Bounded Generalized Kelly Mechanism for Multi-tenant Caching in Mobile Edge Clouds

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Abstract Mobile edge computing represents a promising paradigm for $5G$ telecommunication operators. Among various services that can be provided by this technology, cloud edge caching is receiving increasing attention by network providers. By using cloud technology, in particular, the memory space of mobile edge network devices can be provisioned to OTT content providers over specific areas. They can use cache space in order to serve their customers with improved quality of service figures. We study a competitive scheme where contents are dynamically stored in the edge memory deployed by the network provider. OTT content providers are subject to Kelly mechanism with generalized cost and with bounded strategy set. After proving the uniqueness of the Nash equilibrium of such scheme, a simple bisection algorithm for its calculation is provided. Numerical results characterize the Nash equilibrium.

Key words: Mobile Edge Computing, 5G Networks, Edge Caching, Bounded Kelly Mechanism, Nash Equilibrium

1 Introduction

Currently, ever increasing amount of traffic is generated by mobile devices and large fraction of such traffic is ascribed to over-the-top (OTT) content providers such as, e.g., YouTube. Aggregated figures summed up to nearly 30 Exabytes [1] of mobile traffic at the end of 2014. Capacity shortage has thus become a threat for network operators. The deployment of small cell (SC) base stations has been proposed in literature in order to increase capacity provision. SCs are low power secondary base stations with limited coverage, to which user equipments (UEs) in radio range can associate to. They thus increase spatial reuse, and so network capacity.

However, management costs at the network operators' side may increase due to the larger number of network units to be deployed and controlled. To this respect, *mobile edge computing* [2] technology will ease services and resources management over $5G$ networks, since a network operators' edge-clouds will be able to cover a tagged metropolitan area or a village. Mobile Edge caching, in particular, configures as a basic mobile edge caching service to be offered to OTTs in $5G$ networks. The

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primary goal of edge caching is to improve quality of experience by circumventing the limited backhaul connection of SCs [16]. Moreover, by using SVC technologies, e.g., MPEG-DASH [7], it is possible to perform bitrate adaptation to radio link conditions thus further improving the quality of experience of OTTs' customers.

Contents can be replicated directly on lightweight server facilities attached to SCs and located at the mobile network edge. Recently, edge caching has become an important optimization problem [13, 8, 12, 4, 10, 15, 6]. Finally, using edge cloud technology, caching servers connected to a few SCs will be aggregated into local edge cache units. Those SCs will permit to access a seamless unique caching space for the same local area. Edge caching services can be offered to multiple OTTs at the same time.

In this paper we apply a generalized Kelly mechanism to the problem of sharing the edge cloud cache. Under a multi-tenant caching scheme, the available memory is assigned to OTT content providers who compete for time-limited cache utilization. Also, we assume that part of the caching memory may also be used due to legacy traffic requirements of the network provider.

The proposed model for cloud edge caching accounts for popularity and availability of contents, as well as spatial density of SCs to which UEs may associate to.

In our competitive scheme, OTT content providers purchase cloud edge caching service from mobile operators. Under a spatial Poisson distribution of SCs, we model the competition among content providers using a Kelly mechanism with general costs and bounded bidding space. We hence show that the game admits a unique Nash equilibrium, which can be determined by a simple demand-based bisection search.

2 System Model

The mobile network operator serves C content providers. Content providers serve customers by leveraging on the mobile operator's network. We assume that M classes of contents are available to each content provider based on their popularity. Also, demand rate g_c^i represents the number of contents of class i which are to be served to customers of provider c in the unit of time.

Cache memory is aggregated through several local edge servers connected through a metro area network and managed through the cloud edge service. N is the available memory on each local edge cloud unit and N_0 the total cache space across the whole deployment. Each content is assumed to occupy same unitary memory space, i.e., a slot.

We further assume that the number of contents N_c^i for each class $i = 1, \ldots, M$ is larger than the available memory space. Each content is stored on average a finite amount of time, and it thus erased every $1/\eta$ seconds.

