

Article

New Extensions and Applications of the Modified Chumanov Model for Calculating Entry Capacity of Single-Lane Roundabouts

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Abstract: Over the past decades many models for roundabout capacity have been proposed. Attention to this research sector has never stopped and still today new formulations are always being studied, especially in view of their direct application in design practice. This paper reports the first noteworthy results of a research concerning the Modified Chumanov (MC) model, which can be used to estimate the capacity of single-lane roundabout entrances. After a detailed examination of the original model recommended by the Russian guidelines for small urban roundabouts, the paper proposes some extensions which allow using the revised model even for larger intersections. The MC model also includes some parameterizations that allow its application with different road pavement surface conditions (i.e., dry and wet conditions). The MC entry capacity model, as function of circulating flow and other parameters dependent on geometry and environmental conditions, was compared with 15 widespread models considering a typical medium-sized single-lane roundabout. A validation test was carried out considering four capacity–circulating flow datasets from the literature. The proposed MC model showed good flexibility in adapting to data. This flexibility appears better than the most recent models by Highway Capacity Manual, in the absence of local calibration of the psychotechnical parameters.

Keywords: roundabout capacity model; single lane roundabout; unsignalized intersections; road pavement conditions; roundabout design speed

1. Introduction

Capacity estimation is a fundamental issue in transportation systems analysis, as its values describe the maximum throughput of traffic demand that can be accommodated under current operating conditions. In highway engineering, this is a crucial aspect regarding roadway segments and intersections. Capacity models, in fact, are intimately related to Level of Service (LOS) assessment, a widespread approach for the description of the performances that a transportation facility is able to ensure for users. All over the world, technicians use the methodologies suggested by the Highway Capacity Manual in its various editions (e.g., 2010 [1] or 2016 [2] editions) for calculating LOS at intersections. In particular, for unsignalized at grade intersections, these methodologies focus on estimating time delays in maneuvers, due to the waiting phenomena that occur for a non-priority traffic stream when it encounters a priority traffic stream. Queue and delay evaluations for unsignalized intersections resort to heuristic models for the time-dependent queue, which allow analyzing the whole range of situations that can occur in the real cases. The coordinate transformation method [3], in fact, makes time-dependent queue formulas available for stationary and non-stationary condition treatment, i.e., for non-congested and congested traffic, respectively [4]. From this point of view, capacity is the key variable as it reflects the queue discharge rate [5].

Over the past decades, roundabouts have steadily grown in number among the various types of road junctions, with a huge spread in road transport networks all over the world. Nowadays roundabouts represent an extremely common form of road junction, with high safety standards and economic and social benefits [6]. As for the foundations for traffic quality assessment to support adequate planning and management of mobility infrastructures, roundabout capacity has become a theme of constant interest in highway engineering research and practice.

The performance analyses of roundabouts are founded on the hypothesis that the circulatory carriageway and the exits are always undersaturated. Consequently, congestion phenomenon may occur only at the entries. Thus, roundabout capacity refers commonly to the capacity of each entrance, i.e., the maximum number of vehicles expected to enter the ring in the unit of time. In this paper, we refer particularly to the entry capacity, being this considered in the calculation of intersection performances, namely queue length, delay, and LOS calculations.

Trying to provide a classification of modeling approaches, the entry capacity models for roundabouts are divided into three main categories [5,7–9]: the regression or empirical models, the gap acceptance models, and the microsimulation-based models.

The first models that have been developed were empirical models, which estimate entry capacity through statistical analysis (i.e., multivariate regression and correlations models) with the conflict volume and any other variables affecting capacity, relying on field data. Explicative variables are often related with the geometric characteristics of the intersection [9]. Some entry capacity models of widespread use belong to this group, including Kimber [10], SETRA [11], CETUR [12], VSS/Emch [13], Stuwe [14], Polus and Shmueli [15], Aakre [16], and NRW [17].

Gap acceptance models are widely used in modeling roundabout capacity, describing it by the characterization of driving behaviors through microscopic traffic variables, such as critical gap and follow-up time. Gap acceptance approach, mainly based on the research by Tanner [18], Siegloch [19], and Hagring [20], offers a worldwide used alternative to entry capacity assessment based on theoretical models for circulating and entering vehicles headways. These models use different headway distributions especially for circulating vehicles, from the simple exponential distribution to more complex formulations such as the Cowan M3 distribution [21], with parameters that can be empirically calibrated on real data [22–24]. Widespread use models belong to this group (for an in-depth analysis, see [25]). Purely by way of example, but not exhaustively, we can mention here Troutbeck [26], Explorer [27], Brilon [28], Brilon-Wu [9], Vejdirektoratet [29], HCM 2010 [1], HCM 2016 [2], and others both for traditional or innovative turbo-roundabouts (see, e.g., [30–38]).

The most recently appearing group of models concerns those based on micro-simulations. These models have received support by advances in computer systems, allowing to manage an extremely large number of events, albeit with considerable efforts in calibration and application to real cases, including modern intelligent transport systems and automated vehicles [39]. The recent literature proposes numerous models and solutions based on commercial (e.g., Paramics [40], Aimsun [41], and Vissim [42]) or open source (e.g., SUMO [43]) proprietary microscopic simulation programs, or using for example discrete event simulations (e.g., [44]), game theory (e.g., [45,46]), multi agent systems (e.g., [47]), cellular automata (e.g., [48–50]), and various artificial intelligence learning approaches (e.g., [51–53]).

As considered in [5], each of these approaches shows its strengths and weaknesses: empirical models describe the relationships between capacity and significant input variables, such as geometry, but are heavily influenced by the location and size of the survey samples, as well as by statistical analysis techniques; gap acceptance models are based on driving behavior and traffic characteristics mathematical modeling, but they often offer only weak relationships with the geometry of the intersection and with other boundary conditions; and micro-simulation models provide the greatest flexibility, but they heavily depend on an accurate representation of vehicle-to-vehicle interactions which can be difficult to replicate, and they require case-by-case applications, based on a precise layout

of the intersection and its elements which may not be available in the early stages of planning and design of a roundabout.

From a general point of view, whatever the model, roundabout entry capacity C can be considered as a function $C = f(Q, G, \Theta)$ of some vectors of variables and parameters [8], such as traffic demand variables Q (e.g., entry flow, traffic distributions for exit, and opposing flow); geometric parameters G (e.g., legs number, diameter length, number and width of entry and ring lanes, entry radius, entry angle, etc.); and driver and vehicle parameters Θ (e.g., psychotechnical parameters, speeds, accelerations, vehicle lengths, pavement conditions, localization and surrounding configuration, weather conditions, or other characteristics that may be isolated and considered in the model). Differences in traffic conditions, geometric layout, vehicle and driver characteristics, and boundary conditions lead to different operating characteristics.

In this paper we focus our attention on single-lane and four-leg roundabouts, of the type in Figure 1a, which can be considered the safest form of at-grade intersection [54].

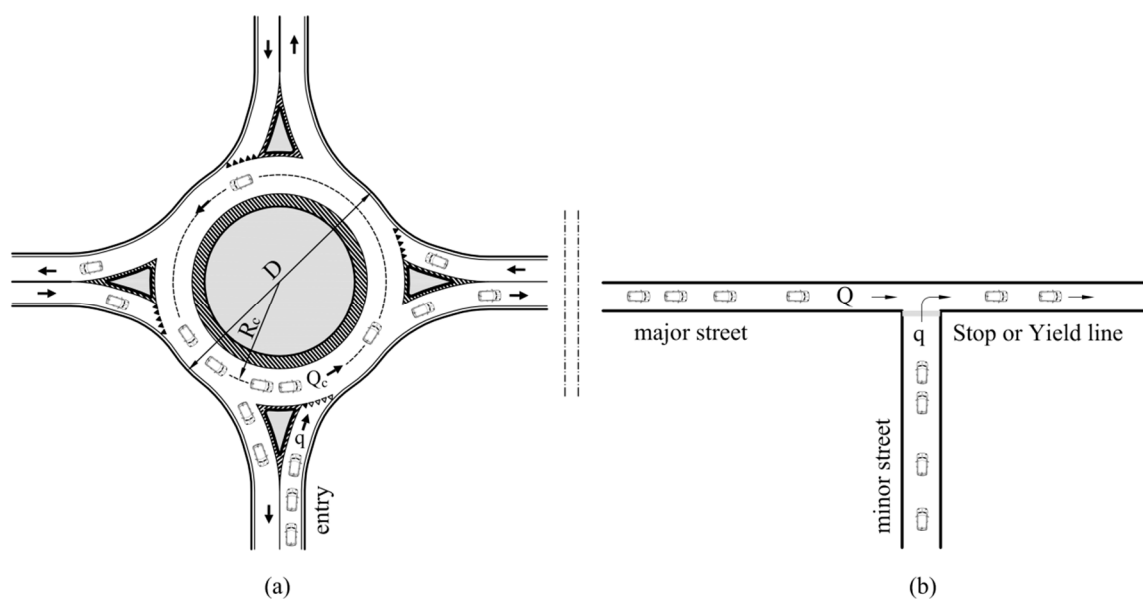


Figure 1. (a) Single-lane four-leg roundabout (typical layout); and (b) basic priority system at a T-intersection (the major flow Q is analogous to the roundabout circulating flow Q_c).

