

Forecasting bridge damage within a predictive Structural Reliability-based DSS

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ABSTRACT

Nowadays there is urgent need for expeditious and efficient approaches to assess the structural reliability of widespread aging bridge stocks. As such, a shift towards predictive maintenance measures is required. This paper addresses such pressing necessity for efficient methods to assess the structural state of aging bridges, centring its research question on developing a Decision Support System (DSS) that integrates an innovative reliability model with Markov Chain projections to prioritize bridge interventions. The proposed DSS effectively forecasts structural risks and computes a Priority Index to guide maintenance decisions. This tool empowers infrastructure managers with timely insights, enhancing bridge safety and optimizing resource allocation. The study's practical example underscores the DSS's applicability, demonstrating its capacity to streamline decision-making in real-world scenarios. By providing a sophisticated yet accessible approach to structural reliability assessment, this research lays the groundwork for future enhancements, including the incorporation of additional risk factors and refined calibration methods.

1. Introduction

The effective management of transportation infrastructure, especially bridges, is crucial for fostering economic growth. Deterioration resulting from factors such as aging, environmental conditions, and increasing traffic loads underscores the necessity for efficient bridge management plans. The global crisis of aging infrastructure further intensifies this challenge, posing a substantial threat to both public safety and economic stability. Notably, 42 % of American bridges aged 50 years or more are structurally deficient [1,2], and the tragic collapse of Italy's Morandi Bridge in Genoa in 2018 accentuates the urgency of addressing vulnerabilities in aging bridges [3]. As we grapple with this challenge on a global scale, the demand for rapid and effective decision-making tools has never been more crucial to ensure public safety and economic stability.

Addressing this crisis poses a formidable challenge, primarily due to budgetary constraints and the environmental and logistical complexities associated with widespread demolition and reconstruction efforts. As observed by Regier and Hoult [4], it is not feasible to replace all structures that have surpassed their intended service lives. Consequently, the most viable strategy involves meticulous assessment and strategic maintenance of the bridges still deemed serviceable [5,6]. Thus,

determining their deterioration level, serviceability performance, and ensuring public safety through inspections, monitoring, and maintenance protocols are of utmost importance [7,8].

Effective maintenance protocols go beyond preserving the structural integrity of bridges; they aim to maximize the utilization of economic resources while safeguarding the lives of those who use these critical infrastructural assets [9,10]. Achieving this delicate balance requires up-to-date knowledge concerning durability, defect evolution, and structural reliability [11]. This knowledge must be continuously updated through regular inspections and the extraction of key data on the asset conservation state. Timely intervention – in the shape of maintenance and reparation works or local/total replacement – in deteriorating structures can yield significant benefits with minimal financial outlay. Given the constraints of infrastructure managers' budgets, it is imperative to prioritize maintenance interventions based on a rigorous assessment of structural reliability [12].

Effective decision-making in bridge management involves optimizing maintenance activities across different scales—network, element, and component—and adopting suitable maintenance strategies. The four primary maintenance strategies—corrective, preventive, condition-based, and predictive—are essential for preserving bridge functionality and safety [9,10]. In traditional approaches to

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infrastructure management, the evaluation of Condition States often takes precedence [13].

Decision-making culminates in optimization methods that allocate resources and prioritize interventions to ensure satisfactory service levels and user safety. Two commonly employed methods for this purpose are the calculation of priority indices and cost-benefit analysis [14]. However, traditional approaches often overlook the direct influence of structural defects on bridge reliability [15]. Despite the expectation that bridges in poor structural health would exhibit lower reliability indexes, a direct correlation between the two variables is not straightforward and thus must be assessed thoroughly. For optimal budget allocation and maintenance scheduling, deterioration models must be employed to properly predict future states. Srikanth and Arockiasamy [16] provide a comprehensive overview of deterministic, stochastic, mechanistic, and artificial intelligence (AI)-based models, emphasizing the multifaceted nature of bridge deterioration modelling. Among these approaches, the most widely employed are the deterministic and stochastic ones. While deterministic models, favored for their simplicity, rely on linear or non-linear regression models for predicting deterioration, stochastic models, particularly Markov Chain models, incorporate uncertainties into the deterioration process [17].

The current landscape of bridge infrastructure management underscores a decisive transition towards advanced probabilistic deterioration models, as revealed in contemporary literature. The studies by Šomodíková et al. [18], Yafei Ma et al. [19], and Ghosh and Sood [20] collectively advocate for the adoption of probabilistic frameworks, treating structural parameters as random variables with probability density functions. Within the realm of probabilistic frameworks, Markov chains emerge prominently. Ferreira et al. [21] stress the utilization of Markov chains for both degradation and maintenance models, providing a consistent framework for the integrated management of bridge components. Bortot et al. [22] exemplify the application of Semi-Markov Processes in APT's Bridge Management System (BMS), accounting for transition probabilities dependent on the time spent in initial condition states. Saheed Hassan et al. [23] and Fang and Sun [24] echo this sentiment, incorporating Markov chains for reliable deterioration prediction. Bayesian methods find prominence in the studies of Widodo Soetjipto et al. [25] and Han et al. [26]. The former introduces Bayesian updating as a crucial element for integrating inspection data into deterioration models. The latter employs Bayesian Markov Hazard models, amalgamating Markov chains, multi-state exponential hazard models, and Markov Chain Monte-Carlo techniques, offering a robust empirical basis for deterioration modelling.

Furthermore, the literature converges on the integration of Life-Cycle Cost Analysis (LCCA) in Bridge Management Systems (BMS), as highlighted by Jean De Dieu et al. [27]. This involves considering not only initial costs but also long-term costs such as operation, maintenance, repairs, and rehabilitation in decision-making processes. Finally, Morcous et al. [28,29] advocate for an integrated BMS architecture that combines state-based/time-based probabilistic models with reliability-based mechanistic models. This approach aims to strike a balance between the practicality of probabilistic models, often reliant on condition ratings, and the accuracy of mechanistic models, which offer precise predictions at a higher cost in terms of data requirements and modelling complexity. Collectively, these methodologies present a holistic foundation for advancing the state-of-the-art in bridge deterioration modelling and Decision Support Systems (DSS).

Despite the availability of sophisticated methodologies for comprehensive structural reliability evaluation and decision-making processes, many prove to be time-consuming and intricate, making swift assessments necessary for urgent maintenance prioritization. Infrastructure managers urgently need rapid tools to identify bridges in critical condition, enabling timely interventions to enhance user safety.

This study proposes a methodology designed to bridge the existing gap by introducing, in Section 2, a Decision Support System tool for swift assessments and prioritization. Focusing on the structural risk associated

with aging bridges, which formulation is presented in Section 3.2, this research introduces an innovative methodology incorporating degradation over various time horizons. The aim is to provide infrastructure managers with a Decision Support System projecting the structural reliability of bridges at specific years, thereby addressing the urgency of assessing and prioritizing maintenance interventions in the face of evolving risk factors. Our methodology centers around a structural reliability model that integrates degradation at different time horizons.

The approach is integrated with a sophisticated degradation model rooted in Markov Chains [30], offering a comprehensive depiction of the evolution of the parameters associated with structural defects. In this framework, the evolution of the defect can therefore be dynamically captured, crystalizing the likelihood of specific states of degradation after a defined temporal interval, facilitating a robust foundation for projecting the system's behaviour [17,31]. The Markov Chain model therefore captures the dynamic nature of structural degradation, allowing for projections of reliability at specific years. Crucially, this methodology presented in Section 4, caters to situations where a defect is identified during the initial inspection (Subsection 4.1), and scenarios where it is not present but could potentially emerge later (Subsection 4.2), forming a comprehensive narrative of the defect's evolutionary trajectory over time. In the former case, the Degradation Model evaluates the deterioration of a studied defect over time, while in the latter case, it assesses the likelihood of the studied defect appearing in the future. It is noteworthy that, within the framework of the Decision Support System, both scenarios lead to the determination of a total Probability of Failure of an asset at a future time. This process lays the groundwork for computing the risk of the structure, in terms of Probability of Failure, at various time horizons, encompassing all conceivable status of defect during the initial inspection and offering a dynamic portrayal of the structural system's health at any given return period. Importantly, Section 5, presents a practical example proving the swift and straightforward applicability of the method.

By focusing on the structural risk factor, the model provides a holistic view of the change of reliability of the structure in time due to degradation. The resulting Decision Support System empowers infrastructure managers to make informed decisions by projecting the evolving reliability of structures. In presenting a simplified structural reliability model designed for expeditious assessment, this article represents a departure from current methodologies, bridging the gap between the need for quick assessments and the comprehensive evaluation of structural reliability.

Glossary

C	Capacity Index
D	Damage Index
E	Extension
Exp	Exposure
G	Permanent Load
i	Projected year
I	Intensity
j	Case study defect
j_k	Triggering defect
LH	Likelihood
P_{app}	Intrinsic Defect Appearance Probability
P_D	Defect Presence Probability
P_{DL}	State Probability Vector
P_F	Probability of Failure
P_{IE}	State Probability Matrix
P_F^i	Forecasted Probability of Failure at year i
P_U	Defect Absence Probability
Q_d	Traffic Load
R	Risk
T_i	Transition Matrix

V_z	Coefficient of Variation
α_{int}	Prioritization Index
β	Reliability Index

2. Decision Support System approach

The decision-making process in bridge management follows two main methodologies: the top-down and the bottom-up approaches. The former optimizes interventions decisions at the network level by considering common features among bridges and minimizing total maintenance costs, whereas the latter prioritizes individual bridge optimization and determining minimum performance standards [32]. This article proposes a methodology within the bottom-up approach framework [33].

The proposed Decision Support System is intended as a distinct and efficient approach to the prevalent limitations of current decision-making methodologies on bridge stocks management. In particular, the DSS was designed on one key principle: having a simplified and swift reliability evaluation, giving way to the formal time-consuming structural reliability analyses (requiring extensive documentation, technical information, and testing data). Furthermore, the DSS is aimed at being as immediate – with the least number of needed inputs – and as versatile as possible, providing rapid and cost-effective solutions applicable to the entire bridge stocks. Such, without compromising the accuracy of the assessment. Having established the goal of the DSS, its function process – presented step by step in the following – is shown Fig. 1.

As displayed in Fig. 1, the main goal of the DSS framework is the

In the specific case of $R_{forecast}$, P_F of Eq. (1) is henceforward referred to as $P_{F,forecast}$. This forecast is an essential part of any asset management strategy, as resource allocation must be planned properly across extended time horizons. Such projection is performed considering the Probability of Failure $P_{F,forecast}$ associated both to the degradation of the defects detected during the inspection, by means of Markov Chains (Section 4.1), and the one associated to the appearance of new defects – absent during the inspection –, by mean of Bayes Theory (Section 4.2). As detailed in Section 3 and Section 4, the combination of these two into a total asset-Probability of Failure is performed as per Eq. (2).

$$P_F \text{ of the asset} = \sum P_{F,defects \text{ present}} + \sum P_{F,defects \text{ not present}} \quad (2)$$

Having determined the above, a post-intervention Risk $R_{post-int}$ is evaluated (square 3) considering the Forecasted Risk with a reduction specific to the intervention of choice. For example, given the reparation of a defect, the Risk associated to it is removed ($R_{post-int} = R_{undamaged \text{ asset}}$) or reduced ($R_{undamaged \text{ asset}} < R_{post-int} < R_{forecast}$). Once again, the risk is evaluated as the combination of the Probability of Failure $P_{F,post-int}$ and the Exposure Exp of the asset, as per Eqs. (1) and (2). Clearly, post-intervention, the $\sum P_{F,defects \text{ present}}^i$ of Eq. (2) will not be including the repaired or removed defects. For clarity's sake, the authors reiterate that Eq. (2) is employed for the calculation of both $P_{F,forecast}$ and $P_{F,post-int}$.

Finally (square 4), a Priority Index α_{int} parameter is evaluated by normalizing the difference between the above risks ($R_{forecast} - R_{post-int}$) by the Intervention Cost $Cost_{int}$ as in Eq. (3).

$$\alpha_{int} = \frac{R_{forecast} - R_{post-int}}{Cost_{int}} = \frac{P_{F,forecast} \cdot Exp - P_{F,post-int} \cdot Exp}{Cost_{int}} = (P_{F,forecast} - P_{F,post-int}) \cdot \frac{Exp}{Cost_{int}} \quad (3)$$

evaluation of a Prioritization Index α_{int} of an intervention i.e., that parameter on the grounds of which one can identify the most efficient intervention scenario/course of action. Possible scenarios are maintenance works, reparations, and replacement of components/full bridge. Let us now go through the various steps of Fig. 1 to determine such α_{int} parameter.

The process starts with the condition assessment of the current state of the bridge through visual inspection. In particular, defects are detected, labelled and recorded accounting for their extent and gravity (square 1), as discussed in Section 4.

Moving on to square 2 of Fig. 1, on the basis of such current day data coming from the inspection and the asset inventory information (design code, material, age, etc.), a Forecasted Risk $R_{forecast}$ is determined for future timeframes. The Risk $R_{forecast}$ is determined following the classic Risk formulation as a function of the Probability of Failure P_F and the Exposure Exp of the asset – discussed in Section 3, as per Eq. (1).

$$R = P_F \cdot Exp \quad (1)$$

This index quantifies the urgency of each possible intervention, permitting their ranking and their chronological distribution from the part of infrastructure manager. The optimal scenario is determined by maximizing the risk reduction ($R_{forecast} - R_{post-int}$) at the same $Cost_{int}$, with the scenario having the highest α_{int} being the preferred choice for resources allocation.

