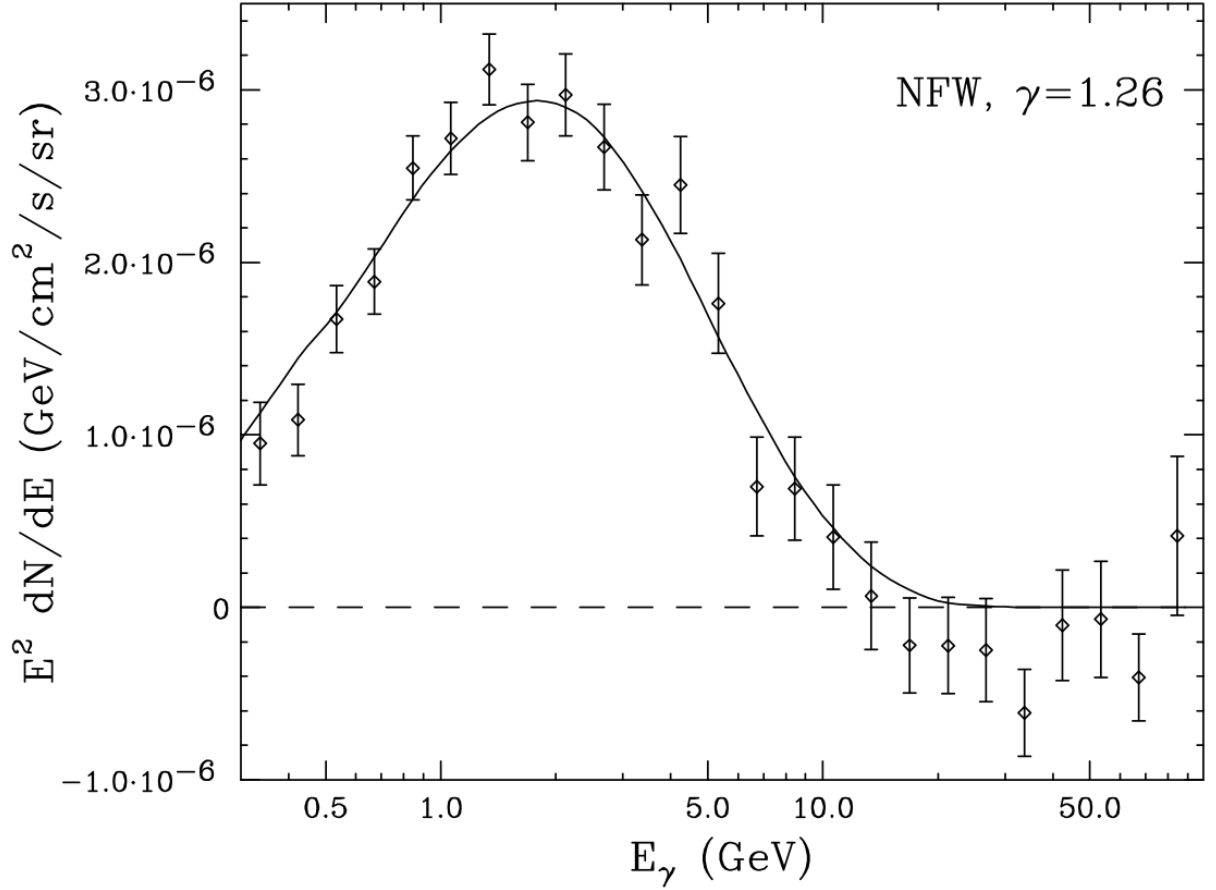


Figure 2.7: The spectral fit of the Galactic Center excess in the γ -ray wavelength, from (Cirelli et al., 2024).

It represents one of the most relevant and actively disputed anomalies currently available in the literature: the distinctive spherical morphological features of this signal were immediately linked to annihilating dark matter with a mass of 45 GeV, a thermal cross-section and quite standard intermediate SM states leading to photons as final states, with DM particles distributed within an NFW halo (Manconi et al., 2024; Ackermann et al., 2017; De Laurentis et al., 2023; Hooper, 2022; Abazajian et al., 2014; Daylan et al., 2016). More elaborate DM models could explain the signal as well and, in general, the observation is robust against additional years of Fermi observations and it has been linked to other gamma ray emissions detected in the galactic plane. At the same time, astrophysical alternatives have been proposed, such as milli-second pulsars (Hooper, 2022; Abazajian et al., 2014; Daylan et al., 2016; Bartels et al., 2016): the explanation of this excess has elicited a still ongoing quite lively debate between the different positions, since milli-second pulsars themselves have not yet been detected in the Galactic Center region, not to mention other astrophysical explanations and the disputed status of similar excesses in other galaxies (Bartels et al., 2016; Lee et al., 2016; Calore et al., 2021). Eventually, relevant model-dependence in the development of theoretical calculations has to be kept in mind (Cholis et al., 2022; Song et al., 2024; Carlson et al., 2016; Gaggero et al., 2015). A very recent similar result has been published in November 2025 by (Totani, 2025): the paper announced the detection of a statistically significant excess in the Milky Way halo, centred around an energy of about 20 GeV. The excess comes from a region rather devoid of competing astrophysical sources and is characterised by a quite isotropic distribution, similar to what might be expected from DM particles distributed

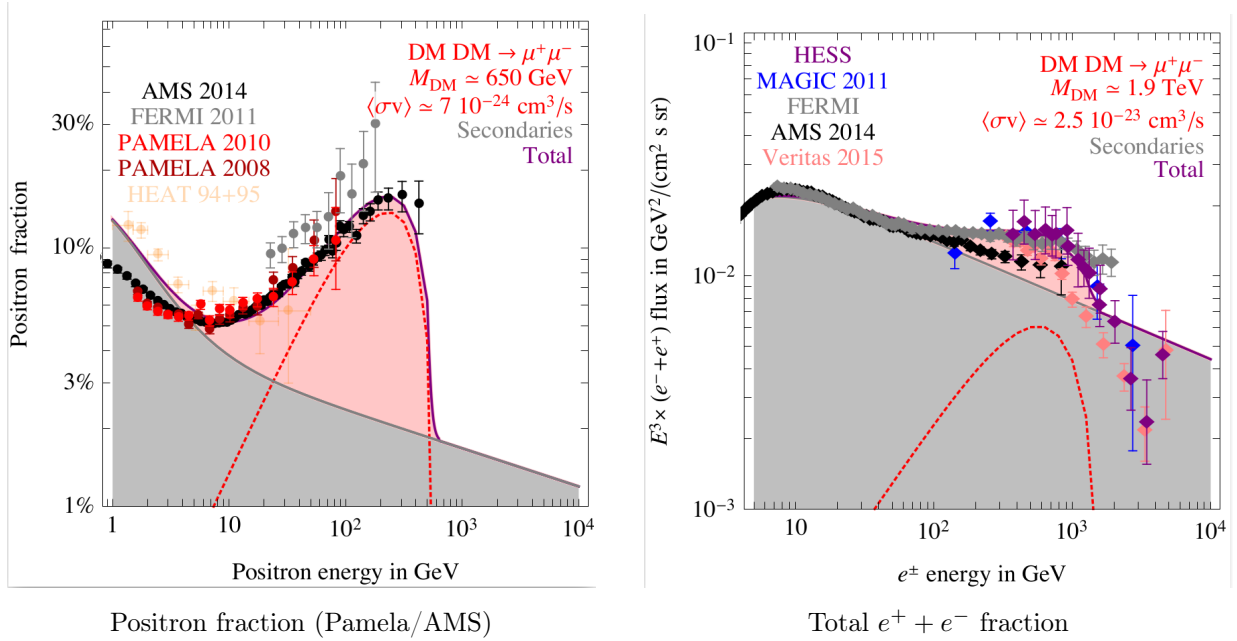


Figure 2.8: The Pamela/AMS lepton excess, with a fit for a dark matter model involving only the positron fraction (left) and the total positron and electron fraction (right), from (Cirelli et al., 2024).

in a spherical halo. The data can be fitted by a rather standard WIMP particle annihilating in high-energy photons through the $\bar{b}b$ channel and endowed with a mass between 500 and 800 GeV. At the same time, the author himself observes how the associated best-fit cross-section rate of $\langle\sigma v\rangle \approx (5 - 8) \cdot 10^{-25} \text{cm}^3 \text{s}^{-1}$ could be exceeding the current upper limits set by competitive instruments. Overall, the methodology and lines of reasoning associated to this last claim are well-known, and additional analyses are but welcome to further update the debate.

There are also several excesses involving other classes of cosmic rays: the most famous one is referred to as the “Pamela/AMS” positron excess (named after the two most important experiments having confirmed its presence) (Adriani et al., 2009, 2010; Aguilar et al., 2013), consisting in an excess of positrons compared to the expectation one obtains when the $e^+/(e^+ + e^-)$ ratio is measured between 10 GeV 100 GeV (fig. 2.8). This excess is also related to observations performed by different instruments similarly showing potentially related excesses in observed fluxes (Ackermann et al., 2012; Borla Tridon, 2011). Its interpretation as a DM signal is controversial because, for general DM models, it should be sourced by very heavy (in the TeV range) and strongly annihilating particles, with consequences not supported by complementary observations, such as being associated to larger photon fluxes produced by Inverse Compton than the bounds Fermi is providing (Arkani-Hamed et al., 2009; Papucci & Strumia, 2010; Cirelli & Panci, 2009; Bergström et al., 2008). At the same time, some properties of signal appear to challenge the more standard available propagation models for cosmic ray produced by baryonic processes (Serpico, 2012; Di Mauro et al., 2016; Cholis et al., 2009). In general, annihilating dark matter models able to fit the excess without the requirement of very heavy particles demand instead rather complex dark sectors, whereas decaying dark matter models tend to be more successful under simpler and more general assumptions. In general, and also due to the progressive acquisition of further data from other probes, the DM explanation for this famous excess has been increasingly

discarded, while theoretical modelling of astrophysical sources has been chiefly concentrating on close and recently-formed pulsars and supernova remnants as accelerators of the particles whose collisions ultimately produce the observed excess positrons, combined to an ongoing effort aimed at determining the spatial distribution of the signal which could help with discriminating between more isotropic DM progenitors or local astrophysical sources (Katz et al., 2010; Serpico, 2012; Di Mauro et al., 2014; Cirelli et al., 2024).

Another interesting excesses worth to be mentioned is the anti-proton Pamela/AMS excess: should it be actually due to Dark Matter effects, it would be more easily relatable to simpler WIMP models pointing at annihilating particles whose mass is in the order of 80 GeV in mass. These conclusions are nevertheless undergoing a strong competition from astrophysical explanations, not to mention the fact that the significance itself of the detected signal has been questioned (Hooper et al., 2015; Cuoco et al., 2017; Heisig et al., 2022, 2020). The AMS anti-helium excess, not associated to collaboration papers, (Cirelli et al., 2024; Carlson et al., 2014) has some interesting properties in that, whatever its astrophysical or BSM explanation, it points towards an excess in antihelium abundance challenging established expectations (Cirelli et al., 2014; Poulin et al., 2019a; Cirelli et al., 2024), whereas the excesses claimed by the CALET (Cannady et al., 2018), DAMPE (Chang et al., 2017) and ANITA (Anita Collaboration et al., 2009) experiments involve more types of leptonic particles. This body of evidence is interesting especially as complementary probes to AMS, which is one of the most powerful and complete cosmic ray detectors currently available, flying on the International Space Station, thus outside of the terrestrial atmosphere and probing primary cosmic rays, as opposed to the by-products of primary cosmic rays interaction with the Earth atmosphere we usually measure with ground-based experiments (see (Cirelli et al., 2024) for further details).

2.3 The WIMP paradigm and its relation to the UGRB

We have now available the main “taxonomic” concepts that allow us to locate this work in the general topic of the study of dark matter as a fundamental particle (Fornengo & Regis, 2014; Feng, 2023; Atwood et al., 2009; Ackermann et al., 2012b, 2015, 2018):

- At the level of *methodology*, we will be performing an indirect detection strategy.
- At the level of *Dark Matter particle model*, we will be considering the simplest weak-scale candidate cold relic known as Weakly Interacting Massive Particle (WIMP), endowed with a mass ranging between 10 GeV and 1 TeV.
- At the level of *observables and estimators*, we will be setting at the interface of statistical cosmology and astroparticle physics, developing forecasts on an enhanced cross-correlation formalism between galaxy survey catalogues of spectroscopic nature and the modelling of the UGRB, the related characterisation of its composition in terms of astrophysical and candidate DM contributions and the salient Fermi-LAT observational features, currently the reference instrument for the UGRB observation.

The last part of this chapter will be mostly devoted to a summary of the so-called WIMP paradigm, together with a more thorough review of more than a decade of extensive research in statistical cross-correlations between the UGRB and a number of LSS tracers of the most disparate nature, from galaxy clustering to voids and HI Intensity Mapping.

2.3.1 WIMP Dark Matter characterisation

As presented in the [Introduction](#), Dark Matter is assumed to be an electrically neutral and most likely extremely long-lived particle, whose two main features are its apparent leading role as mass source for the gravitational potentials determining the dynamics of celestial objects at and above the galactic scale, and a weak level of interaction with other kinds of matter through Standard Model fundamental forces other than gravity. In particular, it seems not to be coupled with the electromagnetic field – it does not emit light – and appears to be “cold” ([Blumenthal et al., 1984](#); [Ostriker & Steinhardt, 1995](#)), that is, its streaming velocity is much smaller than the speed of light c , a feature related to the preservation of small-scale perturbations in the observed clustering of matter in the Universe. Given the size of the smallest scales in the cosmic web not being erased by the free-streaming process, corresponding to the maximum distance DM particles could have travelled from the Big Bang Epoch, 13.7 billions of years ago, their streaming velocity should be no larger than about 200 km/s. Particles of this kind are non-relativistic in the present Universe and had to be already so when, in the Early Universe, they decoupled from the rest of the primordial plasma.

We have to take into account an important bound: if Dark Matter is constituted by particles being synthesised during the Big Bang, any suggested mechanism must predict a resulting abundance in accordance with a number of independent observational measurements of its value spanning from the CMB to LSS statistics to the dynamic and kinematic properties of clusters and galaxies (see e.g. ([Cirelli et al., 2024](#))). These considerations determined the intersection of Cosmology and Particle Physics research lines since the 1970s, when studies appeared on the possibility of connecting the effort, typical of those years, of extending the Standard Model of Particle physics in the weak-scale regime (that is, including particles and physical processes in the GeV-TeV energy regime probed by the weak interaction of the Standard Model of Particle Physics.) with the need for additional matter species required by astrophysicists and cosmologists characterised by long-time stability and heavy mass as responsible of its coldness property ([Lee & Weinberg, 1977](#)). The fundamental argument derived at the time is the following: our goal is to obtain a prediction for the observed Dark Matter abundance parameter today Ω_c , consistent with observations. Recalling the Universe critical density today $\rho_c = \frac{3H_0^2}{8\pi G}$ and the associated number density of DM particles n_0 , we have:

$$\Omega_c \equiv \frac{m_\chi n_0}{\rho_c} \quad (2.29)$$

The equation above can now be manipulated in order to introduce the reference energy and temperature scales in order to ultimately express Ω_c in terms of the main variables we want to use to characterise the dark matter particle: its mass m_χ and its (thermal) annihilation cross-section rate $\langle\sigma v\rangle$. We presented the meaning and reference value of the thermal $\langle\sigma v\rangle_{\text{th}}$ before in [sec. 2.2.3](#), while discussing bounds and exclusion plots. We can now also derive the properties of the mass of the Dark Matter particle that, although the reasoning can be applied quite generally to all thermal relics, can be considered specific of the WIMP class of particles and justifying most of the ongoing interest for this dark matter model. First, we multiply and divide [eq. 2.29](#) by the Universe temperature today T_0 ([Feng, 2023](#)):

$$\Omega_c = \frac{m_\chi T_0^3}{\rho_c} \frac{n_0}{T_0^3}. \quad (2.30)$$

Second, we can introduce the number density of DM particles n_f at their decoupling from the Big Bang plasma and the associated freeze-out temperature T_f . The ratio $\frac{n_0}{T_0^3}$ can be traced back to

the same quantity at decoupling $\frac{n_f}{T_f^3}$: given that in a FLRW Universe the temperature scales as $T = T_0 a^{-1}$ and that the number density scales instead as $n = n_0 a^{-3}$ (Kolb & Turner, 1990), we can replace the ratio by means of $\frac{n_0}{T_0^3} \sim \text{const} \sim \frac{n_f}{T_f^3}$, finding:

$$\Omega_c = \frac{m_\chi T_0^3}{\rho_c} \frac{n_0}{T_0^3} \sim \frac{m_\chi T_0^3}{\rho_c} \frac{n_f}{T_f^3}. \quad (2.31)$$

The last result we invoke is the relation between n_f , T_f and the thermal annihilation cross-section $\langle\sigma v\rangle_{\text{th}}$ that we obtained by solving the Boltzmann equation for the dark matter species in an expanding Universe, which provides us with the number density evolution of our species at a given time once its statistical and dynamical properties are set:

$$n_f \sim (m_\chi T_f)^{3/2} e^{-m_\chi/T_f} \sim \frac{T_f^2}{M_{\text{Planck}} \langle\sigma v\rangle_{\text{th}}} \quad (2.32)$$

Replacing this expression in eq. 2.31, we find that

$$\Omega_c \sim \frac{x_f T_0^3}{\rho_c M_{\text{planck}}} \langle\sigma v\rangle_{\text{th}}^{-1}. \quad (2.33)$$

We are then required to find an expression for the average self-annihilation cross-section rate $\langle\sigma v\rangle_{\text{th}}$ of our particle. Setting ourselves in a weak interaction regime and combined with the request of an heavy and non-relativistic particle, we have that

$$\langle\sigma v\rangle_{\text{th}} = k \frac{g_{\text{weak}}^4}{16\pi^2 m_\chi^2} \quad (2.34)$$

If we set g_{weak} in the order of 0.65 – the weak interaction gauge coupling – and if we account for deviations from its value by letting the additional parameter k vary between 0.5 and 2, inserting eq. 2.34 in eq. 2.33 we have

$$\Omega_c \sim k \frac{x_f T_0^3}{\rho_c M_{\text{planck}}} \frac{16\pi^2 m_\chi^2}{g_{\text{weak}}^4}. \quad (2.35)$$

To match the observed $\Omega_c \sim 0.26 - 0.27$ and depending on the choice of k , we can solve eq. 2.35 for m_χ and find that it can range from some tens of GeV up to about 1 TeV. If we strictly require $k \sim 1$ – that is, if we require that the candidate cold relic is precisely associated to the weak scale of mass of the Standard Model and that a single particle species is able to represent **all** of DM, what is known as a minimal extension of the SM with just one additional particle – then its mass is set to be around 100 GeV. In other words, by combining the observed abundance of Dark Matter provided by astrophysical and cosmological data with the only requirement of weak scale coupling as an additional hypothesis (the requirement characterising the first part of the acronym of WIMP, Weakly Interacting), we determine a mass for the dark matter particle in turn consistent with typical masses for particle associated to the Standard Model weak interaction, such as $W^\pm$ and Z bosons, thus characterising the second part of the WIMP acronym, Massive Particles. The availability of an already known energy scale to which novel particles can be associated, capable of solving a cosmological problem with rather straightforward and ordinary particle physics arguments, without the need to invoke newer or more complicated models, is known as “WIMP miracle” (Feng, 2023) and has determined an extensive research paradigm, known in turn as “WIMP paradigm”. The WIMP paradigm is almost fifty years old: as of today, we dispose of far more DM models, deviating in different ways from those simple original hypotheses, interacting through different modalities with SM models, produced by non-thermal mechanisms. These alternative theories are

equally able to reproduce the required Ω_c : WIMPs correspond to the largest-mass solutions, but alternative mass regimes are also viable and the “coldness” requirement may be satisfied by means of different interaction and decoupling arguments. At the same time, the simplicity of the WIMP paradigm and of theories allowing slight deviations from the base model still make heavy thermal particles among the preferred cold relic candidate to study, associated to an extensive effort aimed at finding observational evidence of their existence.

2.3.2 Properties of the UGRB and anisotropy measurements through statistical correlations

While reviewing the different indirect detection methodologies in sec. 2.2, we have already mentioned the Unresolved Gamma Ray Background, one of the many electromagnetic backgrounds populating the Universe: some of its properties have already been exposed there and in the [Introduction](#), combined with the relevant quantitative modelling collected in chapter 1, especially in sec. 1.3.1 and complemented by additional comments and details throughout chapter 3. Synergies between astroparticle physics and cosmology can start from the consideration that dark matter is expected to cluster into halos representing the scaffolding of the LSS: ignoring galaxies hosted in voids (but see [\(Arcari et al., 2022\)](#)), also astrophysical objects and baryonic gas all reside in such halos and behave therefore as light-emitting tracers of the underlying dark matter distribution. The observed property of the original dark matter field and of its tracers of being organised in the LSS web-like structure translates into a degree of anisotropy and inhomogeneity in their distributions across the Universe. To measure such degree of anisotropy, one can develop a number of estimators: starting from the assumption that the observables are almost uniformly distributed in the three-dimensional or 2D angular space, we can calculate the deviation from the assumed uniform (three-dimensional) or isotropic (two-dimensional) distribution hypotheses between pair of points taken at some metric or angular distance each other, for every pair of object and every distance value. The estimate of such deviation from uniformity or isotropy defines the two-point correlation function that, once Fourier-transformed in the case of tracers for which we dispose of three dimensional information, or decomposed in spherical harmonics if we only dispose of projected, bidimensional maps, gives rise to the three-dimensional power spectrum $P(k, z)$ (this quantity is also a function of the redshift, since we can, to some extent, slice our observables at different radial distances) or the two-dimensional angular power spectrum C_ℓ . The greater the amplitude of the correlation function or of the power spectra, the greater the degree of inhomogeneity and/or anisotropy of the specific observable. This process can be alternatively applied to a single observable, that can thus be autocorrelated, or by cross-correlating distinct observables which we assume or are known to be tracing some shared underlying field (see chapter 1, sec.1.4). Not all cosmic messengers studied by astroparticle physics can provide morphological information: charged particles would typically provide exclusively an energy spectrum, whereas most of the morphology and spacial information is erased because they are deviated and randomised by the action of magnetic fields. But photons are electrically neutral, and from the UGRB it is therefore possible to extract information on both its energy intensity spectrum *and* on its spatial, morphological distribution. This consideration does not still allow to conclude anything concerning the composition of the UGRB (or of other backgrounds allowing similar formalisation) but we can observe that, next to the sheer benefit of having both the intensity and angular power spectra available for our problem, providing at the same time a spatial and an energy breakdown of the UGRB, the angular power spectrum observable stresses more

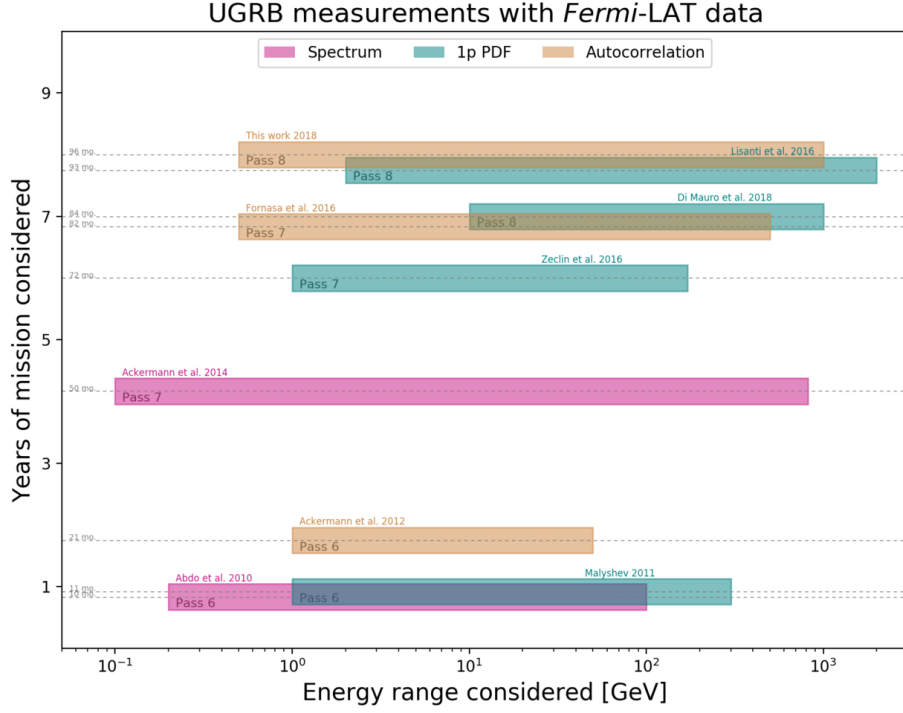


Figure 2.9: Summary of the highly significant detections of the main UGRB properties with Fermi-LAT data, from (Negro, 2018) (Michela Negro’s PhD thesis, corresponding to the “this work” label in the picture). Data are displayed as a function of the energy range and of the different techniques.

the *spatial clustering features* of the UGRB components, possibly enhancing contributions from energetically fainter processes which can be strongly clustered but feebly emitting, to which the intensity power spectrum is consequently less sensitive. Furthermore, morphological information might help with disentangling between astrophysical and DM sources in that astrophysical objects can be considered as point-like sources when observed in the gamma-ray band, since they appear as compact objects given the detector angular resolution, whereas dark matter should generate diffuse signal, tracing the LSS. Likewise, the astrophysical sources differ from DM in their distinct spectral energy and redshift distributions: astrophysical sources do not show a hard energy cutoff at the DM candidate particle mass as DM signals do, while, in terms of redshift distribution, DM gamma-ray emission peaks at low redshift, whilst the unresolved emission from astrophysical sources peaks at higher redshifts and with broader kernels. In this sense, the candidate source kernels (also known as window functions, derived in sec. 3.1) can act as the *trait d’union* between intensity information and angular power spectra, since they are defined such that the integral of the kernel calculated in an energy bin along the radial direction provides the average intensity of the candidate UGRB source in the same energy bin. Angular power spectra act thus as a sort of magnifying glass, or, more generally, they can circumscribe interesting features independently from or complementarily to the intensity information, further supported by another observable, the simple photon count-per-pixel statistics (a one-point statistics known as photon-count distribution, as opposed to the angular power spectrum acting as two-point statistics) (Malyshev & Hogg, 2011; Manconi et al., 2020; Zechlin et al., 2017). Focusing on the two-point statistics, results tell us that the UGRB signal is close to isotropy (Ackermann et al., 2015): yet, if its sources are actually objects clustered in the cosmic web, some degree of anisotropy should be observed and should be

somehow related to the LSS morphology. This statement was indeed confirmed by (Ackermann et al., 2018), reporting the measurement of a non-zero autocorrelation angular power spectrum, too faint in any case to extract significant information telling apart the composition of the UGRB (a graphical summary of the detection significances for exclusively UGRB-based observables as of 2018-2019 is displayed in fig. 2.9, taken from the excellent reviewing effort of (Negro, 2018)). These results, although relevant, are still rather limited, too much burdened by uncertainties and without providing important information other than the general conclusions about the actual presence of some anisotropy in the UGRB, generically probing the coherence of fields at large angular scales but rather insensitive to the properties of the smallest scales (Ackermann et al., 2015, 2018). Here the role of cross-correlations takes up: cross-correlations between electromagnetic signals could point towards shared sources and processes emitting in different channels (an option we are not discussing in this work) whereas, by combining UGRB data with some well-established LSS tracer, e.g. galaxies, weak lensing/cosmic shear, CMB lensing, HI intensity mapping etc., we can enhance by construction the potential LSS-related information contained in the UGRB. Focusing on this second category of studies, weak gravitational lensing signals, for instance, can unbiasedly probe the dark matter distribution in halos and its pure gravitational effects; a cross-correlation with this observable would thus directly point, at the smallest LSS scales, towards the most evident and fundamental feature of DM particles, namely, the gravitational attraction they are exerting (Camera et al., 2013; Fornasa et al., 2016a; Fornengo & Regis, 2014). The theoretical development of such intuition has been laid out in the last decade in works such as (Camera et al., 2013; Fornengo & Regis, 2014; Pinetti et al., 2020) (for HI intensity mapping), and (Camera et al., 2015) for the full deployment of the tomographic-spectral approach and has soon led to many observational results: among the many works (Ando et al., 2014; Fornasa et al., 2016b; Feng et al., 2017; Tan et al., 2020b; Zhou et al., 2024; Pinetti et al., 2025; Fornengo et al., 2015), (Xia et al., 2015) confirmed the detection of the cross-correlation signal and discussed its composition in terms of exclusively astrophysical sources, considering both low and intermediate redshift galaxy catalogues, thus also recovering information about the hypothetical redshift distribution of the UGRB sources; additionally, the work claims that the modelled sources cannot reproduce the 100% of the signal, leaving room for more distant or additional (astrophysical or exotic) sources. Conversely, (Regis et al., 2015) shew that annihilating dark matter alone can explain the cross-correlation with the 2MASS galaxy catalogue at a very low redshift; (Ando et al., 2014) achieved similar conclusions on the 2MASS galaxy catalogue, this time including astrophysical sources and finding they are subdominant if annihilating DM is included. Finally, (Cuoco et al., 2015) discussed the full case of astrophysics and DM either in annihilating or decaying configuration jointly contributing to the UGRB: after having inspected a number of low-redshift galaxy catalogues, a preference was found for a dominant mAGN population with subdominant DM and astrophysical contributions. A relevant astrophysical component, disfavouring dark matter and only posing upper limits on its annihilation and decay cross-sections, was more recently found at low redshifts and with a similar, although not identical, setup by (Paopiamsap et al., 2024). Other works also dealing with galaxies and galaxy clusters we could mention are (Xia et al., 2011; Shirasaki et al., 2015; Cuoco et al., 2017; Ammazzalorso et al., 2018; Branchini et al., 2017; Shirasaki et al., 2018; Hashimoto et al., 2019; Colavincenzo et al., 2020; Tan et al., 2020a). On the other hand, measurements of the shear-UGRB angular 2-point cross-correlation function (i.e., the real-space counterpart of the angular power spectrum) performed by (Ammazzalorso et al., 2020) on DESI data show that the statistics is dominated by a Blazar component at smaller scales, whilst at larger angular separations the size of

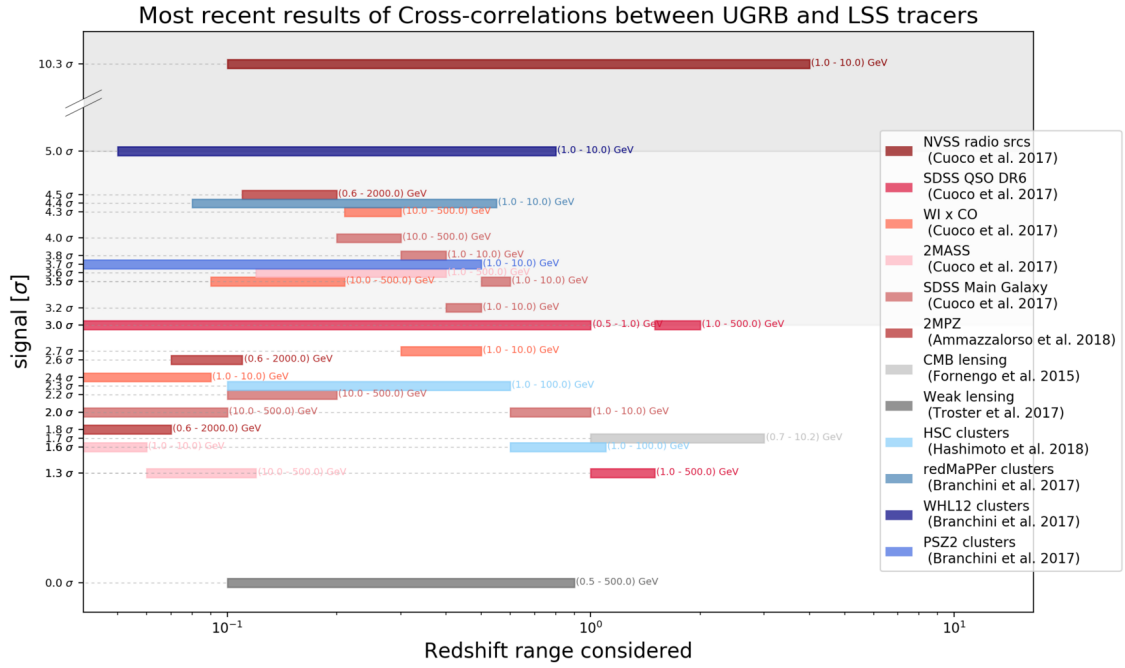


Figure 2.10: Summary of the detections of the main UGRB properties in cross-correlation with LSS tracers. (Negro, 2018). This time the x-axis shows the attained redshift range of the different measurements, potentially informative about the nature of the sources generating the UGRB. Notice how some cross-correlations provide higher signal detection significances with respect to the autocorrelation strategy in fig. 2.9.

the uncertainties prevents us to resolve the DM contribution, potentially dominant on the largest angular scales, from the astrophysical one. Finally, a similar updated analysis setup focusing on astrophysical sources and thoroughly inspecting the cross-correlation with the DES weak lensing datasets was recently performed by (Thakore et al., 2025), supplemented with a multi-tracer study in (Thakore et al., 2026), not discussing the topic of dark matter detection but confirming highly significant detection of the cross-correlation between lensing signals and the UGRB; interestingly, the paper elaborates on potential different compositions between the resolved extragalactic gamma signals and the unresolved one. Very recently, on the other hand, the work of (Zhang et al., 2026) was not able to achieve any statistical claim of a shear-UGRB cross-correlation signal under the hypothesis of a gamma emission being sourced by the combined contribution of BL Lac, FSRQ and annihilating or decaying WIMP Dark Matter particles. Consequently, also the calculated bound on the values of the WIMP annihilation cross-section rate and decay rates are not competitive with the best available results. The authors ascribe the null result to the impact of scale cuts and the consequent rather limited amount of available signal for the KiDS survey. A quite useful result provided by the paper is the independent measurement of the highly significant detection of the cross-correlation between the UGRB and other cosmic shear surveys they achieved while validating their data analysis pipeline against previously published results. Other works that have been dealing with cosmic shear signal cross-correlations have been (Shirasaki et al., 2014, 2016; Tröster et al., 2017). Overall, statistically significant evidence for the existence of cross-correlations has been repeatedly confirmed: with galaxies at the level of 3.5σ (Xia et al., 2015; Regis et al., 2015; Cuoco et al., 2015) up to $8 - 10\sigma$ (Paopiamsap et al., 2024), with galaxy clusters at 4.7σ (Branchini et al., 2017), with the CMB lensing signal at 3.2σ (Fornengo et al., 2015), while the weak

gravitational lensing measurements of (Ammazzalorso et al., 2020) report a signal-to-noise ratio of 5.3σ . A signal-to-noise ratio of 8.9σ is reported in (Thakore et al., 2025) and of 7.9σ in (Thakore et al., 2026) in a multi-tracer framework. Many solid hints about the astrophysical ingredients of the UGRB have been collected in the last years: the assessment of the exact abundance of each component remains conditioned by observational and modelling uncertainties, and similar issues affect the redshift distribution as well as the angular morphology of the spectra; *a fortiori* such burdens reverberate on the quest for more exotic contributions to the UGRB such as DM, leaving most aspects of the topic still undecided. A graphical summary of the detection significances in UGRB-LSS tracers cross-correlation configurations can be seen in fig. 2.10, again obtained from (Negro, 2018) and reporting the status of the measurements as of 2018-2019, and that have therefore to be updated by bearing in mind in particular the results of (Ammazzalorso et al., 2020; Paopiamsap et al., 2024; Thakore et al., 2025).

We have now gathered all materials and context necessary to introduce the research work subject of this first half of the thesis, which will be discussed in chapter 3. There, we will present the properties of cross-correlations between the 2MRS galaxy sample (Huchra et al., 2012) and the UGRB after the introduction of a weighting scheme on the galaxy number counts corresponding to applying a Wiener filter (Wiener, 1949; Rybicki & Press, 1992; Fisher et al., 1995; Lahav et al., 1994; Zaroubi et al., 1995) to the observations. The innovation will move from the results published in (Alonso et al., 2020; Urban et al., 2021; Tanidis et al., 2022), who applied similar techniques to the unresolved gravitational wave background and the high energy cosmic ray background. Both are observables akin in properties to the UGRB, in that they are inherently two-dimensional and lacking direct redshift information, with a high level of isotropy and whose possible residual anisotropic component is of interest for cosmological, astrophysical and astroparticle goals as well, that can be again summarised in what kind of astrophysical sources are producing their signal and, if possible, what they can tell us about either cosmological processes or BSM physics (Agazie et al., 2023; EPTA Collaboration et al., 2023; Christensen, 2019; Muia et al., 2023; Christensen, 2019; Pierre Auger Collaboration et al., 2017; Abbasi et al., 2017; Tinyakov et al., 2016; Kotera & Olinto, 2011). We will combine the Wiener filter formalism with a multi-tracer framework (Seljak, 2009; McDonald & Seljak, 2009; Abramo & Leonard, 2013; Mueller et al., 2018; Karagiannis et al., 2024), a statistical technique widely used in cosmology and consisting in the simultaneous analysis of many tracers, thus overcoming many of the limitations induced by shot-noise and cosmic variance effects one would encounter when analysing every dataset independently from the other ones. We will ultimately test the effects of such technical improvements by considering the bounds we can put on the sensitivity to the annihilation cross-section rate as a function of the cold relic candidate mass and compare them to the best attained limits currently available in the literature and to standard, unweighted approaches in order to understand whether and in what circumstances either the multi-tracer or the Wiener filtering techniques (or both approaches at the same time) might improve the sensitivity to annihilating WIMP.

Concerning the choice of 2MRS as reference cross-correlation galaxy sample, as we said, astrophysical and DM kernel present different redshift and energy dependencies and low-redshift measurements are more apt to probe DM processes with less astrophysical foreground: the extremely low redshift of 2MRS represents the reference sample to test potential WIMP DM features in cross-correlations, overlapping with the bulk of the DM window function and intercepting a very small part of competing astrophysical signals (Ando et al., 2014); in absence of DM sources contributing

to the UGRB at low redshift, the cross-correlation signal between 2MRS and the UGRB should yield a null or extremely weak detection. In addition, 2MRS galaxies are a spectroscopic sample, particularly apt as a benchmark for the Wiener filtering technique that, we will show, corresponds to the use of a redshift-dependent weighting formalism requiring precise identification of the galaxy original position.

Chapter 3

Wiener filtering and multi-tracer techniques for dark matter cross-correlations between gamma-ray emission and galaxy catalogues

This chapter exposes the main research activity published in (Rubiola et al., 2024, 2025). As discussed in sec. 2.3 of chapter 2, we set ourselves in the general framework of statistical cross-correlations between the UGRB and LSS tracers –specifically, galaxy clustering statistics– and we aim at forecasting the possible improvement on the minimum attainable sensitivity of these methodologies to the annihilation cross-section rate of WIMP-like DM candidates in the 10-1000 GeV mass interval. The forecast involves the implementation of a Wiener-filter inspired weighting scheme for the galaxy clustering kernel function, as well as the comparison of the standard cross-correlation methodology with a multi-tracer strategy, in turn subdivided in a study of the effect of the combined galaxy-galaxy and gamma-galaxy angular power spectra on one hand, and a second case leveraging the full data vector comprising the galaxy-galaxy, gamma-galaxy and gamma-gamma correlation power spectra.

Throughout this chapter the analytic expression for the weighting scheme will be derived, followed by an outline and discussion of the main results, according to this subdivision: the general cross-correlations formalism is sketched in sec. 3.1. Sec. 3.2 discusses the Wiener filter and the implementation of optimal weights. Sec. 3.3 presents our analysis, inclusive of the multi-tracer technique. Sec. 3.4 collects the results and sec. 3.5 presents the conclusions. The reference cosmological model for our calculation is the Λ CDM model having the parameters set at the Planck 2018 release (Aghanim et al., 2020).

3.1 Cross-correlation formalism: angular power spectra and window functions

In these pages we will outline how to develop a filtering technique involving an optimal weighting strategy, aimed at improving the chances of extracting a dark matter signal. Operatively, the technique will be based on the manipulation of cross-correlation angular power spectra, starting from their formulation as outlined in sec. 1.4: we will combine a model for the UGRB, including

both astrophysical and WIMP dark matter emission models, with galaxies as tracers of the dark matter distribution in the Universe. The general spirit of the formalism builds therefore upon the vast research framework presented in sec.2.3.2 and we recall that, while the galaxy distribution observations provide the redshift information, the redshift distribution of the gamma-ray emission can only be theoretically modelled. We indeed lack of its observational counterpart, because telescopes can exclusively detect the gamma-ray flux integrated along the line of sight.

Angular cross-correlation power spectra

We start with exposing the angular power spectrum definition: given two observables A and B , their auto- or cross-correlation angular power spectra provide a measurement of the value of the cross-correlation among their harmonic-expansion coefficients. If we are dealing with a statistically homogeneous field projected on the sphere, the spectrum is exclusively a function of the angular multipole ℓ , which corresponds to the angular scale separating any two points on the sky, i.e.:

$$\langle A_{\ell m} B_{\ell' m'} \rangle = \delta_{\ell\ell'}^K \delta_{mm'}^K C_{\ell}^{AB}, \quad (3.1)$$

where the brackets denote ensemble average and δ^K is the Krönecker-delta symbol. We employ the usual Limber approximation (Limber, 1953; Kaiser, 1992, 1998) valid for $\ell \gg 1$, that allows us to formulate the angular power spectrum as:

$$C_{\ell}^{AB} = \int \frac{d\chi}{\chi^2} W_A(\chi) W_B(\chi) P_{AB} \left(k = \frac{\ell + 1/2}{\chi}, \chi \right). \quad (3.2)$$

In this expression, k is the wavenumber for a decomposition in the Fourier space, χ is the radial comoving distance, which can be derived given z for a flat cosmology by means of $d\chi = c dz/H(z)$ with $H(z)$ the expansion rate of our Universe. $P_{AB}(k, z)$ represents the 3D cross-correlation power spectra of the fluctuations of the A, B fields, which, under the assumption of Gaussian fluctuations, are defined as:

$$\langle \tilde{f}_A(\mathbf{k}, z) \tilde{f}_B(\mathbf{k}', z) \rangle = (2\pi)^2 \delta_D^3(\mathbf{k} - \mathbf{k}') P_{AB}(k, z) \quad (3.3)$$

Here, $f_A = (g_A - \langle g \rangle) / \langle g \rangle$ represent the fluctuations of the field g_A around its mean, then normalized to the mean value of the field itself. We know that $P_{AB}(k, z)$ estimates the covariance between two fluctuation-fields $f_{A,B}$ at spatial scale k^{-1} and redshift z and that many theoretical strategies are available in order to properly describe it in different perturbative regimes. For the sake of UGRB anisotropy studies, the general focus is on the medium and small-scale clustering of both dark matter halos and astrophysical sources. We have therefore to go beyond linear perturbation, resorting instead to the halo model formalism (Cooray & Sheth, 2002; Akgari et al., 2023) and its reformulation of the power spectrum $P_{AB}(k, z)$ as a summation of a 1-halo and a 2-halo term:

$$P_{AB}(k) = P_{AB}^{1h}(k) + P_{AB}^{2h}(k). \quad (3.4)$$

More information about the meaning and construction of the two terms is collected in sec. 1.3.1: we remember to the reader that the 1-halo term corresponds to the contributions to the correlation signal between the two fields A, B due to objects residing in the same halo. On the contrary, the 2-halo term traces the contributions to the correlation determined by objects residing on two different halos. For this reason, at the largest scales, the 2-halo term becomes increasingly proportional to the linear power spectrum, the proportionality factors being the bias terms connecting the fields

A, B with the underlying dark matter field. The 1-halo term is the dominant component at the smallest scales, corresponding to the regime we shall be chiefly probing.

Since we assume that the UGRB is composed by four classes of astrophysical sources (BL Lac, FSRQ, mAGN and SFG) plus, when required, by a potential annihilating Dark Matter contribution, to which we have to further add the galaxy clustering signal, we will be actually dealing with combinations of many angular power spectra (see sec. 3.3 for the detailed inventory and the construction of all the required terms to define both signals and covariances) and a large number of corresponding three-dimensional power spectra combinations has to be defined, collected together with additional information in sec. 1.3.1.

Window functions

The second quantity required to build the generic cross-correlation power spectra, and of particular interest in the case of the Wiener filtering techniques, which will directly address their analytic expression, are the window-functions, denoted by $W_{A,B}(\chi)$ in eq. 3.2.

Their general meaning and features have been presented while discussing eq. 1.122: they can be intended as kernel functions that express the redshift dependence of the intensity field of the observables A or B . Here, we will outline their specific modelling applied to the UGRB and galaxy clustering sources: again, since the gamma-ray emission field is composed by a potential contribution from dark matter annihilation in cosmic structures, to be added to the irreducible background emission from astrophysical sources located in the same structures, we have to actually define five window functions. Importantly, we recall that the window functions satisfy the following result, producing the average intensity of the corresponding flux:

$$\langle I_A \rangle = \int d\chi W_A(\chi). \quad (3.5)$$

We eventually remark that the next sections will expose the window functions as differential in energy, whereas all our calculations will further integrate them in each of the energy bins reported in table 3.1.

Annihilating Dark Matter window function We can start by constructing the annihilating dark matter window function, following (Ando & Komatsu, 2006): given the relation between the source kernel and its average intensity (eq. 3.5), we can connect the observed average intensity $\langle I_A \rangle$ per unit energy with the energy at the source E_s as a function of the observed energy E (it holds $E_s = (1+z)E$) by:

$$E \langle I_A \rangle = \frac{c}{4\pi} \int dz \frac{P_\gamma((1+z)E, z, \chi \mathbf{n})}{H(z)(1+z)^4} \exp(-\tau[(1+z)E, z]) \quad (3.6)$$

where $H(z)$ is the redshift-dependent Hubble factor, τ is the absorption factor of the photon flux along the line of sight as a function of the redshift and of the energy at the source and P_γ is the photon energy per unit of volume, time and energy, namely, the photon emissivity. This quantity, in the specific case of photons emitted by annihilating DM particles, can be defined as :

$$P_\gamma(E, z, \chi \mathbf{n}) = E \frac{dN}{dE} \frac{\langle \sigma v \rangle}{2} \left[\frac{\rho_{DM}(z, \chi \mathbf{n})}{m_\chi} \right]^2 \Delta^2(z) \quad (3.7)$$

where $\langle \sigma v \rangle$ is the annihilation rate of a pair of dark matter particles and m_{DM} the dark matter particle mass, $\Delta(z)^2$ the clumping factor, dN/dE the energy spectrum of photons produced

by annihilations as measured in the rest-frame of the annihilation interaction, while $\tau(E, z)$ is the optical depth for gamma-ray absorption along the line of sight, chiefly induced by pair production processes on the extra-galactic background caused by photons emitted by galaxies in the ultraviolet, optical, and infrared wavelengths. To model this latter quantity, we adopt the results from (Franceschini et al., 2008).

We can connect the density of dark matter to its average density and the field density contrast by:

$$\rho_{\text{DM}}(z, \chi \mathbf{n}) = \bar{\rho}_{\text{DM}} \delta(z, \chi \mathbf{n}) = \Omega_{\text{DM}}(z=0) \rho_c(z=0) (1+z)^3 \delta(z, \chi \mathbf{n}) \quad (3.8)$$

where we have made explicit for better clarity that both the critical density $\rho_c = \frac{3H^2(z)}{8\pi G}$ of the Universe and the Dark Matter density contrast are calculated at the present value whereas redshift evolution is factored out in the term $(1+z)^3$. Hereafter, both ρ_c and Ω_{DM} will be intended as calculated at $z=0$. These relations can be inserted in eq. 3.6 after converting the integral in redshift to the integral in comoving distance by $d\chi = c dz/H(z)$, and finding:

$$\begin{aligned} \langle I_A(\mathbf{n}, E) \rangle &= \frac{1}{4\pi} \int d\chi \frac{dN}{dE} [E(1+z)] \frac{\langle \sigma v \rangle}{2} (1+z) \left[\frac{\Omega_{\text{DM}} \rho_c (1+z)^3 \delta(\chi, \mathbf{n}\chi)}{m_{\text{DM}}} \right]^2 \Delta^2(\chi) \\ &\quad \times \frac{1}{(1+z)^4} \exp(-\tau[(1+z)E, z]) \\ &= \frac{1}{4\pi} \int d\chi \frac{dN}{dE} [E(1+z)] \frac{\langle \sigma v \rangle}{2} \left(\frac{\Omega_{\text{DM}} \rho_c}{m_{\text{DM}}} \right)^2 \\ &\quad \times (1+z)^3 \exp(-\tau[(1+z)E, z]) \Delta(\chi)^2 \end{aligned} \quad (3.9)$$

The expression of this last result can be equated to eq. 3.5: the two relations are equal if the integrands are equal, and therefore we have:

$$W_{\text{DM}}(E, \chi) = \frac{dW_{\text{DM}}}{dEd\chi} = \frac{1}{4\pi} \frac{\langle \sigma v \rangle}{2} \Delta^2(z) \left(\frac{\Omega_{\text{DM}} \rho_c}{m_{\chi}} \right)^2 (1+z)^3 \frac{dN}{dE} [(1+z)E] e^{-\tau(E, z)}, \quad (3.10)$$

The expression tells us that the dark matter particle mass m_{DM} and the annihilation rate $\langle \sigma v \rangle$ are the particle physics parameters governing the signal intensity, while the energy spectra govern its evolution in energy, in turn a function of the annihilation processes dark matter particles may undergo. Our reference annihilating dark matter model will thus be that of a WIMP annihilating into a pair of $b\bar{b}$ quarks, which is often the dominant annihilation channel for most of this class of particles. Quantities requiring special care are the energy spectra at the source $\frac{dN}{dE}$: they are modelled from the Monte Carlo simulations published in (Arina et al., 2024).

Astrophysical sources Our usual four astrophysical gamma-ray sources, blazars (BLA), mis-aligned active galactic nuclei (mAGN), flat spectrum radio quasars (FSRQ) and star forming galaxies (SFG) share the same window function functional form:

$$W_{\text{astro}}(E, z) = \frac{dW_{\text{astro}}}{dEd\chi} = \left(\frac{d_L(z)}{1+z} \right)^2 \int_{\mathcal{L}_{\min}}^{\mathcal{L}_{\max}(z)} d\mathcal{L} \frac{dF}{dE}(E, \mathcal{L}, z) \phi(\mathcal{L}, z) e^{-\tau(E, z)}, \quad (3.11)$$

where $\mathcal{L}$ is the source luminosity, $d_L(z) = (1+z)\chi(z)$ the luminosity distance, $\phi(\mathcal{L}, z)$ the gamma-ray luminosity functional form. Each class of object is distinguished by their respective GLF (here noted with $\phi(\mathcal{L}, z)$) by its spectral energy distribution (SED) dF/dE . The parameters and most of the considerations outlined in section 1.3.1 concerning the physics of the astrophysical sources, of their GLFs and about the construction of the halo model power spectra hold for the purpose of

Bin	$E_{\min}$ (GeV)	$E_{\max}$ (GeV)	$\mathcal{N}^\gamma$ ($\text{cm}^{-4} \text{ s}^{-2} \text{ sr}^{-1}$)	f_{sky}^γ	σ_0^{Fermi} (deg)	E_b (GeV)
1	0.5	1.0	1.056×10^{-17}	0.134	0.87	0.71
2	1.0	1.7	3.548×10^{-18}	0.184	0.50	1.30
3	1.7	2.8	1.375×10^{-18}	0.398	0.33	2.18
4	2.8	4.8	8.324×10^{-19}	0.482	0.22	3.67
5	4.8	8.3	3.904×10^{-19}	0.549	0.15	6.31
6	8.3	14.5	1.768×10^{-19}	0.574	0.11	11.0
7	14.5	22.9	6.899×10^{-20}	0.574	0.09	18.2
8	22.9	39.8	3.895×10^{-20}	0.574	0.07	30.2
9	39.8	69.2	1.576×10^{-20}	0.574	0.07	52.5
10	69.2	120.2	6.205×10^{-21}	0.574	0.06	91.2
11	120.2	331.1	3.287×10^{-21}	0.597	0.06	199.5
12	331.1	1000	5.094×10^{-22}	0.597	0.06	575.4

Table 3.1: Table reporting the energy bins for the gamma-ray signal we are employing in our analysis, corresponding to 8 years of Fermi observations (Fermi Pass 8 ([Ackermann et al., 2018](#))). $\mathcal{N}^\gamma$ stands for the auto-correlation noise measured by Fermi and that, in our analysis, it is rescaled down by a factor of 2.5 so as to account for a detector exposure of approximately 20 years. f_{sky}^γ the observed fraction of the sky outside the combined Galactic and point-source masks and σ_0^{Fermi} the 68% containment angle of the PSF and $E_b = \sqrt{E_{\min} E_{\max}}$ the geometric center of each energy bin.

characterising the astrophysical tracers window functions as well. We observe that the window functions for astrophysical objects require an integration in luminosity between the minimum luminosity value $\mathcal{L}_{\min}$, a function of the intrinsic properties of each source class, and the maximal luminosity value $\mathcal{L}_{\max}(z) = \min[\mathcal{L}_{\max}^{\text{intr}}, \mathcal{L}_{\text{sens}}(z)]$ where $\mathcal{L}_{\max}^{\text{intr}}$ again is specific to the source class and $\mathcal{L}_{\text{sens}}(z)$ and a function of the detector sensitivity of the detector. Since we are considering *unresolved* sources, only those sources, located at redshift z and contributing to a photon flux at the detector below the detector sensitivity F_{sens} have to be accounted for.

Galaxy surveys The general expression for the kernel of a galaxy catalogue depends on the comoving number density of galaxies $\bar{n}_g(\chi)$ as:

$$W_g(\chi) = \frac{\chi^2 \bar{n}_g(\chi)}{[\int d\chi \chi^2 \bar{n}_g(\chi)]}. \quad (3.12)$$

This formulation ensures that the window function is normalized as

$$\int d\chi W_g(\chi) = 1, \quad (3.13)$$

with

$$\int d\chi \chi^2 \bar{n}_g(\chi) = \bar{N}_{\Omega, g}, \quad (3.14)$$

$\bar{N}_{\Omega, g}$ being the number density of galaxies per steradian. The specific galaxy survey can then be characterised case by case. Many considerations and the HOD for 2MRS are collected in sec. [1.3.1](#), where also our reference model for $\bar{n}_g(\chi)$ is exposed as reported in ([Huchra et al., 2012](#)).

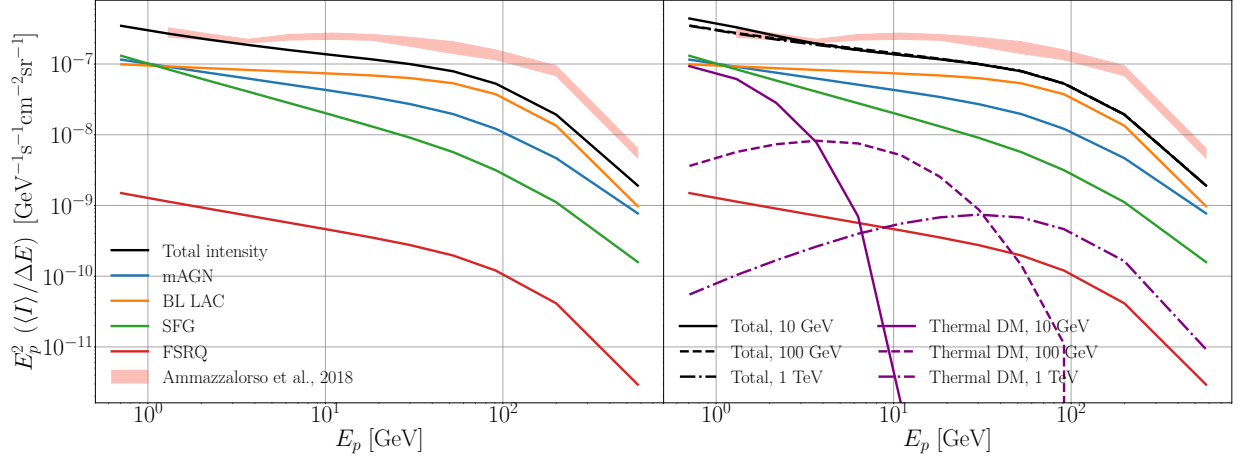


Figure 3.1: A plot of the average intensity for the UGRB autocorrelation signal including exclusively astrophysical contributions (left panel) and different annihilating WIMP models (right panel), compared with available data from (Ammazzalorso et al., 2018). The plotted quantity is the average intensity of 3.5 rescaled by the width of each redshift bin $\Delta E = E_{\max} - E_{\min}$ of table 3.1 and multiplied by the centroid of each redshift bin, the E_p quantity of the same table.

The 2MRS catalogue is characterised as the subsample of galaxies from the wider observational programme of the 2MASS survey (Skrutskie et al., 2006). The positions of the galaxies belonging to this subsample have been measured with spectroscopic, next to photometric, techniques, achieving an extremely better determination of the positions of the object, although at the price of a much reduced catalogue: from the several hundred millions objects of the original survey, the 2MRS sample only has 43812 galaxies. They are all comprised in the redshift range (0.0012, 0.1) associated to a number density of 3920 galaxies sr^{-2} that can be calculated by applying eq. 3.14.

Physical interpretation of the window functions and of some related quantities It is interesting now to refer to some physical interpretations of the window functions and to some of the other quantities we discussed above, that will also further justify the use of the cross-correlation technique, and in particular of the 2MRS survey and its redshift range for the sake of our analyses. We know that we can apply eq. 3.5 to readily extract the average intensity spectrum of the UGRB from the window functions as a function of their energy: this imposes an important physical condition on our models and allows to constrain the associated parameters, since the total modelled intensity, obtained by the summation of the the individual intensities, must not be larger, or, in a weaker sense, has to lay within the uncertainties associated to an experimentally available quantity, the total *observed* intensity. This constraint can be observed in fig. 3.1, where we show the modelled average intensities for the four assumed astrophysical components of the UGRB and for a reference Dark Matter signal for a thermal WIMP having $m_{DM} = 100$ GeV and the thermal annihilation cross-section rate $\langle\sigma v\rangle_{\text{th}} = 3 \cdot 10^{-26} \text{ cm}^3 \text{ s}^{-1}$, evaluated at the energy bins reported in table 3.1, chosen in agreement with the energy binning strategy of the UGRB signal used, among the other works, in (Ackermann et al., 2018; Pinetti et al., 2020; Arcari et al., 2022). In the left panel of the figure, we assume that *all of the UGRB intensity* is of astrophysical origin, and we compare it with the observational values derived by (Ammazzalorso et al., 2018) (showed as a red-pink band in order to account for the uncertainties). This latter study does not exactly match

the corresponding energy intervals used for our models and the coverage of some energy bins is limited or not perfectly overlapping, but we can observe that:

- The total modelled UGRB (black) signal is compatible or even underestimating the observational constraints in the shared energy interval.
- The UGRB is dominated for most of the energies by the BL Lac population (orange), although the contributions coming from the other classes of astrophysical sources is not totally negligible and, in particular, the SFG population (green) becomes the dominating source for $E < 1$ GeV.

Similar considerations apply to the right panel, where we also include the annihilating Dark Matter contributions at various masses (purple, the different masses being associated with different line styles), which acts as a rather subdominant correction with respect to most of the astrophysical sources. Only at masses around 10 GeV does it seem to provide some noticeable corrections to the total predicted UGRB intensity. At the same time, it appears competitive at least with the FSRQ population at low and intermediate energies, while its amplitude displays a typical scaling that we can better understand by looking at eq. 3.10: the window function amplitude is indeed directly proportional to the annihilation cross-section rate $\langle\sigma v\rangle$, a quantity partially degenerate with the mass m_{DM} . At the same time, the proportionality of the window function with respect to m_{DM} is twofold: next to an inverse proportionality quadratic scaling of the amplitude, the window function, and the average intensity consequently, displays as a function of the mass the characteristic cutoff, absent in astrophysical sources, due to the fact that the maximum energy emission from annihilating DM cannot be larger than the associated particle mass^{*}. The results concerning the intensity spectrum can be also compared with some insights derived from the angular power spectra collected in fig. 3.2, where the importance of the morphological information next to the intensity one, and, in particular, the role of cross-correlations, can be appreciated. Fig. 3.2 displays the quantity we define as C_p , i.e., the angular autocorrelation (left) or cross-correlation (right) C_ℓ for the observables of interest, calculated in multiple energy bins and averaged on an associated multipole interval that, on real data, would correspond to the angular scales contemplated in the analyses, typically excluding the very large angular scales because of foreground contamination and the very small ones because of the loss in resolution induced by the instrumental PSF. This observable provides an information about the average level of anisotropy of the signal in a given energy bin: the lower the degree of anisotropy, the less the coherent structure which can be inferred from the UGRB, which instead gets closer and closer to a featureless, uninformative isotropic signal[†]. In the left panel, the C_p for astrophysical sources, DM (again, calculated assuming $\langle\sigma v\rangle_{th} = 3 \cdot 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ and only parametrised as a function of different mass values) and the total C_p under both the assumption of a pure astrophysical UGRB composition (solid black), and astrophysical+DM UGRB composition (depending on the mass, dashed, dotted-dashed and dotted black) are compared with observational results. We notice that the DM contribution is subdominant also at the level

^{*}Notice that the data in the plot are reported as a function of the centroid of the energy bins, the E_p in the last column of table 3.1. Since the bin containing the mass value of each displayed example might be associated to an E_p larger than the mass itself, the intensity curves might display a cutoff artificially placed at a value larger than the mass value.

[†]Importantly, the energy and binning strategies, together with some additional model specifications, do not correspond to the one used in the text, but to the one used by (Ackermann et al., 2018). These modifications are required in order to correctly compare the models with the data reported in that work.

