

Physically based machine learning for the multi-objective optimal design of partially recycled materials: The case study of rubber resources in the automotive industry

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ABSTRACT

This study presents a novel micromechanical-optimization framework that extends the traditional Halpin-Tsai model beyond mechanical properties to incorporate cost and environmental impact indicators, enabling quantitative sustainability assessment of recycled-additive mixtures. The extended multi-phase formulation provides a physically interpretable link between the intrinsic characteristics of recycled matrices, reinforcing additives, and life-cycle and market metrics, while remaining analytically tractable. Coupled with a physics-informed multi-objective optimization algorithm, the framework identifies optimal additive strategies that recover target properties of recycled materials under economic and regulatory constraints. A case study on end-of-life tire rubber demonstrates the model's predictive capability and flexibility for practical calibration. The proposed approach offers a transparent and scalable tool for circular material design and decision-making, with potential applicability to other recycled material systems enhanced with functional additives, such as compatibilizers in recycled polymers, alloying elements in metal scrap, or mineral/polymeric admixtures in construction waste.

1. Introduction

Recycling waste materials is essential for promoting sustainability and environmental protection. The European Commission's "4R" framework, Reduce, Reuse, Recycle, and Recover, aims to optimize resource use and to minimize environmental harm by reintegrating recycled materials into production cycles, fostering a sustainable economic model (Kirchherr et al., 2017; Leong et al., 2023). Recycled waste resources span a diverse spectrum of material classes, including construction and demolition debris (e.g. crushed concrete and masonry), polymers and plastics (thermoplastics, thermosets), metals (e.g. ferrous and non-ferrous scrap), and rubber wastes, notably from the tire industry. The increasing number of vehicles has led to a growing demand for tires, and once tires become unsafe for use, they are classified as end-of-life tires (ELTs) (Sienkiewicz et al., 2012). Although proper recycling can transform ELTs into valuable materials (Abbas-Abadi et al., 2022; Maga et al., 2023; Wu et al., 2024; Chittella et al., 2021), primarily rubber (Quadrini et al., 2019; Gugliemotti et al., 2012), they pose environmental risks such as landfill overflow, fire hazards, and pest breeding (Downard et al., 2015; Valentini and Pegoretti, 2022).

Recycled materials often exhibit inferior properties compared to their virgin counterparts, which limits their ability to fully substitute primary raw materials. Each waste source category involves distinct technical challenges during purification and reintegration into new products. For example, recycled concrete aggregates frequently exhibit higher porosity, lower strength, and weaker interfacial bonding (Wu et al., 2025); likewise, recycled plastics suffer from chain scission, contamination, and heterogeneity during mechanical recycling, reducing molecular weight and mechanical strength (Lamtai et al., 2023). In the case of metals, although recycling is well established, complications such as alloy heterogeneity, contamination with trace elements, and thermodynamic limits on separation restrict the attainment of virgin-equivalent purity (Ferron and Henry, 2015), while modern electronic wastes further complicate the separation of complex metal mixtures (Zhang and Xu, 2016). Rubber crumbs from ELTs may suffer from poor mechanical performance when mixed into composites (Kabakçi et al., 2022a) or weak adhesion in cementitious matrices (Haigh, 2023; Singaravel et al., 2024). Therefore, while recycling offers clear environmental and

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resource-saving benefits, the material-level quality penalty remains a central barrier to broad adoption in high-performance applications.

Ideally, recycling of waste resources aims to *recover the lost properties* of primary raw materials during their transformation into products, allowing them to be reused in the same primary applications. However, the discrepancy between the recycled resources and their virgin counterparts necessitates a careful recycling strategy. A promising path is the incorporation of reinforcing additives to recover the performance of recycled-derived materials. In the case of construction and demolition waste, e.g., recycled aggregate concrete, additives such as nano-silica, silica fume, fly ash, or ceramic powder can be used to refine the mechanical performance, to fill pores, and to reduce water absorption (Younis, 2018; Rao et al., 2024). With recycled plastics, additives and compatibilizers help improve quality, approaching the virgin plastic (Aldahdooh, 2025; Al Rashid and Koç, 2023; Ghosh, 2021; Soni et al., 2022; Romeiro et al., 2024). For metals, alloying and process-level additives can mitigate impurity effects, reduce harmful segregation, and restore mechanical properties (Raabe et al., 2019; Kaliyannan et al., 2024; Mourad et al., 2023). For the waste rubber from ELTs (Wiśniewska et al., 2023), the untreated recycled rubber often has dramatically inferior properties compared to virgin rubber. Vulcanization, which enhances tire properties by creating cross-links in rubber, complicates recycling as it makes reprocessing difficult (Michele Battista and Ramorino, 2021). To address this, devulcanization methods are used to restore the rubber's original form, enabling its reuse in new compounds (Asaro et al., 2018; Imbernon and Norvez, 2016; Bockstal et al., 2019). Mixing devulcanized and virgin rubber offers a cost-effective solution, but the property differences limit its use in high-volume, demanding applications. To meet the required standards, the incorporation of reinforcing additives is necessary. This has led to the development of rubber-based composites, where (nano) particles enhance properties of recycled rubber from ELTs to approach the performance of virgin materials (Feldman, 2012; He et al., 2022; Eugene B. Caldona and Advincula, 2017; Danafar and Kalantari, 2018; Sayfo et al., 2023; Kelnar et al., 2010; Kabakçi et al., 2022b; Jastrzębska and Tokarz, 2021; de Sousa et al., 2015). Despite the advantage of reinforcing additives, they not only increase material and processing cost, but may also add environmental burdens (energy for additive manufacture, potential toxicity, additional processing steps), which could partially undermine the twin goals of recycling: cost-reduction and environmental impact mitigation. Therefore, an approach to determine the optimal amount of additive is essential.

An essential aspect of improving recycled material systems lies in the optimization of processes across the entire value chain, from collection and sorting, through recycling, to integration into new products. The literature demonstrates that optimization in recycling systems, and, more broadly, in circular economy frameworks, is inherently multi-objective (MO), as it involves concurrent trade-offs among cost, environmental impact (e.g., energy consumption, emissions, LCA metrics), logistics and collection efficiency, technical performance (e.g., durability, purity, mechanical response), and even social or regulatory constraints (Chang, 2008; Dobrotă et al., 2020; Ghaleh et al., 2025). Existing research has explored MO optimization at different hierarchical levels within recycling systems. At the logistical and supply-chain level, MO algorithms have been utilized to enhance overall efficiency in the regional and global management of waste flows, such as ELTs (Aydin et al., 2024; Ghasemzadeh et al., 2021; Kiani Mavi et al., 2023; Castañeda-Rodríguez and Espinoza Pérez, 2025; Hu et al., 2024) and photovoltaic panels (Nili et al., 2021). At the environmental level, LCA-based indicators have been integrated into MO frameworks to optimize the distribution and treatment of ELTs across European recycling systems (Duval et al., 2024). MO approaches have also been employed in disassembly line balancing (DLB) problems in end-of-life product recycling and remanufacturing, addressing the simultaneous challenges of cost, performance, and uncertainty across interconnected DLB supply chains (He et al., 2024). At the material-design level, MO

algorithms have been increasingly applied to optimize the composition and performance of heterogeneous mixtures (e.g., composites) partially or fully derived from recycled resources (Liu et al., 2017). For instance, recycled aggregate concrete (Chen et al., 2024; Lu, 2025), waste polymer-filled concrete (Kantasiri et al., 2024; Islam et al., 2022; Ali et al., 2023; Tanhadoust et al., 2023; Lee and Wong, 2023; Tang et al., 2023; Giri et al., 2023), waste rubber-modified asphalt (Liu et al., 2018; Zhou et al., 2025; You et al., 2024), and rubber-sand composite blocks (Doddamani et al., 2024) have employed MO optimization approaches to selectively enhance specific target properties. However, the simultaneous integration of environmental (LCA) and economic objectives within these MO frameworks remains largely unexplored.

In recent years, the integration of machine learning (ML) and MO optimization has emerged as a promising paradigm for advancing sustainable material design and recycling strategies. While traditional MO optimization studies in recycling primarily relied on empirical or response surface models, the incorporation of ML enables data-driven prediction and optimization over high-dimensional, nonlinear design spaces (Zhang et al., 2020; Dabbaghi et al., 2021; Onyelowe et al., 2022). Most existing frameworks have focused on the material-level optimization of concrete, polymer, and rubber-based composites, where ML models serve as predictive surrogates for mechanical or environmental performance, guiding MO algorithms to identify Pareto-optimal mixtures (Chen et al., 2025; Fan et al., 2024; Wang et al., 2024; Amiri et al., 2022). For instance, ML-MO frameworks have been deployed to optimize recycled aggregate concretes, lightweight concretes, and coal waste concretes with competing objectives such as compressive strength, cost, and life cycle indicators (Dabbaghi et al., 2021; Onyelowe et al., 2022; Fan et al., 2024; Wang et al., 2024). Similar hybrid strategies have been extended to other waste-derived materials, including rubber-filled composites (Chen et al., 2025) and pyrolyzed tire products, where ML models have been used to predict oil or limonene yields and guide process optimization (Qi et al., 2023, 2024). Beyond mineral and polymer-based systems, data-driven MO optimization has also been utilized in biological recycling, e.g., enzyme selection for plastic depolymerization, balancing catalytic efficiency and substrate compatibility (Akgüller and Balci, 2025). Despite their growing sophistication, these hybrid ML-MO approaches remain largely localized at the component or process level rather than addressing system-wide trade-offs across the circular value chain. A few review and methodological works have emphasized this gap, highlighting the need for physics-informed or knowledge-constrained ML frameworks to ensure interpretability, scalability, and consistency with sustainability goals (Zou et al., 2025; Zhao et al., 2024; Zhang et al., 2024). Indeed, without explicit coupling to LCA or techno-economic criteria, current implementations risk overfitting to performance-driven objectives and underrepresenting the holistic sustainability of recycling systems. Bridging this gap, by embedding domain constraints and LCA-derived objectives directly into ML-MO frameworks, represents a crucial step toward genuinely intelligent and circular materials design.

In summary, MO optimization and ML have been increasingly applied to recycling and circular economy systems, while most studies address these tools in isolation or within limited layers of the recycling chain, focusing either on supply chain efficiency, environmental performance, or material-level property enhancement. However, the integration of physics-informed ML with MO optimization specifically targeting the property recovery of recycled material systems via reinforcing additives remains largely unexplored. Building upon the identified research gaps, this study introduces a generalized, physics-informed framework for optimizing the recovery and valorization of recycled resources through the strategic addition of reinforcing additives. The core contribution lies in a semi-empirical, micromechanics-based model derived from a multi-phase extension of the Halpin-Tsai (H-T) formulation, where physically interpretable fitting parameters are calibrated using data from experiments or simulations. Unlike conventional micromechanical models that solely predict properties of mixtures,

the proposed formulation systematically incorporates the influence of additives on multiple objective functions, including performance, production cost, and environmental impact. The extended H-T framework can capture non-linear trade-offs among objective functions, beyond linear mixture rules. As a demonstration, the proposed methodology is applied to recycled rubber derived from ELTs reinforced with selected additives. The model-guided optimization illustrates how an appropriate balance between performance and cost can be achieved in practical automotive applications. This general framework can thus be readily adapted to other recycled systems, such as waste plastics, metals, and construction composites, where additive-based reinforcement strategies are employed to restore functional and sustainability-related properties. Overall, the framework provides a transferable, data-informed methodology for designing recycled materials that achieve a balanced compromise among functional performance, economic feasibility, and environmental sustainability.

2. Methods

2.1. Mathematical model for recovery of recycled materials

Here, we formulate the recovery process for recycled materials by incorporating additives. Consider a virgin material (VM) whose number of properties of interest is N_p . Recycling this virgin material degrades its properties as:

$$P'_i = \lambda_i P_i^{(0)}, \quad i = 1, \dots, N_p, \quad (1)$$

where $P_i^{(0)}$ and P'_i are the i th properties of the virgin and the recycled material (RM), respectively, and λ_i is the recycling efficiency which reflects the effect of recycling on i th properties. $\lambda_i = 1$ corresponds to the ideal case of no decrease in the properties. Here, the concept of *recovery* of an RM property is assumed as any post-treatment on the RM to decrease its deviation compared to the VM. As a solution, Additives (ADs) can be introduced in order to recover the lost portion of properties due to recycling to make a recovered recycled material, $\overline{\text{RM}}$. Considering a total number of N_a additives, the relative property of the j th AD with respect to the RM is defined as:

$$r_{ij} = \frac{P_{ij}}{P'_i}, \quad j = 1, \dots, N_a, \quad (2)$$

where r_{ij} is the ratio of the i th property of j th AD, P_{ij} , to the i th property of the recycled material, P'_i . Note that P_{ij} is the effective property of ADs inside the RM and represents all its effects such as introducing defects and interfaces. The ratio of i th property of $\overline{\text{RM}}$, denoted by $\bar{P}_i$, to the corresponding i th property of VM is defined as recovery factor,

$$\bar{R}_i = \frac{\bar{P}_i}{P_i^{(0)}}, \quad i = 1, \dots, N_p. \quad (3)$$

The i th recovery factor, $\bar{R}_i$, is a function, F_i , of the recycling efficiency, λ_i , the vector of the volume fractions of ADs, $\{v\}_{N_a \times 1}$, the packing configuration vector, $\{\xi_{ij}\}_{N_a \times 1}$, and the vector of ADs' property ratios for i th property, $\{r_{ij}\}_{N_a \times 1}$, as:

$$\bar{R}_i = F_i(\lambda_i, \{v\}, \{\xi_{ij}\}, \{r_{ij}\}) \quad (4)$$

The deviation in the recovery of the i th property from its virgin state, δ_i , is defined as,

$$\delta_i = |1 - \bar{R}_i|. \quad (5)$$

Accordingly, the overall deviation, $\bar{\delta}$, is defined as the weighted summation of the individual deviations:

$$\bar{\delta} = \sum_{i=1}^{N_p} w_i \delta_i. \quad (6)$$

The weights, $\sum w_i = 1$, are determined based on the importance of the corresponding deviation. By setting the weights of certain deviations

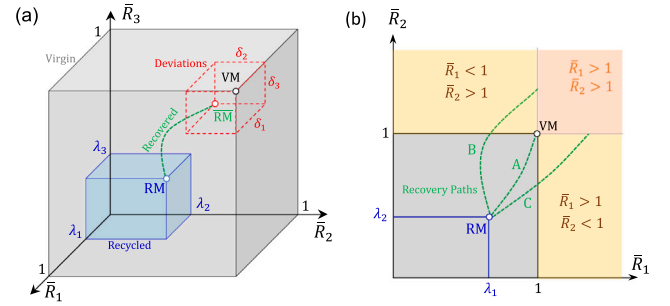


Fig. 1. Ashby plots of the recovery process. (a) The schematic representation of the virgin material (VM), the recycled material (RM), the recovered recycled material $\overline{\text{RM}}$, and the recovery paths in a 3D Ashby plot when $N_p = 3$. (b) The schematic representation of comparative sub-spaces and different recovery paths in a 2D Ashby plot when $N_p = 2$.

to zero, it is possible to exclude them from $\bar{\delta}$. Besides, where all the deviations are of the same priority, the weights are all equal to $1/N_p$. In case all the recovery factors are equal to one, full recovery happens and the RM coincides with the VR, corresponding to no individual and thus overall deviations, $\bar{\delta} = 0$.