In this case, for each entry, we can consider the simple priority system in Figure 1b. This is a fondant queuing system in highway engineering, in which a traffic flow q on a secondary stream (i.e., non-priority) wants to turn right to merge the traffic flow Q on the main stream (i.e., priority) [4]. The priority flow Q is clearly a disturbance for q , and therefore it is called “opposing flow” or “conflict flow”, or more specifically “circulating flow” for roundabouts.

A roundabout’s particularity is that the circulating flow $Q = Q_c$ opposing a certain entry flow $q = q_e$ is a combination of the entry flows from different arms onto the roundabout. Clearly, Q_c shows a negative correlation with the maximum share of q that successfully completes the entry maneuver, i.e., the entry capacity C of the arm. For the generic roundabout of Figure 1a, all the possible components of the vector Q are determined if we know [9]: the traffic demand vector at the entries $q_e = [q_{e,i}]$ being $i = 1, 2, 3, 4$ the generic entry and $q_{e,i}$ its entry flow; and a traffic distribution matrix $P_{O/D} = [P_{i,j}]$ being $i, j = 1, 2, 3, 4$ and $P_{i,j}$ the percentage of $q_{e,i}$ that comes out of the arm j . In fact, from a simple matrix operation, we obtain the origin/destination matrix $M_{O/D} = P_{O/D} \cdot q_e$ of the roundabout. Knowing $M_{O/D}$, we can derive all the interesting flow values for the analysis, including the circulating flows of the vector $Q_c = [Q_{c,i}]$ in opposition to each entry arm i , being $i = 1, 2, 3, 4$. These considerations, which are the basis of the evaluation of the entrance capacity of the roundabout in Figure 1a, are therefore integrated into the different models that the literature proposes with further considerations

regarding the way of competition for entry and opposing flows and some additional parameters (i.e., geometric parameters G , driver and vehicle parameters, local-environmental conditions, or other characteristics Θ).

In this paper, we propose a novel re-visitation of an existing entry capacity model for small roundabouts based on a macroscopic approach, extending its field of application to larger single-lane roundabouts (see Figure 1a). In our opinion, the extension proposed for the base model and provided by this research can significantly improve the model usefulness and its versatility features in technical applications. These derive from taking into consideration, in addition to the entering and circulating traffic data (i.e., Q), geometric characteristics concerning the roundabout design layout and operating parameters related to traffic, drivers, and environmental conditions (i.e., G and Θ). All this can be useful in dealing with real cases, as well as easily described and estimated, in the planning and design phases of a roundabout intersection.

The paper is structured as follow. Section 2 presents the model covered by the paper: first by proposing a brief description of a little-known model for entry capacity estimates suggested by the recent Russian guidelines [55] for small urban roundabouts and then by moving to the presentation of the novel extensions proposed in this paper with a generalization of the original model. The basic relationships are reformulated here to consider the intersection geometry and the traffic operating conditions for the circulating flow through specific variables and parameters. Thus, a complete set of equations for the entry capacity model $C = f(Q, G, \Theta)$ is provided, widening the field of application to single-lane roundabouts with larger diameter. Section 3 presents and discusses some results related to a comparison with some models of practical use for the entry capacity, considering a test case roundabout in Section 3.1, and a first validation of the extended model based on literature data in Section 3.2. Finally, Section 4 highlights some summary remarks and conclusions.

2. Materials and Methods

This paper deals with a novel re-visitation of an existing entry capacity model for small roundabout, based on analytical approach from a macroscopic point of view. The base model was studied by Chumanov [56] and has been proposed by Russian guidelines for the optimization of the geometry of roundabouts [55], published by the Ministry of Transport (MinTrans) and intended for authorities responsible for the road traffic sector within the Russian Federation.

The purpose of this paper is to extend the field of application of the Chumanov's model [56] to larger single-lane roundabouts and to deepen some relationships among the variables that allow evaluating some characteristic parameters of the intersection layout and operation, such as geometric design, ring lane capacity, circulating flow speed, and sight distances between vehicles, and taking into account different road pavement surface conditions.

2.1. Chumanov's Original Model in Russian Roundabout Guidelines

The Chumanov's entry capacity formula for a single-lane roundabout with external diameter varying between 15 and 25 m in [55,56] is:

$$C = Q_c \cdot \left(1 - \frac{4 - t_n}{t_m}\right) \quad (1)$$

where t_n is the so-called "useful headway" (s) between circulating flow Q_c (veh/h) opposing the entry and t_m is the average headway (s) of the same flow. For the calculation of the two different headways t_n and t_m , the following relationships are identified:

$$t_n = \frac{T_n}{Q_c} \quad (2)$$

and

$$T_n = 3600 - t_m \cdot Q_c \quad (3)$$

with

$$t_m = 3.6 \cdot \left(\frac{L_a + L_m}{V_p} \right) \quad (4)$$

where T_n is the “hourly useful fraction” (s), V_p is the design speed of the ring lane (km/h), L_m is the average length of vehicles (m), which can be fixed at 4.5 m, and L_a is the safety distance (m). For the latter, the expression of the stopping sight distance at speed V_p is assumed:

$$L_a = \frac{V_p^2}{25.92 \cdot g \cdot \varphi} + \frac{t_p \cdot V_p}{3.6} \quad (5)$$

where t_p is the drivers’ perception and reaction time (s), g is the gravity acceleration (m/s^2), and φ is the longitudinal friction coefficient.

As for the V_p speed values, the Russian guideline [55] considers values of 25 km/h for external diameters that are less than 25 m. Several tabulated equations are also provided for calculating the average speed V on the ring lane as a function of the volumes of circulating flow ($200 \leq Q_c \leq 800$ veh/h) and the length of the external diameter. Figure 2 shows the speed values calculated with the formulas in the Russian guidelines [55], for variable circulating flow and fixed diameter equal to 25 m, and the interpolating function that is a second-order polynomial, which allows using, with a good approximation of the original points, a unique continuous formula instead of the tabulated ones in [55].

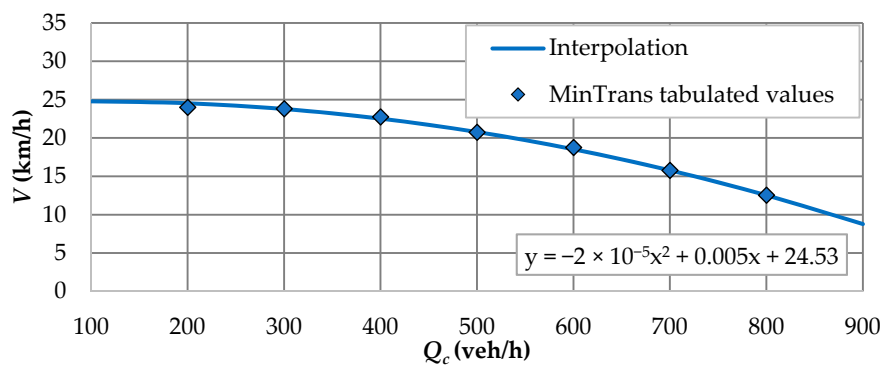


Figure 2. Average speed for the ring lane of a roundabout with $D = 2R = 25$ (m) with variable circulating flow—MinTrans discretized values [55] and polynomial interpolation for continuous values.

Chumanov [11] also indicates an experimental equation for calculating the operating speed V_{85} (km/h) depending on the external radius R of the roundabout (7–13 m):

$$V_{85} = 4.1813 \cdot R - 0.0816 \cdot R^2 - 22.264 \quad (6)$$

2.2. The Proposed Extension for the Modified Chumanov Capacity Model

Below, we present our original insights and modifications to the Chumanov’s entry capacity model [55,56] briefly summarized in Section 2.1, to extend its applicability to larger roundabouts for different environmental and traffic operating conditions. We refer to this new extended model as the Modified Chumanov model (MC model).

