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A simple mechanical model of the drumroll

Stefano Oss

Physics Department, University of Trento, Trento, Italy

E-mail: stefano.oss@unitn.it

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Abstract

We propose an experiment in which the sound of a drumroll is recorded and analysed according to a simple mechanical model in which the inelastic restitution coefficient of the collision between the stick and drum surface, as well as the ongoing kinematics of the drumstick, are considered. The agreement shows that this model is well suited for describing the most relevant features of drumstick behaviour during this type of motion.

Keywords: rigid body mechanics, drumroll, physics and music, coefficient of restitution, time-of-flight

1. Introduction

The scientific study of musical instruments, even if it is not always brought to an appropriate level of attention in many textbooks or in the classroom, can be a valuable benchmark in the physics curriculum from several viewpoints [1–7]. The interpretation of how sound is generated allows one to discover interesting aspects also on the musical side [8]. As a consequence, music students and artists can enrich their experience and acquire new sensitivity when performing with their instruments, whose specific physics mechanisms are subject of study [9–12]. In the case of percussive instruments, and particularly drums, Newtonian mechanics of rigid bodies plays a fundamental role in the description and understanding of the interaction of sticks and drum surfaces in the acoustic process [13–16]. The way in which the drummer hits the instrument is of course determinant in establishing the nature and the character of sound. The specific actions of the player can be indeed quite complex ones, where gravity and friction forces, as well as movements of fingers, wrists and arms are involved. Yet, there are certain 'fundamentals' in the performance of a drum player which can be taken as a reference technique to generate sounds which are somewhat simple and, as such, suited to be treated with relatively straightforward models. We mention, among others, the spontaneous drumroll which consists in letting the stick to bounce on the drum surface without any external interference, besides the almost frictionless support given by the fingers of the drummer holding the stick. The resulting effect is the repetition of a certain number of drumbeats in a sequence with increasing frequency and decreasing sound level. The rebounds of the drumstick can be quite easily described according to a mechanical model in which one includes inelastic collisions of the stick tip with the drum surface. In practice, in order to access some information on the behaviour of this particular drum sound, one has to settle an interesting and intriguing laboratory of Newtonian mechanics, in which the drumstick is treated as a rigid body rotating around the pivot point as a physical pendulum; the associated dynamical equations of motion have to be established and solved. Also, theory can be compared with an actual measurement of the kinematics of rebounds, based on an experiment in which bounce times are obtained by direct acquisition of acoustical signals.

2. The mechanical model

In this simple study, the stick starts to fall with zero speed when its centre of gravity is at a certain height h above the surface of the drum. As shown in figure 1, one can describe the kinematics in terms of the rotation angle θ. The drumstick motion is obtained by solving the rotational Newton's equation for the rigid body,

$$\tau_\mathrm{O} = I_\mathrm{O}\ddot{\theta} \tag{1}$$

in which we introduce the moment of inertia of the stick about the pivot O, which can be conveniently written as $I_\mathrm{O} = mK_\mathrm{O}^2$ in terms of the radius of gyration K_O. We also introduce the torque τ_O of the weight of the stick about the same pivot O. It is possible to recover, solving equation (1), the kinetic rotational energy change of the pendulum which also provides the angular velocity of the stick as a function of the rotation angle according to

$$\omega(\theta) = \frac{1}{K_\mathrm{O}}\sqrt{2g(l_\mathrm{C}-d)\sin\theta}, \tag{2}$$

where l_C denotes the position of the centre of mass of the drumstick. This result also comes directly from energy conservation written in the form

$$\begin{aligned} E &= E_\mathrm{kinetic} + E_\mathrm{potential} \\ &= \frac{1}{2}I_\mathrm{O}\omega^2 - mg(l_\mathrm{C}-d) = 0. \end{aligned} \tag{3}$$

The tip of the stick hits the drum surface when the geometric condition $\sin\theta_\mathrm{C} = h/(L-d)$ is satisfied (θ_C is the contact angle). The corresponding contact angular velocity is given by

$$\omega_\mathrm{C} = \omega(\theta_\mathrm{C}) = \frac{1}{K_\mathrm{O}}\sqrt{\frac{2gh(l_\mathrm{C}-d)}{(L-d)}}. \tag{4}$$

