UNIVERSITY
OF TRENTO

DEPARTMENT OF INFORMATION AND COMMUNICATION TECHNOLOGY

38050 Povo – Trento (Italy), Via Sommarive 14
http://www.dit.unitn.it

WIDEBAND COMMUNICATION SYSTEM SENSITIVITY TO QUANTIZAZTION NOISE

Antonio Moschitta, Dario Petri

June 2004

Technical Report # DIT-04-056

Wideband Communication System Sensitivity to Quantization Noise

Antonio Moschitta, Dario Petri
Dipartimento di Ingegneria Elettronica e dell'Informazione, Università degli Studi di Perugia
Via G. Duranti, 93, 06125 Perugia, Italy
Phone: (+39) 0755853634 Fax: (+39) 0755853654 e-mail: moschitta@diei.unipg.it

***Abstract* – The performances of A/D converters are usually characterized in their granular region, by adopting amplitude limited sine-wave testing signals. However, some applications, like digital telecommunication systems, often require the conversion of signals which noticeably differ from sine-waves and may introduce ADC overloading phenomena. In this paper, the A/D conversion of Gaussian distributed signal is investigated, both for PCM and Sigma-Delta converters. Then, the effects of quantization noise upon the behavior of a Digital Communication System are considered. By extending previous results, a model is introduced, which describes the BER performance of an OFDM system also in presence of overloading quantization noise, both for PCM and Sigma-Delta A/D converters.**

***Keywords* – A/D converters, Testing, Direct Digital Modulation, Orthogonal Frequency Division Multiplexing.**

I. INTRODUCTION

Most of the commonly used ADC characterizing procedures involve the usage of sine wave stimuli, whose amplitude range does not usually saturate the converter under test [1]. However, some application fields may require the conversion of signals whose properties noticeably differ from the sine wave ones and saturate the ADCs. An example are Digital Communication Systems (DCSs) like Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA), which generate Gaussian distributed modulated waveforms [2][3]. Such systems are usually implemented by means of Direct Digital Modulation (DDM), a technique that requires the A/D conversion of the modulated waveforms in the receiver [4]. Moreover, in order to minimize the quantization noise power, the ADCs dynamic range must be properly matched to the input signals, and overloading noise is usually present. In such a case, the test of ADCs by means of Gaussian stimuli may provide more useful results than the traditional sine wave granular noise test.

This paper is focused on the effects of PCM and Sigma-Delta ($\Sigma\Delta$) conversion on the overall performance of an OFDM system. At first, the ADC performance is evaluated in terms of Signal to Noise and Distortion Ratio (*SINAD*). At this regard, $\Sigma\Delta$ converters look as a promising choice for DDM implementation, because they achieve a very high *SINAD* by combining noise shaping with oversampling [5]. Moreover the feedback topology renders $\Sigma\Delta$ converters robust to the internal ADC non-linearity. In particular, $\Sigma\Delta$ conversion with low oversampling ratio (*OSR*) is considered. In fact, when wideband DCSs are considered, like Terrestrial Digital Video Broadcasting (DVB-T) or Universal Mobile Telecommunication System (UMTS), a large *OSR* may require an exceedingly high sampling rate [2],[3].

Section III of this paper analyzes also the effects of quantization noise upon the OFDM *BER* performance. In fact, while *SINAD* conveniently describes the behavior of a standalone ADC, the performance of a DCS is better described by the bit error rate (*BER*), which depends on the interaction between the receiver, the useful signal, and the noise sources. In order to analyze the OFDM *BER* performance as a function of quantization noise, a linear model, based upon the granular noise approximation, has been presented in [4] for both PCM and $\Sigma\Delta$converters. In this paper, the model is extended to keep into account a moderate amount of ADC overloading noise, thus providing a reliable estimate of system *BER* performance in a practical situation. Such a development is relevant, because, in a real environment, the Signal to Noise Ratio (*SNR*) at the input of a DCS receiver may change in time due to fading phenomena [6]. Finally, the range of validity and the limits of the presented model are also discussed.

