Journal of Physics: Conference Series

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Journal of Physics: Conference Series **2647** (2024) 032001 doi:10.1088/1742-6596/2647/3/032001

Flexural Wave Control Using a Far Field Adaptive Vibration Neutraliser

A Menguzzo[1], P M Casado[1], B R Mace[2], M J Brennan[3] and E Rustighi[1]

[1] Department of Industrial Engineering, University of Trento, Trento, Italy
[2] Faculty of Engineering, University of Auckland, Auckland, New Zealand
[3] School of Engineering, State University of São Paulo UNESP, Ilha Solteira, Brazil

E-mail: emiliano.rustighi@unitn.it

Abstract. The subject of this paper is the control of flexural waves on an infinite beam using a vibration neutraliser located in the far field of a forcing input. The neutraliser analytical model consists of a beam carrying three concentrated masses that can be further simplified to a mass-spring-mass system given the system symmetry. Depending on the input frequency, position and value of the masses, the neutraliser can be tuned either to maximise the power reflection or the power absorption. The bandwidth of attenuation is limited so the neutraliser may become ineffective when the forcing frequency changes. To overcome this limitation, shape memory alloy wires are used to design an adaptive beam neutraliser. The wires, made of a Nitinol alloy, exhibit a change with temperature in the elastic modulus of the material caused by a lattice structure reorganisation. This property allows adaptive tuning for different input frequencies via flexural rigidity variation. A preliminary design of the neutraliser is proposed. Analytical and experimental data are compared to assess the validity of the model.

1. Introduction

Neutralisers are well known vibration control devices which are used to suppress the response of a host structure at a particular forcing frequency. They are commonly realised as lumped parameters systems which are clamped to a structure to reduce its vibration level at the contact point. However, they can also be designed for wave propagation control. A neutraliser used for wave propagation control, depending on the design, may present various parameters that must be taken into account to model its effect. The works by Brennan [1], El-Khatib *et al.* [2] and Issa [3] model the neutraliser as a simple spring-mass system clamped to a beam structure and provide tuning strategies to optimise the effects of the attached neutraliser. In this paper their works is expanded by considering also the effect of a base mass on the flexural wave propagation.

Vibration neutralisers' main limitation consists in their narrow frequency range of effectiveness. A drift in the forcing frequency leads in fact to a quick detuning of the system. To counteract the negative effects of variations in the forcing frequency, a tuning method must be implemented. Among the many tuning strategies available, this paper takes advantage of the metallurgical phase change which temperature causes in Shape Memory Alloys (SMA). The principle exploited is the variation of elastic modulus when the alloy temperature is changed, allowing tuning via flexural rigidity control. Rustighi *et al.* [4] were among the first to design a SMA beam-like neutraliser for the suppression of vibrations of a lumped mass structure. Nitinol wires were used as a free-free beam element, kept in place by a central wood mass and epoxy resin. The wires were electrically connected by clips to form a conductive path in order to heat the system through current. The authors reported an increase of 47.5 % in the elastic modulus and 21.4 % in the tuning frequency, that is considered significant for tuning purposes. The authors also proposed a control algorithm for the automatic tuning of the SMA vibration absorber [5]. El-Khatib [6] proposed a different design of an SMA adaptive vibration neutraliser made of Nitinol wires, where two other masses were positioned towards the extremities of the neutraliser in order to improve its performance and versatility. More profoundly, El-Khatib [6, 2] used the SMA beam- like neutraliser to control flexural waves in an infinite beam. The purpose of the system was either to minimise the power transmitted through the absorber or to maximise the dissipated power. The frequency at which this behaviour occurs is called the tuned frequency. The authors reported an experimental variation of about 20% in the tuned frequency obtained by heating the Nitinol wire. In this paper a similar design is proposed. However, the realised prototype avoids the use of epoxy resin whose elastic properties can show some degradation with temperature and ambient exposure. Moreover, two different connections, namely parallel and series, are proposed to heat the Nitinol wire by Joule effect.