To represent the relative status of RM and $\overline{\text{RM}}$ compared to the VM, an N_p -dimensional Ashby plot can be employed, where each axis reflects the relative status of each property in terms of the recovery factor, $\bar{R}_i$. For the VM, all recovery factors are equal to one, while the coordinates of the RM in such an Ashby plot are determined by its given recycling efficiencies, $(\lambda_1, \dots, \lambda_{N_p})$. The more efficient the recycling process, the smaller the disparity between the VM and the RM. Fig. 1(a) visualizes the schematic of the proposed Ashby plot in a three-dimensional Cartesian space, where the number of properties is limited to three, without sacrificing generality. As observed, incorporating ADs modifies the recovery factors through Eq. (4). Consequently, by adjusting the volumetric fraction, packing, and the relative properties of ADs as inputs to the recovery functions, RM can enhance its recovery by minimizing deviations along a recovery path approaching VM. The proposed concept of an N_p -dimensional Ashby plot is introduced as a conceptual framework for mapping multi-property recovery in recycled systems. In practice, when the number of considered properties exceeds three, direct graphical representation becomes impractical. To address this limitation, two complementary strategies can be adopted. First, the properties can be grouped into subsets according to their nature (e.g., mechanical, physical, or electrical), recovery mechanism, priority, or relevance to the target application. This allows for constructing multiple three-dimensional subplots, each representing a specific property category. Second, equivalent or aggregated property indices can be defined for correlated properties to reduce the dimensionality of the system, for instance, by computing averaged or root-mean-square values.

Generally, the comparative space of properties in the proposed Ashby plot can be divided into 2^{N_p} sub-spaces, considering that each recovery factor can be either smaller or larger than unity and there is only one sub-space where all recovery factors, $\bar{R}_i$, are less than unity. To increase transparency, Fig. 1(b) depicts how the division of space into these sub-spaces is accomplished in a two-dimensional Ashby plot. Besides, three schematic recovery paths are compared. In path A, the inputs of recovery functions are adjusted in a way that all properties simultaneously reach their virgin values, where the $\overline{\text{RM}}$ coincides with the VM. However, in paths B and C, the properties are fully recovered one after another. This implies that achieving complete recovery of all properties requires certain properties to surpass even those of the VM, recovery factors greater than one. The difference between *reinforcement*, i.e., $\bar{R}_i > 1$, and *recovery*, characterized by $\bar{R}_i \leq 1$, must be acknowledged. It is crucial to highlight that reinforcement may not

necessarily be beneficial, and it may even be deemed undesirable in certain scenarios.

The key to predicting the effect of ADs on RM, and subsequently achieving optimized recovery, lies in defining appropriate recovery functions, as introduced in Eq. (4). In the context of heterogeneous materials, micromechanical models provide reasonable estimations for the properties of material mixtures through closed-form formulations. Among these models, the semi-empirical Halpin-Tsai (H-T) equation (Afdl and Kardos, 1976) stands out due to its successful application in predicting a wide range of properties, such as Young's modulus, Poisson's ratio, shear modulus (Zare, 2016), hardness (Goyal et al., 2008), thermal and electrical conductivity (Zhu et al., 2024), and friction coefficients (Jalali and Pugno, 2023) of mixtures. This success is attributed to its inherent fitting parameter, ξ (Shokrieh and Moshrefzadeh-Sani, 2016). Here, the $(N_a + 1)$ -phase version of the H-T model is customized to be used as recovery functions for N_a additives within the RM as a matrix:

$$F_i = \lambda_i \frac{1 + \sum_{j=1}^{N_a} v_j \eta_{ij} \xi_{ij}}{1 - \sum_{j=1}^{N_a} v_j \eta_{ij}}, \quad (7a)$$

$$\eta_{ij} = \frac{r_{ij} - 1}{r_{ij} + \xi_{ij}}, \quad (7b)$$

where ξ_{ij} is the fitting parameter representing the effect of the j th AD on the i th property. In the most general case, there are $N_a \times N_p$ independent fitting parameters, which may require extensive experimental and/or simulation studies to be determined. By utilizing the relationship defined in Eq. (7), we propose a physically-informed machine learning calibration approach (Fazio et al., 2024; Henkes et al., 2022; Xiong et al., 2024) to reduce the effort needed to determine the appropriate ξ_{ij} values, particularly when multiple ADs interactively contribute to the recovery function, as illustrated schematically in Fig. 2.

The proposed model provides a versatile basis for estimating the effective properties of mixtures from the characteristics of their individual constituents. Because its fitting parameters are physically interpretable and can be calibrated to experimental or simulated data, the model is readily adaptable to a wide range of recycled material systems, such as concrete with mineral additives, polymer blends with compatibilizers, metallic scrap alloys, and rubber-based compounds, without requiring structural modification of the core equation.

As an extension of the H-T-based formulation developed for mechanical properties, a semi-empirical model is proposed to estimate the overall environmental impact of mixtures composed of recycled rubber and additives. The environmental indicator, denoted by the subscript $\oplus$, can represent any metric derived from a Life Cycle Assessment (LCA), such as the Global Warming Potential (GWP, kg CO₂-eq/kg), Cumulative Energy Demand (CED, MJ/kg), or Water Footprint (m³/kg). The model provides a means to approximate the environmental performance of a recycled material reinforced with N_a additives based on the environmental characteristics of its individual components, as follows:

$$F_{\oplus} = \lambda_{\oplus} \frac{1 + \sum_{j=1}^{N_a} v_j \eta_{\oplus j} \xi_{\oplus j}}{1 - \sum_{j=1}^{N_a} v_j \eta_{\oplus j}}, \quad (7c)$$

$$\eta_{\oplus j} = \frac{r_{\oplus j} - 1}{r_{\oplus j} + \xi_{\oplus j}}, \quad (7d)$$

where $F_{\oplus}$ is the environmental impact ratio of the recovered mixture to the virgin material (RM to VM), $\lambda_{\oplus}$ represents the environmental impact ratio of the recycled matrix to the virgin matrix, and $r_{\oplus j}$ denotes the environmental impact ratio of the j th AD to the RM. The parameter $\xi_{\oplus j}$, which controls the extent of the contribution of the j th additive to the overall environmental impact of the mixture, can be fitted to the data set of LCA outputs for different combinations of RM and ADs.

In the proposed framework, the physically-informed ML scheme is introduced as a hybrid calibration tool linking the analytical micromechanical relation (Eq. (7)) with experimental or simulated data.

Instead of relying on a fully data-driven model, the neural network operates as a constrained regression layer that identifies the coupling coefficients ξ_{ij} and $\xi_{\oplus j}$ by minimizing the deviation between predicted and reference properties or LCA metrics, while preserving the physical form of the multi-phase H-T equation. This formulation ensures that the model remains physically interpretable and analytically consistent. In practice, a shallow feed-forward neural network (typically one or two hidden layers with several tens of neurons per layer) can be employed. The network is trained using gradient-based optimization (e.g., Adam) to minimize the mean-square error between the model predictions and the available dataset. The fitted parameters are subsequently used as input to the MO optimization module to evaluate the trade-offs among performance, cost, and environmental impact. This approach combines the interpretability of micromechanics with the flexibility of ML, improving the model's predictive capability without requiring large datasets or deep architectures.

The H-T version of the recovery functions can be converted to either the direct rule of mixtures (ROM) or the inverse rule of mixtures (IROM) by setting the fitting parameter in Eq. (7) to infinity or zero, respectively (Afdl and Kardos, 1976). The ROM version, which is a linear weighted summation, provides an upper bound for the effect of ADs on properties, as follows:

$$F_i = \lambda_i \left(1 + \sum_{j=1}^{N_a} (r_{ij} - 1) v_j \right) \quad (8)$$

Despite its simplicity, the ROM provides accurate predictions in certain cases, such as Young's modulus when the ADs are in the form of long fibers aligned in the direction of the applied load (Jalali and Pugno, 2024). In contrast, the IROM version of the recovery function provides the lower bound for the effect of ADs on the property of the matrix. It is reported that this simple micromechanics formulation provides an acceptable estimation for some properties of mixtures of materials, such as transverse Young's modulus of aligned ADs perpendicular to the load direction (Jalali and Pugno, 2024), or friction coefficients of mixtures (Ning and Lovell, 2001). The recovery function based on the IROM model is obtained by setting $\xi_{ij} = 0$ in (7) as,

$$F_i = \frac{\lambda_i}{1 + \sum_{j=1}^{N_a} \left(\frac{1}{r_{ij}} - 1 \right) v_j}, \quad (9)$$

The total cost of a blend of RM and ADs depends on component unit cost prices as well as process- and market-dependent factors. Accounting for market variability, supply-chain effects, and potential environmental externalities (e.g. carbon pricing), the cost relation is in general nonlinear. To reflect this, the cost ratio of the RM to VM (recovered to virgin), F_s , is defined in its most general form as a mapping,

$$F_s = f(\lambda_s, \{r_{sj}\}, \{v_j\}), \quad (10a)$$

where λ_s denotes the cost ratio of RM to VM (recycled to virgin), and r_{sj} is the cost ratio of the j th AD to RM. For practical calibration and rapid evaluation, a simplified linear mixing rule can be adopted. Under this assumption, the total cost reduces to the weighted sum of component costs, yielding the ROM-type expression as an updated version of Eq. (8) for the cost:

$$F_s = \lambda_s \left(1 + \sum_{j=1}^{N_a} (r_{sj} - 1) v_j \right) \quad (10b)$$

Eq. (10b) should therefore be regarded as a particular (linear) realization of the general mapping defined in Eq. (10a). The generalized form remains available to incorporate nonlinear cost functions, market-driven dynamics, or environmental levies when such data becomes available.

After defining the recovery problem, the next step is to solve it. This involves identifying the appropriate types of ADs and determining their corresponding volume fractions to achieve the target property

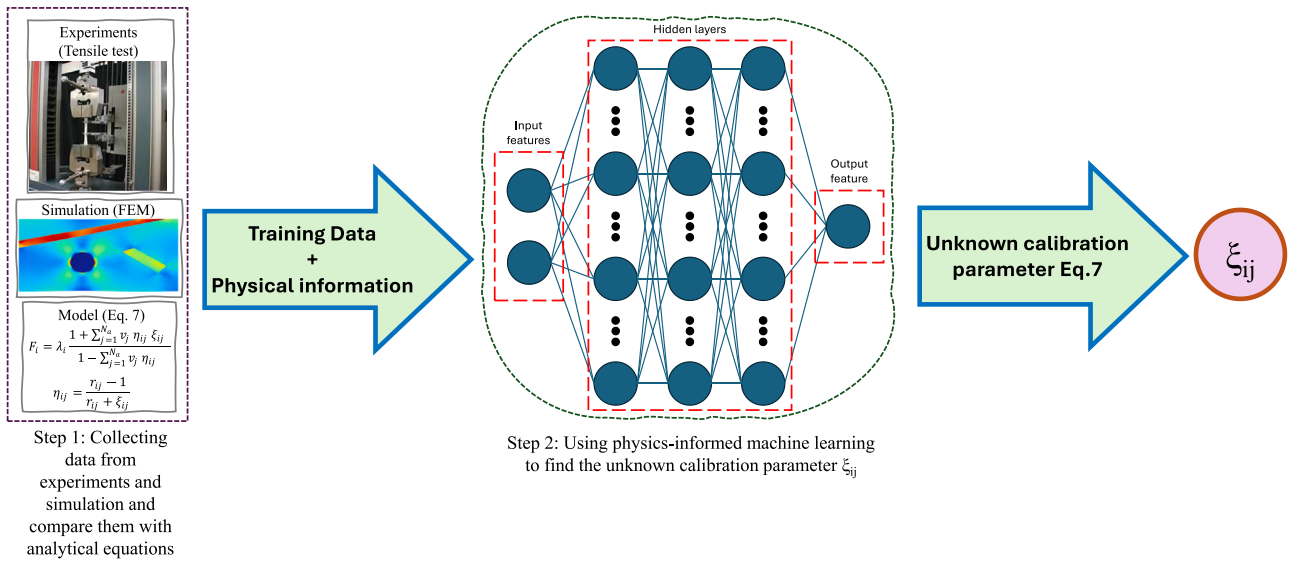


Fig. 2. The schematic of proposed physically-informed machine learning approach. It takes the inputs from experiments/simulations and provides the calibrated fitting parameters, $[\xi]_{N_p \times N_a}$, while physically informed by the recovery model offered in Eq. (7).

deviations. To establish a foundational understanding, consider the simplest case where there is only one AD to recover one property of interest, i.e., $N_a = N_p = 1$. By substituting the recovery function from Eq. (7) into the deviation function, Eq. (5), the volume fraction can be obtained as a function of the deviation, $\bar{v}$, as,

$$\bar{v}(\delta_1) = \frac{\frac{1-\delta_1}{\lambda_1} - 1}{\eta_{11} \left(\xi_{11} + \frac{1-\delta_1}{\lambda_1} \right)}, \quad (11)$$

where $\eta_{11} = (r_{11} - 1)/(r_{11} + \xi_{11})$. The critical volume fraction of AD, $\bar{v}_{cri}$, which causes full recovery, can then be determined by setting the deviation to zero in Eq. (11),

$$\bar{v}_{cri} = \frac{\frac{1}{\lambda_1} - 1}{\eta_{11} \left(\xi_{11} + \frac{1}{\lambda_1} \right)}. \quad (12)$$

Now, let us expand this idea to the case where the number of properties and the number of ADs increases. In this case, each AD has the potential to influence all the investigated properties through the ratio vectors $\{r_{ij}\}$. Considering a deviation δ_i from the VM for each property, by inserting the N_p recovery functions from Eq. (7) (or its simplified versions in Eqs. (8) and (9)) into Eq. (5), a system of algebraic equations in the following matrix expression is obtained:

$$[A]_{N_p \times N_a} \{\bar{v}\}_{N_a \times 1} = \{b\}_{N_p \times 1}. \quad (13)$$

The elements on the i th row of the matrix, $[A]$, specify the efficacy of N_a ADs in the recovery of the i th property, while the element of the vector $\{b\}$ are defined using the predefined deviation, δ_i , and the corresponding recycling efficiency, λ_i . The components A_{ij} , and b_i are presented in Table 1 for H-T, ROM, and IROM recovery functions. Note that when $\delta_i = (1 - \lambda_i)$, then, $b_i = 0$, corresponding to no recovery (RM and RM coincide in Fig. 1(a)). Regarding the algebraic system of equations, three distinct scenarios may arise depending on the comparison between the number of ADs, N_a , and the number of properties considered for recovery, N_p :

Scenario one, $N_a = N_p$: There are exactly enough ADs to recover all the properties. Consequently, the matrix $[A]$ becomes a square matrix, and a unique vector for the volume fraction can be determined as a function of predefined deviations as follows:

$$\{\bar{v}\} = [A]^{-1} \{b\}. \quad (14)$$

Table 1

The elements A_{ij} of the matrix $[A]$ and b_i of the vector $\{b\}$ in Eq. (13) where $i = 1, \dots, N_p$ and $j = 1, \dots, N_a$.

