



Article

An Extended Theory of Rational Addiction

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Abstract: This study extends the rational addiction theory by introducing an endogenous discounting of future utilities. The discount rate depends on habits accumulating over time because of the repeated consumption of an addictive good. The endogeneity of the discount rate affects consumption decisions via a habit-dependent rate of time preference and discloses a patience-dependence trade-off. The existence of a steady state in which habits do not grow and its optimality are proven. The local stability properties of the steady state reveal that the equilibrium can be a saddle node, implying smooth convergence to the steady state, but also a stable or unstable focus, potentially predicting real-world behaviors such as binge drinking or extreme addiction states that may drive to death. The stability of the steady state mostly depends on the habit formation process, suggesting that heterogeneity in habit formation may be a key component to explain heterogeneity in time preferences.

Keywords: addiction; habit formation; endogenous discounting; time consistency

MSC: 34A30; 34A45; 34D20



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1. Introduction

In economics, consumption decisions are the result of an optimizing choice. The consumer chooses the consumption bundle that maximizes their felicity, represented by an utility function, given a budget constraint. Since Ramsey [1], the dynamics of consumption are analyzed considering a representative consumer which maximizes the weighted sum of all their future lifetime utilities, wherein weights decrease with time. In this context, the consumer is seen as a perfectly rational individual, who can exactly predict future earnings and prices and will never change their preferences over time.

However, considering some specific goods, such as alcohol, tobacco or drugs, the phenomenon of addiction may arise and the Ramsey model could be inappropriate for a correct analysis. The state of addiction can lead the consumer to give more importance to the immediate consumption of the addictive good regardless of anything else. In general, addiction creates physical abstinence or withdrawal symptoms when the use of the substance is discontinued, and generates tolerance, which is a physiological phenomenon requiring the individual to use more and more of the substance [2,3]. However, addiction does not arise from one day to another: it is the result of a habitual consumption perpetuated over time. The consumption of possibly addictive goods implies a habit formation process, in which the habit is built day by day through consumption itself. Hence, a habit is formed when past and current consumption are linked by a positive relation. The higher previous consumption was, the larger the habit, and the higher the current consumption level needs to be to deliver the same utility. Pioneer studies on habit formation are due to Gorman [4], Pollak [5], Llach [6] and Boyer [7], but the reference model for applied studies of rational addiction (RA) is proposed by Becker and Murphy [8].

This model is characterized by a constant discount rate for future utilities that implies a constant rate of time preference. The rate of time preference is a subjective indicator of

impatience representing the desire of an agent to anticipate and enjoy the benefits stemming from higher current consumption. A high rate of time preference lowers the propensity towards future utility in determining current consumption choices. This property is known as the time consistency of consumers, as once they have chosen an optimal consumption path in, e.g., time t , they will stick to it even if allowed to reconsider its consumption path in, e.g., $t + x$.

Such a preference structure cannot properly describe situations such as addiction to alcohol (but also in drug use or cigarette smoking), or the existence of goods as holidays and works of art whose benefits continue over the consumption act. For these reasons, several works, starting with Strotz [9], Blanchard and Fisher [10], Deaton [11] and Romer [12], have criticized the assumption of a constant rate of time preference, as suggested more by convenience than economic rationales. Critics have also been made by empirical works, such as Bishai [13] and Laibson et al. [14], who rejected the hypothesis of constant discounting.

One parsimonious attempt at modeling time inconsistency is by introducing the quasi-hyperbolic discounting of future utilities that enable capturing present-biased time preferences. In the formulation proposed by [15], the degree of present bias is modeled through an additional discount parameter $\beta \in (0, 1)$, which reduces the weight of all future utilities accordingly. As a consequence, the optimal future consumption path will never be realized because the intertemporal trade-off changes over time as each future period becomes present and the present bias kicks in.

Present-biased preferences modeled through quasi-hyperbolic discounting have been applied to a variety of choices and situations. For instance, refs. [15–19] studied consumption and saving behavior; ref. [20] studied retirement decisions; ref. [21] applied it to economic growth; ref. [22] analyzed caloric intake; ref. [23] applied it to welfare program participation and labor supply; ref. [24] applied it to job search; and ref. [25] applied it to gym attendance). More closely related to our study, ref. [26] introduced quasi-hyperbolic discounting in an RA setting and applied it smoking behavior (also see [27,28] for similar, more recent applications).

Present-biased preferences, however, are not the only way in which time inconsistency may arise, and such a formulation may not be sufficient to capture more complex forms of time inconsistency. For instance, Piccoli and Tiezzi [29], who developed an empirical test for quasi-hyperbolic discounting within the rational addiction framework developed by [26], found no evidence of time inconsistency of this type for Russian smokers.

Given the conspicuous critics of the constant rate of time preference of the RA model and the possibly insufficient complexity of the quasi-hyperbolic discounting extension, this paper contributes to the literature by proposing an extended theory of rational addiction (ERA) that extends the RA model proposed by Becker and Murphy [8]. The relevance of properly introducing time-inconsistent consumers within the RA framework has also recently been highlighted by [30]. In our model, future utilities are discounted by an endogenous discount rate that depends on the stock of habits accumulated over time and is thus capable of describing the behavior of more complex time inconsistent consumers, but remains encompassing to the RA model itself as a special case. In the present paper, we solve the ERA model, study the existence and uniqueness of a steady state in consumption and habits, and analyze its local stability properties. We then open the way for a possible explanation in the observed heterogeneity of the rate of time preference, which is through the heterogeneity of the habit-formation process: individuals that more rapidly accumulate habits will increase their discount rate faster over time and may behave very differently from other individuals. We study these heterogeneity properties of the rate of time preference by a comparative dynamics exercise.

This approach, first introduced by Epstein and Shi [31], generates a non-constant rate of time preference which depends on habits—an index of past consumption. The consumption decision is influenced by the time at which the decision is taken, as different time periods imply different stocks of habits (which evolve over time) and therefore different discount rates. Thus, a consumer that is allowed to reconsider their consumption plan in a future

period may choose to change plans, because the accumulated habits have increased their discount rate, and hence the weights given to future utilities. In contrast to Epstein and Shi [31], we keep the general utility structure of the RA model since it allows to model the negative effects that habits can have on consumers, for example, through health problems (see [32–36]). A similar approach has recently been applied by [37,38] to the study of health behavior and its consequences for aging and longevity.

The paper is structured as follows. Section 2 introduces the extended rational addiction model, where the consumer maximizes a lifetime utility function with an endogenous discount rate depending on habits. Euler equations resulting from the ERA model are derived and analyzed. Section 3 proposes a definition of the steady state and derives its properties. Section 4 assesses the stability properties of the steady state. Section 5 discusses heterogeneity in the habit formation process. Section 6 presents the comparative dynamics of the rate of time preferences and Section 7 concludes.

2. The Extended Theory Rational Addiction (ERA)

The reference model in the field of addiction is the rational addiction model proposed by Becker and Murphy [8]. This model endows some characteristics that make it an appealing tool for applied works (see, for example, [32,39,40]), being characterized by a linear Euler equation with a simple test for the presence of addiction. This comes at a cost: a constant discount rate equal to the rate of return to savings is assumed. This assumption has never been supported by theoretical reasoning or empirical evidence and has been widely criticized by the literature. We propose to relax this assumption in favor of an endogenous specification of the discount rate determined by the habits accumulated by the individuals.