II. THE SIMULATED OFDM SYSTEM

OFDM is a multicarrier technique [7], adopted for various DCS standards, like DVB-T, Digital Audio Broadcasting (DAB), and Asynchronous Digital Subscriber Line (ADSL), where it is known as Digital Multi Tone (DMT) [8],[9]. Multicarrier systems split data among a large number of carriers, in order to transmit at a very low symbol rate. Coupled with the channel coding [2], such a technique is robust to channel impairments, like multipath and fading. The OFDM modulation is obtained by applying an Inverse Fast Fourier Transform to a sequence of QAM phasors, followed by a D/A conversion [2]. The receiver, after an A/D conversion, digitally demodulates the incoming signal throughout a Fast Fourier Transform (FFT). According to the Central Limit Theorem, an OFDM signal may be modelled as a Gaussian distributed stochastic process, and its power spectrum tends to be brick-wall shaped.

Fig.1 shows the layout of the OFDM system considered in this paper. It employs 2048 QAM carriers, of which only 1705, the closest to the signal center frequency, are active [2],[4]. Such a solution, common in practical systems, reduces the interference on adjacent channels. The transmitter output is a band-pass modulated signal, centered around the $\pi/2$ digital frequency, and up-sampled by a factor *OSR*. The next block models an Additive White Gaussian Noise (AWGN) channel, whose parameters may be tuned to achieve an arbitrary Signal to channel Noise Ratio (*SNR*).

The receiver may operate a b bit PCM or a $\Sigma\Delta$ band-pass A/D conversion prior to digital processing [4]. In particular, a single bit band-pass Sigma-Delta converter, based on a first order loop, has been simulated. The ADC output is then converted to the base-band and down-sampled by a factor *OSR*. Finally, after the FFT demodulation, the *BER* is estimated by comparing the received symbols with the transmitted ones.

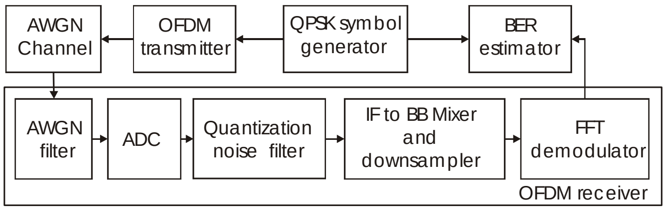

Fig. 1: OFDM system block scheme

III. ANALYSIS RESULTS

The OFDM system described in the previous section has been analyzed by means of both simulations and theoretical modelling. The following subsections are focused on PCM and Sigma-Delta A/D conversion respectively.

3-1. PCM converters

The behavior of a *b* bit PCM, fed with a Gaussian distributed signal, has been considered, in order to find the optimal matching between the ADC Full Scale *FS* and the input signal standard deviation σ_{IN}. The results of this analysis, performed by carrying on software simulations, are shown in Fig.2, where *SINAD* is reported as a function of the ratio σ_{IN}/FS for various converter resolutions. Each curve has a maximum, resulting from a trade-off between granular and overload errors. In fact, granular error power grows with *FS*, while the overload one grows with σ_{IN}. A theoretical exact model, derived in [9], shows a very good agreement with PCM simulation results.

The PCM has been considered also as a part of the OFDM receiver, and the overall *BER* performance has been analyzed in presence of both channel and quantization noises. A linear model has been introduced in [4] to estimate the *BER* when overload effects are negligible. This model, derived under the assumption that the noise at the output of the FFT demodulator is Gaussian distributed, shows a very good agreement with simulation results when the PCM operates in its granular region. However, when optimal PCM dynamic range is used, the linear model does not keep into account overloading contributions to the overall quantization noise power, thus underestimating the *BER*.

The granular noise model has then been generalized, by assuming that a limited amount of overload does not affect the statistical properties of noise at the output of the FFT demodulator, thus obtaining, (see the appendix):

$$BER = \frac{1}{2} erfc\left(\frac{N_A n_B}{N} \left(\frac{1}{SNR} + A \right) \right), \qquad (1)$$

where $A = \dfrac{(1+1/SNR)}{SINAD(\sigma_{IN}/FS)}$, N is the total number of OFDM carriers, N_A is the number of active carriers, n_B is the number of bits conveyed by a QAM symbol, and σ_{IN}/FS is referred to the PCM input signal, that is the OFDM signal corrupted by channel noise. It should be also noticed that, for PCM converters, *SINAD* can be calculated by means of theoretical analysis [10].