As a first consideration, Equation (1) can be generalized in the following form:

$$C = Q_c \cdot \left(1 - \frac{\alpha - t_n}{t_m} \right) \quad (7)$$

The term α (s) can be interpreted as the average headway between the vehicles of the circulating flow Q_c when it reaches the ring lane capacity $Q_{c, max}$ (veh/h), that is

$$\alpha = \frac{3600}{Q_{c,max}} \quad (8)$$

Substituting α according to Equation (8) into Equation (7), we get that $C = 0$ when $Q_c = Q_{c,max}$, whatever the value for α . In the original model [55,56], the author assumed that $\alpha = 4$, which corresponds to a capacity for the circulating flow, for small urban roundabouts, that is $Q_{c,max} = 900$ veh/h. In the presence of a circulating flow $Q_c = Q_{c,max} = 900$ veh/h, therefore, in the original Chumanov's model, we obtain a zero value for the entry capacity. In general, we can observe that, once α has been set and the headways t_n and t_m have been determined, the trend of the capacity $C(Q_c)$ shows a maximum corresponding to a value for Q_c approaching zero and a minimum (equal to zero) for $Q_c = Q_{c,max} = 3600/\alpha$.

2.3. The Capacity of the Ring Lane

Overcoming the restriction of the original Chumanov's model for mini-roundabouts, for larger single-lane roundabouts (with one lane for both the central ring and the entry arms, as in Figure 1a), suitable values for the ring lane capacities are $Q_{c,max} = 1600$ veh/h for medium-sized roundabouts and $Q_{c,max} = 1800$ – 2000 veh/h for large-sized roundabouts [9]. For external diameters ranging from 15 (mini-roundabouts) to 50 m (conventional roundabouts), we can assume values between 900 and 1800 veh/h, identifying the following third-order polynomial relationship with respect to the external diameter D (m):

$$Q_{c,max} = -0.0162 \cdot D^3 + 1.671 \cdot D^2 - 26.7605 \cdot D + 984.524 \quad (9)$$

Equation (9) interpolates the extreme values identified above and returns the intermediate values through a continuous function instead of a set of discretized values, as shown in Figure 3.

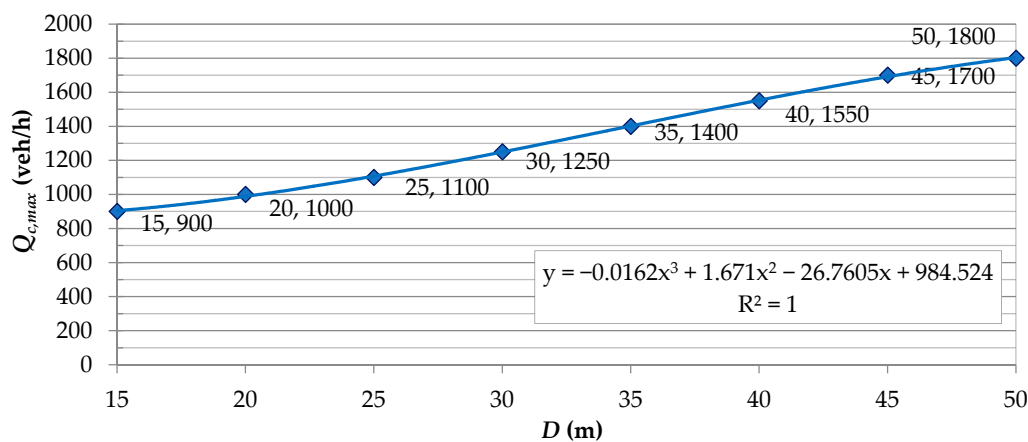


Figure 3. Ring lane capacity as a function of the (outer) diameter of the roundabout ($x = D$).

In view of Equation (9), we have:

$$C = Q_c \cdot \left(1 - \frac{\alpha - t_n}{t_m}\right) \quad (10)$$

with

$$\alpha = 2.00 \times 10^{-5} \cdot D^3 + 1.07 \times 10^{-3} \cdot D^2 - 5.67 \times 10^{-2} \cdot D + 5.02 \times 10^{00} \quad (11)$$

for $15 \leq D \leq 50$ m, as shown in Figure 4.

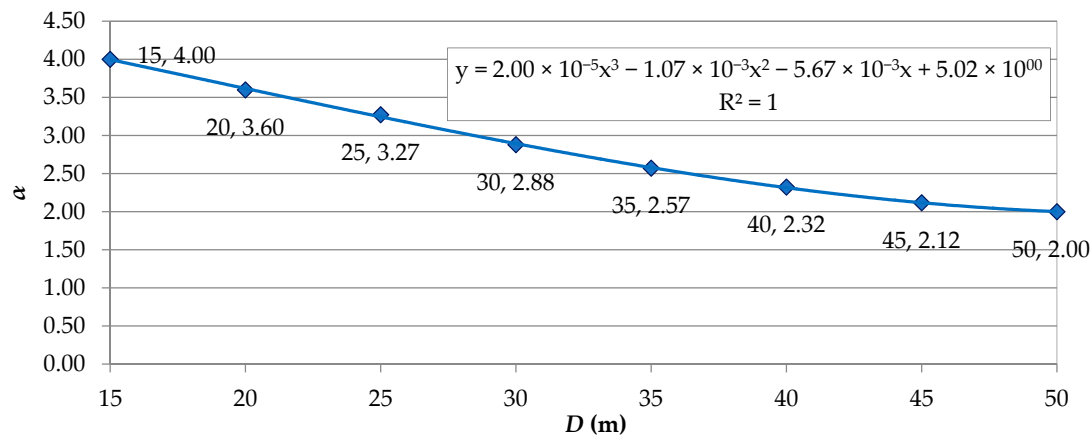


Figure 4. Values for α (s) as a function of the (outer) diameter of the roundabout ($x = D$).

The capacity values of the ring lane in Figure 3 and the consequent values of α in Figure 4 refer to dry weather conditions. To take into account particular situations, a properly calibrated reduction coefficient θ can be introduced, so that $Q_{c,max}^* = \theta \cdot Q_{c,max}$. For example, in the case of rain, and therefore with a wet pavement surface, we can consider $\theta = 0.8$ [57] with $\alpha^* = \alpha/\theta$.

2.4. Vehicle Speed in the Circulating Flow

Taking the expressions for t_n and t_m from the original model, we have

$$t_n = \frac{3600}{Q_c} - t_m \quad (12)$$

and

$$t_m = 3.6 \cdot \left(\frac{L_m + L_a}{V_p} \right) \quad (13)$$

where L_m is the average vehicle length (m), assumed equal to 4.5, and L_a (m) is the safety distance (m). For the latter parameter, in the first analysis, we can consider the value L_{0a} calculated in dependence on the free flow speed for the ring lane, which in turn we can assume equal to the design speed V_p , using the equation for the safety distance between vehicles. For this safety distance, we can consider the expression of the spacing to avoid the collision in the presence of an immediate stop of the leading vehicle and an emergency deceleration of the following vehicle, given by:

$$L_{0a} = \frac{V_p^2}{2 \cdot a_e} + \frac{t_p \cdot V_p}{3.6} + s_0 \quad (14)$$

with t_p drivers' perception and reaction time (s), a_e (m/s^2) emergency deceleration, and s_0 safety distance limit after stopping (assumed constant and equal to 0.9 m). For the emergency deceleration a_e , we assume a value equal to $0.85 \cdot g$ for dry pavement surface condition or equal to $0.41 \cdot g$ for wet pavement surface condition [58], with $g = 9.81$ (m/s^2), i.e., the gravity acceleration. For perception and reaction time, we consider the values obtainable with the equation proposed by the Italian guidelines [59], reduced by 25% to take into account the greater attention of users when driving at an intersection:

$$t_p = (2.8 - 0.01 \cdot V_p) \cdot 0.75 \quad (15)$$

The stopping sight distance L_{0a} by Equation (14), however, depends on the free flow speed V_p and not on the average speed of the flow actually circulating on the ring. We therefore want to identify a formulation of L_a that is dependent on the actual speed on the ring V , which in turn is a function of

the circulating flow Q_c , i.e., $V = V(Q_c)$. In view of this, the dependence on $V = V(Q_c)$ would also result for t_m , that is

$$t_m = 3.6 \cdot \left(\frac{L_m + L_a(V)}{V} \right) \quad (16)$$

For the relation $V = V(Q_c)$, we assume a linear trend, decreasing between the maximum V_p (for $Q_c = 0$) and $V_c = V_p/2$ at the ring saturation flow ($Q_c = Q_{c,max}$). This results in:

$$V = V_p - \frac{V_p}{2 \cdot Q_{c,max}} \quad (17)$$

For the free flow speed V_p , which we assume coincident with the design speed, the point-mass equilibrium model for a vehicle travelling on a horizontal curve can be used. The free flow speed, in fact, is estimated here with the maximum speed that ensures safety and driving comfort on the ring lane, in consideration of the geometry of the roundabout and the driving conditions (current situation for pavement-tires friction). The limiting speed, also termed design speed (for free flow), is obtainable with the well-known simplified formula [60]:

$$V_p = \sqrt{127 \cdot R_c \cdot (r + \varphi_t)} \quad (18)$$

with φ_t the side friction factor, r the transverse slope (superelevation) of the ring lane, and R_c (m) the curvature radius of the same lane (Figure 1a). With regard to r , to allow easier disposal of rainwater, project gradients towards the outside of the ring are generally used with values of -2% . As for φ_t , it is useful to remark that this term represents the portion of lateral acceleration unbalanced by the slope in the equilibrium equation for the vehicle. This term, therefore, represents a request for friction to ensure the equilibrium that must be satisfied by the friction actually available in the tire-pavement interface, which we consider to be φ_{sp} . To avoid vehicle slippage, it must be $\varphi_t < \varphi_{sp}$. A similar condition must be added in consideration of the maximum lateral acceleration that a vehicle can experience without overturning. Therefore, a threshold value φ_r is considered, which depends on vehicle design and load, and the condition that must be met is $\varphi_t < \varphi_r$.