	A_{ij}	b_i
H-T	$\eta_{ij} \left(\xi_{ij} + \frac{1-\delta_i}{\lambda_i} \right)$	$\frac{1-\delta_i}{\lambda_i} - 1$
ROM	$r_{ij} - 1$	$\frac{1-\delta_i}{\lambda_i} - 1$
IROM	$1 - \frac{1}{r_{ij}}$	$1 - \frac{\lambda_i}{1-\delta_i}$

Solving Eq. (14) requires the inverse of the matrix $[A]$, which has dimensions $N_a \times N_p$. Then, one can insert $\{\bar{v}\}$ into Eq. (10b) to obtain the cost ratio of recovery, F_s , as a function of deviations, and consequently assess the cost ratio for full recovery by setting the deviations to zero. **Supplementary Note 1** provides the closed-form equations for determining volume fractions as a function of deviations in the simplest multi-AD recovery scenario, where the number of recovery properties equals the number of additives (both set to 2). This is complemented by **Supplementary Note 2**, which presents a parametric study to analyze the effects of various parameters on recovery performance.

Scenario two, $N_a > N_p$: The number of ADs exceeds the number of properties, resulting in more unknown volume fractions than available equations. In this case, incorporating extra ADs can expand recovery options and enhance decision-making. This flexibility proves particularly advantageous in actual manufacturing conditions where factors such as market availability or complexities in the mixing process impose real-world constraints. With more ADs available than properties to recover, these extra ADs, denoted as ADs⁺, introduce extra degrees of freedom in problem-solving. It is important to note that while this surplus enables more recovery options, it may also increase costs. When $N_a > N_p$, the system of equations in Eq. (13) has more variables than equations, resulting in infinite solutions rather than a unique one. To achieve a unique solution, one approach is to designate a specific number of ADs (equal to the difference between N_a and N_p) as supplemental ADs with predefined fixed volume fractions, $\{\bar{v}\}^+$. This adjustment restores the squareness of the matrix $[A]$, enabling the use of Eq. (14) to determine the volume fractions of the remaining ADs. **Supplementary Note 3** provides a case study illustrating how increasing the number of ADs can diversify choices for solving a recovery problem.

Scenario three, $N_a < N_p$: There are more properties to recover than available ADs. As a result, it may not be feasible to find a solution for

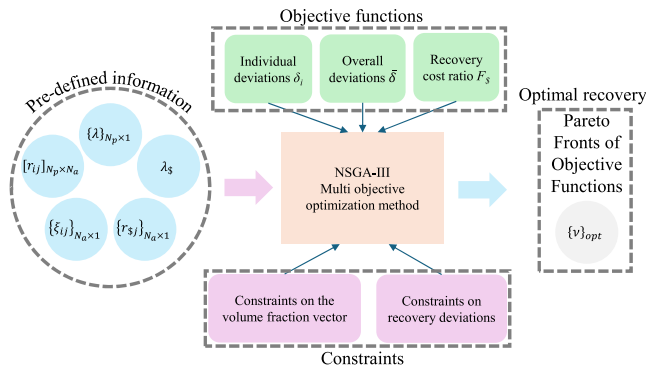


Fig. 3. The schematic of multi-objective optimization algorithm for optimal recovery of RM. The pre-defined information is required as the input. The output is the Pareto Fronts of objectives and the corresponding volume fraction of ADs, while satisfying the imposed constraints on the volume fractions and deviations.

Eq. (13). The only feasible approach to finding a solution is to engineer ADs such that they can simultaneously recover multiple properties to achieve their desired deviations. This consolidation results in some recovery equations in the system of algebraic equations becoming dependent and being eliminated, thereby allowing the matrix $[A]$ to become square and invertible again. Subsequently, the volume fraction vector corresponding to the desired deviations can be determined using Eq. (14).

An extreme case of *scenario three* is to exclusively design a single AD for a specific recovery problem where adding a certain volume fraction of this AD enables the recovery of all investigated properties to reach the target deviations simultaneously. For a single AD ($N_a = 1$) recovering N_p properties, the matrix form of the recovery problem in Eq. (13) is simplified as,

$$v_1 \begin{bmatrix} A_{11} \\ \vdots \\ A_{N_p 1} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_{N_p} \end{bmatrix}. \quad (15)$$

Simultaneous recovery of all properties is feasible by adding the pre-defined chosen volume fraction, v^* , if the relationship between the elements of b_i and A_{i1} remains consistent across all properties:

$$\frac{b_i}{A_{i1}} = v^* \quad i = 1, \dots, N_p \quad (16)$$

From a linear algebra perspective, Eq. (16) condenses the N_p system of equations into a single independent equation that satisfies all properties simultaneously. Indeed, Eq. (16) imposes a restriction to all influencing parameters including ratios of the properties of AD to RM, r_{i1}^* , the recovery efficiencies of properties, λ_{ij} , and the targeted deviations, δ_i . The exclusively designed single AD should be engineered to satisfy the relationship dictated by Eq. (16). The critical predefined volume fraction of the engineered AD, v_{cri}^* , which ensures full recovery of all properties simultaneously, can be determined by setting all the deviations to zero while evaluating the coefficients b_i in Eq. (16). A sample case study on designing such a single AD is presented in **Supplementary Note S4**.

2.2. Establishing the optimization framework for recovery

Here, we describe the setup of the optimization problem for recovering a recycled material using ADs. This includes defining the pre-defined input data, specifying the objective functions, and outlining any constraints that may apply.

required input information:

- The vector of recycling efficiencies of N_p properties under examination (recycled to virgin), $\{\lambda\}_{N_p \times 1}$.
- The cost ratio of RM to VM (recycled to virgin), λ_s .
- The N_p vectors of the property ratio of the ADs to RM (additive to recycled), $\{r_{ij}\}_{N_a \times 1}$
- the N_p vectors of the packing fitting parameter of the ADs, $\{\xi_{ij}\}_{N_a \times 1}$
- The vector of the cost ratio of the ADs to RM (additive to recycled), $\{r_{sj}\}_{N_a \times 1}$.

Possible objective functions to be minimized:

- The individual deviations from the VM, δ_i , as defined in Eq. (5), ideally zero for full recovery of that property.
- The overall deviation, $\bar{\delta}$, as defined in Eq. (6), ideally zero for full recovery of all the properties.
- The relative cost of the recovery, F_s , as defined in Eq. (10b). the smaller the F_s value, the more cost-effective the recovery becomes. In the extreme case where $F_s = 1$, the price of RM equals that of VM, and for $F_s > 1$, the economic justification for the recovery process diminishes.

Possible constraints:

- Constraints on the volume fraction vector: A Constraint can be defined on the total volume fraction of ADs, either via setting a certain total volume fraction, v_T , or setting two maximums and minimums scalars, $(v_T)_{min}$ and $(v_T)_{max}$. Note that the former applied to the total volume fraction is a linear constraint as the summation of the volume fraction of ADs, while the latter imposes nonlinear constraints in the form of inequalities. Additionally, the constraints may be applied as the pairs of maximums and minimums on every individual AD which can be presented via two constrain vectors, $\{v\}_{min}$, and $\{v\}_{max}$ of size $N_a \times 1$.
- Constraints on recovery deviations: It can be applied either as a vector of maximum allowable deviations, $\{\delta\}_{max}$ of size $N_p \times 1$, or as a maximum overall allowable deviation, $(\bar{\delta})_{max}$.

The output of the optimization algorithm is the tuned vector of volume fraction of ADs, $\{v\}_{opt}$, and the corresponding values of the objective functions while satisfying predefined constraints.

From an MO optimization perspective, a set of N_G objective functions can be defined, with the corresponding Pareto fronts determining the optimal solution space. Fig. 3 presents the schematic of the optimization algorithm for optimal recovery.

2.3. The numerical implementation of the MO optimization algorithm

NSGA-III (Non-dominated Sorting Genetic Algorithm III) is a high-dimensional multi-objective optimization algorithm that uses reference points to maintain diverse solutions along the Pareto front (Deb and Jain, 2014; Jain and Deb, 2014). Implemented in Julia through the Metaheuristics package, it capitalizes on Julia's speed and efficiency for complex optimization tasks (de Dios and Mezura-Montes, 2022). In our study, a population of 2000 was used, and the algorithm ran for 1000 iterations to ensure a well-distributed Pareto front, addressing trade-offs between multiple objectives effectively. All computations were performed on an Intel i7-11700 laptop with 64 GB RAM. Typical solving times were approximately 478 s per run. The number of decision variables, objectives, and constraints varies depending on the specific case study. These settings provide reliable convergence and can be readily applied across different case studies. The validation study and the sensitivity analysis for the optimization algorithm is presented in **Supplementary Note 5**.

3. Results

3.1. The targeted virgin and recycled resources

Since the automotive industry is under increasing pressure to adopt more sustainable practices (Bijina et al., 2023), here we explore the feasibility of partially replacing virgin rubbers in two key sectors (Life Green Vulcan, 2024): the production of spring pads in suspension systems, and passenger car tires, referred to as VM1 and VM2, respectively provided by the project partners, Stellantis (<https://www.stellantis.com/en>), and Bridgestone (<https://www.bridgestone-emea.com/>). These components are not only mass-produced but also represent a substantial share of global raw rubber consumption. We utilize a devulcanized recycled rubber from ELTs, referred to as SRC0, provided by Rubber Conversion srl, Italy (<https://www.rubberconversion.com/>). SRC0 is then combined with nine different nanofillers, resulting in nine new nano-filled RM, labeled SRC1 through SRC9. The incorporation of nanofillers significantly modifies the properties of SRC0, thus creating a diverse set of ten choices of replacement with distinct characteristics. The primary objective is to ensure that the final properties of this rubber blend (partial replacement of VM1 or VM2 with SRCs) meet the required technical standards. Such an initiative not only promotes sustainable material usage but also reduces the dependency on natural and synthetic rubber. This strategy can lead to significant environmental and economic benefits by lowering raw material consumption and minimizing waste. The current scenario of replacement of SRCs within the VMs involves a matrix composed of a VM, whose properties are inherently optimal and do not require recovery. The objective, driven by economic and environmental considerations, is to replace a targeted fraction of the VM, v_T , with SRCs. Here, SRCs act as ADs not as reinforcements, since they are (nano-filled) recycled ELTs. As a sequence, the replacement of VMs with SRCs potentially causes deviations from the properties of the VMs. In this context, the proposed model and the associated optimization algorithm are employed to determine the optimal replacement strategy that minimizes costs while effectively minimizing and controlling deviations in properties. This approach ensures that the final rubber blend retains desirable characteristics, promoting the sustainable use of ELTs without compromising performance. Note that it is different from the case studies presented in the **Supplementary Notes S2 to S5**, where the base material (matrix) was an RM with efficiency lower than VM, and ADs were used as reinforcements to recover the properties of RM, reducing its deviation from VM.

For the cost-benefit analysis, it is essential to establish the relative pricing of these SRCs in comparison to VMs. It is assumed that the neat devulcanized recycled rubber, SRC0, which does not contain any nanofillers, is priced at 50% of the cost of either VM1 or VM2, whereas SRC1 to SRC9, which include various nanofillers, are priced equivalently to VM1 and VM2. This reflects the fact that the additional expense incurred from the nanofillers offsets the lower cost of the base SRC0 material. From a cost perspective, substituting VM1 or VM2 with any of the SRC1 to SRC9 does not offer cost savings and SRC0 remains the only cost-effective option for achieving cost reduction while utilizing recycled materials. It is emphasized that cost reduction is not the primary goal of substituting VMs with SRCs. Even if the substitution does not lead to a considerable decrease in the final cost of the product, the incorporation of SRCs into the production cycle represents a significant achievement supporting resource conservation and the sustainable use of recycled materials.

3.2. Experimental measurements of the properties of the virgin rubbers VM1 and VM2 and their blends with recycled rubbers SRC0 to SRC9

To determine the optimal replacement of VMs with SRCs, it is crucial to understand how this substitution affects the material properties.

For VM1, the focus is on six properties: hardness, tensile strength, elongation at break, modulus of elasticity (at strain 100%), tear strength, and compression set. In contrast, for VM2, the evaluation includes four properties: tensile strength, elongation at break, modulus of elasticity at strain 100%, and toughness (the area under the stress-strain curve). We have proposed a detailed micromechanical model for evaluating the impact of nano-filled SRCs on a VM (Jalali et al., 2023). However, in this study, we adopt a direct experimental measurement approach. This methodology is chosen to bypass the uncertainties associated with interphase properties and other unknown variables inherent in micromechanical modeling. By relying on empirical data, we aim to provide an accurate and realistic assessment of the effective modification in the properties of the VMs with SRC additions, thereby inherently integrating all relevant factors such as interphases and defects introduced during the blending. To evaluate the impact of incorporating SRCs on the specified properties of interest, we first measure these properties for two VMs without any SRC addition. Subsequently, we introduce a specific volume fraction of each SRC into the virgin materials — 20% for VM1 and 10% for VM2 — and measure the properties again. We then compute the ratio F_i of the properties of the mixture (VMs plus SRCs) to the corresponding properties of the VM by dividing the measured values. **Supplementary Fig. S6** (a) and (b) illustrate the F_i ratios for the properties of VM1 and VM2, respectively. All mechanical tests were conducted by the industrial partners (Stellantis for VM1 and Bridgestone for VM2) in their certified laboratories, following internal testing standards and quality control procedures. Each reported property represents the mean value obtained from multiple specimens, ensuring statistical reliability of the experimental data used for model calibration. These experimental measurements reveal that the incorporation of SRCs into VMs leads to a diverse spectrum of property modifications. These changes include both enhancements and degradations, depending on the specific SRC and property in question. Nonetheless, due to the intrinsic nature of recycled materials, the addition of SRCs generally results in a degradation of material properties.

For each SRC added to the VM, we insert the measured F_i values and the corresponding volume fraction (0.2 for VM1 and 0.1 for VM2) into Eq. (7). Since there is only one added SRC per mixture, $N_p = 1$. Additionally, given that the matrix is composed of VM, the recycling efficiency is 100%, i.e., $\lambda_i = 1$. The remaining two values in the recovery function, Eq. (7), are the property ratio r_{ij} and the fitting parameter ξ_{ij} . Ideally, the properties of the individual SRCs should also be measured separately, allowing for the calculation of r_{ij} ratios. By inserting r_{ij} , the recovery equations for each property and each blend can be fit to the experimental data to determine the values of the fitting parameters ξ_{ij} . This fitting process can be conducted using a physically-informed machine-learning approach similar to that depicted in Fig. 2. However, the process faces the challenge of a large number of blends and properties to be evaluated. Specifically, here we have a total of 20 blends (combinations of ten SRCs with the two VMs), and six and four properties are of interest, respectively for VM1 and VM2. This results in a total of 100 different recovery functions, necessitating the fitting process to determine 100 fitting coefficients ξ_{ij} , making the fitting process practically infeasible. To overcome, here we make a reasonable assumption for fitting parameter: it is assumed that the SRCs are distributed homogeneously and randomly within the VM as semi-spherical particles by setting the H-T fitting parameter for all blends and all properties equal to $\xi_{ij} = 2$ (Jalali et al., 2023). Then, we solve Eq. (7) for r_{ij} to back-calculate the ratio of the properties of the j th SRC to the i th properties of VMs. In this way, we derive the ratio of the effective properties of each SRC when blended with the VMs, considering all effects such as interface formation or potential defects. Consequently, these r_{ij} values are more practical compared to directly measuring the properties of SRCs and dividing them by the properties of VM since it accounts for possible inaccuracies associated with assuming $\xi_{ij} = 2$ for all properties, providing a more accurate representation of

the SRC's impact on the VM. The total 100 calculated r_{ij} values for the two virgin materials, VM1 and VM2, are presented in **Supplementary Table S3** and **Supplementary Table S4**, respectively. When $r_{ij} = 1$, the property of the SRC is equivalent to that of the VM. Conversely, $r_{ij} > 1$ and $r_{ij} < 1$ signify that the SRC has superior and inferior properties compared to the VM, respectively.