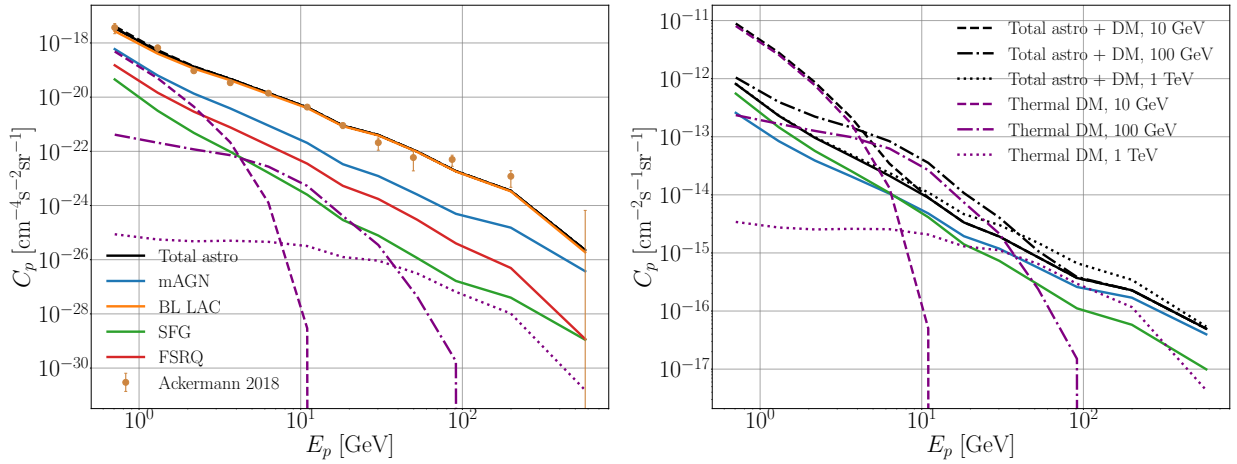


Figure 3.2: A plot of the C_p for the UGRB autocorrelation signal (left) and in cross-correlation with our model of the 2MRs survey (right). In the left panel, the theoretical calculations are compared with available observational data as published in (Ackermann et al., 2018). In the right panel, we can appreciate how the cross-correlation with galaxy surveys better highlights potential Dark Matter contributions, in a specific energy interval, contrasted with the astrophysical counterpart. We can also notice that we can thus eliminate the BL Lac and FSRQ contributions to the total signal. Their kernel starts indeed at higher redshift than the 2MRs one and, since the cross-correlation C_ℓ (eq. 3.2) is a function of the product of the two tracers’ window functions, when one of the two is zero also the correlation signal vanishes.

of spatial anisotropy compared with the astrophysical contributions (although again there is a dependence from the assumed mass and annihilation rates), but the differences among the contributions are less pronounced with respect to the intensity signal. At the same time, we observe that the BL Lac class is more representative of the total morphology of the UGRB – whilst the information coming from the other astrophysical populations is clearly subdominant – and follows rather closely the data and uncertainties we display, derived from those reported in table VII of (Ackermann et al., 2018), binned with respect to the harmonic average of the energy bins limits reported in the same table, and, again, different from the ones we use in table 3.1. The right panel of fig. 3.2 is a purely theoretical result showing the behaviour of the anisotropy power spectrum when instead the cross-correlations of the UGRB with the 2MRs galaxy survey are calculated (to the best of our knowledge, although the cross-correlation between 2MRs and the UGRB has been published (Ando et al., 2014), exact reporting of the corresponding observational C_p data is not available). In this case, the annihilating Dark Matter contribution clearly modifies the predicted level of anisotropy as a function of energy and mass. It typically dominates a specific energy interval – associated to energies of some tens of GeV, suggestive that signals from light WIMP might be more strongly influencing the morphology of the UGRB – with respect to the concurring astrophysical contributions. This is a consequence of the fact that an LSS tracer like galaxies more closely probes the underlying DM halos distribution than the UGRB alone, “extracting”, once correlated with the UGRB, more relevant information about the relation of the UGRB with the same halos. And, since the annihilation or decay signal from WIMP particles is expected to be intrinsically a better tracer of the halos they should be constituting compared to astrophysical contributions, the cross-correlation technique is more beneficial to probe candidate DM effects rather than astrophysical phenomena. This statement encloses redshift-dependence: depending

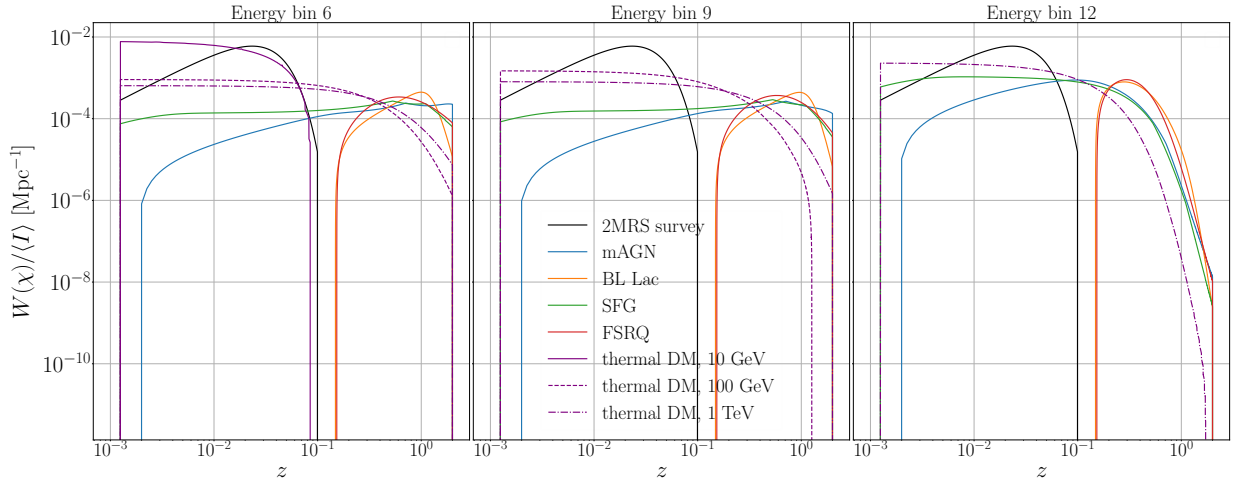


Figure 3.3: A collection of UGRB tracers window functions rescaled to their average intensity and compared with the 2MRS window function of eq. 3.12. The tracers are reported as a function of their redshift, subdivided in three distinct redshift bins and, in each bin, the window functions for three distinct annihilating WIMP models, distinguished by their masses, are displayed.

on the selected redshift survey, we can indeed more effectively avoid competing astrophysical backgrounds. In the right panel of fig. 3.2 we can notice the absence of contributions from BL Lac and FSRQ sources to the cross-correlation signal; they are not expected to be emitters of UGRB photons at the redshifts associated to the 2MRS survey, as we can see by inspecting the window functions of the UGRB superposed to the 2MRS kernel in fig. 3.3. The selection of a galaxy survey at a closer or more distant redshift, combined with the appropriate selection of the more interesting energy channels, is thus part of an observational optimisation strategy: higher redshift surveys probe at the same time astrophysical and WIMP-like kernels (or only astrophysical contributions at even higher z), while very low-redshift surveys such as 2MRS ensure that the annihilating DM signal, if present, only has to compete with mAGN and SFG as astrophysical foregrounds. Further insights about the gamma-ray window functions as a function of redshift and for some representative energy bins can also be recovered from fig. 3.5, and we can contrast the total astrophysical signal, present in every energy bin (represented by the three distinct panels of the image) with the Dark Matter signal, increasingly suppressed at larger and larger energies as a function of the mass. In particular, only the emission associated to the heaviest WIMP model contemplated by our study (1 TeV) is present up to the last energy bin (spanning from 331.1 to 1 TeV) (see fig. 3.4). “Reversing” our perspective, we can instead consider fig. 3.5, where instead the same representative window functions are displayed as a function of all energy bins and sampled at three distinct redshift values. From this other point of view we can better appreciate again how the annihilating dark matter signal tends to dominate the lowest energy bins and redshift against the total astrophysical contribution, showing an additional internal dynamics in terms of the different masses of the candidate particle.

3.2 Wiener filter: the case for optimal weighting

We are now ready to introduce the weighting technique we will employ in our work (Rubiola et al., 2024, 2025), using also arguments that will closely follow the discussions presented in (Alonso et al., 2020; Urban et al., 2021). These latter publications are the seminal contributions from which

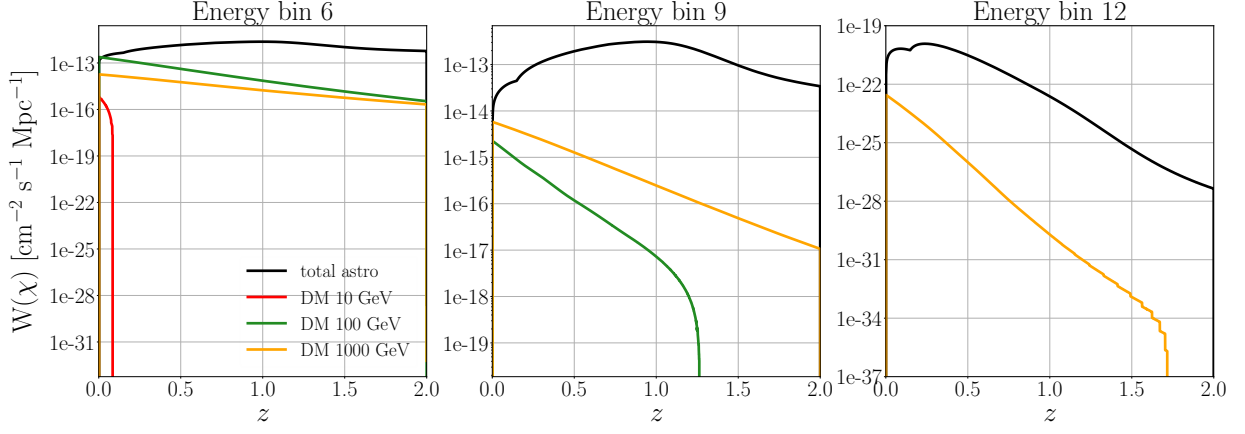


Figure 3.4: UGRB window functions $W_\gamma(z)$ for the different contributors as a function of redshift. We compare the astrophysical sources (blazars, misaligned active galactic nuclei, flat spectrum radio quasars and star forming galaxies) in black, with some representative dark matter emission cases assuming annihilation through the $b\bar{b}$ channel. In particular, we have: $m_{\text{DM}} = 10$ GeV (red), $m_{\text{DM}} = 100$ GeV (green) and $m_{\text{DM}} = 1000$ GeV (blue). The cross-section rate is set at the “thermal” value $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. From left to right, we display in the three panels the different emission components in three different energy bins, corresponding to bins 6, 9 and 12 of table 3.1.

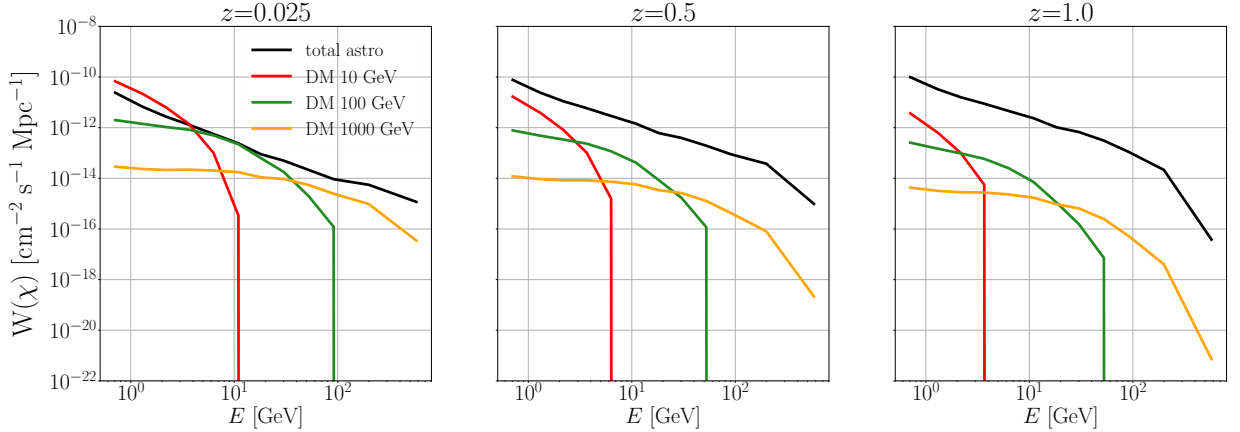


Figure 3.5: UGRB window functions $W_\gamma(E)$ $W_\gamma(z)$ for the different contributors as a function of energy. The total emission from unresolved astrophysical sources (blazars, misaligned active galactic nuclei, flat spectrum radio quasars and star forming galaxies) is in black. We also show some representative dark matter emission cases assuming annihilation through the $b\bar{b}$ channel. In particular, we have: $m_{\text{DM}} = 10$ GeV (red), $m_{\text{DM}} = 100$ GeV (green) and $m_{\text{DM}} = 1000$ GeV (blue). The cross-section rate is set at the “thermal” value $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. From left to right, we display in the three panels the different emission components in three different redshift bins, i.e., $z = 0.025$, $z = 0.5$, $z = 1$.

our work stemmed: they also allow us to present a first operative and conceptual description of our goals and to connect it with the discussion concerning the window functions as exposed in the previous section. We can go back to the definition of the window function for the galaxy clustering signal provided by eq. 3.12: it is the most intuitive formulation, since it substantially constructs a window function based on the actual measured distribution of the number of objects detected by given galaxy survey as a function of their position, expressed in terms of the associated redshift.

As we discussed in sec. 1.4, the C_ℓ can be interpreted as a sum of 3D power spectra, modulated by the window functions of the selected tracers.

By seeing the C_ℓ integrand as a summation of quantities, we might observe that there is some modelling freedom in the definition of our window functions and that we may act on the galaxy kernels according to the specific goals we have in mind. In particular, we can decide to modify the window function and, instead of assigning a weight of one to each galaxy in the survey, we might have good reasons (e.g., a different degree of reliability of the observational data as a function of their distance) to weight galaxies located at different redshifts in different ways. That is, we can rewrite eq. 3.12 in the more general form of:

$$W_{wg}(\chi) = \frac{\chi^2 w(\chi) \bar{n}_g(\chi)}{\left[\int d\chi \chi^2 w(\chi) \bar{n}_g(\chi) \right]}, \quad (3.15)$$

the weights $w(\chi)$ being free functions whose specific analytic expression can be derived once specific weighting strategies and criteria are defined. This said, we can take a first step towards the derivation of the Wiener filter by considering the sky-maps of gamma-ray number counts and galaxies. Each pixel can be assumed to contain N_γ^p photons and N_g^p galaxies. At the same time, every galaxy is associated to a redshift: a vector $\mathbf{N}_g^p$ containing the number of galaxies in each redshift bin can be built. We simplify the notation by calling $\mathbf{x} = (N_{g1}^p, \dots, N_{gM}^p)$ the vector containing the number of galaxies in the different redshift bins (the number of the redshift bins is M) and $y = N_\gamma^p$. In our notation we shall also discard the dependence on energy for the photon counts: our argument can indeed be repeated identically in each energy bin. We remark that boldface symbols denote vector and matrices, non-boldface symbols are scalars.

Here lies the hypothesis at the basis of our Wiener filtering strategy: we said that, generally speaking, we are free to specify a different weighting strategy for the galaxy distribution. Such choice is problem- and data-dependent; since we are considering on one hand UGRB data (gamma photons emitted by objects, either of astrophysical or of more exotic origin, located in the cosmic web) and, on the other one, galaxy clustering observations, we can decide to try to relate the number of photons observed in a given pixel of our UGRB maps with the number of galaxies located in the same pixel observed by some survey. The functional form connecting the two observables can be freely chosen and a simple, intuitive assumption might be that of formulating this connection as a linear relationship:

$$y = \sum_r w_r x_r + n = \mathbf{w}^T \mathbf{x} + n. \quad (3.16)$$

In this expression, $\mathbf{w}$ is a vector representing the coefficients (weights) of the linear regression model while n is the uncorrelated, Gaussian white noise, representing the residuals of the linear regression between the y and $\mathbf{x}$ data (r labels redshift) (Rybicki & Press, 1992; Lahav et al., 1994; Fisher et al., 1995; Zaroubi et al., 1995).

It is exactly the choice of the linear relation – allowing, we will show, for an analytic expression of the regression weights – that leads us to the signal processing technique known as Wiener filter (Wiener, 1949). It was first used as a denoising strategy for Allied radar data in the Second World War[‡] and it is of current use in the signal analysis field, with a number of relevant applications. The original idea was that of extracting an unknown, informative radio signal (the “ $\mathbf{x}$ ” of our formulation) overwhelmed by white noise by knowing, or by modelling some sensible guess about, what frequencies (the “ r ” of our problem) were containing most part the signal and what were

[‡]See for example [this website](#).

instead dominated, or exclusively occupied, by the noise, given the observed combination of noise and information collected by the receiver’s antenna (the y of our problem). Notice that, with respect to the original perspective, we take a slightly different point of view: we are in a theoretical forecast setting, where we are not as much interested in extracting any actual or simulated signal $\mathbf{x}$ from a dataset, as we are instead in finding an expression for the weights which we could then apply to the data and that would allow to denoise/filter potential actual observations. Operationally, once provided an explicit formulation for the $w(\chi)$, given the observationally available redshift distribution of an actual galaxy dataset the quantity can be modified by applying to each object the filter/weight function $w(\chi)$ (we use the two terms interchangeably) which, we will show, is a function of a theoretical model for $W_\gamma(\chi)$. Such re-weighted galaxy distribution might then be cross-correlated with the observed angular gamma-ray map and the operation can be repeated many times while varying the models of gamma-ray emission from dark matter (and, or exclusively of, the astrophysical sources). At every iteration, we can define a figure of merit, for example the signal-to-noise ratio, whose value allows to determine the statistical significance of the agreement, and enabling inference on the underlying theoretical models. The best weighting strategy, associated to the best figure of merit, will thus be associated to the most likely (although not “true” in an absolute sense, since we are not able to observe the redshift distribution of the UGRB sources, but only their cumulative, bidimensional signal projected on the celestial sphere) theoretical functional form for the distribution of the sources composing the UGRB in redshift, energy and inventory of sources, thus possibly answering to questions addressing whether it is fully represented by BL Lac or if it allows for a SFG contribution and, in the context of WIMP searches, whether the weighting scheme with the highest figure-of-merit could allow for a DM contribution.

The optimal weights $\mathbf{w}$ can be found by solving a minimization problem. Under the hypothesis of Gaussianly distributed random variables, the probability for obtaining the photon counts y given the galaxy counts vector $\mathbf{x}$ is (Alonso et al., 2020; Urban et al., 2021):

$$-2 \log \mathcal{P}(y|\mathbf{x}) = \mathbf{z}^\top \mathbf{Cov}^{-1}[\mathbf{z}, \mathbf{z}] \mathbf{z} - \mathbf{x}^\top \mathbf{Cov}^{-1}[\mathbf{x}, \mathbf{x}] \mathbf{x} \quad (3.17)$$

where $\mathbf{z} = (y, x_1, \dots, x_M)$ and $\mathbf{Cov}(\mathbf{a}, \mathbf{b})$ is the covariance between two observables $\mathbf{a}$ and $\mathbf{b}$ (log in our convention is the natural logarithm). Explicitly (Alonso et al., 2020; Urban et al., 2021):

$$\mathbf{Cov}[\mathbf{z}, \mathbf{z}] = \begin{pmatrix} \text{Cov}[y, y] & \mathbf{Cov}^T[\mathbf{x}, y] \\ \mathbf{Cov}[\mathbf{x}, y] & \mathbf{Cov}[\mathbf{x}, \mathbf{x}] \end{pmatrix}. \quad (3.18)$$

It is useful to state the size $\mathbf{Cov}[\mathbf{z}, \mathbf{z}]$ assumes in the different steps and configurations: its size is $(M + 1) \times (M + 1)$, with $\text{Cov}[y, y]$ a scalar (dimension 1), $\mathbf{Cov}[\mathbf{x}, y]$ a vector with dimension $(M \times 1)$ and $\mathbf{Cov}[\mathbf{x}, \mathbf{x}]$ a matrix of dimension $M \times M$.

We can then maximise the likelihood with respect to the variable of interest y by solving $\partial_y \log \mathcal{P}(y|\mathbf{x}) = 0$. We thus obtain the minimum variance estimator for y (Alonso et al., 2020; Urban et al., 2021):

$$y = \mathbf{Cov}^T[\mathbf{x}, y] \mathbf{Cov}[\mathbf{x}, \mathbf{x}]^{-1} \mathbf{x} \quad (3.19)$$

that gives immediately the expression for the filter (Alonso et al., 2020; Urban et al., 2021):

$$\mathbf{w} = \mathbf{Cov}^{-1}[\mathbf{x}, \mathbf{x}] \mathbf{Cov}[\mathbf{x}, y] \quad (3.20)$$

By expressing eq. 3.20 in components, and restoring the notation in terms of number of galaxies and number of photons, we have:

$$w_r = \sum_s \mathbf{Cov}^{-1}[N_{gr}^p, N_{gs}^p] \mathbf{Cov}[N_{gs}^p, N_\gamma^p] \quad (3.21)$$

Assuming a Poissonian statistics for the number counts of both galaxies and photons, a general result holds stating that the covariance of two Poisson samples corresponds to the number of events in the intersections of the two samples (Alonso et al., 2020; Urban et al., 2021), meaning that we can posit:

$$\mathbf{Cov}[N_{gr}^p, N_{gs}^p] = \delta_{rs} N_{gr}^p \quad (3.22)$$

$$\mathbf{Cov}[N_{gr}^p, N_{\gamma}^p] = N_{\gamma r}^p \quad (3.23)$$

where in the last equation $N_{\gamma r}^p$ refers to the number of photons in the redshift bin r . From eqs. 3.5 and 3.14 we have:

$$\mathbf{Cov}[N_{gr}^p, N_{gs}^p] = \chi^2 \bar{n}_g(\chi) d\chi \quad (3.24)$$

$$\mathbf{Cov}[N_{gr}^p, N_{\gamma}^p] = W_{\gamma}(\chi) d\chi \quad (3.25)$$

in the previous expression we reinstated the explicit redshift dependence through the comoving distance χ , whereas $d\chi$ codifies the (small) comoving-distance bin corresponding to the redshift bin selected by the δ_{rs} .

By inserting the expression of the two eqs. 3.24 and 3.25 in eq. 3.21 and by converting the redshift dependence in comoving distance χ dependence for greater consistency with our notation, we can write the explicit expression for the weights/filter functions $w(\chi)$, as follows:

$$w(\chi) = \frac{W_{\gamma}(\chi)}{\chi^2 \bar{n}_g(\chi)} [\Theta(\chi - \chi_{\min}) - \Theta(\chi - \chi_{\max})] \quad (3.26)$$

where $\chi_{\min}$ and $\chi_{\max}$ refer to the boundaries of the redshift bins under consideration, when a tomographic approach is adopted. We have to impose the Heaviside Theta as a “hard cut” because we have to limit the UGRB redshift distribution to the interval covered by the redshift survey (any signal coming from closer or more distant UGRB sources cannot, by definition, correlate with the galaxies in the target survey, see fig. 3.3 to compare the redshift extension of the UGRB sources with 2MRS).

We can then insert the weights of eq. 3.15 and we find:

$$W_{wg}(\chi) = \frac{W_{\gamma}(\chi)}{\int_{\chi_{\min}}^{\chi_{\max}} d\chi W_{\gamma}(\chi)} [\Theta(\chi - \chi_{\min}) - \Theta(\chi - \chi_{\max})] \quad (3.27)$$

We linger a bit again on the qualitative interpretation of the Wiener filter: the filter can thus be understood as a device to align the galaxy kernel to a model of gamma-ray emission, corresponding to the assumption that *all of the halos hosting the galaxies in the redshift interval covered by the survey are also hosting the gamma sources*. The exact composition of $W_{\gamma}(\chi)$ is left free to the modeler and can, in general, be obtained by summing over the set of the selected sources i : for instance, if $i \in \{\text{BL Lac, mAGN, FSRQ, SFG, DM}\}$, meaning that the UGRB is assumed to be emitted by both astrophysical and DM sources, we have that:

$$W_{\gamma}(\chi) = \sum_{i \in \{\text{BL Lac, mAGN, FSRQ, SFG, DM}\}} W_{\gamma, i}(\chi). \quad (3.28)$$

To get a better sense of how the galaxy kernel gets modified under the weighting procedure, we can now look at fig. 3.6, showing the window function $W_g(\chi)$ of the 2MRS galaxy catalogue in the usual unweighted case and compared with three representative cases $W_{wg}(\chi)$, after the use of a filter model based exclusively on the DM emission,

$$W_{\gamma}(\chi) = \sum_{i \in \{\text{DM}\}} W_{\gamma, i}(\chi). \quad (3.29)$$

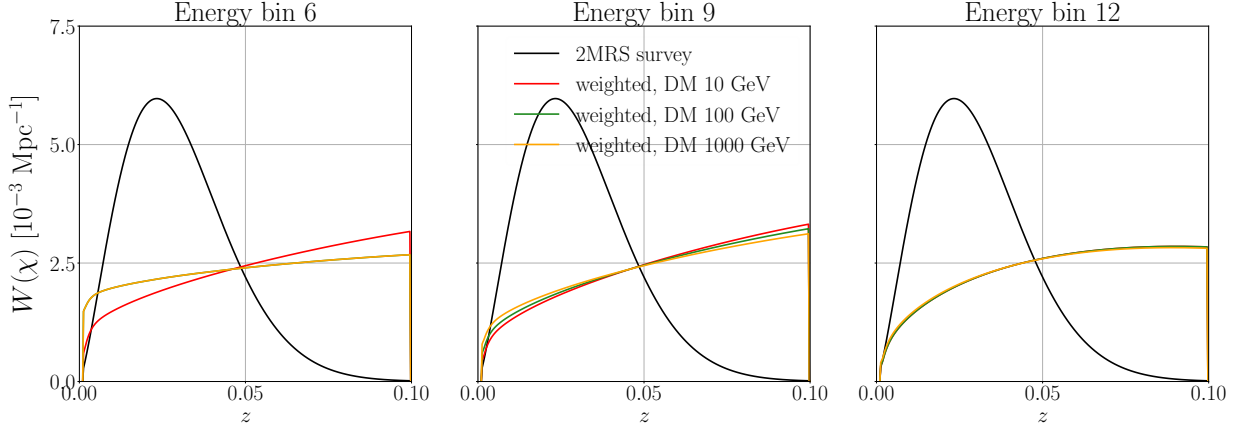


Figure 3.6: The effect of the weighting scheme on the galaxy survey window functions, compared with the unweighted galaxy kernel as a function of the redshift. The standard kernel is in black, showing therefore the 2MRS window function $W_g(\chi)$ in absence of weights (standard case). When we apply the weights, we obtain the $W_{wg}(\chi)$ shown in colours, according to the dark matter mass: $m_{\text{DM}} = 10$ GeV is in red, $m_{\text{DM}} = 100$ GeV is in green, while $m_{\text{DM}} = 1000$ GeV is in yellow. -We have always set the cross section rate value at the “thermal” level $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^{-3} \text{ s}^{-1}$. The three panels refer to the Fermi energy bins number 6, 9 and 12, from left to right.

In other words, for the plotted cases we have

$$W_{wg}(\chi) = \frac{W_{\text{DM}}(\chi)}{\int_{\chi_{\min}}^{\chi_{\max}} d\chi W_{\text{DM}}(\chi)} [\Theta(\chi - \chi_{\min}) - \Theta(\chi - \chi_{\max})] \quad (3.30)$$

with W_{DM} being calculated from eq. 3.10. From the figure we indeed see that the new galaxy window function “inherits” energy and mass dependencies from the original W_{DM} and receives a sharp redshift truncation at the extrema of the original 2MRS redshift survey. On the other hand, fig. 3.7 instead shows the *product* $W_g(\chi)W_\gamma(\chi)$ of the galaxies and gamma-ray kernels, the ultimate quantity determining the amplitude of the cross-correlation power spectrum $C_\ell^{\gamma g}$. In this case, we show the effect of a filter combining in both DM and astrophysical emission. Observe how the integral entering the denominator of eq. 3.27 is calculated only within the support of the considered galaxy redshift bin. The thus modified kernel can then be applied to the measurement of the cross correlation with the observed gamma-ray map, from which we can build some appropriate figure-of-merit on statistical basis (we define it to be a signal-to-noise ratio, as we shall discuss later). Again, we recall that, by varying across different assumptions about the inventory and functional forms of the window functions for the UGRB sources entering in the definition of the weights, we might construct a distribution of the values of the figures-of-merit and select the ones most likely reproducing the unobservable “actual” redshift distribution of the objects emitting the photons detected in the unresolved background (our strategy can thus also be interpreted as a “matched filter” approach).

3.3 Wiener filters and multi-tracers for dark matter searches

We are now ready to apply the described theoretical setup to our forecast: we will test the filtering technique in the cross-correlation configuration and try to understand whether the filter can improve the capability of cross-correlation techniques of placing bounds on the physical parameters

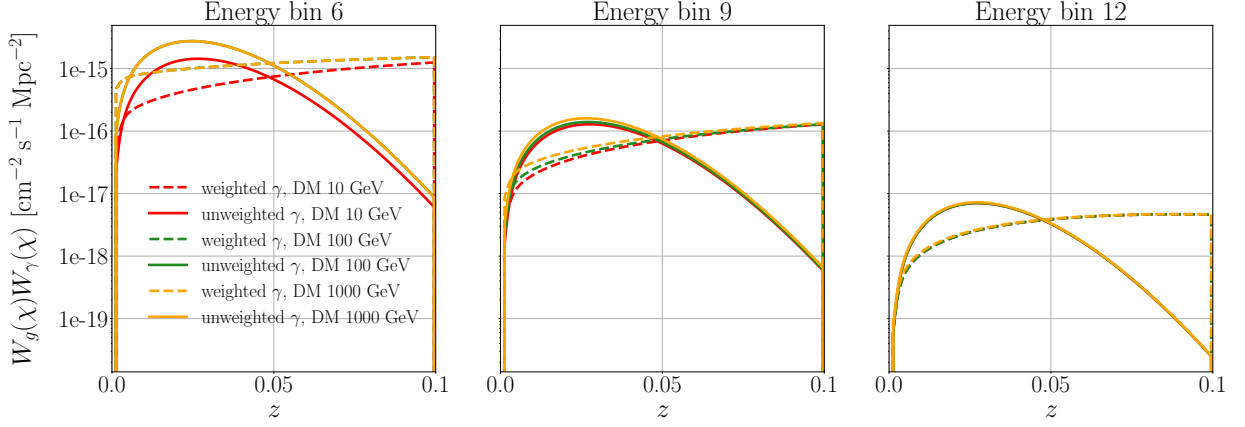


Figure 3.7: Plots displaying the product of the galaxy window function $W_g(\chi)$ with the gamma-rays window function $W_\gamma(\chi)$, the quantity ultimately regulating the amplitude of the unweighted cross-correlation power spectra. We consider both standard unweighted case (solid) and several weighting configurations (dashed). The gamma-ray window function contemplates the combined effect of the astrophysical sources and of the DM signal, in the $m_{\text{DM}} = 10$ GeV (red lines), $m_{\text{DM}} = 100$ GeV (green) and $m_{\text{DM}} = 1000$ GeV (yellow) cases. Again, the cross section is evaluated at the “thermal” amplitude $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^{-3} \text{ s}^{-1}$. The panels from left to right correspond to the Fermi energy bins number 6, 9 and 12

characterising WIMP dark matter particles, namely its mass m_{DM} and its self-annihilation cross section $\langle\sigma v\rangle$. Being a forecast exercise, and not an exploration of actual data, we might already argue in advance some of the results: since, as we said, the Wiener filtering approach corresponds to imposing to the models an Ansatz concerning the correspondence of the halos hosting the galaxies detected in the 2MRS survey with those hosting the astrophysical sources or, in presence of Dark Matter, being also contributed by annihilating WIMPs, we might expect an enhancement of the signal sensitivity in the mass, $\langle\sigma v\rangle$ and energy regimes where the UGRB is dominated by the DM component. If, instead, the UGRB is dominated by the astrophysical component, we might not expect to observe an improvement in the cross-correlation, since the filter might be “diluting” the morphological connection between the original galaxy redshift survey and the DM halos, replacing it with a less tight tracer of the WIMP field. This not granted improvement of the final outcome of the DM detection is due to the peculiar fact that the UGRB model is actually made of two contributions, one corresponding to the signal we are actually interested in and one, the astrophysical component, which is irreducible although detrimental, and both may enter the formulation of the $w(\chi)$. This nuanced result can be interpreted also from another point of view: if, when applying our model to actual data, we will achieve results similar to our forecasts, then we may conclude that our modelling of the galactic and UGRB ingredients used in this study is appropriate although a pessimistic expectation concerning the final results should be considered, due to the intervening unmodelled systematics and limitations of the actual data compared to theoretical quantities. We will be setting **bounds** on an exclusion plot, in the hope that they will be significantly below standard methodologies or current observations. This means that our methodologies might be capable of telling apart a DM contribution to the UGRB with respect to a competitor purely astrophysical explanation of the same signal at a given desired significance, corresponding to the one used as a threshold by the reference observational results we will use for the comparisons, but associated to a *lower* value of the annihilation cross-section rate. Our

methodology could therefore be interpreted as capable of detecting even weaker signatures of Dark Matter compared to current extant experimental evidence. On the other way round, since no positive detection of Dark Matter particles has been unanimously recovered so far, so that bounds are interpreted as excluded limits on the annihilation cross section rate, pointing to a WIMP candidate whose $\langle\sigma v\rangle$, at a given mass, and if existing, must be smaller than the value of the bound, a lower attained bound on $\langle\sigma v\rangle$ would mean that, in any case, our methodology might be able to ensure greater sensitivities to null results without improving the instrumentation, just focusing on the theoretical formalism and the statistical approach. Our approach can be of use also as a more general framework for the study of the UGRB, even limited to the astrophysical part: we may well construct the filter by exclusively considering the astrophysical populations and again check if the modelled redshift distributions can sensibly reconstruct the actual features of the sources of the UGRB implied by observational data and, in particular, whether they can occupy some of the halos also hosting the galaxies of a selected survey.

In general, to evaluate the success and quality of the Wiener filter strategy we have to define a suitable figure of merit: in our case, we will use a signal-to-noise ratio SNR, whose square is defined as:

$$\text{SNR}^2 = \sum_{\ell\ell'} \sum_{EE'} \sum_{zz'} \mathbf{x}_{\ell,E,z}^T \mathbf{\Gamma}_{\ell\ell'EE'zz'}^{-1} \mathbf{x}_{\ell',E',z'} \quad (3.31)$$

where ℓ, ℓ' denote multipoles, E, E' label the energy bins and z, z' the redshift bins. The vector of power spectra is noted with $\mathbf{x}_{\ell,E,z}$. Each power spectrum value is a function of the multipole ℓ , energy bin E and redshift bin z . $\mathbf{\Gamma}_{\ell\ell'EE'zz'}$ denotes the covariance matrix. Since the 2MRS is rather scant and the galaxies all belong to a narrow redshift range (see fig. 3.6, displaying the 2MRS window function and therefore the survey redshift distribution), we use a single redshift bin in our analysis, without introducing tomographic approaches. We can thus discard the redshift label; another important assumption we make is that the covariance matrix is diagonal in multipole space (see eq. 3.36).

The full expression of the covariance in the multi-tracer framework we will eventually adopt in this analysis is rather complicated, given the interplay we will discuss between “standard” configurations and filtered configurations: it can then be useful to expose in full the construction of the generic element for the covariance matrix whose expression can be analytically recovered if the involved fields are Gaussian random variables, as is the case for our density contrast, thanks to the Wick-Isserlis’ theorem (Isserlis, 1916, 1918; Wick, 1950).

By recalling that the angular power spectrum as a correlation among random variables can be defined by

$$\langle A_{\ell m} B_{\ell' m'} \rangle = \delta_{\ell\ell'}^K \delta_{mm'}^K C_{\ell}^{AB}, \quad (3.32)$$

we can derive the more general expression for the covariance that might be applied to our case by letting our fields assume at the same time:

- A capital Latin index ($A, B, C...$) denoting the specific kind of involved field (galaxies, weighted galaxies, gamma rays).
- A lowercase Latin index ($i, j, k...$) denoting an energy bin index.
- A lowercase Greek index ($\lambda, \mu, \nu...$) denoting a redshift bin index.

We can then formulate the covariance between two distinct angular power spectra by resorting to the general statistical result according to which, given two random variables X and Y , their

covariance reads $\text{Cov}(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$, with the angled brackets denoting the ensemble average (also denominated expectation) operator. We thus have:

$$\begin{aligned} \text{Cov}(C_\ell^{A_i^\lambda B_j^\mu} C_\ell^{D_k^\rho E_n^\sigma}) &= \langle C_\ell^{A_i^\lambda B_j^\mu} C_\ell^{D_k^\rho E_n^\sigma} \rangle - \langle C_\ell^{A_i^\lambda B_j^\mu} \rangle \langle C_\ell^{D_k^\rho E_n^\sigma} \rangle \\ &= \frac{\delta_{\ell\ell'}^K}{(2\ell+1)(2\ell'+1)} \left[\langle A_{\ell i}^\lambda B_{\ell j}^{\mu*} D_{\ell' k}^\rho E_{\ell' n}^{\sigma*} \rangle - \langle C_\ell^{A_i^\lambda B_j^\mu} \rangle \langle C_{\ell'}^{D_k^\rho E_n^\sigma} \rangle \right]. \end{aligned} \quad (3.33)$$

It is now possible to apply the Wick–Isserlis’ theorem to the first term. In the case of four fields, we can write

$$\langle x_1 x_2 x_3 x_4 \rangle = \langle x_1 x_2 \rangle \langle x_3 x_4 \rangle + \langle x_1 x_3 \rangle \langle x_2 x_4 \rangle + \langle x_1 x_4 \rangle \langle x_2 x_3 \rangle.$$

We thus identify $x_1 = A_{\ell i}^\lambda$, $x_2 = B_{\ell j}^{\mu*}$, $x_3 = D_{\ell' k}^\rho$, $x_4 = E_{\ell' n}^{\sigma*}$, thus getting

$$\begin{aligned} \langle A_{\ell i}^\lambda B_{\ell j}^{\mu*} D_{\ell' k}^\rho E_{\ell' n}^{\sigma*} \rangle &= \underbrace{\langle A_{\ell i}^\lambda B_{\ell j}^{\mu*} \rangle \langle D_{\ell' k}^\rho E_{\ell' n}^{\sigma*} \rangle}_{\text{(I)}} \\ &\quad + \underbrace{\langle A_{\ell i}^\lambda D_{\ell' k}^\rho \rangle \langle B_{\ell j}^{\mu*} E_{\ell' n}^{\sigma*} \rangle}_{\text{(II)}} \\ &\quad + \underbrace{\langle A_{\ell i}^\lambda E_{\ell' n}^{\sigma*} \rangle \langle B_{\ell j}^{\mu*} D_{\ell' k}^\rho \rangle}_{\text{(III)}}. \end{aligned} \quad (3.34)$$

The first term (I) corresponds to the product of angular power spectra $\langle C_\ell^{A_i^\lambda B_j^\mu} \rangle \langle C_\ell^{D_k^\rho E_n^\sigma} \rangle$, that cancels out with the second term of the difference in eq. 3.33. Concerning the terms (II) e (III), we have

$$\begin{aligned} \text{(II)} : \quad &\langle A_{\ell i}^\lambda D_{\ell' k}^\rho \rangle \langle B_{\ell j}^{\mu*} E_{\ell' n}^{\sigma*} \rangle = (\delta_{\ell\ell'}^K) (\delta_{\ell\ell'}^K) C_\ell^{A_i^\lambda D_k^\rho} C_\ell^{B_j^\mu E_n^\sigma}, \\ \text{(III)} : \quad &\langle A_{\ell i}^\lambda E_{\ell' n}^{\sigma*} \rangle \langle B_{\ell j}^{\mu*} D_{\ell' k}^\rho \rangle = (\delta_{\ell\ell'}^K) (\delta_{\ell\ell'}^K) C_\ell^{A_i^\lambda E_n^\sigma} C_\ell^{B_j^\mu D_k^\rho}. \end{aligned} \quad (3.35)$$

By replacing these expressions in eq. 3.33 and by including, in full generality, the shot-noise terms (Cohn, 2006; Ginzburg et al., 2017; Tessore & Hall, 2025), we obtain:

$$\begin{aligned} \text{Cov}(C_\ell^{A_i^\lambda B_j^\mu}, C_{\ell'}^{D_k^\rho E_n^\sigma}) &= \Gamma_{\ell, \ell'}^{A_i^\lambda B_j^\mu C_k^\rho D_n^\sigma} = \frac{\delta_{\ell\ell'}^K}{(2\ell+1) \Delta\ell f_{\text{sky}}} \left[(C_\ell^{A_i^\lambda D_k^\rho} + N_\ell^{A_i^\lambda D_k^\rho}) (C_{\ell'}^{B_j^\mu E_n^\sigma} + N_{\ell'}^{B_j^\mu E_n^\sigma}) + \right. \\ &\quad \left. + (C_\ell^{A_i^\lambda E_n^\sigma} + N_\ell^{A_i^\lambda E_n^\sigma}) (C_{\ell'}^{B_j^\mu D_k^\rho} + N_{\ell'}^{B_j^\mu D_k^\rho}) \right]. \end{aligned} \quad (3.36)$$

We remark that the UGRB fields, being projected quantities on the celestial sphere, cannot be subdivided in redshift bins, neither, we recall, we will subdivide our galaxy redshift distribution in multiple redshift bins for what concerns this analysis. Yet this is the covariance formulation consistently capable of deriving all the quantities we need in our analysis.

Concerning the other coefficients, $\Delta\ell$ is the width of the ℓ -bin and f_{sky} the sky fraction. The sky fraction for the galaxies f_{sky}^g is taken for the 2MRS galaxy catalogue equal to $f_{\text{sky}}^g = 0.877$ (Ando et al., 2018). For Fermi-LAT, the coverage of the sky represented by the sky fraction is energy-dependent, because different sky maskings are applied at different energies, either because of the energy behaviour of the Galactic foreground emission or because of different contamination levels from resolved astrophysical sources. Again, our reference values are based on the Fermi specifications of (Ackermann et al., 2018), reported in table 3.1. Next to the same energy bins,

we define the same energy-dependent $f_{\text{sky},i}^\gamma$ (i labels the energy bin). We remark that the fully general description of the cross-correlation (our notation is devised in order to consistently cover also the multi-tracer case discussed later on) makes the selection of the sky fraction slightly non-trivial. For each entry in the covariance matrix we set $f_{\text{sky}} = \min[f_{\text{sky}}^g, f_{\text{sky},i}^\gamma]$ for galaxy-gamma cross-correlations and $f_{\text{sky}} = \min[f_{\text{sky},i}^\gamma, f_{\text{sky},j}^\gamma]$ for gamma-gamma cross-correlations. The correct determination of the expressions of the noise terms is worth attention, both because of physical reasons and because of the very general and, for this same reason, potentially misleading notation we are bound to use in order to cover the multi-tracer case and the several multiple dependencies of our variables. We will then describe them in detail below. First, the noise terms for gamma rays depend on energy; the standard treatment is to consider them as diagonal in energy, since the number counts of photons in different energy bins are independent:

$$\mathcal{N}_{ij}^{\gamma\gamma} = \delta_{ij}^K \mathcal{N}^\gamma. \quad (3.37)$$

Again, our reference values for $\mathcal{N}^\gamma$ are the measured values for 8 years of Fermi Pass 8 observations as reported in (Ackermann et al., 2018) and reported in table 3.1. We are now in 2026: in order to adapt our forecasts to 20 years of operations, since the noise scales approximately with the inverse of the exposure (Pinetti et al., 2020; Arcari et al., 2022) we have rescaled the noise values of table 3.1 by a factor of 2.5.

The noise for galaxies is obtained as:

$$\mathcal{N}^{gg} = \frac{4\pi f_{\text{sky}}^g}{N_{\text{survey}}} \quad (3.38)$$

where N_{survey} is the number of galaxies observed in the given survey, while, as usual, the noise associated to the cross-correlation between the unweighted galaxy field and the gamma fields is null:

$$\mathcal{N}_i^{g\gamma} = 0, \quad (3.39)$$

a standard result based on the sensible assumption of absence of correlation between the noise terms in the two fields (Alimi et al., 1988; Blanchard, 2022; Tessore & Hall, 2025; Ginzburg et al., 2017). Now, it is important to notice that, when the filter $w(\chi)$ is applied, we observe a more complex situation, because the weights induce energy dependence in the galaxy window functions (fig. 3.6), so that, in order to correctly express the shot-noise for the cross-correlation between the weighted galaxy field and the UGRB, we have to take a step back and consider the most general expression for the angular shot-noise power spectrum for the galaxies first. The quantity can be derived from the projection of the three-dimensional galaxy shot-noise power spectrum, which is the inverse of the survey number density at a given redshift and characterised by scale independence, just being a function of the redshift: $P_{\text{SN},g}(k, z) = \frac{1}{\bar{n}_g(z)}$. The power spectrum is converted into the angular power spectrum by means of the general galaxy window function formulation of eq. 3.15, including the weights. We thus find:

$$\begin{aligned} \mathcal{N}_{ij}^{gg} &= \int d\chi \frac{1}{\chi^2} W_{wg,i}(\chi) W_{wg,j}(\chi) \frac{1}{\bar{n}_g(\chi)} = \\ &= \int d\chi \frac{1}{\chi^2} \frac{\chi^2 w_i(\chi) \bar{n}_g(\chi)}{[\int d\chi \chi^2 w_i(\chi) \bar{n}_g(\chi)]} \frac{\chi^2 w_j(\chi) \bar{n}_g(\chi)}{[\int d\chi \chi^2 w_j(\chi) \bar{n}_g(\chi)]} \frac{1}{\bar{n}_g(\chi)} \end{aligned} \quad (3.40)$$

leading to the following more general expression of the galactic shot noise:

$$\mathcal{N}_{ij}^{gg} = \frac{\int d\chi \chi^2 w_i(\chi) w_j(\chi) \bar{n}_g(\chi)}{[\int d\chi \chi^2 w_i(\chi) \bar{n}_g(\chi)] [\int d\chi \chi^2 w_j(\chi) \bar{n}_g(\chi)]}. \quad (3.41)$$

which, if we consider eq. 3.14, allows to recover the standard case of eq. 3.38 if we set $w(\chi) = 1$, namely, in absence of filter being applied.

Furthermore, we have also to consider an important observational systematic, the limited angular resolution of γ -rays observations, due to the energy-dependent Fermi point spread function (PSF). We discussed in detail the convolution of beam systematics with the C_ℓ in sec. 1.4. We recall that this prescription requires for correlation functions in the angular space to be corrected: correlation functions of the type $C_\ell^{g\gamma_i}$ are multiplied by the related Fermi PSF $B_{\ell,i}$ calculated in the specific energy bin (i.e. $C_{\ell,i}^{g\gamma} \rightarrow C_{\ell,i}^{g\gamma} B_{\ell,i}$), whereas correlation functions of the type $C_{\ell,ij}^{\gamma\gamma}$ are modified according to $C_{\ell,ij}^{\gamma\gamma} \rightarrow C_{\ell,ij}^{\gamma\gamma} B_{\ell,i} B_{\ell,j}$. The shot-noise terms, instead, are related to Poisson fluctuations in photon number counts. As such, they are not modified by the angular resolution and do not require the beam correction. Concerning galaxies, we substantially assume a “perfect” detector: the angular resolution of galaxy catalogues is indeed much better than the one for gamma-rays, becoming relevant for multipoles which in any case are already excluded because of the effect of the Fermi PSF. It will thus be the Fermi PSF the determinant of our analysis, restrained to maximum multipoles $\ell \sim 1000$, associated to angular scales of the order of $0.1^\circ - 0.2^\circ$, much larger than the typical angular resolution of galaxy catalogues of the order of the arcsec.

The Kroenecker delta entering in the covariance enforces a simplification in the definition of the SNR, that reads:

$$\text{SNR}^2 = \sum_{\ell} \sum_{EE'} \mathbf{x}_{\ell,E}^\top \mathbf{\Gamma}_{\ell EE'}^{-1} \mathbf{x}_{\ell,E'} = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \text{SNR}_\ell^2 \quad (3.42)$$

where we define

$$\text{SNR}_\ell^2 = \sum_{EE'} \mathbf{x}_{\ell,E}^\top \mathbf{\Gamma}_{\ell EE'}^{-1} \mathbf{x}_{\ell,E'} \quad (3.43)$$

the SNR brought by each multipole, cumulated in energy.

The choice of the minimum and maximum multipoles in eq. 3.42 is associated to physical reasons: in order to perform realistic forecasts, $\ell_{\min}$ will be set to 50. Typical real data analyses indeed report that contamination coming from foregrounds and due to the large-angle emission from the Milky Way hinders the measurement of low multipoles, below the one we selected (even after these foregrounds have been subtracted) (Ackermann et al., 2018), while $\ell_{\max}$ will be set to 1000, since at smaller angular scales (larger multipoles) substantially no information is provided given the intervention of the Fermi PSF.

Outline of the analysis setups

We will now discuss the features of the three types of analysis we will develop as distinct test configurations for our filtering strategy: the first case (case I) is the usual standard cross-correlation signal, i.e. $\mathbf{x}_\ell = \{C_\ell^{gg}\}$; in the second case (case II), the data vector also includes the galaxy auto-correlation power spectrum, i.e. $\mathbf{x}_\ell = \{C_\ell^{g\gamma}, C_\ell^{gg}\}$, while the third case (case III) includes in addition the gamma-ray autocorrelation $\mathbf{x}_\ell = \{C_\ell^{g\gamma}, C_\ell^{gg}, C_\ell^{\gamma\gamma}\}$. Importantly, in all configurations each C_ℓ has to be interpreted **as a shorthand for a data vector**: this because we will be simultaneously analysing all the energy bins together and each bin will be a different entry of the data vector.

We can recognise in the last two cases our “multi-tracer” analyses, meaning that we shall combine the filtering formalism with a forecast on the application to the UGRB of the multi-tracer strategy, a widely explored technique proposed in the context of cosmological analyses of the large

scale structure (see also sec. 4.1.2) (Percival et al., 2004; McDonald & Seljak, 2009; Seljak, 2009). This formalism leverages on the inherent correlation of all cosmological observable, which are all assumed to be tracing a unique and shared dark matter field: the multi-tracers act thus in a way capable of using different level of biasing among distinct LSS tracers in order to mitigate effects sourced by the cosmic variance; we refer to (Abramo & Leonard, 2013; Ferramacho et al., 2014; Fonseca et al., 2015) as very interesting studies that investigate in depth the main variables determining the success or the limitations of the multi-tracer approach. Mapping the general idea to the present analysis, the multi-tracer effectively comes into play since the various contributions to the UGRB are related to different galaxy populations — each one with a different level of biasing associated to their distinctive clustering status in the LSS.

Given the complex dependencies of the variables involved, it can be useful to mention explicitly the size and shapes of the vector and matrices we will be constructing. The size of the data vector $\mathbf{x}_\ell = \{C_\ell^{\gamma g}\}$ is the number of the gamma-ray energy bins N (12 in our analysis); the size of $\mathbf{x}_\ell = \{C_\ell^{\gamma g}, C_\ell^{gg}\}$ is $N + 1$, due to the addition of the galaxy-galaxy autocorrelation spectra for a single redshift bin; eventually, the size of $\mathbf{x}_\ell = \{C_\ell^{\gamma g}, C_\ell^{gg}, C_\ell^{\gamma\gamma}\}$ is $N + 1 + N(N + 1)/2$, due to the addition of gamma-ray auto-correlation, including their “internal” cross-correlations calculated among different energy bins.

It is also useful to make explicit the expression of the full cross-correlation and gamma-ray auto-correlation signal power spectra when we are combining the contributions from all the selected species constituting the UGRB: for unweighted galaxies, the total cross-correlation signal in a given energy bin s , reads

$$C_{s,\ell}^{\text{cross,tot}} = \sum_{\alpha} C_{s,\ell}^{\text{gal } \alpha} \quad (3.44)$$

with the summation index α running on the collection of UGRB sources we select to enter our model. For the weighted galaxies case, the galaxy signal this time shows dependence on the energy bin r and therefore the cross-correlation with the UGRB signal reads:

$$C_{\ell,rs}^{\text{cross,tot}} = \sum_{\alpha} C_{\ell,rs}^{\text{rw } \alpha} . \quad (3.45)$$

On the other hand, the angular power spectrum $C_\ell^{\gamma\gamma}$ contains the correlations among all the gamma-ray sources under consideration and reads, including the dependence on the energy bins r and s :

$$C_\ell^{\gamma r \gamma s} = C_\ell^{DM_r \otimes DM_s} + \sum_i C_\ell^{DM_r \otimes \text{astro}_{si}} + \sum_i C_\ell^{DM_s \otimes \text{astro}_{ri}} + \sum_i \sum_j C_\ell^{\text{astro}_{ri} \otimes \text{astro}_{sj}}$$

with i, j running on blazars, mAGN, FSRQ and SFG.

A relevant piece of our SNR calculation is the covariance matrix. For our case I (cross-correlation γg) it can be represented as:

$$\mathcal{A} = \begin{pmatrix} (\gamma_{1g}, \gamma_{1g}) & (\gamma_{1g}, \gamma_{2g}) & \cdots & (\gamma_{1g}, \gamma_{Ng}) \\ (\gamma_{2g}, \gamma_{1g}) & (\gamma_{2g}, \gamma_{2g}) & \cdots & (\gamma_{2g}, \gamma_{Ng}) \\ \vdots & \vdots & \ddots & \vdots \\ (\gamma_{Ng}, \gamma_{1g}) & (\gamma_{Ng}, \gamma_{2g}) & \cdots & (\gamma_{Ng}, \gamma_{Ng}) \end{pmatrix} \quad (3.46)$$

where each entry in the matrix refers to an element of the covariance matrix Γ of eq. 3.36, while the indices i on γ_i label the gamma-ray energy bins. Matrix $\mathcal{A}$ is symmetric and has dimension $\dim(\mathcal{A}) = N \times N$.

Strategy		Criterion
No filter	<i>Unweighted</i>	
With filter	<i>Astro+DM</i>	$W_\gamma(\chi) = W_\gamma^{\text{astro}}(\chi) + W_\gamma^{\text{DM}}(\chi)$
	<i>DM-only</i>	$W_\gamma(\chi) = W_\gamma^{\text{DM}}(\chi) \oplus E \leq m_{DM}$

Table 3.2: Table collecting the analysis strategies corresponding to different weighting schemes. The adopted criteria in each case are also shown. Each strategy is also related to the following combinations of observables: gamma-rays $\otimes$ galaxies cross-correlation (γg), multi-tracer combining gamma-rays $\otimes$ galaxies cross-correlation with galaxies auto-correlation ($\gamma g + gg$) and multi-tracer combining gamma-rays $\otimes$ galaxies cross-correlation, galaxies auto-correlation and gamma rays auto-correlation ($\gamma g + gg + \gamma\gamma$).

For the multi-tracer analysis of case II ($\gamma g + gg$) the covariance matrix will contain $\mathcal{A}$ as a sub-matrix, enlarged by the addition of a column and a row according to:

$$\mathcal{B} = \left(\begin{array}{c|c} \mathcal{A} & \begin{matrix} (\gamma_{1g}, gg) \\ (\gamma_{2g}, gg) \\ \vdots \\ (\gamma_{Ng}, gg) \end{matrix} \\ \hline \begin{matrix} (gg, \gamma_{1g}) & (gg, \gamma_{2g}) & \cdots & (gg, \gamma_{Ng}) \end{matrix} & (gg) \end{array} \right) \quad (3.47)$$

determining a size of:

$$\dim(\mathcal{B}) = \left(\begin{array}{c|c} N \times N & N \times 1 \\ \hline 1 \times N & 1 \end{array} \right) \quad (3.48)$$

We eventually move to the construction of the full multi-tracer analysis of case III ($\gamma g + gg + \gamma\gamma$): the covariance matrix contains matrix $\mathcal{B}$ and is further enlarged. We adopt here a more compact notation for the large additional matrix blocks; i, a, b run from 1 to N and find:

$$\mathcal{C} = \left(\begin{array}{c|c|c} \mathcal{B} & \begin{matrix} (\gamma_{ig}, \gamma_a \gamma_b)_{[a < b]} \\ (gg, \gamma_a \gamma_b)_{[a < b]} \end{matrix} & \\ \hline \begin{matrix} (\gamma_a \gamma_b)_{[a < b]}, \gamma_{ig} \end{matrix} & \begin{matrix} (\gamma_a \gamma_b)_{[a < b]}, gg \end{matrix} & \begin{matrix} (\gamma_i \gamma_j, \gamma_a \gamma_b)_{[i < j, a < b]} \end{matrix} \end{array} \right) \quad (3.49)$$

where we recall that the labels a, b and i, j refer to the energy bins. We summarise the size of the block matrix partitioned by the described blocks below:

$$\dim(\mathcal{C}) = \left(\begin{array}{c|c|c} N \times N & N \times 1 & N \times N(N+1)/2 \\ \hline 1 \times N & 1 & 1 \times N(N+1)/2 \\ \hline N(N+1)/2 \times N & N(N+1)/2 \times 1 & N(N+1)/2 \times N(N+1)/2 \end{array} \right) \quad (3.50)$$

Bound calculation criteria

We anticipated that the main goal of this work is to investigate potential improvements in the upper bounds we can set to annihilating WIMP cross-section rates as a function of the candidate particle mass. The calculation of these bounds can be derived from the SNR by using a $\Delta\chi^2$ statistics:

$$\Delta\chi^2(m_{\text{DM}}, \langle\sigma v\rangle) = \text{SNR}_{\text{DM+astro}}^2(m_{\text{DM}}, \langle\sigma v\rangle) - \text{SNR}_{\text{astro}}^2 \quad (3.51)$$

where $\text{SNR}_{\text{astro}}$ is the SNR we obtain assuming that all of the gamma-ray emission is produced by astrophysical sources, without dark matter contributions, while $\text{SNR}_{\text{DM+astro}}(m_{\text{DM}}, \langle\sigma v\rangle)$ represents the SNR value obtained by adding a dark matter component to the UGRB. The forecasts on the bounds on $\langle\sigma v\rangle$ obtainable in the different configurations are calculated at each given m_{DM} value by setting the criterion $\Delta\chi^2 = 4$. With this position we are substantially looking for the annihilation cross-section rate, as a function of the candidate relic mass, associated to an annihilating Dark Matter signal that would reveal as an excess in the UGRB detectable at about 2σ significance when a combined astrophysical and dark matter explanation of the signal is contrasted with the explanation of the same signal in terms of pure astrophysical contributions. In particular, the hope is that of being able to associate such excess significance to annihilation cross-section rates comparable or lower than other extant methodologies, thus forecasting an improvement in the sensitivity of the formalism.

Exploring different weighting strategies We will therefore perform a case-by-case comparison, in the three multi-tracer configurations, by calculating the SNR, and consequently the $\Delta\chi^2$, with and without the filter applied. In the case *without the filtering*, the implementation of eq. 3.51 requires to construct the signals by including the annihilating DM terms in addition to the astrophysical ones in the modelling of the gamma sources when we define $\text{SNR}_{\text{DM+astro}}$, whereas the signals will only include astrophysical contributions in the $\text{SNR}_{\text{astro}}$ term (equivalently, the DM-dependent terms are included but set all equal to zero); in particular, the gamma autocorrelation power spectrum will be limited to the term $C_\ell^{\gamma_r\gamma_s} = \sum_i \sum_j C_\ell^{\text{astro}_ri \otimes \text{astro}_sj}$

In the weighted cases, the term $\text{SNR}_{\text{astro}}$ will always be constructed in such a way that the UGRB signals are exclusively produced by astrophysical sources, without astrophysical contributions: the W_γ entering the weights *will only include the astrophysical sources (or, equivalently, the DM window will all be set to zero)*. Given the meaning of the weights, we recall that this second weighting scheme corresponds to two combined baseline hypotheses on the UGRB nature: the first is its being exclusively contributed by astrophysical sources (determining the inventory of the gamma-dependent quantities), the second, that those astrophysical sources reside in the same halos occupied by the galaxies we are using for the cross-correlation exercise (because of the definition of the weights).