The time of fall t_C can be expressed in terms of the integral

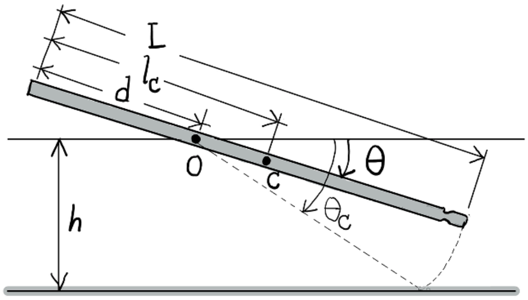

Figure 1. Dimensions and coordinates used in the rigid body model of the drumstick: O is the frictionless pivot; C is the centre of mass.

$$t_\mathrm{C} = \frac{K_\mathrm{O}}{\sqrt{2(l_\mathrm{C}-d)g}}\int_0^{\theta_\mathrm{C}}\frac{\mathrm{d}\theta}{\sqrt{\sin\theta}}, \tag{5}$$

which is related to an elliptic form, not analytically solvable. This last expression is obtained exploiting the angular velocity definition, $\omega = \mathrm{d}\theta/\mathrm{d}t$, expressing it as in equation (2) and integrating over the angular interval which goes from the initial, horizontal position of the stick to the contact angle θ_C.

It is convenient, in a simplified framework also suited to undergraduate students, to linearize the equation of motion in the approximation for small angles or, equivalently, when $h \ll L - d$, see again figure 1. In this case, the integral equation (5) becomes exactly solvable (in a more advanced approach, the integral can be numerically computed and a comparison with the small angle approximation can be done). In any case, the linearization is such that, equivalently, the angular acceleration of the drumstick is constant and is given by

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} \cong \frac{(l_\mathrm{C}-d)g}{K_\mathrm{O}^2}. \tag{6}$$

This immediately leads to the falling or contact time (for small angles)

$$t_\mathrm{C} \cong K_\mathrm{O}\sqrt{\frac{2\theta_\mathrm{C}}{(l_\mathrm{C}-d)g}} = K_\mathrm{O}\sqrt{\frac{2h/g}{(l_\mathrm{C}-d)(L-d)}}. \tag{7}$$

From this last result, it is instructive to observe that the stick behaves as a point mass which falls under the action of gravity from an effective initial height given by

$$h_{\mathrm{eff}} = h\frac{K_{\mathrm{O}}^{2}}{(l_{\mathrm{C}}-d)(L-d)}. \quad (8)$$

One can also point out that, when the pivot is located in the centre of mass ($d = l_{\mathrm{C}}$), the fall time becomes infinitely long as the stick stands in equilibrium, eventually motionless.

We describe now the bounce of the stick according to a simple inelastic collision model in which the angular velocity ω_{i} after the rebound is expressed as a fraction α, to be interpreted as the coefficient of restitution [17, 18], with $0 \leqslant \alpha \leqslant 1$, of the contact value of the angular velocity, i.e.

$$\omega_{\mathrm{i}} = \alpha\omega_{\mathrm{C}}. \quad (9)$$

Within this approximation, the time required for the stick to complete its first bounce after the initial fall will be given by

$$t_{\mathrm{B}}^{(1)} = \alpha t_{\mathrm{B}} = 2\alpha t_{\mathrm{C}} \quad (10)$$

where we introduce the first-rebound time $t_{\mathrm{B}} = 2t_{\mathrm{C}}$. Further bounces happen at every contact with a progressive decrease of both the initial velocity and of the rebound duration since the restitution coefficient α is multiplied by itself to provide, for the Nth collision, a geometric progression as in

$$t_{\mathrm{B}}^{(N)} = \alpha^{N} t_{\mathrm{B}} = 2\alpha^{N} t_{\mathrm{C}}. \quad (11)$$

We thus obtain that, within this approximation, bouncing times decreases according to a simple power law. If the drumstick is treated as a homogenous thin bar with negligible radius in comparison with its length L, the radius of gyration and the centre of mass coordinate are given by

$$K_{\mathrm{O}} = \sqrt{\frac{L^{2}}{3} + d^{2} - Ld}, \quad l_{\mathrm{C}} = \frac{L}{2}. \quad (12)$$

Here, the radius of gyration is obtained via direct application of the Huygens–Steiner (parallel axis) theorem when calculating the moment of inertia according to

$$I_{\mathrm{O}} = I_{\mathrm{C}} + m(L/2-d)^{2} = mL^{2}/12 + m(L/2-d)^{2}. \quad (13)$$