Content provider c will purchase edge caching service by the network provider. Hence, she will issue b_c slot requests per day $0 \leq b_c \leq B_c$, for some $B_c > 0$. Also, the network operator will reserve caching slot for its own purposes at some rate δ which we assume a constant in the rest of the discussion. The network operator reserves space for x_c contents of content provider c according to

$$x'_c = b_c - \eta\, x_c, \tag{1}$$

while the full memory occupation will be ruled by

$$x' = b - \eta\, x, \tag{2}$$

where we let $b := \sum_c b_c + \delta$ the global slot reservation requests per day. If we assume $x(0) = 0$, the resulting governing dynamics is

$$x(t) = N_0 \cdot \max\left\{\frac{b}{\eta}\left(1 - e^{-\eta t}\right), 1\right\}$$

In the rest of the paper, we assume that the network provider aims at fully using the total cache memory by ensuring that $b \geq \delta$. Hence, at time $t_N = -\frac{1}{\eta}\log(1 - \frac{\eta N}{b})$, the cache will be full, i.e., $x(t_N) = N_0$. This will grant for $t \geq t_N$ to each content provider the proportional share of the caching space

$$x_c(t) = N_0 \frac{b_c}{b_c + b_{-c} + \delta}, \tag{3}$$

We assume content requests uniform across the network provider's network. Thus, the same share will be occupied by CP c in each local edge cache unit.

Each CP caches her customer's requests using a weighted round robin scheme, where class i will receive weight t_c^i, and we denote $\mathbf{t}_c = (t_c^1, \dots, t_c^M)$ the vector of weights, where $\sum t_c^i = 1$. If the total cache space is saturated, contents of the same content provider may be overwritten. However, a compatibility condition requires that the number of daily cached contents does not exceed the memory space, i.e., $\sum_c B_c \leq N_0$, which we shall assume throughout the paper.

Finally, x_c^i denotes the edge cloud amount of memory occupied by contents of the i-th class:

$$x'^{i}_c = t_c^i b_c - \delta x_c^i \tag{4}$$

Then, the fraction of local edge cache memory occupied by contents of class i from content provider c is

$$\overline{x_c^i} = \frac{t_c^i bc}{\sum_{v \in \mathcal{C}} b_v + \delta} N \tag{5}$$

A tagged content of class i of content provider c is found in each local cloud edge with probability $P_c^i = \min\{\frac{\overline{x_c^i}}{N_c^i}, 1\}$. In the rest of the paper, we will assume that $N < N_c^i$ for the sake of simplicity.

We assume that fetching a non cached content from the content operator infrastructure through the backhaul has unitary delay cost. Conversely, delay is negligible if the user associates to a SC which is part of local edge cloud and a cached copy of the content is present. But, such a SC must be within the UE radio range $r > 0$. SCs are distributed according to a spatial Poisson point process with average intensity Λ [10]. By a thinning argument we let $\Lambda \cdot P_c^i$ the intensity associated with the distribution of content i of $c \in \mathcal{C}$ within the edge cloud. Hence, given a tagged UE, the probability not to find a content of class i picked at random is $e^{-\pi r^2 \Lambda P_c^i}$.

Thus, if a tagged UE requests a content of class i from content provider c, the missed cache probability depends on the content caching rate chosen by c, i.e., b_c. Finally, the *missed cache rate*

$$U_c^i(b_c, b_{-c}) = e^{-\pi r^2 \Lambda P_c^i} = e^{-\pi r^2 \Lambda \frac{N}{N_c^i} \frac{t_c^i b_c}{b+\delta}} \; ,$$

is the rate at which customers of content provider c do not find a content of class i in the edge cache. Summing over all the contents classes, and weighting each probability by demand rate g_c^i we define the expected missed cache rate, i.e., the actual cost function for content provider c

$$U_c(b_c, b_{-c}) = \sum_i U_c^i(b_c, b_{-c}) = \sum_i g_c^i \, e^{-\Lambda_c^i t_c^i \frac{b_c}{b+\delta}} \tag{6}$$

where $b_{-c} := \sum_{v \neq c} b_v$ accounts for the fact that other content providers share the same cache space. The parameter $\Lambda_c^i := \pi r^2 \Lambda \frac{N}{N_c^i}$ has the meaning of availability of the content per square meter.