Fig. 3 shows *BER* vs. *SNR* simulation results and theoretical estimations, obtained by using the optimal matching between the PCM *FS* and the dynamic range of the ADC input signal, according to Fig. 2. It can be seen that the extended model shows a very good agreement with simulations.

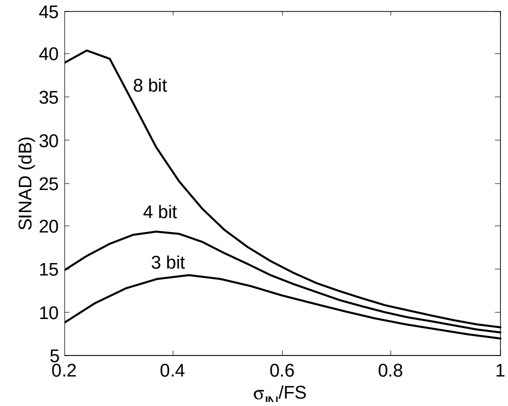

Fig. 2: *SINAD* vs. σ_{IN}/FS, PCM converter

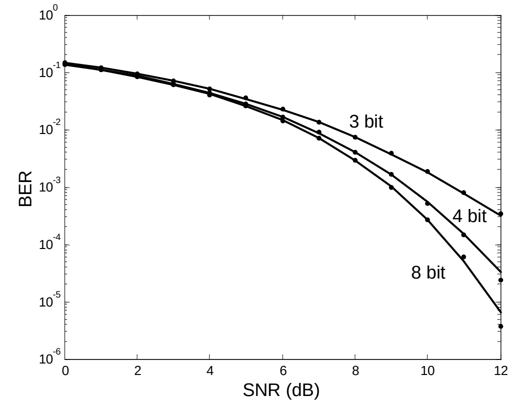

Fig. 3: PCM dynamic range optimized to input signal power. The continuous line represents the analytic model (1), while the dots represent simulation results.

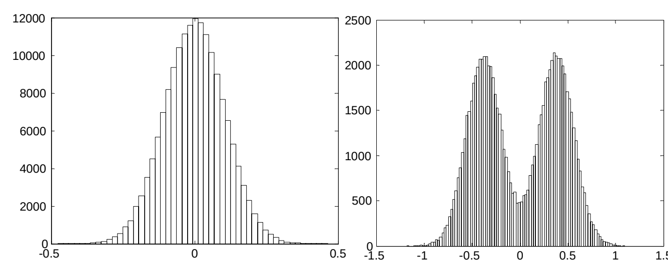

Fig. 4: Noise distribution (real part) after the FFT demodulator, 3-bit PCM, σ_{IN}/FS=0.1 (a) and σ_{IN}/FS=1 (b), obtained from a record of 50 OFDM symbols, made of 2048 samples each. The error samples have been normalized to the QAM symbol amplitude. The imaginary noise component shows the same distribution.

The accuracy of (1) has been evaluated by comparing the predicted *BER* with simulation results, for various levels of quantization noise. In particular, the ratio between the predicted *BER* and the simulation estimate has been evaluated as a function of σ_{IN}/FS, for an OFDM system operating at *SNR*=10 dB. Under such condition of practical interest, quantization noise is not negligible, with respect to channel noise. In particular, for a 3 bit PCM, the estimation error is about 0 dB when σ_{IN}/FS is lower than 0.6. For higher values of σ_{IN}/FS, the theoretical model overestimates the *BER*, leading to a 2 dB error for σ_{IN}/FS=1. Thus, the model provides accurate results for any practical situation. The loss of accuracy is due to the statistical properties of quantization noise. In fact, when overload is introduced, the PCM quantization noise tends to a multi-modal distribution rather than the uniform one of granular noise. Such a noise propagates throughout the OFDM receiver, leading to a non-Gaussian distributed noise at the output of the FFT demodulation block, despite the presence of the Gaussian distributed channel noise. As shown in Fig.4, this behavior is more evident for increasing overloading levels.