2. Flexural wave model of an infinite beam with a neutraliser

As implemented by Rustighi *et al.* [4] a beam-like structure can be approximated as a mass-spring-mass system in the low frequency range, and it is thus possible to approximate the neutraliser as a 2 Degrees-of-Freedom (DoF) system. The neutraliser, modelled as a 2 DoF system, is clamped to a beam in the far-field from any harmonic force, as shown in figure 1. A propagating harmonic flexural wave $b^+\mathrm{e}^{\mathrm{i}(\omega t - k_b x)}$, where the flexural waves wavenumber is $k_b = (\frac{\rho A}{EI})^{1/4}\sqrt{\omega}$, is incident on the neutraliser, attached to the beam at $x = 0$. The power incident on the neutraliser location is transmitted, reflected or dissipated, thus the definition of the power transmission, reflection and dissipation ratios, respectively τ_t, τ_r and τ_d. Transmission and reflections ratios are defined as [2, 3]

$$\tau_t = \frac{P_t}{P_i} = \left|\frac{c^+}{b^+}\right|^2 \tag{1a}$$

$$\tau_r = \frac{P_r}{P_i} = \left|\frac{b^-}{b^+}\right|^2 \tag{1b}$$

where b and c are the wave amplitudes and the superscripts indicate the propagation direction as shown in figure 1. The dissipation ratio is defined consequently as

$$\tau_d = 1 - (\tau_t + \tau_r). \tag{2}$$

Suppressing the exponential time harmonic term for convenience, the displacement of the infinite beam can be described as:

$$w(x \geq 0) = c^+\mathrm{e}^{-\mathrm{i}k_b x} + c_N^+\mathrm{e}^{-k_b x} \tag{3a}$$

$$w(x \leq 0) = b^+\mathrm{e}^{-\mathrm{i}k_b x} + b^-\mathrm{e}^{\mathrm{i}k_b x} + b_N^-\mathrm{e}^{k_b x} \tag{3b}$$

where the subscript N represents the near-field or evanescent waves. The point mobility of the infinite beam is

$$\frac{\dot{W}(x)}{F} = \frac{-\omega}{4EIk_b^3}(\mathrm{i}\mathrm{e}^{-k_b x} - \mathrm{e}^{-\mathrm{i}k_b x}). \tag{4}$$

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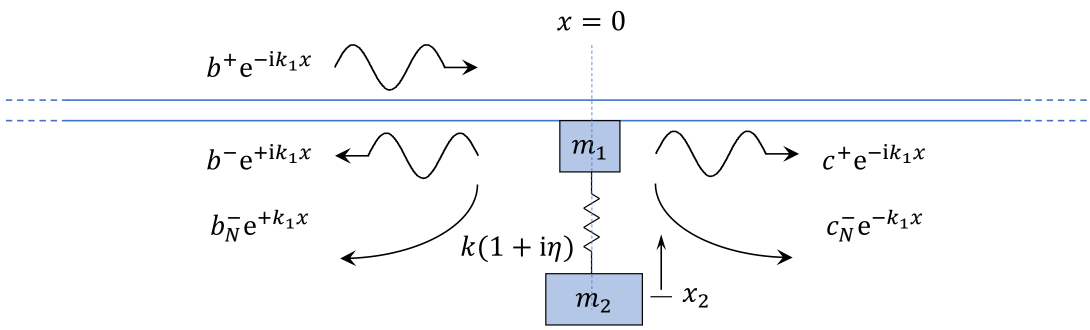

Figure 1: A 2 DoF neutaliser attached to an infinite beam with a positive-going time harmonic travelling incident flexural wave $b^+ e^{i(\omega t - k_b x)}$.

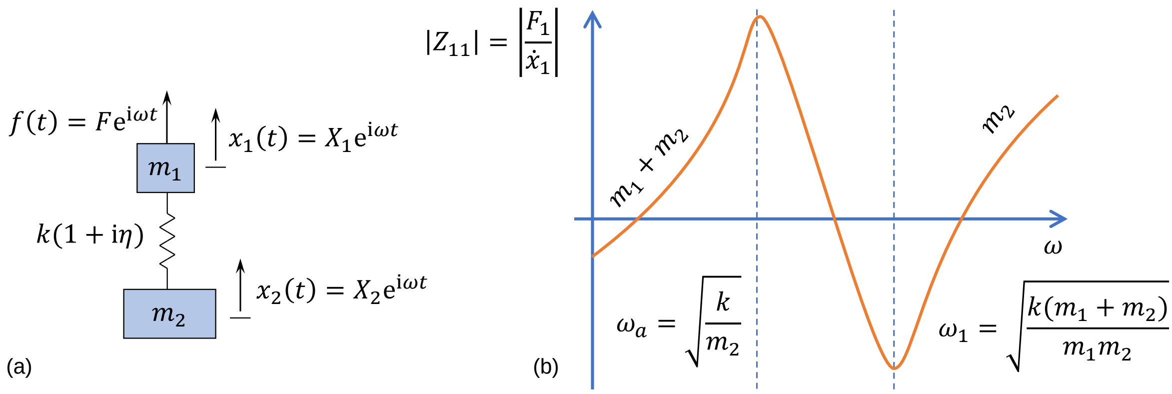

Figure 2: The 2 DoF neutraliser model (a) and its generic point base impedance (b)