The underlying assumption is that habits are likely to influence the discount rate and the rate of time preference inducing increased impatience and, eventually, addiction. In doing so, we follow the work of Epstein and Shi [31], in which the discount rate is a function of the stock of habits, an index of past consumption (Becker and Murphy [8] refers to this concept as the “consumption capital,” in a framework in which “past consumption of c affects current utility through a process of learning by doing...”) We prefer to use the definition proposed by Ryder and Heal [41], which explicitly talks about habits in determining this index of past consumption.) denoted as z . To enlighten notation, we omit the time indication for time varying variables, except when integrating over time. For instance, $z(t)$ would generally be noted as simply z . An individual’s degree of habit is represented by the stock variable z , which accumulates according to a dynamic process. Generally, this process depends on the personal characteristics of the individual and on the consumption level of the addictive good c . The endogeneity of the discount rate implies that the rate of time preference is also endogenous, making the analysis more interesting in terms of optimizing behavior.

In this framework, consumers maximize their felicity, which depends on the consumption of two goods and the stock of habits. The first good, g , is a composite good, which does not cause addiction. The second good, c , is a potentially addictive good (by which we mean a good which generates addiction when consumed under certain conditions, for example, in high quantity) which generates habits in consumption. Alcoholic beverages, for example, are potentially addictive goods. A moderate consumption of alcohol does not lead to addiction, but excessive consumption does. We assume that felicity also depends on the strength of habits, synthesized by the stock of habits z . In line with Becker and Murphy [8], we assume that this effect is negative. Being a potentially addictive good, c generates habits over time, which in turn causes felicity to decrease, so that a larger amount of c needs to be consumed to maintain the same level of felicity.

To keep things simple, we assume that g does not directly interact with c and z in generating felicity, so that the utility function is additively separable and defined by $v(g) + w(c, z)$. In line with Cawley and Ruhm [42], addiction is characterized by enforcement—the marginal utility of current consumption increases with the stock of habits

($w_{cz} > 0$). We denote the derivative of a generic function $f(x)$ with respect to some variable x ($\partial f(x)/\partial x$) as f_x . The second derivative ($\partial^2 f(x)/\partial x^2$) is denoted as f_{xx} .—tolerance—and the stock of habits lowers utility ($w_z < 0$)—and withdrawal—as the current consumption of the addictive good increases utility ($w_c > 0$). Other usual regularity conditions impose $v_g > 0$, $v_{gg} \leq 0$, $w_{cc} \leq 0$, and $w_{zz} \geq 0$.

The endogenous specification of the discount rate closely follows Epstein and Shi [31]. In each instant in time, the discount rate is defined by a discount function $\theta(z)$ which depends on the stock of habits. The discount function $\theta(z)$ is a twice continuously differentiable function assumed to be strictly positive ($\theta(z) > 0$), strictly increasing ($\theta_z > 0$) and concave ($\theta_{zz} \leq 0$).

Regarding the habit formation process, the literature proposes two approaches, which we call the partial adjustment approach and adaptive approach. We propose these names to remind, respectively, the partial adjustment model and the adaptive expectation model, which have evident similarities with the habits specifications under discussion. The partial adjustment approach consists of considering the habit formation as an investment process. The stock of habits z accumulates as if it was capital. Investment is represented by current consumption and the stock of habits depreciates at rate σ . As in Becker and Murphy [8], “the rate of habits depreciation σ measures the exogenous rate of disappearance of the physical and mental effects of past consumption of c .” The dynamic equation which describes this process is

$$\dot{z} = c - \sigma z. \tag{1}$$

In the adaptive approach, the habits accumulation process is due to the difference between the current consumption of the potentially addictive good c and the current stock of habits z , through the rate of habits adjustment λ . If current consumption c exceeds the current stock of habits z , there will be a formation of further habits, at a rate λ ; otherwise, the stock of habits decreases. The dynamic equation which describes this approach is

$$\dot{z} = \lambda(c - z).$$

In this work, we follow the partial adjustment approach, for consistency with the RA model. However, using the adaptive approach, as in [31], would not substantially alter the results.

In the economic literature there is an open discussion if the endogenous discount rate should be considered as increasing or decreasing with respect to consumption c . Koopmans [43] suggests a decreasing rate of impatience, while Lucas and Stokey [44] observe that an increasing rate of impatience is necessary to obtain a single, stable, non-degenerate equilibrium point into wealth distribution in a deterministic horizon with a finite number of agents. According to Blanchard and Fisher [10], the assumption of an increasing rate of impatience is difficult to defend ex ante. On the other side, Epstein [45,46] argues that the more a person consumes, the more she discounts in the future. In line with Epstein, we assume that the endogenous discount rate, $\theta(z)$, is strictly increasing with respect to the stock of habits, and hence consumption. This assumption does not imply an always increasing discount rate. The discount rate may also decrease if, for instance, the consumer quits consuming the addictive good. In this case, the stock of habits z smoothly depreciates at rate σ , and consequently also lowers the discount rate.

This condition is necessary to ensure the stability of the long-run optimal consumption plan, because it guarantees that consumption levels in different dates are substitutes. In this case, as wealth and consumption rise, the marginal private return to further savings, which depends on the marginal utility of future consumption, falls. A discount rate decreasing in consumption would cause consumption in different dates to be complements, so that an increase in present consumption rises the marginal utility of future consumption.

The implication of a discount rate strictly increasing with respect to consumption is that a higher consumption level at time t increases the discount rate applied to utilities after t . An increase in current consumption in t induces an increase in the rate of time

preference: the consumer’s desire to anticipate the effects of future consumption is picked up by a larger consumption in $t + 1$. An increase in current consumption in $t + 1$ rises the stock of habits in $t + 2$, inducing a further increase in the discount rate: the larger the previous consumption is, the larger the habit and the larger the current consumption must be to deliver the same utility. Moreover, an increase in the discount rate rises the degree of adjacent complementarity and hence strengthens the commitment to all habits.

Defining a cumulative subjective discount rate Θ as (We use this formulation because it allows for some mathematical simplifications. This specification is equivalent to using the integral, from period 0 to t , of the discount function. In fact, $e^{-\Theta}e^{-rt} = e^{-\int_0^t (\theta(z(s)) - r) ds} e^{-rt} = e^{rt - \int_0^t \theta(z(s)) ds} e^{-rt} = e^{-\int_0^t \theta(z(s)) ds}$)

$$\Theta = \int_0^t \theta(z(\tau)) - r d\tau, \tag{2}$$

the consumer problem can be written as

$$\begin{aligned} \max_{g(t), c(t)} \int_0^\infty (v(g(t)) + w(c(t), z(t))) e^{-\Theta(t)} e^{-rt} dt \\ \text{s.t. } \dot{a} = ra - g - pc, \quad a(0) \geq 0 \text{ given} \\ \dot{z} = c - \sigma z, \quad z(0) \geq 0 \text{ given,} \\ \dot{\Theta} = \theta(z) - r \quad \Theta(0) = 0 \end{aligned} \tag{3}$$

where

- $v(g)$ and $w(c, z)$ are the instantaneous utility functions;
- g is current consumption of the non-addictive good;
- c is current consumption of the addictive good;
- z is the stock of habits;
- Θ is the cumulative discount rate;
- a is real wealth;
- p is the relative price of the addictive good (price of $g(t)$ is normalized to 1);
- r is the rate of return to savings;
- σ is the rate of habits depreciation;

and the rate of habit depreciation σ is assumed to be bounded between 0 and 1.