It is however possible to adopt a more realistic model for the shape of the drumstick, i.e. a cylinder of finite radius R and length l_1 with a conical tip with length l_2, as sketched in figure 2. It is an interesting exercise—suited for an undergraduate class—to obtain the moment of inertia in terms of the radius of gyration K_{O}' according to the expression

$$K_{\mathrm{O}}' = \sqrt{\frac{3l_1\left[\frac{1}{4}\left(R^{2}+\frac{l_1^{2}}{3}\right)+\left(\frac{l_1}{2}-d\right)^{2}\right]+l_2\left[\frac{3}{80}\left(4R^{2}+l_2^{2}\right)+\left(l_1-d+\frac{l_2}{4}\right)^{2}\right]}{3l_1+l_2}}. \quad (14)$$

In this case, the Huygens–Steiner theorem has been applied twice since there are now two rotating bodies, i.e. the cylinder and the conical tip, at different distances from the common rotation axis. In equation (14), the contribution of the cylinder is in the first term whilst the cone generates the second term. The moments of inertia of simple rigid bodies, including the conical shape, can be found in any introductory mechanics textbook as well as in several sites [19]. More explicitly, equation (14) derives from the following expressions:

$$I_{\mathrm{cylinder}} = m_{\mathrm{cylinder}} l_1^{2}/12; \quad I_{\mathrm{cone}} = 3m_{\mathrm{cone}}\left(4r^{2}+l_2^{2}\right)/80 \quad (15)$$

in which the moments are computed about rotation axes perpendicular to the common longitudinal axis of the bodies and passing through their

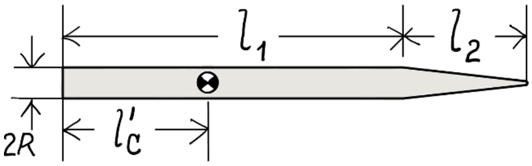

Figure 2. Dimensions of the cylindrical-conical shaped drumstick.

centres of mass. The result of equation (14) will be of a certain practical significance for the specific case of study here. As it will be further discussed in the next section, the size of an actual drumstick, with the conical shaped tip, is such that the moment of inertia computed according to the thin cylinder model of equation (12) differs from that obtained with equation (14) above by about 15% (when the pivot point O is located at around one third of the stick total length). The kinematics of the falling drumstick also changes since the centre of mass is not anymore in the middle of the stick. Its position along the axis is easily obtained according to

$$l'_C = \frac{m_{\mathrm{cyl}} \frac{l_1}{2} + m_{\mathrm{cone}} \left(l_1 + \frac{l_2}{4}\right)}{m_{\mathrm{cyl}} + m_{\mathrm{cone}}} = \frac{6l_1^2 + l_2\,(4l_1 + l_2)}{4\,(3l_1 + l_2)}, \quad (16)$$

which comes directly from the definition of centre of mass, expressed as the mass-weighted position of the two centres of mass coordinates of the cylindrical body and of the conical tip. This is the value to be used in the equations above which give the falling velocity and bouncing times of the drumstick. Numerical differences with the case of a cylindrical shape ($l_C = L/2$) are of the order of 10%.

3. Experimental results

A simple experiment is done and bouncing times are measured and compared with the theoretical model. It is certainly possible to proceed in a traditional way, for example by tracking a video of the bouncing drumstick obtaining the position of its tip as a function of time. This approach is in principle a straightforward one. Yet, it is not immediate and could become quite a tricky one since it requires to shoot a slow-motion video and to proceed with a careful determination of the

Figure 3. The drumstick support and its free-fall bounces on the snare drum.

stick position during its movement. In this work, we acquire directly the bouncing times through the sounds which are generated by the drumroll. This method, which resemble the time-of-flight approach, is quite well known and it can be realized with either a smartphone, a tablet or a personal computer acting as a digital audio recorder [20]. In this work, a standard sound card was connected to a computer to acquire relatively precise signals (the audio card records with a sampling rate up to 96 kHz and a digital resolution of 32 bit, even if such level of accuracy is not really needed for this experiment). A drumstick was drilled with a series of equally spaced holes: a pin was inserted in a chosen hole to allow the free rotation of the stick about the pin itself, see figure 3. The stick is dropped and the drum sounds are digitally recorded. It is possible to vary the distance d by choosing different holes along the stick. It is also possible to change the initial height of the centre of mass or, equivalently, the initial angle of the stick referred to the horizontal surface of the drum. This is quite an important option since it allows to simulate a non-zero initial speed/kinetic energy of the stick: drummers usually play hitting the drum with a wrist movement which in fact provides energy to the stick. In our model, this can be simply accounted by inserting in equations (4) and (7) different