It should be noted that although the same SRCs have been added to both VM1 and VM2, their effects on the same property can differ significantly. A primary reason for this is the intrinsic differences in the compositions of the two virgin materials. VM2, which is utilized in tire production, has a markedly different composition compared to VM1, which is used in the manufacture of spring pads in automotive suspension systems. VM2 inherently contains a substantial proportion of various additives, including carbon black and sulfur, which can significantly influence the efficacy of the SRCs. These additives are designed to enhance specific properties required for tire performance but may interact differently with the SRCs compared to the relatively simpler composition of VM1. Additionally, the process parameters involved in blending SRCs with VMs vary and can affect the final impact of SRCs on the material properties. For instance, when comparing column $i = 2$ in **Supplementary Table S3** with column $i = 1$ in **Supplementary Table S4**, it becomes evident that the addition of SRCs impacts the tensile strength of VM1 and VM2 differently, resulting in distinct r_{ij} ratios.

During the back-calculation, negative values for r_{ij} may be encountered, as seen in **Supplementary Tables S3 and S4**. To interpret these values, it is essential to understand that for a void introduced to the VM, $r_{ij} = 0$. Hence, a negative r_{ij} suggests that the addition of the SRC has a more degrading effect on the properties of the VM than if an equivalent volume fraction of voids were introduced into the VM. Lastly, it is noted that the r_{ij} ratios are calibrated based on experimental test results involving the combination of a single SRC with the VMs. Consequently, applying these ratios to blends containing multiple SRCs does not account for potential interactions among the SRCs. Indeed, in this work, the combined influence of ADs is modeled as the sum of their independent effects under the non-interacting (dilute) micromechanics assumption. This assumption, widely used in composite modeling (Christensen, 1991; Mori and Tanaka, 1973; Mura, 1987), remains valid when the total additive fraction is low, although potential deviations due to local clustering or additive coupling effects cannot be fully excluded.

3.3. Case study: Optimal replacement of recycled ELTs within the virgin rubbers

Here, we examine the replacement of 20% of the virgin rubbers VM1 and VM2 with SRCs. To ensure that this replacement objective is consistently met, we impose a fixed constraint on the volume fraction of SRCs across all results, $v_T = 0.2$. We explore two optimization approaches to determine the optimal recovery strategy: the first involves a three-objective optimization without constraints on property deviations, while the second involves a two-objective optimization with constraints applied to property deviations.

In the three-objective optimization, the objective functions are defined as follows: The first objective function is the ratio of the blend price to the price of the virgin rubber, F_S . Among the SRCs, only SRC0, can reduce the blend price relative to VMs. Since it is assumed to cost half as much as both VMs, F_S ranges between 0.9 and 1, where 0.9 represents the minimum price, when SRC0 fills the entire 20% of replacement, and 1 represents the maximum price, when no SRC0 is used, see Eq. (10b) in the **Methods** section. The second objective function is the overall deviation, $\bar{\delta}$ defined in Eq. (6) in the **Methods** section, which reflects the average performance of the blend compared to VMs. A deviation of zero indicates that the blend's properties match those of VMs exactly. To evaluate the overall deviation, the properties are assigned equal weights, with VM1 properties receiving a weight of

$\frac{1}{6}$ and VM2 properties $\frac{1}{4}$, corresponding to the six and four properties of interest for VM1 and VM2, respectively. The third objective function focuses on the deviation of every individual property, δ_i , see Eq. (5) in the **Methods** section. This allows the optimization process to not only minimize the cost and overall performance deviation compared to VMs but also to target and reduce deviations in a specific property. Thus, the optimization approach aims to balance cost reduction with overall performance improvement and specific property enhancements.

This choice of objective functions leads to six independent optimizations for VM1 and four for VM2. For each optimization study, the Pareto front of the three objective functions is illustrated as a three-dimensional plot in the Cartesian coordinate system ($F_S, \bar{\delta}, \delta_i$). This plot represents a range of optimal solutions, capturing the trade-offs among these three objectives. Additionally, for each displayed Pareto front, the volume fractions of the corresponding SRCs at the boundaries (edges of the 3D surface) are provided. This information helps determine which SRCs should replace 20% of the VMs for each optimal point on the Pareto front.

Fig. 4(a)–(f) shows the Pareto fronts and the corresponding volume fractions of SRCs for optimal replacement with VM1 for six properties of interest. In all scenarios, point A, corresponds to the lowest price, $F_S = 0.9$. At this point, as expected, the optimization algorithm has replaced the entire 20% with SRC0. However, the exclusive use of SRC0 does not necessarily result in the lowest overall deviation, nor does it lead to the lowest deviation for the specific property being investigated. Therefore, to optimize these deviations, other SRCs must be utilized, which increases the cost. This is evident from the gradual decrease in the volume fraction of SRC0 (blue areas in the volume fraction diagrams) as the solution moves away from point A. Since SRC0 is not the best option for reducing the deviation for any of the properties, at the points where deviations reach their absolute minimum values, for example, the BC edge in Fig. 4(a), SRC0 may not be included in the replacement. Consequently, the price of the blend could reach the price of VM1, i.e., $F_S = 1$.

In general, it can be observed that in all six optimal replacements, SRC3 frequently emerges as a significant alternative for reducing deviations since it exhibits acceptable performance across all six properties on average. Therefore, the optimization algorithm is more likely to select it to minimize the overall deviation $\bar{\delta}$. However, when the focus shifts towards achieving the absolute minimum for any specific δ_i deviation, the optimization algorithm tends to allocate a larger share to the SRC that most closely matches the performance of VM1 in that particular property. For instance, for minimizing deviations in hardness, tensile strength, and tear strength, the algorithm assigns higher volume fractions to SRC9, SRC1, and SRC8, respectively, as approved by r_{ij} values in the **Supplementary Table S3**.

Regarding the elastic modulus in Fig. 4(d), although SRC5 exhibits a slightly higher elastic modulus compared to SRC1, both SRCs serve as the suitable choice in bringing the blend elastic modulus closer to VM1. However, their effects on other properties are markedly different and sometimes even opposite. For instance, SRC1 shows the weakest performance in tear strength, whereas SRC5 underperforms in hardness and elongation. In such scenarios, the optimization algorithm smartly employs a combination of SRC1 and SRC5 in specific proportions to enhance the elastic modulus while simultaneously minimizing the overall deviation. This balanced approach ensures that the blend maintains desirable properties across multiple attributes, optimizing the overall performance. A similar interpretation applies to the combination of SRC0, SRC1, and SRC7 in minimizing the deviation in elongation. While both SRC1 and SRC7 exhibit acceptable performance in elongation, comparable to VM1, they both perform poorly in terms of elastic modulus. To address this, the optimization algorithm incorporates SRC2 into the mixture to compensate for the deficiencies in modulus, thereby achieving a balanced improvement across multiple properties.

Lastly, it is noticeable that although SRC1 and SRC9 have compression sets closest to VM1, the optimization algorithm unexpectedly

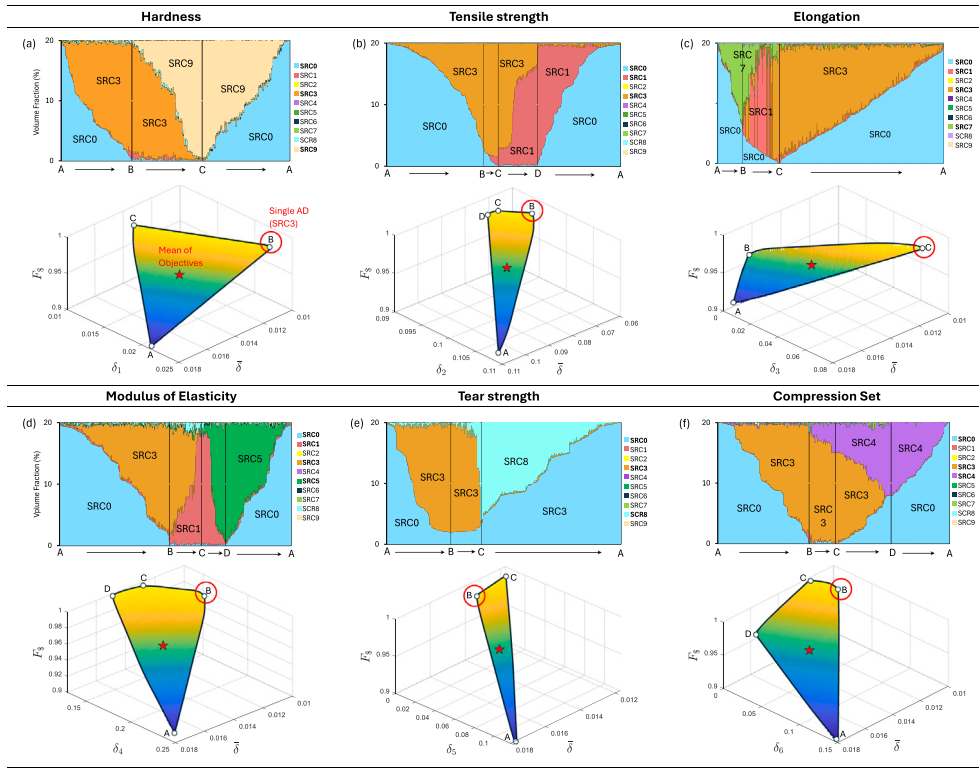


Fig. 4. The three-objective optimization of the replacement of 20% of VM1 with SRCs for the six properties of interest, focusing on every individual property. The objective functions are the cost ratio of blend compared to the VM1, $0.9 < F_s < 1$, the overall deviation of the six properties calculated from Eq. (6) (in the **Methods** section) with the same weight of $w_i = \frac{1}{6}$, and the deviation of the individual under focuses property. The 3D surface representations of the Pareto fronts of objective functions are presented for every optimization study. Besides, the corresponding volume fractions of the contributing SRCs are presented on the boundary of Pareto fronts. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

selects a combination of SRC4 with SRC3 and SRC0 to minimize the deviation. This decision arises because SRC0 and SRC3, which are optimal for minimizing cost and overall deviation, respectively, both exhibit higher compression set values than VM1 (91% and 15%, respectively). Therefore, the algorithm opts to retain SRC0 and SRC3 while incorporating SRC4, which has a compression set of 61% of that of VM1, into the blend to effectively minimize the deviation δ_6 .

The comparison between single-AD and multi-ADs optimal solutions reveals clear trade-offs between cost and performance. For instance, in Fig. 4(a), along the A–C edge of the three-objective Pareto front (cost, overall deviation, and hardness deviation), point A corresponds to the lowest-cost single-AD solution (SRC0), whereas point C represents another single-AD with minimum deviation in hardness (SRC9). Moving from A toward C, intermediate solutions involve a combination of the two ADs, resulting in a moderate cost increase in the range of $0.903 \leq F_s \leq 1.000$ but a reduction in both overall and hardness deviations in the $0.015 \leq \bar{\delta} \leq 0.018$ and $0.012 \leq \delta_1 \leq 0.021$ intervals. Similarly, in Fig. 4(f), progression along the C–D edge shows a transition from two-AD to three-AD mixtures, achieving improved balance between cost and mechanical performance. These observations quantitatively demonstrate how the MO framework identifies mixed-AD formulations that offer enhanced trade-offs compared to single-AD solutions.

Fig. 5(a)–(d) presents the results of four three-objective optimizations for combining SRCs with VM2, depicted as four Pareto fronts along with the corresponding volume fractions of SRCs at the boundaries of these surfaces. In these optimizations, the algorithm predominantly utilizes SRC0 and SRC2 to minimize F_s and $\bar{\delta}$, respectively. For the properties of elongation and toughness, SRC6 is added as a third contributor. For tensile strength, again SRC6 is included as the third SRC, however, a low percentage of SRC1, the highest tensile strength among the SRCs, is added. Regarding the elastic modulus, since SRC0,

as the most cost-effective choice, also provides the highest modulus, the algorithm prefers to easily combine it with SRC2 to minimize overall deviation. Interestingly, in some cases, a small amount of SRC7 is present in the optimal combination despite its overall weak properties. This occurs because the optimization algorithm aims to minimize both positive and negative deviations. Given that the elastic modulus of SRC0 is much higher than that of VM2, achieving minimum deviation requires adding a fractional amount of SRC7 to better match the stiffness of VM2. The algorithm also intelligently excludes SRC3, SRC4, SRC5, SRC8, and SRC9 from the optimal replacement combinations, as at least one of the r_{ij} values for these SRCs is too low, indicating suboptimal performance in the desired properties.

After presenting the three-objective optimizations, a set of two-objective optimizations is introduced where the cost ratio, F_s , and overall deviation, $\bar{\delta}$, are chosen as the objective functions. To focus on the deviation of specific properties, a constraint is defined as $\delta_i < \delta_{i,\max}$. Unlike the three-objective studies, which aim to minimize the δ_i deviation, the goal here is not necessarily to minimize δ_i but to find the optimal solutions for F_s and $\bar{\delta}$ that satisfy the δ_i constraint. The value of the constraint $\delta_{i,\max}$ represents the maximum acceptable deviation for the i th property. This value is typically defined by design standards and operational considerations. The stricter the constraint (i.e., $\delta_{i,\max}$ closer to zero), the more challenging it becomes to find optimal solutions, and the more limited the Pareto front is. If, under stringent conditions, all SRCs for a property are weaker than the VMs (all r_{ij} are less than one), tightening the constraint further may result in no feasible solutions. To systematically explore this, we apply constraints on each investigated property for VM1 and VM2 one at a time, gradually tightening the constraint towards zero until the algorithm fails to find a feasible Pareto front. This process helps identify the SRCs that play the most crucial role in meeting the constraint within the blend.