In (3), consumption goods g and c are control variables, while real wealth a , the stock of habits z and the cumulative subjective discount rate Θ are state variables.

This specification nests several models of habit formation and addiction (for a sketch of the proof, see Appendix A), such as:

- The Ramsey model [1,43,47], characterized by a constant rate of time preference;
- The rational addiction model of Becker and Murphy [8], characterized by a constant discount rate and additive utility function with habits;
- The multiplicative habits model proposed by Carroll [48], characterized by a constant discount rate and a multiplicative utility function with habits;
- The Uzawa [49] or Obstfeld [50] models, that assume an endogenous discount rate depending on current consumption, with no explicit modeling of habit formation;
- The Epstein and Shi [31] model, characterized by an endogenous discount rate depending on habits.

The optimization problem is solved according to the maximum principle of Pontriagin (optimal control theory), and the present value Hamiltonian function (note that the relation between the current value and present value Hamiltonian is $H_d = e^{rt} H$, where H_d is the current value Hamiltonian, H is the present value Hamiltonian and r is the rate of return to savings) is

$$H_d = e^{-\Theta} (v(g) + w(c, z)) + \tilde{q}(ra - g - pc) - \tilde{\varphi}(\theta(z) - r) + \tilde{\Psi}(c - \sigma z),$$

where $\tilde{q} = e^{rt}\hat{q}$, $\tilde{\varphi} = e^{rt}\hat{\varphi}$ and $\tilde{\Psi} = e^{rt}\hat{\Psi}$ are the discounted costate variables.

The first-order necessary conditions for an interior solution are (to save notation, partial derivatives are denoted with a subscript. Thus, for example, $\partial w(c, z)/\partial c = w_c$)

$$\begin{aligned} \frac{\partial H_d}{\partial c} = 0 &\longrightarrow \tilde{q}p = w_c e^{-\Theta} + \tilde{\Psi} \\ \frac{\partial H_d}{\partial g} = 0 &\longrightarrow \tilde{q} = v_g e^{-\Theta} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial H_d}{\partial a} = r\tilde{q} - \dot{\tilde{q}} &\longrightarrow \dot{\tilde{q}} = r\tilde{q} - r\tilde{q} = 0 \\ \frac{\partial H_d}{\partial \Theta} = r\tilde{\varphi} - \dot{\tilde{\varphi}} &\longrightarrow \dot{\tilde{\varphi}} = r\tilde{\varphi} - (v(g) + w(c, z))e^{-\Theta} \\ \frac{\partial H_d}{\partial z} = r\tilde{\Psi} - \dot{\tilde{\Psi}} &\longrightarrow \dot{\tilde{\Psi}} = (r + \sigma)\tilde{\Psi} + \tilde{\varphi}\theta_z - w_z e^{-\Theta}. \end{aligned}$$

It is convenient to re-scale the co-state variables in order to eliminate Θ . Letting $q = \tilde{q}e^{\Theta}$, $\varphi = \tilde{\varphi}e^{\Theta}$ and $\Psi = \tilde{\Psi}e^{\Theta}$, the first-order necessary conditions become

$$q = \frac{1}{p}[w_c + \Psi] \tag{4}$$

$$q = v_g, \tag{5}$$

and, given that $q = \tilde{q}e^{\Theta}$, and

$$\dot{q} = \dot{\tilde{q}}e^{\Theta} + \tilde{q}e^{\Theta}\dot{\Theta} = 0 + q\dot{\Theta},$$

the remaining conditions are

$$\dot{q} = q(\theta(z) - r) \tag{6}$$

$$\dot{\varphi} = r\varphi - (v(g) + w(c, z)) \tag{7}$$

$$\dot{\Psi} = (r + \sigma)\Psi + \varphi\theta_z - w_z. \tag{8}$$

Differentiating Equations (4) and (5) with respect to time, we obtain

$$\dot{q} = \frac{1}{p}(w_{cc}\dot{c} + w_{cz}\dot{z} + \dot{\Psi}) \tag{9}$$

$$\dot{q} = v_{gg}g, \tag{10}$$

and using Equation (10) with (6), we obtain the following Euler equation for \dot{g}

$$\frac{\dot{g}}{g} = \frac{v_g}{v_{gg}}(\theta(z) - r) \tag{11}$$

Note that, by equating Equations (4) and (5), it is possible to obtain a simple analytical expression for Ψ , which is

$$\Psi = pv_g - w_c. \tag{12}$$

The differential Equation (7) gives a continuous time specification of the recursive structure of consumer preferences for every feasible consumption path. If we solve the differential Equation (7) (recall that the solution for a differential equation with no constant coefficients as $\dot{y} + Py = Q$ is $y = e^{-\int P(t)dt} \int Q(t)e^{\int P(t)dt} dt + Ce^{-\int P(t)dt}$. The value that

the solution approaches is referred to as the steady state so the limit for $t \rightarrow \infty$ of the solution is $y = \int Q(t)e^{\int P(t)dt} dt$), we obtain

$$\varphi = \int_t^\infty (v(g(\tau)) + w(c(\tau), z(\tau)))e^{-\int_t^\tau \theta(z(s))ds} d\tau, \tag{13}$$

which is the present value of future utilities at time t which corresponds to the shadow price of the accumulated impatience rate Θ .

By equating the two equations for \dot{q} , (6) and (9), we can solve for \dot{c} and find the Euler equation

$$\frac{\dot{c}}{c} = \eta^c(g, c, z)(\rho^c(g, c, z, \varphi) - r),$$

where, by means of Equations (1), (8), (5) and (12), the rate of time preference for good c is

$$\rho^c(\cdot) = \theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} \tag{14}$$

and the elasticity of intertemporal substitution is

$$\eta^c(\cdot) = \frac{pv_g}{w_{cc}c}.$$

In summary, the resulting Euler equations for g and c are

$$\frac{\dot{g}}{g} = \frac{v_g}{v_{gg}g}(\theta(z) - r) \tag{15}$$

$$\frac{\dot{c}}{c} = \frac{pv_g}{w_{cc}c} \left(\theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} - r \right). \tag{16}$$

In light of Equation (13), the shadow price of the cumulative discount rate Θ may be seen as an index of impatience. In fact, since φ is the weighted sum of all future utilities from time t , the higher future utilities are, the higher the patience is, since the agent is willing to wait for the realization of their desires of consumption. For this reason, from now on, we will refer to φ as the *rate of impatience*.

This formulation, which involves the non-separability of preferences, is suggested by Ryder and Heal [41], who introduced the notion of adjacent complementarity. Adjacent complementarity occurs when the past consumption of a good raises the marginal utility of present consumption. An increase or decrease in consumption at $t - 1$ can induce a variation of the marginal rate of substitution of current and future consumption at $t + 1$. Complementarity is represented by a utility function that depends on both current consumption c and the stock of habits z , which is a weighted average of past consumption levels. Weights decline exponentially in the past at the exogenous depreciation rate σ , which can be interpreted as a measure of permanence of physical and mental effects of past consumption on present consumption c . As σ becomes larger, less weight is given to past consumption in determining z .

The two Euler equations in (15) and (16) implicitly define the rates of time preference and the elasticities of inter-temporal substitution for the two goods. The rate of time preference for g is simply equal to the discount function $\theta(z)$, as for any standard inter-temporal model of consumption. However, the discount rate is not constant and depends on the stock of habits z , implying that the rate of time preference for g depends on the past consumption of c . The inter-temporal elasticity of substitution for g is completely defined by the preference structure, and in particular, by the shape of $v(g)$. The rate of time preference for c , shown in Equation (14), embeds:

1. Memory of past events through the stock of habits z and the rate of habits depreciation σ ;
2. Perception of present events by the current consumption levels of c and g ;
3. The anticipation of future events by the present-value of future utilities φ .