As is known, in practice, the design criteria are not based on formal assumptions about the actual values of φ_{sp} and φ_r but on limiting the value of φ_t so that it is less than or at most equal to a specified value φ_{max} . The latter is selected on the basis of driver's comfort levels (i.e., the comfortable tolerance by the driver for lateral acceleration).

In our case, once a certain value for φ_{max} , a design radius R_c , and a design slope (negative) r for the roundabout are assumed, Equation (18) allows identifying the maximum travel speed around the central island in comfortable conditions. Thus, we take this value as the free flow speed of the ring lane.

It should be remembered that φ_{max} in turn depends on the speed, as well as on the conditions for the pavement/tire contact surfaces. Among the relationships that can be used to determine the transverse friction coefficient, for wet pavement surface conditions, we can consider the relationship in [61]:

$$\varphi_{max,w} = 0.2535 + 2.33 \cdot 10^{-3} \cdot V - 9 \cdot 10^{-5} \cdot V^2 \quad (19)$$

and for dry pavement surface conditions, we can consider the relationship in [62]:

$$\varphi_{max,d} = 0.569 - 0.592 \cdot 10^{-2} \cdot V + 0.198 \cdot 10^{-4} \cdot V^2 \quad (20)$$

where the two quadratic forms for φ_{max} can be expressed in the general form $C + B \cdot V + A \cdot V^2$ and, taking into account Equation (18), the speed is obtainable as the positive root of the second-order polynomial $c + b \cdot V + a \cdot V^2 = 0$, where $c = 127 \cdot R_c \cdot (C - r)$, $b = -127 \cdot R_c \cdot B$, and $a = (1 - 127 \cdot R_c \cdot A)$.

By fixing $r = -2\%$ as usually happens, Figure 5 shows V_p as function of R_c for wet and dry road pavement surface conditions, according to second-order polynomial. Alternatively, with different

values for r and R_c , we can obtain V_p directly as the positive solution of the already mentioned equation $c + b \cdot V + a \cdot V^2 = 0$.

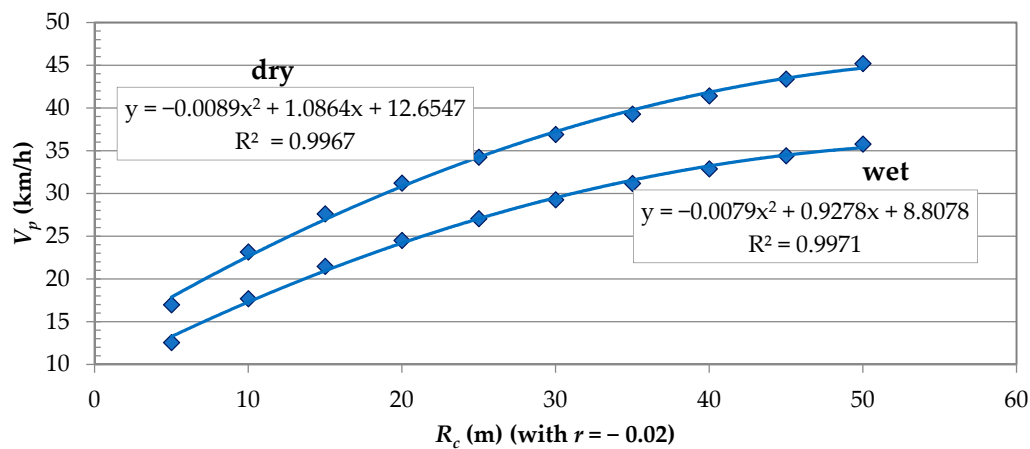


Figure 5. Limit speed values (i.e., free flow speed) on the ring lane as a function of the radius of curvature measured on the lane axis ($x = R_c$).

For $r = -2\%$, we therefore have:

$$V_{p,d} = -0.0089 \cdot R_c^2 + 1.0864 \cdot R_c + 12.6547 \quad (21)$$

and

$$V_{p,w} = -0.0079 \cdot R_c^2 + 0.9278 \cdot R_c + 8.8078 \quad (22)$$

where, if D (m) is the outer diameter length and L_c (m) is the ring lane width, R_c (m) can be calculated as:

$$R_c = \frac{D - 2 \cdot L_c}{2} + 1.50 \quad (23)$$

2.5. Vehicle Distance in the Circulating Flow

The inter-vehicular space distance, assumed equal to the stopping sight distance L_{0a} considering Equation (14), also in consideration of an average circulating flow rate variable according to Equation (17) in place of the fixed value V_p , represents an unrealistic estimate for increasing circulating flow values. In fact, the spacing between two vehicles (L_a) on the ring is usually less than the space required to stop the follower vehicle due to a sudden stop of the leading vehicle (L_{0a}).

A linear trend is therefore assumed for L_a as well, depending on the degree of saturation of the ring $Q_c / Q_{c,max}$ between the stopping sight distance L_{0a} for flows that approach the free flow ($Q_c = 0$) and the minimum spacing L_{min} for saturated conditions ($Q_c / Q_{c,max} = 1$).

L_{min} is calculated considering the net distance (or gap distance) between the vehicles upon reaching the ring lane capacity for a vehicular density $K_c = Q_{c,max} / V_c = Q_{c,max} / (V_p / 2)$, that is:

$$L_{min} = \frac{1000 \cdot V_p}{2Q_{c,max}} - L_m \quad (24)$$

Then, we have:

$$L_a = L_{0a} - \frac{Q_c}{Q_{c,max}} (L_{0a} - L_{min}) \quad (25)$$

Figure 6 shows trends for V_p (km/h), L_{0a} (m), and L_{min} (m) with varying D (m), using Equations (9), (14), (15), and (20)–(24), with: $L_c = 7$ (m); $r = -2\%$; $a_{e,d} = 0.85 \cdot g$ (m/s^2) for dry conditions and $a_{e,w} = 0.41 \cdot g$ (m/s^2) for wet conditions; $s_0 = 0.9$ (m).