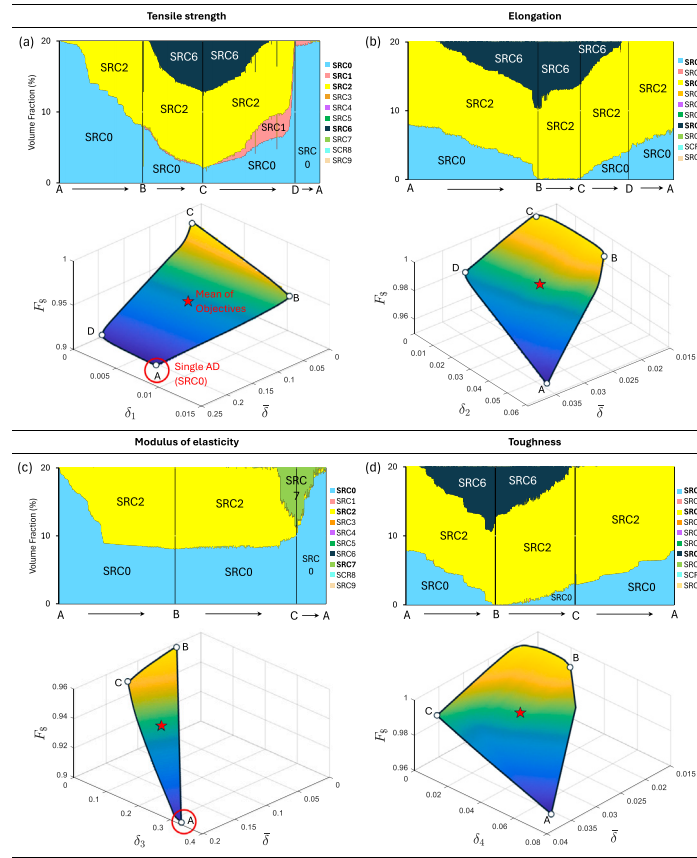


Fig. 5. The three-objective optimization of the replacement of 20% of VM2 with SRCs for the four properties of interest focusing on every individual property. The objective functions are the cost ratio of blend compared to the VM2, $0.9 < F_s < 1$, the overall deviation of the four properties calculated from Eq. (6) (in the **Methods** section) with the same weight of $w_i = \frac{1}{4}$, and the deviation of the individual under focuses property. The 3D surface representations of the Pareto fronts of objective functions are presented for every optimization study. Besides, the corresponding volume fractions of the contributing SRCs are presented on the boundary of Pareto fronts.

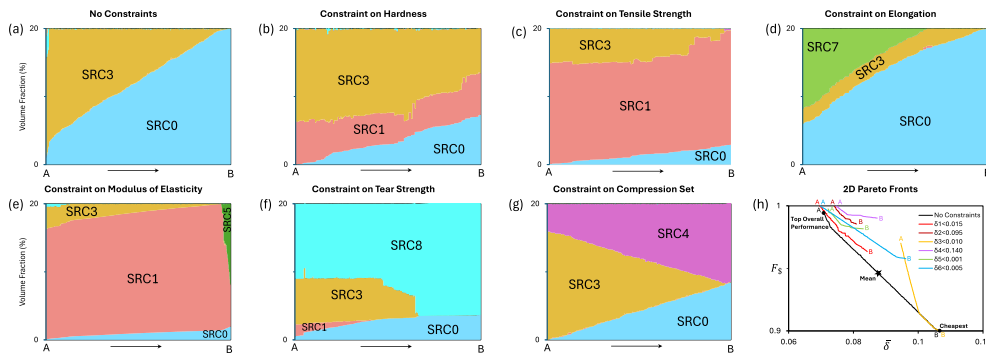


Fig. 6. The two-objective optimization of the replacement of 20% of VM1 with SRCs imposing constraints on the six properties of interest. The two objective functions are the cost ratio of blend compared to the VM1, $0.9 < F_s < 1$, and the overall deviation of the six properties calculated from Eq. (6) (in the **Methods** section) with the same weight of $w_i = \frac{1}{6}$. The 2D Pareto fronts and the corresponding volume fraction of the SRCs for an optimized replacement are presented.

Constrained two-dimensional optimizations for a 20% replacement of SRCs within VM1 are illustrated through the volume fractions of SRCs used (Fig. 6(a)–(g)), and the corresponding 2D Pareto fronts (Fig. 6(h)). Initially, in Fig. 6(a), no constraint is applied to any of the six properties where the optimization algorithm selects SRC0 and SRC3 to minimize F_s and $\bar{\delta}$, respectively. As observed in the corresponding Pareto front, moving from point A to point B, the share of SRC0 increases, and the overall deviation $\bar{\delta}$ gradually increases while F_s approaches its minimum at 0.9.

For hardness with the condition $\delta_1 < 0.015$, the algorithm selects SRC1 to satisfy this constraint. However, in the corresponding three-objective optimization where hardness was the third objective function (see Fig. 4(a)), SRC9 was chosen to minimize δ_1 . This illustrates the difference between applying even a strict constraint on a property and aiming to minimize that property. When the algorithm seeks to achieve the absolute minimum deviation in a property (the state closest to the VMs), it tends to select the SRC with the highest performance in that property (the highest r_{ij}), as seen in the case of SRC9 for hardness. Conversely, when a constraint is applied to the deviation

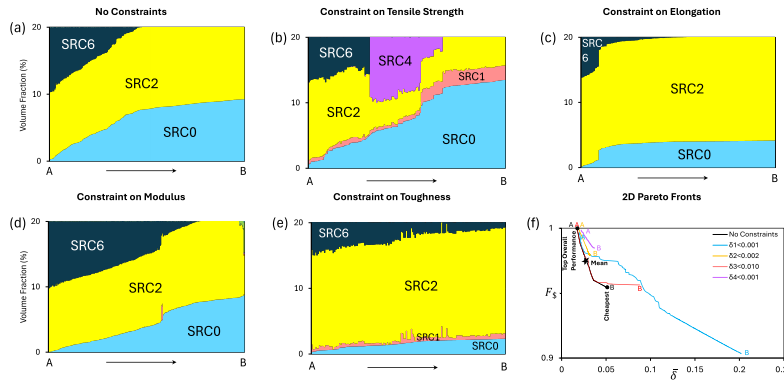


Fig. 7. The two-objective optimization of the replacement of 20% of VM2 with SRCs imposing constraints on the four properties of interest. The two objective functions are the cost ratio of blend compared to the VM1, $0.9 < F_s < 1$, and the overall deviation of the four properties calculated from Eq. (6) (in the **Methods**) with the same weight of $w_i = \frac{1}{4}$. The 2D Pareto fronts and the corresponding volume fraction of the SRCs for an optimized replacement are presented.

of a property, the algorithm prefers the SRC that, while satisfying the constraint, maximizes the overall effectiveness in minimizing both the overall deviation and cost. For tensile strength, elastic modulus, and tear strength, when constraints $\delta_2 < 0.095$, $\delta_4 < 0.14$, and $\delta_5 < 0.001$ are applied, SRC1, and SRC8, which exhibit the best performance in these properties, emerge in the optimal combination. These selections align with the results from the three-objective optimization. The similarity in the choices can be attributed to SRC1 and SRC8, which, despite not having the maximum r_{ij} in other properties, still offer acceptable performance overall. Regarding the elongation at break, since the most cost-effective choice, SRC0, has a very close performance to VM1, the algorithm consistently allocates a major part of the optimal combination to it. To adjust the overall deviation and satisfy the constraint $\delta_3 < 0.01$, a combination of SRC3 and SRC7, similar to the strategy observed in the corresponding three-objective optimization, is used. Applying the constraint $\delta_6 < 0.005$ on the compression set results in the addition of SRC4 to the optimal replacement combination, exactly as seen in the three-objective optimization in Fig. 4(f). As mentioned, SRC4, with a compression set equal to 61% of VM1, is added to moderate the high values of this property for SRC0 and SRC3, thereby bringing it closer to VM1. Among all constraints, the least stringent one is applied to the elastic modulus, $\delta_4 < 0.14$, resulting in the greatest deviation from VM1. This observation is supported by the data in the **Supplementary Table S3**. While for other properties there are SRCs with performance that closely approximate VM1 (indicated by r_{ij} values near one), the best-performing SRC for the elastic modulus, SRC1, remains substantially distant from VM1.

Similarly, Fig. 7 illustrates the optimal combinations and the corresponding Pareto fronts for the constrained two-objective optimization involving a 20% replacement of SRCs in VM2. In the unconstrained scenario, the optimal combination consists of SRC0, SRC2, and SRC6, which effectively minimize both the cost ratio, F_s , and the overall deviation, $\bar{\delta}$. When constraints are applied to elongation, elastic modulus, and toughness with $\delta_2 < 0.002$, $\delta_3 < 0.01$, and $\delta_4 < 0.001$, the algorithm selects the same three SRCs, although in different proportions compared to the unconstrained case, resulting in a higher price but achieving the desired performance. For tensile strength, the algorithm also includes SRC4 in the optimal combination.

In both Figs. 6 and 7, a comparison of the constrained and unconstrained Pareto fronts reveals that the constrained solutions are unable to achieve the minimum cost and overall deviation. This limitation arises because the constraints necessitate the inclusion of alternative SRCs, which prevents reaching the optimal cost associated with SRC0. Consequently, the constrained Pareto fronts are shorter compared to their unconstrained counterparts. However, as the constraints on δ_i are progressively relaxed, the constrained Pareto fronts gradually converge toward the unconstrained ones. This shift occurs because allowing individual properties to deviate more from the VMs. Therefore, the volume

fractions of SRCs in optimal combinations approach the unconstrained solutions in Figs. 6(a) and 7(a) for VM1 and VM2.

4. Discussion

After a detailed discussion on the outputs and the capabilities of the proposed optimization algorithm in the **Results** section, we outline the material selection process followed during the project (Life Green Vulcan, 2024) for integrating SRCs into VM1 and VM2 virgin rubbers. Prior to developing the optimization algorithm, the preliminary identification and selection of superior SRCs were based solely on relative comparisons of the F_i and r_{ij} values derived from the micromechanical model calibrated on the experimental data (**Supplementary Tables S3 and S4**, and **Supplementary Fig. S6**). This oversimplified approach led to the initial selection of SRC3 and SRC0 for spring pads and tires, respectively. While the selection of a single SRC for individual properties may appear justified, it does not account for the lack of absolute superiority across all properties. As a sequence, choosing a single SRC to replace the entire volume fraction, v_T , is suboptimal in addressing the complex requirements of multi-property optimization. To overcome this limitation, our MO optimization algorithm was developed and implemented to tackle the challenge of material selection. As demonstrated in Figs. 4 to 7, the algorithm generates Pareto fronts tailored to specific design scenarios, enabling the selection of optimal solutions based on combinations of objective functions and constraints. Each point on the Pareto front corresponds to a unique SRCs combination, providing a wider spectrum of potential solutions compared to the restrictive approach of selecting a single SRC. For instance, in the Pareto fronts for the spring pad application (Figs. 4 and 6(h)), although SRC3 emerges as a viable single choice for minimizing property deviations, however, depending on technical constraints or competitive market considerations, alternative points on the Pareto front may offer more advantageous trade-offs. Similarly, in the tire application (Figs. 5 and 7(f)), the comparison between selecting a single SRC such as SRC0 and exploring combinations along the Pareto front underscores the flexibility and adaptability of the proposed optimization framework. As a versatile alternative, the *Mean* point on these fronts, representing a balanced compromise across objective functions, could be proposed as a robust and well-rounded selection. It demonstrates the versatility of the optimization algorithm in accommodating diverse design priorities and constraints, facilitating more informed and balanced decision-making in complex material selection problems.

5. Conclusion

This study presented a generalized micromechanical-optimization framework for designing reinforced recycled materials through a multi-phase extension of the semi-empirical H-T model. The developed model

systematically links the intrinsic characteristics of recycled constituents and reinforcing additives to the overall performance, cost, and potential environmental impacts of the resulting blend. By integrating this predictive model within an MO optimization loop, the framework provides a practical route for balancing competing design objectives, offering both interpretability and generality across diverse material systems. Analytical analyses were conducted to validate and interpret the model's behavior before advancing to computational optimization. A sequence of supplementary studies provided physical insights: the analytical two-property trade-off formulation clarified the basic performance-cost relationship (**Supplementary Note S1**); parametric studies revealed the role of recycling efficiency and additive reinforcement capacity (**Supplementary Note S2**); multi-additive analysis demonstrated how increasing the number of additives expands the feasible design space (**Supplementary Note S3**); and the "single tailored additive" concept illustrated a strategy for full recovery of recycled properties (**Supplementary Note S4**). Besides, a practical case study on recycled rubbers from ELTs was used to demonstrate the model-guided optimization of performance-cost trade-offs. Collectively, these results confirm the capability of the proposed framework to provide a physically grounded basis for MO recycled material design.

The limitations of the present work should be noted. First, the current formulation neglects inter-particle interactions among additives, an assumption theoretically justified under the dilute micromechanics approximation, as the total additive content remains below approximately 20% in the presented case study. Although more complex formulations could capture such interactions, they would dramatically increase the model's dimensionality and experimental demand. Second, while the cost function has been defined in a generalized nonlinear form in Eq. (10a), the present validation adopted a simplified linear ROM representation for computational efficiency (Eq. (10b)). With more detailed economic datasets, the model can easily accommodate nonlinear or market-dependent behaviors. Third, the environmental extension of the multi-phase H-T equation, designed to integrate LCA metrics via physics-informed machine learning, has been conceptually introduced in Eq. (7) but not yet implemented in the optimization loop; only performance-cost trade-offs were demonstrated. Finally, the experimental calibration was limited by the industrial project scope, which involved two virgin matrices, ten additives, and fixed additive loadings. Broader datasets would further enhance model robustness and generalizability.

Future research should focus on addressing these limitations through: (i) incorporation of additive-additive interaction effects via multi-scale hierarchical modeling, (ii) adoption of nonlinear and dynamic cost functions, (iii) integration of LCA data within the optimization loop, and (iv) extension of the database through high-throughput testing and simulations for ML-based calibration. The proposed model provides a versatile and physically interpretable basis for estimating the effective properties of mixtures from the characteristics of their individual constituents. Because its fitting parameters can be directly calibrated to experimental or simulated data, the model is readily adaptable to a wide range of recycled material systems, including concrete with mineral additives, polymer blends with compatibilizers, metallic scrap alloys, and rubber-based compounds, without requiring structural modification of the core equation. Overall, the developed framework acts as a decision-support tool for engineers and researchers engaged in sustainable material design and selection. It enables the quantitative evaluation of trade-offs among performance, cost, and environmental impact, guiding the formulation of recycled-additive blends that meet technical and sustainability targets. By providing a unified micromechanical, data-driven, and optimization-based methodology, this work contributes to advancing the predictive design of next-generation recycled materials across industrial sectors.

CRediT authorship contribution statement

S.K. Jalali: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **M.J. Beigrezaee:** Writing – review & editing, Writing – original draft, Visualization, Software, Data curation. **N.M. Pugno:** Writing – review & editing, Supervision, Resources, Project administration, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary information is available in the supplementary file, which includes:

- Supplementary Notes S1 to S5 (analytical equations, parametric studies, case studies, and validation studies).
- Supplementary Figures S1 to S6.
- Supplementary Tables S1 to S4.

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jclepro.2025.147081>.

Data availability

No data was used for the research described in the article.

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Supplementary Information for

Physically based machine learning for the multi-objective optimal design of partially recycled materials: The case study of rubber resources in the automotive industry

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Overview

Supplementary Information

This document contains supplementary information to support the findings of the main paper. It includes:

Five Supplementary Notes:

- Note S1: Closed-Form Solution for the Case of Two Recovery Properties and Two ADs,
- Note S2: Parametric Study on the Required Volume Fractions of ADs for a Full Recovery,
- Note S3: Case Study of Diversifying Choices by Increasing the Number of ADs,
- Note S4: Case Study of Designing a Single Engineered AD for Simultaneous Recovery of All Properties,
- Note S5: Verification of the Optimization Algorithm.

Five Supplementary Figures: Figures S1 to S5.

Four Supplementary Tables: Tables S1 to S4.