3 Game Model

Each content provider pays a cost $\lambda_c(b_c)$ for reserving caching slots at a given rate b_c. Such cost, in turn, is decided by the network operator. The optimal caching rate b_c is settled by content providers accordingly. Content provider c plays strategy b_c, where $0 \le b_c \le B_c$.

The network provider proposes the cost to content providers: they decide the caching rate depending on their contents, and their opponents' strategies. We define a game where each CP c minimizes the cost $U_c(b_c, b_{-c}) + \lambda_c(b_c)$. Hence, the best response of c solves

$$\begin{aligned} \min_{b_c} \; & U_c(b_c, b_{-c}) + \lambda_c(b_c) \\ & 0 \le b_c \le B_c \end{aligned} \tag{7}$$

where $\lambda_c(\cdot)$ is increasing and convex.

Here $b_{-c} = \sum_{v \neq c} b_{-v}$. Also, opponents' strategy profile writes $\mathbf{b_{-c}} = (b_1, \ldots, b_{c-1}, b_{c+1}, \ldots, b_C)$.

The meaning of (7) is that of purchasing a caching rate, e.g., caching slots per day, at a cost $\lambda_c(b_c)$, from the network provider.

By letting $V_c(x_c) := \sum_i g_c^i \, e^{-\Lambda_c^i t_c^i x_c}$: once we denoted $x_c := \frac{b_c}{b+\delta}$, it is immediate to verify that V_c is a convex, strictly decreasing and continuously differentiable function. Hence, the scheme is a generalized Kelly mechanism [9].

The Kelly mechanism prescribes the proportional allocation of a shared resource and the ratio is proportional to the players' bids. In our case, the cache space is the resource and the bids are the required caching rates.

Also, given the constraints on the caching rate $c \in C$, $0 \leq b_c \leq B_c$, the set of strategies is a convex compact subset of $\mathbb{R}^C$, so that the existence of a Nash equilibrium is guaranteed by the result of Rosen [3].

Best response: By differentiating (7), and invoking the convexity of the utility, we can derive the best response $b_c^* = b_c^*(u_{-c})$ of each player by imposing the conditions on the marginal utility, i.e., the increment of the utility derivative for $b_c = 0$. Indeed, we can write

$$V'_c\Big(\frac{b_c}{b+\delta}\Big)\frac{b_{-c}+\delta}{(b+\delta)^2} + \lambda'_c(b_c) = 0 \tag{8}$$

from which it is immediate

Lemma 1 (Best response) *Given opponents' strategy profile $\mathbf{b}^*_{-c}$, let $\lambda_0(b_{-c}) := -\frac{V'_c(0)}{b_{-c}+\delta}$. There exists a unique best response in the form:*
i. $b_c^* = 0$ if and only if $\lambda'_c(0) > \lambda_0(b_{-c})$
ii. $b_c^* > 0$ if and only if $\lambda'_c(0) < \lambda_0(b_{-c})$
iii. $b_c^* = B$ if and only if

$$\lambda'_c(B_c) \leq \lambda_c^B(b_{-c}) := -\frac{B_c \cdot V'_c\Big(\frac{B_c}{B_c+b_{-c}+\delta}\Big)}{(B_c+b_{-c}+\delta)^2}$$

We can obtain explicit expressions from (6): $\lambda_0(b_{-c}) = \frac{\sum_{i=1}^M \Lambda_c^i t_c^i g_c^i}{b_{-c}+\delta}$ and

$$\lambda_c^B(b_{-c}) = \sum_i \frac{(g_c^i \Lambda_c^i t_c^i)(b_{-c}+\delta)}{B_c+b_{-c}+\delta}\, e^{-\Lambda_c^i t_c^i \frac{B_c}{B_c+b_{-c}+\delta}}$$

From the best response we can obtain two trivial Nash equilibria: the null one $\mathbf{b} = \mathbf{0}$ and the saturated one $\mathbf{b} = \mathbf{B} = (B_1, \ldots, B_C)$.