Consumer behavior is non-separable over time, revealing complementarity and time inconsistency. The present consumption of the potentially addictive good c and future consumption (represented by φ) depends on the past consumption of the addictive good through the rate of habits depreciation σ , and does not need to be valued equally along a locally constant consumption path. The rate of time preference expresses the propensity that a person reveals towards future utility in determining current choices. This depends on the ability to anticipate the benefits of future consumption and the related physical and mental consequences of present and past consumption effects.

The Euler equation for c is different from canonical expressions mainly because it endows the complementarity between past consumption z , current consumption c and g , and future consumption by the rate of impatience φ through the endogenous rate of time preference and the elasticity of inter-temporal substitution.

3. The Steady State

Given the Euler Equations (15) and (16), the system of differential equations which describes the dynamic behavior of the ERA model is

$$\dot{z} = c - \sigma z \tag{17}$$

$$\dot{\varphi} = r\varphi - (v(g) + w(c, z)) \tag{18}$$

$$\dot{c} = \frac{pv_g}{w_{cc}} \left(\theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} - r \right) \tag{19}$$

$$\dot{g} = \frac{v_g}{v_{gg}} (\theta(z) - r) \tag{20}$$

$$\dot{a} = ra - g - pc. \tag{21}$$

Defining the steady state as an optimal solution to program (3) in which the stock of habits z and wealth a do not change over time, ($\dot{z} = 0, \dot{a} = 0$), a steady state for system (17)–(21) exists and lies on the optimal consumption path. This definition comes from the consideration that it is physically implausible that a substance continues to generate habits to the infinite, otherwise, to keep a certain level of utility, the consumer should continuously increase the consumption of the addictive good. This, in a world of finite resources, is rather unlikely, although not impossible. If an equilibrium exists where $\dot{z} = 0$, it brings a number of consequences. First, if the stock of habits does not grow, then the consumption of the addictive good c also does not grow at the steady state. In fact, from Equation (17), we find that $c^* = \sigma z^*$ (with the superscript $*$, we denote the variables at their steady state value). Second, if z is constant, Θ is also constant because of Equation (26), and thus $\theta(z)$ is constant and equal to r , as implied by the cumulative discount rate motion equation and $\dot{\Theta} = 0$.

These considerations lead to a first important consequence. At the steady state, the stock of habits z^* is uniquely determined by the discount function $\theta(z)$ and the interest rate r . Hence, the steady state consumption level c^* , which is determined by the stock of habits z^* and the rate of habits adjustment σ , also depends on the shape of the discount factor and the interest rate, but not on the structure of preferences.

The fact that c^* and z^* are constant implies that utility $w(c^*, z^*)$ is constant at the steady state, and so must be the index of impatience φ , which is the discounted value of the future streams of utilities. From Equation (18), considering that $\dot{\varphi} = 0$, we obtain

$$\varphi^* = \frac{v(g^*) + w(c^*, z^*)}{r}. \tag{22}$$

The fact that, at the steady state, $\theta(z) = r$ has another important consequence: g does not grow at the steady state. A condition for this to happen is that marginal utility v_g is constant, whereby Equation (5) implies that q is also constant, i.e., $\dot{q} = 0$. From condition (4), we find that

$$q^* = \frac{1}{p}(w_c + \Psi^*), \tag{23}$$

which in turn, by condition (5), defines the steady state value of g^* .

Finally, assuming that, at the steady state, real wealth a does not grow, from Equation (21), we obtain the steady state value of wealth

$$a^* = \frac{g^* + pc^*}{r}.$$

To date, we determined the steady state levels of the variable involved in the dynamic system of the ERA model. To verify the existence of the steady state and that the steady state is an optimum solution to the maximization program, we can use Equation (19). We know that in the steady state, \dot{c} must be zero. For this to be verified, the term in square brackets must also be zero, and since $\theta(z^*) - r = 0$, we can concentrate on the term

$$w_{cz}^*(c^* - \sigma z^*) + (r + \sigma)(pv_g^* - w_c^*) + \theta_z^* \varphi^* - w_z^*.$$

Considering that, from (17), $(c^* - \sigma z^*)$ is 0 and substituting Equation (12), we obtain

$$(r + \sigma)\Psi^* + \theta_z^* \varphi^* - w_z^*,$$

and using Equation (12), Equation (8) can be set to 0, and

$$\Psi^* = \frac{w_z^* - \theta_z^* \varphi^*}{r + \sigma},$$

which implies that

$$(r + \sigma) \frac{w_z^* - \theta_z^* \varphi^*}{r + \sigma} + \theta_z^* \varphi^* - w_z^* = 0.$$

Then, the steady state lies on the optimal solution to the ERA model. The steady state defined as $(\dot{z} = 0, \dot{a} = 0)$ is unique. To verify the uniqueness of the steady state, we consider the case in which \dot{z} is still 0, but assume that this hypothetical steady state level of $z^\#$ is such that $\theta(z^\#) - r \neq 0$, i.e., $z^\#$ can be any value different from z^* . This implies that at the steady state, the cumulative discount rate Θ constantly changes because $\dot{\Theta}$ is constant. By Equation (20), g also constantly changes at the steady state.

Since z is constant, by Equation (17), the consumption of addictive good c must also be constant. The fact that g grows indefinitely at the steady state while c does not suggests that this hypothetical steady state may not satisfy all optimality conditions, so we need to verify that $\dot{c} = 0$ even when $\theta(z^\#) - r \neq 0$. The steady state levels of φ and Ψ are not constant and will change at a constant rate proportional to \dot{g} . From (19), the necessary condition for $\dot{c} = 0$ is

$$\theta(z^\#) - r - \frac{w_{cz}^\#(c^\# - \sigma z^\#) + (r + \sigma)(pv_g^\# - w_c^\#) + \theta_z^\# \varphi^\# - w_z^\#}{pv_g^\#} = 0,$$

but since $(pv_g - w_c) = \Psi$ and $\dot{\Psi} \neq 0$, using Equation (11), the expression can be written as

$$\frac{pv_{gg}\dot{g} - (r + \sigma)\Psi - \theta_z^\# \varphi^\# + w_z^\#}{pv_g^\#},$$

which, by Equation (20), is 0 if and only if

$$(r + \sigma)\Psi + \theta_z^\# \varphi = w_z^\# + \theta(z^\#) - r. \tag{24}$$

Note that the right-hand side of Equation (24) is constant, which would either imply that both Ψ and φ were constant and take some specific values, which is not the case, or that

$$\Psi = \frac{w_z^\# + \theta(z^\#) - r - \theta_z \varphi}{r + \sigma},$$

which, again, is not the case. This implies that this alternative steady state is infeasible. It follows that, if we define the steady state as an optimal solution to the optimization problem (3) in which the stock of habits is stationary ($\dot{z} = 0$), the unique feasible solution implies that the discount rate $\theta(z^*)$ has to be equal to the interest rate r . Other steady states may exist when the definition of the steady state is different. However, at least we proved the existence and uniqueness of at least one steady state. This, in turns, defines the steady state value of the stock of habits z^* and the consumption of the addictive good c^* purely as a function of the habit-generating process (through σ) and the discount function shape. This sounds intuitively reasonable, since a steady level of the stock of habits could be reached only if the process that generate habits stabilizes and does not affect preferences for the addictive good.