2.1. Model of a 2 DoF neutraliser

The model adopted for the neutraliser, and hence its impedance, has a direct effect on the predicted response. As mentioned and implemented in previous works [4, 2], a beam-like neutraliser can be approximated as a 2 DoF system in the low frequency range. The 2 DoF model of the neutralised, as shown in figure 2(a), is composed of two masses connected by an hysteretically damped spring of stiffness k and loss factor η. The mass m_1 is the mass of the base of the neutraliser, which is attached to the infinite beam. The mass m_2 is the suspended mass, or mass of the neutraliser. An external harmonic force, $f(t) = Fe^{i\omega t}$ is applied to the base mass.

The point impedance at the base of the neutraliser, Z_{11}, is

$$Z_{11} = \frac{F}{V1} = \frac{(i\omega)(m_1 m_2 (i\omega)^2 + k(1 + i\eta)(m_1 + m_2))}{m_2 (i\omega)^2 + k(1 + i\eta)} \tag{5}$$

where $k(1 + j\eta)$ represents the complex stiffness. The impedance, equation (5), shows an anti-resonance, also called the neutraliser frequency, at $\omega_a = \sqrt{\frac{k}{m_2}}$ and a resonance at $\omega_1 = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$. The impedance of a generic 2 DoF system is shown in figure 2(b). The stiffness k varies linearly with the neutraliser elastic modulus, hence a change of the Young's modulus will have a linear effect on k and a square-root effect on ω_a and ω_1.

2.2. Model of a 2-DOF neutraliser on an infinite beam

Once the neutraliser model is known it is possible to include its effect on the wave propagation. It is necessary to find the amplitude ratios defined in equations (1) through the application of continuity (displacement and rotation) and equilibrium (force and moment) conditions at $x = 0$. It is found that

$$c^+ + c_N^+ = b^+ + b_N^+ + b^- + b_N^- \tag{6a}$$
$$-\mathrm{i}c^+ - c_N^+ = -\mathrm{i}b^+ - b_N^+ + \mathrm{i}b^- + b_N^- \tag{6b}$$
$$\mathrm{i}c^+ - c_N^+ - \mathrm{i}b^+ + b_N^+ + \mathrm{i}b^- - b_N^- = \frac{m_1\omega^2 w(0)}{EIk_b^3} + \frac{k}{EIk_b^3}(1+\mathrm{i}\eta)(x_2 - w(0)) \tag{6c}$$
$$-c^+ + c_N^+ = -b^+ + b_N^+ - b^- + b_N^- \tag{6d}$$

where $\frac{m_1\omega^2 w(0)}{EIk_b^3}$ is the result of clamping the neutraliser upper mass to the beam and the term $\frac{k}{EIk_b^3}(1+\mathrm{i}\eta)(x_2 - w(0))$ is given by the force exerted by the remaining spring-mass part of the neutraliser. Also $w(0)$ is the displacement of the infinite beam at the clamping point and x_2 is the displacement of the neutraliser mass m_2 as shown in figure 1.

It is possible to obtain the ratios between wave amplitudes. After some manipulations it is found that

$$\tau_t = \left| \frac{-\Omega^{5/2}\gamma R + \gamma(1+\mathrm{i}\eta)(R+1)\sqrt{\Omega} + \mathrm{i}\eta - \Omega^2 + 1}{-(1+\mathrm{i})\gamma R\Omega^{5/2} + \gamma(1+\mathrm{i}+(-1+\mathrm{i})\eta)(R+1)\sqrt{\Omega} + \mathrm{i}\eta - \Omega^2 + 1} \right|^2 \tag{7}$$

and

$$\tau_r = \left| \frac{\mathrm{i}\Omega^{5/2}\gamma R + \sqrt{\Omega}(R+1)\gamma(\eta - \mathrm{i})}{(1+\mathrm{i})\gamma R\Omega^{5/2} - \gamma(1+\mathrm{i}+(-1+\mathrm{i})\eta)(R+1)\sqrt{\Omega} - \mathrm{i}\eta + \Omega^2 - 1} \right|^2 \tag{8}$$

and τ_d is consequently defined from equation (2). The four characteristic non-dimensional parameters are the loss factor η, the neutraliser's mass ratio R, the ratio between the forcing frequency and the neutraliser frequency Ω and the ratio between the neutraliser's active mass and the beam effective mass γ. The non-dimensional parameters are defined as

$$R = \frac{m_1}{m_2} \tag{9a}$$
$$\Omega = \frac{\omega}{\omega_a} \tag{9b}$$
$$\gamma = \frac{\pi m_2}{2\rho A\lambda_a} \tag{9c}$$

where the beam's effective mass has been taken as that of a length $2\lambda_a/\pi$ of the beam, where λ_a is the beam flexural wavelength at the neutraliser frequency.