In the current article, the focus is on the first two blocks of the Decision Support System process (Fig. 1) i.e., inspection outputs and risk forecasting ($R_{forecast}$). The latter plays a pivotal role in the proposed Decision Support System, serving as the foundation for subsequent managerial decisions. Concerning the third and fourth blocks of Fig. 1, given the critical importance of the post-intervention risk ($R_{post-int}$) and the associated intervention costs ($Cost_{int}$) in decision-making, a comprehensive and extensive analysis of said parameters is required. In light of the above, the authors have decided to tackle these analyses in future publication works.

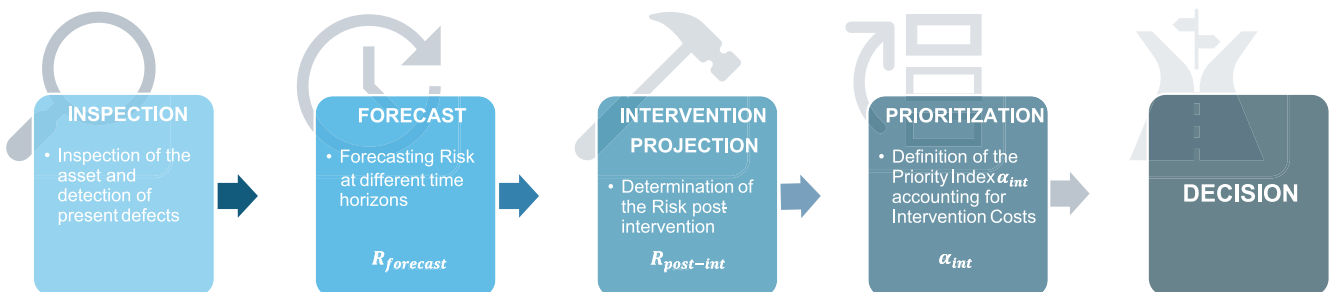


Fig. 1. Decision support system process.

3. Risk assessment in the Decision Support System

The proposed methodology hinges upon the conceptualization of risk, arising from the combination of three fundamental components: Hazard, Vulnerability, and Exposure. In the broader context, risk refers to the likelihood of a specific consequence materializing due to a potential hazardous event or risk factor. Specifically, the Risk (R) is articulated through the expression of Eq. (1). Here, the Probability of Failure P_F is associated to the risk factor x and accounts for both Hazard and Vulnerability.

The Exposure Exp parameter of Eq. (1) is an expression of consequences, encompassing potential loss of human lives, direct and indirect economic damages, as well as environmental and reputational impact. Given that the loss of human lives is often the most severe consequence, Exposure is generally addressed in literature solely on Individual and Societal Risk expressed in terms of FN-curves [34,35]. Our model aligns itself with this approach, though it indirectly also considers environmental impact. Indeed, in those cases where the deficiency of the asset is limited, our model prioritizes those interventions with lower the environmental impact, i.e. maintenance and reparations (versus environmentally-taxing partial / global reconstructions). Finally, the Exposure formulation accommodates the individual operator's attitude towards these main exposure parameters.

Although specific models were created for the Exposure factor, in the primary scope of this article, the authors have chosen not to delve into this parameter, the formulation of which deserves a dedicated publication and will be addressed in future works. The current publication focuses solely on the Probability of Failure P_F . As such, henceforward P_F is considered in the stead of Risk R .

3.1. Probability of Failure in civil engineering

The Probability of Failure can be expressed as the product of hazard and vulnerability: $P_F = P_{(F|x)} \cdot P_x$ Where P_x signifies the Hazard, i.e., the probability of a hazardous event occurring, and $P_{(F|x)}$ denotes the Vulnerability, i.e., the likelihood of an undesirable event (in our case the bridge failure) occurring given the incidence of a hazardous event. However, due to the inherent strong correlation among these risk factors, isolating Hazard and Vulnerability for separate consideration is not always feasible. Consequently, the proposed models incorporate both Hazard and Vulnerability within the unified term P_F .

Such P_F could be evaluated for each of the main risk factors associated with civil engineering structures: risk for Overload and Degradation ($P_{F,D}$), Seismic risk ($P_{F,S}$), Hydraulic risk ($P_{F,H}$) and Landslides risk ($P_{F,L}$). These risk factors are considered as uncorrelated. This assertion is not concerned with the concurrent occurrence of events, such as landslides and earthquakes, but rather centres on the likelihood of a bridge collapse resultant from both these phenomena. It is crucial to delineate between the probability of events occurring simultaneously and the probability of both events concurrently inducing the collapse of the bridge. Despite potential overlap between two risks, the ultimate collapse of the bridge is typically attributable to a singular causative factor. Furthermore, it was imperative to make simplifying assumptions, including the stipulation that the intersection of the Probability of Failure attributable to each risk is considered null. This simplification was deemed requisite for the sake of analytical expediency and to facilitate an effective modelling of the intricate scenario at hand in the scientific investigation.

With the assumption that all risk factors are uncorrelated, a unique value of the Total Probability of Failure P_F can be determined through Eq. (4).

$$P_F = P_{F,D} + P_{F,S} + P_{F,H} + P_{F,L} \quad (4)$$

In the present article the authors proceed to showing only the calculation modus-operandi of $P_{F,D}$, and its associated reliability index β ,

referred to the Overload and Degradation risk, commonly the most impactful and widespread one. Crucially, jointly to the assessment of the $P_{F,D}$ of bridges at the time of inspection, the DSS also includes a Degradation Model capable of forecasting the evolution of the damage state of said bridges.

3.2. Probability of Failure and Reliability

In the present section the authors will display how the reliability of a bridge is assessed and its Probability of Failure determined. Please note, that the following methodology is specifically designed to be a swift and easily employable tool that forgoes detailed and complex bridge reliability analyses for a leaner and generally applicable methodology. The reader should keep in mind, that while the method's reliability evaluation may not precisely coincide with the one extracted from thorough formal assessments, the goal of the present model is not providing an absolute reliability value but rather a relative one intended mainly to rank intervention priority.

The Reliability Index β can be formally defined as the negative value of the inverse of the standardized normal distribution function Φ calculated for the Probability of Failure P_F , as per Eq. (5) [36].

$$\beta = -\Phi^{-1}(P_F) \rightarrow P_F = \Phi(-\beta) \quad (5)$$

Given that in Civil Engineering, Structural Reliability assesses whether the Load Effect S is inferior to the Structural Resistance R , the distinction between a desirable (safe) and undesirable (unsafe) structural condition is defined by the equation: $Z = R - S = 0$ i.e., the so-called Limit State Function or Failure Function [37].

Crucially, several assumptions were made: R and S are considered mutually independent (Coefficient of Correlation = 0) and follow a log-normal probabilistic distribution. Hence, $Z = \ln(R) - \ln(S)$ and $R \sim \mathcal{L}(\mu_R, V_R)$; $S \sim \mathcal{L}(\mu_S, V_S)$ with μ_R and μ_S the mean value of the R and S probabilistic distributions and $V_R = \sigma_R/\mu_R$ and $V_S = \sigma_S/\mu_S$ their coefficients of variation. A log-normal distribution was considered a better description of the probabilistic distribution of R and S compared to a normal distribution for the following reasons. Log-normal distributions begin from a 0 value whilst normal ones from $-\infty$ (less applicable to the semantic concept of Load Effect and Resistance). Furthermore, to account for the variation of μ_R and μ_S from a reference condition (0) with $\mu_{R,0}$ and $\mu_{S,0}$ (e.g., damage-induced deterioration of R) in a normal distribution environment, the absolute values of $\Delta\mu_R = \mu_R - \mu_{R,0}$ and $\Delta\mu_S = \mu_S - \mu_{S,0}$ need to be determined. Differently so, when working with log-normal distributions, the variations of μ_R and μ_S are represented in terms of relative values e.g., $\ln\mu_R - \ln\mu_{R,0} = \ln(\mu_R/\mu_{R,0})$, a more tangible and engineer-friendly sort of assessment. Finally, the reader should note that a better fitting probabilistic distribution would be the Gumbel one [6,38]. Then again, given its complexity and the neglectable nature of the error committed considering a log-normal distribution instead, the authors opted to work with this last one.

Henceforward, by bridges under Reference Condition (0) are intended recently built bridges – thus, completely undamaged – designed according to current construction codes. Now, if to consider a bridge's deviation in Structural Reliability from a Reference Condition (0), its current Reliability Index β and its Reference Condition Reliability Index β_0 can be formulated as in Eq. (6).

$$\beta = \frac{\ln\mu_R - \ln\mu_S}{V_z}; \beta_0 = \frac{\ln\mu_{R,0} - \ln\mu_{S,0}}{V_{z,0}} \quad (6)$$

Being V_z and $V_{z,0}$ the coefficient of variation of a generic Limit State Z under current and reference conditions, respectively. Note that, under reference conditions $V_{z,0} = \sqrt{V_{R,0}^2 + V_{S,0}^2}$ encompasses the general uncertainty of any specific model based on Z (uncertainty in the material and geometry of the structure, uncertainty in the load effects calculation, uncertainty in their parametrical distribution/variance, etc.).

Instead, $V_z = \sqrt{V_{z,0}^2 + \Delta V_z^2}$ encompasses both the general model uncertainties $V_{z,0}$ and an additional component ΔV_z connected to the decrease in confidence on the state of the structure with the passing of time i.e., exposure to environmental factors, damage, etc. It is assumed that the transit load on all current-day bridges (old or new, regardless of the code with which they were designed or their damage state) coincides with the one indicated by reference current day codes i.e., $S \equiv S_0$. Hence, $\mathcal{L}(\mu_S, V_S) = \mathcal{L}(\mu_{S,0}, V_{S,0})$ and the β term of Eq. (6) can be reformulated as Eq. (7).

$$\beta = \frac{\ln \mu_R - \ln \mu_{S,0}}{V_z} \quad (7)$$

Thus, Eq. (7) can be re-written as Eq. (8).

$$\beta = \frac{\ln \left(\frac{\mu_R}{\mu_{S,0}} \right)}{V_z} = \frac{\ln \left(\frac{\mu_R}{\mu_{S,0}} \bullet \frac{\mu_{R,0}}{\mu_{R,0}} \right)}{V_z} = \left\{ \frac{\ln \left(\frac{\mu_{R,0}}{\mu_{S,0}} \right)}{V_z} + \frac{\ln \left(\frac{\mu_R}{\mu_{R,0}} \right)}{V_z} \right\} \bullet \frac{V_{z,0}}{V_z} = \left\{ \frac{\ln \left(\frac{\mu_{R,0}}{\mu_{S,0}} \right)}{V_{z,0}} + \frac{\ln \left(\frac{\mu_R}{\mu_{R,0}} \right)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} = \left\{ \beta_0 + \frac{\ln \left(\frac{\mu_R}{\mu_{R,0}} \right)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} \quad (8)$$

As demonstrated in Poli et al. [39], the term $\mu_R/\mu_{R,0}$ of Eq. (8) can be re-written as $R_d/R_{d,0}$, being R_d and $R_{d,0}$ the percentile of order 5 % related to the log-normal probability distribution of the Resistance of a bridge – its characteristic value – in its current condition (R_d) and in its reference condition ($R_{d,0}$). As the reader may know, the main source of a bridge's Structural Reliability reduction occurs due to the surge and development of structural defects on its components. To account for this, R_d is rewritten as $R_{d,D}$ (where the subscript D stands for Damaged). As such, Eq. (8) can be modified as per Eq. (9) with $R_{d,U}$ the characteristic of the Resistance of an undamaged bridge (with U standing for Undamaged).

$$\beta = \left\{ \beta_0 + \frac{\ln \left(\frac{R_{d,D}}{R_{d,0}} \right)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} = \left\{ \beta_0 + \frac{\ln \left(\frac{R_{d,D}}{R_{d,0}} \bullet \frac{R_{d,U}}{R_{d,U}} \right)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} = \left\{ \beta_0 + \frac{\ln \left(\frac{R_{d,D}}{R_{d,U}} \bullet \frac{R_{d,U}}{R_{d,0}} \right)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} \quad (9)$$

Interestingly, setting aside momentarily the coefficients of variation, Eq. (9) suggests that the current Reliability Index β is equals to its original value β_0 decreased (indeed, $\ln(\dots)$ of an integer comprised between 0 and 1 is negative) by the natural logarithm of two specific components: $R_{d,D}/R_{d,U}$ and $R_{d,U}/R_{d,0}$. These two ratios have specific meanings:

- $R_{d,D}/R_{d,U} \in (0, 1)$ expresses the difference between the resistance of a bridge in its damaged ($R_{d,D}$) and undamaged ($R_{d,U}$) states, hence the damage-induced reduction in carrying capacity of the bridge. Clearly, when dealing with undamaged bridges this ratio would be =1.
- $R_{d,U}/R_{d,0} \in (0, 1)$ expresses the difference between the resistance/carrying capacity of an undamaged bridge designed according to outdated construction codes ($R_{d,U}$) versus its resistance if it were designed with current-day codes ($R_{d,0}$). Indeed, if to assess the reliability of a bridge designed according to current-day codes than this

ratio would be =1. Oppositely, for codes prior to the current-day one the ratio is <1 as the modern-day reference Traffic Loads tend to be more conservative than their outdated counterparts.