4. Local Stability Properties

The system described by (17)–(21) generates a five-dimensional hyperplane which is divided into a number of regions ($2^5 = 32$, to be precise), each of which is characterized by a force leading the system toward the steady state or away from it.

We can reduce the dimension of the system, noting that a is only present in the wealth accumulation equation. Since a does not influence other dynamic equations, we can consider the accumulation equation of wealth (21) as exogenous to the system and drop it from the stability analysis. Because a is recursively determined for each value of c and g by Equation (21), when considered in the system, it would imply an eigenvalue equal to r , which would not influence the stability properties of the system. The new system generates a four-dimensional hyperplane, which allows for an easier mathematical analysis.

Rewriting Equation (19) as

$$\dot{c} = ce^c(\cdot)(\rho(\cdot) - r)$$

where

$$\begin{aligned} \rho(\cdot) &= \rho(g, c, z, \varphi) \\ \theta(z) &= \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} \\ \varepsilon^c(\cdot) &= \frac{pv_g}{w_{cc}c}; \end{aligned}$$

Equation (20) as

$$\dot{g} = g\varepsilon^g(\cdot)(\theta(z) - r)$$

where

$$\varepsilon^g = \frac{v_g}{v_{gg}g};$$

and making each dynamic equation equal to 0 at the steady state (and since every function is intended to be evaluated at the steady state), the system can be written as

$$\begin{aligned}
 (z) \quad & 0 = c^* - \sigma z^* \\
 (\varphi) \quad & 0 = r\varphi^* - w(c, z) - v(g) \\
 (c) \quad & 0 = c^* \varepsilon^c(\cdot)(\rho(\cdot) - r) \\
 (g) \quad & 0 = g^* \varepsilon^g(\theta(z) - r). \\
 (a) \quad & 0 = ra^* - g^* - pc^*.
 \end{aligned}$$

To analyze local stability, we first drop the (a) equation, since the corresponding eigenvalue is trivial and equal to r , and then we linearize the system by a first-order Taylor expansion, obtaining

$$\begin{aligned}
 \dot{z} &\cong c^* - \sigma z^* & (25) \\
 \dot{\varphi} &\cong r(\varphi - \varphi^*) - w_c(c - c^*) - w_z(z - z^*) - v_g(g - g^*) \\
 \dot{c} &\cong A(c - c^*) + B(g - g^*) + C(z - z^*) + D(\varphi - \varphi^*) \\
 \dot{g} &\cong E(g - g^*) + \theta_z(z - z^*)
 \end{aligned}$$

where $A = (\varepsilon^c(\cdot) + c^* \varepsilon_c^c(\cdot))(\rho(\cdot) - r) + c^* \varepsilon^c(\cdot) \rho_c(\cdot)$, $B = c^* \varepsilon^c(\cdot) \rho_g(\cdot)$, $C = c^* \varepsilon^c(\cdot) \rho_z(\cdot)$, $D = c^* \varepsilon^c(\cdot) \rho_\varphi(\cdot)$ and $E = \varepsilon^g(\theta(z) - r) + g \varepsilon_g^g(\theta(z) - r)$. The resulting Jacobian matrix J is

	z	φ	c	g
z	$-\sigma$	0	1	0
φ	$-w_z$	r	$-w_c$	$-v_g$
c	C	D	A	B
g	θ_z	0	0	E

The stability properties of system (25) are determined by the eigenvalues of matrix J , through its characteristic polynomial

$$\alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2 + \alpha_3 \lambda^3 + \lambda^4,$$

where

$$\begin{aligned}
 \alpha_0 &= r\theta_z B + v_g \theta_z D - E(rC + r\sigma A + \sigma w_c D + w_z D) \\
 \alpha_1 &= C(r + E) - AE(r - \sigma) + r\sigma(A + E) + D(\sigma w_c + w_z - w_c E) - \theta_z B \\
 \alpha_2 &= (r - \sigma)(A + E) + AE - r\sigma + w_c D - C \\
 \alpha_3 &= \sigma - r - A - E.
 \end{aligned}$$

The stability of the system is determined by the number of stable and unstable eigenvalues for matrix J , and in particular, if all eigenvalues are real and positive, the steady state is an unstable point; if they are all real and negative, the steady state is a stable point; if they are all real but some are positive and some negative, the steady state is a saddle node; if some of them are complex, the steady state is a saddle focus. The saddle node is the multidimensional equivalent of the saddle point. The system will converge towards the steady state only if the starting point lies on the stable eigenspace. In the saddle, focus convergence (if the real part of the eigenvalue is negative) or divergence (if the real part of the eigenvalue is positive) will be cyclical.

The roots of the characteristic polynomial correspond to the eigenvalues of matrix J . We know that an analytical solution exists for the roots of polynomials of the fourth degree, and that it is given by the Ferrari method. However, the analytical solution and the relative stability analysis are by far too complex to be meaningful, even if we decided to use the Routh–Hurwitz stability criterion.

It is clear that the existence and stability of the steady state depends on the parameters values. For example, if the discount function $\theta(z)$ is always greater than r , a steady state is not possible because of the increasing property of $\theta(z)$. For this reason, to characterize the stability properties of the steady state, we conduct a numerical simulation. The strategy of the simulation is as follows: first, we choose a functional form for the utility functions and the discount function; second, we select plausible numerical values for the parameters of the system and compute the steady state; third, we let the parameters vary and calculate the numerical value of the eigenvalues of matrix J .

In the choice of the functional form of the utility function, we depart from Becker and Murphy [8]. In fact, a quadratic utility function is characterized by a point of maximum and the non-satiation property, commonly assumed in consumption studies, would not hold. The model could still reach the steady state, if lower than the global maximum of utility, but this would imply an additional set of restrictions on the parameters which we prefer to avoid. Instead, we use logarithmic utility functions, and, for simplicity, a linear specification for the discount function:

$$\begin{aligned} \theta(z) &= \theta_1 + \theta_2 z & (26) \\ w(c, z) &= \beta \ln c - \gamma \ln z \\ v(g) &= \alpha \ln g. \end{aligned}$$

The discount function parameters θ_1 and θ_2 are both chosen to be 0.02. Utility parameters α and β are set to 0.8, while γ is 0.2. The relative price of the addictive good p is fixed to 1.2, the rate of returns to savings r is equal to 0.05 and the rate of habits depreciation σ is 0.2. To evaluate the stability of the steady state, we allow each parameter to vary.

The results of the simulations (The table with all eigenvalues is available upon request, together with the program that generates the results.) indicate that the system is a saddle node in the neighbor of the parameters values. The eigenvalues are all real—three are positive and one is negative. This means that the system monotonically approaches the steady state, provided that the starting point lies on the stable eigenfold. This situation is depicted in Figure 1. This phaseplot for the variable current consumption of the addictive good c_t and the current value of future utilities ϕ_t represents in red the stable eigenfold and black arrows represent the forces driving the system away from the steady state if out of the stable eigenfold.