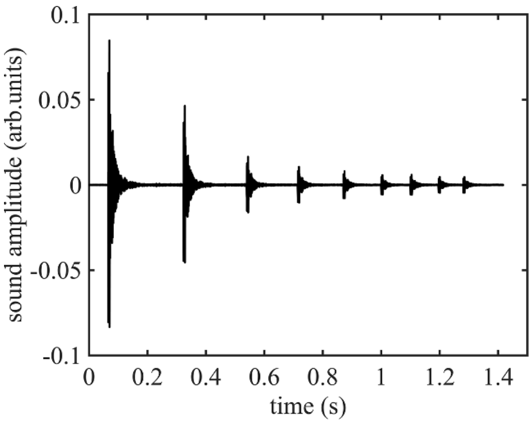

Figure 4. Sound signals of a typical drumroll sequence.

Table 1. Rebound times t_B (theory and fit from the experiment) for three different pivot positions d with fixed height, $h = 80$ mm. The restitution coefficient is only slightly affected.

d (mm)	Computed t_B (s)	exp t_B (s)	Restitution coefficient
110 ± 2	0.22 ± 0.01	0.21 ± 0.01	0.86 ± 0.01
131 ± 2	0.24 ± 0.01	0.26 ± 0.01	0.88 ± 0.01
152 ± 2	0.30 ± 0.02	0.30 ± 0.02	0.84 ± 0.01

Table 2. Rebound times t_B (theory and fit from the experiment) for three different initial heights h of the centre of mass with fixed pivot position, $d = 131$ mm. The restitution coefficient is only slightly affected.

h (mm)	Computed t_B (s)	exp t_B (s)	Restitution coefficient
50 ± 3	0.20 ± 0.01	0.20 ± 0.01	0.90 ± 0.01
80 ± 3	0.25 ± 0.01	0.24 ± 0.01	0.90 ± 0.01
110 ± 3	0.29 ± 0.01	0.29 ± 0.01	0.89 ± 0.01

values of the initial height h. We show in figure 4 a typical audio recording: the sequence of peaks has a regular, decreasing time spacing. We observe also that the sequence of the heights of peaks, which is related to the sound intensity, decreases with time. This can be put in more precise connection with the acoustical characteristics of the drum head [13] but this will not be of further interest in the present work.

The sequence of time intervals can be fitted to the power law of equation (11): the result is particularly instructive in a demonstrative laboratory activity, since it allows to make a prediction of the restitution coefficient α, i.e. of the drum head (in)elasticity, as well as of the collision time t_C, which is related basically to the geometric characteristics of the drumstick, i.e. the initial height and the position of the pivot point, as clearly expressed by equation (7).

For concreteness, we consider three series of data obtained when the drumstick rotates about three different holes (the used stick has a radius $R = 7.5$ mm and lengths $l_1 = 348 \pm 1$ mm, $l_2 = 60 \pm 1$ mm). The measurements show the variation of the bouncing time, whilst the estimated restitution coefficient is much less influenced, as expected. Comparison between fitted and computed bouncing times are reported in table 1. We notice, despite the very simple experimental setup, quite a satisfying agreement. The initial height/angle of the drumstick, with a fixed pivot position, has also been varied. Once again, this will affect the collision time as shown in the measured data reported in table 2. As a further measure, we modified to some extent the restitution coefficient by varying the drum head tension. This can be done by acting on the tensioning keys of the snare drum which are used by the drummer to calibrate the response (and tuning) of the instrument according to his/her needs and preference. In this way, the varying characteristic in our experiment will be the restitution coefficient: in the present setup, with two different tensioning of the drumhead, we obtained, according to the fitting procedure, that the coefficient changes from 0.860 ± 0.005 ('soft' drumhead) to 0.899 ± 0.002 ('hard' drumhead), i.e. an increase by 4%. We summarize and show in figure 5 how rebound times vary under the abovementioned geometric changes (position of the pivot and initial angle/height) as well as a function of the drumhead tension. We observe, in general, at least judging from the overall quality of the numerical fits, that the simple inelastic model based on the restitution coefficient introduced in equation (9), is indeed well suited for the description of the most relevant kinematic features of the drumroll.