Proposition 1 (Trivial Nash equilibrium)
i. $\mathbf{b} = \mathbf{0}$ is the unique Nash equilibrium iff $\lambda'_c(0) > \lambda_0(0)$ for all $c \in \mathcal{C}$
ii. $\mathbf{b} = \mathbf{B}$ is the unique Nash equilibrium iff $\lambda'_c(0) \leq \lambda_c^B(\sum_{v \neq c} B_v)$ for all $c \in \mathcal{C}$

Note that in customary formulations [14, 9], $\mathbf{0}$ is not a Nash equilibrium. Here, it may be the Nash equilibrium of the system since $\delta > 0$.

3.1 Nash Equilibrium

The generalized Kelly mechanism has a unique Nash equilibrium [9]. Even when the strategy set is bounded, the caching game proposed has a unique Nash equilibrium. This is true for the trivial equilibria in Thm. 1. The idea for the proof extends the argument in [11]. There, the existence is proved in the case of $B_c = +\infty$, for all $c \in \mathcal{C}$, and accounts for the case $\delta > 0$.

In our case, given the bounded strategy, we do not require assumptions on the demand function in 0. Nevertheless, some care is required in order to prove the uniqueness for non-interior type of Nash equilibria. The uniqueness for the case when $B_c = +\infty$ for some $c \in \mathcal{C}$ follows immediately.

Theorem 1 *Let V convex monotone and both V and the λ_cs twice continuously differentiable: the Kelly mechanism with bounded strategy sets $[0, B_c]$ has a unique Nash equilibrium.*

Proof. Existence of a Nash equilibrium is given by Rosen's result on concave games [3]. From Prop. 1, we need to prove the statement for $\mathbf{b}^* \neq \mathbf{0}$ and $\mathbf{b}^* \neq \mathbf{B}$. We define $p := \sum b_c + \delta$ and demand function $x_c(p) : [\delta, \infty) \to \mathbb{R}$ where $x_c(p)$ is determined by the unique best response of player c for a given value of p. In particular, the unique minimizer $x_c^\star(p) \in [\delta, +\infty)$ of (8) solves the best response equation

$$V'_c(x_c^\star)(1 - x_c^\star) = -p\lambda'_c(p \cdot x_c^\star) \tag{9}$$

obtained by replacing $b_c = p \cdot x_c^\star$.

The existence of a unique function $x_c^\star(p)$ derives from the implicit function theorem, which requires continuous differentiability of V' and the λ_cs. Uniqueness derives from the convexity of V. Now, by expressing the best response via the demand function x_c

$$x_c(p) = \begin{cases} 0 & \text{if} \quad x_c^\star \leq 0 \\ x_c^\star(p) & \text{if} \quad 0 < p \cdot x_c^\star < B_c \\ \frac{B_c}{p} & \text{if} \quad p \cdot x_c^\star \geq B_c \end{cases} \tag{10}$$

We can define $C_0(p) = \{c \in \mathcal{C} | x_c^\star(p) \leq 0\}$. Also, the set $\mathcal{C}'(p) = \{c \in \mathcal{C} | p \cdot x_c^\star(p) \geq B_c\}$ is unique for every value of $p \in [\delta, \sum B_c + \delta)$. From (9), in the region where $p \cdot x_c^\star(p) < B_c$, the x_cs are strictly decreasing in p; this is showed in [11] by expressing (8) as a function of x'_c due to the convexity of λ_c.