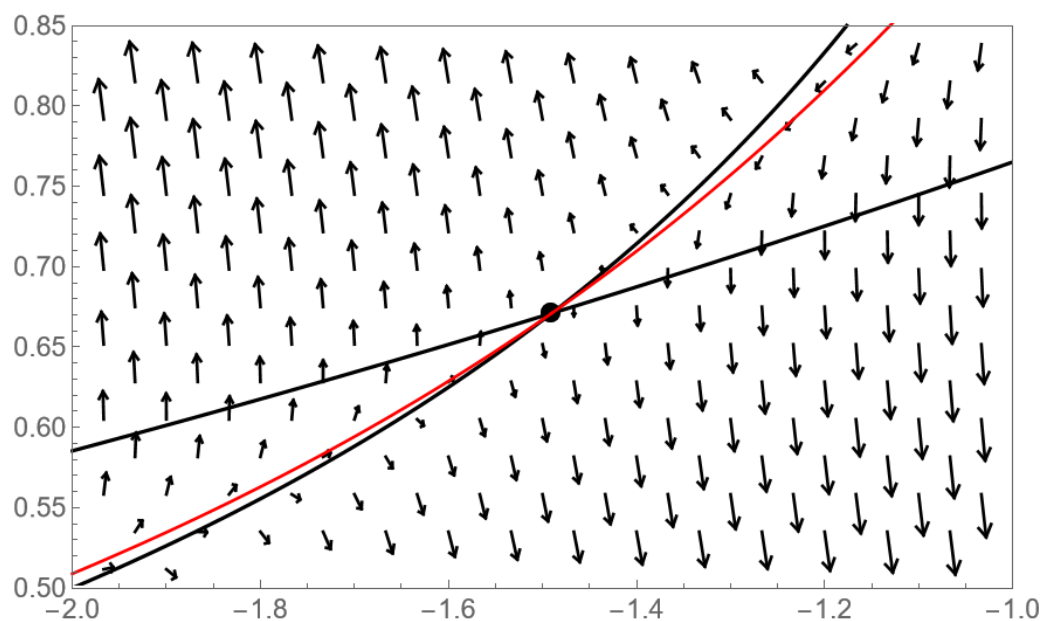


Figure 1. Phaseplot of the steady state.

The stability of the system with respect to the variations of the parameters is less trivial. For example, the steady state keeps being a saddle node for values of σ in the range $(0.1, 0.25)$. For values smaller than 0.1 or greater than 0.25 and smaller than 0.47, the equilibrium is an unstable focus, with a cyclical behavior that diverges from equilibrium. Then, it becomes a stable focus and for values greater than 0.55, again a saddle node. Aside from the numerical values, that depend on the actual choices for the parameters and the consequent steady state's values, this means that the rate of habit depreciation is a key parameter for the behavioral interpretation of the model, and a fundamental ingredient to analyze heterogeneity in habit formation. Normal people with reasonable rates of habit depreciation can find an equilibrium in the consumption of the addictive substance and enjoy a moderate consumption. People with a small σ may be more likely to develop an addiction and for them, a stable equilibrium might not be possible. The cyclical fluctuations of the equilibrium for values in between 0.25 and 0.55 could be interpreted as binge drinking, while a very large rate of habits depreciation may not result in problematic behaviors.

Other parameters that could possibly be of interest from the behavioral point of view are the discount function parameters. In particular, θ_2 determines the degree of dependence of the discount rate on the stock habits z . The discount rate corresponds to the rate of time preference of g and is one of the main determinants of that of c . We observe that for very small values of the parameter, the system becomes an unstable focus. This seems reasonable since one of the equations that must be verified at the steady state is $\theta(z) - r = 0$. If $\theta(z)$ does not grow sufficiently quickly with z , then the equilibrium could be unstable. On the other side, for any value larger than the chosen one, the system behaves as a saddle node.

Furthermore, the disutility caused by the stock of habits, through the utility parameter γ , plays an important role. We observe that the equilibrium becomes an unstable focus with a cyclical path if the disutility of habits is small (below 0.1). This may indicate that people with an insufficient perception of the health issues related to the consumption of alcohol, for example, may fall into binge drinking. For larger values, the equilibrium is always a saddle node. Other parameters' variations do not show unexpected results, and in general, result in a saddle node equilibrium for wide intervals.

A similar result was observed using isoelastic utility functions, while, as mentioned above, the case of a quadratic utility function is slightly more complicated. Our simulations suggest that the model behaves as in the previous cases in several situations, however, depending on the parameters of the utility function, the steady state may be infeasible, since it can lie above the global maximum of the utility functions.

5. Heterogeneity in the Habits Formation Process

Heterogeneity is a necessary feature for almost any work on consumer demand. In fact, it is a common opinion that each individual has their own preference structure and that a correct demand analysis should take this into account. When considering a dynamic context, heterogeneous preferences can be captured through the rate of time preference, as suggested by Fuchs [51] and Lawrance [52]. Following this path, in this section, we concentrate on introducing heterogeneity through the rate of time preference for c .

The definition of an endogenous rate of time preference permits separating the effects of heterogeneity from a generic habit effect. This is achieved through the definition of a rate of habits depreciation σ as a function of personal characteristics d . This implies that each individual develops habits and eventually addiction with a different consumption-habits path and achieve a different steady state. In particular, in Section 3, we have seen how the steady state level of the stock of habit z^* only depends on the discount function $\theta(z)$ and on the interest rate r . As a consequence, the steady state level of consumption of the addictive good c^* , which is equal to $\sigma(d)z^*$, depends on the individual characteristics of the consumer d . Moreover, these characteristics may not be constant over time, as for age, which, as suggested by Bishai [13], may influence the rate of time preference.

An example can help one understand our hypotheses. Define two categories of agents which constitute society, e.g., myopic (m) and forward-looking (f) (any resemblance between the symbols used to indicate the two kind of agents and those used to indicate males and females is purely coincidental). An agent is myopic if they are characterized by strong a preference towards actual consumption, with a large degree of impatience. The impatience is caused by a large discount rate $\theta(z)$, which in turn is caused by a large stock of habits z . A large stock of habits may be caused by a rapid growth of z induced by a small rate of habits depreciation $\sigma(d)$.

An agent is forward-looking if they are characterized by a preference towards future consumption, with a small degree of impatience. The discount rate $\theta(z)$ is small, due to a small stock of habits z , which remains small thanks to a relatively large rate of habits depreciation $\sigma(d)$.

The likelihood that they reveal addiction is picked up by the rate of habit depreciation $\sigma^f(d)$ and their preferences towards alcohol consumption and habits. The larger is $\sigma(d)$, and less weight is given to past habits in determining current habits z . Therefore, even if preferences towards alcohol are strong, the large value of $\sigma^f(d)$ and the strong adversity against health injuries caused by habits, can prevent falling into addiction. Two simulations with a plausible value of $\sigma(d)$ to ensure a saddle node steady state are depicted in Figure 2. Here, the forward-looking agent is characterized by a larger σ , which ensures a lower level of consumption of the addictive good and achieves the steady state later in life.