These two ratios embody the two main aspects behind a reduction in Structural Reliability of a bridge: Damage and Overload. Henceforward, these two ratios will be referred to as Damage Index D and Capacity Index C as per Eq. (10).

$$D = \frac{R_{d,D}}{R_{d,U}} \in (0, 1); C = \frac{R_{d,U}}{R_{d,0}} \in (0, 1) \quad (10)$$

Moving back to the determination of a Reliability Index β of a bridge given potential risks of damage-induced carrying capacity loss and overload, its final form can be obtained by substituting Eqs. (10) in (9),

thus obtaining Eq. (11):

$$\beta = \left\{ \beta_0 + \frac{\ln(D \bullet C)}{V_{z,0}} \right\} \bullet \frac{V_{z,0}}{V_z} \quad (11)$$

It should be noted that, β should be calculated for both the bending limit state (β_M) and the shear one (β_V). Thus, the overall Overload and Degradation Reliability Index β is assumed as their minimum: $\beta = \min(\beta_M, \beta_V)$. Let us now take a step back to the general formulation. Focusing on the Capacity Index C and on its components, $R_{d,0} = G + Q_{d,0}$ stands for the Design Resistance calculated according to the current-day reference code and $R_{d,U} = G + Q_{d,U}$ is the Design Resistance calculated according to an outdated code with G being the Permanent Load, $Q_{d,0}$ the

Traffic Load calculated according to the reference code and $Q_{d,U}$ the Traffic Load calculated according to a specific outdated code. At this point an additional index can be introduced: the Performance Index $I = Q_{d,U}/Q_{d,0}$. Given the above, the Capacity Index C of Eq. (10) can be reformulated as Eq. (12).

$$C = \frac{R_{d,U}}{R_{d,0}} = \frac{G + Q_{d,U}}{G + Q_{d,0}} = \frac{1 + \frac{Q_{d,U}}{G}}{1 + \frac{Q_{d,0}}{G}} = \frac{1 + \frac{Q_{d,U}}{Q_{d,0}} \bullet \frac{Q_{d,0}}{G}}{1 + \frac{Q_{d,0}}{G}} = \frac{1 + \frac{Q_{d,U}}{Q_{d,0}} \bullet \frac{Q_{d,0}}{G}}{1 + \frac{Q_{d,0}}{G}} = \frac{1 + I \bullet \frac{Q_{d,0}}{G}}{1 + \frac{Q_{d,0}}{G}} \quad (12)$$

Focusing on the Damage Index D , in light of the expeditious nature of the proposed model, pre-established values of D are associated directly to the extent and gravity of the defects observed on the structure during routine bridge inspections. These associations are established directly by Infrastructure Managers according to their singular management philosophy and risk-proneness/aversion. Table 1 provides one such association as performed by a medium sized Italian infrastructure manager. Here, six levels of Damage Level (DL) were identified. Each one has an associated Damage Index D (embodying the damage-induced reduction

Table 1

Infrastructure manager-specific correlation of Damage Levels DL and their corresponding values of D , $P_{F,D}$, β and V_z .

Damage Level DL	1	2	3	4	5	6
Damage Index D	1	1	0.99	0.96	0.92	0.8
Probability of Failure $P_{F,D}$	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
Reliability Index β	5.61	5.20	4.75	4.26	3.72	3.09
Coefficient of Variation V_z	0.16	0.17	0.20	0.21	0.23	0.26

in carrying capacity of the bridge), a pre-established Reliability Index β and a Coefficient of Variation V_z (determined through a process of calibration tailored to the concessionary on the basis of Eq. (11) and the assumptions of $V_{z,0} = 0.16$ and $\beta_0 = 5.61$).

Please note that the value of $\beta_0 = 5.61$ (for Damage Level 1) is the outcome of an arbitrary decision of the infrastructure manager to associate a Probability of Failure of $P_{F,D} = 10^{-8}$ to newly built structures (indeed, $\beta_0 = -\Phi^{-1}(P_{F,D} = 10^{-8}) = 5.61$). This may differ between different infrastructure managers and widespread construction codes e. g., the Eurocode (which considers a $\beta_0 = 4.7 \rightarrow P_{F,D} \cong 10^{-6}$). It is crucial to note that the associations of all the values presented in the table with each Damage Level is not only at defect level and element level, but such numerical association will be valid towards the whole article also at a global-bridge level, as we will see afterwards. This comprehensive approach ensures a deep understanding of the structural integrity assessment, encompassing the global perspective while delving into the intricacies of defects and individual elements contributing to the overall bridge condition. In fact, in order to obtain the Probability of Failure of the whole bridge $P_{F,D}$ of the asset, it is possible to sum the contribution of each defect k in terms of Probability of Failure $P_{F,D,k}$. The Probability of Failure of the whole bridge can be therefore evaluated as $P_{F,D}$ of the asset = $\sum_k P_{F,D,k}$ (as per Eq. (2)). The Global Damage Level (GD) of the bridge is the one associated to the order of magnitude of $P_{F,D}$ of the asset by means of Table 1.

For the sake of clarity an exemplary case study is reported below. Let us consider a case study of a prestressed concrete bridge with different span lengths (between 5 m and 100 m) and designed with the 1962, 1980, 1990 and 2018 design codes. Fig. 2a reports the evolution of the Capacity Index C for said bridge for both the bending limit state (C_M) and the shear one (C_V). Fig. 2b, instead, plots the evolution of the Probability of Failure $P_{F,D}$ for different levels of Global Damage Level of

the bridge.

Interestingly, in Fig. 2a all the C curves are somewhat aligned to the reference one, except for the 1962 ones for larger bridge spans. Indeed, according to said code, the Traffic Load is calculated by “fitting” as many vehicles as geometrically possible on the bridge span and summing the load (standard or military) bearing on each axle of each vehicle. Overall, for larger bridge spans, this load setup produces larger transit loads than the current day ones, based on uniformly distributed loads and concentrated loads on the axle of a single vehicle. Additionally, in Fig. 2a it can be observed that for long bridge spans, C tends to 1 as the Permanent Load G is usually superior to the Traffic Load one Q_d (thus, $G \gg Q_d$). In other words, for long bridge spans, the bridge design code – on which Q_d depends – is negligible. Oppositely, for short bridge spans, being G not as significant ($G < Q_d$), the design code Q_d are of fundamental importance. This bridge span-related difference is also the reason behind the horizontal variation in $P_{F,D}$ in Fig. 2b. Finally, as visible in Fig. 2b, the curves shift towards high values of $P_{F,D}$ as the Global Damage Level of the bridge GD increases.

4. Forecasting damage on a bridge

Although the Damage Index D is identifiable at the specific time of the inspection by the inspector himself or by the manager, the degradation of a bridge is a time-varying state, and this should be taken into account when modelling an efficient Decision Support System tool. Effective decision-making extends beyond the current state and involves anticipating the potential evolution of a situation. It recognizes that a seemingly minor defect in the present could progress into a more significant and costly issue in the future. Therefore, strategic interventions in the immediate future, even if modest in scale, can serve as proactive measures, averting the need for more substantial and expensive interventions in the distant future. This forward-looking approach acknowledges the dynamic nature of situations and emphasizes the value of preventative actions to mitigate future risks and costs.

The present section outlines the development of a Degradation Model aimed at evaluating the Probability of Failure of a bridge at a specific point in time. Said methodology provides an assessment encompassing the structural preservation state at the time of inspection, temporal degradation, and the potential future appearance of defects whenever these are currently absent at the time of the inspection. This section will refer solely on the Overload and Degradation Probability of

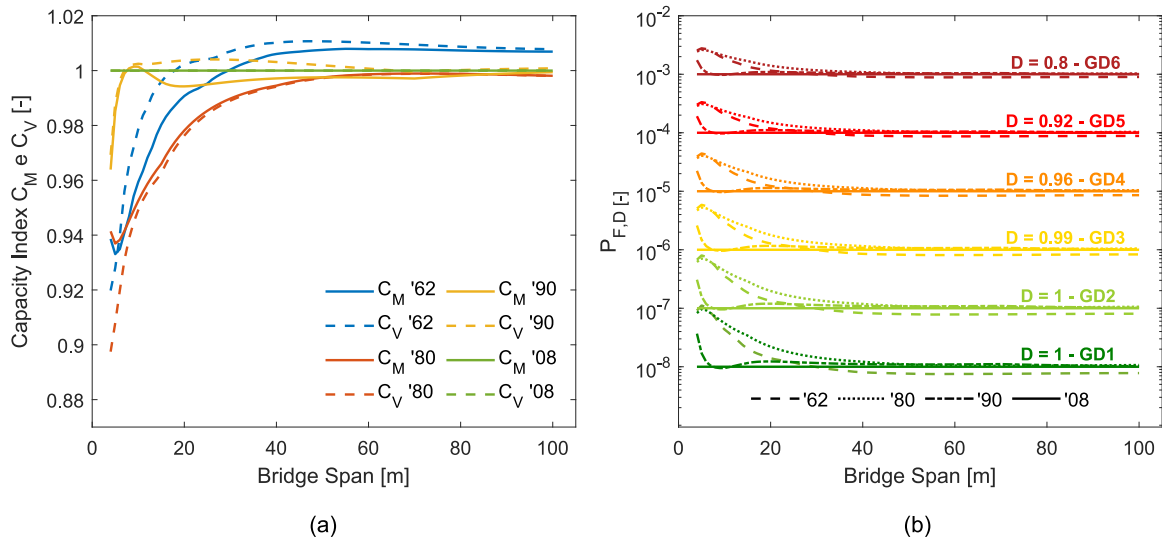


Fig. 2. Given a case study simply supported prestressed concrete bridge, with 3 transit lanes (3.75 m wide transit lanes and 2.5 m wide emergency lane), designed with the 1962, 1980, 1990 and 2018 design codes and with span length 5–100 m: (a) evolution of the Capacity Index C for said bridge for both the bending limit state (C_M) and the shear one (C_V) and (b) evolution of the Probability of Failure $P_{F,D}$ against the bridge span for different Global Damage Level of the bridge (GD1–6 in the Figure).

a. Defect: Corrosion-induced reduction of reinforcement cross-section		
Intensity I Reduction R [%] of the reinforcement in the affected cross-section	I=1	$R < 5\%$
	I=2	$5\% < R < 10\%$
	I=3	$10\% < R < 35\%$
	I=4	$R > 35\%$
Extension E Ratio R [%] between the length of the reinforcement affected by the defect and the total reinforcement length	E=1	$R < 25\%$
	E=2	$25\% < R < 50\%$
	E=3	$50\% < R < 75\%$
	E=4	$75\% < R < 100\%$

b. Defect: Transversal crack opening		
Intensity I Crack width C [mm]	I=1	$C < 0.5$
	I=2	$0.5 < C < 1$
	I=3	$1 < C < 2$
	I=4	$C > 2$
Extension E Ratio R [%] between the height of the crack and the height of the structural component	E=1	$R < 25\%$
	E=2	$25\% < R < 50\%$
	E=3	$50\% < R < 75\%$
	E=4	$75\% < R < 100\%$

Fig. 3. Description of the main parameters – Intensity and Extension – of the following defects: a) corrosion-induced reduction of reinforcement cross-section and of b) transversal crack opening.

Failure $P_{F,D}$, which henceforth will be referred to simply as P_F for the sake of conciseness and clarity.

Before delving into the explanation of how the appearance or evolution of a structural defect is forecasted by the proposed DSS, it is important to clarify how said defects and its magnitudes are represented.

Let us consider a defect characterization system comprising two main parameters, Intensity (I) and Extension (E), each delineated by four discernible levels (a methodology nowadays employed by many highway operators). The Intensity (I) parameter captures the magnitude of the defect, irrespective of the affected area's extent, while Extension (E) parameter encapsulates the spatial scope of the defect, regardless of its magnitude. An example of these parameters for two specific defects is presented in Fig. 3.

Each defect can therefore be effectively encapsulated in a I-E matrix: a Defect Description Matrix (henceforward simply referred to as Defect Matrix). The elements of such Defect Matrix denote the Probabilities of Failure associated with distinct combinations of Intensity and Extension levels.

The Probability of Failure values, corresponding to each combination of Intensity and Extension for every defect, underwent a meticulous calibration process. A key assumption was made, postulating the existence of a Dominant Defect with a Probability of Failure on an order of magnitude higher than any other defects present on the bridge. In essence, this implies that the Probability of Failure for the entire bridge is synonymous with the Probability of Failure of the Dominant Defect. The calibration process of the Defect Matrix of each defect entailed a systematic assessment of the Probability of Failure for a bridge by means

of the Damage Index D as per Table 1, considering each and every defect as the dominant one, and encompassing all possible combinations of Intensity and Extension. As a result, each position within the Defect Matrix signifies the Probability of Failure that a bridge would face if the identified defect, characterized by a specific combination of Intensity and Extension, were regarded as the most critical during the inspection, thereby representing the Dominant Defect. To facilitate interpretation, the values of such matrices are then translated into Damage Levels (DL), expressing the overall severity of a defect, as reported in Fig. 4, leveraging the predefined relationship between Damage Levels and Probabilities of Failure outlined in Table 1. Noteworthy, as visible in Fig. 4, multiple I- E combinations can lead to an identical Probability of Failure and, consequently, an identical DL (e.g., $I = 2$ E = 1 and $I = 2$ and $E = 4 \rightarrow P_F = 10^{-6}$, DL3). It is crucial to acknowledge the inherent subjectivity in associating Probabilities of Failure with Damage Levels, as these linkages may exhibit variations across different operators. To enhance the immediate comprehensibility of the matrix and the severity of each combination of Intensity and Extension, and consequently the associated Damage Level, a colour scale has been assigned to the Damage Level values, hereafter referred to as Colour Codes. It is important to note that these Colour Codes are purely arbitrary and may vary among operators.