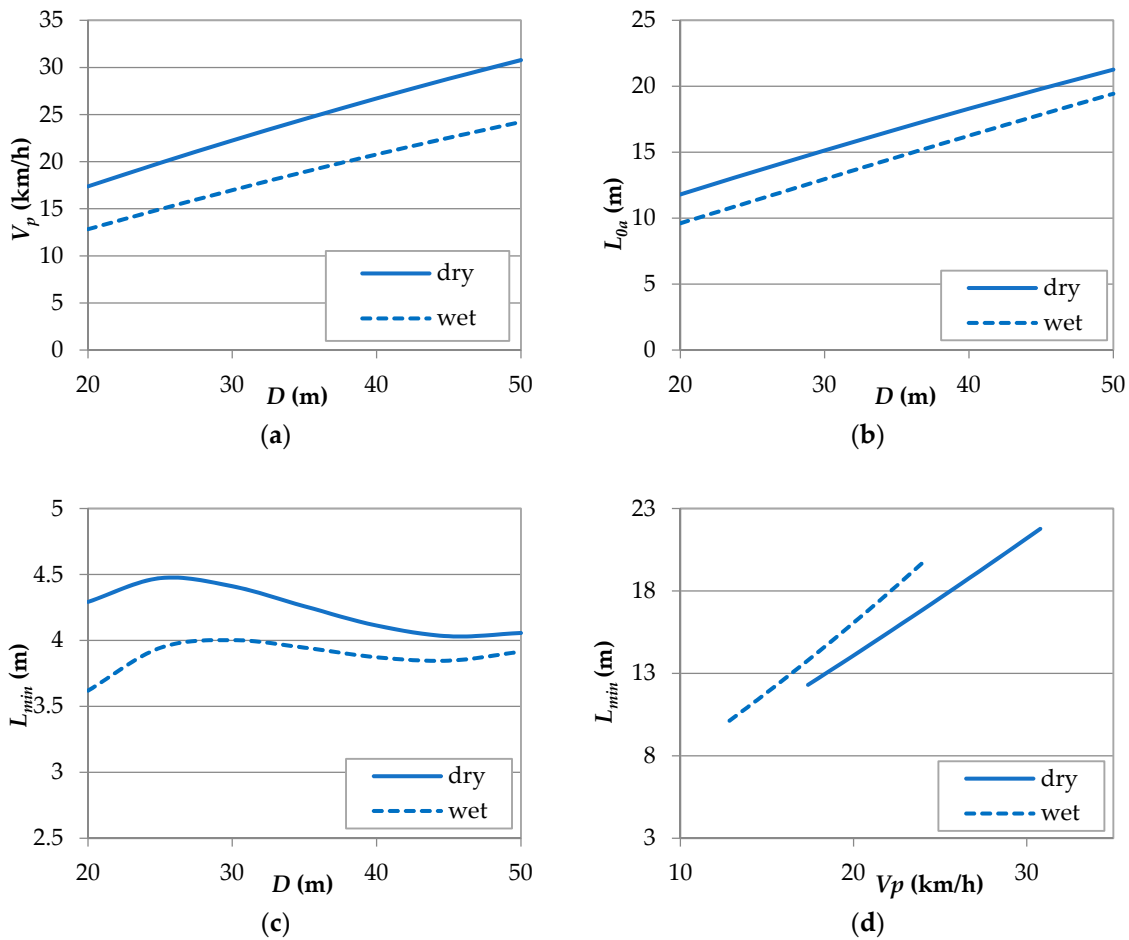


Figure 6. V_p (a); L_{0a} (b); and L_{min} (c) values with varying diameter; and D - L_{min} (d) values with varying V_p .

2.6. The Complete Entry Capacity Formulation for the MC Model

The proposed model allows evaluating the capacity C of each single-lane entry, for roundabout with an external diameter length D and a single-lane ring width L_c , as a function of the circulating flow Q_c opposing the entry flow and considering dry or wet pavement surface conditions. It should be noted that the capacity value refers to a standard entrance width equal to 3.5 m. For larger entry widths, the input capacity can be estimated by multiplying the base capacity by the coefficient $f_e = [1 + 0.1 \cdot (E - 3.5)]$ [9], being E the actual entry width (with $E \geq 3.5$ m).

In summary, the general model provides the following relationships:

$$C = Q_c \cdot \left(1 - \frac{\alpha / \theta - t_n}{t_m}\right) \cdot f_e \quad [veh/h] \tag{26}$$

with

$$\alpha = 2.00 \times 10^{-5} \cdot D^3 + 1.07 \times 10^{-3} \cdot D^2 - 5.67 \times 10^{-2} \cdot D + 5.02E \text{ [s] for } 15 \leq D \leq 50 \text{ m} \tag{27}$$

$$\theta = \begin{cases} 0.8 & \text{wet pavement} \\ 1 & \text{dry pavement} \end{cases} \tag{28}$$

$$f_e = [1 + 0.1 \cdot (E - 3.5)] \tag{29}$$

where

$$t_n = \frac{3600}{Q_c} - t_m \text{ [s]} \tag{30}$$

and

$$t_m = 3.6 \cdot \left(\frac{L_m + L_a}{V} \right) \quad [s] \quad (31)$$

considering

$$L_m = 4.5 \quad [m] \quad (32)$$

$$L_a = L_{0a} - \frac{Q_c}{Q_{c,max}} (L_{0a} - L_{min}) \quad [m] \quad (33)$$

$$L_{0a} = \frac{V_p^2}{25.92 \cdot a_e} + \frac{t_p \cdot V_p}{3.6} + s_0 \quad [m] \quad (34)$$

$$V = V_p - \frac{V_p}{2 \cdot Q_{c,max}} \quad [km/h] \quad (35)$$

with

$$V_p = \begin{cases} V_{p,w} = -0.007 \cdot R_c^2 + 0.927 \cdot R_c + 8.807 & \text{wet pavement} \\ V_{p,d} = -0.008 \cdot R_c^2 + 1.086 \cdot R_c + 12.65 & \text{dry pavement} \end{cases} \quad [km/h] \quad (36)$$

for

$$r = -2 \quad [\%] \quad (37)$$

$$s_0 = 0.9 \quad [m] \quad (38)$$

$$a_e = \begin{cases} 0.41 \cdot g & \left[\frac{m}{s^2} \right] \text{ wet pavement} \\ 0.85 \cdot g & \left[\frac{m}{s^2} \right] \text{ dry pavement} \end{cases} \quad (39)$$

$$R_c = \frac{D - 2 \cdot L_c}{2} + 1.50 \quad [m] \quad (40)$$

$$t_p = (2.8 - 0.01 \cdot V_p) \cdot 0.75 \quad [s] \quad (41)$$

Figures 7 and 8 show the capacity values with varying diameters, with $L_c = 7$ m, $r = -2\%$, and $E = 3.5$ m, depending on the circulating flow Q_c and for dry and wet pavement conditions.

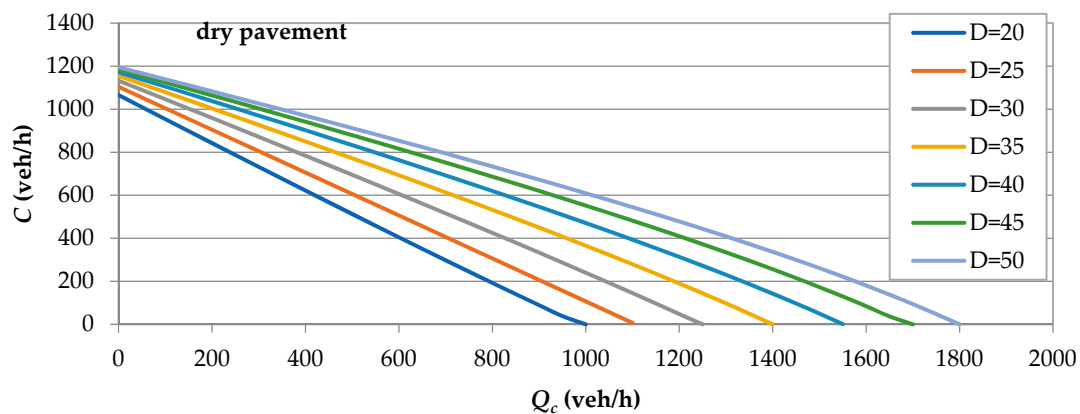


Figure 7. Capacity values with various diameters D (m), with $L_c = 7$ m, $r = -2\%$, and $E = 3.5$ m, depending on the circulating flow Q_c and for dry pavement conditions.

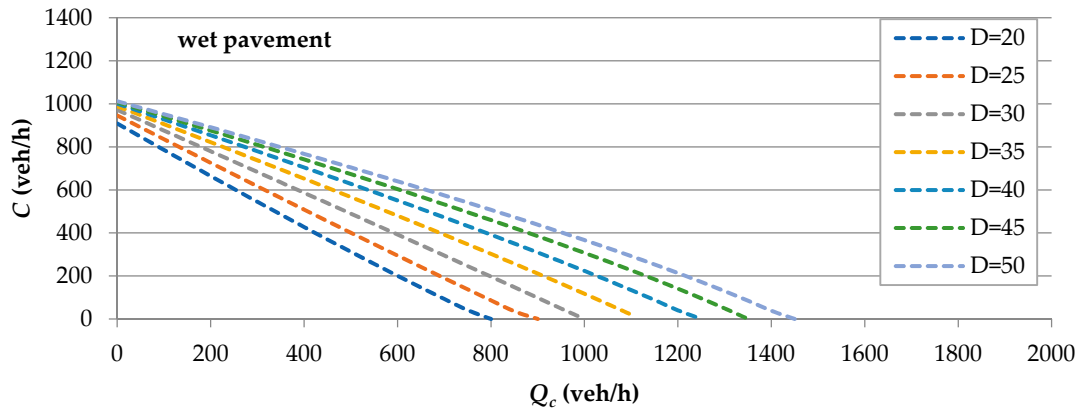


Figure 8. Capacity values with various diameters D (m), with $L_c = 7$ m, $r = -2\%$, and $E = 3.5$ m, depending on the circulating flow Q_c and for wet pavement conditions.

3. Results and Discussion

3.1. A First Level of Validation: The Comparison with Some Models of Practical Use for the Entry Capacity

For the MC model expressed through the set of equations in Section 2.2, we propose a comparison with some widely used models in international practice for the analysis of the roundabout entry capacity belonging to both regression and gap acceptance models. The models considered here are: Kimber [10], SETRA [11], CETUR [12], VSS/Emch [13], Troutbeck [26], Stuwe [14], Explorer [27], Polus and Shmueli [15], Brilon [28], Aakre [16], NRW [17], Brilon-Wu [9], Vejdirektoratet [29], HCM 2010 [1], and HCM 2016 [2].