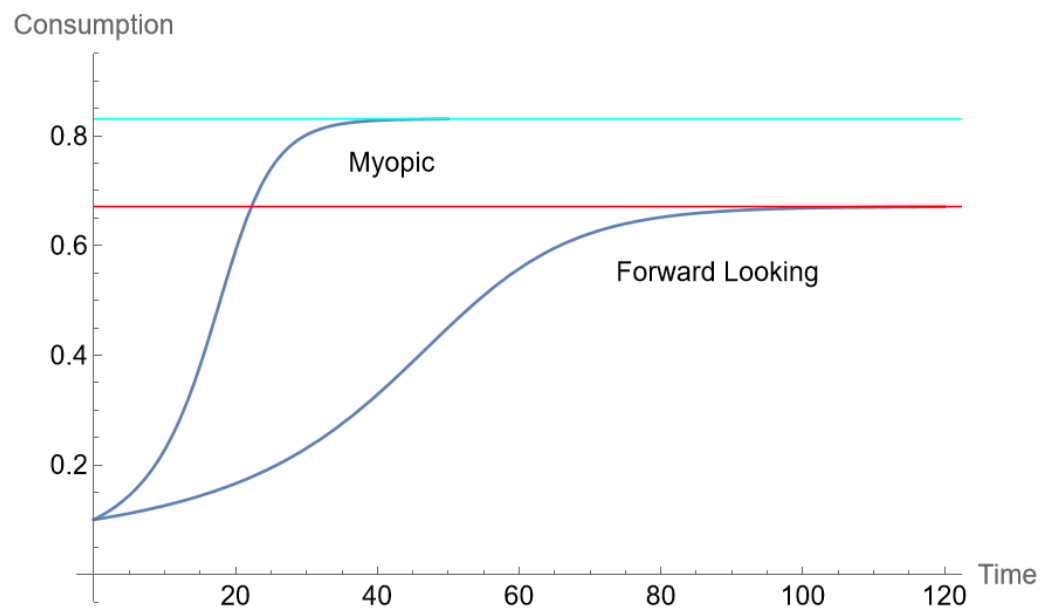


Figure 2. Heterogeneity analysis: paths to the steady states for a myopic (small σ) and a forward-looking (large σ) individuals.

Regarding the shape of the rate of habit depreciation $\sigma(d)$, to ensure the needed regularity properties of the maximization problem, it is necessary to bound it between 0 and 1. In fact, allowing for a negative rate of habit depreciation would rule out the existence of a steady state since habits would arise and continuously accumulate over time even if the consumption of the addictive good happened just once. On the other hand, for values of $\sigma(d)$ greater than 1, the stock of habits z would respond more than proportionally to a change in consumption c , allowing for a negative stock of habits z which would have no sense from a behavioral point of view. The limiting case of $\sigma(d) = 1$ implies that the stock of habits fully depreciates each period, thus excluding the possibility that habits accumulate and addiction ensues.

Other characteristics of $\sigma(d)$, which are not relevant for obtaining an interior solution to the optimization problem, such as slope and concavity, may have important implications for the behavioral analysis.

For example, one appealing feature could be an inverse-U-shape with respect to age, which would imply that an individual is more likely to develop addiction when young or elder, rather than when they are middle-aged. Another example is provided by recent medical evidence that suggests that men and women have different a predisposition to addiction. For example, a research conducted by Mancinelli et al. [53] links the stronger effects produced by alcohol abuse, with a scarce presence of alcohol dehydrogenase (ADH), which is an enzyme involved in the metabolization of alcohol. It is observed that, in general, women have smaller quantities of ADH than men. This may lead to a physiological differences in the process that generates addiction, in which women may be more exposed to the risk of become addicted. However, even if women need less alcohol than men to become addicted to alcohol, they may have different preferences with respect to both alcohol, which may be less valued, and to habits, regarded as the negative health effects that habits may bring, which could be greater valued (in negative). The result may be that there is a smaller probability that women become addicted, since, even if women are more likely to become addicted for a given amount of consumed alcohol, they may be much less inclined to consume than men.

6. Comparative Dynamics

The comparative dynamic analysis investigates how a variation of a variable or parameter of interest, such as c, z, φ and σ , affects the endogenous rate of time preference $\rho(g, c, z, \varphi)$ in the neighborhood of the steady state.

The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to current consumption c . In the neighborhood of the steady state $c - \sigma z \approx 0$, the partial derivative of the rate of time preference with respect to current consumption is thus

$$\frac{\partial}{\partial c} \rho(\bar{g}, \bar{c}, \bar{z}, \bar{\varphi}) = -\frac{1}{pv_g} (w_{cz} - (r + \sigma)w_{cc} - w_{zc}) = \frac{(r + \sigma)w_{cc}}{pv_g} < 0.$$

The rate of time preference is decreasing with respect to an increase in the current consumption of the possibly addictive good $c(t)$. This implies that the consumer is more concerned with immediate consumption rather than future consumption. In such cases, the need to “save against a rainy day” becomes less urgent and there is higher willingness to consume today. In other words, a rise in current consumption causes an increase in the level of impatience. The rate of time preference $\rho(g, c, z, \varphi)$ strictly decreases with respect to the stock of habits z . In the neighborhood of the steady state $c - \sigma z \approx 0$, the partial derivative of the rate of time preference with respect to the stock of habits is impatience

$$\frac{\partial}{\partial z} \rho(\bar{g}, \bar{c}, \bar{z}, \bar{\varphi}) = \theta_z - \frac{1}{pv_g} (-w_{cz}(r + 2\sigma) + \theta_{zz}\varphi - w_{zz}) > 0.$$

The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to the rate of impatience φ . The partial derivative of the rate of time preference with respect to the rate of impatience is

$$\frac{\partial}{\partial \varphi} \rho(\bar{g}, \bar{c}, \bar{z}, \bar{\varphi}) = -\frac{\theta_z}{pv_g} < 0.$$

An increase in the rate of impatience φ indicates that the consumer is less impatient, giving more weight to future consumption. Hence, the rate of time preference is reduced and the consumption of the addictive good grows at a smaller rate. The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to the rate of the habit depreciation

σ if $w_c < pv_g$. In the neighborhood of the steady state $c - \sigma z \approx 0$, the partial derivative of the rate of time preference with respect to the rate of habits depreciation is thus

$$\frac{\partial}{\partial \sigma} \rho(\bar{g}, \bar{c}, \bar{z}, \bar{\varphi}) = -\frac{1}{pv_g} (pv_g - w_c) < 0 \quad \text{iff} \quad w_c < pv_g.$$

An increase in the rate of habits depreciation implies that the rate of time preference declines, i.e., a decrease in the habit effects of the possibly addictive good tends to reduce the growth of consumption of c . This is true provided that the marginal utility of the possibly addictive good w_c is smaller than the marginal utility of the non-addictive good, pv_g . Hence, unless preferences are biased towards the addictive good, an increase in the rate of habit depreciation lowers the rate of time preference, reducing the growth path of c .

The analysis of the variation of the rate of time preference with respect to a variation of the stock of habits is more complex, with no meaningful results.

The analytical and behavioral properties of the rate of time preference allow us to describe the dynamic evolution of an agent from a condition of potential habit to a state of addiction.

Reconsider the case of the myopic and forward-looking agent introduced above. The degree of impatience of the myopic agent is generally higher (lower φ) than the degree of the forward-looking and so will be the degree of habits ($\sigma^m < \sigma^f$). The propensity to exchange current for future consumption becomes less and less considerable. The myopic agent reveals an increasing impatience since their stock of habits is larger.

The subjective rate of time preference of the myopic agent encloses reinforcement and tolerance, two behavioral factors that are closely related to the concept of adjacent complementarity. Reinforcement means that, to obtain the same level of utility, the consumption of the addictive good has to increase when current consumption increases, while tolerance means that given levels of consumption are less satisfying when past consumption has been greater.

The analysis reveals a patience-dependence trade-off. A patient person tends to have a lower stock of habits than an impatient person, since the desire to anticipate future consumption is lower. It is not surprising that addiction is more likely for people who discount the future heavily since they pay less attention to the adverse consequences. Becker et al. [40] suggested that poorer and younger persons discount the future more heavily while Chaloupka [32] found that less educated persons may have higher rates of time preference. The ability of anticipating the consequences of present and past consumption depends on income, education, rank and degree of awareness of dangers.