Now that the description of the defect has been clarified, it is time to tackle the forecasting of the evolution in time. As previously anticipated, at the core of the Decision Support System is a Degradation Model based on Markov Chains. A Markov process, adhering to the Markov property, allows predictions of future outcomes based solely on its present state,

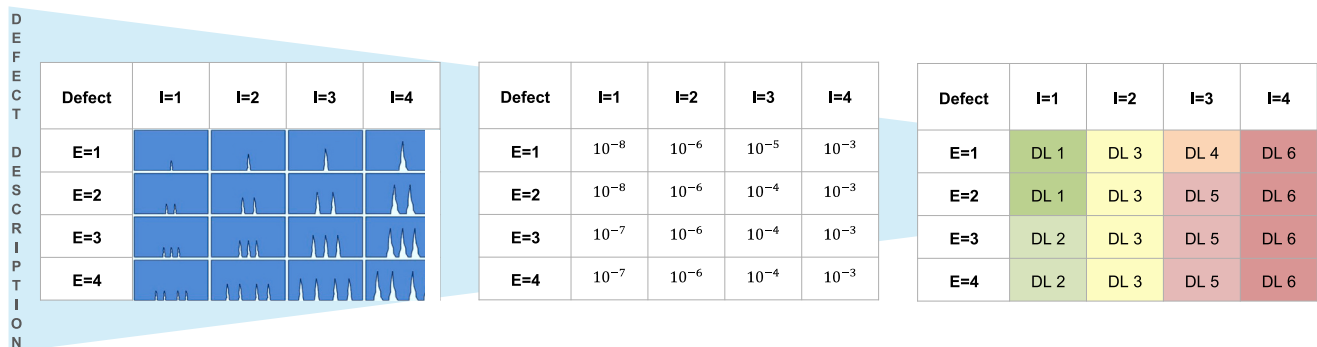


Fig. 4. Example of Defect Matrix for cracks opening defect expressed: graphically, in terms of Probability of Failure and in terms of associated Damage Levels (DL) with Colour Codes.

independently from its history [40]. Markov chains, a type of such processes, involve transitions between states with associated probabilities. These transitions are defined by a transition matrix, and the process is characterized by a state space and an initial state or distribution. Markov processes, like Markov chains, model sequences of events and the probability of each one, finding applications in fields like Bayesian statistics and stochastic simulation methods [30]. As a matter of fact, from literature, Markov chain is the preferred tool for infrastructure deterioration modelling [41,42].

In our case, Markov Chains will be employed to govern the transitions between distinct Degradation Levels and therefore express the probability of maintaining or increasing a particular state condition. In particular, what will be governed is the evolution of the above-described Intensity (I) and Extension (E) parameters of a defect. As thoroughly explained later, the Markov Chain employed is defined by a Transition Matrix T , where the main diagonal values indicate the probability of sustaining the initial state, while off-diagonal values denote the likelihood of transitioning to subsequent degradation states.

Importantly, our Degradation Model not only accommodates the scenario where a defect is encountered during the initial inspection, but also where it is absent (but could potentially appear later). For the former, the Degradation Model assesses the worsening of a case study defect in time through the abovementioned Markov Chains, whilst for the latter the Degradation Model assesses the probability of appearance of the case study defect in the future, accounting also for defect correlations. Notably, with the architecture of the DSS both culminate in a Probability of Failure at a future time of a bridge. These two are in fact crucially combined into a total asset-Probability of Failure by means of Eq. (2). The two scenarios of defect present and not present at the time of inspection are therefore both equally relevant and will be tackled in a separate fashion in the following sub-sections. The first scenario being discussed is the one of a defect being present (Subsection 4.1). Here, once the Damage Level of the defects encountered at the time of inspection are defined and the desired defect evolution timeframes established (i years), specific Transition Matrices T_i will be systematically derived to calculate the probabilities of transition to other Damage Levels. This process contributes to a detailed narrative of the defect's evolutionary journey. The second scenario that will be discussed is the one of the defects being absent (Subsection 4.2). Here a well-defined appearance probability will be established, followed by meticulous computations of subsequent state probabilities for different Damage Levels.

4.1. Scenario 1: Forecasting the degradation of defects detected during an inspection

The present subsection tackles the scenario that a particular case study defect is present on the structure at the time of the inspection. As such, the Degradation Model inside the DSS aims at assessing the evolution of said defect in time and forecasting its entity at an i -th year of choice.

Initially, the defect state encountered at inspection is defined through two four-row column vectors, as Intensity and Extension parameters are ranging from 1 to 4, indicating the current degradation state. In such vectors, the position corresponding to the value of Intensity encountered at time of inspection is set to unity, while the other positions are set to zero. Such vectors, expressing the Intensity and Extension state during the inspection, are defined as Intensity State Vector I and Extension State Vector E respectively. To clarify the concept through examples, let us consider a defect encountered during the inspection with both Intensity and Extension equal to 1, the Intensity and

Table 2

Probability P_{qr} of transitioning from a state q to state r and time period N after which there is a 50 % probability of transitioning to the next state for each of the 4 Intensity and Extension states.

	P_{qr}	N [years]
State 1 to 2	0.034	20
State 2 to 3	0.045	15
State 3 to 4	0.045	15

Extension State Vectors would result in $I = E = [1; 0; 0; 0]$. Again, if a defect was encountered during the inspection with Intensity equal to 2 and Extension equal to 3, the Intensity State Vector would be $I = [0; 1; 0; 0]$, whereas the Extension State Vector would be $E = [0; 0; 1; 0]$.

Having delved into the current-state description of a defect, it is now possible to move on to its future projection, termed as transition. The establishment of transition probabilities from each defect state is governed by the foundational Markovian equation $P_{qr} = 1 - 0.5^{\frac{1}{N}}$. Here, P_{qr} is the probability of transitioning from a state q to state r , and N represents the time period after which there is a 50 % probability of transitioning to the next state. This formulation accommodates the possibility of having different time periods to transition between different states.

In this article, it is assumed that for the case of P_{12} the transition from State 1 to State 2 occurs over a period of 20 years, resulting in an annual transition probability of $P_{12} = 3.4\%$. For subsequent transitions, a shorter period of 15 years has been assumed. Therefore, considering 4 possible states of Intensity and Extension, the aforementioned assumptions are summarized in Table 2.

Now, the different P_{qr} , along with their complementary probabilities, can be collected in a Transition Matrix T_i as in Eq 13.

$$T_i = \prod_{j=1}^i T = \begin{bmatrix} 1 - P_{12} & 0 & 0 & 0 \\ P_{12} & 1 - P_{23} & 0 & 0 \\ 0 & P_{23} & 1 - P_{34} & 0 \\ 0 & 0 & P_{34} & P_{44} \end{bmatrix}^i \quad (13)$$

The matrix cell values of T_i , ranging from 0 to 1, adhere to two foundational assumptions: 1) it is not possible to leap two or more states in a single iteration; 2) it is not possible to transition to a lower state without external maintenance intervention. Furthermore, in the Transition Matrix of Eq. (13) the main diagonal t_{ii} of matrix T denotes the probability of the defect remaining in their initial I-E state, i.e. to not vary its condition from the one encountered during the inspection. The progressive transformation of the defect over time is encapsulated by iteratively multiplying the Transition Matrix T by itself. Hence, at the i -th year, the Transition Matrix T_i emerges as the outcome of raising the initial Transition Matrix T to the power of i .

The evolution of the defect captured by the Transition Matrix (T_i) leads to the formulation of two State Probability Vectors, I_i for Intensity and E_i Extension (see also Eq 14):

- $I_i = T_i \times I$: Intensity State Probability Vector at the i -th year, where T_i is the Transition matrix at the i -th year, and I the above-mentioned Intensity State Vector.
- $E_i = T_i \times E$: Extension State Probability Vector at the i -th year; T_i is the Transition matrix at the i -th year, and E the above-mentioned Extension State Vector.

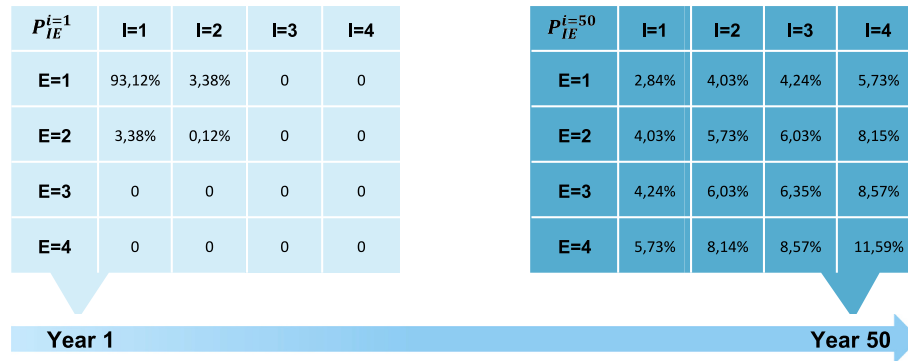


Fig. 5. Example of State Probability Matrix P_{IE}^i at 1 year and 50 years for a starting defect with $I = 1$ and $E = 1$.

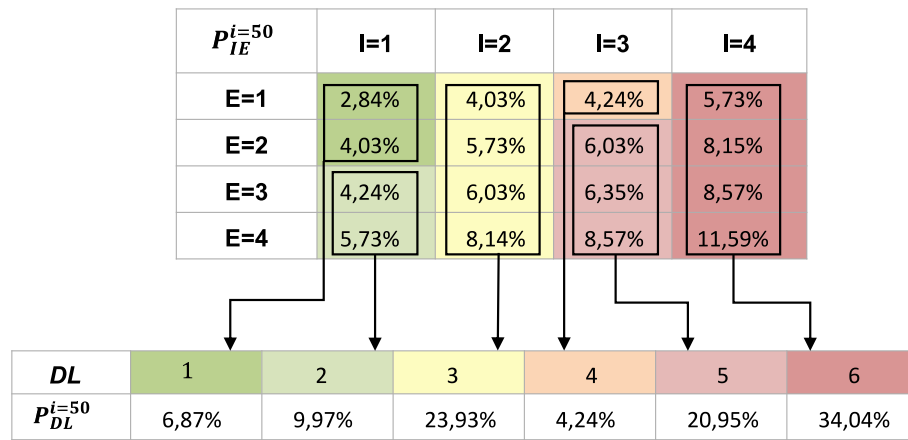


Fig. 6. Example of procedure to pass from the State Probability Matrix P_{IE}^i to the Damage Level Probability Vector P_{DL}^i at $i = 50$ years (given a $I = 1$ and $E = 1$ starting defect).

$$\begin{aligned}
 \text{Defect } I = 1, E = 1 \rightarrow I = & \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} E = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
 \text{After year 1 : } I_1 = T_i \times I = & \begin{bmatrix} 1 - P_{12} \\ P_{12} \\ 0 \\ 0 \end{bmatrix} E_1 = T_i \times E = \begin{bmatrix} 1 - P_{12} \\ P_{12} \\ 0 \\ 0 \end{bmatrix} \quad (14)
 \end{aligned}$$

Each value of the State Probability Vectors provides the probability of the defect under consideration having the level of Intensity and Extension represented by the position in the respective vector after i years (e.g., I_1 suggests that the probability of $I = 1$ after 1 year is equivalent to $1 - P_{12}$ whilst the probability of $I = 2$ is equivalent to P_{12}).

Such results can be employed for the creation of a new matrix, that represents the probability of specific I and E combinations after i years: the State Probability Matrix P_{IE}^i . This can be obtained through the product of the column vector of Extension State Probability E_i and the transposed vector of Intensity State Probability I_i as per Eq. (15).

$$P_{IE}^i = E_i \times I_i' \quad (15)$$

This process crystallizes the probabilities of the defect being in a specific I - E couples after a defined temporal interval i , offering a

comprehensive understanding of the defect's trajectory over time.

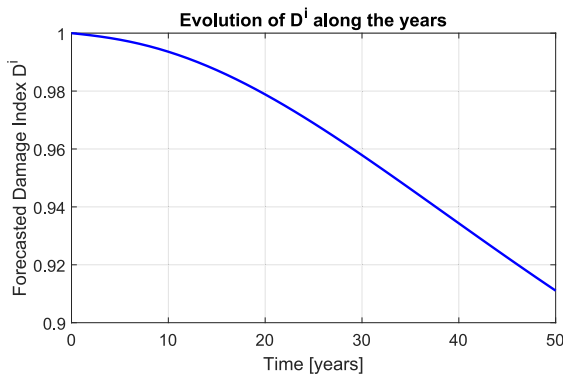
Fig. 5 represents a State Probability Matrix P_{IE}^i (at $i = 1$ year and $i = 50$) for an initial state assessed by the inspector of $I = 1$ and $E = 1$ (similarly to the exemplary scenario of Eq. (14)). The reader should keep in mind that the matrices in Fig. 5 refer only to the scenario of a starting defect of $I = 1$ and $E = 1$, they would be different for any other starting I - E combinations.