3-2. Sigma-Delta converters

Fig.6 shows the *SINAD* optimization results obtained for the single bit band-pass Sigma-Delta converter. This time, σ_{IN}/FS is the ratio between the ADC input standard deviation and the full scale of the ΣΔ inner PCM. By comparing Fig.5 and Fig.2, it can be observed that respect to PCM converters, ΣΔ *SINAD* is maximized for a lower value of σ_{IN}/FS. That's related to the feedback nature of the ΣΔ, which may introduce overload errors even for amplitude limited signals.

As shown in the appendix, the PCM *BER* model can be extended to the ΣΔ ADC, by keeping into account noise shaping effects. By assuming that the ΣΔ quantization noise is locally white with respect to individual carriers, the *BER* of the *i-th* OFDM carrier may be expressed by

$$BER_i = \frac{1}{2} erfc\left(\left(\frac{N_A n_B}{N} \left(\frac{1}{SNR} + B_i \right) \right)^{-\frac{1}{2}} \right), \qquad (3)$$

where $B_i = A \cdot G \cdot |H_N(\omega_i)|^2$. $|H_N(\omega)|$ is the ΣΔ noise transfer function, ω_i is the frequency the *i-th* OFDM carrier, and G is the integral of $|H_N(\omega)|^2$ over the signal bandwidth. The overall *BER* is then obtained by averaging (3) across the OFDM signal bandwidth, that is:

$$BER = \frac{1}{N_A} \sum_{i=0}^{N_A-1} BER_i \qquad (4)$$

Eq. (3) implies that for a noise shaping converter the *BER* performance depends not only on the *SINAD*, but also on the ADC noise transfer function. Consequently, when comparing A/D converters for DCS applications, *SINAD* may not be the best characterizing parameter.

Fig.6 reports *BER* as a function of *SNR*, for a Sigma-Delta converter whose inner PCM *FS* is optimally matched to the incoming signal power. Due to the topology of Sigma-Delta converters, which uses a non-linear device into a feedback loop, the quantization noise power in presence of overload cannot be easily derived by means of a theoretical analysis. Consequently, the *SINAD* simulation results plotted in fig.6 have been used in (3). As for PCM, the extended model provides an accurate *BER* estimation.

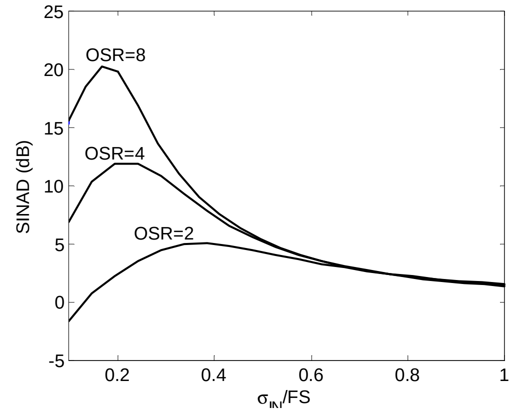

Fig. 5: *SINAD* vs. σ_{IN}/FS, Sigma-Delta converter

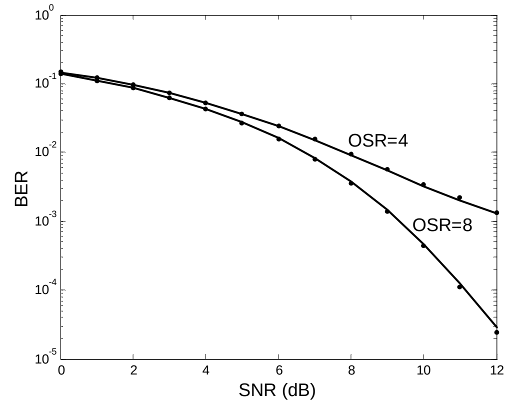

Fig. 6: Sigma-Delta dynamic range optimized to input signal power. The continuous line represents the analytic model (3), while the dots represent simulation results.