The term $\text{SNR}_{\text{astro+DM}}$ in the weighted case leaves us instead the freedom to devise multiple filtering/weighting strategies, that also allow to better highlight how the explicit analytic expression of the filter can be subject to different hypotheses and choices. We shall focus on the ones listed below, also summarised in table 3.2, while the results are presented and discussed in the next section 3.4:

1. *astro+DM* case. In this case, we assume that the UGRB sources residing in the halos also hosting the correlated galaxy field contain the full gamma-ray emission (corresponding to the example of eq. 3.28), i.e., the gamma-ray window function of eq. 3.26 becomes $W_\gamma(\chi) = W_\gamma^{\text{astro}}(\chi) + W_\gamma^{\text{DM}}(\chi)$. In this case, the astrophysical sources are defined in the full energy range of Fermi as outlined in table 3.1 and hence the weights are always non-zero: in our analysis we will therefore use all the available energy bins.
2. *DM-only*. In this alternative case, we filter according only to the DM emission, and we will set $W_\gamma(\chi) = W_\gamma^{\text{DM}}(\chi)$. The choice can be interpreted as a strategy to emphasise and filter specifically for the annihilating DM sources *residing in the 2MRS galactic halos*. We

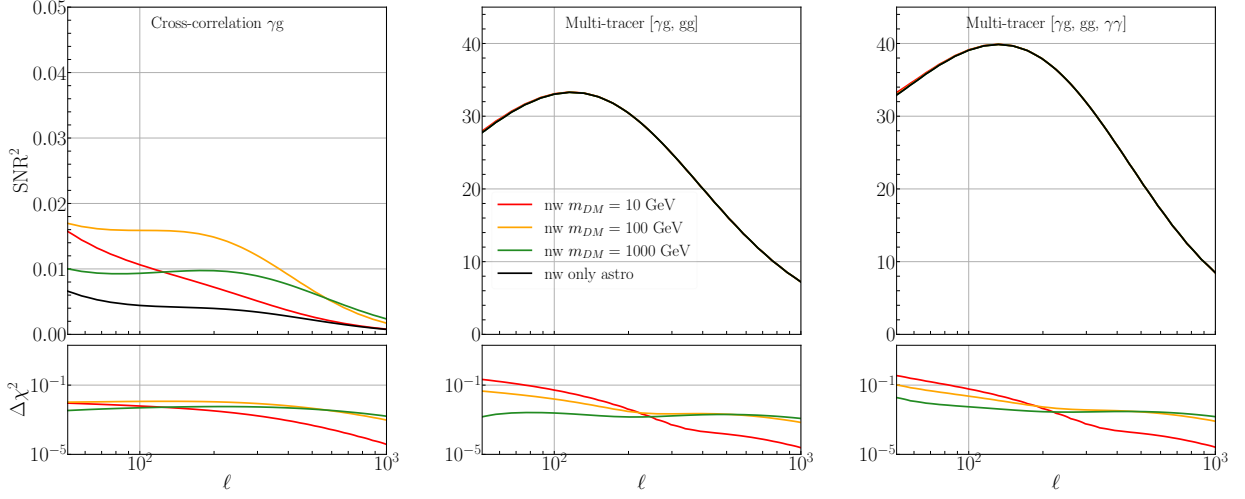


Figure 3.8: Plots reporting the contributions from each multipole to the SNR^2 (the upper panels show the result of eq. 3.43) and to the $\Delta\chi^2$ of eq. (3.51) (lower panels). We consider here the unweighted case. The pair of pictures on the left (upper and bottom row) correspond to the gamma-rays $\otimes$ galaxies cross-correlation (γg) case. The pair in the centre provides the results in the multi-tracer case when gamma-rays $\otimes$ galaxies cross-correlation are combined with the galaxies auto-correlation power spectra (the case we call $\gamma g + gg$), while the rightmost part of the plot shows the results for the multi-tracer case combining gamma-rays cross-correlations with galaxies, the galaxy auto-correlation power spectra and the auto-correlation of the UGRB (that is, the case $\gamma g + gg + \gamma\gamma$). In the upper part of each pair of plots, the gamma-ray emission exclusively from astrophysical sources is displayed in black. On the other hand, we show in colour the gamma-ray emission from the combined effect of astrophysical sources and dark matter. Three dark matter mass values are considered: 10 GeV (red), 100 GeV (yellow) and 1000 GeV (green). We set the value of the annihilation cross-section $\langle\sigma v\rangle$ for the three cases to 0.05, 0.5 and 5 times the “thermal” value $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^{-3} \text{ s}^{-1}$, respectively.

remark that the competing astrophysical foreground is not assumed to be absent, in that we in any case keep the astrophysical contribution to the models of the gamma-ray emission constructing the angular power spectra and, of course, the astrophysical contributions are used to construct the $\text{SNR}_{\text{astro}}$ term in our calculation of the attainable bounds. We make an important remark concerning the *DM-only* setup: since the result of eq. 3.26 that is applied to obtain the galaxy weighted window function $W_{wg}(\chi)$ refers only to the dark matter emission, i.e. to $W_{\gamma}^{\text{DM}}(\chi)$, which in turn is defined only for $E \leq m_{DM}$, accordingly limiting the used energy bins (see also figs. 3.1 and 3.2), we will perform a slight modification to the $\text{SNR}_{\text{astro}}^2$ term (which, we recall, has the weights being defined exclusively as a function of the astrophysical contributors to the UGRB). For consistency and in order not to improperly penalize the $\Delta\chi^2$, instead of using all of the energy bin (although the term is fully defined in each of them), we will only limit the calculation of $\text{SNR}_{\text{astro}}$ to energies smaller than the DM mass value, effectively defining a “capped” $\text{SNR}_{\text{astro}}^2$.

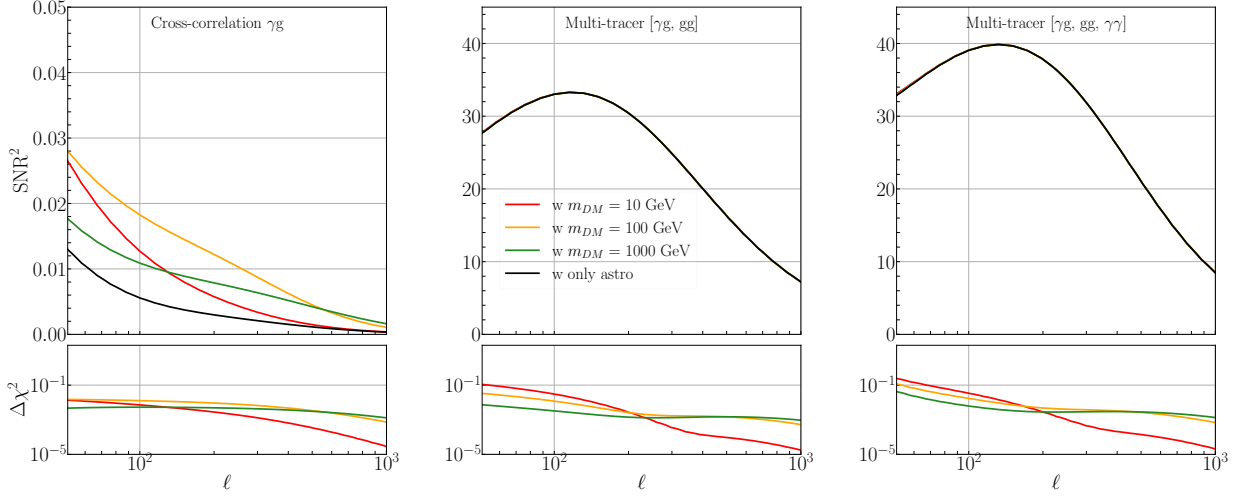


Figure 3.9: The same as in fig. 3.8, for the *DM+astro weighted* case.

3.4 Results and discussion

To begin with, we show in figs. 3.8, 3.9 and 3.10 the contributions from each multipole to the SNR^2 according to the definition of eq. (3.43) (upper panels) and to the $\Delta\chi^2$ defined in eq. 3.51 (in other words, we are plotting SNR_ℓ^2). In each of the mentioned figures, the left panels display the quantities calculated in the gamma-rays $\otimes$ galaxies cross-correlation case (γg), the central one refers to the multi-tracer technique in the case combining gamma-rays $\otimes$ galaxies cross-correlation with galaxies auto-correlation ($\gamma g + gg$), while on the right we are always reporting the outcome of the calculations in the full multi-tracer technique, further including the gamma rays auto-correlation information to the data vector ($\gamma g + gg + \gamma\gamma$). Considering the upper panel, we show in black the gamma-ray emission from astrophysical sources only, that should be compared with the red, yellow and green lines, which are instead referred to the cases where we add a dark matter gamma-ray emission to the emission associated to astrophysical sources. We show the results for three different representative dark matter mass values and annihilation cross sections: 10 GeV and $\langle\sigma v\rangle = 0.05 \times \langle\sigma v\rangle_{\text{th}}$ (red), 100 GeV and $\langle\sigma v\rangle = 0.5 \times \langle\sigma v\rangle_{\text{th}}$ (yellow), 1000 GeV and $\langle\sigma v\rangle = 5 \times \langle\sigma v\rangle_{\text{th}}$ (green), with $\langle\sigma v\rangle_{\text{th}} = 3 \times 10^{-26} \text{ cm}^{-3} \text{ s}^{-1}$ the usual “thermal” value for the WIMP annihilation cross-section rate. These choices are representative of the values for $\langle\sigma v\rangle$ close to the values of $\langle\sigma v\rangle$ corresponding to the predicted sensitivity bounds discussed in fig. 3.11 and can thus be interpreted as showing the values of the SNR_ℓ^2 quantity that, once cumulated in ℓ , construct the $\text{SNR}_{\text{astro+DM}}^2$ and $\text{SNR}_{\text{astro}}^2$ values we would find for two fitting models resolved enough to approximately ensure the desired 2σ detection. Please also remind that in fig. 3.10 we have a different $\text{SNR}_{\text{astro}}^2$ for each DM mass: indeed, the *DM-only* case defines a filter based exclusively on the window function for the DM gamma-ray emission, a quantity defined only for $E < m_{\text{DM}}$ (“capped” $\text{SNR}_{\text{astro}}^2$), therefore limited to a different number of energy bins according to the chosen mass value. We immediately notice how the discrepancy between the various “capped” $\text{SNR}_{\text{astro}}^2$ is extremely small: we thus simplify the picture and only show one case in each upper panel, instead of the required three cases. On the contrary, the lower panels, where the $\Delta\chi^2$ is shown, the required precision obliges us to calculate the figure-of-merit using the actual value of the various “capped” $\text{SNR}_{\text{astro}}^2$ to determine the $\Delta\chi^2$.

We discuss our results by first concentrating on the unweighted strategies displayed in fig. 3.8. We first consider the signal-to-noise ratios (upper row), a measurement of the *absolute*

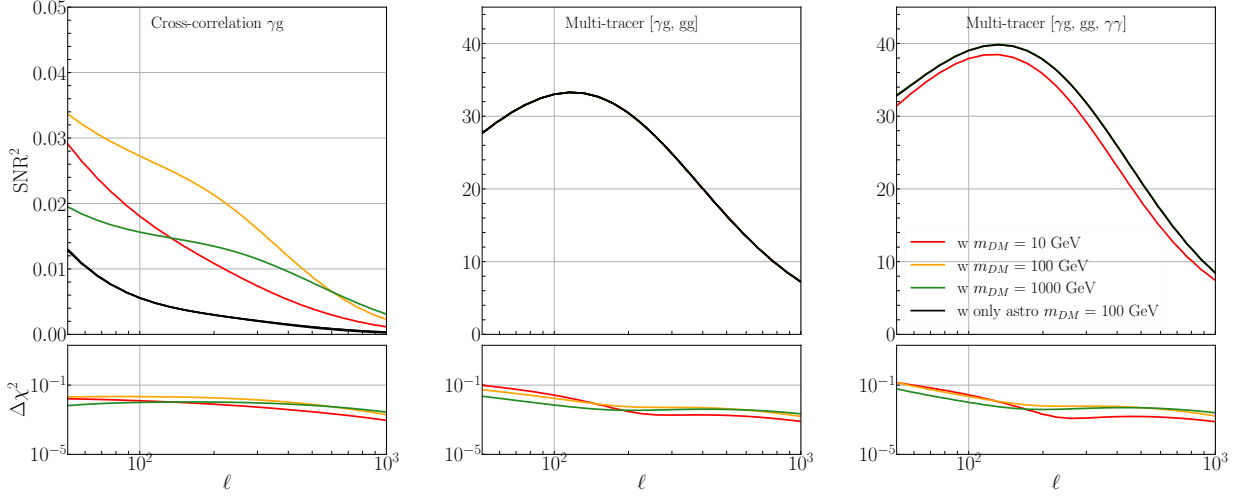


Figure 3.10: A picture analogous to fig. 3.8, this time referring to the *DM-only weighted* case.

prominence of each signal against the competing noise sources. The SNR^2 in the case of the (γg) cross-correlation is noticeably dominated from low multipoles, associated to a decreasing trend with the multipole. We can observe some small features when the gamma-ray DM emission is included, especially at higher masses. The qualitative behaviour of the SNR^2 metric changes when we shift to the two multi-tracer cases: we can notice how the SNR^2 is strongly dominated by a peak located around the multipoles 100-200. Such feature is driven by the properties of the galaxy-galaxy autocorrelation observable.

In other words, the absolute value of the SNR^2 grows importantly under the effect of the inclusion of the highly significant auto-correlation signal and, in the most general case, of the gamma-ray autocorrelation spectra in the data vectors, which “boost” the low SNR of the pure cross-correlation signal. The relevant quantity that allows us to infer bounds for the DM parameters is however the $\Delta\chi^2$, a “relative” metrics, formulated as the difference between two distinct SNR values. Their dynamics can be inferred in the bottom rows of fig. 3.8, where we find that in general the multi-tracer techniques also increases the contribution to the $\Delta\chi^2$, especially by contributions located at the lowest multipoles. These are the largest angular scales, notoriously more affected by cosmic variance, in turn controlled with greater effectiveness precisely by the multi-tracer approach, whose effect is substantially that of reducing the variance. Conversely, the largest multipoles are more affected by the increasing impact of the Fermi PSF, which suppresses the signal below the noise levels, increasing the variance. We notice how the SNR^2 is so strongly reduced that the multi-tracer technique does not provide any relevant improvement to the overall statistical significance of the contrast between the signals.

We can then discuss the analogous metrics after the application of the filters: figs. 3.9 and 3.10 show that they can indeed provide relevant effects, although with some degree of dependence on the specific type of analysis. In the case of the cross-correlation (γg) alone, the *DM-only* filter (fig. 3.10) might enhance the SNR^2 across the full range of multipole we selected (we remark that, although the individual contribution of each multipole to the $\Delta\chi^2$ quantity is small, we deal with cumulative quantities as stated in eq. 3.42. The final amplitude of the figures-of-merit of interest derives from the cumulative contributions along the whole multipole range, from 50 to 1000). Interestingly, also the more conservative weighting strategy represented by the *astro+DM* (fig. 3.9) weighting scheme improves the SNR compared to the unweighted cases of fig. 3.8.

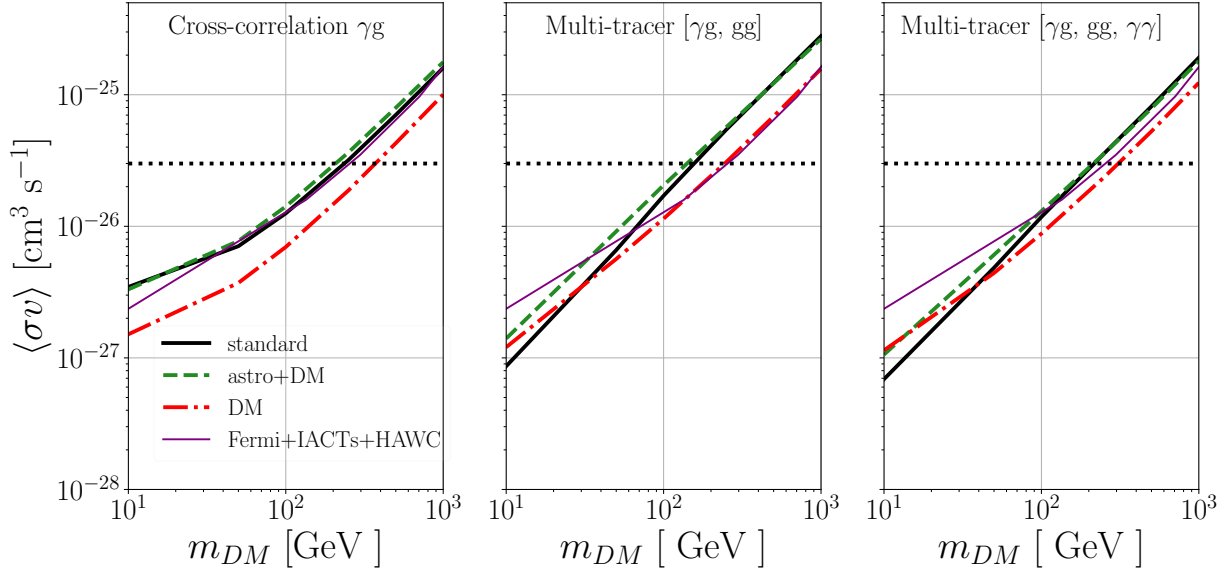


Figure 3.11: Bounds on the minimum detectable dark matter annihilation cross-section rate upon application or not of the weighting scheme combined with the effects of the multi-tracer strategy. We display here our forecasts for $\langle\sigma v\rangle$ as a function of the DM mass m_{DM} once the different analysis strategies are applied. The assumed annihilation channel is the standard $b\bar{b}$. In the three panels we display, from left to right, the results deriving from the adoption of the gamma-rays $\otimes$ galaxies cross-correlation (γg), the multi-tracer that combines gamma-rays $\otimes$ galaxies cross-correlation with galaxies auto-correlation ($\gamma g + gg$) and the multi-tracer that combines gamma-rays $\otimes$ galaxies cross-correlation, galaxies auto-correlation and gamma rays auto-correlation ($\gamma g + gg + \gamma\gamma$). We show the standard unweighted case in black, the green dashed lines refer to the *Astro+DM* weighting scheme while, eventually, the red dot-dashed lines corresponds to the *DM-only* weighting scheme. Our forecasts are compared with the thin purple line, associated to the combined bound obtained from the dwarf spheroidal galaxies observations from the Fermi-LAT, HAWC, H.E.S.S., MAGIC, and VERITAS observatories (Abdalla et al., 2021). The bounds are obtained by the recompilation of (Cirelli et al., 2024). We also display as a reference the “thermal” annihilation cross section $\langle\sigma v\rangle_{th} = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ (horizontal dotted line).

As a consequence, the results obtained in the pure cross-correlation case let us infer that the filtering technique allows us to improve the statistical significance with respect to the standard method of analysis, without any filtering in use. When we instead consider the multi-tracer cases, for ($\gamma g + gg$) the most noticeable improvements arise if the filter is modelled in the *DM-only* case, especially at large DM masses. We see indeed that the high-multipole part of the $\Delta\chi^2$ curve gets enhanced along a wide interval of multipoles. If we then add the gamma-ray autocorrelation power spectra, building the full ($\gamma g + gg + \gamma\gamma$) multi-tracer, we obtain a slight additional improvement to the significance for a DM signal. The further amount of information we are adding in this case is due to the correlations between different energy bins in which the gamma-ray auto-correlation signal is calculated (we refer to the covariance matrix formulation and its dimensions of eqs. 3.49 and 3.50). Such energy-dependent correlated information among different bins might be a further help to the distinction between contributions, namely the gamma emission from DM vs astrophysical sources, which are known to have different and distinctive behaviours.

The analysis in term of SNR and of $\Delta\chi^2$ allows us to have in mind a number of conceptual guidelines to interpret the principal result of this work, that we are now going to discuss: we present the forecasts on the bounds that can be achieved with the different strategies in fig. 3.11, also compared with the experimental bounds achieved by the combined observations of the Fermi-LAT, HAWC, H.E.S.S., MAGIC and VERITAS instruments (Abdalla et al., 2021) (purple line), targeting dwarf spheroidal galaxies (see also figs. 2.4 and 2.3). They are an interesting benchmark, because they are among the best available results in the literature, having been derived from observations of an environment, that of dwarf spheroidal galaxies, expected to be chiefly populated by Dark Matter and rather free from competing baryon processes also emitting γ radiation.

Again, we adopt the convention according to which, from left to right, the panels display the (γg) , $(\gamma g + gg)$ and $(\gamma g + gg + \gamma\gamma)$ cases, whereas the line styles correspond to the three strategies summarised in table 3.2. In particular, the *unweighted* case is shown in solid black; the green dashed lines correspond to the *Astro+DM* weighting schemes, whereas red dot-dashed lines show the results for the *DM-only* weighting scheme. Considering the case of (γg) cross-correlation alone, the use of a *DM-only* filter, based on the predicted gamma-ray DM emission, ensures an improvement to the bounds by a factor as high as 2-2.5 over the whole mass range if we compare the results with the standard analysis in absence of filters. Furthermore, the cross-correlation technique allows for a rather immediate understanding of the final outcome: the best lower bounds are indeed found with the *DM-only* weighting technique, since we are effectively limiting the competition from the astrophysical sources, that do not enter at all in the definition of the weights. Similar results are found in the multi-tracer configurations in most of the mass regimes. Always evaluating in parallel the impact of the weighting scheme and the multi-tracer technique, we find that this latter can actually be very effective, even without filtering, at the lowest DM masses: the $(\gamma g + gg)$ multi-tracer allows to improve the bound for low DM masses by up to a factor of 4, and the $(\gamma g + gg + \gamma\gamma)$ can enhance this improvement up to a factor of 5. At the higher mass end, instead, (γg) , $(\gamma g + gg)$ and $(\gamma g + gg + \gamma\gamma)$ all provide similar bounds, when the *DM-only* filter is applied.

On the other hand, our analysis finds that the *DM+astro* filtering scheme, which simultaneously introduces the DM and astrophysical sources window functions, can in no case improve the bounds on the detectable DM features although it still captures the distinct energy dependence of the astrophysical source and DM window functions as a function of redshift and of energy (see fig. 3.4 and 3.5). Some part of its limited effectiveness might not be related to the competition between “undesired” astrophysical sources and DM sources, and instead could be related to the limited redshift range covered by the 2MRS survey. Although this narrow redshift regime is in many aspects extremely valuable for probing the DM gamma-ray signal at the high prominence compared with the astrophysical contribution, the restricted redshift range may nevertheless not allow to leverage in full on the different dependence of DM and astrophysical sources with redshift, and we might be paradoxically “cutting out” higher redshifts where the annihilating DM contribution might still play a role and improve the contrast against a pure astrophysical signal. Also, 2MRS is both a narrow and a rather small survey, with less than 100,000 objects in its catalogue. In order to achieve high signal detection significances, its dataset must be used all together and we cannot partition it in tomographic slices. On the contrary, galaxy catalogues with a wider redshift range, although still covering the low-redshift regime which is known to be the most recommended one for DM cross-correlation searches, but with richer samples and the possibility to divide the data in different redshift bins, would allow to add the redshift-tomography cross-correlation among the

bins as a further dimension to the analysis. The design of these strategies and the selection of appropriate forthcoming surveys gives us a useful handle to devise future works and investigations.

To summarise, the main conclusion of our forecasts, associated to the strategies in table 3.2, is that, if we adopt a weighting scheme having a filter model based exclusively on the gamma-ray DM emission window function, i.e. on the *DM-only* method, we achieve an improvement in the bounds of a factor of about 2/2.5 covering our whole mass range, when compared to the standard correlation technique, that is, the gamma-rays $\otimes$ galaxies cross-correlation, (γg) and to the best available observational constraints. Our results can be probably better understood, however, in terms of nuances between regimes where *the weighting strategy* is more effective or at least comparable with the results obtained in standard configuration analyses, and regimes where *the multi-tracer technique* acts as the main driving factor for potential improvements. In particular, for DM masses below about 50 GeV, the full multi-tracer method $(\gamma g + gg + \gamma\gamma)$, even in absence of a filter, can improve the bounds for light DM masses up to a factor of 5. It is important to bear in mind that, according to eq. 3.41, also the galaxy shot-noise term in the covariance, together with the galaxy autocorrelation term entering in the covariance as well (eq. 3.36) get reweighed by the implementation of the weighting scheme and possibly boosted, sometimes more effectively than the effect the weighting exerts on the signal terms. Eventually, we can compare the results with the thermal WIMP cross-section rate reported in our exclusion plots as a black-dotted horizontal line at the level $\langle\sigma v\rangle_{\text{thermal}} = 3 \cdot 10^{-26} \text{ cm}^3 \text{ s}^{-1}$: experimental results have already ruled out the possibility of thermal WIMPs in most mass regimes but the largest ones, close to 1 TeV, not finding DM signatures even much below the thermal annihilation cross-section rate, suggestive that, if WIMPs are indeed the “right” DM model and if DM actually exist, its production in the Early Universe is most likely not thermal for most of the possible mass regimes (see sec. 2, in particular sec. 2.2.3 and fig. 2.3 and 2.4); the general improvement in the bound also translates in the potential capability of covering even weaker non-thermal WIMP signatures. Overall, our result might be understood in hindsight if combined with the discussion on the shapes and trends of the window functions, intensity spectra, C_p , SNR and $\Delta\chi^2$ as a function of m_{DM} and energy bins presented in figs. 3.1, 3.2, 3.3, 3.8, 3.10, 3.9, already showing that, typically, smaller masses and lower energy regimes in the GeV-TeV interval are more competitive with the astrophysical background both in terms of signal intensity and of distinctive morphological-summary statistics properties. The results are then suggestive that the ultimate more advisable analysis strategy would be the *combined* use of both the filtering and the full multi-tracer methods, to be understood as complementary and collaborating technical improvements in the wider topic of research about cosmological indirect signatures of WIMP-like DM relics.

3.5 Conclusions

We can collect our conclusions here: we have discussed the application of the Wiener filter, constructed on a model for the gamma-ray emission, translating into an application of a redshift-dependent weighting scheme to the galaxy distribution, then tested according to different strategies. At the same time, we have investigated also the simultaneous use of all available information on the LSS contained in our data, namely the multi-tracer approach, explored in , the two subcases of an analysis of the combined galaxy-galaxy and galaxy-gamma correlation signals on one hand, and of an analysis of the galaxy-galaxy, galaxy-gamma and gamma-gamma correlation signals) testing whether it can help with improving the sensitivity of the baseline cross-correlation analysis.

We thus recognised in the use of a weighting scheme introducing exclusively the dark matter component, excluding the contributions from astrophysical sources, which we call the *DM-only*, the most effective strategy. This configuration can improve the sensitivity to the DM particle parameters already in the simplest cross-correlation configuration, even prior to the application of the full multi-tracer technique. The attained improvements can ensure sensitivity to annihilation cross-section rates as low as about 2/2.5 times smaller than the bounds set by the unweighted (i.e., the standard cross-correlation approach) configuration over the whole mass range. Moving to the full multi-tracers information, we have been able to forecast improvements on the bounds for DM masses below the 50 GeV threshold which might attain up to a factor of 5 if they are compared to the standard analysis (gamma-ray $\otimes$ galaxies), even in absence of a filtering scheme. The *DM-only* filter is again the one yielding the best performances, since it is weighting the galaxy window survey without introducing a competing astrophysical foreground in the Wiener filter formulation, for heavier dark matter masses, with a sensitivity reach comparable to the one obtained using the filter in the gamma-ray $\otimes$ galaxies cross-correlation.

The complex interplay of observables, variables, energy and redshift dependences, together with the implementation of the weights, led us to considerations about the robustness of the techniques we proposed: two aspects should be considered, corresponding to the concurring effects from optimal weights and multi-tracer approaches. The latter aspect has to be considered when referring to auto-correlation spectra. Indeed, cross-correlation spectra are typically robust to systematic errors, thanks to the characteristic property of the cross-correlation of eliminating the effect of additive systematics under the assumption that they do not correlate – an hypothesis largely holding true in our case since the considered fields are usually measured in independent ways. Auto-correlation spectra, on the contrary, require careful modelling, since residual systematics might leak when we build the multi-tracer configuration (Alimi et al., 1988; Blanchard, 2022). At the same time, our results rely on well-known astrophysical and cosmological observables, for which a wide body of literature concerning the separate analyses of pure auto-correlation spectra or cross-correlations (without considering the multi-tracer case) have been performed, with subsequent comparison with available theoretical models in many analyses, validating the employed models (Ackermann et al., 2018; Ando et al., 2018).

On the other hand, we stress once again that the whole Wiener filter formalism of course relies on the assumption that our theoretical model for the experimentally unobservable gamma-ray kernels of both the DM and astrophysical contributors is correct (Fisher et al., 1995; Lahav et al., 1994; Zaroubi et al., 1995; Alonso et al., 2020). At the same time, this observation opens up a further declination and interpretation pathway for our work: should indeed our knowledge of the kernels be unsatisfactory, we would find a very poor agreement between our models and the fit they provide to the real observational data. Such analysis would lead to a suboptimal weighting, without ameliorating the SNR. The model would therefore not be selected by our technique and excluded consequently. But then one could iteratively try to improve the functional forms or the free parameters in the gamma kernels using the weights as a guide to optimise the matching and therefore recovering precious information about the potential actual distribution of the sources, using the Wiener filtering formalism as a guidance criterion to a matched-filter investigation of the UGRB composition.

Limiting ourselves to the fundamental goals we had in mind when starting this analysis, we can conclude that we have been able to show how the two techniques discussed above – a weighting scheme based on a Wiener filter formalism and the multi-tracer – are able to improve on the

standard UGRB-galaxy catalogues cross-correlations, enhancing the attainable measurable bounds on the features of Dark Matter particles: we tested it in the framework of WIMP dark matter models –heavy mass, weakly interacting particles possibly produced in thermal equilibrium conditions in the early Universe– but extension to additional research strands can be considered. By working on other electromagnetic bands (see sec. 2.2.2), such as X-Rays or the radio band, similar cross-correlation formalisms relying on unresolved backgrounds can be implemented in order to test signatures distinctive of other DM candidates, in particular axions and axion-like-particles. Also different LSS tracers, such as galaxy clusters or cosmic shear, might acting as valuable alternatives to galaxy clustering signals, again in line with research being performed in these years, see fig. 2.9 and fig. 2.10 (Branchini et al., 2017; Ammazalorso et al., 2020; Thakore et al., 2025). Altogether, the Wiener filter technique and the multi-tracer analysis applied to the cross-correlation technique is also readily extensible to research programmes focusing on dark matter signals probed in channels different from the electromagnetic one, such as neutrinos and charged cosmic-ray emissions (Urban et al., 2023), and might be of interest also for the investigation of astrophysical sources contributing to the UGRB. If the study of the exclusively astrophysical contribution were the main goal of the analysis, we would recommend to develop the investigation using galaxy catalogues peaked at redshift higher than 2MRS, since unresolved astrophysical sources tend to be emit chiefly at larger redshifts compared to the expected dark matter so (see fig. 3.3).

Part II

Inspecting the growth of structure and
the “ S_8 tension” at the lowest redshifts
with tomographic cross-correlation
techniques

Chapter 4

The S_8 tension: a review of the observational status and of the proposed solutions

4.1 Definition of the tension

We start with recalling the definition of the S_8 parameter:

$$S_8 \equiv \sigma_8 \left(\frac{\Omega_m}{0.3} \right)^\alpha, \quad (4.1)$$

with α usually being set to 0.5. The value of the exponent and the definition of the quantity are constructed in order to break most of the degeneracy between the reference parameter σ_8 and the matter density parameter Ω_m . It is also important to remark that, while Ω_m can be directly measured in a number of ways, σ_8 itself is a derived quantity in the Λ CDM model and that, more than “measuring” its value, it is instrumental to the definition of a normalisation of the power spectrum for both the CMB and the Late Universe probes, according to eq. 1.34. The “tension” on S_8 refers thus to the *observed statistical discrepancy between the measurement of this quantity as derived from the CMB angular power spectrum – both as primary signal and in the form of CMB lensing signal – on one hand (acting as an “Early-Universe” probe of the quantity) and the values inferred from fitting at least some of the statistical and physical observables of cosmological interest in the Late Universe*. The statistical assessment of the degree of compatibility between the measurements is performed through tests depending at the same time on accuracy (the distance between the two reference values) and precision (a function of the size of the uncertainties associated to each measurement) of our estimates: these considerations set the discrepancy at a statistical level from $\sim 2\sigma$ to $\sim 3.5\sigma$ depending on the probe (Di Valentino et al., 2025; García-García et al., 2021; Joudaki et al., 2018; Abbott et al., 2022), see also fig. 4.1. According to usual statistical inference practices, the discrepancy lies therefore at the threshold between full compatibility and the possible existence of a relevant disagreement between methodologies that – if our theories are correct – should provide self-consistent outcomes between measurements at every redshift. This situation has elicited mixed attitudes towards the actual meaning of the discrepancy, from researchers deeming it a simple curiosity, those who consider the discrepancy a stimulus to refine our understanding of observational systematics and of the physical modelling of the smallest scale – although without the requirement of a deep refoundation of the Λ CDM model – and those arguing that it is instead

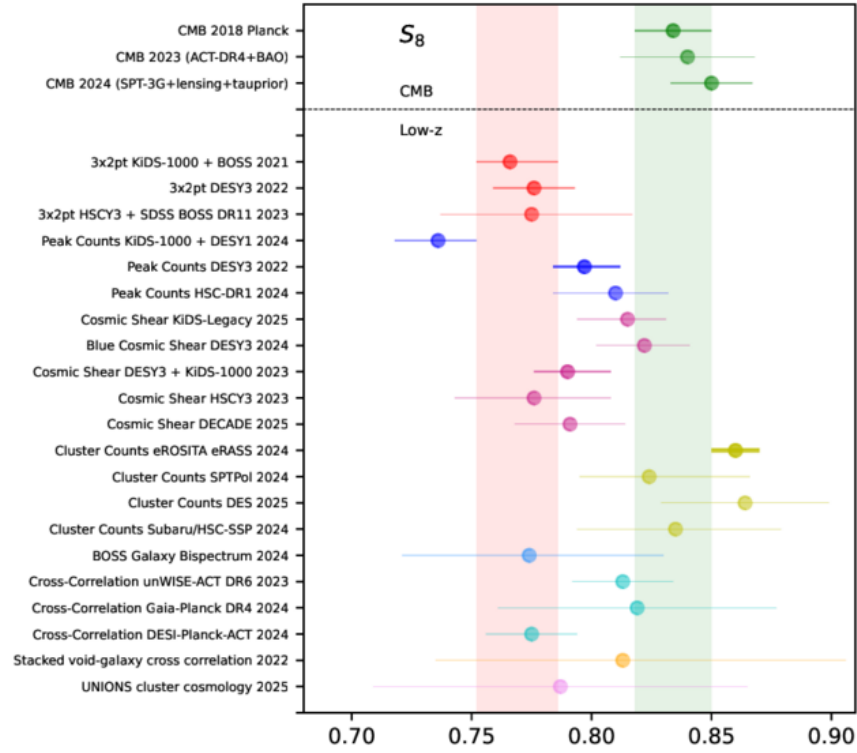


Figure 4.1: Summary table of the measurements of S_8 , from (Di Valentino et al., 2025): the red band encompasses the values among which the main cosmic shear-based measurements of S_8 tend to gather, displaying the greater distance from CMB-based measurements (green band), while the most of the other low- z probes tend to lay in between.

signaling a major flaw in our understanding of the Universe (Madhavacheril, 2025; Binney et al., 2025; Efstathiou, 2025; Douspis et al., 2019; Di Valentino et al., 2025; Scott, 2018; Valentino et al., 2025). According to this latter point of view, the cosmos is far from being completely described by the Λ CDM concordance model and its predictions based on a cosmic inventory of Dark Energy, Dark Matter, baryon matter, radiation, gravitational waves and neutrinos as cosmic fluids combined with the inflationary paradigm setting the initial conditions. Rather, we might be called to a thorough rethinking of the Λ CDM picture in terms of fundamental Physics, introduction of additional fields or through a description of gravitation going beyond General Relativity.

The S_8 tension is often mentioned together with the tension on the expansion rate parameter H_0 (Knox & Millea, 2020; Di Valentino et al., 2021a; Schöneberg et al., 2022): in a similar way, also this second tension testifies a discrepancy between CMB-based observations and Late Universe probes. The attained differences, although again with some degree of variability depending on the used probes, can be this time be as high as 3σ - 5σ . As such, the H_0 tension is deemed as one of the most pressing open problems in the field of Cosmology. The outcomes of H_0 measurements on Late Universe probes seem to go in the opposite direction compared to S_8 , yielding typically *higher* values for the best fit of H_0 compared to the CMB results. Furthermore, while the S_8 tension addresses measurements of perturbative quantities, in the case of H_0 the discrepancy involves both measurements of perturbative and background quantities, touching both LSS statistics and

distance measurements derived by means of standard rulers. The two tensions might be related to some extent and the literature knows attempts of joint solution for both issues together: an outline of these positions and observational results will be provided in sec. 4.2.2. To give the reader a better sense of the variety and typology of measurements now available, we now review the main quantities and related observational campaigns whose results are considered state-of-the art in the endeavour of measuring S_8 , following in particular the results collected in the most updated reviews published in (Di Valentino et al., 2025; Abdalla et al., 2022; Douspis et al., 2019).

4.1.1 High-redshift observables

The Cosmic Microwave Background: primary and lensed signals

Let us start with the Early Universe observable, the CMB. The value of S_8 can be recovered using the photon temperature fluctuation power spectrum (related to scalar perturbations in the matter field, and usually noted with TT), the polarisation (vector) fluctuation power spectrum, that can be decomposed in the E and B modes power spectrum, and their various cross-correlations (TE, EB etc.). We further distinguish between the *primary* CMB signals produced at the Recombination Epoch, and the complementary information provided by the lensed signal of the temperature fluctuations (usually known as CMB lensing *tout court*, regardless of the polarisation data) probing the interaction between pristine CMB photons and the LSS growing along the line of sight throughout the cosmic epochs. This latter observable displays greater sensitivity to smaller scales and is thus of even greater interest for S_8 measurements and will be used in the analyses presented in chapter 5 together with galaxy clustering signals. Given its importance, we discuss more in detail its modelling and observational properties in sec. 5.1.2.

Although affected by complex model dependencies, both lensed and primary signals agree around a higher value for S_8 than the one measured on the LSS (Madhavacheril, 2025). The reference measurements as of 2025 are represented by the almost full-sky observations of the Planck telescope in space and the ground-based observations performed by the ACT observatory, the latter requiring some more contributions from external datasets to achieve its maximum constraining power (Aghanim et al., 2020; Planck Collaboration et al., 2020c; van Engelen et al., 2012; Louis et al., 2025). The outcome is summarised in table 4.1 for the separate primary and lensing results and in table 4.2 for the combined quantities. We shall hereafter consider Planck primary + lensing values as the reference “Early Time” values of the parameters of interest, hence:

- $\sigma_8 = 0.81 \pm 0.01$
- $\Omega_m = 0.32 \pm 0.01$
- $S_8 = 0.83 \pm 0.01$.

Overall, the CMB best-fit parameters tell us that we should expect a higher degree of fluctuations in the matter fields compared to the average density, whence a greater amount of structure at all scales should be observed with respect to the size of the fluctuations we actually measure in the LSS at late times. Some noteworthy main sensitivities specific to the CMB are its dependency on the sum of the neutrino masses, Σm_ν , and on the optical depth at Reionisation, τ , determining the CMB photons absorption rate and both degenerate with the overall power spectrum amplitude (Di Valentino et al., 2025). The parameter τ is known to be the one whose uncertainties have been historically larger, whereas we can only put upper limits on Σm_ν , since the baseline Λ CDM model

Experiment	Primary CMB			CMB Lensing		
	σ_8	Ω_m	S_8	σ_8	Ω_m	S_8
ACT	0.8263 ± 0.0074	0.337 ± 0.013	0.875 ± 0.023	0.820 ± 0.015	0.315 ± 0.016	0.840 ± 0.028
Planck	0.8120 ± 0.0073	0.3166 ± 0.0084	0.834 ± 0.016	0.811 ± 0.019	0.303 ± 0.016	—

Table 4.1: Constraints on σ_8 , Ω_m and S_8 from Primary CMB and CMB Lensing separately, according to ACT and Planck data releases. Concerning Planck, primary CMB results are those quoted in (Planck Collaboration et al., 2020b), Table 2 (TT,TE,EE+lowE) whereas combined lensing results are those quoted in (Planck Collaboration et al., 2020c), page 13 equation 39 (the paper does not provide a value for S_8). Concerning ACT, primary CMB results are those quoted in (Louis et al., 2025), Table 5, while lensing results are those quoted in (Madhavacheril et al., 2024), Table 2 (ACT CMB lensing + BAO).

Experiment	σ_8	Ω_m	S_8
ACT	0.8126 ± 0.0046	0.3032 ± 0.0048	0.8169 ± 0.0087
Planck	0.8111 ± 0.0060	0.3153 ± 0.0073	0.832 ± 0.013

Table 4.2: Constraints on σ_8 , Ω_m and S_8 from combined Primary CMB + Lensing analyses for ACT and Planck. Concerning Planck, the results are those quoted in (Aghanim et al., 2020), Table 2 (TT,TE,EE+lowE+lensing). Concerning ACT, the results are those quoted in (Louis et al., 2025), Table 9, P-ACT-L, actually combining Planck and ACT lensing with DESI BAO data.

assumes neutrinos to be strictly massless, whereas we know from oscillation experiments at least some of the neutrino flavours should have a non-null mass (Di Valentino et al., 2024).

4.1.2 Low and intermediate redshift observables

We can also review the main observables tracing instead the LSS, making the main distinction between measurements based on *clustering* of astrophysical objects in the LSS and measurements based on the *weak lensing* effect exerted by the cumulative LSS mass distribution on photons emitted by the objects clustering in the LSS. The main difference between clustering probes and weak lensing probes is that, while the first observable can be studied both in three-dimensional and two-dimensional power spectra, weak lensing is an inherently projected observable (see sec. 1.4). At the same time, many aspects concerning calibrations, systematics, observables and physical effects involved in both main classes of observables are rather similar and inter-related, so that the boundary concerning some aspects is a bit more blurred and some arguments will be recurring and intertwined, to be partially kept in mind “in parallel” in order to adequately grasp the complexity of the available body of evidence.

Probes of LSS clustering

Under this definition we encompass all statistical measurement of biased tracers of the matter density field relying on photometric and spectroscopic observations of the LSS without involving gravitational lensing signals. They include:

- **Number counts:** involving measurements of the abundance of tracers per unit mass and volume, possibly as a function of the redshift, thus reconstructing the underlying Dark Matter halo mass function or the tracer’s mass function. By looking at eq. 1.51 we see that they display sensitivity to σ_8 , degenerate with Ω_m , through the multiplicity term $\nu = \frac{\delta_{sc}^2}{\sigma^2}$ (Di Valentino et al., 2025).
- **Summary statistics:** especially two-point functions (the power spectrum in the Fourier space or the spherical harmonics space and the correlation function in the real space), and higher-order estimators, especially the three-point estimator, known as the bispectrum. Depending on the specific tracer, multiple dependence on σ_8 and Ω_m are implied; in general, given the relationship between σ_8 and the power spectrum normalisation, the fundamental dependence of summary statistics on this quantity are related to the amplitude of the perturbations at the different angular or metric scales, either in the real space (correlation functions) or in the Fourier and spherical harmonics spaces (power spectra and higher-order spectra) (Madhavacheril, 2025). Dependence of summary statistics on Ω_m is more complex, entering in the setting of the turnover scale, in the growth factor and in the small-scale slope of the power spectrum (see figs. 5.6 and 5.5).

Main tracers used for both number counts and summary statistics analyses are either galaxies (usually considered the clustering observable *par-excellence*), galaxy clusters, and the peculiar velocity field (Tsagas et al., 2025), measured either independently or as a higher-order correction term to the matter tracer power spectra, known as Redshift Space Distortions, RSD (Kaiser, 1987; Tsagas et al., 2025; Huterer, 2023). These tracers are observed in the different bands of the electromagnetic spectrum: standard present or future reference galaxy surveys such as DESI (Raichoor et al., 2023), eBOSS (Avila et al., 2020), Euclid (Amendola et al., 2013), JWST (Gardner et al., 2006) or LSST (Strauss, 2011; Ivezić et al., 2019) observe the sky in the optical and infrared bands. The richness and density of the galaxy sample is the basic fundamental requirement together with the accuracy in determining the redshift position of the objects. Concerning position determination, two main techniques are available, both starting from the characteristic distribution of their electromagnetic emission along the frequencies of the electromagnetic spectrum, a quantity known as spectral energy distribution (SED) of the main observed galaxy populations (in particular, Emission Line Galaxies, ELG, typically spiral objects in early evolutionary stages, and Luminous Red Galaxies, LRG, usually elliptic galaxies in late stages of galaxy evolution (Binney & Merrifield, 1998; Mo et al., 2010)). While spectroscopic redshift estimation concentrates on the observed displacement of very specific emission lines detected in the SED with respect to their rest-frame position (Euclid Collaboration et al., 2025a), photometric redshift estimation is based on the “grosser” measurement of the shift of the whole SED towards longer wavelengths with increasing distance, typically resulting in a signal that, while visible with a specific filter if measured in the rest frame, it would become fainter or absent within the same filter observational band, instead being observe in a “redder” filter, sensitive to higher wavelengths (Salvato et al., 2019). Uncertainties in position measurement obtained through photometric techniques are typically an order of magnitude larger for photometric samples compared to the spectroscopic resolution achieved by current state-of-the-art instruments, with $\delta z/z \sim 10^{-3}$ as a typical figure for spectroscopic measurements and $\delta z/z \sim 10^{-2}$ for photometric measurements (Euclid Collaboration et al., 2025b,a). In terms of sample abundance, spectroscopic measurements from eBOSS and DESI observations stand out among the others with catalogues in the order of 10^7 of galaxies observed up to redshift 1.5, an

abundance expected to be reached also by the operating Euclid telescope, with the further benefit of extending observations up to $z \approx 2$ (Euclid Collaboration et al., 2020), while the proposed 4MOST (Verdier et al., 2025) and WST (Mainieri et al., 2024) observatories on Earth aim at reaching $10^6 - 10^7$ objects spanning a redshift interval from 0 to 4. Such high-precision catalogues are or will be complemented by photometric surveys, covering from 10^7 to 10^8 objects (e.g. 2MPZ (Skrutskie et al., 2006), WISE and SuperCOSMOS (Bilicki et al., 2016), DES (Bechtol et al., 2025), LSST (Strauss, 2011)) together with smaller spectroscopic datasets in the order of 10^5 objects, particularly useful to ensure coverage at the lowest redshifts (e.g., 2MRS (Huchra et al., 2012), and 6dF (Jones et al., 2009)) and detailed tomographic measurements allowing to “unfold” the dynamics of cosmic structure growth in time.

Relatively recently, galaxy cluster detection by means of X-Rays (Ghirardini et al., 2024) observations or by means of the Sunyaev-Zel’dovich effect (Planck Collaboration et al., 2016a; Efstathiou & McCarthy, 2025) (starting from the CMB maps and detecting galaxy clusters as distortions in the reference CMB blackbody power spectrum, induced by Inverse Compton interactions between the CMB photons and the X-Ray photons in the Intracluster Medium (Sunyaev & Zeldovich, 1970, 1972, 1980; Birkinshaw, 1999; Carlstrom et al., 2002)) has reached the level of accuracy and of catalogue abundance capable of ensuring the proper application of statistical techniques to the detected objects. The summary statistics they allow to build are typologically “clustering probes” measurements, since they allow to detect galaxies and clusters in another electromagnetic band, with tracers characterised by some specific features: in particular, the detections are based on properties of the plasma and other effects strictly related with the smallest and non-linear scales and the properties of the plasma in the Large Scale Structure. Moreover, techniques targeting line emissions from the gas residing in the LSS (generally known as Line Intensity Mapping, LIM) are starting to become available and capable of allowing statistics: the most mature result thus far is the observation of neutral hydrogen (HI Intensity Mapping, or HI IM) in the 21 cm radio band (Square Kilometre Array Cosmology Science Working Group et al., 2020; MeerKLASS Collaboration et al., 2025; Cunnington et al., 2023; Eifler et al., 2021; Cunnington et al., 2025) but a richer array of probes is being or will be deployed for multiple cosmological goals (Bernal & Kovetz, 2022). Not all these techniques can ensure good access to the smallest scales and the non-linear regime, or they can do it only in some configurations (for example, HI IM with single-dish observations is more suited for accessing large and very-large scales, whereas interferometric measurements are the reference technique to probe the smallest scales (Square Kilometre Array Cosmology Science Working Group et al., 2020)). In general, the novelty of the field is requiring a wide modelling effort of the tracers’ properties; at the same time, the important expected statistical constraining power and redshift resolution will be of great utility to next-generation cosmological observations, possibly providing us with alternative and independent tracers capable of breaking extant degeneracies or probing different regimes.

Number counts and Halo Mass Function estimation Concerning number counts and the estimation of the HMF, the operation implies instead the derivation of mass estimates of galaxies and galaxy clusters based on the scaling laws between the observed electromagnetic fluxes or luminosities and the Dark Matter halo mass estimates (Di Valentino et al., 2025), often combined with mass measurements relying on weak lensing data from the same dataset (internal calibration), an observable which is known to be the probe driving the tension (Di Valentino et al., 2025; Dark Energy Survey and Kilo-Degree Survey Collaboration et al., 2023; Joudaki et al., 2018;

García-García et al., 2021). Measurements are particularly concentrated in estimating the masses of **galaxy clusters as a whole**, since we are substantially probing the largest Dark Matter halos, corresponding to the high-mass end of the halo mass function (HMF), and most of the HMF dependence on S_8 resides indeed in its intermediate to high-mass tail. Although potentially prone to delicate systematics, cluster counts and fitting of the HMF are extremely valuable as a source of independent estimates of the matter density parameter Ω_m , and the most state-of-the-art measurements are in agreement with the CMB, yielding:

- From eROSITA (X-Rays), $\Omega_m = 0.29 \pm 0.01$ (Ghirardini et al., 2024),
- From XXL (X-Rays) : $\Omega_m = 0.31 \pm 0.03$ (Garrel et al., 2022)
- From WtG (Weak Lensing): $\Omega_m = 0.26 \pm 0.03$ (Mantz et al., 2015),
- From DES (Weak Lensing): $\Omega_m = 0.32^{+0.08}_{-0.07}$ (Costanzi et al., 2021),
- From the SZ-selected clusters from Planck maps: $\Omega_m = 0.33 \pm 0.03$ (Planck Collaboration et al., 2016a).
- SPT (SZ-based): $\Omega_m = 0.29 \pm 0.03$ (Bocquet et al., 2024).

The corresponding measurements on σ_8 are for the most part in agreement with Planck, yielding results in the $0.82 - 0.86$ interval. Consequently, eROSITA, SPT, WTG and XXL also yield satisfying agreement on S_8 , although the HMF-based degeneracy direction between σ_8 and Ω_m suggests to change α from 0.5 to 0.2 – 0.3 (Di Valentino et al., 2025). The main deviant outcome is represented by the Sunyaev-Zel’dovich measurements performed on Planck maps, giving $\sigma_8 = 0.76 \pm 0.03$ and determining a disagreement with Planck in the order of $\sim 2.5\sigma$, and greater agreement with the measurements of S_8 derived from cosmic shear measurements.

Galaxy clustering statistics We can calculate galaxy clustering statistics at different orders: from the basic two-point statistics (galaxy power spectrum) to three-point correlations (the bispectrum), in either the real or the harmonic and Fourier spaces. If available, RSD can be included, thus constraining σ_8 in terms of $f\sigma_8(z)$, possibly using tomographic strategies in order to better measure the time evolution of such parameters (see sec. 1.2.1). Cross-correlation and multi-tracer approaches both at the level of different observables and statistics are often pursued in order to control systematics and augment the constraining power.

Concerning **galaxy clustering autocorrelation statistics** (cross-correlations will be presented more systematically later in sec. 4.2.1, given the closer connection to the studies related with my research) with full-shape modelling of the power spectrum down to small scales and non-linearities described in a non-linear perturbative framework, different results based on the BOSS spectroscopic datasets at rather low redshifts ($0.2 \leq z \leq 0.75$) yield typically a consistent Ω_m in the $0.31 - 0.33$ range. The associated uncertainties ensure compatibility with Planck, but a low σ_8 (ranging between 0.69 and 0.71) propagating to a low S_8 in the $0.71 - 0.75$ range is found: it is the case of (Philcox & Ivanov, 2022; d’Amico et al., 2020) for 3D power spectrum-only data, of (Hahn et al., 2024) only using the multipoles of the bispectrum, of (Ivanov et al., 2023), combining the multipoles of the spectrum and of the bispectrum, on the correlation function (Tröster et al., 2020) and on the angular power spectrum (Loureiro et al., 2019). The results are in agreement with cosmic shear results and, in many cases, the size of the error bars ensure reconciliation with

Planck as well, but can attain a tension as high as 2.5σ with Planck (Ivanov et al., 2023). The eBOSS collaboration published results consistent with Planck, without using power spectra but rather focusing on growth reconstruction through RSD measurements and BAO, finding substantial agreement with Planck ($\sigma_8 = 0.8072 \pm 0.0056$ and $\Omega_m \approx 0.30$ with full degeneracy breaking between the two parameters thanks to the sensitivity of RSD to $f(z)\sigma_8$ (Alam et al., 2021)). Considering statistical quantities, agreement is also recovered with the anisotropic angular correlation function with conservative scale cuts (Semenait et al., 2022). Summary statistics from pure clustering power spectra derived from the observations of galaxies, quasar and Lyman- α observations in the DESI catalogues (Adame et al., 2025a), aimed at covering the redshift regime $0 \leq z \leq 3$ including RSD corrections in the power spectra templates with maximum scale cuts at $k \sim 0.20 \text{ Mpc}^{-1}$, are again consistent with Planck. The collaboration recovered statistical agreement with Planck on σ_8 (0.8121 ± 0.0053), Ω_m (0.3056 ± 0.0049) and S_8 (0.8196 ± 0.0090), with the σ_8 best-fit value found to be *larger* than Planck.

Summary statistics can be applied to galaxy clusters in the same way as we do with individual galaxies, a methodology known as cluster clustering (Miyatake, 2025; Marulli et al., 2021; Moresco et al., 2021; Romanello et al., 2025). Considering that clusters as tracers have a stronger bias with respect to the underlying Dark Matter field and thus typically yield a conveniently stronger signal-to-noise ratio compared to galaxy clustering statistics, we thus obtain a greater sensitivity to the perturbation growth rate, but this is done at the price of smaller samples, which can induce larger uncertainties since galaxy cluster counts have a shot-noise error scaling as $\frac{1}{\sqrt{N}}$ if N is the number of detected objects. Overall, cluster clustering analyses yield values for Ω_m in the $0.22 - 0.3$ interval, compatible with Planck within the uncertainties. Similarly, σ_8 is usually found to be compatible with Planck, although not always studies are able to achieve constraining power on the quantity (Lindholm et al., 2021; Marulli et al., 2018; Lesci et al., 2023; Paech et al., 2017; Lesci et al., 2022).

Weak lensing

Differently from clustering statistics, weak lensing is an inherently projected and weakly redshift-informed two-dimensional observable, consisting in relating the gravitational lensing effect induced by the whole mass distribution of the LSS to the distortions of the geodesics travelled by photons (Bartelmann, 2010; Prat & Bacon, 2025). Weak lensing manifests in two main ways: *cosmic shear*, consisting in a tiny deformation effect of the intrinsic round or elliptical shape of galaxies, whose shape elongation gets enhanced and modified along their axes, proportionally to the features of the interposed lensing mass. The second, independent and usually more difficult to detect, effect, known as *convergence term* – analogous to the action of a convergent optical lens – can under some assumptions be directly connected to the *magnification* and thus interpreted again in an optical analogy as an amplification of the galaxies size in the background (which, strictly speaking, is a function of both convergence and shear) (van Waerbeke, 2010). Most of current measurements are performed on the cosmic shear component, so that the terms weak lensing and cosmic shear are sometimes used interchangeably (Bartelmann, 2010). Differently from strong lensing effects, weak lensing cannot be detected on the individual object with high reliance, so that it is typically measured with statistical methodologies on large samples of galaxies, whose distortions with respect to the “true” shape – that remains unknown to us and must be modelled somehow – is collectively evaluated.

Weak lensing directly accesses the Dark Matter field distribution: the field can be considered as unbiased or, equivalently, to have a bias factor equal to 1. This consideration applies to the CMB lensing signal, for which it is the convergence term the relevant one, as well and it is a major and relevant difference with the clustering signal of biased tracers, since we are directly probing σ_8 as a power spectrum normalisation instead of the degenerate product with some tracer-specific bias $b_i\sigma_8$ (Madhavacheril, 2025). Furthermore, lensing in any form is sensitive to the whole of the cosmic evolution stage between the upper and lower limits of the redshift bin in which we collect our galaxies – in the limit of a single bin, the sensitivity is on the whole cosmic history occurring between the redshift of the galaxy emitting them and the observer. Similarly, specific sensitivity to small-scales, manifesting differently whether we consider the real-space two-point correlation function or the harmonic-space angular power spectrum, is achieved by means of this observable (Ferri et al., 2025). The detected galaxies, at the same time acting as the lensed objects and the lensing sources, i.e., as tracers of the host Dark Matter halo, are associated to a redshift value estimating their distance and consequently binned in a number of tomographic bins. Redshift tomography is often used in weak lensing by resorting to less precise photometric measurements rather than spectroscopic datasets: the choice is due to the high sample richness required to perform robust lensing measurements. We therefore have to trade some precision for more abundant signals. Sub-optimal redshift estimation is one of the two main limiting factors of the analysis, the other being the fact that, although weak lensing is interesting and powerful precisely because it allows to access the smallest scales, an important modelling effort on small-scale non-linearities and, in case of very-high- k cutoffs, of baryonic effects, is required. With these models in hand, one can then build two-point correlation functions considering pairs of galaxies being distorted along two orthogonal directions and, since non-linearity induces non-Gaussianity in the lensing field, also higher-order moment estimators. This allows in particular to build a further quantity, known as peak-count statistics, evaluating the number of peaks in the lensing signal as a function of their signal-to-noise ratio – thus ordering the lensing sources in terms of their prominence against the background and determining the stronger contributors to the lensing effect in every point of the sky – a quantity providing constraints as competitive as two-point correlation functions and power spectra (Jain & Van Waerbeke, 2000). The results from peak-count statistics have precision superior to or competitive with summary statistics, but are quite puzzling, spanning from the combined results of DES-Y1 and KiDS-1000 yielding $S_8 = 0.736^{+0.016}_{-0.018}$ (Harnois-Déraps et al., 2024) to $S_8 = 0.797^{+0.015}_{-0.013}$ for DES-Y3 data (thus a subsequent data release) (Zürcher et al., 2022).

Referring to two-point statistics, the controversial weak lensing results started to arise from the observations of the CFHTLenS collaboration (Heymans et al., 2013; Kitching et al., 2016), finding $\sigma_8 = 0.799 \pm 0.015$ and $\Omega_m = 0.271 \pm 0.010$, thus being found in tension with Planck observations of more than 2σ . Until 2025, state-of-the-art weak lensing measurements were those of the DES-Y3, KiDS-1000, HSC-Y3, reported in table 4.4, quoting the results for the angular correlation function, and in table 4.3, showing instead the results for the angular power spectra, that, either because of lower Ω_m or because of lower σ_8 estimations compared to Planck, yield a S_8 value that can be in a tension as high as 3σ (see fig. 4.4).

They are complemented by the DECADE results on the two-point angular correlation function: $\Omega_m = 0.268^{+0.033}_{-0.050}$ $\sigma_8 = 0.845^{+0.075}_{-0.092}$, $S_8 = 0.791^{+0.026}_{-0.032}$, at about 1.5σ below Planck (Anbajagane et al., 2025a). The situation has slightly changed during the course of 2024 and 2025 because of the publication of updated major or definitive data releases from leading collaboration, that we will better discuss later in section 4.2.1.

Survey	Angular Power Spectrum		
	σ_8	Ω_m	S_8
DES-Y3	$0.863^{+0.096}_{-0.096}$	$0.260^{+0.035}_{-0.057}$	$0.793^{+0.038}_{-0.025}$
HSC-Y3	$0.900^{+0.100}_{-0.139}$	$0.219^{+0.075}_{-0.052}$	$0.776^{+0.032}_{-0.033}$
KiDS-1000	$0.920^{+0.159}_{-0.215}$	$0.202^{+0.127}_{-0.053}$	$0.754^{+0.027}_{-0.029}$

Table 4.3: Constraints on σ_8 , Ω_m and S_8 from the angular power spectra measurements of DES-Y3 (Doux et al., 2022), HSC-Y3 (Dalal et al., 2023), KiDS-1000 (Loureiro et al., 2022).

Survey	Angular Correlation Function		
	σ_8	Ω_m	S_8
DES-Y3	$0.783^{+0.073}_{-0.092}$	$0.290^{+0.039}_{-0.063}$	$0.759^{+0.023}_{-0.025}$
HSC-Y3	$0.818^{+0.089}_{-0.091}$	$0.256^{+0.056}_{-0.044}$	$0.769^{+0.031}_{-0.034}$
KiDS-1000	$0.887^{+0.084}_{-0.107}$	$0.223^{+0.065}_{-0.033}$	0.764 ± 0.018

Table 4.4: Constraints on σ_8 , Ω_m and S_8 from the angular correlation function measurements of DES-Y3 (Amon et al., 2022), HSC-Y3 (Li et al., 2023) and KiDS-1000 (Asgari et al., 2021).

Combined Weak Lensing and Galaxy Clustering measurements

Motivated by the desire of leveraging as much as possible on multi-tracer techniques (see chapter 5, especially sec. 3.3) and on the combined constraining power of weak lensing and galaxy clustering samples, possibly capable of controlling some of the intervening degeneracies and systematics, the KiDS and DES surveys have also inaugurated the cosmological practice of “3x2pt” analyses starting from 2017. This has become a standard terminology to define the combined analyses of the three observables of galaxy clustering autocorrelation, shear autocorrelation and galaxy-shear cross-correlation in two-point correlation functions, either in the real or in the spherical harmonics space using, where possible, galaxy clustering and cosmic shear samples from the same survey (this is the case for DES (Abbott et al., 2018)) or from different surveys with overlapping footprints or complementary features such as spectroscopic galaxy clustering information opposed to photometric shear measurements for KiDS (Joudaki et al., 2018; Heymans et al., 2021; van Uitert et al., 2018) and HSC (Miyatake et al., 2023). The results are all the more important given the inconsistencies that had been appearing in the meantime: the analyses in this configuration have consistently provided low S_8 , always as far as 2.5σ away from Planck reference results, and some variability on σ_8 and Ω_m . The quest for potential mis-modelling and systematics has led even to cross-analyses between distinct weak lensing campaigns, notably KiDS-1000 combined with DES Y3, leading to $S_8 = 0.790^{+0.018}_{-0.014}$ (Dark Energy Survey and Kilo-Degree Survey Collaboration et al., 2023), which reduced the tension with Planck to $\sim 1.7\sigma$. These studies and the need to verify assumptions and systematics, stress-testing as much as possible shear observations, has led to a number of additional tests acting on wider cross-correlation programmes, updated modelling strategies or the use of subsamples of the original dataset. For example, (McCullough et al., 2024) tested cosmic shear measurements only on blue galaxies, less prone to intrinsic alignment effects (see beyond,

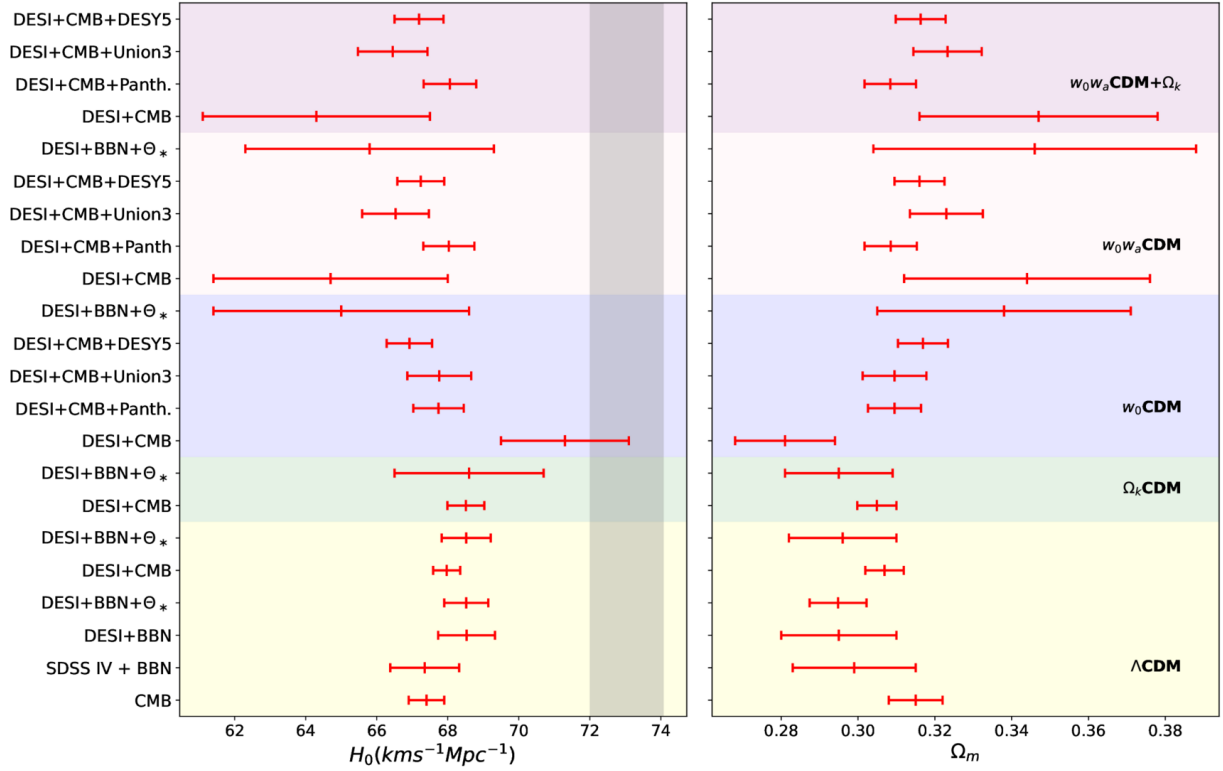


Figure 4.2: Whiskers plot of Ω_m measurements, from (Di Valentino et al., 2025)

sec. 4.2.1) and indeed recovering in some analysis configuration $S_8 \sim 0.82$. In a similar vein, other statistics designed in order to neutralise further systematic contributions, such as the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) statistics (Schneider et al., 2010) can be used, again leading to results in lighter tension with Planck. To further grasp some nuances, we refer to figs. 4.1, 4.2 and to (Di Valentino et al., 2025) for additional details, observing that there is a slight “historical” trend: more recent studies and data releases tend to provide, at least in some cases, a more relaxed degree of tension, see for example the most recent one (D’Amico et al., 2025). Such results usually come along with more comprehensive cross-correlation approaches and more accurate modelling or assessment of non-linear effects, baryon contributions and intrinsic alignment systematics. Also for 3x2pt analyses we therefore prefer to quote the most recent results in the next section 4.2.1, together with the discussion of the main systematics these more recent studies have tackled, combined with the simple enlargement of the dataset due to the longer acquisition times compared to the above-mentioned KiDS-1000, HSC-Y3 and DES-Y3 data releases, usually considered as the most representative sources of the S_8 tension debate.