The comparative case study concerns a roundabout with diameter $D = 42$ m, ring lane width $L_c = 7$ m, slope $r = 2\%$, and entrance width $E = 4$ m. Figures 9 and 10 show the results obtained with the control models.

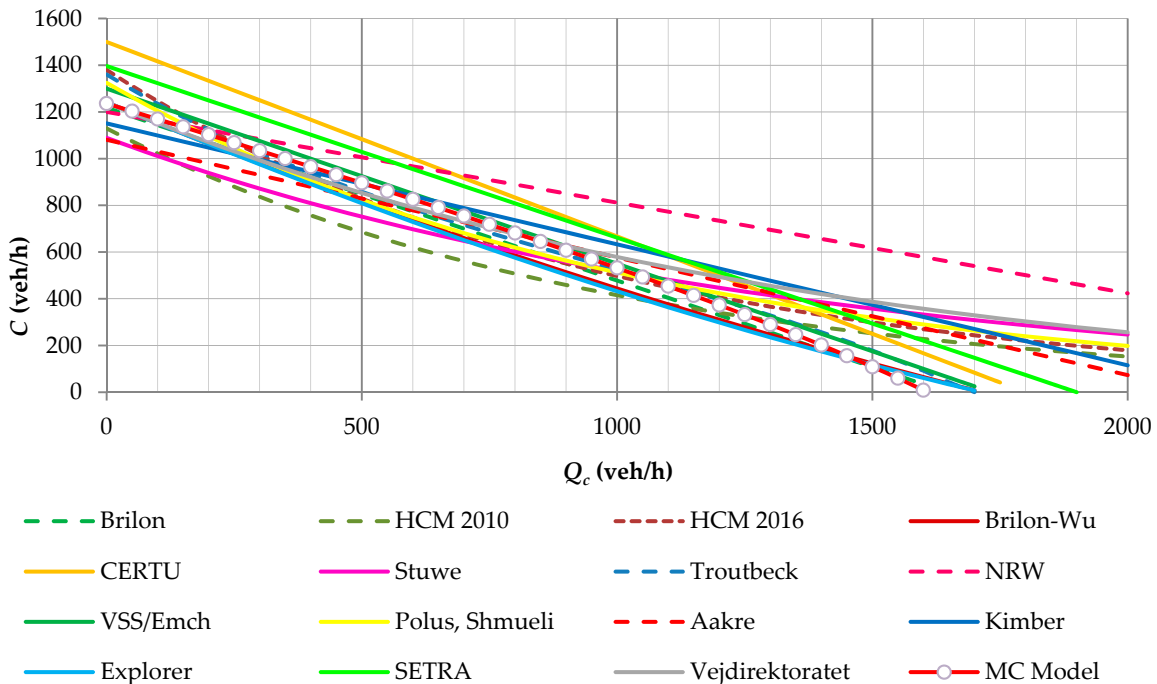


Figure 9. Extended comparison with 15 control models.

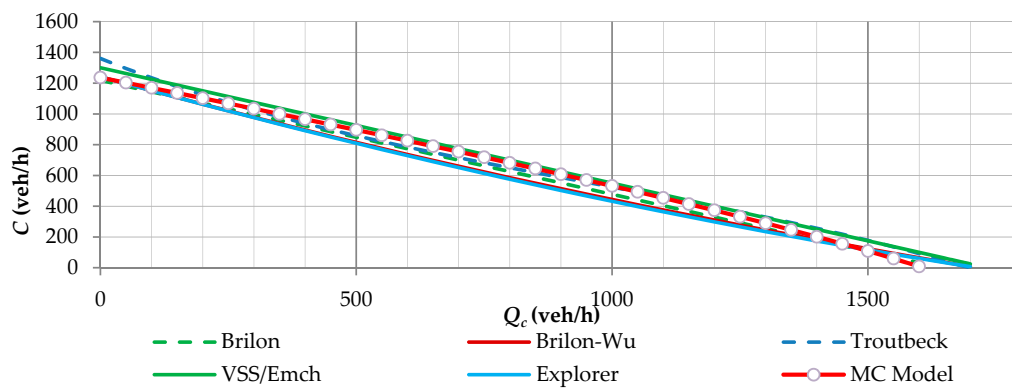


Figure 10. Restricted comparison with five control models.

As shown in Figures 9 and 10, the MC model produces results completely in the range of variability of the 15 models. Even if at first for a single test roundabout, it allows validating the MC model calculation and results with the most popular entry capacity models belonging to both regression and gap acceptance approaches. Figure 10 also identifies a smaller group of models, all considering gap acceptance except the VSS/Emch one, with respect to which the MC model produces closer results.

3.2. A Second Level of Validation: The Goodness of the Fitting Compared to Some Experimental Situations

In this section, we propose some analyses aimed at verifying whether the MC model is able to correctly estimate the capacity of the roundabout entrances in the various operational situations of the intersections. For this purpose, comparisons were made between the results obtained with the MC model and the capacity values taken from the technical-scientific literature for single lane roundabouts. Several studies that have provided dataset for the estimation of capacity, using micro simulation models [62] or deriving from real cases [63,64], were selected. Specifically, the reference cases used in this study and the related datasets are described below:

- Dataset A: About 300 pairs of circulating flow values—simulated entry capacities for a single-lane roundabout with a small diameter ($D = 75' = 23 \text{ m}$; $L_c = 5 \text{ m}$; $E = 3.5 \text{ m}$) in [63]
- Dataset B: About 260 pairs of circulating flow values—monitored entry capacity for a single lane roundabout with a medium diameter ($D = 105' = 32 \text{ m}$; $L_c = 14' = 4.25 \text{ m}$; $E = 12' = 3.65 \text{ m}$) at New York Glens Falls-US 9/ Warren Street in [64]
- Dataset C: About 280 pairs of circulating flow values—monitored entry capacity for two single-lane roundabouts with a medium-large diameter ($D = 140\text{--}148' = 44 \text{ m}$; $L_c = 23, = 7 \text{ m}$; $E = 13\text{--}15' = 4.5 \text{ m}$) at 116th–106th Street/Spring Mill Road Carmel in [64]
- Dataset D: Approximately 740 pairs of circulating flow values—monitored entry capacities for single-lane roundabouts of medium-large diameter in [65] assuming a standard geometry ($D = 45 \text{ m}$; $L_c = 7 \text{ m}$; $E = 3.5 \text{ m}$)

For the different sets of pairs (\hat{Q}_c, \hat{C}_i) , the average values of \hat{C}_i were calculated for Q_c binning intervals of amplitude 100 veh/h. Each empirical mean value \hat{C}_i was centered in its averaging interval for Q_c . In these terms, we obtained the trends of \hat{C}_i for n steps i of 50 veh/h for Q_c .

The trend of the entry capacity with varying circulating flow was estimated using the MC model, considering for all the four cases a standard slope of -2% and dry pavement conditions.

The results were compared with those obtained by applying the two most recent models, namely those indicated by the Highway Capacity Manual in the 2010 [1] and 2016 [2] editions. Specifically, the HCM 2010 [1] and HCM 2016 [2] models for estimating the entry capacity for single-lane roundabout are, respectively:

$$C = 1130 \cdot \exp(-0.0010 \cdot Q_c) \quad (42)$$

$$C = 1380 \cdot \exp(-0.00102 \cdot Q_c) \quad (43)$$

The results obtained with the MC, HCM 2010 and HCM 2016 models were evaluated in terms of Round Square Mean Error (RMSE) and Normal RMSE (NRMSE) with respect to the average values observed, considering:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\hat{C}_i - C(50 \cdot i))^2}{n}} \quad (44)$$

$$NRMSE = RMSE / \frac{\sum_{i=1}^n \hat{C}_i}{n} \quad (45)$$

where \hat{C}_i is the average of the measured values of capacity \hat{C} for the interval of Q_c between $50 \cdot (i - 1)$ and $50 \cdot (i + 1)$ and centered at $(50 \cdot i)$ veh/h of circulating flow, for $i = 1, \dots, n$ and $C(50 \cdot i)$ is the value estimated with each model. The best approximation of the data is ensured by the model that has the lowest RMSE and NRMSE values.

Figures 11–14 show the trends of the empirical averages \hat{C}_i and the estimates obtained with the three models, superimposed on the monitoring data relating to each of the four datasets. Tables 1 and 2 show the values obtained for the two fitting indicators, relating to the three models for each of the four datasets.