Supplementary Note S1: Closed-Form Solution for the Case of Two Recovery Properties and Two ADs

Here the closed-form expressions for the simplest case of multi-AD recovery, where the number of recovery properties and the number of ADs are both equal to 2 are presented. In this case, the matrix $[A]$ in Eq. 14 becomes the smallest possible square matrix, giving a simple expression for $[A]^{-1}$. Consequently, the expression for the volume fractions $\bar{v}_1$ and $\bar{v}_2$ in terms of deviations δ_1 and δ_2 can be derived as follows:

$$\bar{v}_1 = \frac{Q_1\delta_1 + Q_2\delta_2 + Q_3\delta_1\delta_2 + Q_4}{\chi}, \quad (\text{S1a})$$

$$\bar{v}_2 = \frac{Q_5\delta_1 + Q_6\delta_2 + Q_7\delta_1\delta_2 + Q_8}{\chi}, \quad (\text{S1b})$$

$$\chi = Q_9\delta_1 + Q_{10}\delta_2 + Q_{11}\delta_1\delta_2 + Q_{12}. \quad (\text{S1c})$$

where,

$$Q_1 = \eta_{12} \frac{q_2}{\lambda_1} - \eta_{22} \frac{q_4}{\lambda_1}, \quad (\text{S2a})$$

$$Q_2 = \eta_{12} \frac{q_6}{\lambda_2} - \eta_{22} \frac{q_1}{\lambda_2}, \quad (\text{S2b})$$

$$Q_3 = \frac{(\eta_{22} - \eta_{12})}{\lambda_1\lambda_2}, \quad (\text{S2c})$$

$$Q_4 = \eta_{22}q_1q_4 - \eta_{12}q_2q_6, \quad (\text{S2d})$$

$$Q_5 = \eta_{21} \frac{q_5}{\lambda_1} - \eta_{11} \frac{q_2}{\lambda_1}, \quad (\text{S3a})$$

$$Q_6 = \eta_{21} \frac{q_1}{\lambda_2} - \eta_{11} \frac{q_3}{\lambda_2}, \quad (\text{S3b})$$

$$Q_7 = \frac{(\eta_{11} - \eta_{21})}{\lambda_1\lambda_2}, \quad (\text{S3c})$$

$$Q_8 = \eta_{11}q_2q_3 - \eta_{21}q_1q_5, \quad (\text{S3d})$$

$$Q_9 = \frac{\eta_{12}\eta_{21}q_5 - \eta_{11}\eta_{22}q_4}{\lambda_1}, \quad (\text{S4a})$$

$$Q_{10} = \frac{\eta_{12}\eta_{21}q_6 - \eta_{11}\eta_{22}q_3}{\lambda_2}, \quad (\text{S4b})$$

$$Q_{11} = \frac{(\eta_{11}\eta_{22} - \eta_{12}\eta_{21})}{\lambda_1\lambda_2}, \quad (\text{S4c})$$

$$Q_{12} = \eta_{11}\eta_{22}q_3q_4 - \eta_{12}\eta_{21}q_5q_6, \quad (\text{S4d})$$

in which,

$$q_1 = \frac{1}{\lambda_1} - 1, \quad (\text{S5a})$$

$$q_2 = \frac{1}{\lambda_2} - 1, \quad (\text{S5b})$$

$$q_3 = \xi_{11} + \frac{1}{\lambda_1}, \quad (\text{S5c})$$

$$q_4 = \xi_{22} + \frac{1}{\lambda_2}, \quad (\text{S5d})$$

$$q_5 = \xi_{21} + \frac{1}{\lambda_2}, \quad (\text{S5e})$$

$$q_6 = \xi_{12} + \frac{1}{\lambda_1}. \quad (\text{S5f})$$

$$(\text{S5g})$$

Then, one can substitute these volume fractions into Eq. 10b to obtain the corresponding cost ratio of recovery as a function of deviations, δ_1 and δ_2 , as well:

$$F_{\$} = \lambda_{\$} (Q_{\$1}\delta_1 + Q_{\$2}\delta_2 + Q_{\$3}\delta_1\delta_2 + Q_{\$4}), \quad (\text{S6})$$

where,

$$\begin{Bmatrix} Q_{\$1} \\ Q_{\$2} \\ Q_{\$3} \\ Q_{\$4} \end{Bmatrix} = \begin{bmatrix} Q_1 & Q_5 \\ Q_2 & Q_6 \\ Q_3 & Q_7 \\ Q_4 & Q_8 \end{bmatrix} \begin{Bmatrix} q_{\$1} \\ q_{\$2} \end{Bmatrix}. \quad (\text{S7})$$

and,

$$q_{\$1} = r_{\$1} - 1 \quad (\text{S8a})$$

$$q_{\$2} = r_{\$2} - 1 \quad (\text{S8b})$$

Note that the coefficients Q_i and $Q_{\$i}$ are functions of $\{r_{ij}\}$, $\{\xi_{ij}\}$, $\{\lambda_i\}$, and $\{r_{\$j}\}$.

Supplementary Note S2: Parametric study on the required volume fractions of ADs for a full recovery

Here a parametric problem of full recovery of two properties, utilizing two ADs ($N_p = N_a = 2$) is examined. To evaluate the required volume fractions of AD1 and AD2, $\bar{v}_1$, and $\bar{v}_2$, it is assumed that:

- *The property ratios:* Properties 1 and 2 of AD1 and AD2 are 10 times those of the recycled matrix, respectively, i.e., $r_{11} = r_{22} = 10$. The mutual ratios r_{12} and r_{21} , representing the effect of AD2 on property 1 and AD1 on property 2, are equal and vary within $0.5 \leq r_{12} = r_{21} \leq 1$. At $r_{12} = r_{21} = 1$, AD1 has no impact on property 2 and AD2 has no effect on property 1. At $r_{12} = r_{21} = 0.5$, property 1 of AD2 and property 2 of AD1 are 50% weaker than the recycled matrix.
- *The recycling efficiency:* λ_2 varies across values of 0.5, 0.6, 0.7, 0.8, and 0.9. For each specific λ_2 , the recycling efficiency λ_1 of property 1 is progressively reduced from its maximum value of 1 until the condition $\bar{v}_1 + \bar{v}_2 \leq 1$ is met.
- *The packing fitting parameter:* Since the H-T model spans between the ROM and IROM extremes, properties 1 and 2 are assumed to adopt either ROM or IROM conditions, resulting in three extreme combinations: both ROM (all four ξ_{ij} set to infinity), both IROM (all four ξ_{ij} set to zero), and ROM-IROM where property 1 follows ROM ($\xi_{11} = \xi_{12} = \infty$) while property 2 obeys IROM ($\xi_{21} = \xi_{22} = 0$).
- *The deviation:* Full recovery is assumed, resulting in both deviations being set to zero: $\delta_1 = \delta_2 = 0$.
- *The solution:* Since there are two ADs, we use Eqs. S1 to S5 to obtain $\bar{v}_1$, and $\bar{v}_2$.

Fig. S1 plots $\bar{v}_1$ versus $\bar{v}_2$ for three combinations of ROM and IROM models. Consider the dashed horizontal lines in the graphs, indicating when $r_{12} = r_{21} = 1$. Across all combinations, as λ_1 decreases from 1 (moving left to right in the graphs), $\bar{v}_1$ gradually increases to compensate for the drop in property 1. With $r_{21} = 1$, the addition of AD1 doesn't affect property 2, keeping $\bar{v}_2$ constant. Notably, for a given λ_1 , ROM requires a lower $\bar{v}_1$ for complete recovery compared to IROM. As r_{12} (and r_{21} , assumed equal here) gradually decreases, increasing each volume fraction aids in one property's recovery but leads to a corresponding decline in the other property. For a given λ_2 , to offset the loss in property 1, $\bar{v}_1$ needs to increase, causing a further decline in property 2. This mutual weakening effect can be counteracted by re-adjusting $\bar{v}_2$, resulting in optimal $\bar{v}_1$ and $\bar{v}_2$ values for complete recovery of both properties.

Since in this parametric study, property enhancement with a factor of 10 and reciprocal property reduction with a maximum factor of 0.5 are considered, the addition of ADs improves the properties of the recycled matrix, in an overall sense. However, if the enhancing effects of ADs are minimal and/or the mutual weakening effect is significant, an optimal solution may not exist (resulting in negative optimal volume fractions or a sum exceeding one), indicating that the selected ADs are unsuitable. In Fig. S1(b), where the micromechanics model is IROM, yielding a significantly lower recovery effect compared to ROM, the mutual weakening ratios are limited to 0.8 instead of 0.5 to ensure the feasibility of the optimal volume fractions. As a system of linear

equations, a solution is not feasible as well when $\det(A) = 0$ in Eq. 14, unless all equations become independent, indicating identical recovery functions for all the properties. This implies an infinite number of optimized combinations. To illustrate, consider the special case where matrix $[A]$ is 2×2 and both properties follow the ROM. In this scenario, if $(r_{11} - 1)/(r_{22} - 1) = (r_{12} - 1)/(r_{21} - 1) = (1/\lambda_1 - 1)/(1/\lambda_2 - 1)$, infinite pairs of optimal $\bar{v}_1$ and $\bar{v}_2$ exist, resulting in full recovery of both properties. When, $(r_{11} - 1)/(r_{22} - 1) = (r_{12} - 1)/(r_{21} - 1) \neq (1/\lambda_1 - 1)/(1/\lambda_2 - 1)$, there is no answer for Eq. 14.

Supplementary Note S3: Case Study of diversifying choices by increasing the number of ADs

This case study illustrates how incorporating additional ADs broadens recovery options, enhancing decision-making. It is assumed that:

- *The property ratios of ADs:* Five ADs, namely AD1 to AD5, can potentially contribute whose $\{r_{ij}\}$ vectors are listed in Table S1 for recovery of two properties (property 1 and 2) of the recycled matrix.
- *The recycling efficiency:* are both fixed to 50% recovery efficiency, i.e., $\lambda_1 = \lambda_2 = 0.5$.
- *The packing fitting parameter:* effect of all ADs on both properties obeys ROM meaning that all ξ_{ij} is set to infinity.
- *The deviation:* Deviations for both the properties are assumed to be equal ranging from no recovery to full recovery, $0 \leq (\delta_1 = \delta_2) \leq 0.5$.
- *Relative cost of RM to VM:* is assumed to be 70%, resulting in $\lambda_s = 0.7$.
- *Relative cost of ADs to RM:* is assumed to be twice for all the ADS, i.e., $r_{sj} = 2$.

Since we are recovering two properties ($N_p = 2$), selecting two ADs among five from Table S1 is sufficient which results in 10 possible pairs. Among them, the pairs AD1 + AD3 and AD2 + AD4, which do not contribute to the recovery of property 1 or property 2 respectively, do not lead to any solution. Additionally, since both properties are strengthened via the same micromechanical model, ROM, when AD5, which equally enhances both properties, is involved, the volume fraction of the other AD in the pair gets zero. Thus, all pairs including AD5 present the same result, and only the pair AD1 + AD5 is considered. Furthermore, AD2 + AD3 is excluded because it yields a solution identical to AD1 + AD4. These considerations reduce the number of AD pairs with feasible solutions from ten to four. To determine $\bar{v}_1$, $\bar{v}_2$, and the corresponding cost ratio of recycling, F_s , for these four selected pairs, one can utilize Eqs. S1 to S8.

Fig. S2(a) plots the calculated recovery cost ratio, F_s , as a function of deviations for the selected pairs of ADs. For all pairs, when the deviations are at their maximum value ($\delta_i = 0.5$), no ADs are introduced, and no recovery is performed. Consequently, the mixture solely consists of the RM, and $F_s = \lambda_s = 0.7$. As ADs are gradually added to the RM, deviations decrease, and the recovery cost ratio, F_s , increases. Complete recovery is achieved when both deviations reach zero. The required volume fraction of ADs for full recovery is shown in Fig. S2(b). Although full recovery is cost-saving for all AD pairs ($F_s < 1$), the pair AD3 + AD4, which requires the least volume fraction of ADs, is the most cost-effective choice.

After emerging AD3 + AD4 pair as the most cost-effective option, this absolute optimal pair is selected as the base, and a specified volume fraction v_3^+ from either AD5 or AD1 is gradually added to the base as the third supplemental AD⁺. Then, the recovery problem can be expressed as follows:

$$\begin{Bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} b_1 \\ b_2 \\ v_3^+ \end{Bmatrix} \quad (\text{S9})$$

Solving Eq. S9, one can obtain the updated volume fractions $\bar{v}_1$, and $\bar{v}_2$ of AD3 and AD4 which are plotted in Fig. S2 (c) and (d) versus the volume fraction v_3^+ of supplemental AD5 and AD1, respectively. It is observed that adding AD5 to the AD3+AD4 pair leads to an equal decrease in both, and eventually, adding 25% of AD5 results in complete substitution. This leads to a 10.8% cost increase, as evident from the total volume fraction of ADs in Fig. S2(b). On the other hand, adding AD1 primarily replaces AD3 due to their similar role in recovering property 1. However, it reduces the volume fraction of AD4, albeit partially, because, unlike AD3, AD1 does not negatively affect property 2. Nevertheless, since AD1 has a lower recovery factor than AD3 (10 vs. 20 in Table S1), the complete substitution results in a 5.3% increase in cost. This example demonstrates how substituting the optimal cost-effective combination with other alternatives can serve as a backup strategy, enabling full recovery with a minor increase in cost.

In the next example, all five ADs in Table S1 are assumed to participate in the recovery process of two properties. It results in two recovery equations against five unknown volume fractions. In addition, here, we define a constraint on the price ensuring that the cost of the recovered material is precisely 80% of the VM ($F_{\$}=0.8$). Given the equal pricing of ADs, this condition equates to restricting the sum of volume fractions of ADs, leading to the third equation for the problem. Since the three available equations are fewer than the five variables, to find a solution, the AD1 and AD2 in Table S1 are assumed as supplements and their volume fractions, v_1^+ and v_2^+ , are directly inserted into the problem within the range of 0 to 1 with a step of 0.025 leading to the following matrix form:

$$\begin{Bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \\ \bar{v}_4 \\ \bar{v}_5 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{\$1} & A_{\$2} & A_{\$3} & A_{\$4} & A_{\$5} \end{bmatrix}^{-1} \begin{Bmatrix} v_1^+ \\ v_2^+ \\ b_1 \\ b_2 \\ b_{\$} \end{Bmatrix} \quad (\text{S10})$$

where A_{ij} , b_i , and $b_{\$}$ are calculated from Table 1 (main text) for ROM model. By solving Eq. S10, the volume fraction of the three remaining ADs, i.e., AD3, AD4, and AD5 are determined to achieve full recovery of two properties while satisfying the specified cost constraint. Despite the fact that for $N_a > N_p$ there could be an infinite number of solutions, since a step of 0.025 for entering the volume fractions of AD1 and AD2 is chosen, six available distinct combinations are obtained which are depicted in Fig. S2(e) as representatives of multiple choices for the same recovery problem considering more ADs than properties. The smaller the steps of AD1 and AD2 changes are chosen, the number of possible choices increases.

Supplementary Note S4: Case Study of designing a single engineered AD for simultaneous recovery of all properties

This case study focuses on designing an exclusive single AD for the recovery of N_p properties. Consider the full recovery (no deviations) of four properties of an RM, with $N_p = 4$, each having a 50% recycling efficiency, $\lambda_i = 0.5$. The objective is to design an engineered AD that fully recovers all properties for a given critical volume fraction, v_{cri}^* . By substituting λ_i and the packing fitting parameters, ξ_{i1} , listed in Table S2 for each property, into Eq. 16 (main text), the corresponding required property ratio of the AD for that property, r_{i1}^* , can be calculated for three different critical volume fractions of AD, $v_{cri}^* = 0.1$, 0.3, and 0.5, as presented in Table S2.