4.2 Proposed solutions to the S_8 tension: refining our models or introducing new physics?

As we anticipated and as the previous review of the observational status might have suggested, it is actually unclear whether the S_8 tension should be seen as an actual source of concern or not, and, if so, whether it is just requiring refinements to the Λ CDM model – that holds however true overall – or a deeper rethinking of our understanding of the Universe (Efstathiou, 2025; Binney et al., 2025; Madhavacheril, 2025). Whatever the final answer, a first step to take – representative of an approach to which my research work also belongs – might be that of at least persevering

in the analysis of current and future datasets, in order to better understand the trend of the discrepancy broken down in different redshift bins, thus possibly circumscribing the cosmic epoch driving the discrepancy, or to figure out what observables, systematics and scales are the main drivers of such disagreement, or, conversely, what probes are self-consistent. The easiest pathway would be finding that, in the end, all probes will be reconciled in one fell swoop. However, also recognising that the observables driving the tension all share some set of characteristics would be a desirable result on which future work could be laid out. Another relevant theme in current research on S_8 is the collections of approaches wherein we try to keep ourselves as much model-free as possible, performing “exploratory” analyses circumscribing phenomenological trends that could be later addressed by theoretical effort capable of reproducing them from first principles, and those setups where we instead consistently derive and test specific models and assess whether they are able to eliminate the tension (Di Valentino et al., 2025). Of course, the theoretical derivations elaborated in this second approach must not simply be able to solve exclusively the open tensions, but also to produce a coherent description of the Universe competitive with the Λ CDM picture.

We might thus distinguish between two main research areas:

- **Solutions proposed fully within the Λ CDM model**, that can be achieved through the introduction of known physical phenomena thus far discarded or subdominant, or more refined treatment thereof: such frameworks can be further classified between those focusing on the description of non-linear clustering and baryonic effects, having a more theoretical flavour, and those theoretical-observational contributions based instead on reviewing in greater detail the status of the observational systematics, their impact on the error budget and, if they prove to be relevant drivers of the discrepancy, finding out ways to control them in full. It can perfectly be that the two approaches may equally contribute to the solution of the puzzle and that some empirical consideration correcting for observational systematics can ultimately be remapped, or be degenerate, with a more accurate description of the non-linear LSS regime. In some sense, a refinement of our current models and the possible introduction of so far unaccounted for effects could be understood as “New Physics”, although of course in the weaker sense of a refinement of our current understanding of structure formation history and the inclusion in the calculations of contributions we believed to be subdominant.
- **Solutions introducing actually novel physics**: on the other polarity of the alternatives, we can instead consider the possibility that the tension is suggestive that not only are our perturbative theories unable to describe the non-linear clustering regime because of a lack of sufficient accuracy but, more radically, because processes beyond the Λ CDM model are occurring, either in the form of additional fields or as extensions of General Relativity. In this latter case, the S_8 tension and its direct link to the growth of structure would be a signal that we need a better understanding of gravity. Since it is Gravitation the driving force of the Universe expansion and LSS clustering, many of these approaches also ponder the possibility that solving the S_8 tension could also give us some clues for a positive resolution of the H_0 tension and, in some cases, to shed light on the existence and properties of Dark Energy and Dark Matter. This latter scenario, more revolutionary than the previous one, is also more delicate to manage in strictly technical terms, in that effects also at the background level might be implied next to adjustments in the description of the perturbative regime.

4.2.1 Proposed solutions within the Λ CDM framework: the S_8 tension as a stimulus to better understand modelling and instrumental systematics

More accurate modelling of the smallest scales, of LSS tracers, physical effects and summary statistics

The general idea behind this first framework is that we have to more accurately describe the features of the LSS at high k , without the requirement of including new fields or exotic physical processes in the Λ CDM cosmological model: in this scenario, the observed small value of S_8 is caused by the high- k power spectrum trend not being damped enough to match the data. The lower S_8 resulting from the fits “compensates” thus for the theoretical limitations in our models. If we are able to construct a more strongly suppressed power spectrum, the S_8 value should automatically assume a larger best-fit value, closer to the one found by Planck (we recall that the statistical discrepancy between the two best-fit parameter classes is currently no larger than $\sim 3.5\sigma$). The description provided below tries to classify the main research directions in this field, not to be understood as rigidly compartmentalised, but as often intertwined and simultaneously introduced analysed aspects.

Non-linear clustering regime and the role of baryons

Even at the smallest scales (up to around $k \sim 0.5 \text{ Mpc}^{-1}$), most of the properties of the LSS can be described almost exclusively as the outcome of gravitational effects, sourced by the combined effect of dark and baryon matter masses. This general statement must nevertheless come along with two caveats: in general, at the smallest scales the amplitude of the density contrast becomes much larger than one; we have therefore to abandon the linear perturbation theory and replace it with a more appropriate description of virialised structures, as described in sec. 1.3, even if we were to consider exclusively the gravitational interaction. Baryon physics, however, cannot be in general reduced to gravitation only; the additional effects due to plasma flows, thermal and non-thermal pressure and magnetic fields cannot be completely neglected, especially at the smallest scales and in the most recent epochs in cosmic history, where the impact of non-linearity grows along with the increasing growth of structure. Hydrodynamical simulations – and in particular reference suites such as FLAMINGO or Magneticum (Schaye et al., 2023; Dolag et al., 2025) – are the most important tool at our disposal in order to handle such more refined description: their codes include radiative effects, due to photon emission by baryonic matter, next to gas heating due to gravitational effects, or plasma processes and matter transfer from galaxies to the LSS and vice versa known as AGN feedback, although with some limitations related to the limited attainable resolution in spatial scales. This is a well-known aspect, referred to in the literature as “subgrid effects”: neglecting them must be added to the general budget of systematics. The impact of baryon effects is usually quantified in the percentage power spectrum suppression they induce with respect to a reference power spectrum free from baryonic effects. Their contributions, together with other small-scale corrections induced by neutrinos and non-linear gravitational contributions typically disrupt the process of structure formation at scales up to about $k \sim 10 \text{ Mpc}^{-1}$, reducing the abundance of clusters as a function of halo masses and concentration, since in the densest, innermost regions of halos, baryons sum up to about 6-7 % of the total halo mass, whereas in the outskirts of the halos – whose definition in terms of overdensity might be also slightly corrected by the presence of baryons compared with the standard definition in Dark Matter-only models – their contribution to the

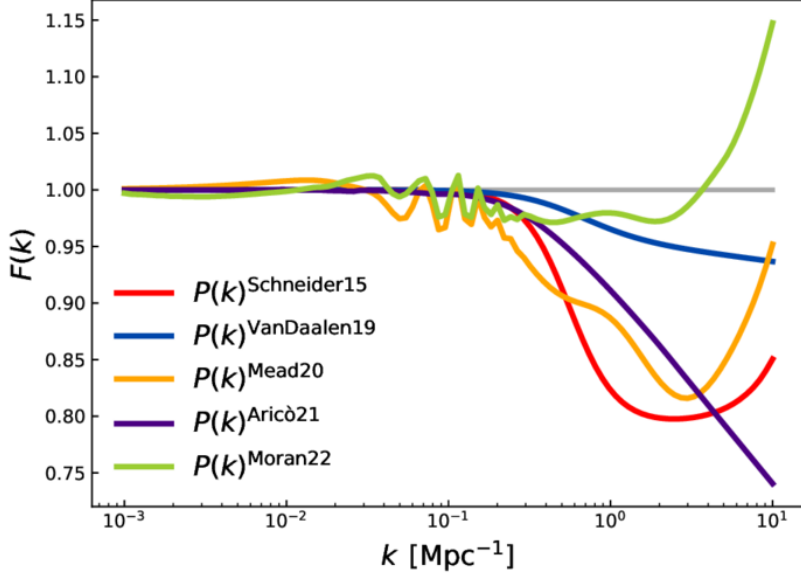


Figure 4.3: Impact of baryonic effects as percentage small-scale power spectrum suppression with respect to a reference baryon-free prediction, as a function of different models available in the literature, and the subsequent upturn on the power spectrum at even larger k (smaller scales). The picture is taken from (Di Valentino et al., 2025)

mass budget is around 1%; eventually, and interestingly, at even larger k , corresponding to galactic scales, baryon matter clustering switches the regime, *enhancing* structure growth (Di Valentino et al., 2025; Sorini et al., 2022, 2025). Altogether, when we account for baryon effects and combine them with neutrinos and gravity-induced non-linearities, we find that, at the lowest masses and redshifts, halo abundancies get reduced up to about 15%, suppressing the power spectrum at scales around $k \sim 1 - 10 h \text{ Mpc}^{-1}$, extremely local in nature. As anticipated, the situation changes for $k > 10 h \text{ Mpc}^{-1}$, translating in star formation and the induction of non-Gaussianity in the statistical properties of cosmological fields (fig. 4.3). As such, the use of higher-order statistics such as the bispectrum statistics, sensitive to non-Gaussianity, or the study of exclusively or chiefly baryonic probes such as the Sunyaev-Zel’dovich effect or X-Ray observations, become increasingly justified, allowing for increased constraining power on cosmological parameters (Di Valentino et al., 2025). The use of those probes gives us also the possibility of fitting for the baryonic contributions and then marginalise over the associated astrophysical parameters in order to obtain unconditional best-fit posteriors for the cosmological parameters, the ultimate quantities we want to study in order to understand whether S_8 deviates or not from the CMB value (Madhavacheril, 2025; Chisari et al., 2019b; Sunseri et al., 2023). The potential increase in statistical information induced by the introduction of more scales requires however the establishment of a trade-off with the additional modelling burden due to baryonic feedback as well as the inherent increase in the size of uncertainties in those data points. We will see below that, overall, the community shows a general agreement on the fact that a more careful modelling of non-linearities, translated into a contribution of the baryonic effects at the smallest scales, could help with relieving the issues with S_8 , although specific conclusions on the size of the impact on the final solution of the tension are different, based on the distinct data analysis exercises. Reviewing the outcome of the analyses published in the last years and displayed in fig. 4.4, we see that works such as (Terasawa et al.,

2025; Grandón et al., 2024), based on the Subaru Hyper Suprime-Cam (HSC), conclude that accounting for baryon effects, is **not resolute**: the S_8 tension is persistent even when data are fitted against models generated by emulators including non-linear effects of both gravitational and baryon origin. In particular, (Grandón et al., 2024) concentrates on the correction of weak lensing measurements by accounting for the information on baryon feedback provided by hydrodynamic simulations (Schaye et al., 2023; McCarthy et al., 2017, 2023). Accounting for baryons is found to be overall an important conceptual point in order to refine the theoretical templates against which statistics are calculated and indeed they are found to slightly move the value of S_8 in the final fits, but in a subdominant way if compared with other systematics. Other works observe that, in order to reconcile the best-fit value of S_8 with the Planck value, it is necessary to require very strong baryonic feedback rates, considered at odds with both the results of reference simulations and by the upper limits imposed by X-Ray observations, that directly constrain baryonic processes by observing the hot intracluster medium (Akino et al., 2022); further theoretical studies have also been performed in the spirit of (Terasawa et al., 2025), where the phenomenology of structure growth is compared against different candidate driving factors. These works conclude that other kinds of effects, especially time-dependent growth dynamics, should be the leading sources of the observed discrepancies, while baryon feedback models are not associated to relevant consequences (see also (McCarthy et al., 2023)). An extensive body of work represented by the papers (Amon & Efstathiou, 2022; Preston et al., 2023; Efstathiou, 2025), addresses the problem from both a theoretical and observational approach, suggesting a non-negligible impact of baryons in solving the tension and providing also important guidelines concerning the inspection of the smallest scales. The main point the authors make starts from the consideration that we can capture the necessity of accurate modelling of non-linearities by fitting various datasets with a power spectrum defined as

$$P_{\text{tot}}(k, z) = P_{\text{lin}}(k, z) + A_{\text{nonlin}} [P_{\text{nonlin}}(k, z) - P_{\text{lin}}(k, z)]. \quad (4.2)$$

P_{nonlin} is a power spectrum term constructed in a way that only includes *gravitational* nonlinearity. This fitting template has two effects: the closer to 1 the value of A_{nonlin} is, the greater the need for proper inclusion of *gravitational* non-linearity in our model; what is more, however, if A_{nonlin} turns out to be *smaller* than 1, this implies that *additional, non-gravitational*, power suppression mechanisms are required. In these works, the authors analyse KiDS21 and DES DR3 photometric observations of cosmic shear, finding that A_{nonlin} is in the order of 0.7-0.9 and significantly different from 1 in statistical terms. This is therefore interpreted as a hint for an actual role of non-gravitational effects at non-linear scales, further suppressing the power spectrum, on top of gravitational effects. The above-cited works do not conclude that baryon effects are the *only* actual source of this correction, but make a number of relevant points, trying to make order in the current body of evidence. Among the main conclusions, we can quote:

- The acknowledgement of the **existence of a discrepancy**, possibly driven by cosmic shear data, in the measurement of S_8 ; this result is not as alarming as the tension on H_0 for what concerns the foundations of the Λ CDM model, but must be taken into consideration.
- **The role of the non-linear scales.** This is connected with the role of scale cuts in the data analyses pipelines: the tension decreases with decreasing values of the Fourier mode k_{max} at which we perform the scale cuts, namely, the larger the last spatial scale at which we measure the power spectrum. Measurements exclusively including rigorously linear scales are found not to show a tension. Rather, a possible falsification of the insight that the S_8 tension

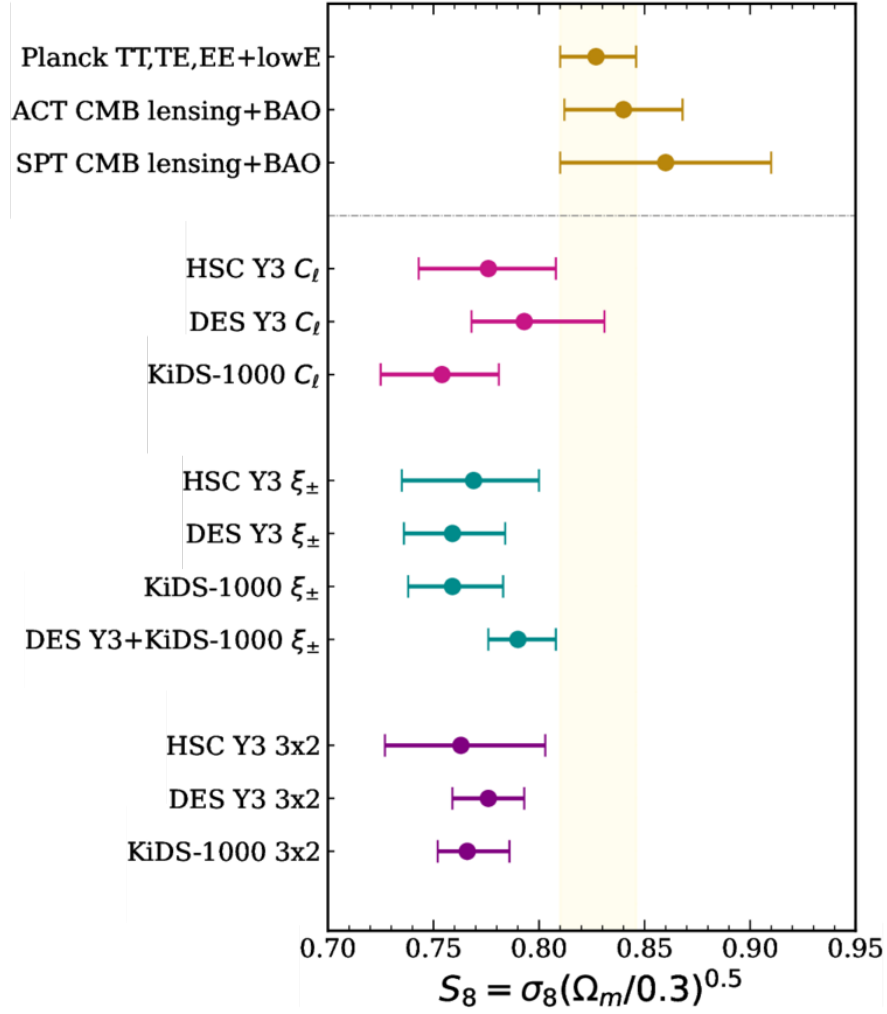


Figure 4.4: A summary of different weak lensing-based measurements of S_8 , from (Di Valentino et al., 2025)

is driven by inaccurate modelling of non-linearity could be finding *a single case of tension* on S_8 derived from measurements involving exclusively linear regimes and probes (taking into account the fact that linear scales have an intrinsically weaker constraining power on the parameter, so that we should usually expect larger uncertainties on the outcome of our measurements).

- The time-evolution of such suppression through redshift is also explored, but with less solid conclusions.
- Remarkably, the simple more accurate accounting for non-linearity pushes S_8 to a value closer to CMB measurements, even without imposing priors on parameters based on the reference Planck results, although at the expense of larger uncertainties, which “help” with the final reconciliation (fig. 4.5). If Planck-based priors are imposed, reconciliation is complete, but, of course, this is chiefly due to the strong constraining power of CMB data.
- The role of baryons: the aforementioned works tackle the topic of non-linearity regardless of its physical origin, as emphasised many times in the papers. At the same time, baryon feedback models are extensively investigated as among the principal explanations. The conclusion is

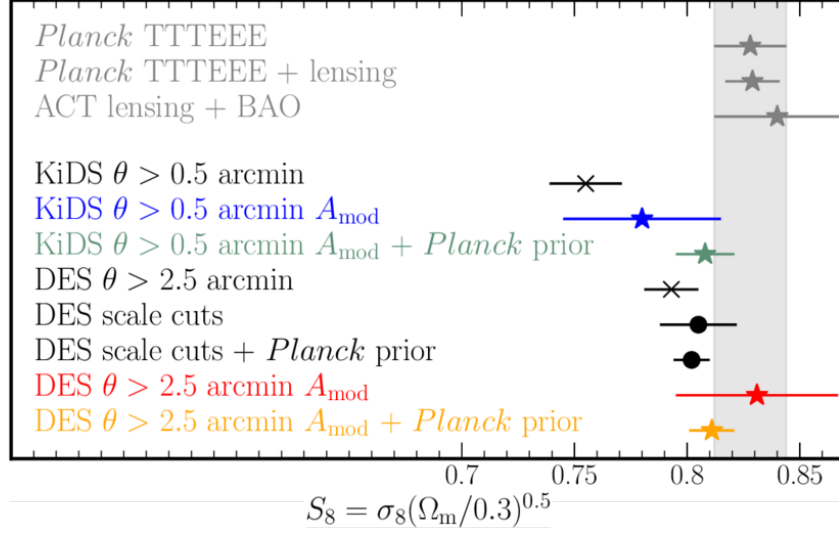


Figure 4.5: S_8 tension mitigation and comparison of measurement outcomes by different probes sensitive to the non-linear LSS, from (Preston et al., 2023)

that, overall, if baryons play a role in the discrepancy between the Planck best-fit S_8 value and the LSS best-fit S_8 value, the impact of baryon feedback should be rather large, larger than usually estimated.

- The papers point out that small-scale power spectrum suppression can be also mimicked by exotic physics associated to particle Dark Matter models: in particular, “warm” Dark Matter particles, satisfying most of the standard cold Dark Matter properties but with more complex interaction channels with baryons could determine a power spectrum suppression mechanism.

Going deeper into the role played by baryons, the work of (Efstathiou & McCarthy, 2025) performs an analysis in the same spirit of the above-mentioned papers, this time focusing on the thermal Sunyaev-Zel’dovich effect autocorrelation power spectrum, a probe endowed with strong sensitivity to small-scales and to baryon processes, obtained from combined maps of the CMB from Planck and ACT. The paper appeared in March 2025: around the same time period, a number of works concerning the measurement σ_8 using newer datasets or improving modelling appeared, tending to reconcile the best-fit value with the Planck measurements. In particular, tSZ measurements performed on the new ACT DR6 data release got published (Louis et al., 2025), agreeing with (Efstathiou & McCarthy, 2025), which was using a previous ACT dataset. Both works conclude that the small-scale suppression of the tSZ power spectrum must be rather high and could be related to strong baryon feedback effects. Moreover, while (Efstathiou & McCarthy, 2025) does not point at a unique cause for such suppression, (Louis et al., 2025) directly states that such strong tSZ power spectrum suppression at small scales is compatible with enhanced baryon feedback, also found as an intermediate-strength suppression by studies on previous ACT data (Dalal et al., 2025). Since previous literature based on simulations excluded that baryon feedback effects were capable of strongly suppressing the power spectrum (McCarthy et al., 2023), or put extreme suppression among the alternative, but not the preferred, cases, observations suggesting that baryon-induced suppression of the power spectrum is larger than what usually observed in reference hydrodynamic simulations are sometimes deemed as “surprising”, although at the same time already in 2023 the

work of (Aricò et al., 2023) reported the viability of reconciling DES Y3 S_8 measurements by relying on a combined modelling of intrinsic alignments affecting cosmic shear and an intermediate, yet non-negligible, baryonic feedback, in the order of about 10% additional power spectrum suppression at $k \sim 1 \text{ Mpc}^{-1} h$. As of the second half of 2025, the reality of stronger levels of baryon feedbacks are starting to be more often reported in the literature (Siegel et al., 2025), although not with unanimous consensus: instead, the combined analysis of DES-Y3 MAGLIM weak lensing and galaxy clustering catalogues with Planck CMB lensing observations recovers parameters in agreement with Planck Early Universe results (apart from the pure weak lensing analysis), associated with a very low degree of baryon feedback (Xu et al., 2025). The work of (Wright et al., 2025), studying the other important dataset apparently affected by a strong tension on S_8 , KiDS, provides a very recent and strong claim about the resolution of the S_8 tension, based on improving the intrinsic alignment model and accounting for baryonic feedback, assessed to be in the range of a 1% – 10% contribution to the small-scale power spectrum, hence comparable, although milder, to the work of (Aricò et al., 2023), whereas the work of (Broxterman et al., 2025) always on KiDS data claims a baryon suppression up to $\sim 30\%$ in the same scale regime. The different configurations explored in those works typically still underestimate the value of S_8 compared with Planck, but at much lower statistical significance. Recently, the work of (Fabbian et al., 2025) suggested the use of the CMB spectral distortion - related quantity $\langle y \rangle$, associated to the Sunyaev-Zel'dovich effect, as a new observable linked to estimations of baryon feedback and the related power spectrum suppression at $k \sim 1 \text{ Mpc}^{-1}$; the results are suggestive of a feedback intensity comparable with what is eventually, always in 2025 a work using SNIa weak lensing signal (Shah et al., 2025) appeared, using the same A_{nonlin} - based conceptual setup of eq. 4.2. They find $A_{\text{nonlin}} = 0.77^{+0.69}_{-0.40}$: uncertainties make the result compatible with 1, which would imply no need for further power suppression mechanisms other than gravitational non-linearities. The authors claim that most of the posterior probability mass is actually below 1.09, interpreting that the result is however suggestive of an actual need for non-gravitational contributions to the power spectrum suppression at small scales. Regardless of the conclusions, the paper provides an interesting study of the information provided by a probe of the non-linear Universe not yet accounted for.

On the direction of an increased tension, among the last results to be quoted, are however different statistics being tested on HSC and DES: both collaborations published works on higher-order statistics calculated from cosmic shear data (Sugiyama et al., 2025; Gomes et al., 2025) at intermediate redshifts, finding a 3.2σ tension for HSC data and a 2.3σ tension for DES in the value of S_8 with respect to Planck. Both studies, although exploring the linear regime, try to perform scale cuts conservative enough to avoid the impact of baryon effect and other poorly understood contributions from deep non-linear regimes. Other results to be quoted are those appeared in September 2025 combining DES and DECADE (Anbajagane et al., 2025b; Gatti et al., 2025), characterised by a very large sky fraction ($13,000 \text{ deg}^2$, about one-third of the full sky) and recovering agreement with Planck within 2σ with baryon-induced suppression of the power spectrum being quantified to about 10 % at $k \sim 1 \text{ Mpc}^{-1}$ providing about 0.3σ further shifts in the final outcome of the parameter measurement. Data from (Aymerich et al., 2025) combining weak lensing observations from DES, X-Ray measurements from Chandra and hydrostatic bias calibrations from Planck are found to be in agreement with Planck, although with a low, error bar-driven S_8 in tension with eROSITA at 2.9σ . Proper calibration of baryon probes is however a delicate issue (Aymerich et al., 2025), as we will outline better further in the chapter. Synergies

between state-of-the-art galaxy clustering surveys such as DESI and cosmic shear measurements from several surveys have been also proposed (Krolewski, 2021; Lange, 2024; Emas et al., 2025): their results cannot be but desirable, since they will eventually allow to probe small-scale linear regimes having most likely a high constraining power across a large redshift interval.

Some of the above-mentioned works belong to the “family” of approaches, to which my own research work (that will be exposed in greater detail in the next chapter 5) also belongs: studies aimed at measuring σ_8 or S_8 by dividing spectroscopic and photometric datasets in bins and trying to circumscribe what phases of the cosmic history provide more constraining power in driving the best-fit value of σ_8 to the outcome we observe, either considering its value at different redshifts (proper and complete tomographic approaches) or by limiting the measurement to its present value if the single bins do not provide enough constraining power. The study of the evolution of σ_8 in time, and of the general problem of the S_8 tension, can be also rephrased in terms of a study of the growth factor $D(z)$ evolution in time (see sec. 1.2), comparing the outcome with the prediction one obtains by using the Λ CDM model with its parameters set at the Planck best-fit values, so that problems addressing the growth factor are also relevant for this research lines. In general, all these works aim at fitting as many cosmological parameters as possible on their own, with the least dependence on Early Universe probes, at precision and accuracy levels competitive with Planck. Not always do the utilised observables allow the full breaking of all degeneracies, but some of them can be controlled and, in general, the achieved constraining power can be rather high if multi-tracer approaches are devised.

In the general setup, CMB lensing maps (see section 5.1.2) are used as high-redshift anchors, combined with cross- as well as autocorrelation data from galaxy clustering and cosmic shear measurements, usually covering the redshift interval between 0 and 3. In every redshift bin, a value for $\sigma_8(z)$ can be recovered in terms of σ_8 , a procedure usually known as growth reconstruction, using the generic relation

$$\sigma_8(z) = \frac{D(z)}{D(0)} \sigma_8 \quad (4.3)$$

(with $D(0)$ usually set to 1). $\sigma_8(z)$ is recovered by propagating the recovered value of σ_8 and can be approached through two main strategies:

- **Model-dependent strategies:** we set a cosmological model and we recover from the derived perturbation theory the functional form for $D(z)$. If we use the reference Λ CDM model in a FLRW metric, $D(z)$ is a function of the cosmological parameters that can be partially fitted against the data and partially set to Planck values or fully fitted with low-redshift probes, depending on the constraining power of our datasets. This approach allows to compare the degree of discrepancy between the predictions of the Λ CDM with the parameters set to Planck and the equivalent predictions based on Intermediate and Late Universe probes, testing the sensitivity to specific parameters and the redshift intervals where discrepancies localise.
- **Model-independent strategies:** in this case, the analytic form of $D(z)$ is defined phenomenologically, for example as a spline or as a Gaussian process, without dependence on any cosmological model and parameters. The goal here is to find a functional form fitting the trend observed in the data across the redshift bins that can be then simply ascertained in an “agnostic” way, or compared with the analytic prediction of $D(z)$ in alternative frameworks (e.g. by Modified Gravity theories). Notice that the degree of model-independence and

agnosticity can vary, based on the strategy used to recover $D(z)$. For example, although the growth factor is treated in a model-free way, the power spectrum templates it multiplies might be however derived from a perturbative theory, in more model-dependent conditions, or calibrated on simulations and translated in a model-free, phenomenological functional form. Similarly, the assumption itself that the growth factor is exclusively a function of the redshift, but not of the scale, might be questioned, so that the degree of “radicality” in model independence can vary based on the approach.

Similar results are found in the full multitracers study of (Maus et al., 2025a), combining DESI LRG and BGS samples on 3D galaxy clustering power spectra combined with galaxy clustering angular power spectra and the cross-correlation with CMB lensing, centred around $z \sim 1$. In the work of (Karim et al., 2025) the DESI ELG sample is combined with the CMB lensing signal in order to have additional constraining power at small scales, although an actually conservative scale cut at $\ell \sim 400$ is performed: substantial agreement with Planck results is also found in this work. A greater disagreement, although never reaching extremely high significances, is instead observed when the data are binned in narrow redshift bins. The works of (García-García et al., 2021) and (Sailer et al., 2025b) use a combined model-dependent test to assess the tension with Planck-based Λ CDM predictions and two distinct model-independent strategies: (García-García et al., 2021) recovers the model-free $D(z)$ by fitting a first- and third-order spline whose nodes are left free to vary, while the $P(k)$ templates are based on HALOFIT (Takahashi et al., 2012), a matter power spectrum template without excessive model-dependence, whereas (Sailer et al., 2025b), defines first

$$\sigma_{8,\text{fid}}(z)\alpha_8(z) = \frac{C^{\kappa g}}{\sqrt{C^{gg}}}, \quad (4.4)$$

with $\sigma_{8,\text{fid}}(z)$ the value of the quantity in the fiducial cosmology, e.g. a Λ CDM model. Overall, the quantity is weakly cosmology-dependent in that it can be considered to “simplify” the power spectrum template contribution in the ratio at the right-hand side. Deviations from the fiducial model can be traced back to the possible discrepancies in the linear galaxy bias factors, indeed degenerate with the growth factor by construction, as recovered by the galaxy-CMB lensing cross-correlation as measured by $C^{\kappa g}$ and the one measured by C^{gg} , since α_8 can be written as

$$\alpha_8 \equiv \frac{1 + b_1^{L,x}}{1 + b_1^{L,a}}, \quad (4.5)$$

where $b_1^{L,x}$ is the linear bias factor as measured in galaxy-CMB lensing cross-correlation and $b_1^{L,a}$ is the linear bias factor as measured in the galaxy autocorrelation signal. The model-independent trend on $\sigma_{8,\text{ind}}(z)$ is thus recovered as the trend on $\sigma_{8,\text{fid}}(z)\alpha_8(z)$ once the ratio $\frac{C^{\kappa g}}{\sqrt{C^{gg}}}$ is calculated in different redshift tomographic bins. The measurement outcome and, in particular, the recovered trend against the Λ CDM prediction for $\sigma_8(z)$ can then be compared with the prediction based on the parameters measured by the model-dependent CMB fit. Being a main practical and conceptual reference for the research work exposed in chapter 5, further details on (García-García et al., 2021) are provided in the next lines: the work uses eBOSS, KiDS, DESI and DELS galaxy clustering and cosmic shear datasets together with CMB lensing in the redshift interval 0.25-1.5. Five cosmological parameters $\{\Omega_m, \Omega_b, h, n_s, A_s/10^{-9}\}$ are fitted from the data, thus making the work substantially independent from Planck reference values: notice that the data fit the amplitude $A_s/10^{-9}$ as normalisation of the power spectrum, not σ_8 directly, which will be recovered as a derived parameter by using eq. 1.34 and employing all the other parameters in order to calculate

the quantities required to perform the integral. We highlight that, instead, the covariance among the data is calculated under the assumption of Gaussian data and with a fiducial cosmology set to Planck, but in general the dependence of the fitting parameters on the fiducial cosmology imposed on the covariance can be considered sufficiently weak not to be a concern (Rybicki & Press, 1992; Tegmark, 1997; Dodelson & Schneider, 2013; Taylor & Joachimi, 2014). The model accounts for intrinsic alignments through a non-linear model based on (Hirata & Seljak, 2004; Bridle & King, 2007) and redshift evolution of their amplitude based on (Abbott et al., 2018; Troxel et al., 2018). Non-linearities in the small-scale power spectrum are modelled based on the simulation-calibrated HALOFIT power spectrum (Takahashi et al., 2012), that, as we said, ensures some generality in the derived $D(z)$, decoupled from strictly Λ CDM-based models. Galaxy bias description is kept at a linear level and the performed scale cuts are such that there should not be need to account for baryon effects. The main results of this work are:

- The S_8 best-fit value when all probes (galaxy and quasar clustering, cosmic shear and CMB lensing) and bins are combined is $S_8 = 0.7781 \pm 0.0094$. This value is at a 3.4σ tension level with Planck. In terms of “chronological” evolution, the discrepancy with CMB-based prediction is at most a 2σ deviation concentrated in the redshift interval 0.25-0.7. At lower redshifts, no measurements are performed, but only extrapolated with large uncertainties that determine agreement with Planck-based predictions. In a similar way, data at higher redshifts, driven by quasar clustering statistics, agree with the Planck prediction.
- The magnitude of the deviation of S_8 from Planck results (and, consequently, of Ω_m and σ_8) shows some dependence on the specific survey among the ones the work makes use of. Nevertheless, this effect is of secondary importance compared to the role played by specific probes: the tension is high *as long as cosmic shear data are included*. When only galaxy clustering signals are retained, reconciliation with Planck results is achieved or the tension is relieved to fewer σ thanks to a shift of the best-fit S_8 parameter towards a higher value. Overall, the growth factor still tends to *underestimate* the Planck prediction, but at a lower level.

The role played by cosmic shear as a main driver for the tension on similar datasets was also investigated in a dedicated paper (García-García et al., 2024). This time, data from highly non-linear scales were included and accurate modelling of baryons was taken into account. As a result, S_8 is found to be in statistical agreement with Planck within 1.8σ , although at the price of a 3.5σ tension on Ω_m . The authors highlight that, once all dependency of the data on H_0 is made explicit, such deviation seems to be linked to the value of the Hubble parameter one chooses in order to break degeneracies, the one preferred by Planck and the one recovered from the LSS, recasting the behaviour of S_8 in terms of a manifestation of the H_0 tension, regardless of the σ_8 behaviour and the fact that both S_8 and H_0 share a degeneracy with Ω_m (fig. 1.2).

The previous conclusions are similar to the ones achieved by the other two works we can see as important references for this research, (White et al., 2022) and (Sailer et al., 2025b). They performed a study subdividing the data in redshift intervals comparable with (García-García et al., 2021), combining galaxy clustering samples only (namely, the DESI Luminous Red Galaxies sample) with CMB lensing from Planck and ACT observations. Since both works do not have as much constraining power as the multiple datasets used by (García-García et al., 2021), the baryon abundance is not fitted, but, rather, imposed from Planck through the parameter the CMB

is constraining, $\omega_b \equiv \Omega_b h^2 = 0.02236$ and, for (Sailer et al., 2025b) also by fixing $\omega_m h \equiv \Omega_m h^3 = 0.09633$, a quantity well constrained by BAO measurements on LSS clustering statistics acting as a prior on the acoustic scale calibrated exclusively on low- redshift measurements.

In these works, the statistical discrepancy on S_8 against CMB measurements is below 3σ and, although the growth factor tends to **underestimate** the Planck prediction, this is always within the levels of statistic compatibility. While (White et al., 2022) works exclusively on σ_8 (i.e., the parameter measured at $z=0$) as it can be inferred combining all redshift bins or at different redshift bins, (Sailer et al., 2025b) performs full-fledged tomography, also recovering the full trend on $\sigma(z)$, in analogy with (García-García et al., 2021). In general, measurements of σ_8 or of $\sigma_8(z)$ show an increasing disagreement with Planck with decreasing redshift, staying constantly below the Planck-based predicted trend. A similar result and on a comparable redshift interval is obtained by (Abbott et al., 2023a) that, using instead **cosmic shear** data in a tomographic framework, underestimates the growth factor and the S_8 value at about 1σ statistical significance.

Very-low redshift measurements of S_8 and of the growth factor Concluding this paragraph, I separately address the works performing tomographic growth factor measurements at very low redshifts, namely, having the majority of the data points in bins between $z \sim 0$ and $z \sim 0.5 - 1$. This is the most specific class of works my research also belongs to. In particular, we anticipate from the next chapter 5 that my framework will include:

- Three redshift bins between $z = 0$ and $z = 0.6$.
- Treatment of nonlinearities using a combination of emulator-based non-linear power spectra and a non-linear modelling of galaxy bias.
- Maximum Fourier modes, set at $k_{\max} = 0.42 \text{ Mpc}^{-1}$, associated to scales large enough, however, not to require inclusion of baryonic feedback effects in the fitting templates.
- CMB lensing data in cross-correlation with exclusively **galaxy clustering**-based datasets, namely the 2MPZ photometric sample (Skrutskie et al., 2006) and the samples derived from the combination of the WISE and SuperCosmos (Bilicki et al., 2016) catalogues, aimed at understanding the effect of high non-linearity (both because of the inclusion of small scales and of the low redshift at which measurements are performed) without the complex modelling of cosmic shear signals.

Other measurements of this kind, always including CMB lensing, are available. The most recent one is the work of (Sailer et al., 2025a) that also includes the effects of magnification bias. The result tends to **overestimate** the growth rate compared to the Planck result, but recovers a general agreement with S_8 as measured by Planck, with discrepancies within 1.1σ . Whilst my work aims at including three data points at low redshifts, all previous works concentrating on low-redshift data typically add one single very low- z point to the tomographic growth curve. In particular:

- The work of (Balaguera-Antolínez et al., 2018) uses the 2MASS sample to fit for Ω_m and σ_8 , non-linearity is introduced through HALOFIT and no tomographic study is performed: no significant tension on the σ_8 parameter is found, also due to the rather large size of the uncertainties.
- The work of (Peacock & Bilicki, 2018) is an actual tomographic study, but the whole of the 2MPZ sample is used to introduce a single point at low-redshift combined with higher-redshift

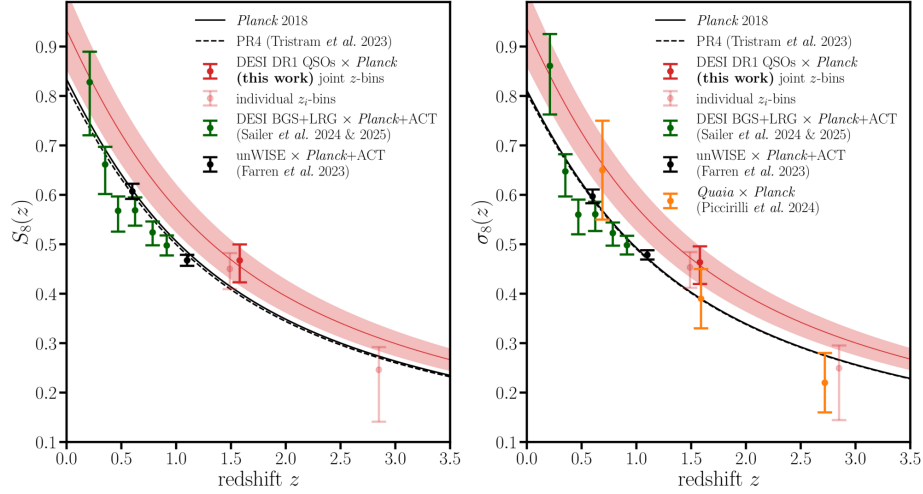


Figure 4.6: Summary of the most recent growth reconstruction measurements, involving galaxy clustering and CMB lensing results. The picture is taken from (de Belsunce et al., 2025)

measurements coming from other surveys. The focus is slightly different from the other works, i.e., it concentrates on the growth function $f(z) = \Omega_m^\gamma$, where γ is a parameter that, in standard General Relativity, should be equal to 0.55 (Linder, 2017, 2005; Huterer, 2023). The 2MPZ-based datapoint tends to overestimate the growth predicted with Planck parameters and the γ best-fit value is found to be 0.77 ± 0.18 , statistically compatible with the General Relativity prediction.

- The work of (Marques & Bernui, 2020) is again a tomographic study, but limited to $z \geq 0.2$, again using 2MPZ data to construct a single redshift bin. The outcome is a slight overestimation of the growth factor compared to the Λ CDM prediction with Planck parameters, but no significant statistical deviation from Planck values is found.
- The work of (Bianchini & Reichardt, 2018) focuses on testing deviation from General Relativity predictions at low redshift: it is based on 2MPZ data and can be understood as an assessment of the growth rate behaviour compared to Λ CDM predictions. The authors define an estimator, called D_G , as a ratio between observed and Λ CDM-predicted power spectra. The setup is conceived in order to eliminate the role of the galaxy bias factors and of their potential mismodelling, that could be confused with deviations from General Relativity. The expected value of D_G , if General Relativity holds, is equal to one, and this is the result authors find within the uncertainties.
- The work of (Rauhut et al., 2025), appeared in July 2025 and, in the same spirit of (Bianchini & Reichardt, 2018), performs a test for General Relativity and its growth prediction, confirming agreement with the Λ CDM model.

Finally, we report the recently published forecast on intermediate-high redshift growth and gravity testing measurements based on Euclid (Ocampo et al., 2025), together with its capability of testing the non-linear scales (Bel et al., 2025); the work on LSST and its capability of testing the E_G statistics is in a similar vein (Leonard et al., 2025). From the observational side, we quote the tomographic studies always using the CMB lensing signal but instead probing the high-redshift regime (Sailer et al., 2021) and complementing the results from (García-García et al., 2021; Sailer et al., 2025b), using quasar samples: we mention among them (Alonso et al., 2023) combining

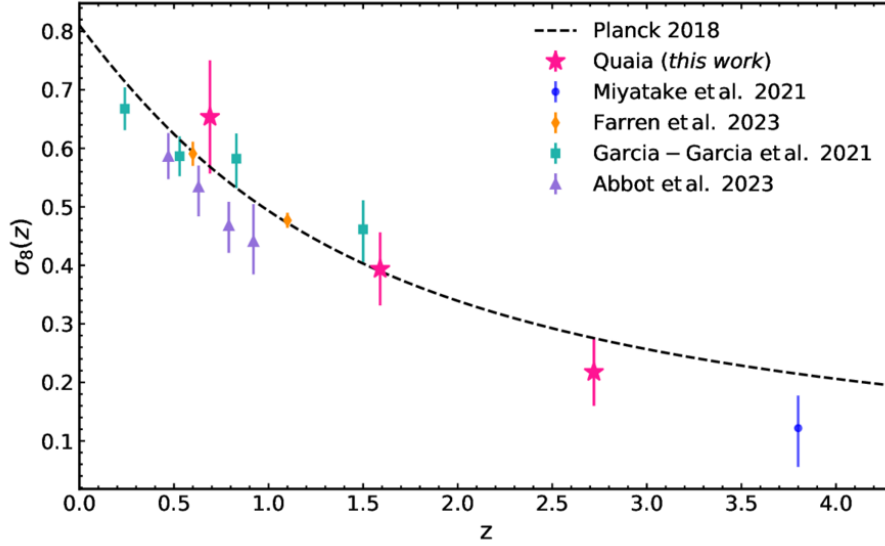


Figure 4.7: Summary of recent growth reconstruction measurements, involving galaxy clustering, quasar clustering, cosmic shear and CMB lensing results, and the combined DES and KiDS autocorrelation datasets (without CMB lensing) noted as “Abbot et al. 2023” (Dark Energy Survey and Kilo-Degree Survey Collaboration et al., 2023). The picture is taken from (Alonso et al., 2023)

exclusively quasar clustering and Planck lensing data (binned and tomographically studied from redshift 0.8 to redshift 3), and (Embil Villagra et al., 2025), performing a similar analysis using also ACT CMB lensing data, both relying on the Quaia quasar clustering catalogue (Storey-Fisher et al., 2024). Such results are partially confirmed by the measurements on the Lyman- α signal, against relevant to probe high-redshifts (Rogers & Poulin, 2025; Palanque-Delabrouille et al., 2020), although these latter works come along with claims of tension and provide different interpretation of the results when different models are explored. On the other hand, the DESI spectroscopic quasar sample (characterised by high-quality redshift evaluation and thus particularly valuable for tomographic studies) has been used combined with the CMB lensing signal in (de Belsunce et al., 2025) in a vein similar to (Alonso et al., 2023): quasars have been binned in the $0.8 \leq z \leq 3.5$ interval and used to measure S_8 and Ω_m first keeping all the other cosmological parameters fixed and, in a subsequent analysis, also constraining H_0 by including information coming from ACT, Planck and the power spectrum turnover scale (but, remarkably, without the use of the BAO and the acoustic horizon information). The paper finds a value for S_8 about 1.5σ larger than Planck ($0.922^{+0.059}_{-0.073}$) combined with $\Omega_m \sim 0.29$ and $\sigma_8 \sim 0.9$. This leads, consistently with (Alonso et al., 2023), to some *overestimation* of the growth function with respect to the Planck-based prediction, differently from the usual *underestimation* obtained by clustering and lensing probes at least until $z \sim 0.5$ (where some overestimation is observed (Sailer et al., 2025a; Marques & Bernui, 2020)) and a value for $H_0 = 69.1^{+2.2}_{-2.6}$. When also H_0 is measured, S_8 is recovered to be around 0.81, in even closer agreement with Planck. Altogether, nuances aside, consistence with Planck cannot be rejected; the work is among the most recent ones in the field of growth reconstruction and the synoptic figure published there and reproduced in fig. 4.6 is also useful as an updated graphical reference for the topic together with fig. 4.7 including KiDS data and hence allowing comparison between galaxy or quasar based results on one hand and weak lensing-based observations on the other hand. Eventually, the two studies of (Harscouet et al., 2025; Verdiani et al., 2025) analysed within the general tomographic galaxy clustering-CMB lensing cross-correlation framework, the

impact and peculiarities of different formulations of the three-point correlation function, the bispectrum, either in isolation (Harscouet et al., 2025), or combined with the power spectrum (Verdiani et al., 2025). Their results, focusing at intermediate and low redshifts (between 0.2 and 1) and involving the DESI Luminous Red Galaxy sample as galaxy clustering signal, tend to recover a value for Ω_m in the order of 0.29-0.31, S_8 at about 2.5σ below the Planck value (in (Verdiani et al., 2025)), quite important underestimation of σ_8 in (Harscouet et al., 2025) (possibly burdened by modelling systematics, the authors remain rather cautious in their conclusions) and general agreement with the Planck constraints (Verdiani et al., 2025); the results can then be mapped in terms of growth reconstruction, with good agreement with the Planck-based prediction in the latter work and, inevitably, important deviations from it in the former. Much attention in the two works is devoted to the novelty represented by the introduction of the bispectrum, which proves capable of reducing the size of the uncertainties of a further 10-20 % and shifting slightly the value of σ_8 downwards and Ω_m upwards in the combined power spectrum-bispectrum configuration of (Verdiani et al., 2025).

Results published during and after November 2025 This paragraph briefly reviews some of the most recent results, that appeared after the first submission of the thesis and are associated to publications that appeared in the winter 2025-2026. For what concerns **weak lensing-based** results, data analyses leveraging on DES (Salcedo et al., 2025; Bigwood et al., 2025), that implement updates on sample selection, stronger baryon feedbacks, intrinsic alignment modelling and additional calibrations, tend to significantly relieve the tension with Planck, which, for S_8 becomes as low as $1.5 - 2.8\sigma$, an overall good agreement for what concerns σ_8 and somewhat larger discrepancies for Ω_m . The same collaboration also extensively discussed the 3x2pt analysis of the two-point angular correlation functions in (Legnani et al., 2026; DES Collaboration et al., 2026; Weaverdyck et al., 2026; Giannini et al., 2026), obtaining $\{\sigma_8, \Omega_m, S_8\} = \{0.751^{+0.034}_{-0.036}, 0.333^{+0.023}_{-0.028}, 0.789^{+0.012}_{-0.012}\}$ in the base analysis configuration. The results are quite similar to those obtained with DES Y3 data (Abbott et al., 2018) (see sec. 4.1.2) with a remarkable reduction of the uncertainties on S_8 by 50% and a quoted discrepancy with Planck between 1σ and 2.8σ depending on the analysis configurations. The works of (Choppin de Janvry et al., 2025a,b) deal instead with the HSC Y3 lensing observations and perform a calibration of the survey’s redshift distribution by cross-correlating the HSC galaxy sample with the one of the DESI spectroscopic survey in narrow redshift bins (a methodology known as “clustering redshifts” technique). The better distance estimation associated to the DESI spectroscopic dataset leads to a strongly reduced tension on S_8 ($S_8 = 0.805 \pm 0.018$), to be compared with the typically high discrepancy recovered in HSC data analyses when compared with Planck. At the same time, the parameters building S_8 , namely σ_8 and Ω_m , are found to be somewhat more distant from Planck results than their combination, although overall within compatibility. Another important observational result is collected in the series of papers of (Turner et al., 2025; Qin et al., 2025; Lai et al., 2025), that instead present the measurement of $f\sigma_8$ and of the growth rate Ω_m^γ (see sec. 1.2.1) derived from the galaxy clustering autocorrelation power spectrum, from the peculiar velocity fields autocorrelation power spectrum and from the clustering-peculiar velocity cross-correlation power spectrum, all based on the data from the DESI collaboration. The three strategies, associated to measurements within $z = 0.1$, are all mutually consistent and eventually yield a growth rate in agreement with Planck and General Relativity predictions (which states $\gamma \approx 0.57$), associated to values for S_8 , σ_8 and Ω_m largely in agreement with Planck and with results derived from galaxy clustering. Later on in

December 2025, a series of works also taking advantage of the high-quality DESI datasets has provided further contributions: the effort reported in (Blanco et al., 2025; Ruggeri et al., 2025) has presented the positive results of cross-calibration studies of the KiDS-1000 photometric redshift sample by means of the galaxies of this latter survey also belonging to the DESI survey (see also the description of the DIR methodology in sec. 5.1.1). The DESI collaboration has also published a number of results concerning the measurement of the cosmological parameters: the work of (Lange et al., 2025), working on the two-point angular correlation functions for the DESI-based clustering auto-correlation data (including an HOD modelling for the smallest scales and k values between about 0.6 and 1 Mpc^{-1}), combined with the weak lensing angular correlation functions from each of the three reference cosmic shear surveys (DES, HSC and KiDS), in a 2x2pt analysis configuration. The work keeps all cosmological parameters but σ_8 and Ω_m to the Planck-based values (similarly to what we discussed in sec. 4.2.1 and to what we shall discuss in chapter 4). When using DES and KiDS data, the paper reports as results $S_8 = 0.794 \pm 0.023$ and $\Omega_m = 0.29 \pm 0.012$; when using HSC data, the results shift to $S_8 = 0.793 \pm 0.017$ and $\Omega_m = 0.303 \pm 0.010$ (no values for σ_8 are provided). The results are mutually consistent as well as in agreement with Planck at about 2σ . Potential “lensing is low” effects (Di Valentino et al., 2025) – i.e. an underestimation of the lensing signal when results are extrapolated to even smaller scales – are also reported. The full 3x2pt analysis in the configuration space is presented in (Semenaitte et al., 2025), again, combining DESI with, at a time, HSC, KiDS and DESI data. Having more constraining power available, the collaboration could thus fit for $\{\Omega_b h^2, \Omega_c h^2, h, A_s \cdot 10^9, n_s\}$, and then derive S_8 , Ω_m and σ_8 as final published results. Data up to about $k \sim 35 \text{ Mpc}^{-1}$ were used and RSD measurements were included among the different analysis configurations. The work extensively reviews the data analysis setup, bias modelling, measurements outcomes and previous literature, eventually finding high mutual consistence between each dataset and substantially recovering the measurements based on weak-lensing only data published by the HSC, KiDS and DES collaborations; this translates, consequently, in a 1.9-2.9 σ difference with the values of S_8 reported by Planck ($S_8 = 0.787 \pm 0.020$ for HSC data, $S_8 = 0.791 \pm 0.016$ for DES data and $S_8 = 0.771 \pm 0.017$ for KiDS data; the other parameters are not systematically quoted with the associated errors). At the same time, general agreement with other measurements of the same quantities involving DESI data are reported. The analogous study of (Porredon et al., 2025), this time not including RSD observations, again performing extensive sensitivity and stress tests, reports self-consistency among weak lensing surveys and other extant DESI-based measurements, at a lower discrepancy with Planck (1.5-2.5 σ , associated to $\{\sigma_8, \Omega_m, S_8\} = \{0.777^{+0.045}_{-0.039}, 0.316^{+0.043}_{-0.032}, 0.760^{+0.064}_{-0.057}\}$ for DESI combined with DES; $\{\sigma_8, \Omega_m, S_8\} = \{0.770^{+0.030}_{-0.035}, 0.336^{+0.043}_{-0.043}, 0.732^{+0.057}_{-0.045}\}$ for DESI combined with KiDS; $\{\sigma_8, \Omega_m, S_8\} = \{0.753^{+0.036}_{-0.034}, 0.357^{+0.038}_{-0.044}, 0.694^{+0.063}_{-0.045}\}$ for DESI combined with HSC). A joint analysis of BOSS clustering data (particularly sensitive to $f\sigma_8$ by means of the peculiar velocities) and of DES+HSC weak lensing data aimed at constraining the cosmological parameters in a 2x2pt (clustering autocorrelation and clustering-lensing cross-correlation) configuration is reported in (Gao et al., 2026): the work reports some sensitivity to systematics in the modelling of the weak lensing signal, and emphasises in particular the modelling of the smallest scales by means of an HOD approach. Several distinct analysis configurations are inspected, all involving real-space correlation functions, covering both pure clustering statistics and clustering and lensing in distinct configurations. All of them, regardless of the surveys and modelling choices, are substantially self-consistent, statistical compatibility being ensured either because of precision or of accuracy reasons, or both. The recovered values of S_8 are found to be at a distance from Planck spanning

from 1σ to 4σ ; the work does not provide a main reference result, rather a synoptic comparison of different choices, reflecting on the need of future denser samples and on the impact of different systematic mitigation strategies as sources of variation. Similarly, the paper does not take a strong position concerning the existence and the nature of the tension, although tends to refer to the recovered values for S_8 as “low” compared to Planck.

Systematics on the Halo Mass Function

We saw that another main piece of theory entering the modelling of the non-linear regime both in terms of number counts observables or non-linear power spectra modelling in terms of halo model, displaying particular sensitivity to S_8 and to possible deviations from standard General Relativity, is the halo mass function (HMF) (Press & Schechter, 1974; Jenkins et al., 2001; Tinker et al., 2008; Cui, 2024), a fundamental prediction in every cosmological theory, since it directly addresses the numerical abundance of structures in the Universe across redshifts and masses. Concerning cluster count analyses, we saw that it is necessary to relate the statistical abundance of Dark Matter halos the HMF predicts to the observable tracer we are using to indirectly measure it, remapping the properties of the observable astrophysical objects (e.g., dark and baryonic dynamic mass, luminosity in some electromagnetic band...) into the underlying Dark Matter halo mass. Typically, such conversions are based on scaling relations between the observable of interest and the mass, either available from the literature as a “universal” astrophysical relations, or by means of dataset-specific conversions based on internal calibration (Truong et al., 2018; Pop et al., 2022). This definition refers to the fact that some of the objects targeted by the survey are accompanied with a mass measurement, usually performed by means of weak lensing techniques. The outcomes of such measurements act as calibrators for the subsequent calculation of masses for all the other objects in the same survey, so that the general astrophysical scaling laws are thus accompanied or superseded by those obtained specifically in the survey (Di Valentino et al., 2025). The σ_8 parameter enters in the HMF with a strong degeneracy with Ω_m : especially at the largest masses; in general, when one of the two quantities increases, the number of formed structures is expected to grow, either because of greater availability of matter or as an effect of larger density fluctuations. The actual measurement of HMF statistics is however particularly difficult, because of the delicate modelling of the above-mentioned proxies; the general expectation is that scaling relations should display a power-law behaviour, corrected with some scatter induced by baryonic effects. An important detail to mention in this case is that the mass estimation based on X-Ray luminosity relies on assuming that the tracer is in hydrostatic equilibrium within the galaxy cluster. This is not always the case, and a part of the total baryon matter may not be in hydrostatic equilibrium; we can account for this by introducing a “hydrostatic bias” b_H (Gianfagna et al., 2023; Muñoz-Echeverría et al., 2024; Kay et al., 2024; Braspenning et al., 2025; Butt et al., 2024) such that the total mass of the cluster (Dark Matter and baryons) can be connected to the X-Ray based estimation by means of

$$M_{\text{DM+baryons}} = \frac{M_{\text{X-Ray}}}{(1 - b_H)}.$$

The value of b_H can be estimated in the local Universe by means of X-Ray surveys or by means of tSZ effect measurements (through Planck, ACT and SPT CMB maps) and is typically in the order of 0.2, meaning that about 80 % of the X-Ray estimated mass of the cluster is in actual hydrostatic equilibrium within the structure’s gravitational potential. However: measurements assuming cosmological parameters from Planck require b_H value up to 0.3-0.4 with a lower degree

of hydrostatic equilibrium in the observed baryonic structures. Redshift evolution and tensions on σ_8 can be thus also considered as at least partially sourced by a disagreement on the value of b_H as measured in the local universe compared with the one predicted by Planck parameters. Heavy model dependence is currently limiting our understanding of this quantity: b_H and related parameters are found to even be sample-dependent (Wicker et al., 2022), although the value of $b_H \sim 0.8$ is consistently retrieved. The work of (Koukoufilippas et al., 2020a,b) finds a nuanced result, in partial disagreement at least with the previous one: combining local LSS tracer and Planck-based tSZ samples, b_H is recovered to be 0.75 ± 0.03 , thus in statistical agreement with Planck-based predictions and not significantly away from the LSS measurements, without hints for redshift evolution. Further calibration of the halo mass function and related scaling relations can be performed by considering independent cluster mass estimates, starting from models validated on hydrodynamical simulations down to additional measurements based on the kinematics of cluster galaxies within their gravitational potential, although weak lensing remains the preferred methodology. Altogether, however, the expected percentage contributions of systematics on S_8 estimations induced by mass calibrations and related quantities are not exceedingly large, in the order of some unit (Di Valentino et al., 2025).

Impact of shear-related systematics

We have repeatedly observed how weak lensing, and especially its cosmic shear component, appears to be the main driver of the S_8 tension, or, at least, the probe whose signal is more often associated to a S_8 value displaying the greatest distance from the CMB values. Overall, cosmic shear as an observable is quite delicate to handle, displaying sensitivity to a number of systematics (Mandelbaum, 2018). Moreover, weak lensing is at the same time utilised as the most important observable to the task of measuring S_8 and as a fundamental tool for properly estimating the mass of Dark Matter halos hosting the sources used in other techniques aimed at measuring this parameter, with systematics potentially leaking into many other quantities of interest.

The very measurement of galaxy shapes, the observable at the heart of weak lensing, and the evaluation of the redshift distribution by means of photometric techniques, can be heavily affected by errors, to which we must add the intrinsic variability of the observed cluster mass profiles, contrasting with the assumption of a spherical or ellipsoidal shape on which most of our models are based. Recent testing on KiDS-1000 of the two main methodologies currently available to determine the elliptic shape of the observed galaxies based on fitting its image against shape templates defining its projected profile, radius and photometric profile (the *lensfit* approach (Miller et al., 2013)) or by means of comparison between the statistical properties of simulated shear surveys in controlled conditions and the real data (the METACALIBRATION approach, (Huff & Mandelbaum, 2017)) have been made available in the literature. Ellipticity measurements and the subsequent statistics are found to be consistent between *lensfit* and METACALIBRATION at the same time confirming the similar level of tension (Yoon et al., 2025). We mentioned already that the last data release of KiDS, KiDS-Legacy (Wright et al., 2025), provided a relieved degree of tension compared with previous datasets such as KiDS-1000, and indeed the work proposes to perform soon a similar shape-fitting test on the final survey dataset as a further decisive test.