Now, we observe that the actual best responses of players in a Nash equilibrium need to satisfy the condition

$$\sum_{c=1}^{C} x_c(p) = 1 - \frac{\delta}{p} \tag{11}$$

Because we assumed $\mathbf{b}^* \neq \mathbf{0}$, it holds $0 < \sum_{c=1}^{C} x_c(\delta)$. Also, $\delta < p < \sum B_c + \delta$ since we assumed $\mathbf{b}^* \neq \mathbf{B}$: hence, there exists always a strictly decreasing $x_c(p)$ for $p \in (\delta, \delta + \sum B_c)$ and so it is the sum appearing in the left-hand term. But, for the right hand term is increasing, if a non identically zero or non identically saturated solution $\mathbf{x}^*$ exists, it must correspond to a unique value of p, which we denote p^*.

Finally, the Nash equilibrium $\mathbf{b}^*$ is derived by the bijection $\mathbf{b} = \phi(p^*)$, where
i. $\phi_c(p^*) = 0$ for $c \in \mathcal{C}_0(p^*)$;
ii. $\phi_c(p^*) = B_c$ for $c \in \mathcal{C}_B(p^*)$;
iii. the $\phi_c(p^*)$s for $c \in \mathcal{C}'(p^*) = \mathcal{C} \setminus (\mathcal{C}_0(p^*) \cup \mathcal{C}_B(p^*))$ solve the full rank compatible linear system

$$b_c^*(1 - x_c^*) - \sum_{\substack{v \in \mathcal{C}'(p^*) \\ v \neq c}} x_c^* b_v^* = x_c^* \delta + x_c^* \sum_{v \in \mathcal{C}_B(p^*)} B_c, \quad c \in \mathcal{C}'(p^*)$$

which concludes the proof.

3.2 Calculation of the Nash equilibrium

The above proof suggests an algorithm for the calculation of the Nash equilibrium as reported in Tab. 5. `NBKG` (Generalized Bounded Kelly Nash) uses a simple bisection search for the optimal demand value p^*.

The search is operated on the total demand interval $[0, \sum B_c + \delta]$, and the algorithm performs a preliminary check for the null Nash equilibrium (line 1). The usage of bisection is suggested by the fact that (11) provides negative values for $p > p^*$ and positive values for values $p < p^*$. At each step it solves in the x_cs the set of the best responses (9) for a given value of p.

Because of convexity, the solutions of the system of equations (9) can also be calculated using a simple bisection algorithm. Hence, we can characterize the computational complexity of the algorithm under the assumption that we impose a precision parameter $\epsilon > 0$ both for the search of the optimal total demand p^* and the calculation of the best response.

Proposition 1. *The time complexity of* `NBKG` *is* $O(\epsilon^{-2} \log_2(\sum B_c + \delta) \log_2(1 + \lambda_c \max_c B_c))$

Proof. The number of iterations of the bisection search in the main `WHILE` loop (lines 1 to 10) is $O(\epsilon^{-1}(\sum B_c + \delta))$. The fact follows from elementary properties of bisection search [5][Ch. 4, pp. 145]. Moreover, at each iteration the calculation of the best response appearing at line (2) requires to compute $x_c^\star(p_{\text{mid}})$ as in (9). The latter operation can be done at a cost $O(\epsilon^{-2}(1 + \lambda_c \max_c B_c)$.

3.3 Interior Nash with Linear Costs

Let us consider a linear cost $\lambda_c(b_c) = \lambda_c b_c$. When the Nash equilibrium is an interior one, i.e., $b_c^* < B$ for all $c \in \mathcal{C}$, a simple extension a result of Hajek [9] determines the Nash equilibrium as the solution $\mathbf{x}^*$ of the optimization problem

$$\begin{cases} \text{Minimize} & \sum_{c \in \mathcal{C}} \widehat{V}_c(x_c) \\ \text{subject to} & x_c \geq 0 \\ & \sum x_c \leq \frac{\sum B_c}{\sum B_c + \delta} \end{cases} \tag{12}$$

where $\widehat{V}_c(x_c) = \dfrac{1}{\lambda_c}\left(V_c(x_c)(1 - x_c) + \displaystyle\int_0^{x_c} V_c(z)dz\right)$.