At this point, the reader is reminded that multiple cells of a Defect Matrix (containing different I - E combinations) can lead to an identical Probability of Failure and that each Probability of Failure within defect matrices are linked to a distinct Damage Level. The same is applicable to a State Probability Matrix P_{IE}^i . As such, a Damage Level Probability Vector P_{DL}^i can be obtained for an i -th year as per Eq. (16). In particular, as illustrated in Fig. 6, P_{DL}^i is obtained by summing the single probabilities of the P_{IE}^i matrix (noted in Eq. (16) as $P_{IE,DLn}^i$ for the i -th year and for the n -th Damage Level DL) corresponding to the same Damage Level in the original Defect Matrix (seen in Fig. 4). The Damage Levels of the original Defect Matrix are referenced in the matrix presented in Fig. 6 through Colour Codes to facilitate the readers. This process encapsulates in a

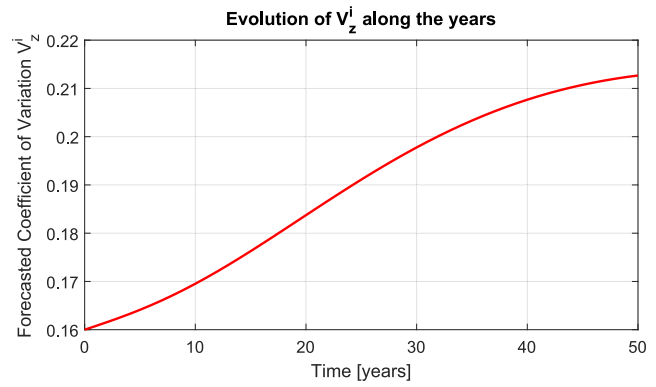
Table 3

Damage Level Probability Vector P_{DL}^i at $i = 1, 10, 30, 50$ years (given a $I = 1$ and $E = 1$ starting defect).

DL	1	2	3	4	5	6
P_{DL}^1	96,50 %	0 %	3,50 %	0 %	0 %	0 %
P_{DL}^{10}	66,02 %	4,01 %	24,25 %	3,53 %	1,51 %	0,68 %
P_{DL}^{30}	22,87 %	11,47 %	32,26 %	7,16 %	13,69 %	12,54 %
P_{DL}^{50}	6,87 %	9,97 %	23,93 %	4,24 %	20,95 %	34,04 %



(a)



(b)

Fig. 7. The evolution over time of the (a) Forecasted Damage Index (D^i) and (b) Forecasted Coefficient of Variation (V_z^i) given a $I = 1$ and $E = 1$ starting defect. The D^i scalar value diminishes over time, indicating structural deterioration-induced capacity reduction. In contrast, V_z^i increases over time due to the increase in uncertainty of the state of the bridge caused by the combined effects of model projection uncertainty and defect degradation.

single Damage Level Probability Vector P_{DL}^i the defect's evolutionary trajectory over successive years.

$$P_{DL}^i = \left[\sum P_{IE,DL1}^i; \sum P_{IE,DL2}^i; \sum P_{IE,DL3}^i; \sum P_{IE,DL4}^i; \sum P_{IE,DL5}^i; \sum P_{IE,DL6}^i \right] \quad (16)$$

Such P_{DL}^i vector can be evaluated for each service year, as showed in Table 3, providing a dynamic depiction of the structural system's integrity over time.

At this point, we move on to the forecasting of those critical structural parameters providing a dynamic depiction of the structural system's degradation, on the grounds of which a proper bridge management can be performed. Specifically, we proceed evaluating the Forecasted Damage Index (D^i), which expresses the mean residual capacity expected after i years, and the Forecasted Coefficient of Variation (V_z^i) at the i -th year. The computation of D^i involves determining the summation of the probabilities P_{DLn}^i (for the i -th year) of all the Damage Levels n contained in the Damage Level Probability Vector, and the Damage Index corresponding to each Damage Level n (D_{DLn}) – accordingly to Table 1 – as in Eq. (17). The above operation corresponds to the determination of the weighted mean value of the Damage Index (D) at the i -th year, namely the Forecasted Damage Index D^i .

Meanwhile, the calculation of the Forecasted Coefficient of Variation (V_z^i) at the i -th year, associated with the updated D^i values, employs the variance expression (σ^2) and the Damage Index D as in Eq. (17).

$$\begin{cases} D^i = \sum_{n=1}^6 P_{DLn}^i \cdot D_{DLn} \\ V_z^i = \sqrt{\frac{\sigma^2}{D}} = \sqrt{\sum_{j=1}^6 P_{DLn}^i \cdot \left[V_{z,DLn}^2 \cdot \left(\frac{D^i}{D_{DLn}} \right)^2 + \left(1 - \frac{D^i}{D_{DLn}} \right)^2 \right]} \end{cases} \quad (17)$$

Please note that $V_{z,DLn}$ and D_{DLn} – the single values of V_z and D inside Table 1 – do not depend on the year i but only on the n -th Damage Level DL. Therefore, drawing from diverse probability values associated with each Damage Level at different i years, the Forecasted Damage Index (D^i) and the average uncertainty quantified by the Forecasted Coefficient of Variation (V_z^i) are meticulously computed for each of the i years through Eq 17, as represented in Fig. 7.

Note that, the gradual decline of D^i over time implies an ongoing structural deterioration, while the ascent of V_z^i symbolizes the evolving uncertainties in both the model projection and defect degradation.

As we are currently investigating a temporal evolution that can be

associated with various return periods, it is imperative to express the Reliability formula referencing a 1-year return period. This adjustment allows for the assessment of contributions on an annual basis, deviating from the original model's 50-year reference period. The calculated Forecasted Damage Index (D^i) and Forecasted Coefficient of Variation (V_z^i) play a crucial role in the computation of the Forecasted Reliability Index (β_i) and Forecasted Probability of Failure (P_F^i) at any specific temporal juncture i . Clearly, the formulation of the formers builds on Eq 11, altering those values that depend on the passing of time (β_0 , D and V_z), as in Eq. (18).

$$\beta_i = \left\{ \beta_0^1 + \frac{\ln(D^i \cdot C)}{V_{z,0}} \right\} \cdot \frac{V_{z,0}}{V_z^i} \quad (18)$$

Where β_0^1 is the Scaled Reliability Index of the structure in undamaged state, gauged over a 1-year Reference Period. The determination of β_0^1 establishes the foundation for calculating the Forecasted Probabilities of Failure (P_F^i) across various timeframes, encapsulating all possible combinations of defect Intensity and Extension during the initial inspection. Its formulation entails a scaling process of an initial distribution designed for a 50-year reference period (β_0) to one tailored for a 1-year timeframe (β_0^1). This calibration is executed through an inverse cumulative distribution operation, as detailed in Eq. (19), culminating in the extraction of the specific variable Scaled Reliability Index β_0^1 from a Scaled Probability of Failure P_F^1 derived through a Poisson process [43].

$$\beta_0^1 = -\Phi^{-1}(P_F^1) = -\Phi^{-1}(1 - e^{(-\lambda \cdot T)}) = -\Phi^{-1}\left(1 - e^{\frac{\ln(-P_F^{Tr})}{T_r} \cdot T}\right) \quad (19)$$

Where T represents the New Reference Period considered for evaluating P_F^1 , which in this case is 1 year; T_r , is the Reference Period of the mathematical model, assumed to be 50 years; and P_F^{Tr} denotes the Probability of Failure corresponding to the reference period T_r considered to be $P_F^{Tr} = 1,00 \cdot 10^{-8}$. These last two parameters are key to define λ , designated as the Annual Rate within the framework of the model's Reference Period T_r . The Annual Rate λ , was necessary for the evaluation of the Scaled Probability of Failure at 1 year $P_F^1 = 2,00 \cdot 10^{-10}$ and the associated Scaled Reliability Index $\beta_0^1 = 6.2540$.

Once all the pertinent parameters are acquired, the ensuing step involves computing the Forecasted Probability of Failure at i years $P_{F,\Delta i} = \Phi(-\beta_i)$ where β_i is given by Eq. (18). Crucially, to obtain the final value of the Forecasted Probability of Failure $P_{F,i}$ on the basis of the Law of Total Probability of Uncorrelated Events [44], we cumulate the single values of $P_{F,\Delta i}$ according to Eq. 20.

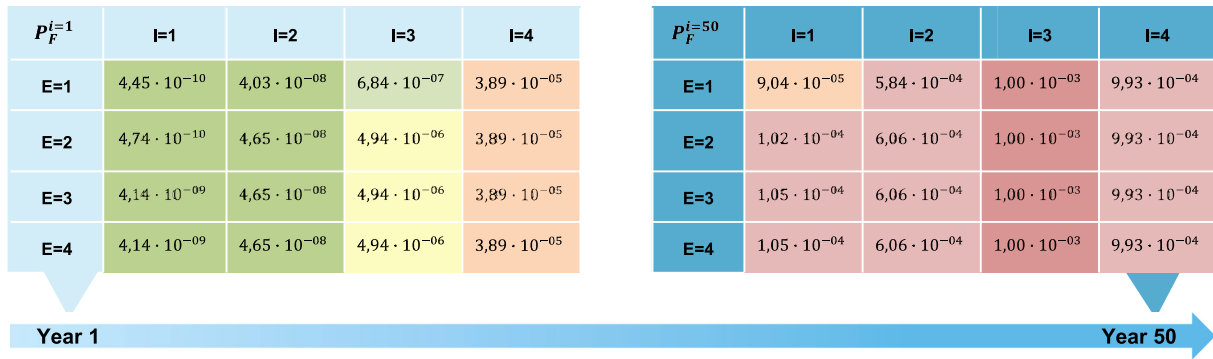
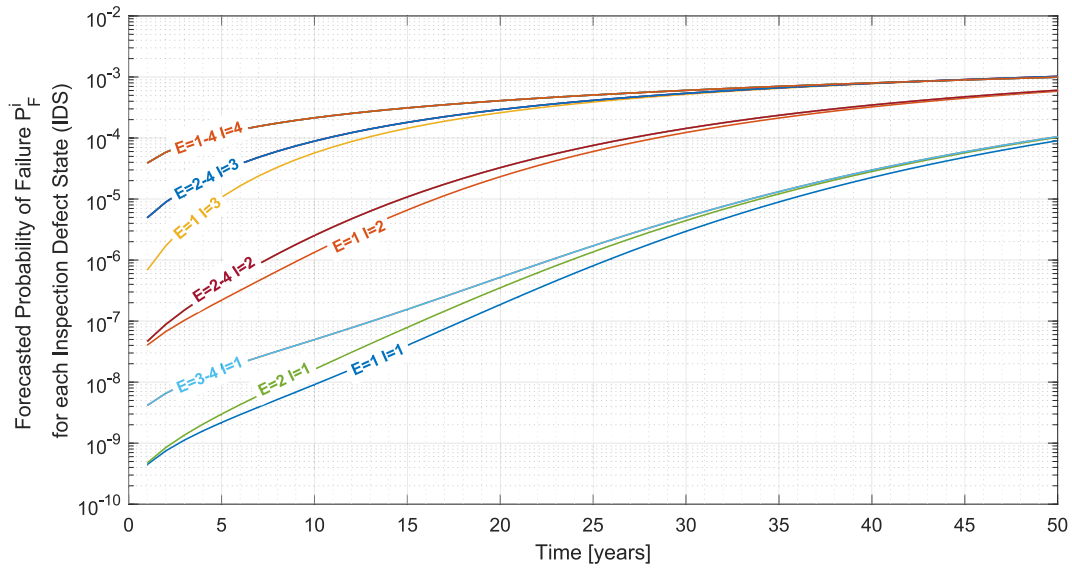


Fig. 8. Example of Forecasted Probability of Failure matrices after 1 and 50 years.

Fig. 9. Forecasted Probability of Failure P_F^i curves for each initial state of the defect.

$$P_{F,i} = \sum_{i=1}^n P_{F,\Delta i} \quad (20)$$

Such Forecasted Probability of Failure $P_{F,i}$ can be obtained for every conceivable combination of Intensity and Extension of the defect at the time of inspection. Consequently, matrices depicting the Forecasted Probability of Failure at the i -th year for each potential starting position of the defect can be systematically crafted in Forecasted Defect Matrixes (see Fig. 8) and represented in their globality by the curves of Fig. 9. The Forecasted Defect Matrixes express the Forecasted Probability of Failure value that a given defect will entail at the i -th year for each I-E combination identified during the inspection. For instance, considering a defect detected during the inspection with $I = 1$ and $E = 1$, the Forecasted Probability of Failure value attributed to this defect after 1 year can be found in the first cell (position 1-1) of the Forecasted Defect Matrix $P_{F,i=1}$. Similarly, the value of Forecasted Probability of Failure resulting from the same defect after 50 years is found in the first cell (position 1-1) of the Forecasted Defect Matrix $P_{F,i=50}$. Likewise, the position 4-4 in matrices $P_{F,i=1}$ and $P_{F,i=50}$ contain the Forecasted Probability of Failure values associated with a defect identified with $I = 4$ and $E = 4$ during the inspection after 1 and 50 years, respectively. Please note that the I and E used to enter the Forecasted Defect Matrixes are not the final I and E combination of the defect after i years, but the initial ones detected at the time of inspection. Once again, in Fig. 8, the Colour Codes associated with each Damage Level, thus the Forecasted Probability of Failure values, were employed to facilitate the comprehension

of the worsen in terms of increase of Forecasted Probability of Failure of each I-E combination in time.