As of today, observational systematics represent the main component in the error budget and the object of most of the effort aimed at reducing uncertainties in cosmic shear data (Di Valentino et al., 2025): the determination of the the shape of the galaxy depends on the telescope PSF and on

all the instrumental sources of variability entering the imaging process of the instrument. In order to perform observations at the precision level required by contemporary surveys, such effects need to be modelled and deconvolved from the raw data in a way capable of ensuring that systematics in the measurement of shapes are in the order of per mille or less. This implies attentive deployment of algorithms modelling the telescope PSF and of seeing effects in general, disturbances induced by overlapping galaxies (an effect known as blending) as well as increasing our knowledge on galaxies photometric profiles – determined by their astrophysical properties (Mandelbaum, 2018; Yin et al., 2025).

Besides observational systematics, the whole data analysis and statistics extraction pipeline is burdened by other potential sources of biases: the potential confusion between a galaxy own ellipticity and the deformation induced by weak lensing, known as shape noise, is among the most distinctive issues of cosmic shear. Importantly the lensing-induced deformation effect we want to measure is an actually subdominant correction to the intrinsic elliptic shape of the galaxies (Niemi et al., 2015; Mandelbaum, 2018). Furthermore, intrinsic ellipticity of galaxies is also a function of the structure where they originated, i.e., a function of the galactic environment, and of their evolutionary status, hence changing with redshift along the line of sight. Fortunately, since we are or we will be soon able to collect extremely large samples of galaxies, in the order of $10^7 - 10^8$ objects, statistical power should no longer be a major concern, whereas observational limitations will continue to be an issue to deal with (Ma et al., 2006; Newman et al., 2015; Hartley et al., 2020; Masters et al., 2015). The last systematic specific to cosmic shear measurements is represented by intrinsic alignments (IA): galaxies forming in a shared host halo environment and gravitational field tend to show preferred and coherent alignments, regardless of the lensing effect induced by the foreground LSS (Lamman et al., 2024; Chisari, 2025). Modelling and removal of IA represents one of most important research topics in the broader field of studies on cosmic shear. Studies oriented on IA themselves as a valuable source of cosmological information have been performed (Chisari & Dvorkin, 2013; Shim et al., 2025; Joachimi & Bridle, 2010), but, whether they are removed or fitted simultaneously to the cosmological signal, the requirement is always that of carefully controlling their impact. Multiple improvements to the basic summary statistics estimators have been developed to this aim, developing kernel weighting techniques and linear combination of the window functions of the observables entering the measurements – the latter is a methodology known as nulling, aiming at “zeroing out” sources of disturbance to the signal we want to measure, combined with scale cuts or redshift distance (Bernardeau et al., 2014; Fronenberg et al., 2024; Camera, 2022; Piccirilli et al., 2025; Faggioli et al., 2020; Tanidis & Camera, 2023); a combined higher-order perturbation theory and nulling-like approach is discussed in (DeRose & Chen, 2025) which recovers agreement with Planck while using DES Y3 data, while very recently a comparison of more nulling techniques and their effects in controlling possible biases on the measurement of the S_8 , σ_8 and Ω_m parameters is collected in (Robertson & Hall, 2025).

Next to the sheer necessity of removing IA as a main source of signal disruption, the main issue with IA is that S_8 measurement outcomes were proven to display dependence on the chosen Intrinsic Alignment model, as one can see in fig. 4.8. In particular, the torque effects due to the tidal forces determining the IA effect can be described either at a linear or non-linear level. Linear models systematically yield lower S_8 value, inducing a drift in the tension up to 0.9σ further or closer to the Planck value. Correct management of intrinsic alignment effects can also be performed by considering galaxy colours. All galaxy typologies undergo some form of alignment due to their interaction with the environment’s gravitational potential; however, whilst elliptically-shaped “red”,

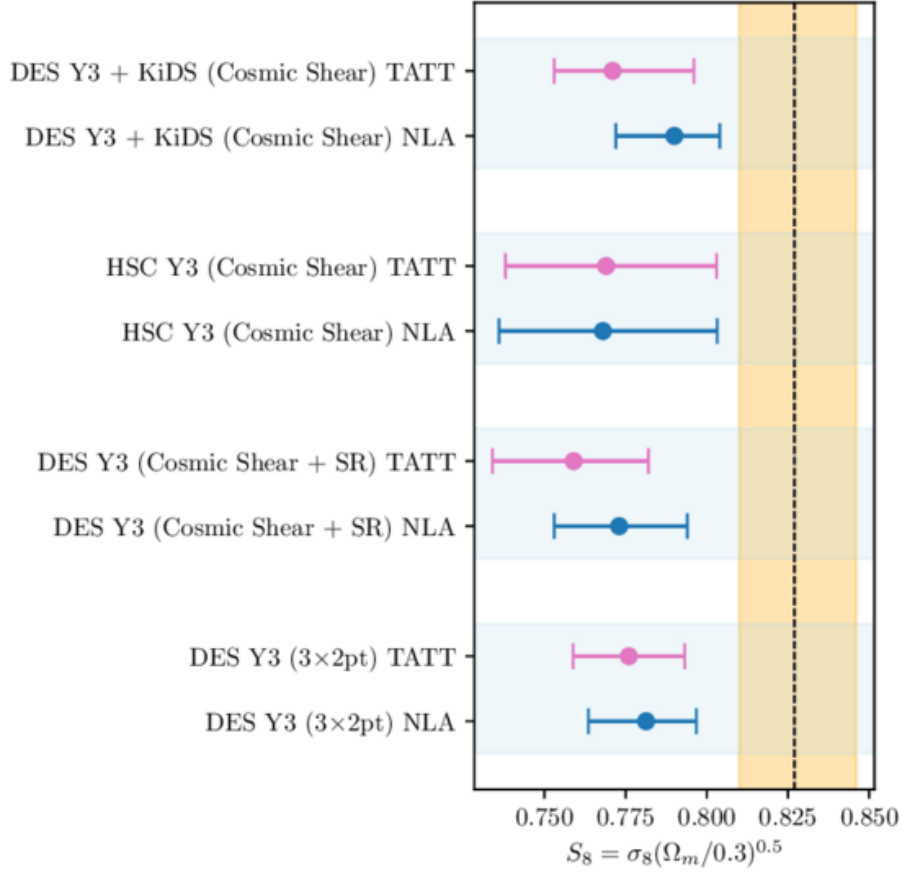


Figure 4.8: Impact of different IA models on the S_8 measurements, from (Di Valentino et al., 2025)

usually more distant, galaxies modify their very shape to adapt to the gravitational potential, making IA effect quite prominent, spiral and rotating “blue” galaxy, more often found at lower redshift, react to the environment in terms of spin direction alignment rather than in shape. This effect is more prominent at very small scales (Mandelbaum et al., 2018; Kiessling et al., 2015; Johnston et al., 2019; Samuroff et al., 2019), but gets erased at the typical cosmological scales currently accessed by instruments, so that the greatest concern at the moment is about shape effects affecting elliptic galaxies. For all practical purposes, therefore, blue galaxies can act as IA-immune galaxy samples for cosmic shear measurements.

Systematics in the data analysis pipeline

Multi-tracer techniques can be leveraged in order to mitigate part of cosmic shear-related systematics by combining them with more robust observables such as galaxy clustering (Abbott et al., 2022), that can compensate for limited knowledge of photometric redshifts through more accurate spectroscopic measurements, and breaking the degeneracy between Ω_m and σ_8 . The ultimate outcome of these analysis strategies is that the tension is typically reduced to about $1-2\sigma$, suggestive that the galaxy clustering signal compensates for those effects we observe in the most deviant measurements based on shear; similar effects can be obtained through CMB cross-correlations. To conclude, an increasing attention has been also dedicated to the the proper modelling of the likelihoods, going beyond the simplest assumption of data being Gaussianly distributed and on more accurate calculation of covariances: although not as impactful as IA, these studies are also relevant in term

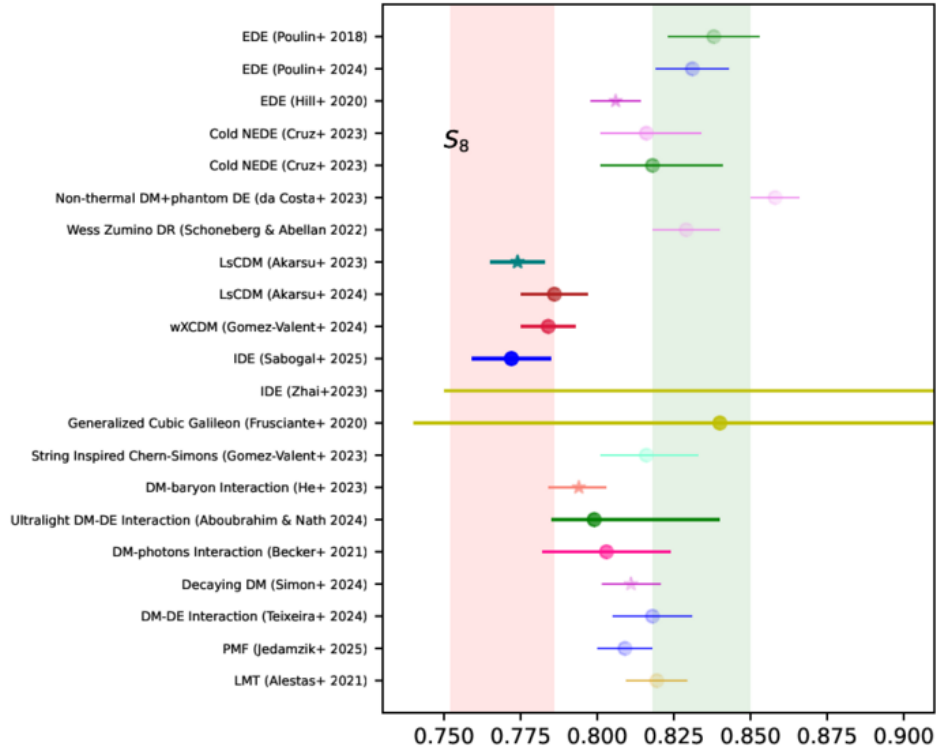


Figure 4.9: Proposed solutions to the S_8 tension based on the introduction of New Physics intervening in the cosmic history and modifying the base Λ CDM picture, from (Di Valentino et al., 2025)

of good practices for data analysis, leading to better treatment of non-Gaussianity in data and covariances (Gatti et al., 2024; Reischke et al., 2025; Sellentin & Heavens, 2016; Sellentin et al., 2018).

4.2.2 Proposed solutions extending the Λ CDM framework: the S_8 tension as a hint of new physics

In this section we will briefly review the other end of the spectrum of proposed solutions to the S_8 tension, those based on the assumption that the observed tension between Early-Time and Late-Time S_8 measurements is a hint of a deeper limitation of the Λ CDM model in correctly describing our Universe and that new physical fields and mechanisms must be included (see fig. 4.9 from (Di Valentino et al., 2025) for a summary of the main proposed theoretical frameworks). To set the stage, we recall that the value of S_8 measured on the CMB (according to (Aghanim et al., 2020) it corresponds to 0.834 ± 0.016) is *larger* than the one retrieved in the local Universe (where the measured values can be as low as about 0.776, determining tension levels typically comprised between 1σ and 3.8σ). We also remind that the CMB power spectrum does not allow to directly measure neither σ_8 nor S_8 , nor Ω_m .

Concerning Ω_m , the CMB actually measures the product between this quantity and the dimensionless Hubble parameter h , a quantity defined as $\omega_m \equiv \Omega_m h^2$. The CMB has also sensitivity

(recall also table 1.1) to the angular acoustic scale θ_s , that can be related to the quantity $\Omega_m h^3$ (note the cubic exponent, not quadratic!) (Percival et al., 2002). Eventually, it is also sensitive to the baryon content in the Universe through the similarly defined parameter $\omega_b \equiv \Omega_b h^3$. ω_b can be externally constrained from the BBN (Schöneberg, 2024) independently from the CMB through measurements based on the abundance of the light elements, while from the LSS statistics we can impose sensitivity on $\Omega_m h$ and on $\Omega_m h^3$ through BAO measurements. Degeneracies can thus be broken in full, and the values of Ω_b , Ω_m , and h be singled out.

Concerning σ_8 , the CMB does not strongly probe it directly, whereas the CMB lensing signal does. The primary CMB signal actually provides the set of best-fit parameters that, once inserted in the calculation of eq. 1.34, eventually allow for an estimate for such quantity. For LSS-based measurements, we generally fit the summary statistics as a function of the cosmic inventory parameters Ω_i and of h (either individually, or degenerate among each other, different probes allowing to break degeneracies) and eventually derive σ_8 from eq. 1.34. LSS statistics has as an additional advantage the possibility of directly accessing the value of σ_8 for example through the measurement of RSD, that are a function of the product $f(z)\sigma_8$, where $f(z)$ is the logarithmic growth function defined in eq. 1.39, that can be further recovered also by means of astrophysical and cosmographic distance indicators sensitive to the peculiar velocities of galaxies (Stiskalek, 2025; D’Agostino & Nunes, 2023). In this context, time- and scale-dependence have also been pointed out as delicate properties to be considered (Madhavacheril, 2025), since the precise definition of σ_8 (eq. 1.34 in chapter 1.2.1) implies the introduction of a filtering scale at $z = 0$. Measurements at different redshifts might determine extrapolation issues or miss decisive and unaccounted for physical phenomena, because, for example, the employed observables do not probe the scales at which these phenomena are active and that yet affect the filtering scale $R = 8 \text{ Mpc } h^{-1}$ at present redshift. Vice versa, suitable scale-dependent and redshift dependent tracers might allow us to test for the presence of such effects and propagate the prediction up to $z = 0$, at which σ_8 is computed, possibly solving the tension.

To conclude this introductory part, another important aspect to keep in mind is that we do not dispose of a prediction for σ_8 based on more fundamental assumptions: we only dispose of its “operative” definition and we can only require the consistency between independent measurements of its value, without possessing a “ground truth” reference value. The general research programme shares many aspects with the efforts aimed at solving the H_0 tension, considering the inconsistencies in the measurements of both cosmological parameters and solutions thereof as test beds for fundamental physics, of both phenomenological and microphysical nature, and the mutual interactions of Dark Matter and Dark Energy.

As for the tension on H_0 , usually assuming a fixed null curvature of the Universe and the Harrison-Zel’dovich primordial power spectrum with $n_s = 0.96$ (Poulin et al., 2024), two main distinctions can be drawn:

- **Early-Time solutions.** Models belonging to this class assume in this case that the “correct” value for σ_8 is the lower one, as measured in the local Universe. According to this approach, we have to modify the description of the Early Universe and the CMB theoretical model in such a way that the CMB angular power spectrum template is still able to fit the data but with a reduced σ_8 . This requirement translates in particular into damping the small angular scale (large- ℓ) end of the CMB angular power spectrum: a delicate task, since the increased slope of the high- ℓ trend must still be able to intercept the observational data within small

uncertainties. Such an excess damping would translate in a smaller σ_8 value, close enough to the one measured on the LSS to solve the tension.

- **Late-Time solutions.** If, conversely, we assume that the “correct” value for σ_8 is the one measured in the Early Universe, we have to retrieve a larger value of σ_8 from the fit of LSS statistics data. In other words, we assume that the low σ_8 value we measure on some LSS tracers is actually a “compensation” effect for an actually more dampened shape of the power spectrum, so that a more correct model would accommodate a σ_8 value consistent with the CMB value while showing a more suppressed trend. This class of models leaves somewhat larger room for corrections, considering the larger uncertainties in the small-scale end of the power spectrum as measured on the LSS.
- Solutions mixing Early and Late Time modifications of the standard Λ CDM picture also exist. What is more, and as we will outline in the next section, there are consequences of pure Early Time effects on Late Time effects and vice versa that must be accounted for and that act as predictions or constraints on the viability of the models. A summary of the effects of the proposed solutions on the retrieved best-fit S_8 parameter and the degree of reconciliation they ensure on either the Planck-based or LSS-based value is displayed in fig. 4.9.

The connection between S_8 and H_0 in the parameter space

The focus of my research work and of this chapter is always on S_8 , hence the current research on H_0 is treated and will be presented as instrumental to the main topic (Knox & Millea, 2020; Di Valentino et al., 2021a; Schöneberg et al., 2022; Di Valentino et al., 2025; Abdalla et al., 2022). At the same time, the connection between these two parameters in the parameter space has been pointed out not to be negligible as well; it is thus interesting to discuss a bit also the H_0 tension as orientative to understanding the modelling effort (Secco et al., 2023; Poulin et al., 2024; Pedrotti et al., 2025).

At first sight, measurements based on clustering or cosmic shear statistics aimed at constraining S_8 do not show constraining power on h if they are not informed by additional datasets such as BAO, BBN or CMB. Only in presence of vast and diverse simultaneously fitted datasets is it possible to almost fully constrain the Λ CDM model with Late Time probes (García-García et al., 2021). In presence of insufficiently constraining data, the value of h or of related parameters must be set to a fiducial value, usually corresponding to the constraints imposed by Planck or by other LSS measurements, either in the form of the exact parameter value or by means of the degenerated combinations measured by the chosen probes (García-García et al., 2024; White et al., 2022; Sailer et al., 2025b).

Nevertheless, S_8 and h “talk to each other” by means of their mutual connection with Ω_m , with a typical dependence (Pedrotti et al., 2025) scaling as

$$\frac{\delta S_8}{S_8} \approx 2 \frac{\delta h}{h}. \quad (4.6)$$

Modifications of h aimed at either increasing or reducing its value – depending on whether we want it to attain the CMB value as the fiducial one or the one calibrated in the local Universe – might determine rather large changes on the uncertainties and/or of the value of S_8 , leading to a number of possibilities, from a less concerning loosening of the constraints to a potential full modification of the fiducial outcomes of the analyses. Furthermore, from eq. 4.6 we see that S_8

and h are directly proportional, but we know that, while the H_0 tension is between a **low Early Universe** H_0 value and a **high Late Universe** H_0 value, the S_8 tension goes in the opposite direction: we find a **high Early Universe** S_8 value and a **low Late Universe** S_8 value. Fixing one tension without acting correctly on the functional dependencies of the other parameter could thus increase the other tension (without considering possible effects increasing the uncertainties that might still ensure an adequate degree of statistical compatibility).

The effect of h has an impact on the very definition of σ_8 and of S_8 (Sánchez, 2020; Forconi et al., 2025): the filtering radius R entering in the definition of σ_8 provided by eq. 1.34 is indeed a function of the Hubble constant, $R = 8 \text{ Mpc } h^{-1}$. Given the extant tension on H_0 , it has been remarked that the definition of σ_8 could be ambiguous (Sánchez, 2020); furthermore, when in an analysis pipeline it is required to infer a value for h or to convert the scale from $\text{Mpc } h^{-1}$ to Mpc (an operation known sometimes in the literature as passing from “dimensionless” to “dimension-full” quantities), a choice must be effectively done. The choice makes determines any subsequent analysis and associated outcome conditional on the Hubble tension (Secco et al., 2023; Poulin et al., 2024; Pedrotti et al., 2025). It can thus be useful to compare the results achieved in the parameter space $S_8 - \Omega_m$, as customary in the analyses, with the ones recovered in the alternative parameter space defined by the variables

$$S_{12} \equiv \sigma_{12} \left(\frac{\omega_m}{0.14} \right)^{0.4}, \quad (4.7)$$

with ω_m defined as above. If we assume that the densities of neutrinos and baryons are either negligible or well-measured independently, we can concentrate on the sole cold Dark Matter density $\Omega_c = \Omega_m - \Omega_b - \Omega_\nu$, define $\omega_c = \Omega_c h^2$ and introduce this further sensitivity

$$\frac{\delta \omega_c}{\omega_c} \approx 2.38 \frac{\delta h}{h}, \quad (4.8)$$

and substantially reason in terms of the connection between S_{12} and the Dark Matter density Ω_c . This definition derives from having selected σ_{12} as a reference power spectrum normalisation obtained from the measurement of the variance filtered at $R = 12 \text{ Mpc}$ (**without** h -rescaling). This quantity is selected because, if we set h to 0.67, roughly corresponding to using the Planck-based cosmology to calculate distances and lengths, the value of σ_8 would then substantially correspond to σ_{12} and be discrepant otherwise. This helps with the effort of understanding whether or not, and to what extent, the tension on S_8 can be also phrased in terms of tension on H_0 and how much of it is instead “intrinsic” to the measurement of the clustering variance in the Universe. We finally recall that, **in the Late Universe**, Ω_m is rather well constrained by many independent probes, spanning from summary statistics measured on different tracers to direct observations based on supernovae or clusters and baryon observables, that also help with independently calibrate Ω_b (Riess et al., 2022; Ettori et al., 2003; Mantz et al., 2014, 2022). These measurements set the density parameter to a value comprised between 0.28 and 0.35, with good agreement among the datasets within the error bars: modifications on S_8 , S_{12} and h should therefore leave Ω_m untouched or, at least, ensure the same degree of cross-dataset consistency across datasets. The works (Secco et al., 2023; García-García et al., 2024) elaborate on this by performing a number of interesting tests: keeping fixed the pair $[S_{12}, \omega_m]_{Pl}$ derived from Planck observations and having set h to the Planck reference value, one then calculates the pair $[S_{12}, \omega_m]_{LU}$ measured in the Late Universe a first time using h_{Pl} from Planck and a second time setting h_{LU} at a reference value derived from Local Universe observations. It is then found that **the tension on S_{12} is relieved with h set to the Late Universe values, and increased when h is set to the Planck values** : the impact is

of few decimals in σ for HSC observations, and up to 1.1-1.2 σ for DES and KiDS observations **without worsening the fits on lensing data** (Secco et al., 2023). Regardless of the effects of modelling systematics at the smallest scales when working with cosmic shear, a fraction of the S_8 tension should actually be attributed to the underlying H_0 tension, influencing the final result either in terms of widening of the uncertainties or shifting of the best-fit values, due to conversions acting in a way not immediately noticeable in the $S_8 - \Omega_m$ plane. On the other way round, fitting h by means Late Universe probes also sensitive to $S_8 - \Omega_m$ or to $S_{12} - \omega_m$ would determine a potential tension with h as measured by Planck.

This consideration leads us to the another research direction focused on solving the H_0 and involving – intentionally or not – the tension on S_8 . The works of (Poulin et al., 2024; Pedrotti et al., 2025) start from the consideration that, given the general agreement on Ω_m displayed by many independent probes in the Late Universe, model building should try not to modify the outcome of this measurement (the authors rather suggest the community to perform further measurements and devise additional methodologies to make the estimate of Ω_m even more solid from multiple independent probes), and work instead on the parameter $\omega_m = \Omega_m h^2$, which is measured by the CMB, and enlarge its value acting exclusively on h . Models belonging to the Early-Time solutions class are thus split in two subsets: both subsets assume as “true” H_0 the one measured in the local Universe and thus require a modification of the pre-CMB Universe evolution capable of yielding a larger H_0 even from the CMB measurements. The first class is represented by those models acting on the Early Universe *growth* history, addressing a modified redshift evolution between the Big Bang and the formation of the CMB, are capable of producing a CMB spectrum capable of fitting a higher H_0 and determining a larger ω_m **without requiring further modifications in the Late Universe**, and, in particular, leaving Ω_m untouched. Such models, however, usually require a larger value of σ_8 , and thus of S_8 as measured from the CMB, further increasing the tension with the low S_8 value recovered from the LSS. Therefore, models “trusting” the Late Universe value of H_0 and trying to keep the remaining Late Universe phenomenology as much untouched as possible might exacerbate the S_8 tension.

The second class of models acts *on the recombination redshift* z_{rec} only, as a point in time event, aiming at increasing its value and thus setting the CMB further away, a procedure equally granting a larger value for H_0 . These models typically tend to automatically decrease S_8 as measured on the CMB, although not enough to completely solve the tension, but are found for the most part unable to accommodate a reconciliation on H_0 without predicting a value of Ω_m inconsistent with the local Universe measurement. To solve this issue, **additional new physics** in the Late Universe should be invoked to recover the desired Ω_m value. For the same reason, though, the introduction of novel physical processes acting on the structure growth history, or modifying gravity beyond General Relativity, would be a helpful device to solve the S_8 tension without compromising the solution to the H_0 these models find.

The whole discussion also calls for attentive cross-checks with independent cosmological constraints, such as the age of the Universe or the baryon abundance parameter ω_b and can be a relevant point of view even if we consider the S_8 **not** to be a fundamental problem, but just a consequence of systematics or mis-modelling within the Λ CDM model. Even deeming the tension on H_0 as the only actual inconsistency in Cosmology measurements of the fundamental parameters, we must nevertheless be aware of the consequences that solving the Hubble constant puzzle might have on the treatment of quantities associated to the growth of the LSS and to the characterisation of the small scales.

Outline of the main proposed solutions introducing new physics beyond the Λ CDM model

Early-time solutions Early-time solutions (Abdalla et al., 2022; Di Valentino et al., 2025) typically try to increase the power spectrum suppression by introducing mechanisms in the earliest phase of the Universe, as early as the inflation epoch, capable of producing particles that act in a way similar to Dark Matter particles of the axion class, with a $\rho \sim a^{-3}$ density scaling and a phenomenology ensuring the dissipation of power at the smallest scales required to obtain a more damped CMB power spectrum with respect to the Λ CDM calculation and thus recovering a lower value of S_8 , compatible with LSS measurements. At the same time, these solutions ensure that the associated field energy density decays soon enough not to be detectable in the present Universe (Rogers et al., 2023; Rogers & Poulin, 2025). Another popular class of solutions belongs to the Early Dark Energy (EDE) paradigm (Gómez-Valent et al., 2022; Reeves et al., 2023; Poulin et al., 2019b, 2018; Karwal & Kamionkowski, 2016): they stem from conceptual frameworks aimed at solving the H_0 tension, consisting in a number of mechanisms which have been postulated to occur in the Universe between the epoch of radiation-to-matter domination transition ($z \sim 3400$) and the CMB formation redshift ($z \sim 1100$). The general idea is that of constructing a field behaving like Dark Energy (whence its name) or as an inflaton potential. Differently from Dark Energy (or, more precisely, its specific formulation as a Cosmological Constant), whose domination seems to have started around $z \sim 2$ and is currently ongoing, the existence of the EDE field is extremely localised and limited in time, imposing a sudden acceleration in the expansion of the Universe and reducing the scale of the sound horizon r_s , that, when measured in absolute metric units, corresponds to an increase of H_0 . After having exerted its effect on the cosmic history, the field is conceived in order to ensure it decayed away very quickly, long before the CMB formation epoch, without influencing the Late Universe structure evolution. As we said, basic models of this sort could even be detrimental to S_8 , since they change the primordial perturbation spectral index so that smaller scales are further enhanced, translating into an even higher clustering variance, whereas we would like to reduce it in order to match to the Late Universe measurements. Nevertheless, more complex models based on this general idea are able to handle the H_0 and S_8 tensions together.

CMB data often show preference for Early Dark Energy scenarios against standard Λ CDM descriptions; yet, measurements of perturbation growth at different redshifts put stringent upper limits on the amplitude of an Early Dark Energy field, reducing evidence for its existence once LSS data and, particularly, weak lensing statistics, are included in the analysis (Abdalla et al., 2022; Di Valentino et al., 2025). Other Early Time solutions challenge standard physics requiring that fundamental constants such as the mass and charge of the electron evolve in time, or by imposing non-standard CMB temperature-redshift relations. The existence of primordial magnetic fields influencing baryon clustering has also been postulated: they would partially dissipate the growth of structure at the smallest scales, thus reducing the power spectrum variance in that regime. Such a solution might have far-reaching predictions, implying, among the other features, inhomogeneous recombination mechanisms at the CMB formation epoch, a hard-to-detect signature given the current state of things. At the same time, they are required to modify the Λ CDM standard picture both in the pre- and post-CMB formation epochs (Abdalla et al., 2022; Di Valentino et al., 2025; Secco et al., 2023; Poulin et al., 2024; Pedrotti et al., 2025).

Late-time solutions The array of solutions to the S_8 tension addressing the growth of structures in the local Universe starts from the assumption that the “correct” value for S_8 is the one measured at the CMB, hence a higher one, and that we have thus to modify the power spectrum at low and intermediate redshifts in such a way that its normalisation can be reconciled with the Early Time measurement. A delicate requirement for these models is that they need to match observations of the structure growth history in a large redshift interval. Since a good part of this temporal extension can be accessed tomographically by observations, strong constraints on the timing for the onset and dissipation of new physics effects can be put, combined with the request that the tension, as observed from the data seems to grow with decreasing redshift, i.e. the further away from the CMB we go in time (García-García et al., 2021). Unless these models are also able to solve the H_0 tension (Vagnozzi, 2023), they are generally required not to modify the background expansion, since large-scale perturbations observed at the CMB must be kept preserved when propagated to the Late Universe, as standard linear theory, with its strongly predictive power of the observed data, does (Linder, 2017). Furthermore, effects involving both smaller and larger scales should happen at $z < 1$, because at larger redshift data can be anchored to the CMB lensing signal, whose amplitude and constraining power strength peaks between $z \sim 2$ and $z \sim 3$ and we know from available measurements that they agree with the Early Universe probed by “primary” CMB signal (Planck Collaboration et al., 2020c).

Suggested mechanisms (Schmidt, 2016) span from modified baryon-Dark Matter interaction mechanisms, for instance relaxing the claim of Dark Matter to be strictly non-interacting through channels other than gravitation, a requirement obtained by imposing that CDM has a longer streaming length, compatible with a particle that has (or had for a longer time than the standard CDM case) relativistic properties, adding a behaviour which can be interpreted in terms of “warm” Dark Matter.

Several classes of models assume a more complex phenomenology for the cosmic fluids: for instance, by assuming some form of viscosity, or interactions between Dark Energy and Dark Matter, sometimes even unifying both “Dark” components in a single substance. As such, Dark Energy can be endowed with clustering properties, in a way similar to standard matter fluids in the Universe: an additional matter field in the universe would conveniently add suppression to the power spectrum but, at the same time, this would require that the Cosmological Constant properties of the Dark Energy model utilised in the standard Λ CDM description, with a constant negative energy density associated to a barotropic index $w = -1$, must be relaxed, allowing to its density to evolve with time. These scenarios are associated to the class of models known as Dynamical Dark Energy (DDE) scenarios (Abdalla et al., 2022; Di Valentino et al., 2025).

Alternatively, Interacting Dark Energy (IDE) (Murgia et al., 2016; McDonough et al., 2022; Wang et al., 2024; Bolotin et al., 2015; Barros et al., 2023, 2019) classes of models have been explored. They can be generically understood either in terms of models for which the energy flows from Dark Energy to Dark Matter (requiring Dark Energy to have $w < -1$) and models for which energy flows from Dark Matter to Dark Energy (requiring Dark Energy to have $w > -1$). Such theoretical frameworks, either phenomenological or derived from fundamental physics, can alleviate the tension because the CMB power spectrum gets shifted and the normalisation adjusted consequently without changing its shape, thus maintaining the desired consistency with the CMB data. At the same time, these models are to some extent also able to relieve the much larger H_0 tension.

Another relevant class of Late-Time solutions are those directly addressing Einstein’s General Relativity, extending his theory and thus belonging to the class of Modified Gravity models. They are important for Cosmology even at a more fundamental level, since they are aimed at understanding the wider array of open problems gravitational dynamics poses at cosmic scales. Modifications can be achieved either by endowing the Einstein-Hilbert action dependence on curvature through higher-order powers or non-linear functions of the Ricci Scalar R – while General Relativity can be recovered by requiring a dependence for the Lagrangian density on the Ricci Scalar only – or by adding further properties to the manifold such as torsion. These approaches are however closely following the General Relativity paradigm and are indeed known as “extended Modified Gravity” approaches. Otherwise, it is also possible to challenge the foundations themselves of General Relativity, for instance going beyond the equivalence principle. These are actual “alternative” theories of gravity (Saridakis et al., 2021; Bajardi & Capozziello, 2023; Di Valentino et al., 2016; Planck Collaboration et al., 2016b; Joudaki et al., 2017). Modified Gravity theories address the nature of the gravitational interaction in a number of ways, from the introduction of screening mechanisms (Cai et al., 2021; Khoury & Weltman, 2004; Karwal et al., 2022) modifying the Newtonian inverse-square trend of the gravitational attraction, transitioning to a weaker, exponentially suppressed strength when distance scales reach galactic or cosmic sizes, to predicting the evolution of the gravitational constant G (Sola et al., 2020; Xue, 2015). Particularly relevant to the topic of the tension on S_8 is the fact that we can connect the growth of structure (Wen et al., 2022) to tests potential deviations of gravity from the description provided by General Relativity by writing the growth function $f(a)$ as $f(a) = \Omega_m^\gamma(a)$ (Wang & Steinhardt, 1998; Linder, 2005; Linder & Cahn, 2007; Wen et al., 2022). In this parametrization, the γ exponent can be predicted from first principles: in particular, General Relativity requires $\gamma = 0.55$. The growth function can be directly fit against data from RSD measurements, sensitive to $f(a)$, and translated into a measurement of γ ; in this case, the combined fit of Planck, galaxy clustering and DES lensing data performed by (Nguyen et al., 2023) yields $\gamma > 0.55$ at about 3.7σ significance, while allowing for a S_8 value simultaneously in agreement with both Planck and LSS measurements, which can indeed be interpreted as a deviation from General Relativity. We recall nevertheless on the other hand the results on DESI peculiar velocity and galaxy clustering statistics (both in autocorrelation and cross-correlation) that appeared in November 2025 (Turner et al., 2025; Qin et al., 2025; Lai et al., 2025) that instead find substantial agreement with a Planck cosmology and with General Relativity.

Chapter 5

Low-redshift constraints on structure growth from CMB lensing tomography

The results collected in this chapter are the exposition of my research activity on the topic of low-redshift measurements of the S_8 and Ω_m parameters by means of the combined study of galaxy clustering and CMB lensing cross-correlations as published in (Rubiola et al., 2025).

We recall from the previous chapter that this approach, known as “CMB lensing tomography” (Madhavacheril, 2025), has been used to study the growth of cosmic perturbations in a wide redshift range: using quasars and other high-redshift samples, the history of structure growth could be reconstructed as early as $z \gtrsim 2$ (Miyatake et al., 2022; Piccirilli et al., 2024; de Belsunce et al., 2025; Farren et al., 2025; Embil Villagra et al., 2025), while, when combined with high-density optical and infrared samples, allowed to measure percent-level constraints in the redshifts range ($0.5 \lesssim z \lesssim 1.5$) (Krolewski et al., 2021; García-García et al., 2021; Farren et al., 2024; Pellejero Ibáñez et al., 2024). In general, for high-density enough galaxy clustering samples, this approach allows to probe relatively small scales, as low as $k \lesssim 0.7 h \text{ Mpc}^{-1}$ at the smallest redshifts, without depending on the complex and more systematic-prone information provided by galaxy weak lensing observations, suffering from the many systematics outlined in sec. 4.2.1 and known to be the main observable leading to potential important tensions on the measurements of S_8 and the consequent predictions of the LSS growth.

At the same time, below $z \simeq 0.3$, tomographic measurements are still rather scarce: this is partially caused by quite low CMB lensing signal, as we can see in fig. 5.3, and to increasing complexity in the adequate modelling the clustering of galaxies at the small, non-linear scales (see sec. 1.3), whose effects gets mixed and projected onto large angular scales when considering angular power spectra in this redshift range (Bianchini & Reichardt, 2018; Peacock & Bilicki, 2018; Sailer et al., 2025a). Targetting these redshifts is therefore relevant in order to achieve a complete view on the topic of the “ S_8 tension” across all cosmic epochs, complementing extant work, and to address potential issues specific of low-redshifts measurements. This effort, we mention in passing, might also be instrumental to other open problems in low-redshift cosmology, not discussed in this thesis, pertaining to the search for potential signatures from non-standard dark energy models, departing from the Cosmological Constant model (a brief bridging between the two topics can be found in sec. 4.2.2) as suggest by most recent BAO and supernova measurements (Abdul Karim et al., 2025; Gu et al., 2025).

Armed with these considerations and by utilising a suitable HEFT-based non-linear galaxy bias model following the theory described in section 1.3.2, we will complement extant constraints having as a main goal the improvement of the resolution of extant measurements and to push the body of results towards even lower z : we will see that the use of the 2MPZ catalogue effectively permits to add a data point at redshift as low as 0.074. Given the delicate role of systematics of various nature in the discussed topic, we will be carefully modelling both the data and their covariances, adding corrections to the most standard hypotheses of Gaussianity of the fields (the more standard setup exposed in chapter 1 and supporting the forecasts of chapter 3). As a further validation effort, robustness of current constraints to the sample of galaxies used, as well as the discussion of other analysis choices, has been a major subject of investigation: a good part of the work has thus been dedicated to checks and comparisons with the results on similar data achieved by, among the others, (Balaguera-Antolínez et al., 2018; Bianchini & Reichardt, 2018; Peacock & Bilicki, 2018; Sailer et al., 2025a,b; García-García et al., 2021). In particular, we will directly show comparisons with the measurements published in (Sailer et al., 2025a): employing low-redshift photometric data from the DESI Legacy Survey, combining CMB lensing and including galaxy magnification. This latter dataset will be our reference in the final part of the work, where we will be able to present our growth reconstruction predictions in terms of $\sigma_8(z)$. This quantity can be recovered after having measured the reference Λ CDM parameter σ_8 , as measured at redshift 0, substantially by calculating the normalisation of the power spectrum at the desired redshift always according to eq. 1.34, and can be used as a proxy for the measurement of the perturbation growth factor $D(z)$, regulating the time evolution of the LSS according to the theory exposed in chapter 1.2.1 and in particular in eq. 1.33.

The discussion of our results will be articulated as follows. Section 5.1 discussed the galaxy and CMB lensing data we used. Theoretical models, with the methodologies used to analyse the data and to measure the cosmological constraints, are described in section 5.2. The results of our analysis, presenting Λ CDM constraints, growth factor measurement and the investigation of the properties of galaxy bias are presented in section 3.4, leading to the concluding remarks collected in section 5.4.

5.1 Data

5.1.1 2MPZ and WIXSC

We start with presenting the galaxy clustering datasets we will be employing and the preliminary reduction steps performed in order to extract from them the measured power spectra: figure 5.1 shows the redshift distribution of our data (also shown in fig. 5.3 to allow a comparison with the CMB lensing kernel) allowing the reader to immediately get a sense of the investigated redshifts.

The first and lowest redshift dataset we use is the 2MASS photometric redshift survey (2MPZ (Bilicki et al., 2014)) that combines optical and infrared photometry observations recovered from the observational programmes of the 2-Micron All-Sky Survey (2MASS (Jarrett et al., 2000)), the Wide-field Infrared Survey Explorer (WISE (Wright et al., 2010)), and the SuperCOSMOS survey (Hambly et al., 2001). The resulting magnitude-limited photometric sample (having magnitudes limited to $K_s \leq 13.9$) cover the celestial sphere in full, resorting to highly accurate photometric redshifts (the typical redshift uncertainty being in the order of $\sigma_z \simeq 0.015$). Concerning sample richness, we will use substantially the full 2MPZ sample, using the galaxies whose photometric

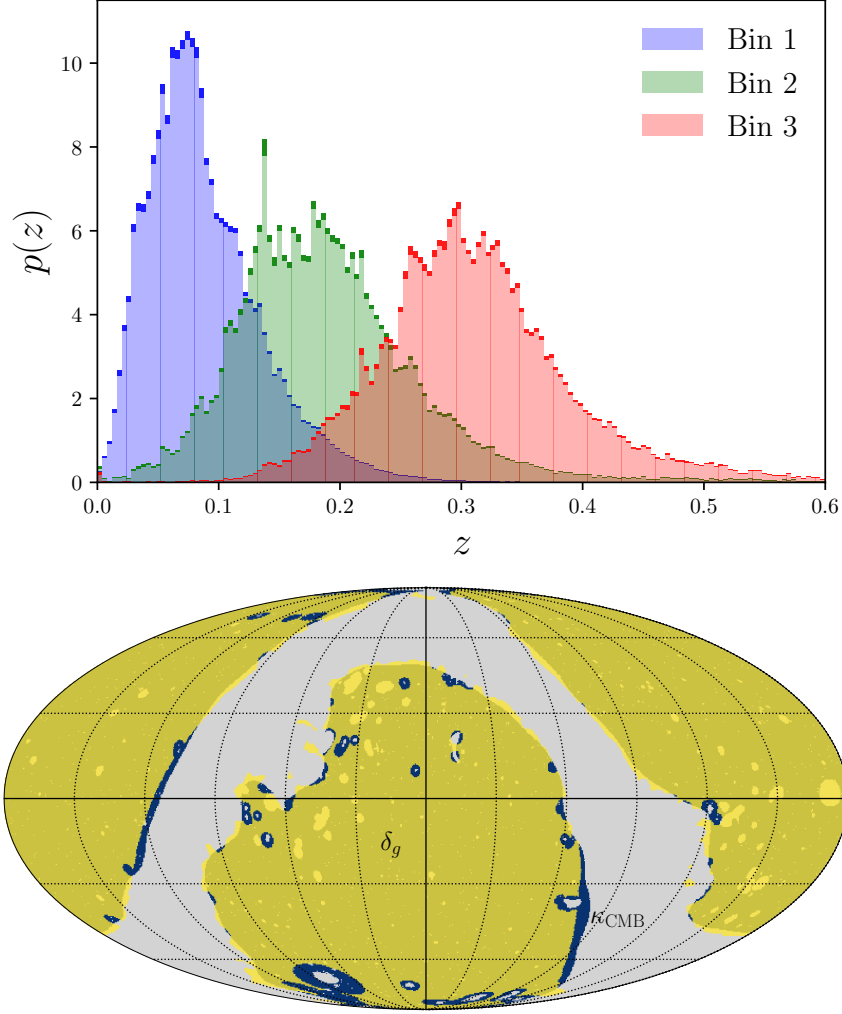


Figure 5.1: *Top*: we show the redshift distributions for the three galaxy samples we have employed in our analysis. The darker shade allows to appreciate the uncertainties obtained after the bootstrapping procedure *Bottom*: the footprint for the galaxy tracers (yellow) upon application of the masks, together with the CMB lensing convergence map, displayed in dark blue.

redshifts are limited by $z_p < 0.5$, corresponding to our lowest-redshift tomographic sample. In total, we can thus comprise a total of 718,020 galaxies within our analysis mask (more details below), that will be collected in a single tomographic setup with a median redshift $z_m = 0.074$. Working in a tomographic redshift setup for angular power spectra, the correct measurement of the redshifts associated to our objects is extremely important in order to properly collocate them in our bins, all the more so given the known limitations of photometric redshift determination (Ma et al., 2006; Newman et al., 2015; Salvato et al., 2019). We take care of this issue for all the employed galaxy samples, succeeding with an improved estimation of their redshift as we shall outline below. In particular, we estimate the redshift distribution of the sample using the direct calibration (DIR) approach of (Lima et al., 2008): the idea behind this method is that, instead of trying to estimate the photometric redshift of all galaxies belonging to the observed catalogue directly, one uses a (possibly quite large) spectroscopic subsample (usually all photometric galaxy surveys make available a spectroscopic subsample) in the same dataset, for which a more reliable and accurate redshift distribution can be extracted, with redshift uncertainties typically smaller by about a factor 10, hence with $\sigma_z \sim 0.001$. Once the redshifts for this subsample are recovered, they represent the

galaxies for which we have the “true” position information: it is thus possible to construct a redshift distribution function dN/dz in which every galaxy position is multiplied by a free weight coefficient, an approach quite similar to the weighting schemes and matched filters strategies described in chapter 3, sec. 3.2. The values of the weight are obtained by fitting the spectroscopic redshift distribution with the free parameters against the full photometric distribution of the survey: under the assumption that the two catalogues are comparable (i.e., that the spectroscopic subsample contains objects representative of the full redshift extension and features of the full photometric survey, and that the very large systematic error are not intervening neither in the spectroscopic nor in the photometric samples), the weights can be interpreted as coefficients taking care of the additional uncertainties and imperfections of the photometric survey. The spectroscopic survey acts thus as a training set, to which the photometric data adapt and get consequently calibrated. The second galaxy clustering dataset is represented by the WISE×SuperCOSMOS survey (Bilicki et al., 2016), constructed in a manner similar to 2MPZ but excluding precisely the 2MASS data, thus selecting exclusively a higher-redshift sample. It is thus possible to build a catalogue, of about 20 million sources at redshifts $z \lesssim 0.4$, whose photometric redshifts is known at moderate precision ($\sigma_z \simeq 0.033$), that we partition in two tomographic bins, the first being represented by 11,288,037 galaxies with photometric redshifts in the ranges $0.1 \leq z_p < 0.25$, collectively identified by a median redshift of $z_m = 0.18$, while the second bin collects 5,213,088 objects at $z_p \geq 0.25$, with median redshift $z_m = 0.302$ and extending up to $z \sim 0.5$ - 0.6 . Again, calibration is performed by means of the DIR methodology, using a spectroscopic sample constructed from a variety of existing data sets, as described in (Paopiamsap et al., 2024).

The DIR methodology of course introduces its own uncertainties on the redshift distribution, combining the underlying instrumental uncertainties of photometric data with the errors induced by the manipulation of the data: their estimation via bootstrap resampling and the subsequent final estimation of our errors will be described in section 5.2.3. Given the limited quality of redshift determination in photometric data, objects observationally assigned to one tomographic bin but actually belonging to another one may determine leakage of information and induce correlated uncertainties between the redshift distributions of the different redshift bins, but we shall ignore this effect assuming it is subdominant; the final statistical uncertainties on the resulting data at the end of the calibration pipeline and bootstrap estimations can also be seen in fig. 5.1 together with the redshift distributions. We establish here our naming convention: we will refer to our three tomographic samples as “Bin 1”, “Bin 2” and “Bin 3”, in order of increasing redshift. In order to produce the angular power spectra, we need to construct the maps of the galaxy overdensity for each of these samples: for one thing, we bin each catalogue into sky maps, referring to the standard HEALPix pixelation (Gorski et al., 2005) having a resolution parameter $N_{\text{side}} = 1024$, associated to a pixel size $\delta\theta \simeq 3.4'$. The number count map we derive is then converted into an overdensity map; this can be done by subtracting the mean value for the number of sources in each pixel, $\bar{N}_p$, comprised in our analysis mask. The quantity is then divided by $\bar{N}_p$, thus recovering the observational $\delta_g = \frac{N_p - \bar{N}_p}{\bar{N}_p}$. A main systematic known in the literature (Koukoufilippas et al., 2020a,b) is the contamination in the overdensity map induced by stars and instrumental fluctuations in the different SuperCOSMOS plates. Typically, it is a large-scale effect limited to multipoles $\ell < 10$, excluded from our analysis. We further control such effect by subtracting a third-order polynomial function of the local density of stars as mapped by the WISE survey. This compensates for the specific systematic of star-driven fluctuations and can be estimated by tabulating the observed galaxy overdensity against the star density measured by the WISE survey.

The sky fraction actually available for our study is thus further reduced compared to the nominal f_{sky} of our reference surveys: the mask we use here is described in (Koukoufilippas et al., 2020a,b) and it is constructed by removing the areas most contaminated by Galactic dust and stars, as well as the regions close to the Large and Small Magellanic Clouds, resulting in the observational patches shown in the bottom half of fig.5.1.

5.1.2 Planck CMB lensing: a valuable observable connecting the Early and the Late Universe

The CMB weak lensing convergence field, a consequence of the presence of matter overdensities located between the last scattering surface and the observer is chiefly contributed by structures located in the $0.5 \lesssim z \lesssim 3$ redshift interval (Zaldarriaga & Seljak, 1999; Lewis & Challinor, 2006). Theoretical work on its signal – usually noted with the Greek letter κ – has been laid out starting from the 1980s, with works such as (Blanchard & Schneider, 1987; Seljak, 1996; Linder, 1990; Bernardeau, 1997) prospecting the rich array of cosmological applications it allows to develop, that eventually translated into active research programmes after the announcement of the first positive detection of its autocorrelation signal by ACT in 2012 (Das et al., 2011), followed by the extremely high-quality observations of Planck (Planck Collaboration et al., 2014b, 2016c, 2020c). The maps constructed by the two collaborations represent the standard reference datasets in the field, further backed by the results achieved by other CMB observatories such as SPT, PolarBear and BICEP (van Engelen et al., 2012; Ade et al., 2014; BICEP2 Collaboration et al., 2016). The CMB lensing effect manifests itself on both the scalar temperature fluctuation field and on the polarisation maps; phenomenologically, its main properties are (Hanson et al., 2010):

- Preservation of the original CMB black-body spectrum.
- Important smoothing of the acoustic oscillation peaks in both scalar and polarisation signals.
- Induction of secondary B-mode polarization in the polarisation signal, whereas primary polarisation should only involve E-modes in absence of primordial gravitational waves. Conversely, should primordial gravitational waves signatures be detected in the CMB signal, it will be imperative to clearly distinguish their signal from the secondary, lensing-induced B-mode polarisation (Kamionkowski & Kovetz, 2016; Chiueh & Ma, 2002; Fidler et al., 2014).
- Introduction of relevant non-Gaussianity and non-linearity in the radiation field (while the CMB is a highly linear and Gaussian signal) at small angular scales in even-correlation functions, such as the power spectrum; odd-order correlation functions are less affected. Large-scale non-Gaussianity is also induced in all summary statistics.

Our discussion shall focus on the scalar perturbation field, the one we use in this analysis, but the availability and importance of work on polarisation data should not be discarded (Zaldarriaga, 2004; Rahimi & Reichardt, 2024). It is interesting to observe that the CMB lensing is the actual “primary” CMB signal we have access to: indeed, assuming we have been able to deconvolve the cosmic background photon maps from the many systematics by which they are affected *, we are however unable to directly access the photon distribution as they were emitted at the

*These include both contamination from astrophysical sources emitting in the microwaves, coupling with the Cosmic Infrared Background, and the complicated mixing between the non-Gaussian, hence inherently complex, cosmological signal with instrumental effects (Madhavacheril, 2025).

recombination epoch, so that to recover the CMB signal at recombination further de-lensing by means of suitable procedures has to be performed, making the pristine CMB an even more indirect observable (Hotinli et al., 2022).

CMB lensing kernels and power spectra properties

During their travel to the observer, the CMB photon geodesics get distorted by effect of the gravitational interaction between the electromagnetic radiation and the LSS forming and growing along the line of sight: the large size of the lenses, corresponding substantially to the whole structure in the Universe, induces deflection angles that are not necessarily large in magnitude, rather, they are typically in the order of arcminutes, yet showing coherence even on large angular scales precisely because they are the effect of large lensing masses. The integrated distorting effect on the photon trajectories from $z \sim 1100$ to $z = 0$ can be described in terms of the gravitational weak lensing formalism, adapted to some of the specific properties of the CMB. Starting from the definition of the general lensing convergence field for any tracer L (CMB being a projected observable, only angular power spectra can be constructed), we have (Madhavacheril, 2025; Hanson et al., 2010; Lewis & Challinor, 2006)

$$\kappa_L(\mathbf{n}) = \int_0^\infty dz W_L^\kappa(z) \delta_m(\chi(z)\mathbf{n}, z). \quad (5.1)$$

In the expression above we recognise the observable's specific window function, or lensing kernel,

$$W_L^\kappa(z) = \frac{3}{2} \Omega_m H_0^2 \frac{(1+z)}{H(z)} \frac{\chi(z)}{c} \int_z^\infty dz' n(z') \frac{\chi(z') - \chi(z)}{\chi(z')}, \quad (5.2)$$

that can be further adapted to the CMB case by observing that the CMB is generated, discarding the finite time, and hence redshifts interval Δz , it took to the process for completely realising, at a substantially unique redshift value $z_\star$, so that the integral in eq. 5.2 can be approximated by a Dirac delta function, yielding:

$$W_{CMB}^\kappa(z) = \frac{3}{2} \Omega_m H_0^2 \frac{(1+z)}{H(z)} \frac{\chi(z)}{c} \left[\frac{\chi(z_\star) - \chi(z)}{\chi(z_\star)} \right]. \quad (5.3)$$

It is now possible to develop the angular power spectra of the CMB lensing in autocorrelation or in cross-correlations with other tracers following eq. 1.131. A specific characteristic of the CMB lensing signal, related to its probing the LSS from its seeding about 300,000 years after the Big Bang up to today is that, depending of the tracers with which it is combined, different properties of it get more or less highlighted and allow to test different regimes. Indeed, the CMB lensing kernel peaks between $z \sim 1 - 3$ and receives contributions even for $z > 5$ (fig. 5.2). The contributions along the line of sight translate into angular dependence by virtue of the relation $k \sim \ell/\chi$, allowing to measure different parameters and scales for disparate goals, spanning from structure growth, background curvature measurements to testing dark energy models and neutrino effects.

The main analysis strategies (Hanson et al., 2010; Lewis & Challinor, 2006; Madhavacheril, 2025) can be listed below:

- **CMB lensing autocorrelation:** the pure CMB-lensing autocorrelation statistics is a particularly complex observable, subject to instrumental noise and systematics, foregrounds and higher-order contributions. Typically, low multipoles carry more information about

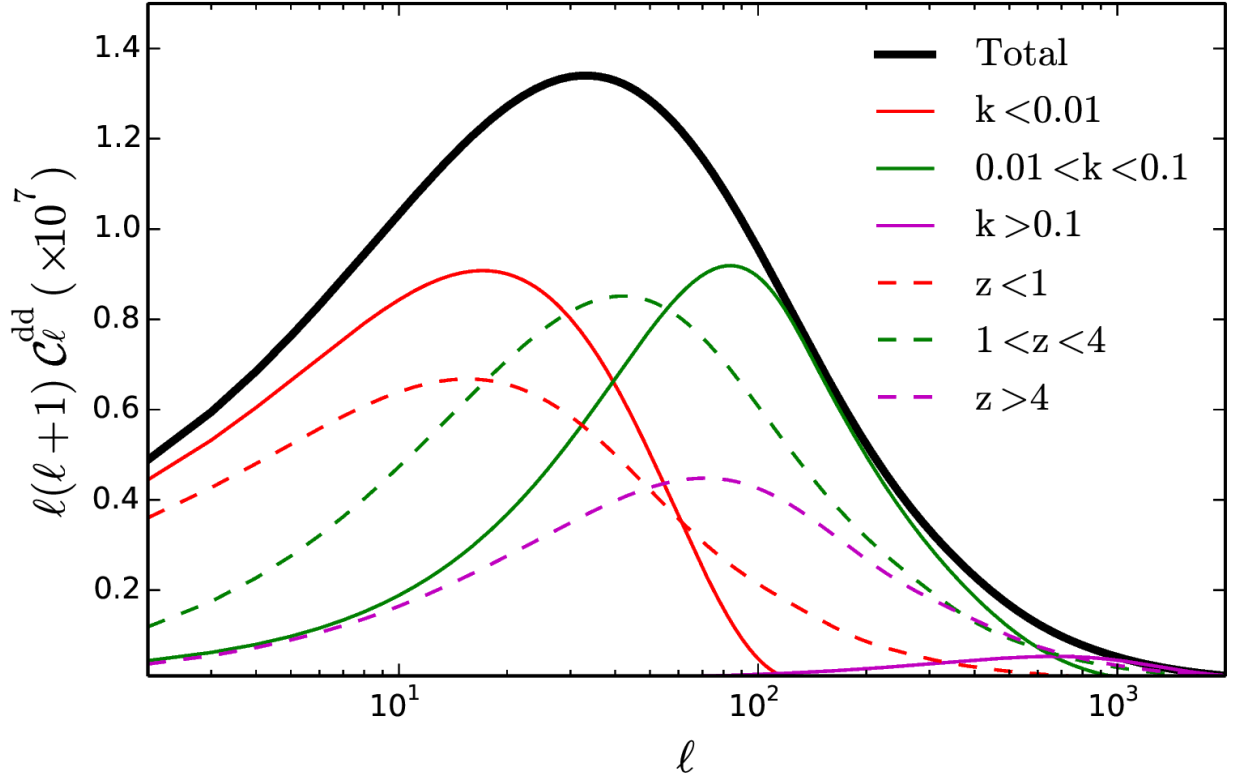


Figure 5.2: The CMB lensing angular autocorrelation power spectrum, displaying the contributions to the total signal broken down in terms of redshift contributions (dashed coloured lines) and scale contributions (solid coloured lines). The picture is taken from (Hojjati & Linder, 2016).

lower redshifts and larger scales, with $\ell < 20$ being dominated by $z < 0.5$ effects, whereas higher redshifts are usually probed by $\ell > 100$, translating in particularly useful information about growth evolution (fig. 5.2). Since high multipoles are related chiefly to high redshifts, associated to less non-linear dynamics compared to small scales sampled at later epochs, the CMB autocorrelation signal does not provide too much information about deeply non-linear regimes. For all its potential usefulness, the CMB autocorrelation signal is particularly hard to deal with because it is more burdened by systematics that typically get erased or mitigated in cross-correlation studies. Astrophysical foregrounds and observational systematics are treated either by directly modelling or by fitting to the data suitable estimators constructed in order to define a signal component template and a noise component template which can thus be extracted a posteriori (Hu & Okamoto, 2002). The shot-noise contribution to the signal is also difficult to manage: it is highly non-Gaussian, due to its non-negligible coupling with the CMB lensing signal so that, to avoid the effort of modelling and subtracting the intervening systematics, Monte Carlo techniques are often used in order to produce several stochastic realisations of them and estimate their impact based on the average of such simulations. Remarkably, this is the approach used in the case of Planck maps (Madhavacheril, 2025; Planck Collaboration et al., 2016c).

- **CMB lensing-LSS weak lensing (cosmic shear) cross-correlation:** in this case, two lensing kernels with the form of eq. 5.1 get combined, with galaxies typically displaying a broad redshift distribution compared to the very narrow one associated to the CMB lensing signal. Both observables are unbiased tracers of the matter field, providing us direct access

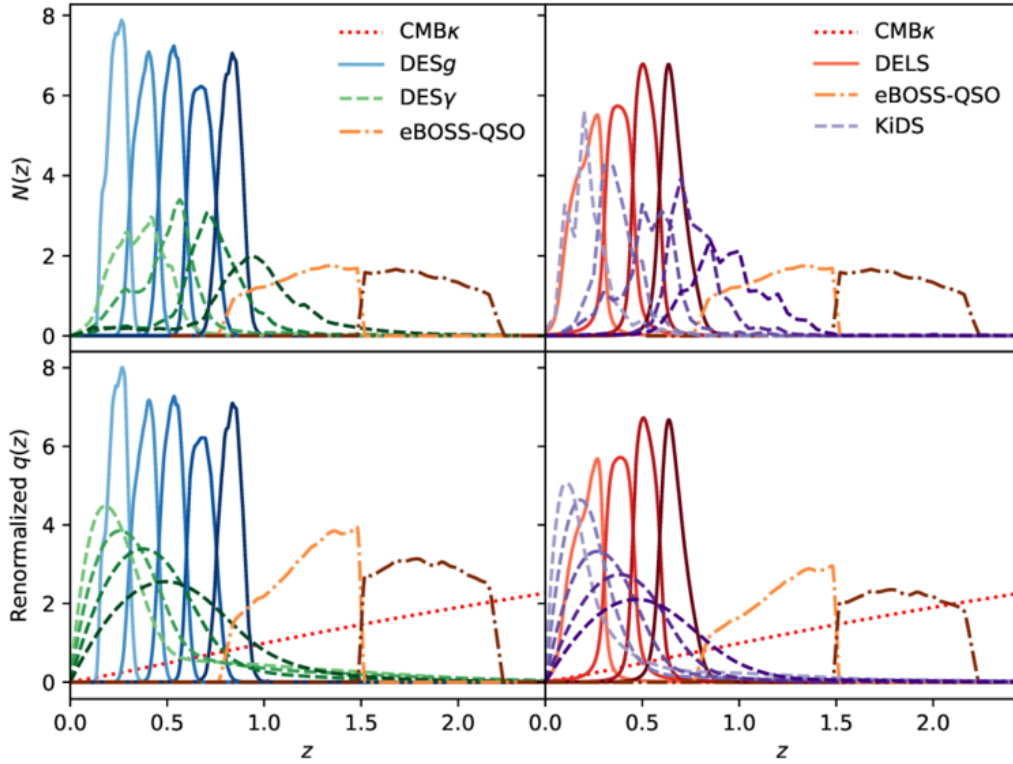


Figure 5.3: CMB lensing kernel and galaxy redshift distributions of some of the main current galaxy clustering and weak lensing redshift surveys. On the upper panel, the actual redshift distribution of the observables is displayed, i.e., dN/dz , count of galaxies per unit redshift, in the case of the galaxies and the quantity q_κ of eq. 5.5 for the CMB lensing (red dotted line). Because of the units of measure, the CMB lensing redshift distribution is barely visible. The lower panel shows the same quantities rescaled to their integrals (hence $q(z) = \frac{dN/dz}{\int dz dN/dz}$ and $q(z) = \frac{q_\kappa}{\int dz q_\kappa}$), allowing a better visualisation of the same quantities. The CMB lensing kernel can be seen extending on a broad redshift interval: the degree of overlap with representative galaxy surveys as well as its distinctive and independent features make it a particular useful anchor between the Late and the Early Universe. The picture is taken from (García-García et al., 2021).

to σ_8^2 . Cosmic shear is burdened by additional systematics but it is also interesting in that, peaking at redshifts much smaller than the CMB lensing kernel, it is associated to scales corresponding to rather high k (García-García et al., 2021). Cross-correlations between the CMB lensing and weak lensing signals are therefore more sensitive to non-linearities and baryon feedback compared to the CMB lensing autocorrelation (see fig. 5.3). At the same time, the cosmic shear kernel provides more weight at redshifts around the onset of dark energy domination with respect to the CMB window function, thus being more useful for testing modified gravity and dark energy.

- **CMB lensing-galaxy clustering cross correlation** Combining the CMB lensing signal with galaxy clustering tracers characterised by a specific redshift distribution dn/dz and bias b_g (either linear or non-linear), we typically have sensitivity to the degenerate combination

$b_g\sigma_8^2$ (García-García et al., 2021; Karim et al., 2025; White et al., 2022; Sailer et al., 2025b). Multi-tracer studies combining CMB lensing-galaxy clustering cross-correlations with the galaxy clustering autocorrelation signal, that instead probes b_g^2 , are often performed, breaking the degeneracy with σ_8 . Non-linear regimes – although less deeply so than in the weak lensing case – are usually tested and, depending on whether we use a spectroscopic or a photometric galaxy sample, modelling might be characterised by different degrees of precision in distance measurements and additional systematics, influencing the final quality of the outcomes of binning and tomographic measurement exercises (refer again to fig. 5.3).