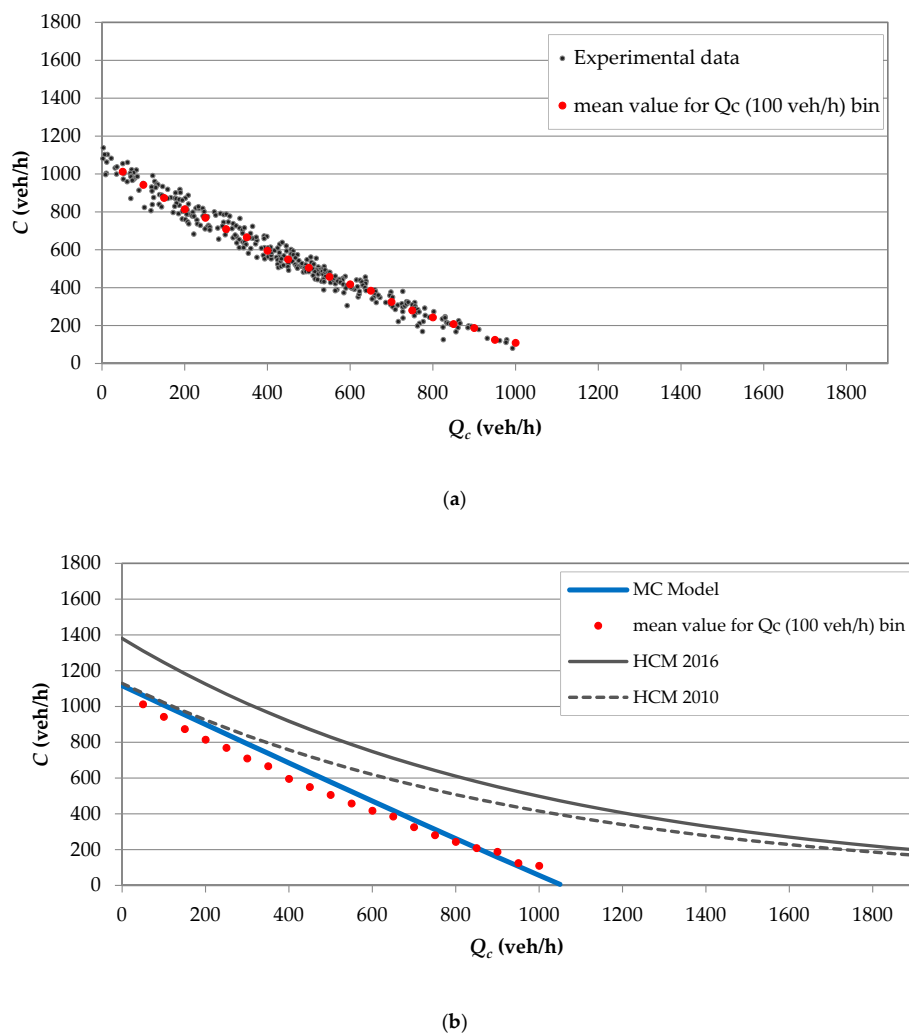
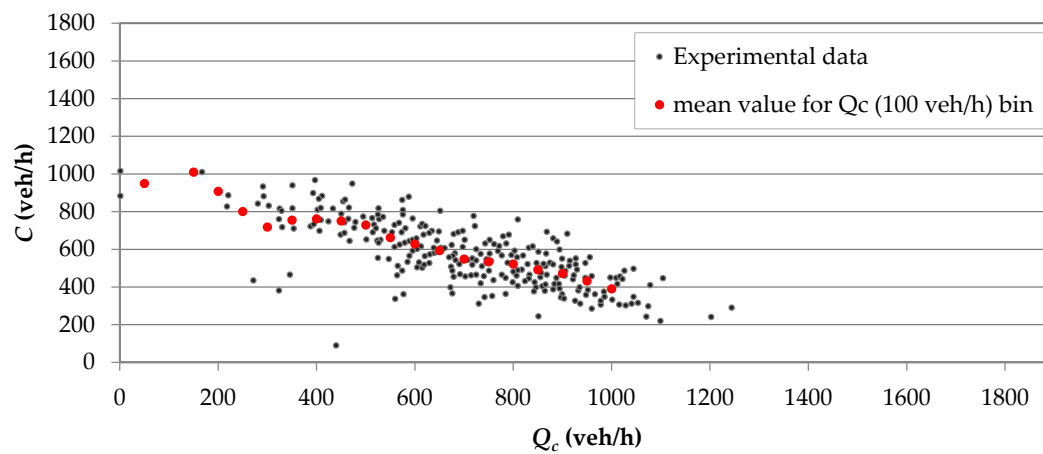
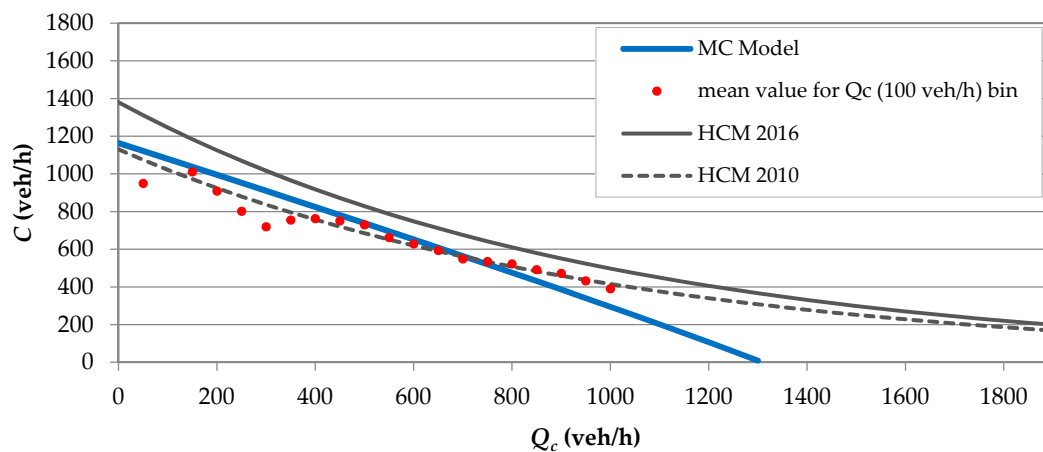


Figure 11. Dataset A: (a) scatterplot of experimental data and trend of average capacities; and (b) comparison between trends in empirical mean capacities and values resulting from the MC model and from the HCM 2010 and HCM 2016 models.



(a)



(b)

Figure 12. Dataset B: (a) scatterplot of experimental data and trend of average capacities; and (b) comparison between trends in empirical mean capacities and values resulting from the MC model and from the HCM 2010 and HCM 2016 models.

Table 1. RMSE for the three models in the four test cases.

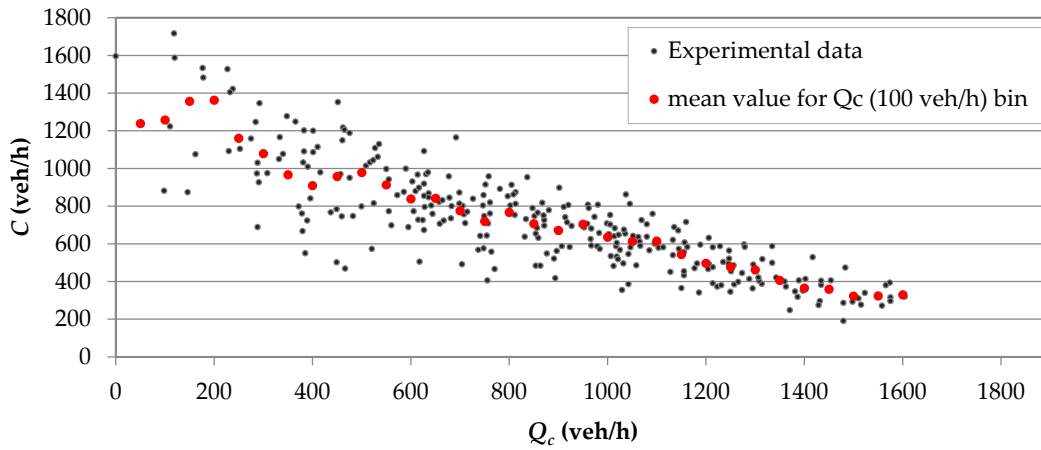
Model	Dataset A	Dataset B	Dataset C	Dataset D
MC	61	111	102	104
HCM 2010	336	170	111	68
HCM 2016	202	51	222	123

Table 2. NRMSE for the three models in the four test cases.