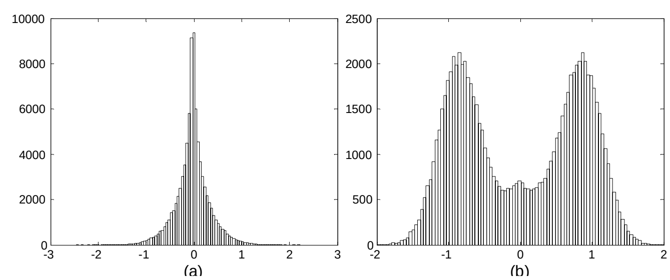

Fig. 7: Noise distribution (real part) after the FFT demodulator, single-bit Sigma-Delta, σ_{IN}/FS=0.1 (a) and σ_{IN}/FS=1 (b), obtained from a record of 50 OFDM symbols, made of 2048 samples each. The error samples have been normalized to the QAM symbol amplitude. The imaginary noise component shows the same distributions.

The accuracy of the extended Sigma-Delta *BER* model has been also evaluated, for an OFDM system operating at *SNR*=10 dB. In particular, it has been verified that the model does not provides reliable results for *OSRs* lower than 4. For such a value, the estimation error is negligible for the optimal value of σ_{IN}/FS, while it quickly grows for higher values of σ_{IN}/FS. Such a behavior is due to the higher sensitivity of ΣΔ converters to overloading effects, with respect to PCM converters. It is worth of notice that when overloading noise is present, the Sigma-Delta model tends to underestimate the *BER*. This is due to the Sigma-Delta quantization noise, whose properties, as shown in Fig.4, are noticeably different from the PCM ones. Simulations show that the ratio between the predicted *BER* and the *BER* obtained by means of simulations is about –5 dB for σ_{IN}/FS=1. Thus, also the Sigma-Delta *BER* model provides accurate results for practical applications.

IV. CONCLUSIONS

The A/D conversion of Gaussian distributed signals has been investigated, and the effects of PCM and Sigma-Delta conversion upon the performances of an OFDM DCS have been analyzed. A model has been presented, which conveniently describes OFDM *BER* behavior when overload effects are introduced. It has been shown that *SINAD* may not adequately characterize the behavior of a noise shaping ADC, when applied to an OFDM DCS. The model accuracy has been evaluated, with respect to both PCM and Sigma-Delta A/D conversion. Further development of the research activity are the extension of the analysis to other DCSs and to other ADC non-idealities.

APPENDIX: DERIVATION OF THE ANALITICAL MODEL FOR THE OFDM *BER*

As the carriers of an OFDM signal are orthogonal to each other, the overall *BER* is expressed by

$$BER = \frac{1}{N_A}\sum_{i=0}^{N_A-1} BER_i \tag{A1}$$

where BER_i is the *BER* of the *i-th* carrier [7]. According to [6] and [7], the *BER* of a QPSK modulated carrier, operating on an AWGN channel, may be described by

$$BER = \frac{1}{2}erfc\left(\sqrt{\frac{E_B}{\eta_0}}\right), \tag{A2}$$

where E_B is the energy per bit, η_0 is the level of the AWGN spectrum in the useful signal bandwidth, and *erfc*() is the complementary error function. Eq. (A2) may be used to describe the *BER* performance of the OFDM carriers, provided that at the output of the FFT demodulator the overall noise is white and Gaussian distributed. Simulations show that such condition is verified, as far as deep quantizer overload is avoided. This may be considered a consequence of the Central Limit Theorem, because in the receiver the noise is repeatedly filtered.

In order to derive the *BER* model, E_B and η_0 have been evaluated at the output of the quantization noise rejection filter, and their propagation to the FFT demodulator has been analyzed [6],[7]. In particular, E_B is the ratio between the energy of an OFDM symbol and the number of conveyed bits, which can be expressed by

$$E_B = \frac{\sigma^2_{OFDM}\cdot N\cdot OSR}{N_A\cdot n_B}, \tag{A3}$$

where σ^2_{OFDM} and $N{\cdot}OSR$ are the power of the band-pass OFDM signal and the number of samples in an OFDM symbol respectively. N_A is the number of active carriers and n_B, which for a QPSK modulation scheme equals 2, is the number of bit transmitted by a single carrier during an OFDM symbol. By assuming that channel noise and quantization noise are uncorrelated, the noise level η_0 may be expressed by $\eta_0 = \eta_{CH} + \eta_Q$, where η_{CH} is the channel noise level, and η_Q is the quantization noise level. The channel noise level is described by