- As usual, more extended multitracers approaches combining clustering, weak lensing and CMB lensing signals can and have been explored in the last years (Abbott et al., 2023b) as well as combinations with other probes, in particular of baryonic nature, such as Sunyaev-Zel’dovich effect (proposed and measured) (Hill & Spergel, 2014; McCarthy & Hill, 2024; Hurier & Angulo, 2018), X-Rays (measured) (Hurier et al., 2017) and Intensity Mapping (forecast and discussed) (Marins et al., 2025; Shen et al., 2025; Schaan & White, 2021). These studies involving baryonic probes typically have the main goal of calibrating the parameters characterising the dynamics of baryons at small scales, but can also pursue fundamental cosmology objectives using independent and complementary observables to the more standard galaxy clustering and lensing tracers. As usual in cosmology, next to the use of specific tracers, research is also oriented towards exploring the features of the CMB lensing signal in higher-order statistics and their best fields of application (Robertson, 2025; Fabbian et al., 2018; Schmittfull & Seljak, 2018).

Overall, estimates of fundamental Λ CDM parameters derived from CMB lensing analyses display particular sensitivity to σ_8 or A_s , Ω_m and H_0 , thus on S_8 , and a weak dependence on n_s and $\Omega_b h^2$. CMB lensing provides increasing constraining power on $\sigma_8(\Omega_m/0.3)^{0.25}$ rather than on the properly defined S_8 (with $\alpha=0.5$ instead of $\alpha=0.25$) (Madhavacheril, 2025). Measurements are typically in agreement with Planck (van Engelen et al., 2012; Planck Collaboration et al., 2020b), with or without the inclusion of Late Universe information coming from the BAO, a strategy often utilised in order to break the degeneracies among the parameters measured by the CMB alone (Madhavacheril et al., 2024). Back to observational considerations and to the data preparation effort, we will specifically use the convergence map made available as part of the Planck 2018 data release (Planck Collaboration et al., 2020c), covering a sky fraction $f_{\text{sky}} = 0.671$ and thus overlapping spatially with the galaxy samples outlined in sec. 5.1.1. As we said, the CMB weak lensing signal is a bit delicate to handle and various techniques are available to construct the final lensing maps given the raw CMB observations. We choose the maps generated with the “Minimum Variance” (MV) technique (Maniyar et al., 2021) to extract the spherical harmonics coefficients containing the fundamental information in order to map the lensing convergence field on the sphere and transformed into an HEALPix map at $N_{\text{side}} = 1024$ resolution. The map is generated after having excluded contributions associated to harmonic coefficients having $\ell > 3N_{\text{side}} - 1 = 3071$, then combined with the binary sky mask provided by the Planck collaboration alongside this map. Further data manipulation involve the treatment of the lensing reconstruction noise power spectrum, characterised by a strong rise with ℓ at small scales. The mask is apodised with a 0.2° “C1” kernel (Alonso et al., 2019), partially degrading the resolution but at the same time also diluting the effect of the shot noise against the signal at the smallest scales. In the end, the final usable sky fraction is $f_{\text{sky}} \simeq 0.66$. Following (Planck Collaboration et al., 2016c, 2020c), we remove

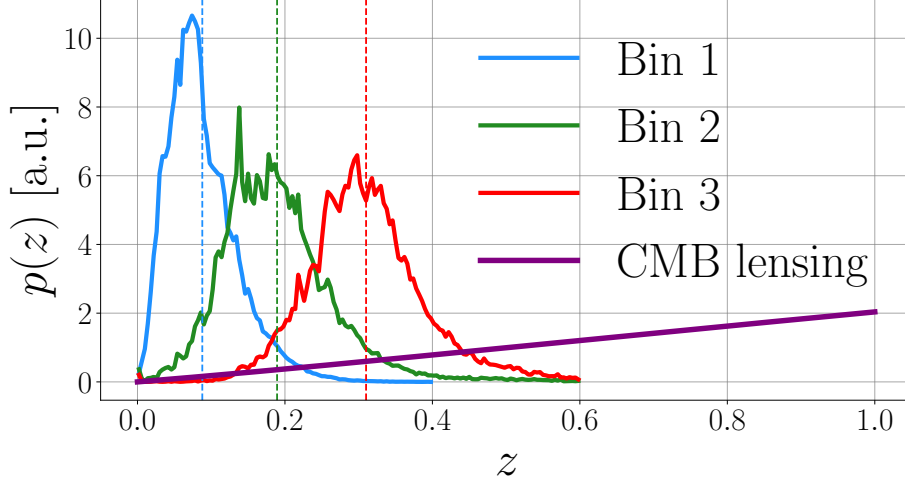


Figure 5.4: Adimensionalised kernels for the three galaxy redshift bins (respectively in blue, green and red with increasing redshift) and for the CMB lensing signal (purple) using the same definitions of the lower panels of fig. 5.3, in order to better visualise the degree of overlap between the two observables. The three vertical dashed lines correspond to the mean redshifts for each galaxy survey redshift bin.

from the angular power spectra the multipoles at $\ell < 8$, too sensitive to the subtraction of the mean value of the field throughout the application of the lensing reconstruction algorithm. The cutoff at $\ell_{\max} = 2000$, corresponds to the *aggressive* scale range determined by (Planck Collaboration et al., 2020c). Given the above-mentioned complexity of the CMB lensing auto-correlation signal, we do not include it in our analysis: we can thus avoid more rigorous modelling of the several noise terms included in the CMB lensing likelihood. At the same time, however, the CMB lensing noise must be included in the covariance matrix for all those power spectra correlating with the CMB lensing signal, determining a non-negligible contribution that will be dealt in sec. 5.2.3.

5.2 Methodology

5.2.1 Projected statistics: galaxy clustering and CMB lensing

We can now concentrate on the theoretical modelling of the estimators we will fit against the observational data (again referring to sec. 1.4 for details). We have to construct the theoretical templates as functions of the angular galaxy overdensity (in three different redshift bins) and of the CMB lensing convergence as projected tracers of the large-scale structure. Both of these tracers can be written in general as

$$a(\mathbf{n}) = \int d\chi q_a(\chi) A(\chi\mathbf{n}, z(\chi)). \quad (5.4)$$

Here, $\mathbf{n}$ is a unit vector along the line of sight, χ stands for the radial comoving distance, and z is the corresponding redshift. We note with A the three-dimensional field traced by the projected quantity a , and $q_a(\chi)$ the associated radial kernel. When A corresponds to δ_g , the radial kernel is proportional to the redshift distribution of the galaxy sample, while the CMB lensing kernel stretched from $z = 0$ to the recombination epoch, characterised by a peak at $z \sim 2$. We fix the notation we shall use hereafter below:

$$q_g(\chi) = H(z)p(z), \quad q_\kappa(\chi) = \frac{3}{2}\Omega_m H_0^2 (1+z)\chi(z) \left(1 - \frac{\chi(z)}{\chi_{\text{LSS}}}\right) \Theta(z < z_{\text{LSS}}). \quad (5.5)$$

Here $H(z)$ is the expansion rate[†], $H_0 \equiv H(0)$, χ_{LSS} is the distance to the surface of last scattering, at redshift $z_{\text{LSS}} = 1100$, and Θ is the Heaviside function. We assume a flat Λ CDM model for this work. In order to appreciate the extent and magnitude of the overlap between the two observables, we refer to fig. 5.4, where the two quantities $q_g(\chi)$ and $q_\kappa(\chi)$, rescaled for better visibility to their integral according to what described in fig. 5.3, are shown. The three-dimensional fields we model the 3D galaxy overdensity δ_g , and the matter overdensity δ_m (of which every lensing signal, and the CMB in particular are unbiased tracers) in the case of κ . As customary, the angular power spectrum for such two projected fields, a and b , can be related to the 3D power spectrum of their associated 3D quantities $P_{ab}(k)$ via

$$C_\ell^{ab} = \int \frac{d\chi}{\chi^2} q_a(\chi) q_b(\chi) P_{ab} \left(k = \frac{\ell + 1/2}{\chi}, z(\chi) \right) \quad (5.6)$$

As usual, the use of this expression implies the use of the Limber approximation (Limber, 1953): the general interest for intermediate and low scales makes its use reliable and justified. We also checked that this approximation is not significantly affecting the reported results, indeed modifying our constraints on S_8 by less than $\sim 2\%$, which is significantly below statistical uncertainties.

The last building block required to model the analysed power spectra C_ℓ^{gg} and $C_\ell^{g\kappa}$ power spectra are the underlying three dimensional power spectra for the galaxy clustering $P_{gg}(k, z)$ signal and of the the matter-galaxy cross-correlation $P_{gm}(k, z)$. Their theoretical modelling will be discussed below.

5.2.2 Perturbative bias expansion

As presented in sec. 1.2.1, the measurement of σ_8 and of S_8 requires the choice of suitable tracers and models sensitive to intermediate and small scales of the LSS, corresponding to non-linear regimes calling for a more complex treatment of the perturbative regime compared to the standard linear perturbation theory; the use of very low-redshift observations, non-linearity of the LSS increasing with decreasing redshift, further justifies this consideration (see sec. 1.3). The formalism we shall employ is the hybrid Lagrangian bias expansion, also called Hybrid Effective Field Theory (HEFT) (Matsubara, 2008b; Modi et al., 2020; Zennaro et al., 2023; Kokron et al., 2021; Hadzhiyska et al., 2021), that we presented in sec. 1.3.2.

We recall that this theoretical approach treats the galaxy overdensity field by combining a formulation in Lagrangian coordinates with a description of the non-linear δ_g as a function of operators, collected in a weighting function $F(\mathbf{q})$, that acts on the underlying Lagrangian matter field – which can be equally modelled by HEFT as associated to $F(\mathbf{q}) = 1$, i.e., a homogeneous field. Cutting our expansion to the second order, the involved fields are δ, δ^2, s^2 and $\nabla^2 \delta$, where δ is the linear matter density and s^2 is the traceless contracted tidal tensor (Modi et al., 2020). Moreover, to recover observable quantities, we have to convert the Lagrangian positions into the final, Eulerian coordinates. This is made possible by defining a suitable advection law as a function of the displacement field applied to the Lagrangian galaxy density propagated to the initial conditions to its late-time coordinates. Eventually, the galaxy overdensity field expressed in Eulerian coordinates $\mathbf{x}$ reads

$$1 + \delta_g(\mathbf{x}) = \int d^3q [1 + b_1 \delta(\mathbf{q}) + \frac{b_2}{2} \delta^2(\mathbf{q}) + \frac{b_s}{2} s^2(\mathbf{q}) + b_{\nabla^2} \nabla^2 \delta(\mathbf{q})] \delta_D(\mathbf{x} - \mathbf{q} - \boldsymbol{\Psi}(\mathbf{q})), \quad (5.7)$$

[†]Note that we use natural units, with $c = 1$, throughout.

with b_1, b_2, b_{s2} , and b_{∇^2} all free parameters (the galaxy bias parameters), $\mathbf{q}$ are the Lagrangian coordinates, while Ψ represents the displacement field vector that transforms the Lagrangian to the Eulerian coordinates. We omit the average of the fields ($\langle \delta^2 \rangle, \langle s^2 \rangle \dots$) since in any case, once the fields are transformed in the Fourier space, they would only contribute to the value of the power spectrum at the unobservable $k = 0$ scale and be 0 otherwise. The treatment of the displacement field represents one of the peculiarities of the HEFT formalism: while in a fully perturbative approach Ψ is derived by expanding the density field, if we use the HEFT framework we can obtain it through a non-linear N -body solver (see sec. 1.3.2 for more details on this theoretical framework). From these considerations we can derive the galaxy auto-power spectrum and galaxy-matter cross-power spectrum formulations:

$$P_{gg}(k) = \sum_{i,j} b_i b_j P_{ij}(k), \quad P_{gm}(k) = \sum_i b_i P_{1i}(k). \quad (5.8)$$

In this expression, the indices i, j run on the set of operators involved by the expansion, whereas $P_{ij}(k)$ is the cross-power spectrum of the i -th and j -th operators after advection in the Eulerian coordinates frame is performed. The specific implementation of HEFT we use is the one presented in (Zennaro et al., 2023), where the cross spectra P_{ij} are measured in the BACCO N -body simulations (Angulo et al., 2021; Aricò et al., 2021b), providing the base dataset for the `baccoemu` emulator, described in sec. 1.3.2.

HEFT is capable of providing unbiased predictions for the 2-point clustering for many classes of galaxies and their clustering regime, up to relatively small scales, $k \lesssim 0.7 h\text{Mpc}^{-1}$ (Zennaro et al., 2022; Nicola et al., 2024; Euclid Collaboration et al., 2024a). and also to provide insights on the correlation between the higher-order bias coefficients, b_2, b_s, b_{∇^2} and the linear bias b_1 , the so-called “coevolution relations” (sec. 1.3.2) (Zennaro et al., 2022), phenomenological laws, with a relatively broad scatter that we will also discuss later in the text in sec. 5.3.3.

5.2.3 Calculation of the summary statistics from the data and estimation of the covariances

We now outline the methodologies employed in order to extract the power spectra from our maps and their associated covariance matrix.

Extraction of the C_ℓ

The correct evaluation of the C_ℓ from the data is notoriously a delicate and far from immediate task to perform (Bonvin & Durrer, 2011; Asorey et al., 2012), since actual observations lack the well-behaved properties we assign to the theoretical estimators. In particular, they typically invalidate the assumption of the absence of correlation in the ℓ and m modes underlying the usual calculations that lead us to the formulation of the C_ℓ estimators as derived in sec. 1.4. This complication can be due to a number of reasons: for example, because of observational and astrophysical systematics contaminating the signal and coupling different scales, or because of the masking procedure, that reduces the effective size of the available celestial sphere and deforms the footprint of the sky regions we populate with our data during the map-making phase. Furthermore, even in presence of ideally clean data, most computational pipelines aimed at deriving the actual power spectra might be very slow and time consuming. Alternative approaches not striving to immediately recover the actual C_ℓ from the data, but an approximation, non-Gaussian perhaps

in nature but sufficiently decontaminated, are often pursued and have led to the definition of the so-called pseudo- C_ℓ methodology, usually calculated from the data by means of the **MASTER**, **PolSpice** and **NaMaster** codes (Hivon et al., 2002; Challinor et al., 2011; Alonso et al., 2019). In this framework, and in particular considering the **NaMaster** suite we shall use [‡], the observed version counterpart of any project field a^v can be expressed as a product in the real space, of the true underlying field a multiplied by a mask v :

$$a^v(\mathbf{n}) = v(\mathbf{n}) a(\mathbf{n}). \quad (5.9)$$

This relationship maps into a convolution when we move to the harmonic space, and similar result can be obtained whe moving to the ensemble average of the power spectra for fields a^v and b^w after the application of the respective masks v and w :

$$\langle \tilde{C}_\ell^{ab} \rangle = \sum_{\ell'} M_{\ell\ell'}^{vw} C_\ell^{ab} + \tilde{N}_\ell^{ab}. \quad (5.10)$$

Here, C_ℓ^{ab} is the underlying, not directly accessible, angular power spectrum of the fields, and $\tilde{C}_\ell^{ab}$ is the so-called “pseudo- C_ℓ ”, or “coupled C_ℓ ” that we can instead measure from the data,

$$\tilde{C}_\ell^{ab} \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \text{Re}(a_{\ell m}^v (b_{\ell m}^w)^*), \quad (5.11)$$

while $\tilde{N}_\ell^{ab}$ is the noise pseudo-spectrum (namely, the noise spectrum of the a and b fields, which is more relevant when auto-correlations are calculated). Finally, $M_{\ell\ell'}^{vw}$ stands for the mode-coupling matrix, which describes the correlation between pairs of angular multipoles ℓ (thus violating the assumption of absence of correlation among multipoles we require to calculate the theoretical angular power spectra) caused by the presence of the mask. The **NaMaster** algorithm is based on the fundamental result that the mode-coupling matrix allows for an analytic estimation with efficient algorithms, and written as a function of the pseudo- C_ℓ we derive from the data. Thanks to this property, we recover an intermediate quantity between the original data and the actual C_ℓ , known as “bandpowers”, that can be estimated through a binning of the pseudo- C_ℓ and performing an inversion of the binned mode-coupling matrix. By performing a further decoupling step (see (Alonso et al., 2019)), again made possible thanks to the availability of the mode-coupling matrix, we can eventually recover from the bandpowers the true angular power spectra, which can be defined as a function of the bandpowers and of suitable projection operators, the “bandpower window functions”, also a function of the mode-coupling matrix.

Estimation of the multi-tracer covariance matrix

Our analysis is effectively a multi-tracer setup (McDonald & Seljak, 2009; Seljak, 2009; Abramo & Leonard, 2013) (see also chapter 3, sec. 3.3), combining

- Three galaxy clustering autocorrelation data vectors, corresponding to the three employed bins.
- Three CMB lensing-galaxy clustering cross-correlation data vectors, one for each redshift bin of our tomographic subdivision of the galaxy clustering data.

[‡]See (Alonso et al., 2019) for the complete description of the algorithm, and [this website](#) for documentation.

We recall once again that we are not using, as data, neither the CMB lensing autocorrelation power spectra, too prone to systematics, nor the galaxy clustering cross-correlations among different bins, which are assumed to be substantially null. The combined analysis calls for the definition of the associated covariance among tracers and redshift bins, which we estimate analytically starting from the individuation of the four contributions we define below:

$$\text{Cov} = \text{Cov}_G + \text{Cov}_{\text{cNG}} + \text{Cov}_{\text{SN}} + \text{Cov}_{p(z)}. \quad (5.12)$$

In detail, the four terms are:

- Cov_G : the disconnected Gaussian covariance, is a more sophisticated version of the standard analytic covariance among Gaussian fields whose derivation again follows the reasonings of eq. 3.33 in sec. 3. This term dominates the total uncertainty and it is here calculated in the more accurate formulation based on the Narrow Kernel Approximation (iNKA) as described in (García-García et al., 2019; Nicola et al., 2021), which takes into account the correlations between different angular scales determined by the effect of survey geometry combined with masking procedures, again resorting to the pseudo- C_ℓ formalism of (Alonso et al., 2019). The CMB lensing autocorrelation term entering in this component of the covariance also calls for the associated shot-noise contribution, that we evaluate by using the estimate of the noise power spectrum N_ℓ^κ provided with the Planck data release.
- Cov_{cNG} : the connected non-Gaussian trispectrum contribution, relevant exclusively for the terms involving the galaxy signals, either in auto or cross-correlations, without involving the CMB lensing signal. This term would vanish by virtue of the Wick-Isserlis theorem if our fields were strictly normal random variables (see eq. 3.34), but here we keep it because the pure Gaussianity assumption is not realistic for our observational data. It is possible to estimate it by means of a derivation available in Appendix A of (Krause & Eifler, 2017), which obtains the expression of Cov_{cNG} in the spherical harmonics space as a function of higher-order statistics, estimated in the halo model formalism (see section 1.3.1), in turn a function of a halo occupation distribution for the galaxy clustering signals that we fit against the angular auto-correlations of each redshift bin. The data are fitted in the range of scales $k < 1 h \text{ Mpc}^{-1}$, with the HOD parametrisation of (Zheng et al., 2005). This HOD allows to express the mean number of central and satellite galaxies as

$$\bar{N}_c(M) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\log(M/M_{\min})}{\sigma_{\ln M}} \right) \right], \quad \bar{N}_s(M) = \Theta(M - M_0) \left(\frac{M - M_0}{M_1} \right)^\alpha.$$

We leave only two parameters free, $M_{\min}$ and M_1 , while the other parameters are kept fixed at $\alpha = 1$, $M_0 = M_{\min}$, $\sigma_{\ln M} = 0.4$. Once the halo model has been fit, a power spectrum accounting for non-Gaussianity can be calculated and used to generate realisations of a non-Gaussian matter field from which the cNG contribution can be estimated. They are eventually found to be subdominant in the scale regime relevant for our analysis.

- Cov_{SN} : this is the contribution to the covariance from uncertainties in the effective shot noise (a sort of “uncertainty on the uncertainty”) of the sample for the terms involving the galaxy signals, limited to the redshift bin autocorrelations (it is null for both galaxy-galaxy terms involving the cross-correlation of different bins or in galaxy-CMB lensing contributions) terms and does not involve the CMB lensing signal. All our galaxy auto-spectra are corrected analytically for the shot-noise contribution related with the discreteness of the tracer sample

(the usual Poissonian shot-noise term described in 3.3, real data are affected by other stochastic contributions to the galaxy overdensity. These contributions could provide further terms in the auto-spectrum, summarised in the term we are describing but we can rather easily treat them by observing that they behave as white noise, without correlation on the probed scales. To account for this effect we can therefore define an additional amplitude parameter A_{SN}^i that rescales the magnitude of the standard Poissonian shot-noise formula (eq. 3.38) entering the galaxy autocorrelation spectra in the i -th bin, modelling thus such additional effects (i.e., we set $N_\ell^{gg,i} \rightarrow A_{\text{SN}}^i N_\ell^{gg,i}$). This would introduce three additional free parameters to be fitted in the final analysis, and, rigorously speaking, we should therefore fit it in the data analysis pipeline. We can devise a strategy to overcome this complication by formally configuring the analysis as if we were actually fitting it, introducing a Gaussian prior on the A_{SN}^i terms, centred around the value of 1 (corresponding to the absence of additional amplitude corrections to the Poissonian term), and with standard deviation σ_{SN} , for which we can set a reasonably wide uncertainty, such as $\sigma_{\text{SN}} = 0.1$, meaning that we are allowing for $\sim 10\%$ corrections to the purely Poisson contribution. But then, we can treat σ_{SN} as a proxy for the free parameter itself, reformulating the covariance term as

$$\text{Cov}_{\text{SN}}(C_\ell^{gg}, C_{\ell'}^{gg}) = \sigma_{\text{SN}}^2 N_\ell^{gg} N_{\ell'}^{gg}, \quad (5.13)$$

where N_ℓ^{gg} is the standard Poisson shot noise contribution to the power spectrum, estimated as described in e.g. (Nicola et al., 2020). Once including these terms in the diagonal covariance blocks associated to each galaxy auto-correlation power spectra, substantially selecting a value for the A_{SN}^i without the need of actually fitting these parameters in the final data analysis runs, formally marginalising over them, that no longer require estimation. The amplitude of the prior, and hence the potential size of the correction to the pure shot-noise term, can be tuned and made larger or smaller, according to how conservative we want to be.

- $\text{Cov}_{p(z)}$: this latter term accounts for contributions to the covariance deriving from possible uncertainties in the redshift distributions of our galaxy datasets, induced by the limited quality of photometric redshift estimation and on the DIR calibration methodology. It does not involve the CMB lensing term and, since we assume lack of correlation between different galaxy clustering redshift bins, it is a term only entering entries to the covariance associated to autocorrelations in redshift bins. Given our redshift distributions, estimated as described in section 5.1.1, we propagate the additional uncertainties according to the methodology of (Hadzhiyska et al., 2020) and validated in (Ruiz-Zapatero et al., 2023), exclusively assuming that every redshift bin has its own uncertainties that do not correlate with those affecting another redshift bin. The idea is to further develop the DIR calibration procedure and the estimation of the weights matching the photometric redshift distribution of the full sample of galaxies belonging to the dataset with the redshift distribution estimated from the reference spectroscopic training subsample. Once we have calibrated the full redshift distribution, we still have to assign suitable uncertainties to the derived redshifts. Instead of providing an analytic, possibly heavily parametrised model for such uncertainties, also fitted against the calibrated redshift distribution, we can start from the decomposition of the recovered calibrated photometric distribution against a suitable basis function set. These functions will depend on the redshift and will be associated to the binning of the calibrated samples in redshift intervals. The free coefficients of the regression of the redshift distribution against

Parameter	Prior
σ_8	$U(0.4, 1.2)$
Ω_m	$U(0.1, 0.5)$ or $\mathcal{N}(0.2975, 0.0086)$
$b_{1,i}, b_{2,i}, b_{s,i}$	$\mathcal{N}(0, 5)$
$b_{\nabla^2,i}$	$\mathcal{N}(0, 10 \text{ Mpc}^2)$

Table 5.1: Table reporting the priors for the set of parameters we employed in the analysis. The notation $U(a, b)$ represents a uniform distribution between a and b , while $\mathcal{N}(a, b)$ stands for a Gaussian distribution having mean a and variance b . Depending on the analysis configuration discussed in the main text, we put on Ω_m , both the reported non-informative uniform priors and the narrow, Gaussian informative prior based on the BAO data from DESI (Abdul Karim et al., 2025).

the basis function can be recovered by fitting the basis functions to the data: in a Bayesian framework (see beyond, sec. 5.3.1), this means that we actually recover a full probability distribution for the regression coefficients. Drawing different realisations of the posteriors allows to define a bootstrap resampling procedure by which we simulate many possible redshift distributions more or less close to the best-fit estimator of the unknown actual redshift distribution of the photometric sample, that we can never access.

We model this covariance contribution in a way similar to Cov_{SN} , obtaining:

$$\text{Cov}_{p(z)} = \mathbf{T}^T \mathbf{P} \mathbf{T}, \quad (5.14)$$

we condensate in $(\mathbf{T})_{\alpha j} \equiv \partial t_\alpha / \partial [p(z)]_j$ the sensitivity of the position of the α -th element in the best-fit redshift distribution (the *predicted*, supposedly calibrated, redshift) to the corresponding j -th redshift element of the *observed* redshift distribution (the methodology substantially provides low uncertainties as long as the sensitivity of the model to variations of the data is not excessive, and checks for the dependence of every predicted position on every observed position), while $(\mathbf{P})_{ij}$ is the Gaussian prior covariance of the redshift distributions, which can be estimated numerically from the covariance of the many bootstrapped redshift distribution realisations we can generate starting from the posterior distribution of the regression parameters. Putting all together, we have thus recovered an estimate for the contribution to the covariance due to the intrinsic observational systematics, to be added to the pure, unavoidable statistical uncertainties proper of the power spectra estimators.

We compared the fiducial model (the “All bins” case), which implements all these innovations with the simpler model discarding $\text{Cov}_{\text{SN}} + \text{Cov}_{p(z)}$ (the “No $p(z)$, N_ℓ marg.” case) and summarised the outcome in table 5.2, where we can observe that the results are substantially identical, suggestive that the impact of these correction is limited (given the amplitude of the additional shot-noise contribution we assume), a reassuring fact concerning the soundness of our redshift calibration methodology.

5.2.4 Likelihood, priors, and scale cuts

The model we can construct with all the outlined ingredients available will allow us to measure two main classes of parameters:

- The two cosmological parameters of the Λ CDM model that can be more properly constrained at low redshift from galaxy clustering and CMB lensing data, namely the σ_8 normalisation of the power spectrum and the total matter (DM + baryon matter) density parameter Ω_m , from which it will also be possible to measure S_8 .
- The HEFT bias parameters $\{b_1, b_2, b_s, b_{\nabla^2}\}$, summing to four parameter for every redshift bin, thus twelve in total. These parameters can be considered at a first glance as “ancillary”, “nuisance” terms. They nevertheless possess some cosmological and physical relevance related to the potential coevolution relations they imply and will be subject of a specific discussion in sec. 1.2.1.

The data analysis proceeds following a rather standard pipeline: we set ourselves in a Bayesian framework (see sec. 5.2.6 for more detail) assuming a Gaussian likelihood. Given the scales of interest, we can safely assume that our data are indeed Gaussianly distributed, the normality of the probed galaxy and CMB fields being supported, next to their intrinsic properties, by the widesky area covered by the data sets used) (Hamimeche & Lewis, 2008) and by the scale cuts. Potential non-Gaussian fluctuations derived from some regions in the sky get averaged out by the multiple samplings of each multipole ensured by the large sky fraction, while intrinsic non-Gaussianity in the LSS is not yet relevant at the scales of interest.

We can then construct the log-likelihood:

$$-2 \log p(\mathbf{d}|\vec{\theta}) = \left(\mathbf{d} - \mathbf{t}(\vec{\theta})\right)^T \text{Cov}^{-1} \left(\mathbf{d} - \mathbf{t}(\vec{\theta})\right) + K. \quad (5.15)$$

In this expression, $\mathbf{d}$ is the data vector (made by our galaxy auto-spectra and galaxy-lensing cross-spectra in the different redshift bins), $\mathbf{t}$ is the parameter-dependen theoretical model, where the parameter vector is condensed in $\vec{\theta}$, and constructed using the model described in sections 5.2.1 and 5.2.2, Cov is the covariance matrix of $\mathbf{d}$, whose estimation we outlined in section 5.2.3, while the negligible K term acts as a normalisation constant.

The posterior distribution for our parameters will be recovered as a function of the data likelihood and of a number of priors we introduce on the model parameters (summarised in table 5.1). Concerning the nuisance parameters, we assume Gaussian priors for the linear, quadratic, and tidal bias parameters, centred around 0 with a standard deviation of 5, and a broader Gaussian prior for the non-local bias term b_{∇^2} with zero mean and a standard deviation of 10 Mpc^2 ; the choice is due to the greater potential variability range and minor reliability of a guess we may reasonably perform on the values of a parameter accounting for the “blurring” and loss of coherence at small scales induced by the combined effect of baryon processes and systematic smoothing (Zennaro et al., 2023). The use of Gaussian priors allows us to use the approximate analytic marginalisation method (AAM), whose formalism is outlined in section 5.2.6, the large variance ensuring that we are not exceedingly driving our constraints towards prior-dominated converged values. We now concentrate on the two cosmological parameters, σ_8 and Ω_m . We can thus impose a uniform flat prior for σ_8 , while two cases for Ω_m are contemplated:

- Free Ω_m : in this case, a flat prior in the range $0.1 \leq \Omega_m \leq 0.5$ is set. This way, a substantially free parameter space exploration is allowed and we can recover the value of Ω_m our data prefer. In general, we find that these constraints are chiefly influenced by the overall shape of the galaxy auto-spectrum, not by specific features.

- Ω_m upon use of BAO constraints: in this second case, a Gaussian prior on Ω_m is imposed. We derive it from the the BAO reported by DESI (Abdul Karim et al., 2025): $\Omega_m = 0.2975 \pm 0.0086$ (see also chapter 4 and the discussion of the constraining power of the LSS data on Ω_m , σ_8 and S_8 . The prior is so narrow to be almost equivalent to keeping fixed the value of Ω_m . We can thus break the partial degeneracy between σ_8 and Ω_m generated by measuring in the harmonic space clustering and lensing data.

In our data analysis process, the MCMC sampling of the parameter space might reach, because of the broad priors, values that are beyond the **nominal training range** of the **baccoemu** emulator. To prevent the code from stopping, we will have therefore to extrapolate the calculation of the power spectra according to a strategy described in section 5.2.5 and relying on some approximations we will outline. Controlling and limiting the excessive variability of the value of Ω_m will thus also allow us to avoid potential important errors that might be introduced if, when the HEFT power spectrum templates are extrapolated, we were sampling small values for Ω_m , lying outside of the **baccoemu** parameter training range. It can be observed that the tight prior on Ω_m , associated to a value for the parameter larger than the one we obtain in the prior-free analysis, can be interpreted as selecting a value for the matter density that might be artificially corresponding to a value of the density parameter at higher redshifts (since the density of matter decreases with redshift), influencing the measurement of σ_8 that we are instead aiming to measure at low redshift through our datasets. This is however not the case, since the DESI prior on Ω_m is however derived from BAO measurements derived from multiple samples, including data at $z < 0.6$, and it is compatible with other estimations of Ω_m in the local Universe (see sec. 4.2.1). The resulting constraints on σ_8 will therefore still be chiefly informed by low-redshift data, but at the same time basing our measurements on a more robust power spectrum feature capable of constraining Ω_m ; when instead the prior on Ω_m coming from the BAO is not used, the overall functional form of the power spectrum determines the measurement of Ω_m , at a quite weaker constraining power. To better understand this concurrence of effects, it is interesting to look at figs. 5.5 and 5.6, where the dependence of the three-dimensional power spectrum $P(k)$ and of the CMB are shown, comparing the Λ CDM prediction with some extreme combinations of the parameters both at the level of spectral shapes (upper panels) and of ratios with respect to the Λ CDM power spectrum (lower parameters). Now, we see from the left panel that, in general, variations in Ω_m modify both the shape and the slope of the power spectrum, but chiefly at very large scales (small k), while intermediate and non-linear scales are quite comparable with the Λ CDM prediction even for important variations of the Ω_m value. On the other hand, σ_8 acts as a normalisation of the power spectrum, shifting it consistently up and down with respect to the reference Λ CDM cosmology (middle panel). When modifying both parameters together, corresponding to our case (right panel) we see again that variations in σ_8 more decidedly distinguish among models, whereas large variations on Ω_m still maintain similar power spectra. Similar reasonings hold for the CMB signal of fig. 5.6, but we can appreciate the greater sensitivity to Ω_m provided by the BAO (the oscillatory pattern in the power spectra). Our sampling might thus converge towards unphysical values for Ω_m , not compatible with any other probe, even measuring the actual σ_8 . With the BAO prior, we ensure instead that, in any case, also the recovered value for Ω_m is physically meaningful and concordant with other independent probes of its value.

We can then calculate constraints on $S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$, defined, as we know, in order to break the remaining degeneracy of σ_8 with Ω_m . Concerning the cosmological parameters we will

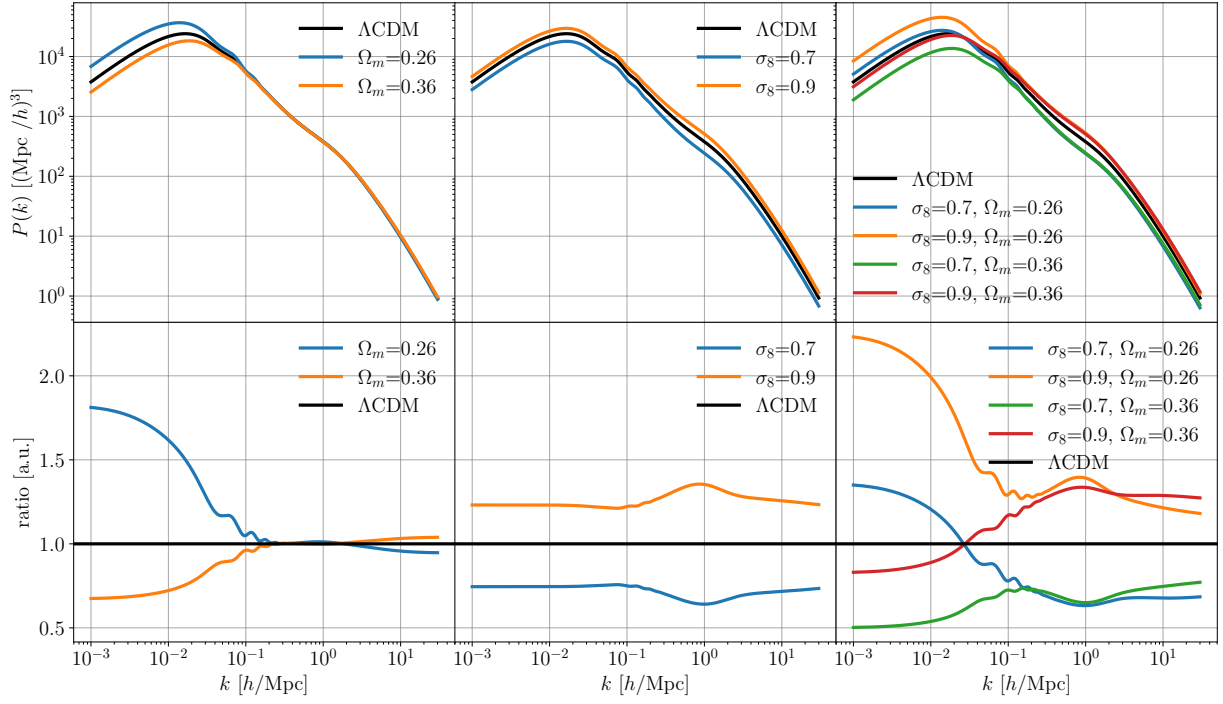


Figure 5.5: Sensitivities of the three dimensional power spectra (calculated at $z = 0$) to the σ_8 and Ω_m parameters compared with the reference Λ CDM power spectrum (black). The power spectra are displayed in the upper panels: from left to right we can recognise the typical modification in the shape and slope induced by Ω_m (left panel), the modification in the normalisation as induced by σ_8 (middle panel), and the combined parameters together (right panel). In the lower panels the ratio of the displayed power spectra with respect to the Λ CDM model are displayed (black).

keep fixed at reference values provided by CMB and LSS measurements because impossible to constrain with our data, we decide to work in the baseline Λ CDM cosmological mode, with massless neutrinos ($\sum m_\nu = 0 \text{ eV}$). Since also neutrinos induce processing at the smallest scales that might be degenerate with the parameters we aim to measure (Di Valentino et al., 2024), we checked that our cosmological constraints vary by less than 1% if setting the sum of neutrino masses to the lower bound from neutrino oscillation experiments $\sum m_\nu = 0.06 \text{ eV}$. The remaining cosmological parameters are in turn kept at values which are either Planck-compatible or directly derived from CMB measurements. In particular, the physical baryon density parameter is kept fixed together with the scalar spectral index. For both cases we use the reference parameters from Planck, namely, $\Omega_b h^2 = 0.02236$, $n_s = 0.9649$. We further set $\Omega_m h^3 = 0.09633$, a combinations akin to the the angular acoustic scale, constraining the value of the rescaled Hubble constant h at every given Ω_m .

Other important details regarding the construction of our analysis configuration are represented by the scale cuts on the power spectra: we will consider only measurements corresponding to physical scales $k < k_{\text{max}}^{\text{HEFT}} = 0.42 \text{ Mpc}^{-1}$ (corresponding to $\sim 0.6 h \text{ Mpc}^{-1}$ for $h \simeq 0.7$), where the HEFT theory is known to remain accurate (Kokron et al., 2021; Zennaro et al., 2023; Hadzhiyska et al., 2021; Pellejero Ibañez et al., 2023; Nicola et al., 2024; ThBeyond-2pt Collaboration et al., 2025). These get translated into a maximum angular multipole $\ell_{\text{max}} \equiv \chi(\bar{z}) k_{\text{max}}^{\text{HEFT}}$, with $\chi(\bar{z})$ the comoving distance calculated at the mean redshift for our galaxy samples, having found the best-fit Planck cosmology. Concerning the largest scales, we also remove the largest angular scales, with $\ell < \ell_{\text{min}} = 20$ to avoid biases in the galaxy auto-correlation from large-scale systematics, which

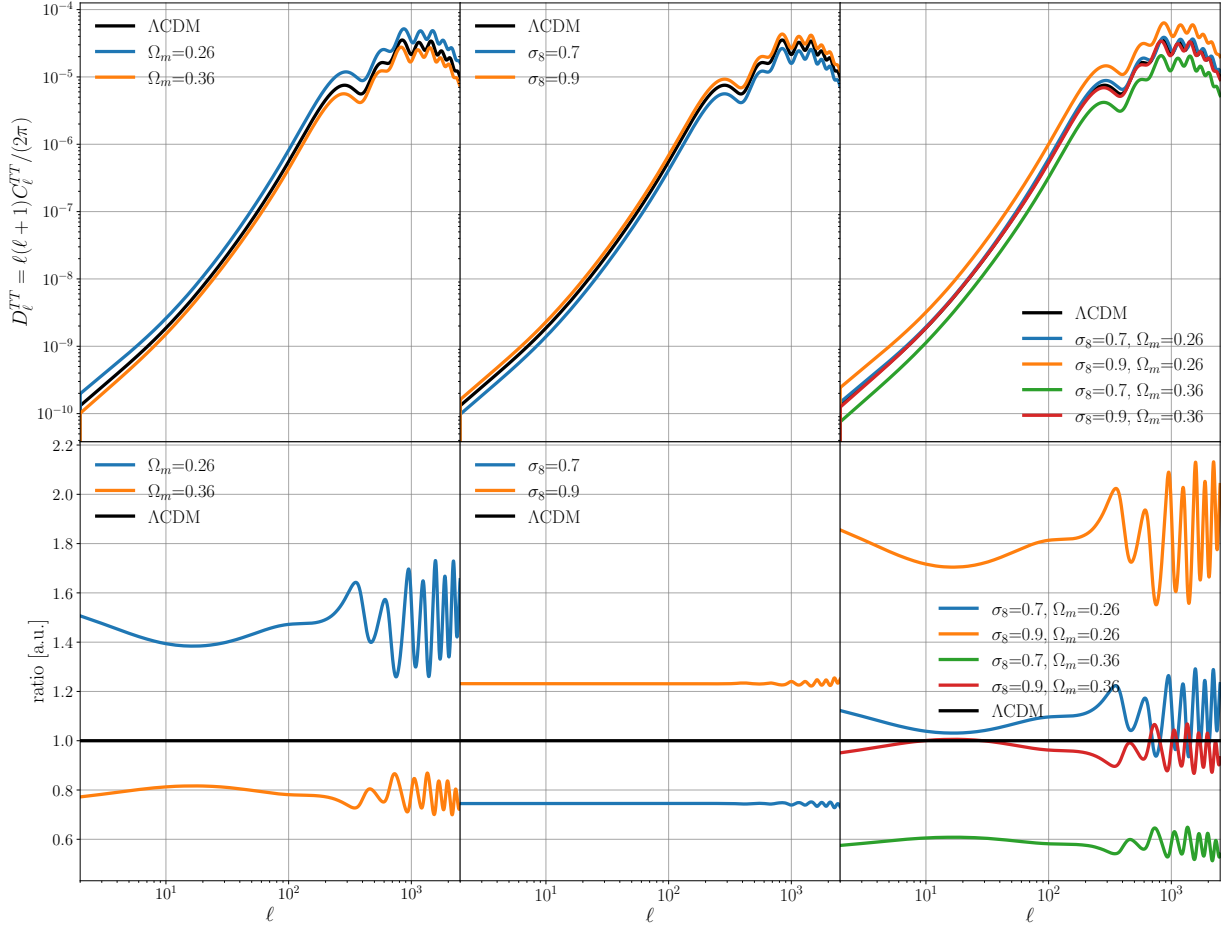


Figure 5.6: Sensitivities of the CMB temperature fluctuation power spectrum to the σ_8 and Ω_m parameters compared with the reference Λ CDM power spectrum (black), analogously to fig. 5.5

are particularly relevant for the WISE $\times$ SuperCOSMOS sample (Bilicki et al., 2016). In general, upon cross-correlation analysis between galaxies and CMB lensing, the scale cuts for the galaxy power spectra are the dominating ones, as they are more conservative than those in section 5.1.2.

5.2.5 HEFT emulator extrapolation

The choice of wide, uninformative flat priors for Ω_m and σ_8 comes at the price of being beyond the parameter range attainable by the **baccoemu** emulator combined with its training on the HEFT power spectrum templates (see sec. 1.3.2). In order to obtain reliable estimates of the HEFT templates outside this range we extrapolate beyond this range using a controlled Taylor expansion at first order (Hadzhiyska et al., 2021). Namely, if $\vec{\theta}$ lies outside of the **baccoemu** training range, the P_{ij} template can be estimated as

$$P_{ij}(k, \vec{\theta}) = P_{mm}(k, \vec{\theta}) \left(\mathcal{R}_{ij}(k, \vec{\theta}_b) + \nabla_{\vec{\theta}} \mathcal{R}_{ij}|_{k, \vec{\theta}_b} \cdot (\vec{\theta} - \vec{\theta}_b) \right), \quad (5.16)$$

having set $\vec{\theta}_b$ as the closest point to $\vec{\theta}$ still included in the emulator range, while we construct the ratio

$$\mathcal{R}_{ij}(k, \vec{\theta}) \equiv \frac{P_{ij}(k, \vec{\theta})}{P_{mm}(k, \vec{\theta})}. \quad (5.17)$$

We define $P_{mm}(k, \vec{\theta})$ as the matter power spectrum and we observe that we decide to perform the extrapolation on the ratio $\mathcal{R}_{ij}$, whose dynamic range and parameter dependence is weaker

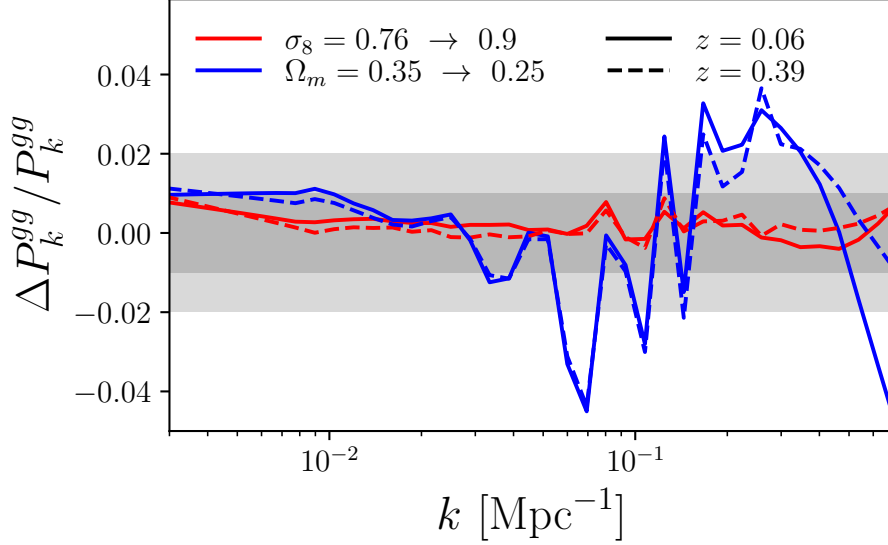


Figure 5.7: Validation and relative error on the the galaxy power spectrum obtained when applying the extrapolation scheme to the HEFT power spectrum templates, as a function of the cosmological parameters. We display with red and blue lines the extrapolation results on σ_8 and Ω_m , respectively, while the gray bands corresponds to the 1% and 2% limits in the potential extrapolation error. The legend displays the starting and end values for the extrapolation on each of the two cosmological parameters. The solid and dashed lines correspond instead to the extrapolation outcomes calculated respectively at $z = 0.06$ and 0.39

than P_{ij} (for instance, most of the dependence on σ_8 gets absorbed in the ratio with P_{mm}). Similarly, it shows a less prominent scale dependence. To make sure that $P_{mm}(k, \vec{\theta})$ will be correctly evaluated outside the **bachoemu** training range, its template is based on the HALOFIT parametrisation (Takahashi et al., 2012), having the linear power spectrum calculated by means of the parametrisation proposed by (Eisenstein & Hu, 1998) to ensure a fast evaluation of the likelihood. We made a further comparison between the use of the HALOFIT prescription and the result coming from the use of a Boltzmann solver, but only small, sub-percent deviations, in the recovered cosmological constraints are found. An additional improvement of the quality of the extrapolated templates can be obtained by modifying the definition of $\mathcal{R}_{ij}$ for the HEFT templates that are positive-definite (e.g. $P_{\delta^2\delta^2}(k)$). In this case, $\mathcal{R}_{ij}$ is replacing by the logarithm. In this second case, the extrapolation scheme would read:

$$P_{ij}(k, \theta) = P_{mm}(k, \vec{\theta}) \exp \left[\left(\tilde{\mathcal{R}}_{ij}(k, \vec{\theta}_b) + \nabla_{\vec{\theta}} \tilde{\mathcal{R}}_{ij}|_{k, \vec{\theta}_b} \cdot (\vec{\theta} - \vec{\theta}_b) \right) \right], \quad (5.18)$$

with $\tilde{\mathcal{R}}_{ij} \equiv \log \mathcal{R}_{ij}$.

Validation of the extrapolation

The extrapolation accuracy is quantified by comparing the exact value of the galaxy power spectrum as calculated at one end of the **bachoemu** training range with the extrapolated power spectrum calculated at the opposite end of the range, after applying the above-outlined methodology: we are thus checking the validity of the first-order Taylor extrapolation using the maximum allowed excursion within **bachoemu**. We might expect the width of the training range to be larger than the maximum distance between the emulator’s limit and the maximum and minimum sampled

values reached by the MCMC algorithm, and therefore a validation strategy of this kind is quite orientative of the maximum error with respect to the unknown actual power spectrum we would calculate at those parameter values. The corresponding bias value is thus $1 + b_1 = 1.3$, found to be consistent with the values of the large-scale bias parameters in the samples studied here, whereas the higher-order bias terms are set to the values one would obtain by using the coevolution relations of (Zennaro et al., 2022) for this value of b_1 . Fig. 5.7 shows the percentage errors (relative difference) between the true and extrapolated power spectra if we extrapolate in an extreme range for σ_8 and for Ω_m . A general observation is that the extrapolation shows more variability in terms of scales k rather than of the redshift z . The extrapolation on σ_8 appears more stable, with very high accuracy in most scale regimes. The extrapolation in Ω_m appears to be weaker, although at most displaying discrepancies of $\sim 4\%$, in particular close to the BAO scale ($k \sim 0.1 \text{ Mpc}^{-1}$). Most likely, such fluctuations should not have strong consequences on the precision of the constraints we will find, but can be considered a further justification for the imposition of a BAO prior on Ω_m discussed in sec. 5.2.4 to ensure a more robust estimation of the power spectra, compensating for errors in the templates extrapolations beyond `baccoemu` limits.

5.2.6 Improving marginalisation on nuisance parameters with profile likelihood techniques

Before proceeding with the description of our analysis and the exposition of the physical results and of some additional technical innovations next to the ones we described concerning the processing of the raw data and the extrapolation of the theoretical power spectra templates, it is interesting to present a broader discussion of some foundational methodological aspects. As we have seen above, current data analysis pipelines in Cosmology rely of preference on Bayesian methodologies, usually backed by Monte Carlo Markov Chain (MCMC) computational techniques. These methods and approaches are not unique, although predominant and usually also the first exposed as textbook introductions to data fitting tasks: reviews of the broader landscape about the topic, sometimes reflecting different “historical” stages of the discipline and the availability of computational power at the time can be found in the following works (Trotta, 2017; Hobson et al., 2014; Abdalla et al., 2022; Di Valentino et al., 2025; Heavens, 2009; Martinez et al., 2004), presenting a broader and deeper discussion of conceptual frameworks, comparing Bayesian and frequentist approaches, model selection procedures and different algorithms.

Bayesian methods in astrophysics and cosmology have started to encounter a particular favour at the turn of the Millennium, motivated by a number of scientific reasons (Trotta, 2017). In particular, research has started to focus on very weak signals and effects, for which sometimes multiple concurring explanatory models are available, so that decisions have to be taken based on external information and criteria beyond basic goodness-of-fit metrics. Furthermore, models in astrophysics and cosmology have started to rely on rather heavy parametrisations. Free parameters can be partitioned in two main classes: the subset of quantities of actual physical interest (e.g., in a cosmological context, the Λ CDM model parameters) on one hand and, on the other hand, the often larger subset of auxiliary parameters, spanning from quantities capturing instrumental systematics to those describing concurring astrophysical phenomena intervening in the modelling of the cosmological tracer. All these parameters, although required for the complete model specification, are of little or no interest for the main analyses and are indeed usually known as “nuisance parameters”. A relevant problem related to this is that large parameter sets often induce

degeneracies between nuisance parameters and parameters of interest, non-Gaussian behaviours of the data likelihood and other spurious effects (Trotta, 2017; Hobson et al., 2014; Di Valentino et al., 2025; Hadzhiyska et al., 2023), sometimes even hindering the convergence of the likelihood minimisation process, or driving the likelihood towards local minima rather than as close as possible to the global minimum.

A Bayesian inference framework comes thus in handy since it allows to readily implement degrees of belief and sensibility assumptions on the values attained by parameters, on their probability distributions and uncertainties size in terms of “prior” probability distributions that can be combined with the usual data likelihood distribution by means of Bayes theorem. In this alternative formulation of statistics and probability, the outcome of a Bayesian analysis is a probability distribution for the data – known as the “posterior” distribution $p(\boldsymbol{\theta}|\mathbf{d}, \mathcal{M})$ – corresponding to the probability of the parameters of some model $\mathcal{M}$ to belong to a collection of values $\boldsymbol{\theta}$, given the set of data $\mathbf{d}$. The parameter posterior distribution can be written as a function of the priors $\pi(\boldsymbol{\theta}, \mathcal{M})$ and of the likelihood $\mathcal{L}(\mathbf{d}|\boldsymbol{\theta}, \mathcal{M})$ by means of Bayes theorem

$$p(\boldsymbol{\theta}|\mathbf{d}, \mathcal{M}) = \frac{\mathcal{L}(\mathbf{d}|\boldsymbol{\theta}, \mathcal{M}) \pi(\boldsymbol{\theta}, \mathcal{M})}{\mathcal{Z}(\mathbf{d}, \mathcal{M})}, \quad (5.19)$$

where $\mathcal{Z}(\mathbf{d}, \mathcal{M})$ is the model evidence, i.e., the probability of the data distribution unconditional on the parameters. As such, this quantity is usually treated as an overall normalization of the posterior and neglected in usual data analysis tasks (its logarithm is a function of the constant negligible term K defined above in eq. 5.15) although it is a relevant quantity in *model-selection* procedures (Heavens, 2009, 2016).

It is important to remark that the Bayesian approach avoids *in full* the problem of minimising the likelihood: we **do not** look for a stationary point in the parameter space associated to a single value of the likelihood function; rather, we recover for each parameter its full probability distribution, *interpreting* some of its statistical moments (such as the mean or the median) or some feature of the associated likelihood probability distribution as related to a form of *best-fit* values for the parameters, as well as the quantiles of the posteriors can be connected to the traditional parameter uncertainties, not necessarily corresponding to Gaussian standard deviations if the posterior distribution does not follow a normal density function. At the same time, the Bayesian approach increases our control over biasing effects and convergence towards undesired local minima: the use of priors can indeed be interpreted computationally as a conditioning strategy of the likelihood, ensuring that the final posterior distribution is more likely centred around the values (and, in particular, it should allow to recover those points we deem as best-fit values according to some criterion) we can either safely consider as “correct”. This can be understood especially in a forecasting task, where we want to check that a given analysis strategy or model recovers the fiducial quantities from which synthetic data are generated: since we know the ground truth values for the parameters, we can implement priors in order to control convergence of the data analysis towards them. Similarly, when actual data analyses are performed, like in this chapter, not necessarily do we know the true values of the parameters (or we might be happy to challenge established consensus), but we could rely on priors in order to help the outcome of often non-linear and complex fitting functions dealing with noisy data to converge towards parameter values we are allowed to deem as “more reasonable” in the deepest spirit of Bayesian statistics. At the same time, the size and functional form of the prior probability densities is left free to the researcher, so that informing the likelihood does not necessarily mean heavily restricting the parameter space, and it is also possible not to introduce any prior at all on some parameters. The kind and degree of

information conveyed by priors can come from different sources, and we have again a ready example in the previous sections of this chapter: for example, by combining different datasets, observables and experiments. If, for instance, we know that our measurements are unable to properly measure some parameter, but, rather, a combination of it with other quantities, external likelihoods derived from different observables and instead capable of constraining the target quantity can be used, in order to break some of the parameter degeneracies or reduce the variance of the posterior distribution within more physically justified limits.

The use of Monte Carlo Markov Chain techniques for data analysis in Cosmology

The usual final step of a Bayesian data analysis exercise is the marginalisation procedure over eq. 5.19 for each parameter of the dataset: this step allows one to calculate the posterior distribution for every single parameter Θ conditional on the data but unconditional of all the other parameters, or, at least, of all those parameters we deem as “nuisance parameters”. If we condensate the nuisance parameters in the vector α of size N , the marginalisation procedure reads:

$$p(\Theta | D) = \int p(\Theta, \alpha | D) d^N \alpha = \int p(\Theta | \alpha, \mathbf{d}, \mathcal{M}) d^N \alpha = \int \frac{\mathcal{L}(\mathbf{d} | \Theta, \alpha, \mathcal{M}) \pi(\Theta, \alpha, \mathcal{M})}{\mathcal{Z}(\mathbf{d}, \mathcal{M})} d^N \alpha. \quad (5.20)$$

Eq. 5.20 closes the circle between Bayesian inference, likelihood minimisation and computational techniques in cosmological data analysis. Indeed, the evaluation of the integral of eq. 5.20 can be performed by means of suitable analytic (in the rare cases of a closed-form solution for the problem) or, more often, numerical, techniques. In particular, it is known that Monte Carlo techniques are suitable tools to evaluate complex and multidimensional integrals (given the connection of this strategies with random numbers, the integration procedure is often called “sampling”) (Press et al., 1986; Hammersley & Handscomb, 1964; Rubinstein, 1981). The problem becomes thus that of individuating fast and reliable methods to perform the computation. Monte Carlo Markov Chain (MCMC) algorithms are among the favourite ones: these techniques represent a whole and diverse landscape, as shown in fig.5.8.

Out of them, the most standard algorithms, associated to many packages of daily use in the field, are those belonging to the Metropolis-Hastings (Metropolis & Ulam, 1949; Hastings, 1970) technique implemented for instance in `cobaya` (Torrado & Lewis, 2019, 2021) and `MontePython` (Brinckmann & Lesgourgues, 2019) and its affine-invariant version (Goodman & Weare, 2010; Christen & Fox, 2010) implemented in `emcee` (Foreman-Mackey et al., 2013) – devised to better cope with highly degenerate parameters thanks to the property of affine invariance. Also relevant are the Hamiltonian (Duane et al., 1987; Hoffman & Gelman, 2011) methodology implemented e.g. in `numpyro` (Phan et al., 2019; Bingham et al., 2019) and the nested sampling algorithm (Skilling, 2004) available for example in `Polychord` (Handley et al., 2015) and allowing for an interface with `Cobaya`.

We can briefly sketch the simplest and seminal Metropolis-Hastings algorithm, that can be understood as follows: given an initial proposed starting point for each parameter value, the parameter space is explored sequentially, one step after the other, in a *chain* of states. The goal to be achieved step after step is to obtain a decorrelated posterior distribution, insensitive of initial conditions and step selection choices. Once this posterior distribution is reached, the posterior probability distribution is stationary: that is, it does no longer vary after further sampling steps, effectively becoming time-invariant and therefore converged. Assessment of convergence can be

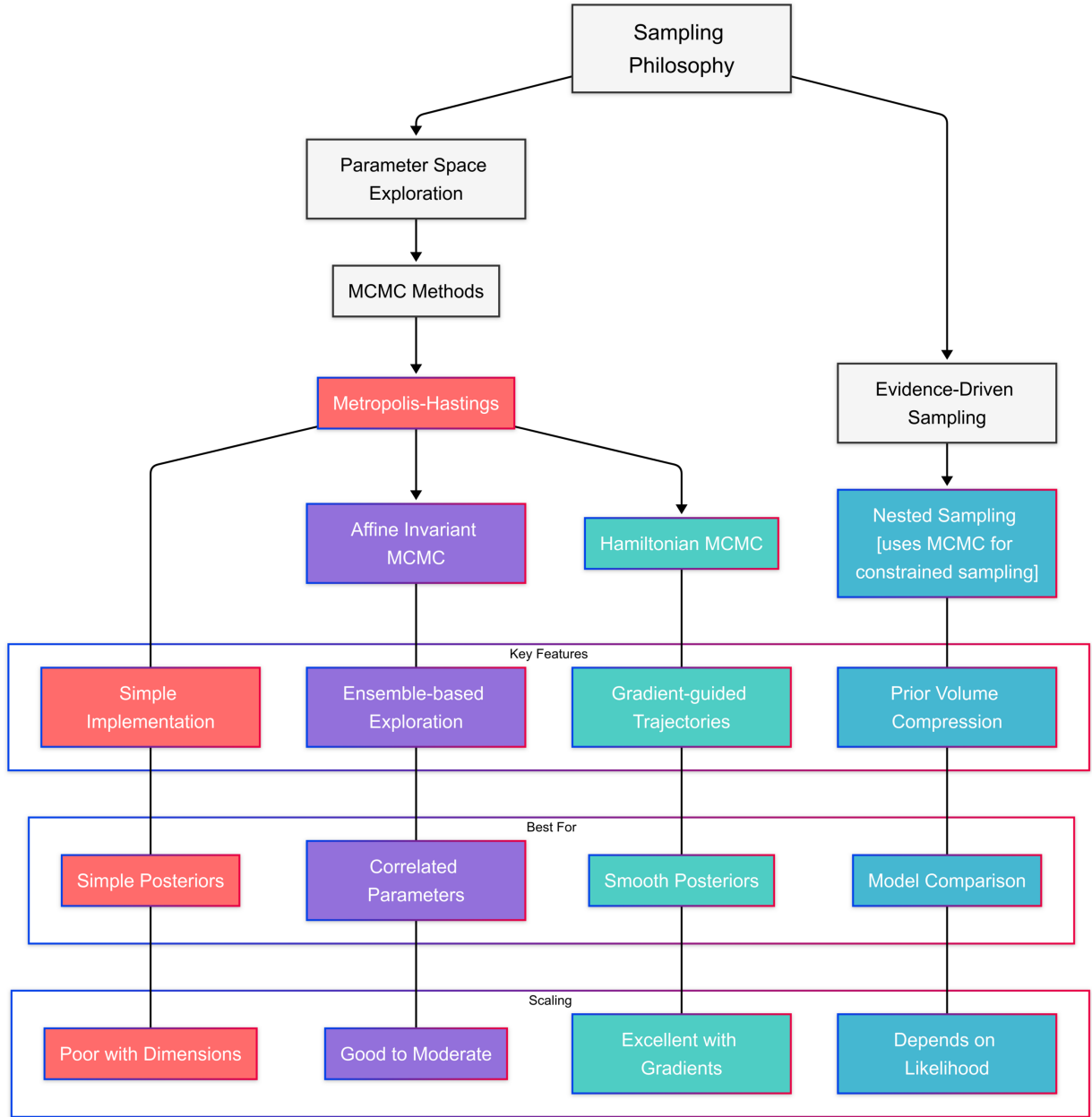


Figure 5.8: The MCMC techniques landscape in cosmology. The flow chart is drawn from (Di Valentino et al., 2025)

usually performed with suitable stopping conditions, such as the rather standard Gelman-Rubin criterion (Gelman & Rubin, 1992).