Hence, this methodology, tailored for scenarios where the defect is identified during inspection, culminates in the systematic generation of robust Defect Matrixes portraying the Forecasted Probabilities of Failure projected at any given year of each and every defect. These matrixes serve as a comprehensive tool enhancing our insights into the structural system's reliability under diverse conditions and timeframes.

4.2. Scenario 2: Forecasting the appearance of defects not present at the time of inspection

The following section addresses the situation where a specific defect is not initially observed during an inspection. When a defect has not been detected during an inspection, one must still consider that the defect has an intrinsic probability of manifesting itself in the coming years. This key aspect must be considered in a DSS as the appearance of new defects might worsen already present defects and singularly or jointly compromise the structural reliability of the asset. The present subsection intends to analyse these aspects and, in particular, whether the effect on the overall system reliability is significant or negligible. Clearly, the probability of appearance of a defect significantly depends on its surrounding conditions, such as the presence of other defects that might promote its appearance. Then again, before studying how said conditions increase the probability of appearance of a defect, this last one needs to be determined in an isolated manner with an assumption of

non-correlation with other defects.

4.2.1. STEP 1: Assumption of non-correlation between defects

The present subsection explains how to determine the probability of a defect manifesting itself after i years assuming that no correlation exists among neighbouring defects, essentially an Intrinsic Defect Appearance Probability (henceforth noted as P_{app}). To determine the P_{app} of individual defects, a statistical analysis was conducted on a set of bridges. The process involved: (1) analyzing the present and historic inspection reports of each asset of the set, (2) cataloging the identified defects jointly with the year when they first appeared – compared to the year the bridge was constructed in – and (3) extracting the Intrinsic Defect Appearance Probability P_{app} . As can be evinced, the process at hand is a reverse problem i.e., determining a probability of appearance of defects P_{app} based on a set of observations of existing defects (that have clearly already appeared). Given the nature of said problem, its solution requires the use of Bayes' Theorem [45] as described in Eq. (21).

$$P(P_{app}|y) = \frac{P(y|P_{app}) \cdot P(P_{app})}{P(y)} \propto LH(P_{app}) \quad (21)$$

Where $P(P_{app}|y)$ is the posterior probability distribution, y is the observation – the year of appearance of a defect –, $P(y|P_{app})$ the likelihood probability distribution, $P(P_{app})$ the prior probability distribution and $P(y)$ the Marginal Likelihood (or model evidence). Assuming the prior knowledge to be described by uniform probability distribution and the Marginal Likelihood is a constant in respect to P_{app} given a specific value of y , then it can be stated that the posterior $P(P_{app}|y)$ is proportional to the likelihood $P(y|P_{app})$ – henceforth noted as $LH(P_{app})$ – as noted in Eq. (21). In light of the above, using the Principle of Maximum Likelihood [46], the sought-after Defect Appearance Probability P_{app} , is that value of P_{app} that maximizes the likelihood $LH(P_{app})$ i.e., that value that best describes the observation y . Now, having established the need of studying the $LH(P_{app})$ to assess P_{app} , let us analyse how $LH(P_{app})$ was determined.

Starting from a first step with the assumption of no correlation between defects, $LH(P_{app})$ is composed by:

- The likelihood of a structural element not being affected by a specific defect $P(U|P_{app}, n)$ where U stands for Undamaged and n for the years of service of the bridge.
- The complimentary likelihood of that same structural element being affected by the same specific defect $P(D|P_{app}, n)$ where D stands for

Damaged. If to express $P(U|P_{app}, n) = (1 - P_{app})^n$ for n years, then its complimentary $P(D|P_{app}, n) = 1 - P(U|P_{app}, n) = 1 - [(1 - P_{app})^n]$.

Let us now consider a single defect whose P_{app} needs assessment. Furthermore, let us consider a single bridge with n years of service composed by several Structural Elements (SE) affected by said defect SE_D and others not affected by it SE_U . In this case, the likelihood $LH(P_{app})$ can be determined as $LH(P_{app}) = \prod_{SE_D} \{1 - (1 - P_{app})^n\} \cdot \prod_{SE_U} \{(1 - P_{app})^n\}$. Now, if to expand this formulation to a larger set of k -bridges, then the likelihood $LH(P_{app})$ can be determined as $LH(P_{app}) = \prod_{SE_D} \{1 - (1 - P_{app})^{n_k}\} \cdot \prod_{SE_U} \{(1 - P_{app})^{n_k}\}$ with n_k the age of the k -th bridge. Please note, that this time both $\prod_{SE_D}(\dots)$ and $\prod_{SE_U}(\dots)$ refer to the Structural Elements of all the k -bridges. Clearly, the larger the bridge stock, the more accurate the evaluation of P_{app} . However, as in this formulation we deal with extremely small values of P_{app} , the assessment of the likelihood $LH(P_{app})$ leads to a numerical zeroing situation. To circumvent this issue, a logarithmic reformulation was performed, as per Eq. (22).

$$\ln[LH(P_{app})] = \sum_{SE_D} \ln \{1 - (1 - P_{app})^{n_k}\} + \sum_{SE_U} \ln \{(1 - P_{app})^{n_k}\} \quad (22)$$

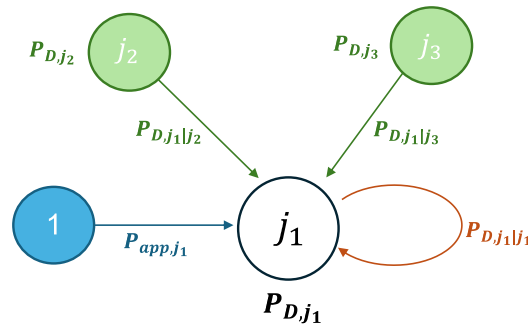
Crucially, the maximum positions of the two distributions – $LH(P_{app})$ vs. $\ln[LH(P_{app})]$ – are aligned, which guarantees that the P_{app} that maximizes both likelihoods are the same one i.e., $\maxarg \{LH(P_{app})\} = \maxarg \{\ln[LH(P_{app})]\}$. Finally, the Intrinsic Defect Appearance Probability P_{app} is determined, offering an intrinsic probability for each defect of manifesting after i years, assuming, in this phase, that no correlations exist among defects.

4.2.2. STEP 2: Assumption of correlation between defects

Let us now move to the second assumption which considers that a correlation between defects is in fact possible. Within such hypothesis, it becomes necessary to quantify the influence that a potential “triggering defect” has on the probability of appearance of another defect.

In order to jointly compute in the DSS (1) the intrinsic P_{app} , (2) its alteration caused by the presence of triggering defects and (3) the possibility that the defect has already been triggered, a new probability is introduced: the Defect Presence Probability $P_{D,j}$ (once again D stands for Damaged and j represents the specific defect under consideration). Said $P_{D,j}$ embodies the annual probability that a specific defect is now “present on the structure” whilst being absent in the previous year or at the time of the latest visual inspection.

Quantifying $P_{D,j}$ entails quantifying the consequential connections



$$\begin{aligned} P_{D,j_1} &= 1 - P_{U,j_1} = \\ &= 1 - \{P_U(\text{intrinsic appearance}) \cdot P_U(\text{other triggering defects}) \cdot P_U(\text{self presence})\} = \\ &= 1 - \{[1 - P_{app,j_1}] \cdot [(1 - P_{D,j_1|j_2} \cdot P_{D,j_2})(1 - P_{D,j_1|j_3} \cdot P_{D,j_3})] \cdot [1 - P_{D,j_1|j_1} \cdot P_{D,j_1}]\} \end{aligned}$$

Fig. 10. Appearance processes for a given defect j_1 in view of Triggering Defects j_2, j_3 and use of the exploiting complimentary probability to determine P_{D,j_1} .

between defects through a Correlation Probability $P_{Dj|jk}$, where j represents the specific defect under consideration and j_k the defect that could potentially expedite the appearance of defect j i.e., the Triggering Defect. Furthermore, the reader should keep in mind that each defect j_k is characterized by its own Defect Presence Probability $P_{D,jk}$. Having said all the above, P_{Dj} can be derived by exploiting the complimentary probability that the defect does not appear P_{Uj} as per $P_{Dj} = 1 - P_{Uj}$. In particular, as plotted in Fig. 10, P_{Uj} only occurs if the defect is not already present from the previous year (in orange), if it does not intrinsically appear (in blue), and if it is not triggered by any influencing defects (green).

Crucially, the simplified formulation of P_{Dj} represented in Fig. 10, refers to the annual probability of having the given defect j . Hence, to account for the evolution of P_{Dj} in the following years, it is necessary to undertake an iterative process. Such iterative process now operates in a vector space to account for the fact that the evolution of each and every defect of the catalogue, and therefore their own Defect Presence Probability, is now correlated to the Defect Presence Probability of all the other defects of the catalogue as well. This process enables the computation of the vector P_{Dj}^i at a given year i , which in its terms compute the Defect Presence Probability of each defect at any time horizon, as presented in Eq. (23):

$$P_{Dj}^i = 1 - \left\{ (1 - P_{app,j}) \cdot \prod_{k=1}^n (1 - [P_{Dj|jk}] \cdot P_{D,jk}^{i-1}) \right\} \quad (23)$$

Let us now study the different components of Eq. (23). Starting from vector $(1 - P_{app,j})$, the Intrinsic Appearance Probability of each defect j i.e., each term of $P_{app,j}$ coincide with what explained in the Section 4.2.1 for each defect j . Moving on to the second component $\left[\prod_{k=1}^n (1 - [P_{Dj|jk}] \cdot P_{D,jk}^{i-1}) \right]$, this one encapsulates the influence of all the Triggering Defects j_k . In particular, the term $[P_{Dj|jk}]$ represents a symmetric matrix, where: along the main diagonal values are set to 1, signifying $P_{D,jj}$ equal to 100 % to represent a complete influence of the defect on itself. In other positions values denote the Correlation Probability $P_{Dj|jk}$ expressed as percentual amplification of $P_{D,j}$ associated to the j -th defect caused by the presence of the defect j_k . The authors considered three possible levels of percentual increase given by $P_{Dj|jk}$:

- High Influence: a 5 % increase;
- Medium Influence: a 2 % increase;
- Low Influence: a 0.5 % increase.

Note that these values are preliminary arbitrary values that might vary depending on each infrastructure manager and require a thorough understanding of the specific cases under examination. Finally, the vector $P_{D,jk}^{i-1}$ encapsulates the evolution of the Defect Presence Probability of each defect j_k from the moment the bridge was put into service ($P_{D,jk}^{i-1} = 0$) up until the present day. As the iterations progress, $P_{D,jk}^{i-1}$ terms increase up until a maximum value of 1 i.e., the given defect has a 100 % probability of being present. Please note that, if the defect j_k is detected during an inspection, the associated term within $P_{D,jk}^{i-1}$ becomes =1 regardless of the value extracted from the iteration.

The iteration functions and to stops for each term whenever the defect j is detected during an inspection or has reached 100 %

Table 5

State Probability Vectors at a year $i = 1, 10, 30, 50$ for a defect j not present at the time of inspection.

DL	0	1	2	3	4	5	6
P_{DL}^1	99,900 %	0,097 %	0 %	0,003 %	0 %	0 %	0 %
P_{DL}^{10}	99,000 %	0,660 %	0,040 %	0,200 %	0,035 %	0,015 %	0,068 %
P_{DL}^{30}	97,040 %	0,680 %	0,340 %	0,950 %	0,210 %	0,400 %	0,370 %
P_{DL}^{50}	95,120 %	0,340 %	0,490 %	1170 %	0,210 %	1020 %	1660 %

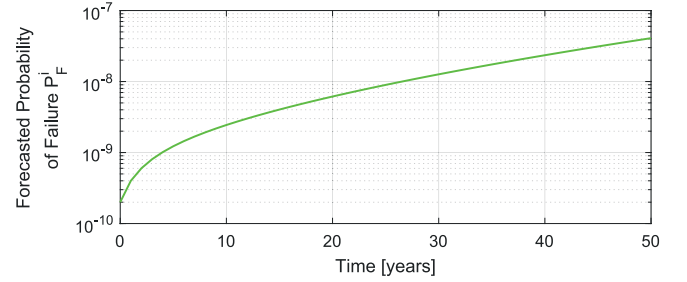


Fig. 11. Values of P_F^i obtained for each year in the case of a defect not present at the time of inspection.

probability of being present $P_{Dj}^i = 1$. The vector P_{Dj}^i encapsulates the evolution of the Defect Presence Probability of all defects j . When the defect is absent during the latest inspection, then the respective term of $P_{Dj}^i \neq 1$ and encompasses the contributions of the growing Intrinsic Defect Appearance Probability and Correlation Probability accrued over the years of service of the bridge.