The upper constraint follows immediately from $\sum x_c + \frac{\delta}{\sum b_c + \delta} \leq 1$. The equivalence with the proposed optimization problem is seen by writing the Lagrangian

$$L(\mathbf{x}, \xi, \mu) = \sum_{c \in \mathcal{C}} \widehat{V}_c(x_c) - \sum_{c \in \mathcal{C}} \xi_c x_c - \mu(1 - \frac{\sum B_c}{\sum B_c + \delta} + \sum_{c \in \mathcal{C}} x_c)$$

and apply the KKT conditions. Indeed, first order optimality requires

$$\frac{\partial L}{\partial x_c} = \frac{V'_c(x_c)}{\lambda_c}(1 - x_c) - \xi_c - \mu = 0$$

Trasversality condition $\lambda_c x_c = 0$ brings

$$\frac{V'_c(x_c)}{\lambda_c}(1 - x_c) = \mu \quad \text{if} \quad x_c > 0$$
$$\frac{V'_c(0)}{\lambda_c}(1) < \mu \quad \text{if} \quad x_c = 0 \tag{13}$$

Hence, assume that $\mathbf{b}^*$ is a Nash equilibrium: all the components x_c^* such that $0 \leq b_c^* < B$ verify (13) by letting $\mu = \sum b_c^* + \delta$ because of Lemma 1, which proves the equivalence. The explicit form of the Nash equilibrium for the problem described in (12) is easily derived from (6) and omitted here for the sake of space.

4 Numerical results

Fig.2 reports on the numerical description of the Nash equilibria in various cases. We have considered first the effect of the upper bound B_c in Fig.2a. As seen there, before a critical value ($B_c = B = 10$), the Nash equilibrium is the saturated one and all players equally share the total cache memory. We observe that the share of the total memory in the first part of the graph increases: this is due to the fact that as B is initially small with respect to the parameter δ. In turn, δ represents by construction a small fraction of the cache memory available to content providers for their own operations: as B becomes bigger with respect to δ, the portion of the cache that can be used by the players increases. In particular, at first, player 2 settles on the optimal best response when the others play B. Finally, for $B \geq 15$ the Nash settles onto the value corresponding to the one when the strategy set is unbounded.

We have considered next the case of a two players game as depicted in Fig.2b and c. Both players have two contents classes. For the sake of notation it is convenient to express the two players' strategy as $\mathbf{t}_c = (t_c, 1 - t_c)$, $c = 1, 2$. In particular, in Fig.2b the ratio of the cache occupied at the Nash equilibrium is plotted against the possible content allocations chosen by the two players. In Fig 2c the total utility function at Nash equilibrium is plotted against the content allocations chosen by the two players. The graph suggests that the two players could further enhance their own utility by optimizing allocations, i.e., acting on parameters t_c^i.

Finally, Fig.2d reports on the revenue of the network operator in the case of two players. We have consider the cost in the form $\lambda_c b_c^\alpha$, where $\alpha = 1.1$. By changing the parameters λ_c, $c = 1, 2$, we have explored the total revenue of the network operator. From the image it is not clear if a global maximum exists. Actually, in the linear cost case, it is actually possible to show that even for equal prices, not always there exists a unique price able to maximize the revenue of the network provider. In turn this suggests that the network provider may have slack in order to search for optimal prices satisfying other metrics, e.g., guaranteeing largest possible caching rates to the content providers.

5 Conclusions and Discussion

Edge caching in the radio access network represents a convenient technology for $5G$ network providers. Cache space of the edge network can be provisioned to

OTT content providers requesting cache space in order to serve their customers with improved quality of experience. We study a competitive scheme where contents are dynamically stored in the edge memory deployed by the network provider. We have proposed a generalized Kelly mechanism to model the cache utilization on the edge cloud. While doing so, we have proved the uniqueness of the Nash equilibrium of the Kelly mechanism on a bounded strategy set and provided an algorithm for its calculation.