Most of the differences among the algorithms reside in the kind of criteria behind the step choice, the ways to achieve decorrelation among the steps and the way parameters are accepted or rejected step after step, and thus the assessment of whether the algorithm is moving towards the stationary posterior distribution or not. The Metropolis-Hastings prescription updates the state σ (corresponding to a set of values for the model parameters and, possibly, of other hyperparameters required to fully evaluate the posterior distribution) of the parameters' sampling paths to a new state σ' , that is, a new draw of parameters values extracted from the proposal density $q(\sigma'|\sigma)$. The proposal density is conditional, namely, a function, exclusively of the previous step σ and of no other previous steps in the chain. Since Markov Chains are defined as stochastic processes whose transition probability from a state A at the time-step $t = n$ to a state B at the time-step $t = n + 1$

is at most a function of the state in t and of no “older” states in the chain, the construction of the algorithm matches the Markov Chain condition (Chib & Greenberg, 1995). The new state is accepted with probability

$$\Pi = \min \left(1, \frac{\phi(\sigma') q(\sigma | \sigma')}{\phi(\sigma) q(\sigma' | \sigma)} \right), \quad (5.21)$$

where $\phi(\sigma)$ is the target posterior distribution. The value of Π becomes the weight of the posterior distribution $\phi(\sigma')$ (loosely speaking, the posterior probability to be assigned to the value that each model parameter receives at the state σ'). The ratio in eq. 5.21 can be interpreted as a rejection rate, since, if the quantity is smaller than 1, we can consider the value assumed by the parameter in the new state less likely than the one it assumed in the previous state. In the end, the posterior distribution can be obtained as a weighted sum of all explored states σ'_t at the time-step t with weight Π_t (and again, we can see that states associated to a $\Pi_t \sim 1$ will be more represented in the posterior, and thus associated to the peaks of their distribution. This kind of stochastic processes is usually ensured to converge to a stationary distribution, namely, that the steps have substantially sampled the whole posterior distribution and that, by collecting the weighted values of the states σ , we have obtained a representative estimator of the actual posterior distribution (which we will never actually obtain) (Chib & Greenberg, 1995). The proposal distribution can also be defined by the user, e.g. as a multivariate Gaussian distribution, – together with other algorithm-specific hyperparameters. Similarly, the basic algorithm requires the steps to be updated in terms of draws taken from a normal distribution for which we specify average and variance (or a vector of mean values and a covariance matrix in case we sample multiple parameters), although more sophisticated techniques to accelerate convergence and shorten correlation length in the chain – namely, the number of steps it takes to the chain to no longer be influenced by the initial proposal distribution – can be devised. MCMC algorithms do not exhaust the inventory of techniques, but can still be considered as the standard tool for most of cosmological research. The number of approaches has grown in the last years to face their limitations, especially in terms of convergence speed and, most importantly, in robustness of the outcome, independence from the prior and other reliability requirements, a pressing requirement since the reliability of likelihood minimisation algorithms and model interpretation or selection criteria could play a role among the systematics affecting current open problems and tensions in the field (Di Valentino et al., 2025).

Combining MCMC sampling and likelihood minimisation: the profile likelihood approach

As we described above, cosmological models can quickly get overparametrised: this is particularly true when dealing with non-linear LSS statistics based on LPT or SPT approaches, since the number of free parameters, either to be interpreted as bias terms or as free coefficients in the series expansion, can scale up rather quickly, inducing undesired effects spanning from the sheer increase in chain convergence time to heavy degeneracies among parameters and possible strong biases in the inferred best-fit values due to dependence on the size and shape of the nuisance parameter prior or on “ancillary” cosmological parameter priors one has to define to break degeneracies (Simon et al., 2023; Carrilho et al., 2023), as well as other classes of drawbacks known as “volume effects”. Volume effects (Hamann et al., 2007) are typically induced by complex likelihood shapes, by the model parametrisation and choice of functional form itself, or by the interplay between the unconditional likelihood and the prior distributions it gets combined with: they can heavily bias the inference of cosmological parameters, preventing one to recover the correct, fiducial

value of a parameter, or, more generally, hindering the convergence towards the global best-fit value of the analysis by pushing the parameters towards regions where the bulk of the posterior probability (hence the name), but not its maximum, resides: a typical case for this undesired phenomenon is when the maximum is enclosed in a narrow and strongly peaked region or in a noisy one whose integration measure is rather small. The research work described in this chapter indeed uses LPT to evaluate the non-linear galaxy bias parameters: it has therefore proved to be a suitable workbench for testing a computational improvement of the data analysis pipeline based on the marginalisation technique presented in (Hadzhiyska et al., 2023) and known as “profile likelihood” methodology. The idea is that of *completely* avoiding the MCMC sampling of the nuisance parameters – corresponding in this case to the subset of HEFT-LPT bias parameters, but additional parameters might be contemplated in other contexts –, and to marginalise over these latter quantities through a semi-analytic approach. The exact procedure will be described below, but the basic intuition can be exposed as follows: the MCMC sampling is performed as usual **exclusively on the cosmological parameters** of interest, σ_8 and Ω_m ; as long as the chain progresses, it converges towards the stationary distribution for the cosmological parameter posteriors, at the global likelihood maximum, conditional on the values the bias parameters will attain. We then have to find the actual global maximum: we therefore have to complete the set of best-fit parameters by considering the nuisance parameters, and, eventually, to recover the cosmological parameters posteriors marginalised over (made unconditional of) the best-fit values we find for the nuisance parameters. To do so, the methodology we employ partially falls back again to a maximum likelihood perspective: we indeed look for the nuisance parameter values that solve an actual maximisation problem. To do so, instead of using the MCMC algorithm, the likelihood is semi-analytically maximised, keeping the cosmological parameters fixed at the sampled value and looking for the values of the bias parameters associated to the zeros of the likelihood first derivatives with respect to the nuisance parameters. Once this step is done, we can fill the vector of parameters of interest θ (that, after a number of steps, will have reached their stationary, converged values) and the vector of nuisance parameters α . The marginalisation step of eq. 5.20 can be implemented together with the minimisation step, having as a final outcome the Bayesian posterior distribution for the cosmological distribution combined with a frequentist maximum likelihood point-like estimate of the nuisance parameters. Since the estimation of the best-fit value of the nuisance parameters switches from the calculation of a posterior distribution to the calculation of a unique, point-like, optimum value, volume effects that can be described in terms of a second-order Taylor series expansion of the likelihood around the cosmological parameters values are largely neutralised.

To explicitly show this approach, we start again from Bayes theorem with the vector of model parameters θ and the vector of data $\mathbf{d}$. Recalling the posterior distribution

$$p(\theta|\mathbf{d}) \propto p(\mathbf{d}|\theta)p(\theta), \quad (5.22)$$

with $p(\mathbf{d}|\theta)$ our likelihood distribution and $p(\theta)$ the prior distribution for all of our parameters, we split the model parameters as $\theta = \{\Omega, \mathbf{n}\}$, where Ω is the set of parameters we care about and $\mathbf{n}$ is a vector of nuisance parameters. We then define the Gaussian log-likelihood, the logarithm of eq. 5.22, expressed in the following way:

$$-2 \log p(\Omega, \mathbf{n}|\mathbf{d}) = (\mathbf{d} - \mathbf{t})^T \mathbf{C}^{-1} (\mathbf{d} - \mathbf{t}) + \chi_{p,\Omega}^2(\Omega) + \chi_{p,n}^2(\mathbf{n}), \quad (5.23)$$

where $\mathbf{t}(\boldsymbol{\Omega}, \mathbf{n})$ is the theory vector, which depends on the model parameters, $\mathbf{C}$ is the covariance matrix of the data, which we assume to be calculated at a fiducial set of parameter values or model-independent, in both cases not undergoing sampling in our calculations and $\chi_{p,\Omega}^2$ and $\chi_{p,n}^2$ are the parameter priors. Notice that the presence of the minus sign in eq. 5.23 makes us actually *minimise* the log-likelihood with a minus sign in front of it, but this is a standard practice in maximisation problems: for convex functions such as a Gaussian log-likelihood, the maximisation procedure corresponds to minimising the opposite of the log-likelihood, in agreement with the theoretical and computational literature, usually solving minimisation rather than maximisation problems.

The methodology, in order to ensure analytic calculation, assumes that the nuisance parameters have Gaussian priors, and that, therefore, it holds:

$$\chi_{p,n}^2(\mathbf{n}) = (\mathbf{n} - \mathbf{n}_p)^T \mathbf{C}_n^{-1} (\mathbf{n} - \mathbf{n}_p), \quad (5.24)$$

where $\mathbf{C}_n$ is the prior covariance: the assumption can be considered however general enough given the availability, in the case of non-Gaussian priors, of transformation methods leading to Gaussianisation of the prior distributions (Hadzhiyska et al., 2023). We now aim to obtain the marginalised posterior distribution for $\boldsymbol{\Omega}$: this requires to integrate the conditional posterior distribution on the nuisance parameters space $\mathbf{n}$:

$$p(\boldsymbol{\Omega}|\mathbf{d}) = \int d\mathbf{n} p(\boldsymbol{\Omega}, \mathbf{n}|\mathbf{d}). \quad (5.25)$$

To proceed, it is useful to first define the chi-squared distribution depending on the whole model parameter set $\chi^2(\theta)$ in terms of the posterior distribution

$$\chi^2(\boldsymbol{\Omega}, \mathbf{n}) \equiv -2 \log p(\boldsymbol{\Omega}, \mathbf{n}|\mathbf{d}) + K, \quad (5.26)$$

where K is an arbitrary constant independent of $\boldsymbol{\theta}$, so that we can refer to the marginalisation problem as a marginalisation over the χ^2 :

$$p(\boldsymbol{\Omega}|\mathbf{d}) \propto \int d\mathbf{n} e^{-\frac{1}{2}\chi^2(\boldsymbol{\Omega}, \mathbf{n}|\mathbf{d})}. \quad (5.27)$$

Assuming that we are MCMC sampling the cosmological parameters of interest $\boldsymbol{\Omega}$ and that they are converging towards their stationary posterior distributions, to complete the set of best-fit parameters, we have to maximise the posterior distribution with respect to the remaining nuisance parameter subset $\mathbf{n}$ according to:

$$\mathbf{n}_*(\boldsymbol{\Omega}) \equiv \arg \max_{\mathbf{n}} p(\boldsymbol{\Omega}, \mathbf{n}|\mathbf{d}). \quad (5.28)$$

Thanks to the link between the χ^2 and the posteriors defined in eq. 5.26, the problem of eq. 5.28 can be restated in terms of solving the following problem for the χ^2 function:

$$\left. \frac{\partial \chi^2}{\partial \mathbf{n}} \right|_{\mathbf{n}_*} = 0. \quad (5.29)$$

To proceed, we assume we can Taylor expand the integrand of eq. 5.27 to the second order around $\mathbf{n}_*$ keeping $\boldsymbol{\Omega}$ fixed. This translates into a Taylor expansion of the exponential as a function of the χ^2 evaluated in $\boldsymbol{\Omega}, \mathbf{n}_*$:

$$\chi^2(\boldsymbol{\Omega}, \mathbf{n}) \simeq \chi_*^2(\boldsymbol{\Omega}) + \Delta \mathbf{n}^T \mathcal{F}_* \Delta \mathbf{n}, \quad (5.30)$$

where $\chi_*^2(\boldsymbol{\Omega}) \equiv \chi^2(\boldsymbol{\Omega}, \mathbf{n}_*)$, its first derivative is null in $\mathbf{n}_*$ by construction, $\Delta\mathbf{n} \equiv \mathbf{n} - \mathbf{n}_*$, and we have introduced the matrix

$$\mathcal{F}_{*,ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial n_i \partial n_j} \Big|_{\mathbf{n}_*}. \quad (5.31)$$

This second order expansion is the so-called *Laplace approximation* (Kass et al., 1990). that will provide a simpler expression to integrate in order to recover the marginalised posterior distribution for the parameters belonging to $\boldsymbol{\Omega}$.

There is also a theoretical benefit, in that the Laplace approximation allows greater insight into the parameter behaviours: the magnitude of the different terms in eq. 5.30 allows indeed to circumscribe what effects drive the likelihood convergence towards possible incorrect parameter posteriors. We can further develop the first-order term of the Taylor expansion (eq. 5.29) and the second-order term (eq. 5.31) as a function of the data and theory vectors and of the covariances as

$$\frac{\partial \chi^2}{\partial n_i} = -2\partial_i \mathbf{t}^T \mathbf{C}^{-1}(\mathbf{d} - \mathbf{t}) + 2 \sum_j [\mathbf{C}_n^{-1}]_{ij} (n_j - n_{p,j}), \quad (5.32)$$

and introduce

$$\mathcal{F}_{ij} = \partial_i \mathbf{t}^T \mathbf{C}^{-1} \partial_j \mathbf{t} + [\mathbf{C}_n^{-1}]_{ij} + \partial_i \partial_j \mathbf{t}^T \mathbf{C}^{-1}(\mathbf{t} - \mathbf{d}), \quad (5.33)$$

with the shorthand $\partial_i \equiv \partial/\partial n_i$. We can then define:

$$F_{ij} \equiv \partial_i \mathbf{t}^T \mathbf{C}^{-1} \partial_j \mathbf{t} + [\mathbf{C}_n^{-1}]_{ij}, \quad (5.34)$$

$$\Delta \mathcal{F}_{ij} \equiv \partial_i \partial_j \mathbf{t}^T \mathbf{C}^{-1}(\mathbf{t} - \mathbf{d}), \quad (5.35)$$

$$\mathcal{F}_{ij} \equiv F_{ij} + \Delta \mathcal{F}_{ij}, \quad (5.36)$$

thus identifying two contributions to the χ^2 : a first order term (eq. 5.32, whose zeros we will be looking for), known as the *profile likelihood* term and a second-order term, known as *Laplace term* or *Occam's razor* term, in turn split into two addenda. The profile likelihood can be related to frequentist inference practices: since it does not depend on the nuisance parameter priors and the possibly volume effects they induce, it is by construction centred around the **correct** best-fit parameters (Cousins, 1995) values, that define the global maximum of the joint unmarginalised distribution of nuisance and cosmological parameters, free from volume effects (Hamann et al., 2007; Herold et al., 2022; Campeti & Komatsu, 2022). The conditional distribution being free from volume effects, we are therefore also sure that the marginal distributions of each parameter we can recover after the joint maximum-likelihood estimation will be unbiased and centred around its unconditional best-fit value. Concerning the Laplace term of eq. 5.36, the second addendum, $\Delta \mathcal{F}_{ij}$, which depends on both the theory vector $\mathbf{t}$ and the data $\mathbf{d}$, is usually negligible, regardless of the presence or absence of volume effect. The first contribution to $\mathcal{F}$, F , can be identified with the Fisher matrix (Heavens, 2009; Wittman, 2025; Heavens, 2016; Ly et al., 2017) of the problem under the assumption of a Gaussian likelihood and, as such, it is positive-definite, therefore invertible, and independent of the dataset $\mathbf{d}$. This latter property determines its connection with the volume effects: the term depends on the theory vector $\mathbf{t}$, in turn a function of the parameters, but not on the distance between the theory vector and the data $\mathbf{d}$: in other words, even if our sampling is not converging towards a model adequately close to the data, F , and hence the Laplace term $\mathcal{F}$, will take a non-zero value, possibly magnified by additional contributions from $\Delta \mathcal{F}_{ij}$. This is exactly the kind of fitting distortion effect sourced by volume effects: under their contribution, the sampling

strategy always yields a parameter set, typically located towards the region hosting most of the probability mass, but not necessarily towards the parameter region yielding the maximum of the likelihood function, that instead is reached when the *distance* between data and theory vectors is minimised. In presence of volume effects, the analysis would eventually converge towards a posterior distribution, but its maximum would be shifted with respect to the actual maximum likelihood value. Under the assumption of a Taylor-expanded posterior distribution, we can conclude this section providing a general analytic expression for the marginalisation problem of eq. 5.27: the marginalised posterior distributions for each parameter of interest belonging to the subset Ω will read

$$\chi_m^2(\Omega) \simeq \chi_*^2(\Omega) + \log \{ \det [\mathcal{F}_*(\Omega)] \} + \text{const.}, \quad (5.37)$$

with the further typical property

$$\log \det \mathcal{F}_* = \log \det (F_* + \Delta \mathcal{F}_*) \simeq \log \det F_*. \quad (5.38)$$

Looking at these results, positive-definiteness of the Fisher matrix ensures that the expression of eq. 5.38 is always a real number. In a typical data analysis exercise, a better fit is associated to smaller χ^2 values, so that the less the contribution from eq. 5.38, the sounder our conclusion that we have actually reached the best-fit set of parameter values, since a vanishing Laplace contribution implies a vanishing contribution from volume effects, which is what we are aiming for.

In general, the Laplace approximation is effective for sufficiently well-behaved distributions and can be numerically recovered in a rather straightforward way by means of well-established minimization and root-finding algorithms, such as the Newton-Raphson algorithm, aimed at solving eq. 5.32, thus finding the subset of nuisance parameters $\mathbf{n}_*$ that allow us to evaluate eq. 5.37. And now we come to the fundamental methodological improvement: if we assume, by experience or through previous tests, that the Laplace term *will* be large and that volume effect will actually occur in the fitting procedure, we can take the expansion in eq. 5.30, and only retain the first addendum. Discarding the second term means that we are thus *subtracting* and hence neutralising the volume effect captured by the Laplace term, performing an analysis that will provide unbiased or much more less biased results. This operation corresponds to more formal approaches such as the introduction of specific prior functional forms, like the *Jeffreys prior* (Jeffreys, 1946):

$$p_J(\theta) = \sqrt{\det \{F\}}, \quad (5.39)$$

where F is the Fisher information matrix, a specific conditioning strategy defined such that it cancels the contribution from the Laplace term if $\Delta \mathcal{F}$ is smaller than F . At the same time, while the Jeffrey prior approach requires a full Bayesian analysis for the full parameter set, including nuisance parameters, our approach effectively reduces the number of parameters to be sampled. This may translate into a reduced computational effort for certain types of nuisance parameters, such as bias factors, as we shall describe below.

Application to bias parameters

The procedure outlined above provides several simplifications to the parameter sampling problem, providing semi-analytic expressions of rather general application. In some cases, further simplifications might be achieved, in particular, depending on the functional form of the nuisance parameters $\mathbf{n}$ and how they enter the theory vector $\mathbf{t}$. Recalling eq. 5.32, the general problem we have to

solve in order to find a generic nuisance parameter n_i at its best-fit value in the profile likelihood configuration is

$$\frac{\partial \chi^2}{\partial n_i} = -2\partial_i \mathbf{t}^T \mathbf{C}^{-1}(\mathbf{d} - \mathbf{t}) + 2 \sum_j [\mathbf{C}_n^{-1}]_{ij} (n_j - n_{p,j}) = 0 \quad (5.40)$$

Considering the expression, we observe that, in the second addendum, nuisance parameters enter linearly, so that the actual complexity of the expression depends on the first addendum and on how the the nuisance parameters combine in $\partial_i \mathbf{t}$ and $\mathbf{t}$. For fortunate enough cases, the expression might be reduced to a polynomial equation, ensuring a particularly simple solution, not necessarily analytic but at least of fast convergence and reliable stability. This happens for example for bias parameters, a particularly useful result for complex multi-tracer or multi-bin approaches. In a higher-order, perturbative theory formulation of the bias parameters, their number quickly increases, leading to potential important enhancement of distorting volume effects in a typical data analysis task. In the LPT formalism used in this work and introduced in chapter 2 we have four bias parameters – $\{b_1, b_2, b_s, b_{\nabla^2}\}$ – for each of the three redshift bins, giving in total twelve nuisance parameters against only two cosmological parameters to fit, Ω_m and σ_8 . It is clear that the robustness of our results might be limited by volume effects, and indeed volume effects are known to cause potential incorrect inference on aspects such as the degree of tension between the constraints on growth place by large-scale structure measurements on one hand and by CMB on the other hand (Hadzhiyska et al., 2023; Maus et al., 2025b). Such issues make therefore our study an ideal case for applying a profile likelihood strategy to the bias parameters (Rubiola et al., 2025). The bias parameters act at most as quadratic factors multiplying the angular power spectra templates: they therefore have a particularly simple form that allows us to further simplify the calculation of the terms entering the profile likelihood strategy: we start by defining a generic cosmological tracer field u^α measured in the α -th redshift bin δ^α expressed in the $\chi, \mathbf{n}$ coordinates we used to derive the angular power spectra in sec. 1.4, already multiplied by its window function $W^\alpha(\chi)$:

$$u^\alpha(\chi, \mathbf{n}) = W^\alpha(\chi) \delta^\alpha(\chi, \mathbf{n}). \quad (5.41)$$

In full generality, the field can be represented as a superposition of biased fields (we hereafter neglect the dependence on χ):

$$u^\alpha(\mathbf{n}) = \epsilon_u u_M^\alpha(\mathbf{n}) + \sum_k b_{\alpha,k}^u u_k^\alpha(\hat{\mathbf{n}}), \quad (5.42)$$

with k running on the set of bias parameters defined in our LPT formalism $\{b_1, b_2, b_s, b_{\nabla^2}\}$, assumed to be also redshift-dependent and therefore labelled with α as well, while ϵ_u is a Boolean variable that is either 1 if the tracer contains an unbiased contribution (e.g. for cosmic shear and CMB lensing), and 0 otherwise (e.g. for galaxy clustering). From eq. 5.42 and recalling the theory exposed in section 1.4 we can construct the generic angular cross-correlation power spectrum between two such tracers u^α and w^β in two distinct redshift bins α and β as a quadratic function of the bias parameters:

$$\begin{aligned} C_\ell^{u^\alpha, w^\beta} &= \epsilon_u \epsilon_w C_\ell^{u_M^\alpha, w_M^\beta} + \sum_i b_{\alpha,i}^u \epsilon_w C_\ell^{u_i^\alpha, w_M^\beta} \\ &+ \sum_j \epsilon_u b_{\beta,j}^w C_\ell^{u_M^\alpha, w_j^\beta} + \sum_{i,j} b_{\alpha,i}^u b_{\beta,j}^w C_\ell^{u_i^\alpha, w_j^\beta} \end{aligned} \quad (5.43)$$

The profile likelihood and the Laplace terms for the bias parameters can be derived following eqs. 5.32 and 5.33, requiring the calculation of the first and second derivative of the power spectra.

Bias parameters being linear prefactors of the angular power spectra, we obtain the first derivative of our templates as a linear polynomial in the bias parameters:

$$\begin{aligned} \frac{\partial C_\ell^{u^\alpha, w^\beta}}{\partial b_{\gamma, k}^r} = & \delta_{\alpha, \gamma}^K \delta_{u, r}^K \left[\epsilon_w C_\ell^{u^\alpha, w^\beta} + \sum_j b_{\beta, j}^w C_\ell^{u^\alpha, w_j^\beta} \right] \\ & + \delta_{\beta, \gamma}^K \delta_{w, r}^K \left[\epsilon_u C_\ell^{u_M^\alpha, w_k^\beta} + \sum_i b_{\alpha, i}^u C_\ell^{u_i^\alpha, w_k^\beta} \right] \end{aligned} \quad (5.44)$$

where δ^K is the Kronecker delta, and $b_{\gamma, k}^r$ denotes the k th bias term of tracer type r (i.e., galaxy overdensity or shear) in redshift bin γ , while for the Hessian we have :

$$\frac{\partial^2 C_\ell^{u^\alpha, w^\beta}}{\partial b_{\gamma, k}^r \partial b_{\sigma, m}^s} = [\delta_{\gamma, \alpha}^K \delta_{\sigma, \beta}^K \delta_{r, u}^K \delta_{s, w}^K + \delta_{\gamma, \beta}^K \delta_{\sigma, \alpha}^K \delta_{r, w}^K \delta_{s, u}^K] C_\ell^{r^\gamma, s^\sigma}. \quad (5.45)$$

Eq. 5.43 corresponds to the entries of the $\mathbf{t}$ vector of eq. 5.32, while 5.44 corresponds to the entries of the $\partial_i \mathbf{t}$ term in the same expression. The equation of eq. 5.40 becomes thus a polynomial problem in the bias parameters values, whose zeros can be suitably found numerically and then used to evaluate eq. 5.45, required to compute the Laplace term in eq. 5.33. This is one of those particularly simple cases in which the benefits of the profile terms can be observed particularly well also at the level of computational efficiency: indeed, using a Gauss-Newton algorithm, the solution of eq. 5.40 for n_i having functional forms such as those of the bias terms typically requires $O(10 - 100)$ iterations, with a significant acceleration in the whole fitting procedure compared to the use of an MCMC algorithm extended to the whole parameter space (Hadzhiyska et al., 2023). This is of course not the unique available approach to tackle volume effects: other approaches are available in the literature, including the use of simulation-based priors on the bias parameters (Ivanov et al., 2024; Chen & Ivanov, 2025), the use of well-educated reparametrisations (Tsedrik et al., 2025), or the already mentioned use of Jeffreys priors (Hadzhiyska et al., 2023). Hereafter, we shall call our profile-likelihood based methodology, formulated following (Hadzhiyska et al., 2023), approximate analytic marginalisation (AAM). To recapitulate, the results in the AAM configuration are obtained by a marginalised likelihood where the MCMC chain is exclusively performed on the cosmological parameters. In the AAM case, by construction the marginalised posterior distributions is centred around the best-fit values for the model parameters (that is, they are not undergoing volume effects). As such, the effect is analogous to the use of a Jeffreys prior to control the variability of the bias parameters, and further benefitted by much faster exploration of heavily parametrised models, such as in our case with many redshift bins and multiple bias parameters (Hadzhiyska et al., 2023). Computationally, the AAM formulation is implemented in the *Cosmotheke* likelihood[§], which interfaces with *cobaya* (Torrado & Lewis, 2019, 2021) to sample the posterior distribution, and makes use of the Core Cosmology Library (Chisari et al., 2019a) for most theoretical calculations. Eventually, we will base our convergence criterion on the Gelman-Rubin test (Gelman & Rubin, 1992): starting from the four MCMC chains we will be running independently in order to ensure greater independence in our sampling strategy, the Gelman-Rubin compares the variance of each chain with the variance of all chains together, and calculates an aggregate ratio R between the variances. The closer this value is to 1, the more similar the variances of the four independent chains taken individually are to the variance of the combined chain. This means that internal correlation effects due to the initial conditions specific

[§]Source codes, full tutorials and other materials are available [at this link](#).

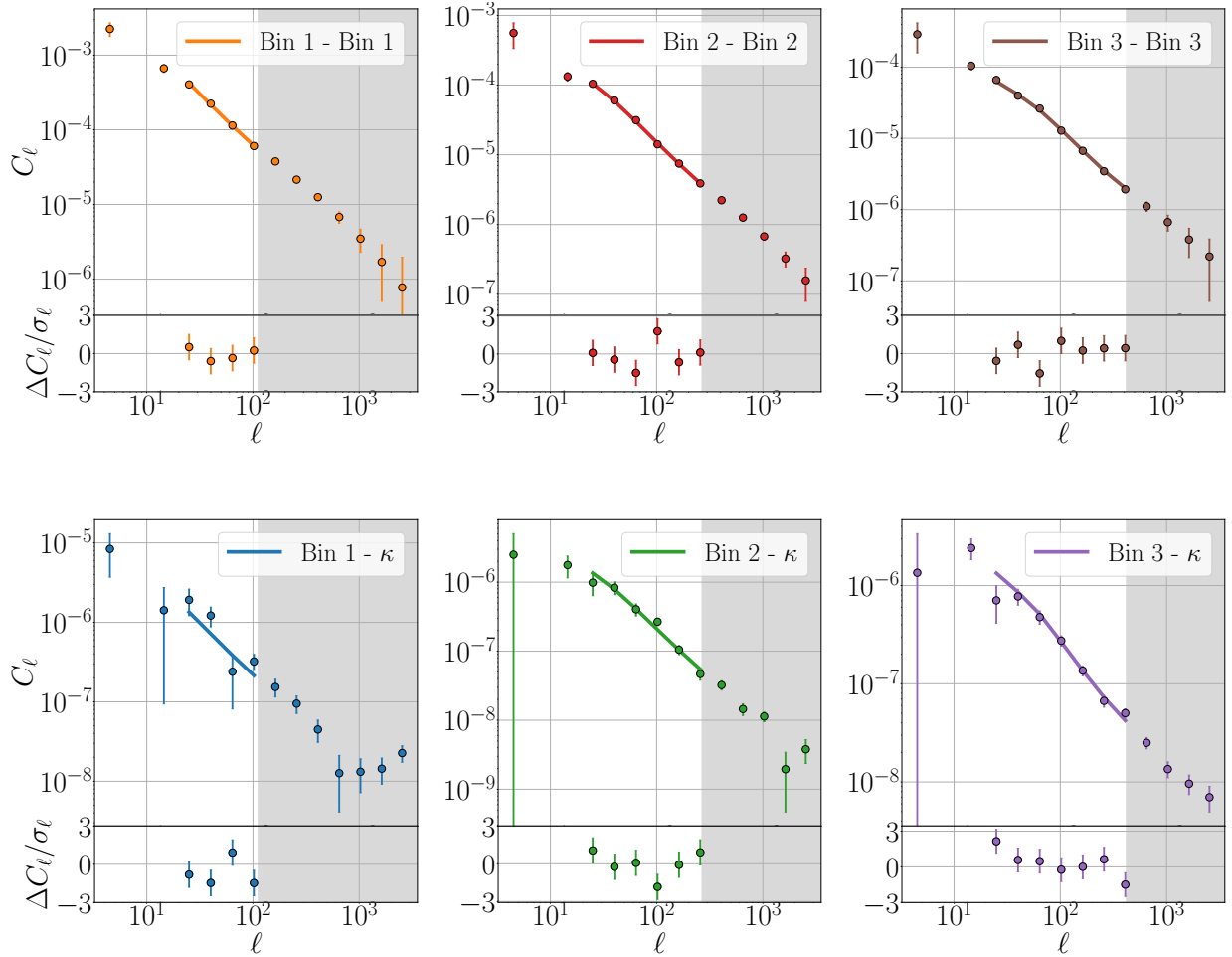


Figure 5.9: Power spectrum measurements together with our best-fit spectrum templates. Along the columns the galaxy auto-correlation C_ℓ^{gg} (top) and the cross-correlation with CMB lensing $C_\ell^{g\kappa}$ (bottom) are displayed, while Bins 1, 2, and 3 can be seen by moving from left to right. The differences between the data and the fiducial best-fit model (solid lined in each of the main panels) are collected in the sub-panels and calculated in terms of fraction of the errors. The angular scales not included in the analysis correspond to the shaded bands.

to each chain and other deviations to a perfect Brownian motion (also known as a Markovian, white noise process) in the sampling process have become negligible and all chains can be assumed to be representative realisations of the same white noise process oscillating around the stationary distribution of the posteriors, centred around the values of the parameters we may deem as “best-fit”. In particular, we consider the MCMC converged when the difference between R and one, the Gelman-Rubin statistics, becomes equal to $R - 1 = 0.01$.

5.3 Validations and results

5.3.1 Power spectra and constraints on Λ CDM

We are eventually ready to discuss the results of our data analysis. To begin with, fig. 5.9 shows our power spectrum data together with the best-fit theory templates in a fiducial Λ CDM analysis (described below). Having in mind the applied scale cut, the model appears a relatively good fit (the quality of the fit and the χ^2 metrics reported in Table 5.2 will be subject of further discussion

below). First, we confirm the possibility of detecting the galaxy-CMB lensing cross-correlation signal, found at a high significance in each of our redshift bins. In particular, within the scales used for our analysis, $C_\ell^{g\kappa}$ is detected associated to SNR of 6.5, 14, and 19 for bin 1, bin 2 and bin 3. Note that the information we can recover from Bin 1 is limited to only four data points in C_ℓ^{gg} and $C_\ell^{g\kappa}$, due to the low mean redshift associated to those data (also recall fig. 1.6 and 1.7).

Distinction between Maximum Likelihood Estimation and Maximum a Posteriori parameters

We also take advantage of the previous discussion to distinguish between two distinct interpretations of the parameters that could be interpreted as best-fit values and that influence the potential calculation of tension metrics: the proper **best-fit** values (also known as Maximum Likelihood Estimation, MLE) for a fit, also in the case of a Bayesian data analysis, correspond to the combination of marginalised parameters that *maximise the value of the likelihood* (in agreement with the usual maximum likelihood procedure, regardless of the preferred inference paradigm we adopt, be it frequentist or Bayesian inference). These values are reported in the rows of table 5.2 between parentheses and named “b.f.”. They correspond to the quantities we use to calculate the χ^2 , to estimate the degrees of freedom and to draw the curves in fig. 5.9, because they are the values of the parameters that, once inserted in our models, minimise the *distance* of the theoretical templates from the data.

However, tension metrics and the parameters usually reported in the cosmological literature, as well as the ones typically used for comparisons and discussions, are those associated to the *peaks* of the marginalised posterior distribution of each parameter and correspond to the values quoted together with their uncertainties in the same table. The peak posterior distribution values are known as Maximum a Posteriori (MAP) and they act as a reference in the literature since they are more influenced by the effect of the prior and set ourselves in a fully Bayesian inference framework. They are therefore the quantities we mention in the abstract and in the discussions: these values correspond exactly to the best-fit values only for very Gaussian posterior distributions, associated to absent or uniform priors for the parameters, while for non-uniform priors (such as the Gaussian ones we use here) and possibly non-Gaussian likelihoods or posteriors the two might not coincide. For sufficiently symmetric and Gaussian-like posterior distributions, MAP values correspond to the mean (expected value) or median (maximum of the distribution) of the posterior, and their uncertainties can be similarly calculated starting from the quantiles of the posterior (accounting for possible residual asymmetries in the posterior, and allowing for a different upper and lower uncertainty) or from their standard deviation (that is instead symmetric and might be improper in case of non-Gaussian posteriors). We will quote as Maximum a Posteriori value the mean value (assuming that the posteriors are symmetric enough to ensure that the mean lies at the peak of the posterior) of the posterior distribution for each of our parameters and we calculate the values of the distribution at the 16th and 84th quantile in order to calculate the upper and lower uncertainty as difference between the values at those quantiles and the mean as lower and upper uncertainty (thus encompassing the 68% of our nearly-Gaussian posterior distribution), allowing for asymmetric uncertainties. In general, the rather symmetric values of the upper and lower uncertainties (see table 5.2) suggest that the posteriors are quite Gaussian in shape.

Fig. 5.10 shows the constraints on Ω_m and S_8 obtained by the combined measurement of C_ℓ^{gg} and $C_\ell^{g\kappa}$ in each of our three redshift bins, using scales up to $k_{\max} = 0.42 \text{ Mpc}^{-1}$. We can compare

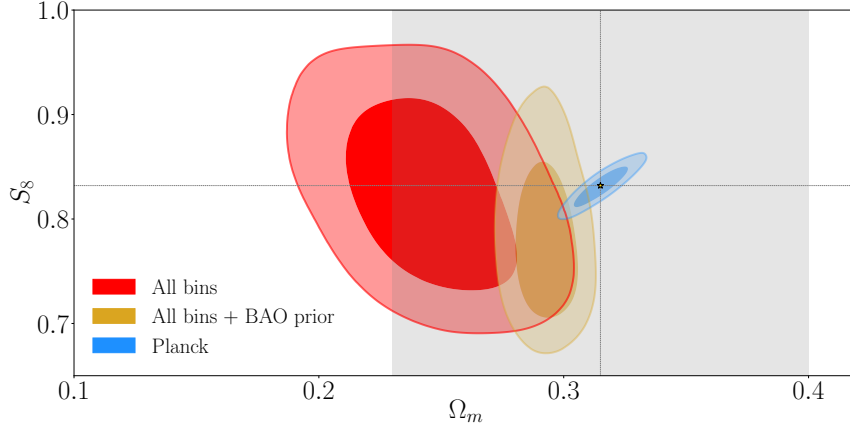


Figure 5.10: The figure reports the two-dimensional constraints in the (Ω_m, S_8) plane, showing the results for what we define our fiducial analysis configuration, the combination of all redshift bins, having let the Ω_m prior completely free (red). The fiducial case is thus compared with the analysis combined with the imposition of a narrow BAO prior from DESI on Ω_m (yellow), eventually compared to Planck (blue). The gray strip allows to see the Ω_m variation range allowed by the **baccoemu** HEFT emulator; it can be observed how a part of the posterior probability mass for Ω_m is beyond it at lower values.

the results in the case when Ω_m is left completely free (red contour) and when instead we include the DESI BAO prior (yellow contours). Instead, the Planck constraints are the quantities shown in blue. The numerical constraints on Ω_m , σ_8 , and S_8 associated to the plots are collected in table 5.2, including as well the constraints we recover in the other analysis choices we explored, and visually compared in fig. 5.11. Our constraints on S_8 are thus:

$$\boxed{S_8 = 0.82 \pm 0.07 \text{ (free } \Omega_m), \quad S_8 = 0.79 \pm 0.06 \text{ (DESI BAO prior).}} \quad (5.46)$$

As such, our results are compatible with Planck TT,TE,EE+lowE+lensing data ($S_8 = 0.832 \pm 0.013$), and comparable with most available CMB lensing tomography analyses. Namely, (García-García et al., 2021) find $S_8 = 0.825 \pm 0.023$ in the absence of cosmic shear data, using galaxy samples in the redshift range $0.25 \lesssim z \lesssim 1.5$. Since we use only galaxy clustering data combined with the CMB lensing, that is the value we use as a reference for our comparisons, while their value of $S_8 = 0.7781 \pm 0.0094$ reported in the abstract of (García-García et al., 2021) and acting as their fiducial analysis output is associated to the full dataset including both clustering and weak lensing signals. The comparison would not be accurate since (García-García et al., 2021) concludes indeed that cosmic shear data, no matter what the underlying reason is, are the observable mainly driving the “tension”, and our work indeed does not use cosmic shear data in order to perform a first analysis in possibly more controllable experimental conditions.

Further comparisons can be made with the analyses upon the quasars collected in the *Quiaia* catalogue, spanning the range $z \lesssim 3$, (Piccirilli et al., 2024) find $S_8 = 0.841 \pm 0.044$ and of (Farren et al., 2024), quoting $S_8 = 0.810 \pm 0.015$ using lensing maps from Planck and ACT with galaxies from the unWISE sample in the range $0.5 \lesssim z \lesssim 1.5$, in any case compatible with the two results we report, with and without the BAO prior. The work most closely covering our redshift range explored here is the paper of (Sailer et al., 2025a), which combines DESI Legacy photometric bright galaxy and luminous red galaxy samples (BGS and LRG) with CMB lensing, including the lensing magnification effect. The combined constraint they report is $S_8 = 0.788 \pm 0.020$, becoming,

Configuration ($N_{\text{dof,eff}}$)	Ω_m (b.f.)	S_8 (b.f.)	σ_8 (b.f.)	χ^2 (PTE)
All bins (22)	$0.25^{+0.02}_{-0.02}$ (0.24)	$0.82^{+0.07}_{-0.07}$ (0.82)	$0.91^{+0.10}_{-0.10}$ (0.92)	31.2 (9%)
All bins + BAO (23)	$0.29^{+0.01}_{-0.01}$ (0.29)	$0.79^{+0.06}_{-0.06}$ (0.75)	$0.80^{+0.06}_{-0.06}$ (0.76)	35.6 (4.5%)
No AAM (22)	$0.27^{+0.02}_{-0.02}$ (0.25)	$0.75^{+0.04}_{-0.04}$ (0.80)	$0.79^{+0.06}_{-0.06}$ (0.90)	33.4 (6%)
No $p(z)$, N_ℓ marg. (22)	$0.25^{+0.02}_{-0.02}$ (0.24)	$0.82^{+0.06}_{-0.06}$ (0.84)	$0.91^{+0.09}_{-0.10}$ (0.93)	32.0 (8%)
Linear HEFT (13)	$0.25^{+0.03}_{-0.03}$ (0.25)	$0.70^{+0.06}_{-0.06}$ (0.70)	$0.76^{+0.09}_{-0.08}$ (0.77)	14.3 (36%)
$k_{\text{max}} = 0.25 \text{ Mpc}^{-1}$ (16)	$0.25^{+0.03}_{-0.03}$ (0.25)	$0.84^{+0.08}_{-0.08}$ (0.82)	$0.93^{+0.14}_{-0.13}$ (0.90)	20.4 (20%)
Bin 1 + BAO (3)	$0.30^{+0.01}_{-0.01}$ (0.30)	$0.98^{+0.14}_{-0.13}$ (1.10)	$0.98^{+0.14}_{-0.13}$ (1.10)	4.00 (26%)
Bin 2 + BAO (8.5)	$0.30^{+0.01}_{-0.01}$ (0.30)	$0.88^{+0.12}_{-0.12}$ (0.94)	$0.89^{+0.12}_{-0.12}$ (0.95)	16.6 (4.4%)
Bin 3 + BAO (10.5)	$0.29^{+0.01}_{-0.01}$ (0.29)	$0.75^{+0.05}_{-0.05}$ (0.73)	$0.76^{+0.05}_{-0.05}$ (0.74)	13.7 (22%)

Table 5.2: Summary table for the Ω_m , S_8 and σ_8 constraints obtained in our several analysis configurations. The posterior mean and 68% confidence interval are reported in each analysis strategy, alongsied with the best-fit value, which is instead reported in parentheses. A better graphic visualisation of the constraints can be appreciated in fig. 5.11. The p -value (or PTE) we associate to each analysis configuration is obtained after calculating the effective number of degrees of freedom (summarised in the first column), using the method exposed in the main text.

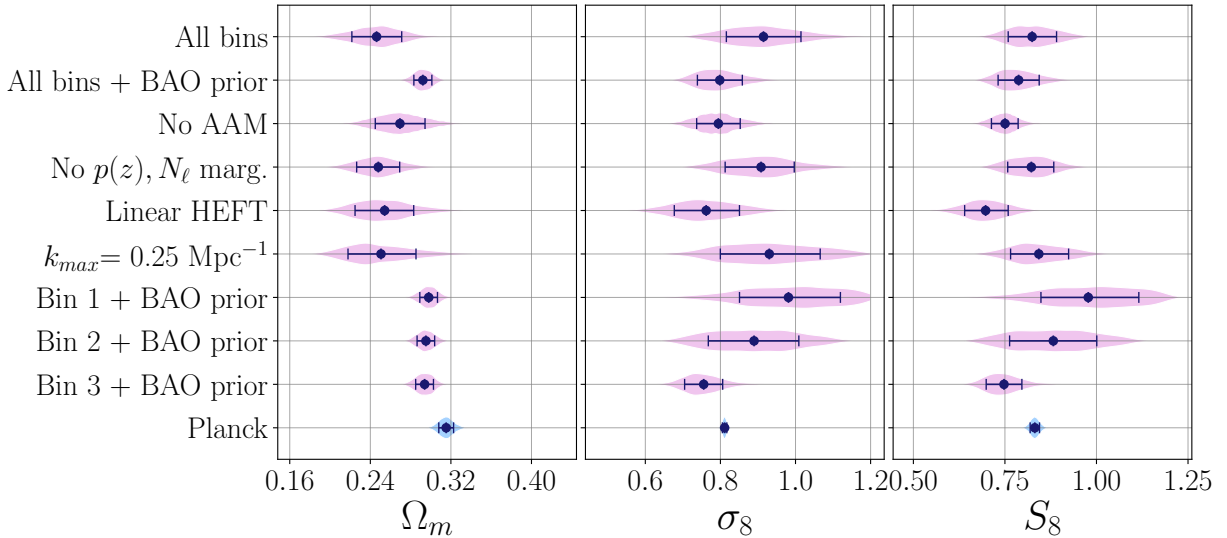


Figure 5.11: Synoptic constraints on Ω_m , S_8 and σ_8 in the different analysis choices and compared with the Planck measurement, corresponding to the results reported in tab. 5.2. The mean value of each parameter is in black together with the error bars at 68% confidence level. This quantities and statistics are the same we will employ to estimate the possible levels of tension on S_8 . In pink we show the marginalized posterior distributions for each parameter.

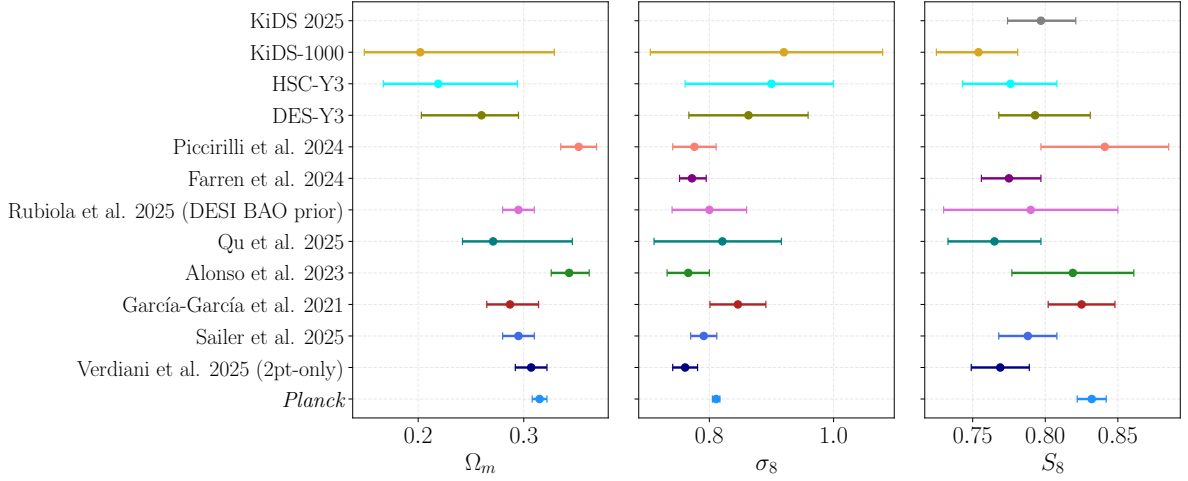


Figure 5.12: Comparison of the measurements of the parameters Ω_m , σ_8 and S_8 (from left to right) as obtained in the main reference works performing CMB lensing-clustering analyses, contrasted with Planck primary CMB analyses and the results obtained by angular power spectra of cosmic shear data. Missing data correspond to values not clearly reported in the papers, which I thus preferred not to report in order to limit the discussion exclusively to clearly comparable results. The first four results are associated to the main available weak lensing datasets (from top to bottom, they correspond to the publications of (Wright et al., 2025; Loureiro et al., 2022; Dalal et al., 2023; Doux et al., 2022)). From the fifth results up to the one denoted with *Planck*, the measurements are associated to the CMB lensing-galaxy clustering results discussed in the text as the works more suitable for comparison to the one discussed in this chapter. In order from top to bottom, they correspond to the following works (Piccirilli et al., 2024; Farren et al., 2024; Rubiola et al., 2025; Qu et al., 2025; Alonso et al., 2023; García-García et al., 2021; Sailer et al., 2025a; Verdiani et al., 2025). Eventually, *Planck* refers to the TT,TE,EE+lowE+lensing results published in (Aghanim et al., 2020).

when using only BGS at $z < 0.4$, $S_8 = 0.870^{+0.050}_{-0.064}$. Our statistical uncertainties are larger than the one in the compared literature, but we emphasise how we have been able to confirm the available measurements in complementary redshift ranges ($z \lesssim 0.3$).

We already observed how our data might fit better a low Ω_m value if this parameter is let varying freely: we find indeed $\Omega_m = 0.245 \pm 0.024$, that, compared with Planck TT,TE,EE+lowE+lensing, is about 2.8σ lower, while, when we impose the DESI BAO prior, the fit of the Ω_m converges to the prior mean value, as expected, corresponding to a significantly lower (1.6σ) distance from Planck. We will discuss in detail the different effects concurring at the determination of the best-fit value for Ω_m in sec. 5.3.1. We already anticipated that the constraint on Ω_m in our cross-correlation exercise and in absence of the DESI prior depends on the overall functional shape (the “broadband”) of the power spectrum. This kind of measurement is nevertheless much more error-prone compared to what it is possible to infer from a measurement based on a well-defined standard ruler like the BAO. Hereafter, since all the other constraints we recovered in our analysis are broadly compatible with DESI (Adame et al., 2025b,a) ($\sigma_8 = 0.842 \pm 0.034$ and $S_8 = 0.836 \pm 0.035$ in the combined DESI (FS+BAO)+BBN+ n_{s10} analysis of (Adame et al., 2025b)). We will therefore report most of our results as a results of analyses including a DESI BAO prior, a decision ensuring, as we said, a better control of potential extrapolation errors of the *bachoemu* HEFT templates when Ω_m

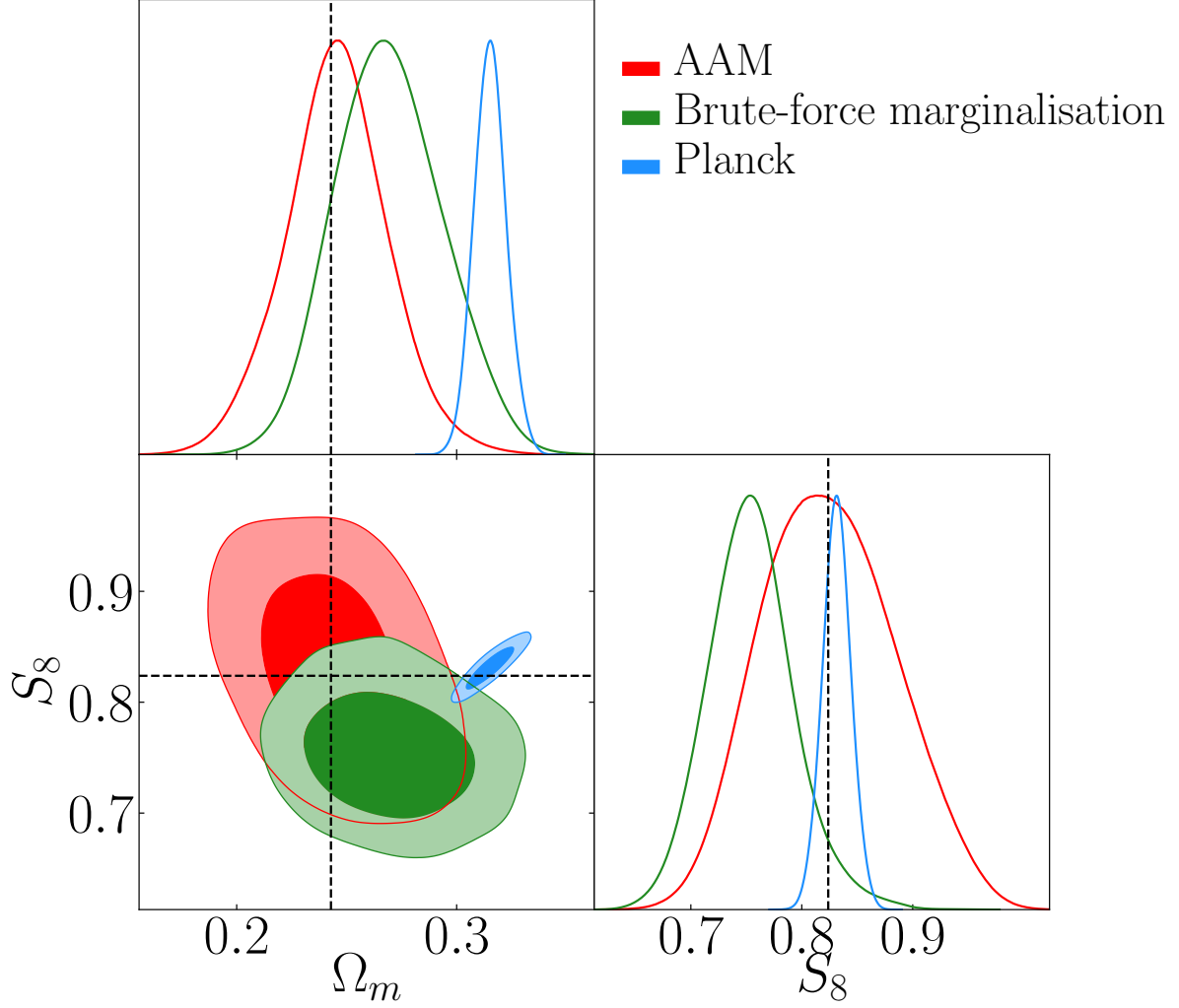


Figure 5.13: S_8 and Ω_m constraints in the fiducial analysis (AAM strategy, marginalising over the HEFT bias parameters in order to eschew volume effects, red). Such results are compared with brute-force MCMC marginalisations over the bias parameters (green). We observe how the the posterior contours shift under the influence of volume effects if we compare the position of the posterior distribution with the value of the best-fit parameters (dashed lines). The AAM technique is free from volume effects, since it corresponds to the imposition of a prior whose effect is analogous to the Jeffrey prior (see sec. 5.2.6).

attains low values. Bearing in mind this choice, we refer to fig. 5.12 for a synoptic collection of the discussed comparisons, which also includes Planck and shear-based reference measurements for Ω_m , σ_8 and Ω_m , from left to right. The figure can be further contrasted with fig. 4.1 for additional measurements of S_8 .

Estimation of the degrees of freedom for the χ^2 test The best-fit model for our fiducial case has a χ^2 of 32.1 with a free Ω_m and a χ^2 of 35.5 when including the DESI prior. The quantitative evaluation of the goodness of fit should be carried out via the usual χ^2 hypothesis test, requiring the correct definition of the model’s degrees of freedom. At a first sight, we should account for 12 nuisance parameters in our model (i.e., 4 bias parameters for each of the three redshift bin), to which the two cosmological parameters have to be added. Nevertheless, our model is quite non-linear and the same non-linearity affects how our model parameters enter the model,

so that the introduction of the priors aimed at controlling the model flexibility limits indeed the property of the parameters of being fully “free” (Andrae et al., 2010; Bernhard, 2023; Das, 2024; Gelman et al., 2013; Robert & Rousseau, 2010); as such, the standard calculation of the number of degrees of freedom $N_{\text{dof}} = N_{\text{data}} - N_{\text{param}} = 34 - 14 = 20$ required to perform the χ^2 test for the goodness of fit might not provide robust estimates of the effective number of degrees of freedom. We therefore estimate the number of degrees of freedom of our model starting from the statistical properties of the χ^2 probability distribution: 1000 Gaussian realisations of our data vectors are generated, using the best-fit parameters found using the DESI prior as mean value for our data distribution, while the covariance is the one of the original dataset. The best-fit model prediction is then calculated for each synthetic data realisation performing a χ^2 minimisation. The corresponding χ^2 values we recovered are, by construction, distributed as the underlying χ^2 probability distribution, parametrised by the actual number of degrees of freedom of our models, that can be recovered by fitting the random variables distribution to the χ^2 analytic functional form. We thus find that our fiducial model corresponds to a χ^2 distribution characterised by $N_{\text{dof,eff}} = 22$ instead of the 20 one would calculate using $N_{\text{dof}} = N_{\text{data}} - N_{\text{param}} = 34 - 14 = 20$. The associated probability-to-exceed (PTE, also known as p -value) of the χ^2 ’s thus calculated in the data is 0.09 in the free Ω_m case, and 0.045 when using the DESI BAO prior. This latter is worse than the fully free fit, a result that can be interpreted as an effect of the DESI prior that, although providing greater physical sensibility to the results, may not completely reflect the global minimum preferred by the data alone. Both values appear to be relatively low, yet acceptable enough, since their p -values correspond to potential $\sim 1.7\sigma$ and $\sim 2\sigma$ fluctuations. We will apply this procedure to any other data combination used here, and report the results consistently in table 5.2, showing the number of effective degrees of freedom for every fit, the associated χ^2 (not the reduced one) and the related PTE.

Profile likelihood validation and additional robustness tests

Recalling what we discussed in sec. 5.2.6 and in (Ruiz-Zapatero et al., 2023), we are already using as fiducial result the AAM strategy obtained by marginalising over the HEFT bias parameters and avoiding volume effects that would be induced by the large nuisance parameter space. We find indeed as a validation that these distorting effects caused by the HEFT bias parameters are important: when our constraints are recovered upon application of the “brute-force” marginalisation scheme over the bias parameters, letting them act as free parameters in the MCMC chain, the posteriors become those displayed in green in fig. 5.13, compared again with the Planck reference posteriors (blue) and with the AAM best-fit results (red), that also allows to derive the best-fit values, reported in the figure by the black dashed line.

The magnitude of the volume/projection effects translates in the important shift of the green marginalised posterior distribution for S_8 to a lower value, more than 1σ away from the best fit, associated to an artificial shrinkage in the associated uncertainties. We recall that the AAM strategy only samples the two cosmological parameters by means of the MCMC algorithm, that we have imposed a uniform prior on them and that the profile likelihood, by removing the Laplace term (corresponding to the introduction of a Jeffrey prior), compensates for the volume effect: therefore, by construction, the peak (the MAP) of the AAM posterior distribution is centred around the best-fit (MLE) value (MAP and MLE indeed agree when the prior is uniform) and can be interpreted as the “correct” best-fit value. If volume effects are large, we are expecting

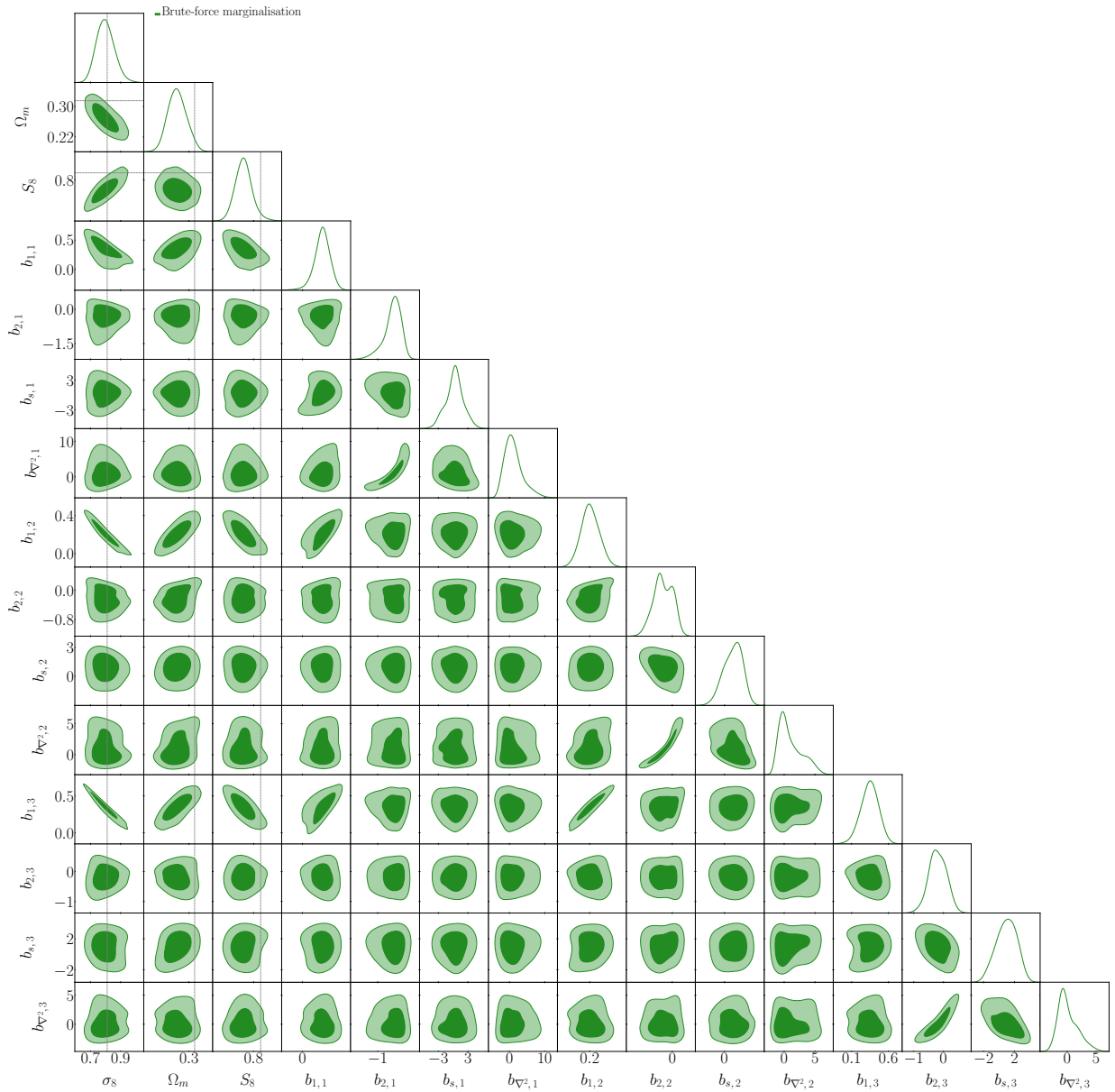


Figure 5.14: Full model parameters posteriors as recovered by using the standard MCMC (“brute force”) approach. Planck 2018 best fit (Planck Collaboration et al., 2020b) values are in grey (dashed) for the measured cosmological parameters. No comparison with other reference values is possible for the bias parameters.

the posterior distribution recovered in the “brute-force” case to shift with respect to the “correct” best-fit value, and this is indeed the case of fig. 5.13, suggestive of an important impact of volume effects and that, therefore, we cannot fully trust the recovered cosmological parameters in absence of an AAM implementation. As a further piece of information, we display in fig. 5.14 the full constraints corner plot, for bias and cosmological parameters included in our model under brute-force MCMC, interesting in that they provide information on the full degeneracy scheme of the nuisance parameters and the cosmological ones. We emphasise once again that the nuisance parameters posterior distributions are not defined in the AAM case, since the nuisance parameters are evaluated each at a single point in the parameter space, the one solving the maximisation problem of eq. 5.40. The other important topic requiring validation is testing the stability of our results under potential uncertainties in the modelling of non-linear bias. Such tests are performed

by repeating our fiducial analysis, excluding the DESI BAO prior, after having applied a more conservative scale cut, $k_{\text{max}} = 0.25 \text{ Mpc}^{-1}$, lying midway between a fully linear regime and the scale cuts in the fiducial analysis, corresponding to $k_{\text{max}} = 0.25 \text{ Mpc}^{-1}$. We show the results in fig. 5.11 and in table 5.2. Overall, the uncertainties on both Ω_m and S_8 increase (by 50% and 14.5%, respectively), yet the preferred values do not significantly shift from the constraints we had found in the fiducial analysis configuration; this means that at $k_{\text{max}} = 0.25 \text{ Mpc}^{-1}$ (already quite linear as a regime) there is still enough information coming from the small scales of the LSS, fundamental to properly constrain Ω_m and σ_8 , to recover what we can consider as the “correct” best-fit value, at the price of an increased size of the uncertainties. By further reducing the k_{max} value, constraining power might be reduced accordingly, with potential additional biasing of the posterior means (MAP) or best-fit values (MLE) next to an increase of the uncertainties.