4.2.3. STEP 3: Forecasting the appearance of defects

Having defined the Defect Presence Probability P_{Dj}^i of a given defect j at a given year i and its complimentary probability P_{Uj}^i , it is now possible to forecast the extent of said defect and its consequent Forecasted Probability of Failure. Importantly, when forecasting the initiation of a

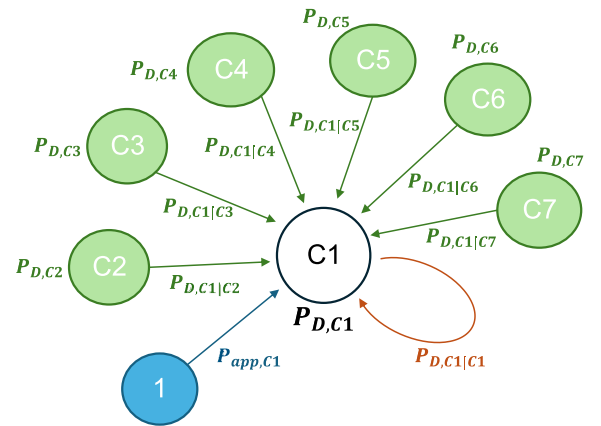


Fig. 12. Defect C1 and parameters that influence its appearance.

Table 4

State Probability Vector at a year i for a defect j not present at the time of inspection.

DL	0	1	2	3	4	5	6
P_{DL}^i	P_{Uj}^i	$P_{DL1} \bullet P_{Dj}^i$	$P_{DL2} \bullet P_{Dj}^i$	$P_{DL3} \bullet P_{Dj}^i$	$P_{DL4} \bullet P_{Dj}^i$	$P_{DL5} \bullet P_{Dj}^i$	$P_{DL6} \bullet P_{Dj}^i$

defect ($P_{D,j}^i = 1$), this one assumes the conditions $E = 1/I = 1$, which then evolve in the following years. The reader is reminded (Subsection 4.1) that each defect encompasses different Damage Level Probabilities P_{DL} (collected in a single State Probability Vector) that vary year per year. Henceforward, the undamaged state with probability of occurrence $P_{U,j}^i$ is associated to a Damage Level 0 DL0. If defect j is absent at the time of inspection, to assess its Damage Level Probabilities at the i – th year, all the P_{DL} referred to DL1 to DL6 need to be multiplied by the Defect Presence Probability $P_{D,j}^i$, expressing the probability of the bridge being in the actual damaged condition, as in Table 4.

Once again, such vector can be evaluated for the reference defect at each reference year, as showed in Table 5.

The calculated values of P_{DL}^i represent the probabilities of being in each Damage Level when the defect is not present at the time of inspection after i years. The values of D^i , V_z^i , β_i can then be obtained starting from each Damage Level Probability Vector using the previous procedure involving Eq. (17), and Eq. (18). Subsequently, for each year, a single value of P_F^i is obtained in the case of an undetected defect, as illustrated in Fig. 11. Importantly, P_F^i is calculated by aggregating individual $P_{F,\Delta t} = \Phi(-\beta_i)$ values using a cumulative approach akin to Eq. 20 in Section 4.1. The resultant P_F^i reflects the Forecasted Probability of Failure across all time horizons under the assumption of undetected defect.

Remarkably, as reported in Table 5, even after 50 years, P_{DL}^{50} exhibits a high probability of remaining in the undamaged state (95.120 %) and therefore predominantly indicates a DL0 state. This implies that the Forecasted Probability of Failure, when a defect goes undetected during inspection, primarily aligns with the magnitude associated with the undamaged state. This section, along with the new P_F^i values, enriches the temporal damage forecast, complementing the projected P_F^i matrix from Section 4.1 (see Fig. 8). It extends the analysis by accounting for the possibility of the structure remaining undamaged over time.

4.2.4. Exploring the influence of triggering defects on the forecasted probability of failure

The authors will now present a practical and comprehensive example to investigate the relevance of Triggering Defects on the Defect Presence Probability $P_{D,j}^i$ of a given defect j not detected at the time of inspection, as well as on its final Forecasted Probability of Failure $P_{F,j}^i$ at various time horizons i . Let us assume a defect “C1 – Water infiltration” which

from statistical analysis result having a Probability of Appearance $P_{app,C1} = 1,4 \cdot 10^{-3}$. It is also considered that such defect is potentially influenced by the presence of 6 other Triggering Defects C_k in Fig. 12 and Table 6. Please note that Table 6 also reports the Probabilities of Appearance P_{app,C_k} of the Triggering Defects j_k (established by the authors on the basis of statistical analysis on several bridge stocks) as well as their yearly Correlation Probabilities in respect of defect C1 $P_{D,C1|Ck}$. Furthermore, the current example includes seven different hypotheses that clarify the impact of one or more Triggering Defects C_k on the probability $P_{D,C1}^i$. Please note that, as explained above, when a specific defect C_k is present in a hypothesis HP, its Defect Presence Probability is =1 (e.g. $P_{D,C2} = 1$ in HP2).

The last hypothesis reported in Table 6 is a crucial one, aimed at maximizing the impact of the triggering defects on the Defect Presence Probability $P_{D,C1}^i$. This strategic approach enables the exploration of a worst-case scenario, serving as a prerequisite to evaluating the significance of Triggering Defects on non-present defects at the time of inspection and, ultimately, on the Probability of Failure.

By applying Eq. (23) for each HP, the Defect Presence Probability for the given defect $P_{D,C1}^i$ can be determined for each case. This last one allows for the evaluation of the State Probability Vector P_{DL} for each HP as per Table 4, and therefore of the crucial parameters D^i , V_z^i , β_i through Eq. (17), and Eq. (18). Finally, a single value of P_F^i is obtained for each year. The Defect Presence Probabilities and Forecasted Probabilities of Failure of defect C1, $P_{D,C1}^i$ and $P_{F,C1}^i$ respectively, were calculated for all the given HPs at different time horizons, as shown in Table 7 and Fig. 13.

Fig. 13a unmistakably illustrates that the presence of Triggering Defects markedly expedites the Defect Presence Probability of defect C1. This, in fact, ensures its manifestation in under 35 years in the most adverse scenario HP7, as opposed to the nearly 50-year timeframe observed when no Triggering Defects are present as in HP1. While the existence of Triggering Defects may indeed expedite the Defect Presence Probability, however, the visual evidence presented above in Fig. 13b illustrates that despite the presence of Triggering Defects leads to a slight variation in the Forecasted Probability of Failure, the order of magnitude of such remains unchanged, showing a negligible impact of the Triggering Defects Failure on the ultimate Forecasted Probability of Failure for the structure within a 50-year time horizon for this specific case study. This is attributed to the prerequisite condition for the Probability of Failure to undergo alteration—namely, an actual capacity loss in the structure originating from a DL3. Consequently, in the

Table 6

Probabilities of Appearance and Correlation Probabilities for the Triggering Defects k of defect C1 – Water infiltration and their presence in different exemplary hypothesis (presence is marked with •).

DEFECT k	$P_{app,k}$	$P_{D,C1 Ck}$ (per year)	HP1	HP2	HP3	HP4	HP5	HP6	HP7
C2: Concrete segregation	$7,60 \cdot 10^{-5}$	2 %		•	•	•	•	•	•
C3: Exudation cavities, efflorescence	$5,10 \cdot 10^{-5}$	2 %			•	•	•	•	•
C4: Pavements cracks and anomalies	$6,30 \cdot 10^{-5}$	5 %				•	•	•	•
C5: Water infiltration through the joints	$1,52 \cdot 10^{-4}$	5 %					•	•	•
C6: Joint drainage elements permeable, clogged or absent	$1,30 \cdot 10^{-5}$	5 %						•	•
C7: Bridge water disposal malfunction	$8,90 \cdot 10^{-5}$	5 %							•

Table 7

Resulting Defect Presence Probability $P_{D,C1}^i$ and Forecasted Probabilities of Failure $P_{F,C1}^i$ of defect C1 for each hypothesis HP and different reference times.

$P_{D,C1}^i$	5 years	15 years	30 years	50 years	$P_{F,C1}^i$	5 years	15 years	30 years	50 years
HP 1	0.0070	0.0219	0.0463	0.0824	HP 1	1,20e-09	3,21e-09	6,33e-09	1,09e-08
HP 2	0.1024	0.2775	0.4794	0.6652	HP 2	1,20e-09	3,37e-09	8,04e-09	2,15e-08
HP 3	0.1886	0.4663	0.7159	0.8779	HP 3	1,21e-09	3,51e-09	9,45e-09	3,01e-08
HP 4	0.3722	0.7527	0.9389	0.9906	HP 4	1,21e-09	3,75e-09	1,15e-08	3,86e-08
HP 5	0.5142	0.8853	0.9869	0.9993	HP 5	1,22e-09	3,90e-09	1,23e-08	4,03e-08
HP 6	0.6241	0.9563	0.9972	0.9999	HP 6	1,22e-09	3,98e-09	1,25e-08	4,07e-08
HP 7	0.7728	0.9754	0.9994	1	HP 7	1,23e-09	4,03e-09	1,26e-08	4,08e-08

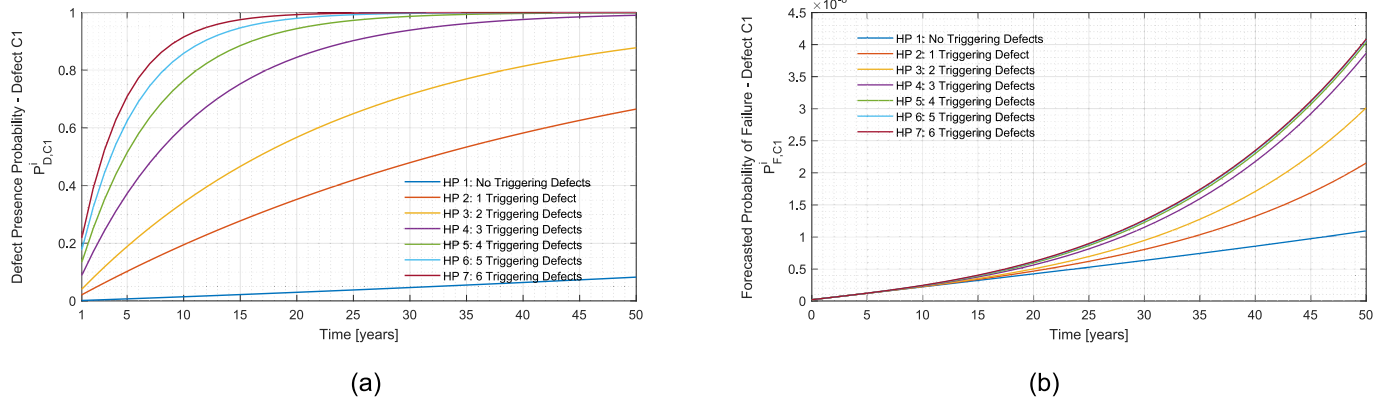


Fig. 13. (a) Defect Presence Probability of defect C1 $P_{D,C1}^i$ for each case scenario and (b) Forecasted Probabilities of Failure of defect C1 $P_{F,C1}^i$ for each case scenario.

scenario where the defect C1 is not detected during the inspection, it not only needs to manifest but must also undergo evolution up to a DL3, a process that necessitates more time than the 50-year horizon under consideration for the specific defect analysed. However, it is essential to bear in mind that the negligibility of this result in this specific case study does not render the method as a whole negligible. Indeed, if analyzing a defect with a higher Defect Appearance Probability, or a scenario where the Correlation Probabilities of the considered Triggering Defects are higher, the appearance of a given defect could indeed influence the Forecasted Probability of Failure of the structure. Understanding the forecasted state of damage of structures over time is therefore crucial for making informed decisions regarding infrastructure stocks.

5. Practical example

The present section aims at walking the reader through each step of the DSS, giving a clear overview of the contribution each of the above-described terms within the DSS framework. Let us consider a bridge operator having a hypothetical catalogue composed of 10 total defects

Table 8

Exemplary catalogue of the infrastructure operator with indication of defect presence on the asset, defect Intensity (I) and Extension (E)

Defects catalogue	Presence	I	E
C1 – Water infiltration			
C2 – Concrete segregation			
C3 – Exudation cavities, efflorescence			
C4 – Pavements cracks and anomalies			
C5 – Water infiltration through the joints	X	1	1
C6 – Joint drainage elements permeable, clogged or absent			
C7 – Bridge water disposal malfunction			
C8 – Corrosion-induced reduction of reinforcement cross-section	X	2	4
C9 – Transversal crack opening	X	3	1
C10 – Generic defect			

which can be present on an asset, illustrated in the first column of Table 8. The simulation sees the bridge operator finding the bridge damaged after a visual inspection and having to choose between different courses of action (henceforward defined as Scenario) to guarantee the maximum reliability of a specific bridge at a time frame of choice.

After conducting the inspection on the case study bridge, the operator identified the presence of 3 defects on the asset: (1) C5 – Water infiltration through the joints, (2) C8 – Corrosion-induced reduction of reinforcement cross-section, (3) C9 – Transversal crack opening. The defects were catalogued by the inspector with the Intensity and Extension characteristics reported in Table 8 (last two columns). Given the exemplary defects correlation introduced in Section 4.2.4, the C5 defect may trigger the appearance of defect C1, whereas defect C8 and C9 do not trigger any defect.