The numerical results (Fig.2b and c) suggest a variant of the proposed scheme. A CP might optimize her utility by a) bidding first for her best response, and then b) optimize the allocation among her own content classes. If this optimization is operated iteratively, this would result in successive rounds of optimization. The convergence properties of such process are part of future work.

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$\mathbf{b}$ =NBKG$(\delta, \mathbf{B}, C, \{V_c(\cdot), \lambda_c(\cdot)\}_{c\in\mathcal{C}})$

Receives: $\mathbf{B}, C, \{V_c(\cdot), \lambda_c(\cdot)\}_{c\in\mathcal{C}}$
Initialize: $p_L = 0, p_R \leftarrow \sum B_c + \delta$
$\quad T_{\text{tmp}} \leftarrow +\infty, T \leftarrow 0$
$\quad p_{\text{mid}} \leftarrow p_R$
`IF` $V'_c(0) + \delta\lambda'_c(0) = 0$ for all $c \in \mathcal{C}$
`RETURN` $\mathbf{0}$
`WHILE` $|T - T_{\text{tmp}}| > \epsilon$
$\quad$ Calculate $x_c^\star(p_{\text{mid}})$, for all $c \in \mathcal{C}$ according to (9)
$\quad x_c^* \leftarrow \max\{0, \min\{B_c/p_{\text{mid}}, x_c^\star(p_{\text{mid}})\}\}$
$\quad T_{\text{tmp}} \leftarrow T$
$\quad T \leftarrow \sum x_c^\star(p_{\text{mid}}) - \left(1 - \frac{\delta}{p_{\text{mid}}}\right)$
$\quad$ `IF` $T > 0$
$\qquad p_L \leftarrow p_{\text{mid}}$
$\quad$ `ELSE`
$\qquad p_R \leftarrow p_{\text{mid}}$
$\quad$ `END`
`END`
$\mathbf{b}^* \leftarrow p_{\text{mid}} \cdot \mathbf{x}^\star(p_{\text{mid}})$
`RETURN` $\mathbf{b}^*$

Fig. 1 NBKG: algorithm computing the Nash equilibrium for the game.

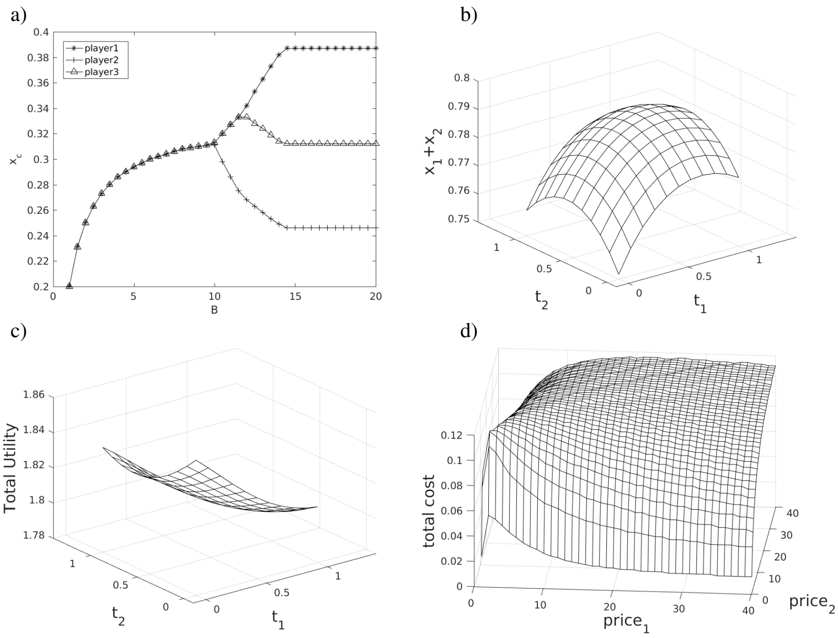

Fig. 2 a) Cache occupation for two CPs for two content classes b) corresponding total utility c) Nash equilibrium for increasing $B = B_i$, $i = 1, 2, 3$ d) network operator's revenue (linear cost) in the two players case as a function of λ_1 and λ_2.