A check of this second argument corresponds to the analysis we perform using the simplified bias model with $b_2 = b_s = b_{\nabla^2} = 0$. Only b_1 acts as a free parameter, while the range of scales is limited to $k < 0.11 \text{ Mpc}^{-1}$. We can call this configuration “linear HEFT”. For comparison, this is similar to the “model independent” setup of (Sailer et al., 2025b)[¶]. We observe a shift of about $\sim 1.1\sigma$ downward in σ_8 next to an increase in relative terms of the uncertainties, confirming the expectations we described. A similar shift is found in (Sailer et al., 2025b) in their model-independent analysis, results suggestive that we have to move to very linear scales to observe important deviations between the fiducial constraints on the parameters σ_8 and S_8 from the one recovers in a linear regime. On the other hand, the preferred value for Ω_m under the assumption of a linear bias and linear scale cuts does not significantly move relative to the fiducial analyses in the non-linear regime.

A remark on our stress testing strategy and on the topic of “compatibility” among our constraints This point allows us to make a remark: strictly speaking, all our results are compatible among each other and with Planck. **No hint of tension** can be found in terms of strict statistical criteria: all parameters are compatible with Planck and with other concordant results in the literature within 3σ and, *a fortiori*, within 5σ . Looking at fig. 5.11, we see that the previous statement is particularly true for S_8 and σ_8 , whilst Ω_m shows a bit more variability – concerns about its stability indeed called for the introduction of the external BAO prior – although always with a maximum deviation of 2.6σ from Planck in the fiducial case without BAO prior. At the same time, the studies of sensitivity we are reporting are important in order to understand “in the safe conditions” ensured by the general concordance of our results the impact of the modelling of non-linearity and other complexities of the cosmic web at low redshift and small scales, all the more important should greater deviations arise in other studies.

Sensitivity study of our results on Ω_m

The checks on the non-linear and linear regimes we performed are also useful to back the analysis we are now going to outline, which shows that the constraints on Ω_m are dominated by the overall shape of the matter power spectrum in the highest redshift bin (Bin 3). Aiming at identifying what are the data more strongly governing the recovered preference for a low value of Ω_m , we apply a reanalysis of our data in a more model-agnostic setting. That is, we isolate the information

[¶]This work (Sailer et al., 2025b) considers a linear bias model too. However, they instead marginalise over counterterm parameters in order to account for residual systematics induced by non-linear effects

coming from the shape of the power spectrum from other kinds of informations. To do so, we use the empirical bias model published in (Maleubre et al., 2025). The work models the galaxy-galaxy and galaxy-matter power spectra as

$$P_{gg} = (A_{gg}^0 + k^2 A_{gg}^2) P_{mm}(k) + N_{gg}^0, \quad P_{gm} = (A_{gm}^0 + k^2 A_{gm}^2) P_{mm}(k) + N_{gm}^0. \quad (5.47)$$

The model has $\{A_{xy}^n, N_{xy}^n\}$ as free parameters, while $P_{mm}(k)$ is the matter power spectrum. The shot-noise contributions, together with potential additional other stochastic contributions to the fields are absorbed in the parameters N_{xy}^0 , whereas the amplitude parameters A_{xy}^0 and A_{xy}^2 track the linear and the scale-dependent bias, respectively. In the previous work, this model was able to accurately describe the galaxy clustering auto-correlation power spectra, as well as their correlation with the matter overdensity, performing well up to a scale of $k_{\max} = 0.3 \text{ Mpc}^{-1}$. In addition, A_{gg}^0 and A_{gm}^0 are not directly associated to linear galaxy bias formulations, and are degenerate with all the cosmological parameters determining the amplitude of the matter power spectrum, in particular σ_8 , and can absorb further potential model-dependence, combined with the use of a rather universal HALOFIT template for $P_{mm}(k)$, weakly cosmology-dependent. The information contained in the shape of the power spectrum can be thus isolated, and mapped to constraints on Ω_m sourced by different correlators in our data vector, while we marginalise over the free amplitude and the noise parameters. Following the main analysis scheme, we control volume effects by using the AAM technique as marginalisation strategy. Indeed, since all nuisance parameters are linear factors multiplying the power spectra, we can apply the arguments applied to the bias parameters in sec. 5.2.6 also to the parameters of eq. 5.47, $\{A_{xy}^n, N_{xy}^0\}$, recovering analytic expressions for the $\mathbf{t}$, $\partial_i \mathbf{t}$ and the Fisher matrix entering eq. 5.40, ensuring that the analytic marginalisation is exact. Each redshift bin is associated to specific nuisance parameters $\{A_{xy}^n, N_{xy}^0\}$, together with the same large-scale cuts of the reference configurations, while we set $k_{\max} = 0.3 \text{ Mpc}^{-1}$, smaller than our reference analysis setup but corresponding to the maximum scale for which the model is known to be sufficiently precise (Maleubre et al., 2025).

We use this methodology in order to understand how much information is present in C_ℓ^{gg} and $C_\ell^{g\kappa}$ separately. We fit independently the C_ℓ^{gg} data on one hand and the $C_\ell^{g\kappa}$ data on the other hand, and compare the results with those of the analysis of the combined dataset in the framework of (Maleubre et al., 2025). The result is collected in the left panel of fig. 5.15, where we infer that C_ℓ^{gg} provides most of the constraining power on Ω_m : the posterior for C_ℓ^{gg} substantially overlaps with the posterior for the fit with the combined C_ℓ^{gg} and $C_\ell^{g\kappa}$ data, consistently with the much larger significance in the detection of the C_ℓ^{gg} signal with respect to that of $C_\ell^{g\kappa}$ (see figs. 5.9 and 5.3, where we can notice that the CMB-lensing kernel at the very low redshifts we are considering is associated to a rather low signal). Focusing then on the galaxy auto-correlation statistics as the one bringing the actual information on Ω_m , the right panel of Fig. 5.15 shows instead the constraints we can obtain by looking at the galaxy auto-correlation in each individual redshift bin, to be compared with the combined constraint. We see how these are dominated by the highest-redshift bin (Bin 3, in red), substantially overlapping with the fit derived from all the bins together (black, dashed), deriving from the fact that at larger z the scale cuts we impose correspond to a great interval of scales (we recall the discussion associated with fig. 1.6). Conversely, Bin 1 (lowest redshift) is substantially insensitive to Ω_m (we are indeed only able to fit four data points given the scale cuts), while Bin 2 (green) has a loose constraining power.

We collect the results of such analysis in table 5.3, where we can observe the range of variability on Ω_m associated to the constraining power provided by the different bins alone and in combined

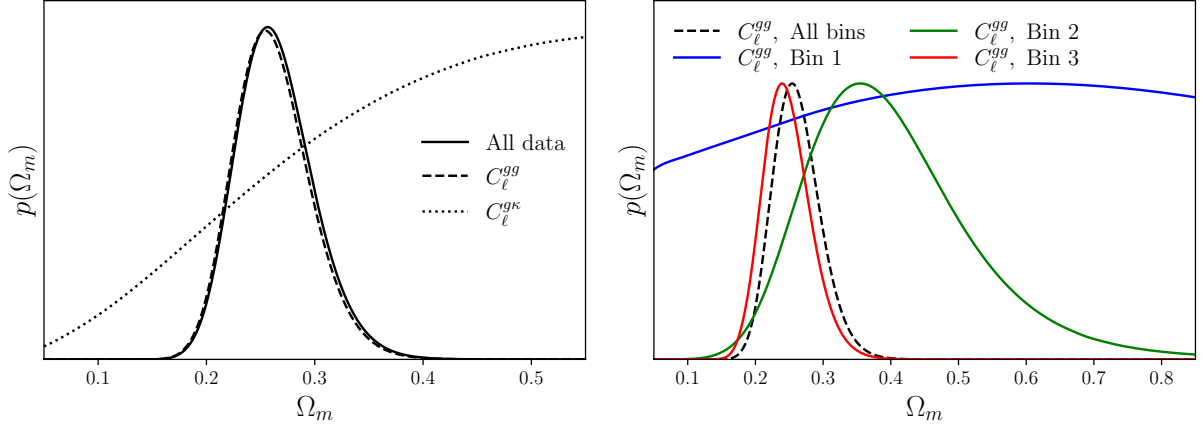


Figure 5.15: Constraints on Ω_m recovered in the agnostic model of (Maleubre et al., 2025), reported in eq. 5.47. *Left*: we show the constraints obtained by fitting the full data vector (solid), by exclusively fitting the galaxy auto-correlations (dashed), and by exclusively fitting the galaxy- κ cross-correlation angular power spectra (dotted). *Right*: we show the constraints obtained upon fitting the full set of galaxy auto-correlations (dashed black), together with the results obtained on the galaxy auto-correlation measurements performed in each bin respectively (blue, green and red for bins 1, 2, and 3, respectively). We can thus conclude that the constraints on Ω_m are chiefly driven by the functional trend and shape of the galaxy autocorrelation power spectrum measured, and, in particular, by its value in the third redshift bin.

Configuration	Ω_m	χ^2	N_{dof}	PTE
All bins, $gg + g\kappa$	0.264 ± 0.035	9.00	11	62%
All bins, gg	0.262 ± 0.035	4.59	5	47%
Bin 2, $gg + g\kappa$	0.41 ± 0.12	1.81	3	61%
Bin 3, $gg + g\kappa$	0.250 ± 0.036	5.28	5	38%
Bin 2, gg	0.40 ± 0.12	0.7	1	41%
Bin 3, gg	0.248 ± 0.036	2.65	2	27%

Table 5.3: Ω_m obtained by using the agnostic bias model of (Maleubre et al., 2025) described in eq. 5.47, throughout the different data configurations explored in section 5.3.1. Configurations that cannot provide finite constraints on Ω_m (e.g. those using $C_\ell^{g\kappa}$ alone or the first redshift bin), are not shown.

configuration and to be compared with the results of our fiducial analyses in table 5.2. Our fiducial MAP value for the “All bin” case is $\Omega_m \sim 0.25$, indeed quite close and compatible with the result of this second reanalysis ($\Omega_m \sim 0.26$ for All bins, $gg + g\kappa$ and All bins, gg) and comparable with the results obtained from the fit of Bin 3, gg . We do not display, instead, those configurations that cannot place relevant constraints on Ω_m , e.g. those using Bin 1 only in the $C_\ell^{g\kappa}$ configuration. We can observe that also the model fits are quite good, with PTE values comprised in the 27%-62% range. Since, in this case, all model parameters are linear, the number of degrees of freedom can be simply calculated as $N_{\text{dof}} = N_{\text{data}} - N_{\text{param}}$.

5.3.2 Low-redshift tomographic growth reconstruction

Given the available data and measurements, we might then perform a measurement of the growth of matter perturbations at different redshifts. We can thus reconstruct the growth history at late times, capturing the effects that, at the linear level, are described by the growth factor $D(z)$ (eq. 1.33, a function of cosmological parameters and in particular of Ω_m) and that, more generally, may combine in a redshift, scale and cosmology dependent quantity modifications of the amplitude of the primordial fluctuations and therefore of the LSS. This quantity, potentially suggestive of deviations of the gravitational dynamics from General Relativity, or of the intervention of New Physics in some redshift interval determining the deviations of the data from the Λ CDM prediction, has been repeatedly measured in these years both in model-dependent ways and in agnostic configurations aimed at just deriving the trends to be compared with the Λ CDM formulation of $D(z)$ and with other extant theories (García-García et al., 2021; Bianchini & Reichardt, 2018; Sailer et al., 2025b,a) (see chapter 4, especially secs. 4.2.1 and 4.2.2). As we discussed above in sec. 5.3.1, our constraints on Ω_m it is the galaxy auto-correlation signal associated to last redshift bin the main driver of the signal: the lower-redshift correlations are measured at a lower range of scales, a fact that limits the possibility of simultaneously measuring both σ_8 and Ω_m with decent accuracy in the first two redshift bins. It is again therefore safer to reconstruct the growth of the LSS by referring to the analysis including the DESI BAO prior on Ω_m , recovering the three value of σ_8 (at $z=0$) we obtain in its three per-bin constraints. In other words, our growth reconstruction task will be based on the data reported in the 7th, 8th and 9th row of table 5.2: the interpretation is substantially that the reconstructed growth history, associated to the value of $\sigma_8(z)$ calculated in a given redshift bin is exclusively constrained by the measurement of σ_8 (i.e., the Λ CDM parameter, associated to the amplitude of fluctuations at $z=0$) performed in the same redshift bin. We start by considering fig. 5.16, which shows the constraints on S_8 obtained in the reference fiducial setup, in both cases where the BAO prior is absent and present, together with the constraints recovered independently in each redshift bin, corresponding to the results also listed in table 5.2. Consistently with the discussion of the constraining power of the different probes and redshift intervals described above, we can indeed compare the constraints on S_8 as a function of the different redshift bins with the constraints on Ω_m in fig. 5.15, finding again that the fit results are dominated by the measurement in the highest redshift bin. On the contrary the measurements obtained in the two lower-redshift bins appear broader and therefore compatible with our fiducial measurement. The breath of the posterior is such that, however, larger values of S_8 are also allowed. Overall, the constraints found are substantially agreeing with the Planck measurements (orange posterior), with most of the posteriors overlapping for many quantiles. For better visual comparison, we show

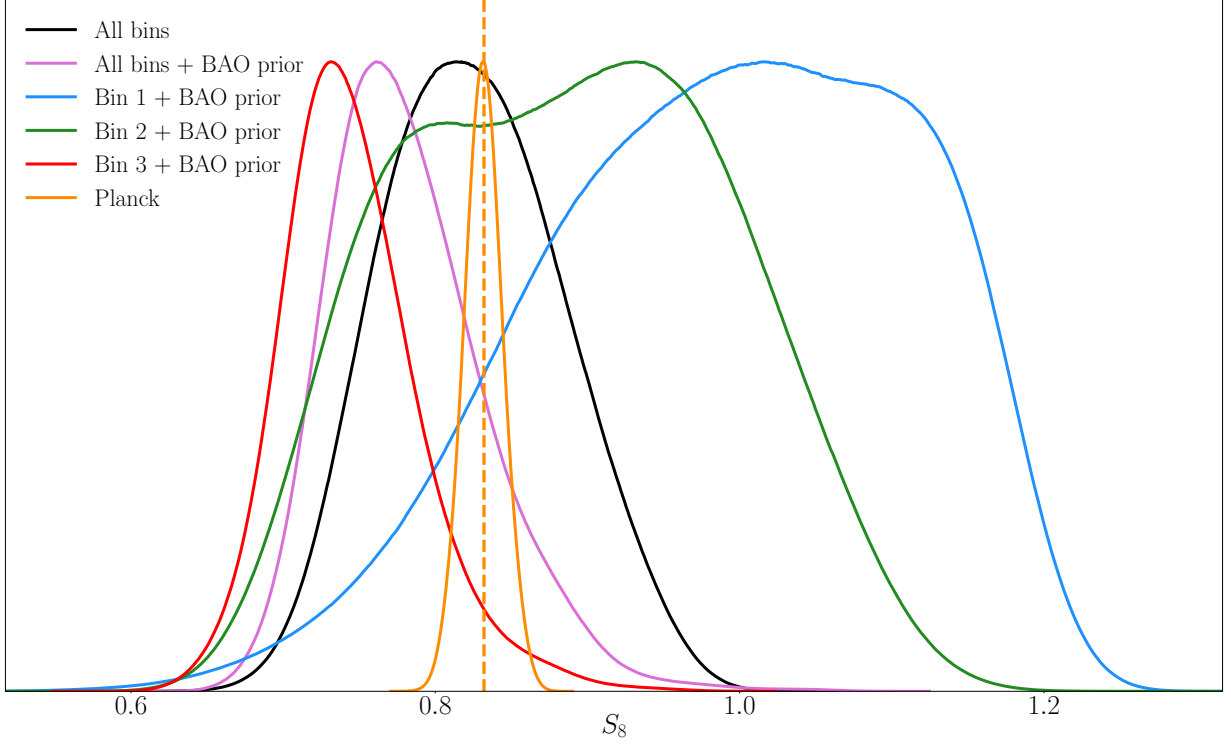


Figure 5.16: The marginalised posterior for S_8 obtained by using all redshift bins and letting Ω_m free to vary (black). If instead the BAO prior on Ω_m from DESI is applied, we recover the pin posterior for the joint fit of the three redshift bins, while the posteriors for the single redshift bins are in blue (Bin 1), green (Bin 2), and red (Bin 3). The Planck constraints are also displayed for comparison (orange). The constraining power on the data comes chiefly from Bin 3, whereas the information in bins 1 and 2 is limited as an effect of the scale cuts we applied, together with the lower signal-to-noise ratio with which the CMB lensing signal is detected there.

the Planck results in the figure in terms of both the reference value and, equally important, of the width of the posterior distribution, thus allowing to compare the uncertainties.

The focus of our discussion on growth reconstruction is the result displayed in fig. 5.17, where these constraints, converted into the redshift-dependent $\sigma_8(z)$ and defined as $\sigma_8(z) = \sigma_8 D(z)$, are displayed. In this expression, $D(z)$ is the linear growth factor, having been chosen as normalisation $D(0) = 1$. We calculate $\sigma_8(z)$ by using the best-fit cosmological parameters, σ_8 and Ω_m , recovered by fitting the data in every bin and keeping the other cosmological parameters set to the configuration values we described in sec. 5.2.4. Overall, the three values we find for the parameter, in the respective redshift bin, are:

$$\sigma_8(z = 0.074) = 0.99 \pm 0.12, \quad \sigma_8(z = 0.18) = 0.82 \pm 0.11, \quad \sigma_8(z = 0.30) = 0.635 \pm 0.038.$$

By comparing the size of the error bars for each data point with the growth history predicted by Planck, we observe that they are in substantial agreement: a somewhat sharper growth might be preferred (the lowest- and highest-redshift measurements are respectively 1.8σ above and 1.4σ below the Planck prediction). As a further consistency check, we observe a substantial agreement with the constraints of (Sailer et al., 2025a), representing as far as we know the only other lensing tomography analysis having as a target the low-redshift regime and also implementing HEFT. Even that work (Sailer et al., 2025a) appears to recognise a slight upward trend in the growth function at low redshifts, which is nevertheless compatible within the uncertainties with the Planck prediction.

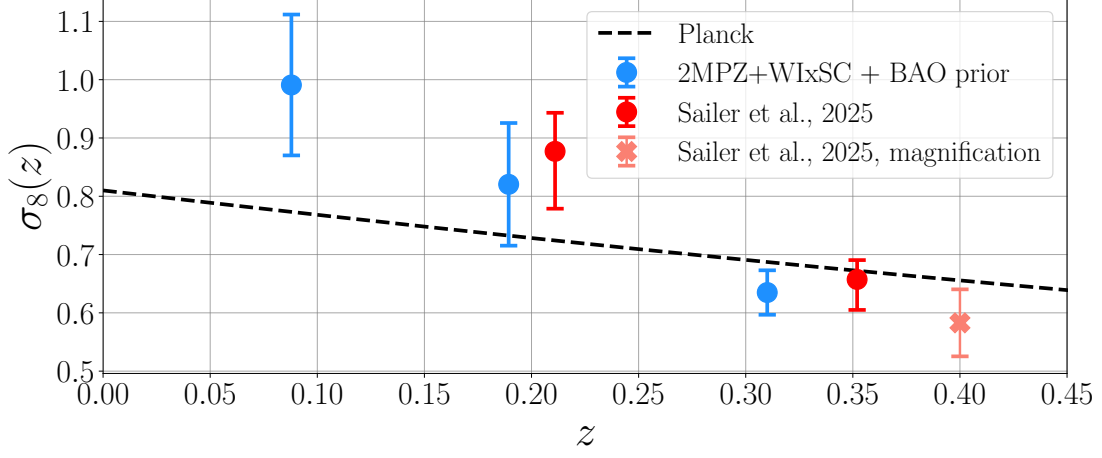


Figure 5.17: We display here our reconstruction of the structure growth function at different redshift (in terms of $\sigma_8(z)$), deriving from our measurements of C_ℓ^{gg} and $C_\ell^{g\kappa}$ in each redshift bin, obtained in the configuration which includes the BAO prior on Ω_m from DESI (blue). Our results are in substantial agreement with the prediction of $\sigma_8(z)$ based on the Λ CDM parameters set at the values measured by Planck (dashed line), and with previous works also performing low-redshift measurements, in particular (Sailer et al., 2025a) (red and pink).

In general, the reasonings of sec. 5.3.1 apply: no actual “tension” is found, but it is important to stress-test and understand the driving factors of the complicate process of structure growth at low redshift and non-linear regimes. We recall and briefly review the other low-redshift tomographic results we presented also in sec. 4.2.1: in general, our results show equally good agreement with them. Using 2MPZ, (Bianchini & Reichardt, 2018) measures a late-time fluctuation amplitude rather compatible with the Planck prediction, analogous to the findings of (Peacock & Bilicki, 2018), which combines 2MPZ and WISE×SuperCOSMOS, another work reporting some trend for larger σ_8 at decreasing redshifts, and similar considerations hold for (Marques & Bernui, 2020). An important difference to bear in mind is that those works used quite different bias models and analysis choices compared to the later publications of the last couple of years, (Sailer et al., 2025b,a) representing more direct terms of comparison for our work.

5.3.3 Constraints on bias parameters and coevolution relations

The primary goal of this work was to measure fundamental cosmological parameters and to reconstruct the growth function at low redshift: thus far, we have always been treating the HEFT galaxy bias parameters as secondary nuisance parameters. As we discussed above and as we presented in greater detail in sec. 1.3.2 there is nevertheless an intrinsic interest in their values, redshift evolutions and other properties. More in detail, it has been reported that higher-order bias parameters tend to display a level of correlations with the associated linear bias value in terms of empirical laws often called “coevolution relations” (Lazeyras et al., 2016; Desjacques et al., 2018; Philcox & Ivanov, 2022; Lazeyras & Schmidt, 2019; Zennaro et al., 2022; Shiferaw et al., 2025). By putting constraints on such correlations, the parameter space to be explored could be reduced, since those higher order terms could be written as a function of the linear term b_1 only, with clear benefits both at a conceptual and practical level (probably reducing, among the other things, the volume effects we found to be so important in our analysis). Importantly, for analyses including scales $k > 0.1\text{-}0.2 \text{ Mpc}^{-1}$, it could be recommendable to consider the effect of

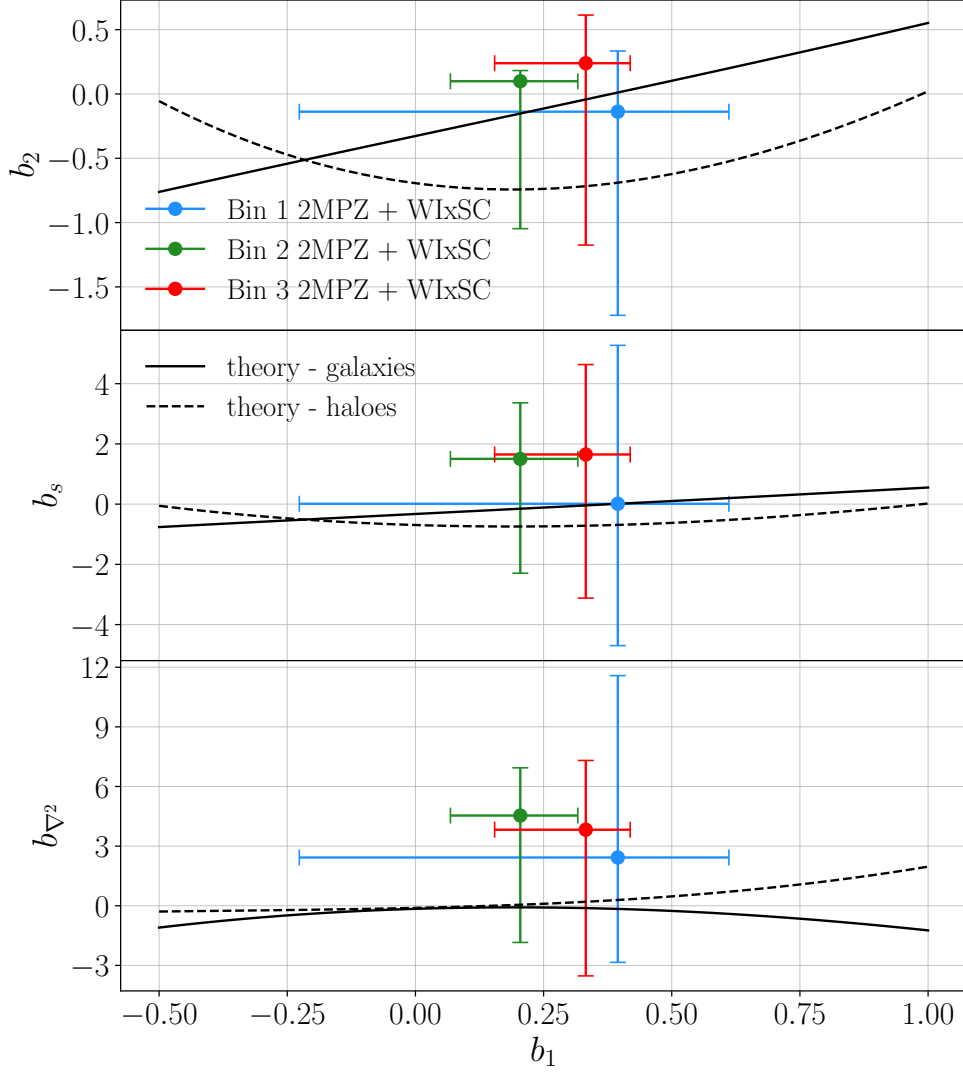


Figure 5.18: We display here the constraints on the higher-order HEFT bias parameters (b_2 , b_s , b_{∇^2}) as a function of the linear Lagrangian bias b_1 . The results are compared with the coevolution relations reported in (Zennaro et al., 2022) for both haloes and galaxies (dashed and solid lines, respectively). We remark that, while Bins 2 and 3 contain data from the WISE×SuperCOSMOS catalogue, the Bin 1 galaxy sample is made of galaxies belonging to the 2MASS survey. They are effectively a distinct galaxy populations so that, while b_1 grows with redshift, as expected, for the data measured in Bins 2 and 3, Bin 1 has a larger and independent value.

baryons on the cross-correlations between matter and galaxy fields (Zennaro et al., 2024) (see also sec. 4.2.1). Nevertheless, the size of the error bars in our dataset, combined with the fact that we are not pushing ourselves to extremely small scales (those in the order of $k = 5 - 10 \text{ Mpc}^{-1}$), leads us to the conclusion that it would not be possible to constrain baryonic models, supporting our conclusion to discard their effects in our analyses. Our results are exposed in fig. 5.18; the specific analysis configuration we use is that of the simultaneous analysis of all three redshift bins in the “brute-force marginalisation” configuration: our predominant interest is in reporting the inferred marginalised posterior distributions of the bias parameters; this prevents us from using the AAM methodology, for which a posterior distribution for each bias parameter is not defined. Should we leave the cosmological parameter free, as we said, we might end up with volume effects distorting both the cosmological parameters and, by interaction among the bias parameters and

Haloes, $k_d = 0.75h \text{ Mpc}^{-1}$, $k_{\text{max}} = 0.7h \text{ Mpc}^{-1}$	
$b_2(b_1) =$	$2 \times \left[-0.09143 (b_1)^3 + 0.7093 (b_1)^2 - 0.2607b_1 - 0.3469 \right]$
$b_s(b_1) =$	$2 \times \left[(0.02278(b_1)^3 - 0.005503 (b_1)^2 - 0.5904b_1 - 0.1174) \right]$
$b_{\nabla^2}(b_1) =$	$-0.6971 (b_1)^3 + 0.7892 (b_1)^2 + 0.5882b_1 - 0.1072$
Galaxies, $k_d = 0.75h \text{ Mpc}^{-1}$, $k_{\text{max}} = 0.7h \text{ Mpc}^{-1}$	
$b_2(b_1) =$	$2 \times \left[0.01677 (b_1)^3 - 0.005116 (b_1)^2 + 0.4279b_1 - 0.1635 \right]$
$b_s(b_1) =$	$2 \times \left[-0.3605 (b_1)^3 + 0.5649 (b_1)^2 - 0.1412b_1 - 0.01318 \right]$
$b_{\nabla^2}(b_1) =$	$0.2298 (b_1)^3 - 2.096 (b_1)^2 + 0.7816b_1 - 0.1545$

Table 5.4: Fitting functions for the phenomenological bias coevolution relations as reported in (Zennaro et al., 2022). We show them for both haloes and galaxies, having set the two smoothing scales of $k_d = 0.75 h \text{ Mpc}^{-1}$ and $k_{\text{max}} = 0.7 h \text{ Mpc}^{-1}$. We remark that we used different pre-factors, in order to account for our distinct definition of b_2 and b_s as quoted in eq. 5.7 with respect to (Zennaro et al., 2022).

the cosmological parameters, possibly the HEFT bias terms, too. We therefore investigate the coevolution relations keeping the cosmological parameters fixed at the values of Planck. Since the bias parameters act effectively as amplitude coefficients degenerate with σ_8 , we have further checked that the qualitative results remain the same for two different cosmologies, with higher and smaller σ_8 .

Eventually, we are able to show the relations between the higher order bias parameters (b_2 , b_s and b_{∇^2}) and the linear bias b_1 , as inferred in our analysis, compared with the fitting functions of the coevolution relations present in the literature. Different works have explored these relations in the context of Eulerian bias expansions both for haloes (e.g. Lazeyras et al., 2016; Lazeyras & Schmidt, 2018) and for galaxies (Barreira et al., 2021), but, in general, it is difficult to translate Eulerian results in a Lagrangian framework. This limit has been overcome recently in (Zennaro et al., 2022; Shiferaw et al., 2025). The authors have been able to recover different fitting functions for haloes and various types of galaxies calibrating results of simulations with hybrid Lagrangian bias expansions and descriptions of the perturbative fields. We employ the fitting functions reported in (Zennaro et al., 2022), which uses our same HEFT implementation, also including a smoothing of the original Lagrangian fields. We however had to correct those fitting functions since we had to account for the different numerical prefactors between the expansion assumed in eq. 5.7 and in (Zennaro et al., 2022): the modified functions are shown in table 5.4. Fig. 5.18 display thus the observed relations between higher-order bias parameters and the associated linear bias term. They are ordered from top to bottom, while the comparison is performed against both the coevolution relation as calibrated on the galaxies (black-solid lines in fig. 5.18) as well as on the Dark Matter haloes (black-dashed lines). All our measurements lie within about 1σ from the theoretical predictions. We do not aim at recovering exact relations: some level of scatter is expected in all these kinds of analyses, because coevolution relations are strongly dependent on the specific galaxy sample, particularly affected by galaxy assembly bias effects and equally specific selection effects amplifying the scattering of the datapoints around the fitting functions, which

remain phenomenological in nature. Further agreement is recovered with (Pellejero Ibáñez et al., 2024), in which HEFT was also used in order to constrain galaxy bias parameters (this time, using three-dimensional clustering measurements from the SDSS-III BOSS survey): the work analogously observed that the measured values are scattered around – although generally compatible with – the same coevolution relations we used. An interesting progress is represented by the combined use of power spectrum and bispectrum discussed in (Verdiani et al., 2025) that, next to the main cosmological goals of the analyses, also measures the coevolution relations for b_2 and b_s , observing a significantly increase of constraining power compared to the results found with the power spectrum only. A last observation considering fig. 5.18: one may notice that $b_{1,1} > b_{1,3} > b_{1,2}$, which might be a bit surprising. The general expectation would be that the galaxy bias is a uniform growing function of the redshift when samples with a similar apparent magnitude limit are used. This is indeed the hierarchy between the b_1 values in Bin 2 and Bin 3. The reason why $b_{1,1}$ displays a larger value compared with the other ones, the expectation being instead that Bin 1 should take the lowest values, is that it is probing a different population of galaxies. Indeed, the sources from 2MPZ contained in Bin 1 are selected after having performed a cross-match between that catalogue and the 2MASS mass catalogue, in addition to the WISE and SuperCOSMOS datasets. Instead, the galaxies from bins 2 and 3 have all been selected from the WISE×SuperCOSMOS survey, without contaminations from other catalogues.

5.4 Conclusions

This chapter has described in even increased detail the results published in (Rubiola et al., 2025), a work where I was able to measure the growth of structure at late times, resorting to the combined photometric galaxy clustering samples collected in the 2MPZ and WISE×SuperCOSMOS surveys, cross-correlated with the CMB lensing signal maps provided by Planck. We wanted to add further results and constraints on the evolution in the amplitude of matter fluctuations at late times ($z \lesssim 0.3$), relatively less explored in the literature and important as a phase of the cosmic history characterised by domination of dark energy, which, at those z , has widely become the dominating fluid in the cosmic inventory, determining the accelerating expansion we observe, and accessing a redshift range complementary to other comparable lensing tomography studies (García-García et al., 2021; Sailer et al., 2025b; Farren et al., 2024; Piccirilli et al., 2024; Sailer et al., 2025a).

By imposing a prior on Ω_m derived from the BAO measurements of (Abdul Karim et al., 2025), we find $S_8 = 0.79 \pm 0.06$ and $\sigma_8 = 0.80 \pm 0.06$, in agreement with Planck and with most part of comparable analyses in the lensing tomography configurations. There is a certain degree of prior-dependence in the constraint on Ω_m : we found indeed that, upon removal of the BAO prior, a low value of $\Omega_m = 0.245 \pm 0.024$ is preferred, having a discrepancy with the Planck measurements at about 2.8σ . Statistically, this is not a concern, and can be included as an additional result on the debate surrounding the “ S_8 tension” – a parameter related to Ω_m by means of its formulation in terms of this quantity and of σ_8 . The absence of a tension between our Late Universe measurements and the high-redshift results of Planck, and therefore the assumption of being able to safely work in a self-consistent Λ CDM model, invites to concentrate on the careful investigation of potential systematics in the analysis techniques (from extrapolations in the emulated power spectra templates to profile likelihood parameter marginalisation techniques) and of the choices concerning the delicate topic of the modelling of the smallest scales of the LSS at low redshifts. This urgency is shared with those works that inspected the observational results providing the strongest tensions on

S_8 (Amon & Efstathiou, 2022; Efstathiou, 2025; Efstathiou & McCarthy, 2025; García-García et al., 2021), which addressed our degree of understanding of a number of phenomena, from baryonic effects and gravitational and clustering non-linearities to cosmic shear and intrinsic alignments, the properties of Dark Matter or even of new physics effects (see sec. 4.2.1 and sec. 4.2.2).

The characteristic of a low Ω_m is shared with analogous harmonic-space based measurements of LSS statistics at intermediate redshifts such as (García-García et al., 2024) using cosmic shear, and (Hang et al., 2021; Farren et al., 2024; Qu et al., 2025) using CMB lensing tomography. Less differences between the measured value of the parameters and the results obtained by Planck are obtained at similar redshifts by (Sailer et al., 2025b; Harscouet et al., 2025)). We discuss that, in the context of galaxy clustering power spectra, the information on Ω_m is dominated by overall functional shape of the galaxy auto-correlation (specifically informed in our case by Bin 3, at the highest redshift), less robust a probe of the matter density parameter if compared with the use of the BAO as a standard ruler. The broadband shape might be more sensitive to clustering systematics or to a not properly specified non-linear bias model. The clustering signal would thus be affected by distinct types of galaxies in different ways: in particular different classes of galaxies may undergo different bias regimes and satisfy different models. Such considerations could explain the differences and variability between these analyses (for instance, (Maus et al., 2025a; Harscouet et al., 2025) which was employing Luminous Red Galaxies samples, different from the galaxy classes used in our and other works). We refer to fig. 5.12 for a synoptic comparison among analogous works in the literature together with shear-only based measurements and Planck results, to be further complemented with fig. 4.1.

By performing an independent analysis of each redshift bin, we were also able to reconstruct the late-time growth of density fluctuations, recovering a good agreement with predictions based on Planck best-fit parameters. Slight hints for a somewhat steeper growth trend at late times was observed, but, indeed it is a general trend, characterised by weak evidence without statistically significant departures from Planck, also present in previous analyses (García-García et al., 2021; Sailer et al., 2025a). Our statistical uncertainties are too large to draw solid definitive conclusions from our constraints, yet precisely because of this fact they are stimulating and make us look at lensing tomography measurements as an important probe to consider in the future. Being the most natural and most constraining probe for the smallest scales of the LSS, if these kind of data will be able to provide sufficiently precise measurements of $\sigma_8(z)$, they could provide tighter constraints on the growth factor translating in important bounds on potential deviations of gravitational dynamics from the General Relativity prediction and the physics of dark energy, possibly deviating from the standard cosmological constant model entering in the Λ CDM description of the Universe (see the Introduction and sec. 4.2.2). There is some connection between this aspect and the more recent pieces of evidence for dynamical dark energy obtained by the combined analysis of CMB, BAO, and supernova datasets (Gu et al., 2025). This is a topic not discussed in this thesis, but we can remark here that potential discrepancies between the growth of structure as predicted under the assumption of a simple cosmological constant and a more general dark energy model would have a direct impact on $D(z)$. This quantity is therefore extremely useful for verifying possible evidence for new physics, and, possibly, to understand the actual nature of what is driving the observed late-time accelerated expansion. Other important observables instrumental to this effort would be redshift-space distortion measurements, that may succeed with testing the evidence for new physics using the observations from DESI and Euclid (Laureijs et al., 2011), in combination with lensing tomography measurements provided by forthcoming photometric imaging surveys, such

as the Rubin Observatory (Ivezić et al., 2019; The LSST Dark Energy Science Collaboration et al., 2018), and ground-based CMB experiments, such as the Simons Observatory (Ade et al., 2019): all of them should be able to reach the precision we need to answer these topics (Ade et al., 2019). Already available data could be leveraged as well: in particular, CMB lensing could be used or replaced with cosmic shear measurements from reference surveys such as DES, KiDS, and HSC (Secco et al., 2022; Wright et al., 2025; Dalal et al., 2023), adding sensitivity to $\sigma_8(z)$ in the $z \lesssim 0.3$ range and the bounds we can place on them.

We devoted much discussion to the proper modelling of non-linear galaxy bias: the low redshifts intervals probed require the use of non-linear formalisms to properly describe the clustering of galaxies and matter since they inevitably contribute to the measured angular statistics and provide most of the interesting information on σ_8 , Ω_m and S_8 .

Also in the vein of comparisons between results obtainable in the linear and non-linear regimes, we can meditate on the viability of different approaches: in particular, we can reflect on the level of cosmological information we can actually recover from small-scale galaxy clustering measured in the non-linear regime, and whether it can be *actually* important to probe those complicated regimes, or whether we might be able to perform “enough” cosmological analysis by limiting ourselves to larger, linear and better understood scales, for which we dispose of simpler and maybe more reliable models. Past analyses have indeed found that, in the case of projected clustering without redshift-space distortions), the gain in information from the mildly non-linear scales is limited by the increased complexity in the modelling effort (Nicola et al., 2024; Farren et al., 2024; Sailer et al., 2025b), suggesting that the effort to be applied should be adequately pondered. Also “physics-agnostic” approaches like (Maleubre et al., 2025) achieved similar conclusions. For instance, the results obtained at $k < 0.25 \text{ Mpc}^{-1}$ are substantially comparable with those we obtain in the fiducial configuration at $k < 0.25 \text{ Mpc}^{-1}$; even the measurements and the precision on σ_8 associated to the “linear HEFT” bias model for $k < 0.11 \text{ Mpc}^{-1}$ are, all things considered, still quite comparable with those found in the fiducial analysis configuration, although the k_{max} attained by using the full HEFT parametrisation is about four times larger – we notice however that, while the comparison between the $k < 0.25 \text{ Mpc}^{-1}$ and the $k < 0.42 \text{ Mpc}^{-1}$ is more straightforward and appropriate, the maxima of the posteriors in the analysis with scale cut $k < 0.11 \text{ Mpc}^{-1}$ tend to shift, although still keeping mutual statistical compatibility as well as with Planck, see tab. 5.2 and fig. 5.11). Given this consideration, gains in the sought-after precision and accuracy might be made, at least partially, instead of further complicating the non-linear modelling of the power spectrum, by leveraging higher-order statistics, in particular the bispectrum. These additional tests of correlations might contribute to the self-calibration of the bias parameters and increasing the retrieval of information from the smallest scales, this time effectively reaching a situation in which the constraining power on quantities of interest for cosmology increases enough to justify the effort against spurious correlations, volume effects and other checks similar to the ones we presented in the previous sections. The studies dedicated to the projected galaxy bispectrum alone (Harscouet et al., 2025) or the combined effects of power spectrum and bispectrum (Verdiani et al., 2025) have found the cosmological parameters can be measured with errors reduced by a factor of $\sim 10\text{-}20\%$, with a tendency towards important underestimation with respect to Planck and other measures when using the bispectrum only (Harscouet et al., 2025), results possibly affected by modelling systematics about which the authors themselves spend words of caution, and general agreement with the Planck prediction in traditional analyses involving the power spectrum alone when power spectra and bispectra are used together (Verdiani et al., 2025). The interesting results concerning

the fundamental cosmological parameters come along with an increased control of the uncertainties on the coevolution relations for the non-linear galaxy bias terms (Weekes et al., 2002). Future data showing greater sensitivity to the quantities of interest might be also helpful with the improvement of our results, as well as by the introduction of more complex bias models, addressing on one hand the reference galaxy power spectrum statistics but also capable of predicting the bispectrum shape at the smallest scales.

General Conclusions

Coming to the end of this work, we can recapitulate the general spirit and conclusions of the research exposed in the previous chapters.

Part I has presented the results of analyses belonging to the field of astroparticle physics and, in particular, to the research strand aiming at circumscribing the properties of dark matter particle candidates by developing cross-correlation statistics between tight cosmological and astrophysical tracers of the LSS – in our case, the 2MRS galaxy survey – with unresolved electromagnetic backgrounds permeating the Universe. These are observables that might conceal, combined with the cumulative effects derived from astrophysical sources not individually resolved, signatures of Beyond the Standard Model physics. In particular, those backgrounds might help with the task of indirectly detecting evidence of the particle phenomenology of Dark Matter, supplementing with independent clues the body of gravitational dynamics puzzles that back the assumption of its existence in the context of astrophysics and cosmology, the disciplines in which the hypotheses about its existence originated, and whose current better-established scientific paradigms and models heavily rely on it (see chapter 2). We concentrated in particular on heavy dark matter particles, known as Weakly Interacting Massive Particles (WIMP) (see sec. 2.3): their large mass (in the GeV-TeV range) should imply that, if these particles do actually constitute Dark Matter and if they undergo annihilation or decay processes in the LSS, the associated photon emission should belong to the gamma band and therefore contribute to the Unresolved Gamma Ray Background (UGRB), vastly explored in the last seventeen years thanks to the reference observations provided by the Fermi-LAT satellite. Throughout the years, the study of the UGRB has provided evidence of its statistical correlation with the LSS in terms of an anisotropic component in its morphology: photons are not randomly scattered on the celestial sphere, but seem to show some form of spatial coherence detectable at the level of autocorrelation power spectra (Ackermann et al., 2018). Such insight is relatable to the fact that they might, at least partially, be sourced by objects residing in the cosmic web, thus following its peculiar network-like distribution, a conclusion further backed by the fact that the anisotropic component is better recognised precisely when cross-correlations with LSS tracers are performed (Fornengo & Regis, 2014; Ammazzalorso et al., 2020; Xia et al., 2015; Regis et al., 2015; Thakore et al., 2025). At the same time, whether the UGRB photons are emitted by astrophysical objects only, or they also contain dark matter signatures, it is a still undecided conclusion (Camera et al., 2013; Fornengo & Regis, 2014; Ammazzalorso et al., 2020; Xia et al., 2015; Paopiamsap et al., 2024; Ando et al., 2014; Cirelli et al., 2024; Cuoco et al., 2015) (see section 2.3.2).

Building upon these general research ideas, we elaborated on the angular cross-correlation power spectra formalism by introducing two ingredients (chapter 3 and (Rubiola et al., 2024, 2025)):

1. A Wiener filter-inspired weighting scheme of the redshift distribution of the galaxy catalogue we were correlating with the UGRB (see sec. 3.2). The idea is that of assuming that all of the halos hosting the galaxies belonging to the reference survey are also hosting the UGRB sources in the same redshift interval. The weighting translates into replacing the original redshift distribution in the galaxy window function with a rescaled version of the UGRB window function (the main result concerning the development of the weights in our case is summarised by eq. 3.26), containing all of the sources, of astrophysical and/or dark matter nature, we assume are sourcing this electromagnetic background.
2. Following a widespread statistical cosmology practice, we have extended to the astroparticle-cosmological approach of UGRB studies the so-called “multi-tracer” framework (Seljak, 2009; McDonald & Seljak, 2009; Abramo & Leonard, 2013), see sec. 3.3: next to the baseline study of the cross-correlation power spectra between the UGRB and the galaxy clustering signal, we have also performed the combined study of the two subcases of a simultaneous statistical analysis of the galaxy-galaxy and the galaxy-gamma correlation signals on one hand and, on the other hand, the simultaneous analysis of the galaxy-galaxy, galaxy-gamma and gamma-gamma correlation power spectra, developing the related formalism, with and without the further introduction of the weighting scheme on top of the multi-tracer.

Armed with this formalism, we have thus been able to forecast the sensitivity these various correlation configurations will have on the smallest WIMP annihilation cross-section rate $\langle\sigma v\rangle$ as a function of the WIMP mass between 10 GeV and 1000 GeV capable of ensuring the detection of an excess dark matter signal against the irreducible astrophysical background at the 2σ level – mass of the particle and annihilation cross-section rates representing the two main factors modulating the intensity and amplitude of the Dark Matter signal and the possibility of properly recognising against the irreducible astrophysical competitor background (sec. 3.4). We assessed the possibility of contrasting the (astrophysical + DM) description of the signal against the (astrophysical only) modelling of the same sources at the 2σ significance in terms of the differences between the respective χ^2 calculated from the signal-to-noise ratios, finding that, overall, multi-tracer and weighting schemes work well especially when considered together, and with different degrees of effectiveness depending on the mass values. In particular, the use of a weighting scheme built on the dark matter window function alone is the most effective, improving the sensitivity to the DM annihilation cross-section rate, both in the cross-correlation and in the multi-tracer case. This weighting scheme can ensure sensitivity to annihilation cross-section rates as low as about 2/2.5 times smaller than the bounds set by the unweighted (i.e., the standard cross-correlation approach) configuration over the whole mass range, while, when moving to the full multi-tracers information, the *DM-only* weighted multi-tracer approach is still the best configuration for most mass ranges: we forecast that the attainable lower bounds are below or comparable with those set by observations on the most sensitive available probes, gamma-ray fluxes from dwarf galaxies (Abdalla et al., 2021). If the weights are defined as the sum of dark matter and astrophysical window functions, improvements with respect to current bounds can be recovered *in the multi-tracer, unweighted configuration* and therefore probably driven by the combined effect of weighting and multi-tracer effects. The *astrophysical+DM* weighted case for the basic cross-correlation provides bounds comparable or worse than the standard, unweighted cross-correlation but we find at the same time that the associated multi-tracer configurations *have worse bounds than the unweighted case*. Especially at the lowest masses, in both the $(gg, g\gamma)$ and $(gg, g\gamma, \gamma\gamma)$ unweighted multi-tracer

approaches provided the best forecasts, ensuring that, at 50 GeV, a WIMP particle with $\langle\sigma v\rangle$ almost a factor of ten below the thermal cross section ($3\cdot 10^{-26}\text{ cm}^3\text{ s}^{-1}$) could still be disentangled from the astrophysics-only explanation of the UGRB at the 2σ level. In general, we remarked that there is a complex interplay of observables, variables, multipole range (“scale cuts”) involved in the analysis, energy and redshift dependencies in the multi-tracer setups, combined with the fact that in the *astrophysical+DM* weighted case we are bound to introduce the competing astrophysical signal in the weights, which complicates the analysis and limits the actual attainable improvements. Moving to larger masses, weighted and unweighted schemes yield rather comparable results and the multi-tracer configurations do not seem to improve on the simple cross-correlation results, neither to forecast sensitivities below current best results, a result probably related to the strongest suppression of the DM signals at masses above 100 GeV compared with astrophysical explanations, in which the DM would more helplessly “drown”. In general, that gamma signals would contain more distinguishable signals of DM if associated to low-mass WIMP particles, below 100 GeV, is a rather well-known results (Regis et al., 2015; Ando et al., 2014).

Pondering these results, we can devise future perspectives, that can be summarised in the first place in terms of the possibility of applying our methodology, thus far applied in a forecasting configuration, to the context of real data, combining the better-suited galaxy surveys with gamma-ray maps: as time goes by, the Fermi-LAT instrument keeps being operational, accumulating statistics and possibly granting higher level of signal detections against the noise just because of augmented exposures – we partially addressed it already in our model, by setting the photon shot-noise values at about the levels associated to 20 years of Fermi-LAT operations, that will be reached around 2028. Concerning more theoretical approaches and forecast studies, the possibilities can span from refining the Wiener filter approach with different, more sophisticated formalisms and Ansatz concerning the link between the host halos of the UGRB components and the ones hosting the galaxies, to the development of analogous weighting schemes and multi-tracer approaches in the context of other electromagnetic bands: we have indeed other unresolved cosmic backgrounds available and there is always interest towards a study of their correlation properties with alternative DM particle models and the associated phenomenology. Especially for what concerns the multi-tracer, cosmological literature has time and again proven the power of this technique and its use is substantially standard practice in the field (the second part of this work itself, in chapter 5, is substantially a multi-tracer data analysis exercise) (Dark Energy Survey and Kilo-Degree Survey Collaboration et al., 2023; Karagiannis et al., 2024; Kopana. Sheean Jolicoeur & Maartens, 2023; Barberi Squarotti et al., 2024; Novara et al., 2025; Montano & Camera, 2024; Euclid Collaboration et al., 2024b). In addition to this, even the understanding in pure astrophysical terms of the UGRB is an interesting topic: it could allow to cross-check the conclusions achieved on the resolved objects belonging to the same classes of galaxies we assume should chiefly contribute to the UGRB, in particular Blazars, Flat Spectrum Radio Quasars, misaligned AGN and Star Forming Galaxies (Fornasa & Sánchez-Conde, 2015; Xia et al., 2015; Zhou et al., 2024; Korsmeier et al., 2022; Manconi et al., 2020), improving therefore the constraints we can put on the many free parameters of the models. Working in terms of weighting schemes might in particular allow to, at least partially, “invert” the projection of the unknown redshift distribution of the sources yielding the observed angular cross-correlation power spectra (see sec. 1.4) and therefore better match our models with the actual location of these candidate astrophysical sources along the line of sight.

Part II concentrated instead on the topic of the study of the low-redshift and small-scale dynamics

of the Large Scale Structure of the Universe. This is a particularly complex clustering regime, in which the interplay of many effects requires careful modelling of a number of variables, especially concerning the non-linear gravitational dynamics, modifying the simplest description of perturbations and clustering provided in linear theory, especially for what concerns the biasing of the tracers and the scale and redshift evolution of the underlying matter field and the statistics we can infer from them (see chapter 1, especially sec. 1.3 and sec. 1.3.2). Such complexity translates in particular into a number of delicate topics, involving both the redshift and, possibly, scale-dependent evolution of the growth factor of the matter perturbation amplitudes predicted by the Λ CDM model. Since the growth factor behaviour is a direct consequence of General Relativity, significant deviations from the Einsteinian paradigm would reveal possible signatures of the need of an updated theory of gravity. Moreover, even in the case of the General Relativity still holding true, relevant inconsistencies with the Λ CDM predictions would be suggestive of the presence of fields and new physics activating in the Late Universe and leaving imprints of their existence in the growth history of the LSS, requiring use to update the cosmic inventory of fields and matter components we currently assume to constitute the whole of matter and energy in the Universe: a cosmological constant, dark matter, baryon matter, neutrinos, electromagnetic radiation and gravitational waves.

A particular manifestation of the complexity of the dynamics at small scales, even prescinding from the very small regimes influenced by baryonic plasma effects but already manifesting at scales where we should at least take care with modelling gravity and including non-linearities is the “ σ_8 ” or “ S_8 tension”, the discordance, up to about 3.5σ , in the measurements of the typical variance of matter perturbations at the scale of $8 \text{ Mpc } h^{-1}$ as derived on one hand from the cosmological parameters measured by the CMB in the Early Universe and, on the other hand, by the results derived from the measurements of cosmological parameters performed with multiple cosmological probes both by means of summary statistics and physical measurements in the Late Universe (Di Valentino et al., 2025). The subject is complex and controversial, as we have outlined in chapter 4. It indeed can be interpreted under two main different lights: the first alternative is to interpret it as an inconsistency directly questioning or falsifying the Λ CDM model, in combination with other known tensions – especially, the H_0 tension on the cosmic expansion rate, again due to a discrepancy, this time up to 5σ , in the measurement of this quantity as found in the CMB and at low redshift —, foundational inconsistencies spanning from the nature of dark matter and dark energy, and signatures of potential deviations from homogeneity and isotropy of the metric (Binney et al., 2025). The other alternative interpretation, the possible future falsification of the Λ CDM model aside, interprets the S_8 tension more cautiously, recognising and inviting to investigate in depth the possible limits of our current modelling of the smallest scales, chiefly addressing effects spanning from clustering non-linearity to baryon effects. In particular, the studies point at the delicate modelling of cosmic shear, the observable better suited to estimate σ_8 , Ω_m and S_8 , as the main possible source of the inconsistency (Amon & Efstathiou, 2022; Efstathiou, 2025; Efstathiou & McCarthy, 2025; García-García et al., 2021).

Taking a rather agnostic stance on fundamental problems, we exposed in chapter 5 an elaboration of the results published in (Rubiola et al., 2025). Our main goal was to perform and report a measurement of σ_8 , Ω_m and S_8 , from which we could derive an estimate of the growth factor evolution in redshift combined with accurate checks of the main potential modelling and observational systematics that might be intervening even in other measurements, such as dependence on the maximum power spectrum scales included in the analyses, non-linear bias, photometric

errors in the measurement of galaxy distributions, volume effects in the posterior sampling. We worked on low-redshift tomographic measurements ($0.1 \leq z \leq 0.6$, with bins centred at $z = 0.07$, $z = 0.18$ and $z = 0.30$) of the cross-correlation between the galaxy clustering photometric samples obtained by combining the 2MPZ and WISE×SuperCOSMOS surveys datasets and the CMB lensing signal maps obtained by Planck. We have thus been considering a redshift interval relatively less explored in the literature yet interesting as a complement of analogous studies of structure growth and potential tension assessment performed at larger redshifts on a number of observables, from galaxies to weak lensing and quasars, all using the CMB lensing signal as an “anchor” to gauge the Late and the Early Universe. Our probes were chosen in order to be adequately sensitive to non-linear scales and capable of measuring the desired parameters without being reliant on the correct modelling of the complex effects affecting the formalism describing the weak lensing of galaxies (see secs. 4.1.2 and 5.1.2) (García-García et al., 2021; Sailer et al., 2025b; Farren et al., 2024; Piccirilli et al., 2024; Sailer et al., 2025a; Hang et al., 2021; Farren et al., 2024; Qu et al., 2025; Harscouet et al., 2025; García-García et al., 2024).

We thus found that, after imposing a prior on Ω_m whose distribution is based on the the baryon acoustic oscillation (BAO) measurements of DESI (Abdul Karim et al., 2025), S_8 was measured to be $S_8 = 0.79 \pm 0.06$, about 1.5σ away from the Planck constraints, while we recovered the Ω_m value corresponding to the prior. Without using the prior, we recover $S_8 = 0.82 \pm 0.07$, compatible with Planck, and of $\Omega_m = 0.245 \pm 0.024$, about 2.8σ smaller than the Planck best-fit parameters.

Our results are not concerning from a strictly statistical point of view (for synoptic comparisons with analogous CMB lensing-galaxy clustering results, Planck TT,TE,EE+lowE+lensing and cosmic shear angular power spectra results, see fig. 5.12 and, for a broader comparison of results and probes fig. 4.1), but they stimulated careful investigation of systematics in the analysis techniques (from calibration of photometric galaxy distances and specification of covariances to the extrapolations in the emulated power spectra templates to profile likelihood parameter marginalisation techniques, see secs. 5.1.1, 5.2.3, 5.2.5 and 5.2.6). All of these aspects have been extensively discussed, finding in particular that, in the context of galaxy clustering power spectra and without a prior, the information on Ω_m is dominated by the overall functional shape of the galaxy auto-correlation power spectrum and by information sourced in the higher redshift bin. The power spectrum functional shape is however known to be a weaker probe for Ω_m compared to the specific sensitivity that the BAO exhibit on the same quantity, this reason justifying the use of the Ω_m prior, indeed based on BAO measurements. In addition, we were modelling our non-linear galaxy clustering power spectra templates by means of the Hybrid Effective Field Theory (HEFT) approach, a re-formulation of the Lagrangian Perturbation Theory (LPT, see sec. 1.3.2), and some limits in the predictive power could have been involved as well, again better controlled by a tight and reliable prior in the context of a Bayesian analysis. Non-linear biased clustering in the context of HEFT Lagrangian Perturbation Theory, the role of scale cuts and the associated degree of information and constraining power have been also investigated, on one hand finding that scale cuts at the lower value of $k_{\max} = 0.25 \text{ Mpc}^{-1}$, associated to a more linear regime, still provide unbiased and competitive measurements of the cosmological parameters, and that even at $k_{\max} = 0.11 \text{ Mpc}^{-1}$ the outcome of the measurement on Ω_m does not change significantly compared to the fiducial case, whilst σ_8 tends to move a further 1σ below the results found in the non-linear regime, however still compatible with Planck. This can be an interesting insight and a first possible suggestion for future work, since it might hint at the possibility of performing valuable measurements without committing to complex modelling of very small scales in the

power spectrum. The constraints recovered on the HEFT non-linear bias parameters have also been discussed for their intrinsic value as measurements of the potential existence of coevolution relations, i.e., the possibility of formulating the higher-order bias terms as a function of the linear bias term b_1 only. Such insight, explored in the literature both in terms of theoretical foundations, outcomes of numerical simulations and observational results (Lazeyras et al., 2016; Desjacques et al., 2018; Zennaro et al., 2022, 2023; Philcox & Ivanov, 2022) would be particularly interesting in establishing a tighter connection among the different driving factors of galaxy clustering and the well-comprised theory of linear bias (Kaiser, 1984, 1987) and in simplifying the description of HEFT terms, possibly avoiding the introduction of volume effects in the MCMC fitting pipelines induced by the large number of nuisance parameters in LPT and SPT models of non-linearity (see secs. 5.3.1 and 1.2.1 (Hadzhiyska et al., 2023)). Overall, we recovered the loose, phenomenological trends published by extant literature. Our data are characterised by large uncertainties, but that was rather expected: the many concurring effects influencing these kind of measurements typically lead to the establishment of a sensible range of variability for the parameters rather than exact results. Back to fundamental cosmology, the independent analysis of each redshift bin allowed us to reconstruct the growth of density fluctuations at late times, substantially consistent with the growth prediction based on Planck best-fit parameters (sec. 5.3.2). There is a slight evidence for an “upturn” of the growth function at late time, which we describe as a general trend, without statistically significant departures from Planck, also present in previous analyses (García-García et al., 2021; Sailer et al., 2025a; Bianchini & Reichardt, 2018; Peacock & Bilicki, 2018).

It is precisely the commitment to further inspection of the topic of structure growth, by means of future datasets or additional probes, that leads us to an outline of the perspectives this work opens. The most relevant one is the introduction of weak lensing data as an additional fundamental probe to test and for which adequate stress-testing methodologies should be devised, in order to understand its impact on the measurements of S_8 and Ω_m as well as on growth reconstruction analyses. Given, in addition, the important flow of recent publications analysing and reanalysing the main available weak lensing datasets, further modulating the topic of tensions (Yoon et al., 2025; Wright et al., 2025; Gomes et al., 2025; Sugiyama et al., 2025), we know that we cannot prescind from weak lensing in this branch of cosmology, but the observable must be approached having the “experimental” conditions under control, which is what we aimed to do in this analysis. Other important observables instrumental to this effort and that future analyses should include would be redshift-space distortion measurements, ground-based CMB experiments, additional galaxy clustering observations, involving both forecast exercises and analyses of existing or soon available datasets. It would also be interesting to consider the extensive combination of the current framework of tomography and cross-correlations, relying on two-point statistics, with higher-order statistics, in particular the bispectrum. Instead of further complicating the non-linear modelling of the power spectrum, given the above considerations on the possibility of attaining comparable precision and accuracy on the parameters of interest without going to extremely small scales, these additional correlators would also allow to calibrate the nuisance parameters (especially non-linear bias terms), possibly relieving volume effects and better balancing the research effort between gaining information on fundamental parameter measurements and taming volume effects and other spurious disturbances in the analyses. These are, for example, the main results obtained by (Harscouet et al., 2025; Verdiani et al., 2025) leading to a reduction (~ 10 -20%) in the final cosmological parameter errors (with different levels of tension with the Planck reference values and possible intervention of modelling systematics) that could be further improved by future,

more sensitive data. The deeper comprehension of the Universe at the smallest scales, be it from a fundamental cosmological point of view or by taking a stance closer to the astrophysical-cosmological interface (Maleubre et al., 2025; Koukoufilippas et al., 2020a; Broxterman et al., 2025; Aricò et al., 2023) appears thus as a stimulating research direction, enclosing a variety of interesting perspectives for future studies reflecting the rich physics of those dynamic and eventful regions of our Universe.

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Doctoral studies are almost a matter of the five senses: readings, faces, places and details, lights of African sunrises and colours in Capetonian downtown, iron-grey Bavarian mornings, skyscrapers and monuments in visited cities, all of them impress your sight. Swift typing on a responsive and agile keyboard gives your touch a remote idea of what playing an instrument should feel like. Ground coffee scents in silent Oxonian departmental mornings, exotic odours in streets from another hemisphere, plain pizzas or unexpected foreign delicacies, all impress your smell and taste. And then it comes to listening, and its counterpart, speaking: from casual conversations to questions and debates. There, I’d say, it lies the actual sense of intellectual work, of, if achieved, the intellectual growth ensuing doctoral studies. The constant murmur of knowledge and doubt, of opinions and point of views, the colourful variety of people around you that science makes one. From all of you, people of the Physics group in Turin, new friends met at schools and older fellows out of the academic sphere, casual speakers at conferences and

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