The decision of the manager is to intervene within 5 years, choosing the best possible intervention through the evaluation of the Priority Index. The owner of the asset is considering 3 possible scenarios, each characterized by a specific cost:

- (1) Scenario 1: replace the joints for a cost of 30.000 €;
- (2) Scenario 2: replace the joints (30.000 €) and repair the transversal crack opening (15.000 €) for a total scenario cost of 45.000 €;
- (3) Scenario 3: replace the bridge for 600.000 €.

5.1. Determining the pre-intervention $P_{F,forecast}$

As elucidated in Section 4, The Forecasted Probabilities of Failure P_F^i of the asset is composed of two elements: the contribution of defects present during the inspection (Section 4.1) and the contribution of defects not present during the inspection but that might appear later (Section 4.2). Both contributions converge in the determination of the P_F^i of the entire asset through Eq. (2).

C5	I=1	I=2	I=3	I=4	C8	I=1	I=2	I=3	I=4	C9	I=1	I=2	I=3	I=4
E=1	DL 1	DL 2	DL 2	DL 3	E=1	DL 3	DL 3	DL 5	DL 6	E=1	DL 1	DL 3	DL 4	DL 6
E=2	DL 1	DL 2	DL 2	DL 3	E=2	DL 3	DL 4	DL 6	DL 6	E=2	DL 1	DL 3	DL 5	DL 6
E=3	DL 1	DL 2	DL 3	DL 4	E=3	DL 3	DL 5	DL 6	DL 6	E=3	DL 2	DL 3	DL 5	DL 6
E=4	DL 1	DL 2	DL 3	DL 4	E=4	DL 3	DL 5	DL 6	DL 6	E=4	DL 2	DL 3	DL 5	DL 6

Fig. 14. Defect Matrixes for the defects detected during the inspection, namely C5 – Water infiltration through the joints, C8 – Corrosion-induced reduction of reinforcement cross-section, C9 – Transversal crack opening.

Starting from the former contribution, the Forecasted Probabilities of Failure of the three defects at 5 years was determined, following the steps presented in Section 4.1 and by means of the Defect Matrices illustrated in Fig. 14 and transition probabilities of Table 2.

The resulting Forecasted Probabilities of Failure at 5 years are: $P_{F,C5}^5 = 1,62 \cdot 10^{-08}$, $P_{F,C8}^5 = 1,98 \cdot 10^{-05}$ and $P_{F,C9}^5 = 1,12 \cdot 10^{-05}$. Moving on to the evaluation of the contribution of defects absent during the inspection, as anticipated above, the defect “C5 – Water infiltration through the joints” could trigger “C1 – Water infiltration”. Hence, the contribution of the latter was determined, following the steps presented in Section 4.2.4: $P_{F,C1}^5 = 1,22 \cdot 10^{-9}$.

Using Eq. (2), it is now possible to obtain the Forecasted Probability of Failure of the bridge for the desired timeframe as per Eq. (24):

$$P_{F,forecast}^5 \text{ of the asset} = \left[P_{F,C5}^5 + P_{F,C8}^5 + P_{F,C9}^5 \right]_{\text{defects present}} + \left[P_{F,C1}^5 \right]_{\text{defects not present}} = \left[1,62 \cdot 10^{-08} + 1,98 \cdot 10^{-05} + 1,12 \cdot 10^{-05} \right]_{\text{defects present}} + \left[1,22 \cdot 10^{-9} \right]_{\text{defects not present}} = 3,10 \cdot 10^{-5} \quad (24)$$

$$P_{F,post-int1}^5 \text{ of the asset} = \left[P_{F,C8}^5 + P_{F,C9}^5 \right]_{\text{defects present}} + \left[P_{F,C5}^5 + P_{F,C1}^5 \right]_{\text{defects not present}} = \left[1,98 \cdot 10^{-05} + 1,12 \cdot 10^{-05} \right]_{\text{defects present}} + \left[4,56 \cdot 10^{-10} + 1,20 \cdot 10^{-9} \right]_{\text{defects not present}} = 3,10 \cdot 10^{-5} \quad (25)$$

It is important to keep in mind that the owner's defect catalogue includes additional defects other than the ones analysed above (C2-C4; C6;C7;C10), each with their own Forecasted Probability of Failure P_F^i powered by their Intrinsic Defect Appearance Probability P_{app} (see Section 4.2). Technically, their P_F^i should be listed jointly with $P_{F,C1}^5$

$$P_{F,post-int2}^5 \text{ of the asset} = \left[P_{F,C8}^5 \right]_{\text{defects present}} + \left[P_{F,C1}^5 + P_{F,C5}^5 + P_{F,C9}^5 \right]_{\text{defects not present}} = \left[1,98 \cdot 10^{-05} \right]_{\text{defects present}} + \left[1,20 \cdot 10^{-9} + 4,56 \cdot 10^{-10} + 1,03 \cdot 10^{-10} \right]_{\text{defects not present}} = 1,98 \cdot 10^{-5} \quad (26)$$

among the defects not present. However, since their P_F^i would remain unchanged between the two projections $P_{F,forecast}$ and $P_{F,post-int}$, their contribution is cancelled out whenever evaluating $(P_{F,forecast} - P_{F,post-int})$ of the Priority Index α_{int} – see Eq. (3) –, thus can be neglected.

5.2. Determining the post-intervention $P_{F,post-int}$

Once the Forecasted Risk of the asset is obtained, it is necessary to evaluate how this risk changes in each of the three different intervention scenarios:

1. In Scenario 1, the bridge joint is replaced, thus defect C5 is eliminated. The Forecasted Probability of Failure of defect C5 is then determined as the one of a defect not present and is powered only by its Intrinsic Defect Appearance Probability (Section 4.2.1): $P_{F,C5}^5 = 4,56 \cdot 10^{-10}$. Analogously, having eliminated defect C5, defect C1 would have no more Triggering Defect, bringing its Forecasted Probability of Failure for 5 years down to $P_{F,C1}^5 = 1,20 \cdot 10^{-9}$. On the other hand, the Forecasted Probability of Failure of the other defects

would remain unvaried (several orders of magnitudes higher than the one associated to the defect removed by the repair: $10^{-05} \gg 10^{-08}$). The Post-intervention Risk after 5 years of Scenario 1 is explicated in Eq. (25).

It can be noticed that the total Probability of Failure of the asset did not change in this specific case. In fact, the defect C5 and the consequent defect C1 are orders of magnitude lower than the Probabilities of Failure of defects C8 and C9, and their contribution is therefore not significant.

2. In Scenario 2, the transversal crack opening would be repaired and the joint replaced, leading to $P_{F,C9}^5 = 1,03 \cdot 10^{-10}$; $P_{F,C5}^5 = 4,56 \cdot 10^{-10}$ and $P_{F,C1}^5 = 1,20 \cdot 10^{-9}$, i.e. the ones caused by their sole Intrinsic

Defect Appearance Probability after 5 years. The Forecasted Probability of Failure of defect C8, instead, remains unvaried. The Post-intervention Risk after 5 years of Scenario 2 is explicated in Eq. (26)

3. In Scenario 3, the entire asset would be replaced, thus bringing the total Forecasted Probability of Failure P_F^5 after 5 years to be the one associated to a new asset without any defect: $P_F^5 = 10^{-8}$. The Post-intervention after 5 years would therefore result in $P_{F,post-int3}^5 \text{ of the asset} = 1,00 \cdot 10^{-8}$.

5.3. PriorityIndex evaluation

With both the asset's Forecasted Risk $R_{forecast}$ and the Post-intervention Risk $R_{post-int}$ determined for each scenario, it is possible to evaluate the Priority Index through Eq. (3) of each. Please note that for this particular example, being the Exposure Exp of the asset, as stated before, out of the primary scope of the current article, it is assumed to be equal to 1 [life] – the Exposure is measured in lives saved, making the Priority Index an expression of lives saved for each Euro spent –. Table 9 displays the Priority Indexes of each intervention scenario along with their intermediary parameters.

Hence, being Scenario 2 characterized by the highest Priority Index, it qualifies as the most advantageous one for the Infrastructure Manager,

Table 9
Intervention Scenarios priority ranking

α_{int} Ranking	Scenario	Time horizon	$Cost_{int}$	$P_{F,forecast}$	$P_{F,post-int}$	α_{int} [life/€]
1st	Scenario 2: repair the crack and replace the joints	5 y	45.000 €	$3,10 \cdot 10^{-5}$	$1,98 \cdot 10^{-5}$	$2,49 \cdot 10^{-10}$
2nd	Scenario 3: replace the bridge	5 y	600.000 €	$3,10 \cdot 10^{-5}$	$1,00 \cdot 10^{-8}$	$5,17 \cdot 10^{-11}$
3rd	Scenario 1: replace the joints	5 y	30.000 €	$3,10 \cdot 10^{-5}$	$3,10 \cdot 10^{-5}$	$3,50 \cdot 10^{-13}$

suggesting that repairing the crack and replacing the joints is the most cost-effective intervention. Indeed, transversal cracks are the defect that affects the Probability of Failure the most and, by repairing it, the risk reduction is maximized whilst avoiding the considerable bridge replacement cost.

The example reported in this section shows the practical applicability of the method, which allows for a swift and simple assessment of the most cost-effective choice among the possible intervention scenarios.

6. Conclusions

In the face of global challenges posed by aging infrastructure, particularly the prevalence of structurally deficient bridges, the demand for efficient and rapid assessment and management tools is becoming increasingly urgent. This paper offers a pragmatic solution to the gap between the urgency of practical assessments and the comprehensive yet intricate evaluation of structural reliability for infrastructure management. Such gap is exacerbated considering the size of the asset stocks, the non-negligible computational complexity of formal structural reliability evaluations, the frequency of the analyses and the time-sensitive nature of intervention planning.

This work addresses the above problematic by introducing a Decision Support System (DSS) that provides infrastructure managers with an efficient tool for optimization. The DSS suggests optimal actions and resource allocation based on swift, simplified structural reliability evaluations. It requires minimal input parameters and relies on straightforward, readily available inventory data without compromising accuracy. The system ranks assets by urgency for intervention, supports both reactive and preventive maintenance approaches, and offers versatile, cost-effective management solutions for entire asset stocks. This automatically translates into an optimized resources allocation, cost-effective maintenance scheduling and, consequently, increased user safety.

Specifically, the proposed DSS introduces a simplified structural reliability model focused on the Overload and Degradation risk associated with aging bridges. The innovative methodology integrates (1) a degradation model grounded on Markov Chain processes capable of projecting the worsening of already present damage and (2) a Bayesian Theory-based model to predict the rise of new structural defects in the years to come. Crucially, the model acknowledges that even minor defects can worsen over time, sometimes even surpassing issues more severe at the time of the visual inspection. This proactive approach enables the projection of bridge structural reliability and its associate Forecasted Probability of Failure over specific time horizons, responding to the need of infrastructure managers for proper bridge stock upkeep.

The main findings of the article are embodied in a practical example simulating the every-day pursuit of an infrastructure manager. In particular, the example demonstrated the notable performance of the DSS tool whenever tasked with ranking three possible intervention scenarios in a 5 years' timeframe. The most cost-effective intervention among the three scenarios was calculated by means of the Priority Index, on the basis of a Forecasted Structural Reliability parameter and intervention cost. When comparing two intervention scenarios with similar costs (i.e., Scenario 1: replace the joints for 30.000 € vs. Scenario 2: repair the crack and replace the joints for 45.000 €) the DSS assigns higher priority to the scenario with the highest Risk reduction (Scenario 2), despite the slight cost surplus. Similarly, when comparing the

efficiency of Scenario 1 (risk reduction close to zero), against the one of an expensive bridge reconstruction (Scenario 3, 600.000 €), the DSS still prioritizes the Scenario which embodies the best cost-investment in terms of future risk (Scenario 3). The analysis, other than demonstrating the applicability of the DSS, highlights the important role of predicting future damage deterioration for present decision-making.

While the methodology simplifies structural reliability evaluations to prioritize interventions efficiently, several assumptions were made to render the model fast and user-friendly. These include treating different civil engineering risks—structural, seismic, hydraulic, and landslides—as uncorrelated. Additionally, the accuracy of the model's projections relies heavily on Correlation Probabilities (requiring extensive historical data for proper calibration) and on visual inspections, which can introduce subjective uncertainties.

Future research is addressed to further refining the model by addressing the aforementioned criticalities and incorporating several key improvements. The authors are currently developing models for intervention effects on risk and intervention costs to achieve a comprehensive Priority Index for decision-making. Furthermore, the authors are working on integrating additional major risk factors pertinent to civil engineering structures, such as seismic, hydraulic, and landslides risks. An Exposure model is also being developed, integrating critical data related to social, political, economic, and environmental impacts. Addressing these key areas of improvement can enhance the effectiveness and applicability of the proposed methodology, ultimately contributing to more robust and efficient bridge management practices.

CRedit authorship contribution statement

Francesca Brighenti: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Mattia Francesco Bado:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Francesco Romeo:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Daniele Zonta:** Resources, Project administration, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Data availability

No data was used for the research described in the article.

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