As expected, generally, a higher critical volume fraction of $v_{cri}^*=0.5$ necessitates smaller recovery ratios, r_{i1}^* , compared to a relatively low critical volume fraction of 0.1 while $v_{cri}^*=0.3$ stays between. When opting for a generous volume fraction of 0.5, the engineered AD formulation should be designed to ensure that properties 1 to 4 of the AD are respectively 3, 4, 3.02, and 9 times those of the RM. It is revealed that property 1, governed by the robust ROM recovery model demands the least reinforcement. Conversely, property 4, following the weakest recovery model (IROM), necessitates the highest level of reinforcement, reaching up to 9 times that of RM. For properties 2 and 3, both recovered by the HT model, increasing the fitting parameter from $\xi = 2$ to $\xi = 100$ reduces the need for reinforcement. This occurs because higher ξ values approach the H-T model to the ROM. Besides, it is highlighted that when v_{cri}^* is as low as 0.1, properties 2 and 4, following weaker recovery models of HT with $\xi = 2$ and IROM, cannot achieve full recovery whereas properties 1, and 3 can provide a full recovery with the engineered AD presenting reinforcing ratios as high as $r_{11}^*=11$, and $r_{31}^*=11.98$ even with $v_{cri}^*=0.1$.

Supplementary Note S5: Verification and Sensitivity Analysis

of the Optimization Algorithm

To validate the algorithm a sample recovery problem of an RM with two properties of interest, $N_p=2$, is defined as follows:

- *The recycling efficiencies:* They are 50% for both properties, meaning RM properties are half of the VM properties, $\lambda_1 = \lambda_2 = 0.5$.
- *The cost of the RM:* It is assumed to be half of VM, leading to a recycling cost ratio of $\lambda_{\S}=0.5$.
- *The property ratio of ADs:* Five ADs namely AD1 to AD5, can potentially contribute to the recovery process whose property ratios to RM, $\{r_{ij}\}$, are listed in Table S1. AD1 and AD2 are engineered to exclusively recover one property without affecting the other. AD3 and AD4 provide high recovery in one property by weakening another. Lastly, AD5 uniformly and moderately recovers both properties.
- *The cost of ADs:* All ADs are assumed to be twice as expensive as the RM, i.e., $r_{\S j} = 2$.
- *The packing fitting parameters:* Three distinct values, 2, 20, and 200, are considered for the fitting parameter of each property, ξ_{1j} and ξ_{2j} . These values represent a progressive increase in the influence of ADs on the recovery of properties. Consequently, a total of 9 possible combinations are obtained for the recovery of the two properties. In every combination of ξ_{1j} and ξ_{2j} , it is assumed that the packing fitting parameters of all ADs are identical for each property.
- *Objective Functions:* Three objective functions are considered, including the two individual deviations from VM, δ_1 and δ_2 , as well as the recovery cost ratio, $F_{\S}$.
- *Constraints:* No constraints are imposed for the feasibility of comparison to the analytical solution obtained from Eqs. S1 to S8.

Since we are interested in two properties, any pair of ADs from Table S1 may be sufficient to recover them. Choosing two among five yields 10 possible pairs. However, AD1 + AD3 and AD2 + AD4 do not contribute to the recovery of property 1 and property 2, respectively, and are therefore excluded from the analysis. This reduces the number of pairs to 8. As the reference for validation, we first extract the analytical solution for each pair of ξ_{1j} and ξ_{2j} following these four steps:

1. For every available pair of ADs, the recycling cost ratio is evaluated using Eq. S6 as a function of deviations, $F_{\S}(\delta_1, \delta_2)$.
2. The recycling cost ratio, $F_{\S}(\delta_1, \delta_2)$, is visualized as a 3D surface plot in Cartesian coordinates, where deviations δ_i vary from 0, indicating 100% recovery, to $(1 - \lambda_i) = 0.5$, indicating no recovery.
3. In every (δ_1, δ_2) point, among eight surfaces corresponding to eight AD pairs, the one with the minimum cost ratio, $F_{\S}$, is chosen, to identify the surface with minimum recovery cost across the entire range of deviations.

4. The volume fraction of ADs corresponding to the emerged surface with minimal recovery cost are calculated using Eq. S1.

On the other hand, for the numerical optimization algorithm, there is no limit on the number of ADs; for each pair of ξ_{1j} and ξ_{2j} , the algorithms free to use any combinations of five ADs from Table S1 to minimize three objective functions, including δ_1 , δ_2 , and F_s . No constraints are applied to the objective functions, except that the sum of the volume fractions of ADs does not exceed one, $v_T \leq 1$. Fig. S3(a)-(i) displays the optimized objective functions from the numerical optimization algorithm, represented as 3D Pareto fronts (green dots), for nine different combinations of the H-T fitting parameters, ξ_{1j} and ξ_{2j} . For comparison, the corresponding 3D surface plots derived from the reference analytical solutions are also presented in the same 3D Cartesian coordinates. A high level of agreement is observed between the numerical Pareto fronts and the minimum analytical surfaces, confirming the accuracy and efficiency of the MO optimization algorithm for the recovery of recycled resources through the addition of ADs. Note that in Fig. S3 the percentage of recovery for property i -th is defined as, $(1 - \frac{\delta_i}{1-\lambda_i}) \times 100$.

For further investigation, Fig. S4(a)-(i) compares the volume fractions of AD pairs used by the analytical approach, corresponding to the minimal surfaces (solid bars), with the volume fractions of ADs obtained by the numerical optimization algorithm (hatched bars), corresponding to the Pareto front of minimum objective functions. Interestingly, despite the numerical algorithm having the freedom to use all five ADs to minimize the objective functions, it consistently selects the same pair of ADs corresponding to the minimal surfaces obtained from the analytical solution. Moreover, the volume fraction of each AD in the selected pair closely matches the analytical solution with acceptable accuracy. This demonstrates the numerical algorithm's capability and precision in identifying suitable ADs for optimal and cost-effective recovery from a diverse library of available ADs.

After validation of the optimization algorithm, let us analyze the results further. As shown in Fig. S3, with an increase in the recovery percentage of either of the properties (decreasing the deviation from VM), the recovery cost increases. Therefore, in all Pareto fronts, the highest recovery cost occurs at point A, where both properties are fully recovered. Conversely, the lowest recovery cost is at point I, where no recovery has taken place (no AD has been added) and the mixture consists only of RM, resulting in $F_s = \lambda_s = 0.5$. Moreover, the results indicate that, in general, the cost of recovery decreases with an increase in the H-T fitting parameters, ξ_{1j} and ξ_{2j} . This is because higher ξ_{ij} values enhance the effectiveness of the ADs in the recovery of RM, resulting in a lower required volume fraction of ADs and consequently reducing the overall cost of recovery. It is also evident that when the fitting parameters are not equal for two properties, $\xi_{1j} \neq \xi_{2j}$, the property with the larger value requires less cost for full recovery than the other property, resulting in an asymmetrical recovery process. For example, in Fig. S3(g) where $\xi_{1j} = 200$ and $\xi_{2j} = 2$, point C, which corresponds to the full recovery of the property 1 without recovering the property 2, requires a lower volume fraction of AD and, consequently, a lower recovery cost compared to point G and vice versa.

Since all ADs are priced equally, the recovery cost is proportional to the total volume fraction of ADs, represented by the height of the bar graphs in Fig. S4. Consequently, as the H-T fitting parameters increase, the height of these bars gradually decreases. Moreover, it is observed that among the five available ADs, only AD3, AD4, and AD5 are utilized for optimal recovery, while AD1 and AD2 are always excluded. Analysis from Table S1 reveals that AD3 and AD4 exhibit significant capability in enhancing proper-

ties 1 and 2, respectively, despite their mutual weakening impact on the recovery of the other property. In contrast, AD1 and AD2 do not present any weakening recovery but offer lower recovery potential compared to AD3 and AD4 (10 vs 20). The optimization algorithm therefore favors AD3 and AD4, utilizing AD5 to offset their mutual weakening effect since AD5 contributes moderately to the recovery of both properties equally. Consequently, AD5 participates in optimal recovery when the fitting parameter of H-T is small. As ξ_{1j} and ξ_{2j} increase, enhancing the impact of ADs in recovery, the need for AD5 diminishes (blue bars in Fig. S4), and AD3 (Green bars) and AD4 (yellow bars) independently achieve the desired recovery across the entire range of target deviations.

After verifying the robustness of the optimization algorithm, a sensitivity analysis was conducted to examine the influence of key model parameters on the optimization outcomes. The analysis was performed for the representative case of $\xi_{1j} = \xi_{2j} = 200$, where both properties exhibit a strong reinforcing response to the ADs. The goal was to assess how variations in recycling efficiencies, λ_1 , and λ_2 , and the cost ratio, $\lambda_{\S}$, affect the average values of the three objective functions, δ_1 , δ_2 , and $F_{\S}$, as well as the corresponding ADs volume fractions. Three sensitivity analyses were defined as follows:

1. **Analysis 1:** $\lambda_1 = [0.4, 0.45, 0.5, 0.55, 0.6]$, while $\lambda_2 = 0.5$, $\lambda_{\S} = 0.5$, and $r_{\S1} = r_{\S2} = 2$ were kept constant.
2. **Analysis 2:** $\lambda_2 = [0.4, 0.45, 0.5, 0.55, 0.6]$, while $\lambda_1 = 0.5$, $\lambda_{\S} = 0.5$, and $r_{\S1} = r_{\S2} = 2$ were fixed.
3. **Analysis 3:** $\lambda_{\S} = [0.4, 0.45, 0.5, 0.55, 0.6]$, while $\lambda_1 = \lambda_2 = 0.5$ and $r_{\S1} = r_{\S2} = 2$ were constant.

For each analysis, the optimization algorithm was executed for all parameter values, and the Pareto front corresponding to the three objective functions was obtained. To enable quantitative comparison, the average point of each Pareto front (point E in Fig. S3(i)) was extracted, and the mean values of δ_1 , δ_2 , $F_{\S}$, and the associated volume fractions of ADs were reported. The results are summarized in Fig. S5(a–c). In Analyses 1 and 2, where the recycling efficiencies λ_1 and λ_2 were varied independently, the objective functions showed minimal sensitivity to these parameters, as all three curves of δ_1 , δ_2 , and $F_{\S}$ remained nearly flat. However, notable changes were observed in the fractions of ADs: when λ_1 increased, the required fraction of AD3 (the additive enhancing property 1) gradually decreased, while AD4 remained nearly constant. Conversely, in the case of increasing λ_2 , the opposite trend was observed, AD4 gradually decreased while AD3 remained unchanged. These trends confirm that the optimization algorithm systematically adjusts the proportion of ADs based on the efficiency of property recovery. In Analysis 3, the increase of $\lambda_{\S}$ (the relative cost of the recycled matrix) caused a proportional rise in the normalized cost objective $F_{\S}$, while δ_1 and δ_2 were largely unaffected. The fractions of ADs were nearly constant across the range, indicating that cost variations of the base recycled material influence the total recovery cost without altering the optimal ADs composition. Overall, the results indicate that the MO optimization framework exhibits strong stability against moderate variations in recycling efficiencies and costs. The physical consistency of the model response, particularly the compensatory behavior between AD3 and AD4, further validates the robustness and interpretability of the proposed optimization scheme.

Supplementary Figures

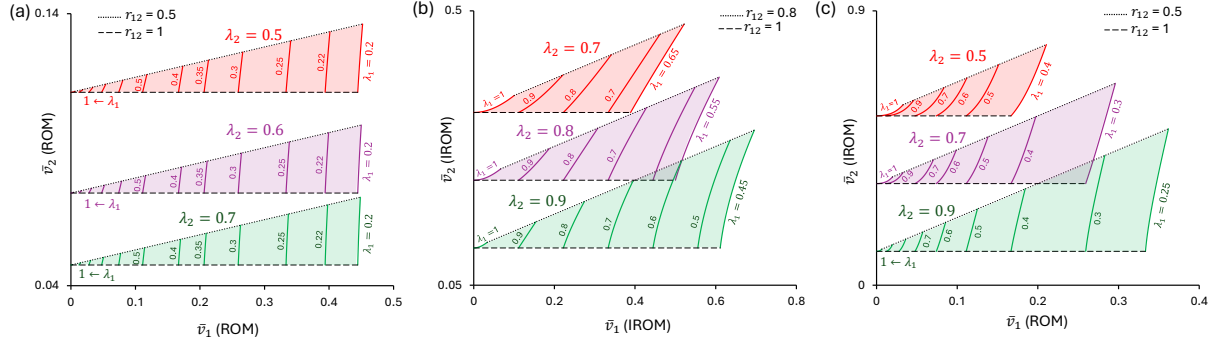


Figure S1: **A parametric study for recovery problem of two ADs and two properties of interest.** The volume fraction of ADs, $\bar{v}_1$, and $\bar{v}_2$, for a wide range of recycling efficiencies, λ_1 and λ_2 and different combinations of ROM and IROM micromechanics models. It is assumed that $r_{11}=r_{22}=10$ and $0.5 \leq r_{12} = r_{21} \leq 1$ while targeting a full recovery with no deviations, $\delta_1 = \delta_2 = 0$.