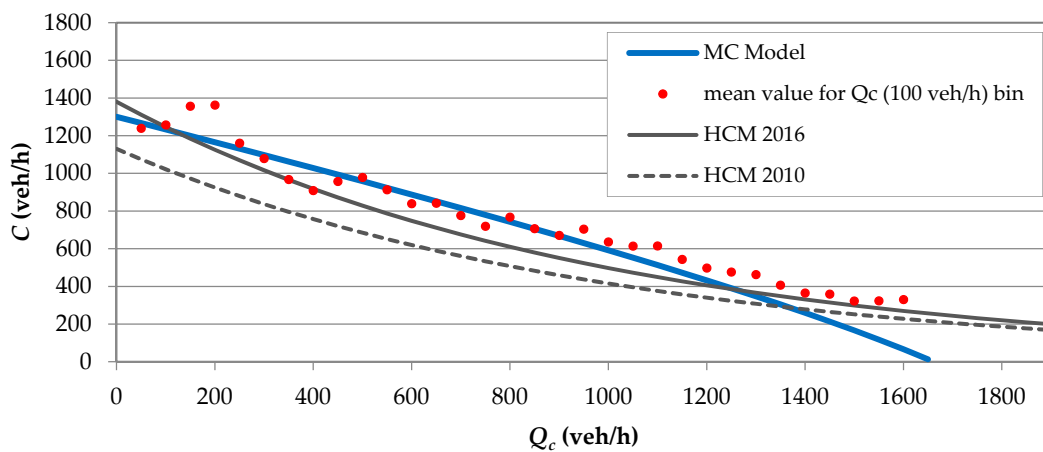
Model	Dataset A	Dataset B	Dataset C	Dataset D
MC	12%	18%	13%	16%
HCM 2010	66%	27%	15%	10%
HCM 2016	40%	8%	29%	19%

The results in Tables 1 and 2, as well as the trends in Figures 11–14, generally show a good degree of fitting for the MC model, both in absolute and in relative terms, in the comparison with the HCM 2010 [1] and HCM 2016 [2] formulas.

In conclusion, the MC model shows very good adaptation flexibility on the four test cases, which cover a range of external diameters varying between 23 and 45 m (small to medium-large roundabouts). This flexibility is overall greater when compared with that of the standard HCM models considered here.

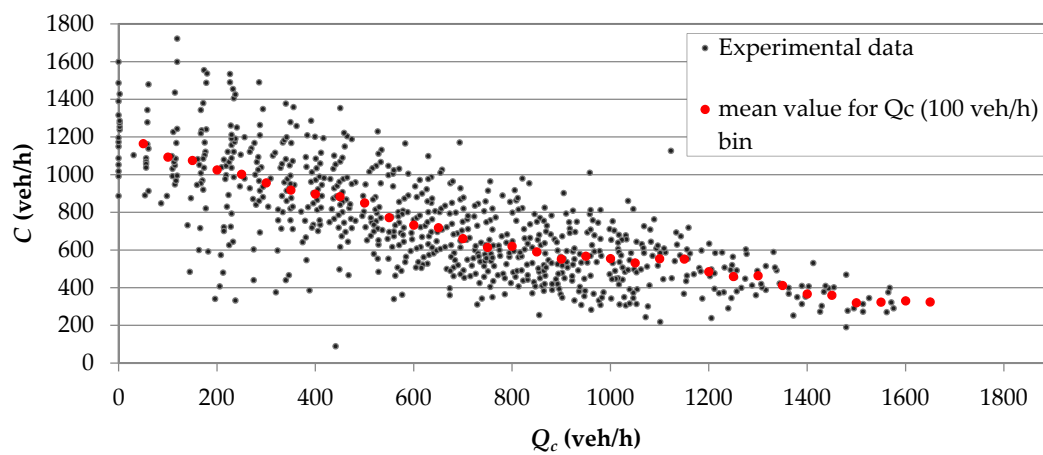


(a)

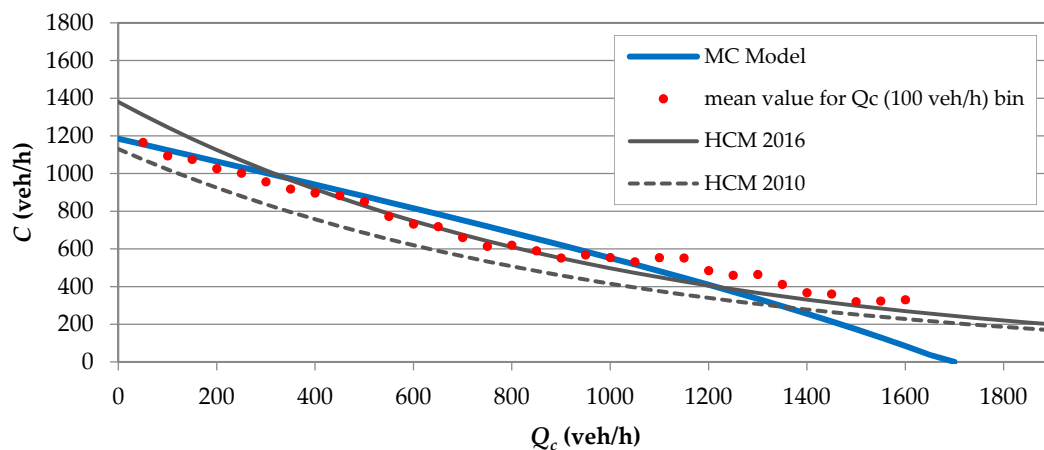


(b)

Figure 13. Dataset C: (a) scatterplot of experimental data and trend of average capacities; and (b) comparison between trends in empirical mean capacities and values resulting from the MC model and from the HCM 2010 and HCM 2016 models.



(a)



(b)

Figure 14. Dataset D: (a) scatterplot of experimental data and trend of average capacities; and (b) comparison between trends in empirical mean capacities and values resulting from the MC model and from the HCM 2010 and HCM 2016 models.

4. Conclusions

Entry capacity calculations for roundabout are a constant concern for traffic planners and road and highway designers. For this reason, over time, numerous models have been produced to assist technicians in these analyses, on which the evaluations on the functionality of the intersections are based in terms of queues and delays. In this constantly updated panorama, this paper proposes an extension of an existing and little-known model for calculating the capacity of single-lane roundabouts, i.e., the Chumanov's model.

The original Chumanov's model concerns small roundabouts and the authors found it useful to present an extension of its field of application. Compared to the original model, the paper proposes a generalization that makes it applicable to roundabouts of larger diameter, up to 50 m, also introducing new mathematical relationships for the in-depth description of the model parameters.

The proposed MC model, which considers an analytical approach from a macroscopic point of view, consists of a series of equations that allow evaluating the entry capacity depending on the circulating and opposing traffic flow and average values for lane capacity, speed, headway, and sight distances on the ring—all this considering the geometric characteristics of the intersection and the

environmental boundary situation, mainly represented by the pavement surface conditions (i.e., dry or wet). Then, compared to the other models of widespread use based both on the regression of empirical data and on the microscopic gap acceptance, the MC model allows us to consider further parameters of great importance in the design and analysis of roundabouts, besides the circulating traffic. It should be noted that the MC model, while not considering the aspect of the drivers' behaviors from the usual point of view of the gap acceptance models, that is the probability distribution of the psychotechnical time intervals among the drivers, nevertheless considers their representation on average by the mean values for perception and reaction times, speed, deceleration, and driving comfort conditions in the ring curve.

Having defined and presented all the equations of the extended MC model, the authors propose the first noteworthy results regarding two validation levels: a first level, with the comparison between the results obtained for a medium-sized test roundabout with MC model and with 15 models of widespread use, based on both empirical regression and gap acceptance; and a second level, with goodness-of-fitting assessment considering four dataset from experimental cases (real or simulated) in the literature and the most recent HCM formulas.

At the first level of validation, the MC model produced results that fall in the range of variability of the control models and do not reveal outlier situations with respect to these consolidated procedures. At the second level of validation, the MC model showed very good flexibility in adapting to the data of the four test cases, which covered a range of external diameters varying between 23 and 45 m (small to medium-large roundabouts). This flexibility appears greater when compared with the most recent standard models proposed by the most recent HCM editions, in the absence of local calibration of the psychotechnical parameters.

It is necessary to specify that, at this stage of the research, no validation checks were carried out on real or simulated data relating to small roundabout, being the smallest tested diameter equal to 23 m (i.e., Dataset A). Although the uncovered ranges actually belong to the action field of the primary Chumanov model [55,56], in the continuation of this research, the validation database will be extended to consider a greater number of cases (including also small roundabouts) under different environmental and boundary situations.

The progress of the research, in addition to further validation tests for the entry capacity, may also allow considering real data for headways, ring capacity, speed, and safety distances between vehicles to investigate extensively the model compliance with real cases. In any case, and without prejudice to further in-depth analyses which will concern the future progress of the research, the MC model can represent a useful contribution to the general discussion as well as a further and effective working tool for technicians engaged in the planning and design of roundabouts.

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