In line with Becker and Mulligan [54], we find that "... the analysis of endogenous discount rates implies that even fully rational utility-maximizing individuals who become addicted to drugs and other harmful substances or behavior are induced to place less weight on the future, even if the addiction itself does not affect the discount rate."

7. Conclusions

Traditionally, the economic literature represents the structure of preferences in a dynamic context through utility functions discounted by a constant rate. This choice, often adopted for the sake of mathematical tractability, does not allow explaining situations where the discount rate changes over time for the same individual (see, for example [10–14]).

Starting from the rational addiction model proposed by Becker and Murphy [8], this paper develops an extension which allows for time-inconsistent consumers. Assuming an endogenous discount rate depending on past consumption as adopted in Epstein and Shi [31], this study develops a formulation of intertemporal preferences that generalizes several rational models of habit formation and addiction. The proposed rate of time preference supports a subjective structure of preferences that comprehends the memory of past events, the perception of present events and the anticipation of future events, revealing adjacent complementarity. The behavioral contents delivered by the ERA model are consistent with the results of the theory of rational addiction proposed by Becker and Murphy [8]

but introduce a further dimension in the analysis—an endogenous discount rate—that allows the consumer to be time inconsistent in a more complex way with respect to the quasi-hyperbolic discounting used by [26].

The proposed model presents a steady state in which the stock of habits, consumption of the possibly addictive and non-addictive goods, the shadow price of habits and the index of impatience do not grow. This steady state shows the interesting property that the consumption level of the addictive good and the stock of habits are not determined by the corresponding preferences, but only by the discount function parameters, the interest rate and the rate of habit depreciation.

Numerical simulations show that the steady state tends to be a saddle node, but some parameters variations, in particular the rate of habits depreciation, can introduce instability in the equilibrium by being cyclical. This introduces a strong motivation for analyzing heterogeneity in the consumption of addictive goods, with the possibility of finding the conditions under which habits generate addiction or even binge drinking behavior. A further feature of our specification is that it allows us to analyze how heterogeneity in time preferences may in part arise by how easily individuals accumulate habits over time.

The main limitation of the current study is that the presented model is not suitable—as it currently is—to be used in empirical applications. To overcome this limitation, our future research agenda foresees an econometric implementation of our extended rational addiction model in the form of a discrete time/finite time model which will produce an estimable Euler equation allowing us to test the hypotheses derived in the comparative dynamics analysis of the model and to estimate the subjective individual specific rate of time preferences. This parameter would then be used to calibrate our ERA model and run dynamic simulations reproducing the competitive evolution leading to the prevalence of the patient and forward-looking sub-population over the impatient and myopic sub-population that are more exposed to the risk of addiction. From a policy perspective, this evolutive exercise would show the importance for society to invest in the “production” of young people with a greater endowment of non-cognitive skills such as patience, which may increase the share of forward-looking individuals in the population, much in the spirit of the so-called Heckman equation line of research [55].

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Abbreviations

The following abbreviations are used in this manuscript:

RA Rational addiction
ERA Extended rational addiction

Appendix A

Here, we present a sketch of the proof that the ERA model is encompassing with respect to the models listed in the introduction. We show how imposing restrictions on the parameters or on the utility function of the proposed model is reduced to each of those models.

We start specifying the representative consumer optimization problem for the ERA model, i.e.,

$$\begin{aligned} \max \int_0^\infty u(g(t), c(t), z(t)) e^{-\int_0^t \theta(z(\tau)) d\tau} dt \\ \text{s.t. } \dot{a} = ra - g - pc, \\ \dot{z} = c - \sigma z. \end{aligned}$$

This specification is different from (3) in two ways: the utility function that here is not divided into the non-addictive and addictive parts, and the discount rate, which, as shown in footnote 14, is perfectly equivalent to the alternative specification.

Epstein and Shi. The ERA model reduces to the endogenous discounting model with habits proposed by Epstein and Shi [31] by setting $u(g, c, z) = u(c)$ and assuming an adaptive habits accumulation process. Under these restrictions, the consumer maximization problem is

$$\begin{aligned} \max \int_0^\infty u(c(t)) e^{-\int_0^t \theta(z(\tau)) d\tau} dt \\ \text{s.t. } \dot{a} = ra - c, \\ \dot{z} = \lambda(c - z), \end{aligned}$$

which corresponds to Epstein and Shi [31] (page 63 Equation (2.1)).

Uzawa and Obstfeld. The model proposed by Uzawa [49] and Obstfeld [50] is obtained by assuming that the utility function only depends on the consumption of a non-addictive good, i.e., $u(g, c, z) = u(g)$, and that the endogenous discount rate depends on the current consumption of the non-addictive good. This would be equivalent to set $\theta(z(t)) = \theta(g(t))$. The maximization program, taking into account that the habits accumulation equation must not be considered, becomes

$$\begin{aligned} \max \int_0^\infty u(g(t)) e^{-\int_0^t \theta(g(\tau)) d\tau} dt \\ \text{s.t. } \dot{a} = ra - c, \end{aligned}$$

which correspond to Obstfeld [50] (page 14, Equations (20) and (21)).

Carroll. The multiplicative habits model by Carroll [48] is characterized by a constant discount rate, such that $\theta(z) = \theta$, by a utility function which depends on the ratio between the consumption of the addictive good c and the stock of habits z , so $u(g, c, z) = u(c, z)$, and by an adaptive habits formation process. Even though the author explicitly sets up a CES-like utility function from the beginning and uses a capital investment equation rather than a wealth equation, the following program

$$\begin{aligned} \max \int_0^\infty u(c(t), z(t)) e^{-\theta t} dt \\ \text{s.t. } \dot{a} = ra - c, \\ \dot{z} = \lambda(c - z), \end{aligned}$$

can be considered equivalent to Carroll [48] (page 7, Equations (4) and (5)).

Rational addiction. The rational addiction model proposed by Becker and Murphy [8] is characterized by a constant discount rate and by an utility function which depends on the current consumption of an undifferentiated good g , a possibly addictive good c and the stock of habits z . Hence, the RA model can be easily obtained by setting $\theta(z) = \theta$ and taking into account the fact that the authors prefer to use a lifetime budget constraint rather than a motion equation for wealth. It is worth noting that the authors initially consider an

additional term, the expenditure on endogenous depreciation D , which is not present in the ERA model. Since the authors soon assume $D = 0$, we consider the program

$$\begin{aligned} \max \int_0^{\infty} u(g(t), c(t), z(t)) e^{-\theta t} dt \\ \text{s.t. } \dot{a} = ra - g - pc, \\ \dot{z} = c - \sigma z \end{aligned}$$

to be equivalent to Becker and Murphy [8] (page 677, Equations (2)–(4)).

Ramsey. The Ramsey model, often referred to as the basic consumption model, is characterized by a constant discount rate and a utility function depending on an undifferentiated good. Hence, we can recover it by assuming $u(g, c, z) = u(g)$ and $\theta(z) = \theta$. The resulting maximization problem is

$$\begin{aligned} \max \int_0^{\infty} u(g(t)) e^{-\theta t} dt \\ \text{s.t. } \dot{a} = ra - g, \end{aligned}$$

which is one of the possible representations of the Ramsey model.

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