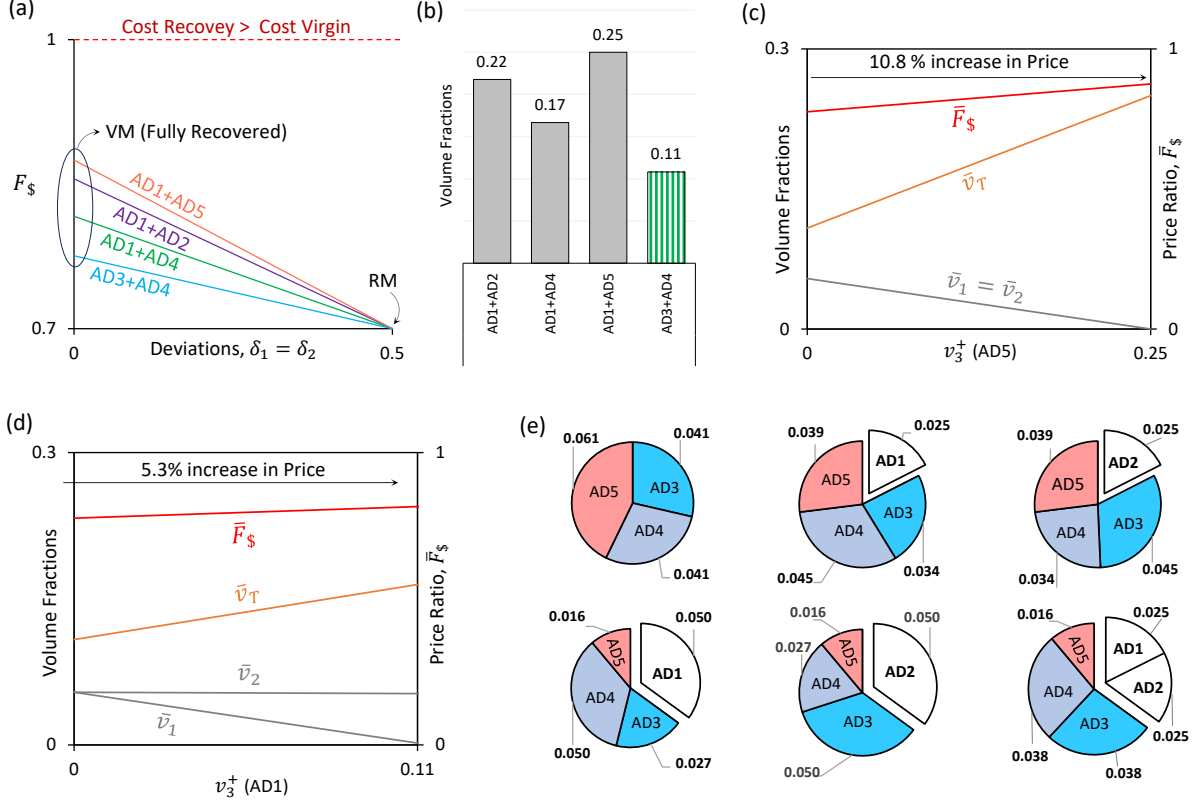


Figure S2: **Expanding the circle of choices by including supplementary ADs.** (a) The recovery cost ratio, $F_{\S}$, versus the deviations, $0 \leq (\delta_1 = \delta_2) \leq 0.5$, for the four selected pairs of ADs, identifying AD3 + AD4 as the most economical option. (b) The corresponding total volume fractions of AD pairs for full recovery of properties. As all the ADs are assumed to have a price twice that of RM, a higher total volume fraction results in a higher recovery cost. (c) Gradually addition of AD5 to the optimal AD2+AD3 pair from volume fraction 0 to 25 obtained by Eq. S9 resulting in a full replacement of ADs with 10.8% increase in cost, (d) Gradually addition of AD1 to the optimal AD2+AD3 pair resulting a the replacement of AD3 by AD2 at a cost increase of 5.3%, (e) six possible ADs combinations of the same price ratio of $\bar{F}_{\S}=0.8$ obtained from Eq. S10. Note: $\lambda_{\S} = 0.7$, and $\lambda_1 = \lambda_2 = 0.5$.

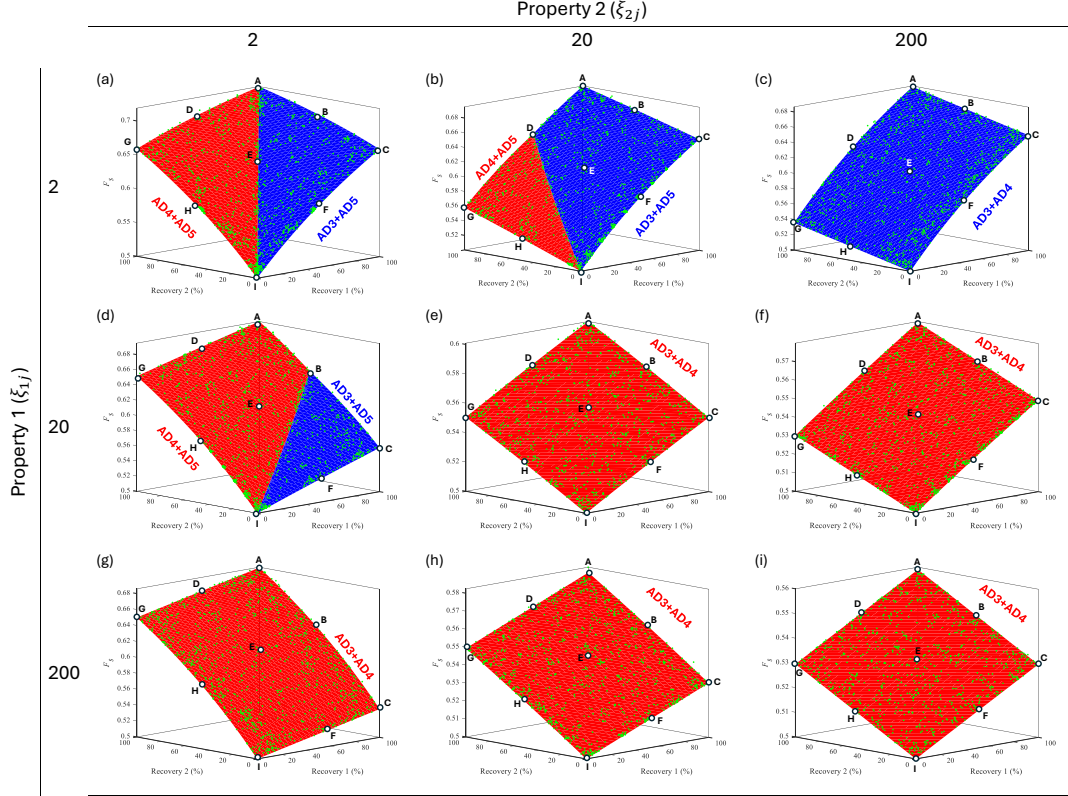


Figure S3: **Validation study of the numerical optimization algorithm, Pareto Fronts of the objective functions.** Comparison between the Pareto fronts of three objective functions, deviations δ_1 and δ_2 , and the recovery cost ratio F_s , shown as green dots, with the analytical minimal surfaces obtained from Eqs. S1 to S8, depicted as solid blue and red surfaces. The study examines nine different combinations for the two recovered properties, assigning three values (2, 20, and 200) to the H-T fitting parameters ξ_{1j} and ξ_{2j} . Note: the recovery percentage of property i -th is defined as, $(1 - \frac{\delta_i}{1-\lambda_i}) \times 100$.

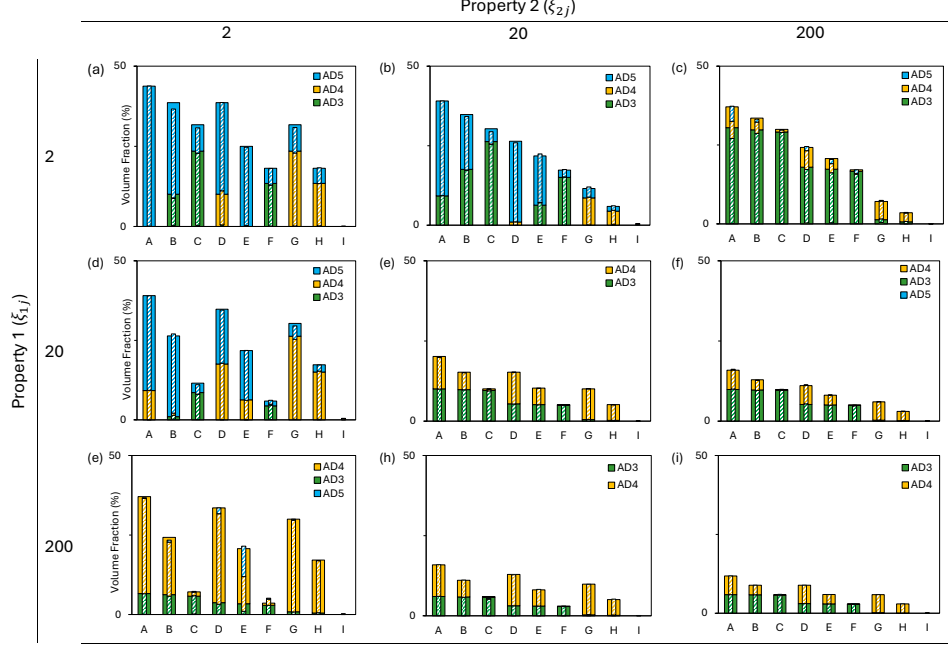


Figure S4: **Validation study of the numerical optimization algorithm, the volume fractions of ADs.** The comparison between the volume fractions of AD pairs calculated from the analytical approach, corresponding to the minimal surfaces (solid bars), with the volume fractions of ADs obtained by the numerical optimization algorithm (hatched bars), corresponding to the Pareto front of minimum objective functions for nine different combinations of ξ_{1j} and ξ_{2j} , presented in Fig. S3.

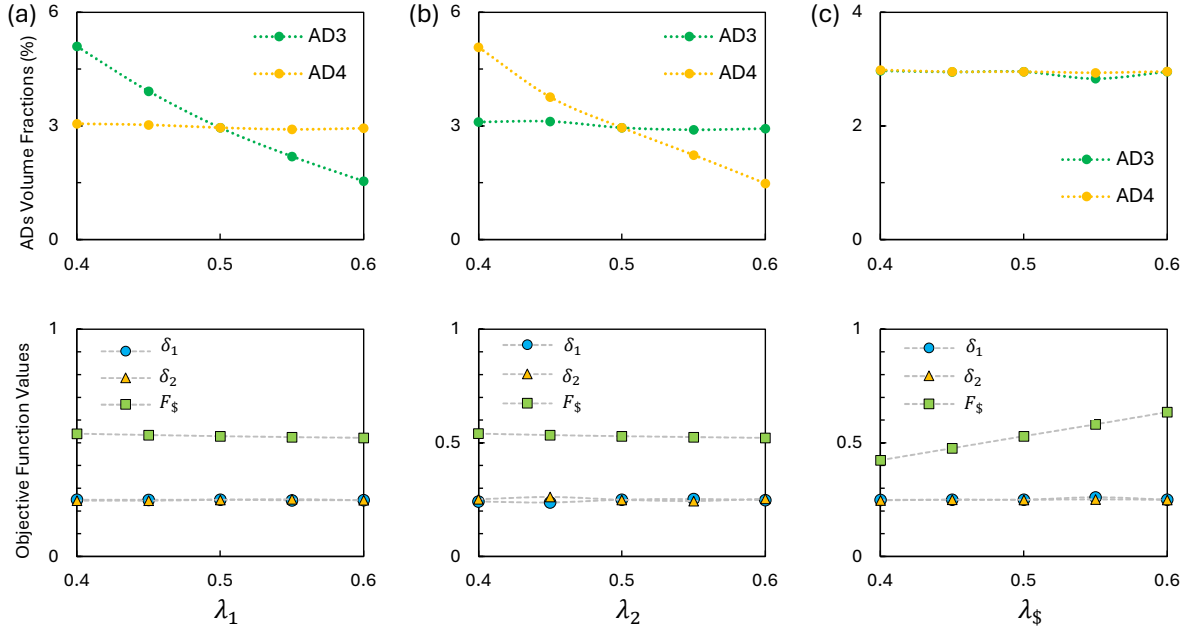


Figure S5: **Sensitivity analysis of key parameters for the case of $\xi_1 = \xi_2 = 200$.** (a)–(c) Variations of the three objective functions (δ_1 , δ_2 , and $F_\S$) and the corresponding volume fractions of ADs with respect to the recycling efficiencies λ_1 , λ_2 , and the cost ratio $\lambda_\S$, respectively. Each data point represents the average value of the Pareto front (point E in Fig. S3(i)) obtained for the specified parameter set.

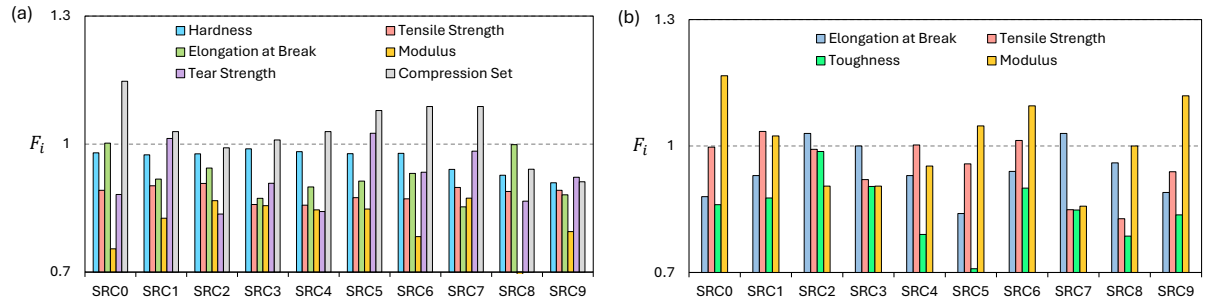


Figure S6: **The evaluated recovery factors from experiments.** The recovery functions, F_i , (the property ratio of SRC-VM blend to VM) calculated from experimental tests. (a) SRCs blended to VM1, (b) to VM2.

Supplementary Tables

Table S1: **The property ratios, r_{ij} , of sample ADs (AD1 to AD5) for two sample properties of interest.** It is defined to be used as the input of the case study in Supplementary Note S3 and the verification study in Supplementary Note S5.

j	ADs	r_{ij}	
		i	
		1	2
1	AD1	10	1
2	AD2	1	10
3	AD3	20	0.5
4	AD4	0.5	20
5	AD5	5	5

Table S2: **A sample of an engineered single AD for multiple full recovery of properties..** The property ratio, r_{i1}^* , of the exclusively designed AD in Supplementary Note 4, for the simultaneous full recovery (no deviations) of four properties, each with a 50% recycling efficiency ($\lambda_i = 0.5$). The property ratios, r_{i1}^* , are listed for three different critical volume fractions of AD ($v_{cri}^* = 0.1, 0.3$, and 0.5), given the packing fitting parameter ξ_{i1} for each property.

Property		v_{cri}^*		
		0.1	0.3	0.5
i	ξ_{i1}	r_{i1}^*		
1	∞ (ROM)	11	4.33	3
2	2	-	16	4
3	100	11.98	4.41	3.02
4	0 (IROM)	-	-	9

Table S3: The back-calculated property ratios, r_{ij} , for the SRCs blended with VM1, used in the case study of spring pads.

		r_{ij}					
		i					
		1	2	3	4	5	6
j	SRCs	Hardness	Tensile strength	Elongation	Modulus	Tear strength	Comp. Set
1	SRC0	0.90	0.53	1.01	0.08	0.49	1.91
2	SRC1	0.89	0.59	0.74	0.44	0.33	0.96
3	SRC2	0.89	0.45	0.69	0.16	0.70	1.50
4	SRC3	0.88	0.57	0.63	0.29	1.07	1.15
5	SRC4	0.60	0.53	0.49	0.19	0.65	0.61
6	SRC5	0.73	0.55	0.39	0.46	0.92	1.50
7	SRC6	0.91	0.40	0.56	0.36	0.35	1.15
8	SRC7	0.67	0.52	0.99	-0.09	0.43	0.73
9	SRC8	0.89	0.46	0.61	0.37	1.13	1.44
10	SRC9	0.94	0.40	0.46	0.39	0.59	1.05

Table S4: The back-calculated property ratios, r_{ij} , for the SRCs blended with VM2, used in the case study of car tires.

		r_{ij}			
		i			
		1	2	3	4
j	SRCs	Tensile strength	Elongation	Modulus	Toughness
1	SRC0	0.97	0.12	4.33	0.11
2	SRC1	1.38	0.42	1.25	0.66
3	SRC2	0.92	1.33	0.26	1.24
4	SRC3	0.36	1.00	0.26	0.36
5	SRC4	1.03	0.42	0.58	0.44
6	SRC5	0.62	-0.08	1.56	-0.23
7	SRC6	1.14	0.49	2.33	0.58
8	SRC7	-0.04	1.33	0.00	0.09
9	SRC8	-0.14	0.64	1.00	-0.27
10	SRC9	0.48	0.17	2.85	-0.10