$$\eta_{CH} = \sigma^2_{OFDM}\cdot OSR/SNR \tag{A4}$$

For a flat noise A/D converter, such as a PCM in granular region, η_Q is described by

$$\eta_Q = \frac{\sigma^2_{IN}}{SINAD(\sigma_{IN}/FS)}, \tag{A5}$$

and is equal for all of the OFDM carriers. Eq. (A5) can be obtained by expressing the ADC quantization noise power as the integral across the [-π,π] bandwidth of the noise power spectral density and in terms of *SINAD*, which is reported a function of the σ_{IN}/FS ratio.
The ADC input signal is the sum of the useful OFDM signal and the AWGN. As both signals are white, Gaussian and uncorrelated, the input to the ADC is still a white Gaussian process, and may be expressed in terms of OFDM signal power only, by using the *SNR* definition. Consequently, (A5) may be rewritten as follows:

$$\eta_Q = \frac{(1+1/SNR)\sigma^2_{OFDM}}{SINAD(\sigma_{IN}/FS)} \tag{A6}$$

Eq. (A6) shows that the quantization noise and the channel noise are uncorrelated only for high *SNRs*. It should be also noticed that η_Q is minimized when σ_{IN}/FS equals the optimal values shown in Fig.2.

By analyzing the signal propagation throughout the receiver, the energy per bit E and the noise level η at the input of the FFT demodulator are expressed by $E=E_B/OSR$ and $\eta=\eta_0/OSR$. By substituting such results in (A2) and in (A1), and keeping in mind that for PCM converters all carriers show the same *BER*, equation (2) is obtained.
The presented model can be extended to noise shaping converters, by keeping into account the noise transfer function. In particular, the unfiltered noise level η_U can be expressed as

$$\eta_U = \frac{\sigma^2_{IN}}{G\cdot SINAD(\sigma_{IN}/FS)}, \tag{A7}$$

where $G = \int_{BW} |H_N(\omega)|^2\, d\omega$, and *BW* is the useful signal bandwidth. By assuming that the ADC noise transfer function $H_N(\omega)$ is approximately constant across the bandwidth of each OFDM carrier, Eq. (A6) is then replaced by the following

$$\eta_{Qi} = | H_N(\omega_i) |^2 \frac{(1+1/SNR)\sigma^2_{OFDM}}{G \cdot SINAD(\sigma_{IN}/FS)}, \qquad \text{(A8)}$$

Eq. (A7) expresses the quantization noise level for each OFDM carrier, whose center frequency is ω_i. In particular, for the ΣΔ ADC considered in this paper, $|H_N(\omega)|^2=2(1+\cos(2\omega))$.

References

[1] "IEEE Standard for terminology and test methods for Analog to Digital Converters," *IEEE std*. 1241-2000.

[2] ETS 300 744, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital Terrestrial television (DVB-T)", available on the Internet at www.etsi.org.

[3] T. Ojampera and R.Prasad, "An overview of air interface multiple access for IMT-2000/UMTS," *IEEE Comm.. Mag.*, no. 9, pp 82-86, 91-5, Sept. 1998.

[4] A. Moschitta, D. Petri, "Analysis of Bandpass Sigma-Delta conversion in OFDM Systems, " proc. of the 8th IEEE Int. Conf. on Electronics, Circuits and Systems, Malta, 2-5 September 2001.

[5] S. R. Norsworthy, R. Schreier, G. Temes, *Delta-Sigma Data Converters*, IEEE press, 1997.

[6] J. G. Proakis, *Digital Communications*, Mc. Graw Hill, 1983.

[7] I. Kalet, "The Multitone Channel," *IEEE Trans. Communications*, vol. 37, no. 2, February 1989.

[8] ETS 300 401, "Radio Broadcasting systems; Digital audio Broadcasting (DAB) to mobile, portable and fixed receivers" available on the Internet at www.etsi.org.

[9] ANSI T1.413, "Asymmetric Digital Subscriber Line (ADSL) Metallic Interface," 1995.

[10] F.H. Irons, K.J.Riley, D. M. Hummels, G. A. Friel, "The Noise Power Ratio," *IEEE Trans. Instrumentation and Measurement*, vol. 49, no. 3, June 1999.