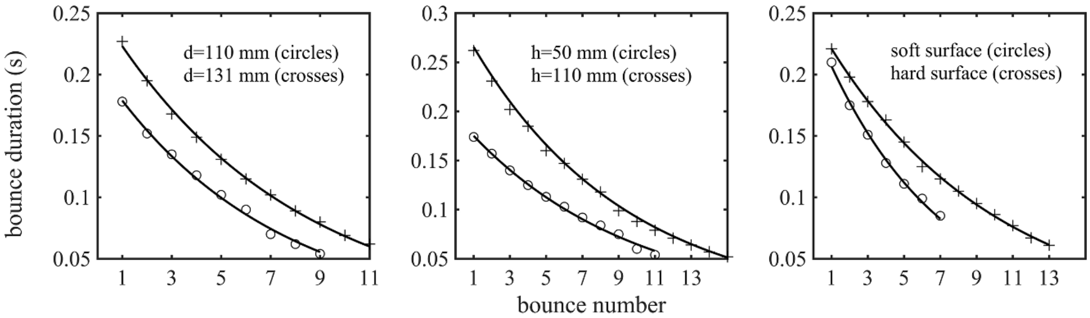

Figure 5. Bounce duration series: two different pivot positions, fixed height $h = 80$ mm (left); two different fall heights, fixed pivot $d = 131$ mm (centre); two different drumhead tensions, $h = 80$ mm, $d = 131$ mm (right). Continuous lines are the best-fit curves based on the inelastic mechanical model of equation (11).

4. Conclusions

The sound of an actual snare drum was digitally recorded to discuss the validity of a simple mechanical model for the drumroll generated by spontaneous rebounds of the drumstick. Despite its simplicity, this experiment and the associated theoretical interpretation constitute quite a concrete example of how a demonstration laboratory activity can be carried out. Here, the inelastic collisions of the drumstick with the drumhead have been shown to be fairly well reproduced in terms of a single restitution coefficient which has been estimated in our measurements. Also, the Newtonian dynamics of the rigid body rotation has been put in evidence and adapted to describe the observed bounce sequence of the drumstick and to provide support for a practical connection between physics and music.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iD

Stefano Oss https://orcid.org/0000-0001-9427-2151

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References

[1] Andreotti E and Frans R 2019 The connection between physics, engineering and music as an example of STEAM education *Phys. Educ.* **54** 045016
[2] Neilsen T B, Vongsawad C T and Onwubiko S G 2020 Teaching musical acoustics with simple models and active learning *J. Acoust. Soc. Am.* **148** 2528
[3] Ramsey G P 2012 The physics of music course as an introduction to science *J. Acoust. Soc. Am.* **132** 1957
[4] Ramsey G P 2015 Teaching physics with music *Phys. Teach.* **53** 415–8
[5] Hechter R P and Bergman D 2016 In harmony: inquiry based learning in a blended physics and music class *Phys. Educ.* **51** 065015
[6] Hoffman G L and Jouny I 1996 Using music to teach physics *Am. J. Phys.* **64** 1304
[7] Shonle J I 1976 Implementing a course on the physics of music *Am. J. Phys.* **44** 240
[8] Rossing T D 1971 Physics for musicians *Am. J. Phys.* **39** 1480
[9] Boysen E and Ruiz M J 2017 Flute physics from a flutist's perspective *Phys. Educ.* **52** 045004
[10] Suits B H 2001 Basic physics of xylophone and marimba bars *Am. J. Phys.* **69** 743
[11] LoPresto M C 2014 A theoretical trombone *Phys. Educ.* **49** 580
[12] Bonard J-M 2001 The physicist's guide to the orchestra *Eur. J. Phys.* **22** 89
[13] Neville H. F and Rossing T D 1998 *The Physics of Musical Instruments* (New York: Spinger)
[14] Obata J and Tesima T 1935 Experimental studies on the sound and vibration of drum *J. Acoust. Soc. Am.* **6** 267–74

[15] Rossing T D 1976 Acoustics of percussion instruments—part I *Phys. Teach.* **14** 546
[16] Rossing T D 1977 Acoustics of percussion instruments—part II *Phys. Teach.* **15** 278–88
[17] Gonzalez M A, Gonzales M A, Vegas J and Llamas C 2017 Measuring the coefficient of restitution and more: a simple experiment to promote students' critical thinking and autonomous work *Phys. Educ.* **52** 055002
[18] da Silva M F 2007 Meaning and usefulness of the coefficient of restitution *Eur. J. Phys.* **28** 1219–32
[19] List of moments of inertia (available at: https://en.wikipedia.org/wiki/List_of_moments_of_inertia)
[20] Nunn J 2014 The bounce meter *Phys. Educ.* **49** 303

Stefano Oss is professor at the Department of Physics, Trento University. He is tin charge of the Laboratory of Physical Science Communication. His main interests are in the field of educational applications of classical mechanics and thermal physics.