2.3. Optimum tuning of the neutraliser

Tuning a neutraliser for the control of wave propagation means defining the parameters of the system that accomplish either minimum transmission or maximum dissipation [2, 3] at the forcing frequency. Closed form solutions can be found only by introducing approximations as reported in previous works [1, 2, 3]. This work expands previous results by the inclusion of the base mass m_1. Such an inclusion increases the complexity of the system, thus a numerical approach to analyse the parameters effect has been chosen. In particular, the mutual effects of the parameters on the propagation has been studied by changing one parameter at a time while leaving the others constant. The parameters' reference values used to show variation in transmission, reflection and dissipation ratios have been chosen as $\gamma = 0.05$, $R = 1$ and $\eta = 0.01$. The frequency ratio at which the performance of the neutraliser is optimum is called the tuned frequency ratio [2]. The tuned frequency ratio for optimal τ_t is denoted by Ω_{tt} and corresponds to a function minimum, whereas the tuned frequency ratios for optimal τ_r and τ_d are Ω_{tr} and Ω_{td} respectively correspond to a function maximum. Figure 3 shows the effect that a parameter variation has on the tuned frequencies and the respective tuned values ($\min(\tau_t)$, $\max(\tau_r)$ and $\max(\tau_d)$).

The effect of the base mass is shown by varying R, and it is shown in figures 3a and 3b. For each value of R the optimal values have been calculated for all frequency ratios. Figure 3a shows the tuned frequencies while optimal transmission and dissipation are presented in figure 3b. Figure 3b shows that small transmission values and large reflections are found at small ratios: $R < 10^0$ and $\Omega_t \approx 1.025$. Dissipation presents a maximum-like behaviour at $R \approx 2 \times 10^1$ (and $\Omega \approx 1.018$). At the extremes of R the system becomes either a spring-mass system or a blocking mass where $\min(\tau_t)$ and $\max(\tau_r)$ converge to 0.5.

The effect of the neutraliser mass ratio is presented in figures 3c and 3d when varying the non-dimensional parameter γ. Varying γ produces a plateau in the transmission minimum and a high reflection for values $\gamma \approx 10^0$ and $\Omega \approx 1.2$. Increasing γ to large values has a pinning effect. The maximum dissipation presents two peaks that strongly depend on the damping value used. In this case they are present at $\gamma = 1 \times 10^-2$, $\gamma = 2 \times 10^1$ and $\Omega \approx 1$, $\Omega \approx 1.4$ respectively. Depending on the optimisation chosen, trade-off considerations must be made.

The last parameter investigated is η. The results are shown in figures 3e and 3f. To obtain minimum transmission it is best to use small values of loss factor, $\eta < 10^-2$ and $\Omega_t \approx 1.025$. For maximum dissipation there is an optimal loss factor at $\eta \approx 4.5 \times 10^-2$ and $\Omega_t \approx 1.025$. Since Nitinol may present both a variation in elastic modulus and damping behaviour depending on temperature [2], the latter must be taken into account when modelling the neutraliser.

3. A beam-like SMA adaptive vibration neutraliser

The design of a beam-like neutraliser is a straightforward choice to easily include Nitinol wires. The frequency response of a beam-like structure can be approximated as a mass-spring-mass system for frequencies below the second natural frequency, thus converging to the already presented approximated model for wave propagation control. SMAs are typically known for superelasticity and the shape memory effect produced by a phase change with varying stress or temperature [7]. The martensite-austenite transition also produces a change in the elastic modulus of the alloy. This property gives the possibility to tune the neutraliser in the presence of an excitation frequency drift.

3.1. Design of the SMA adaptive vibration neutraliser

Before designing and producing the neutraliser prototype an analytical model has been developed to assess the frequency response functions of the system. As mentioned, the neutraliser comprises a beam-like structure on which three masses are clamped. The approach used is described in [8] and [9] where lumped masses are included with Dirac's Delta functions in the Euler-Bernoulli beam differential equation. This approach produces analytically correct mode shapes with the use of the Laplace transform. To approximate the beam-like model as a 2 DoF configuration the steps in [4] were followed. If the first natural frequency ω_1, the first mode shape Φ at the mid-span ($x = L/2$) and the the total mass of the device m_t are known, the parameters of the

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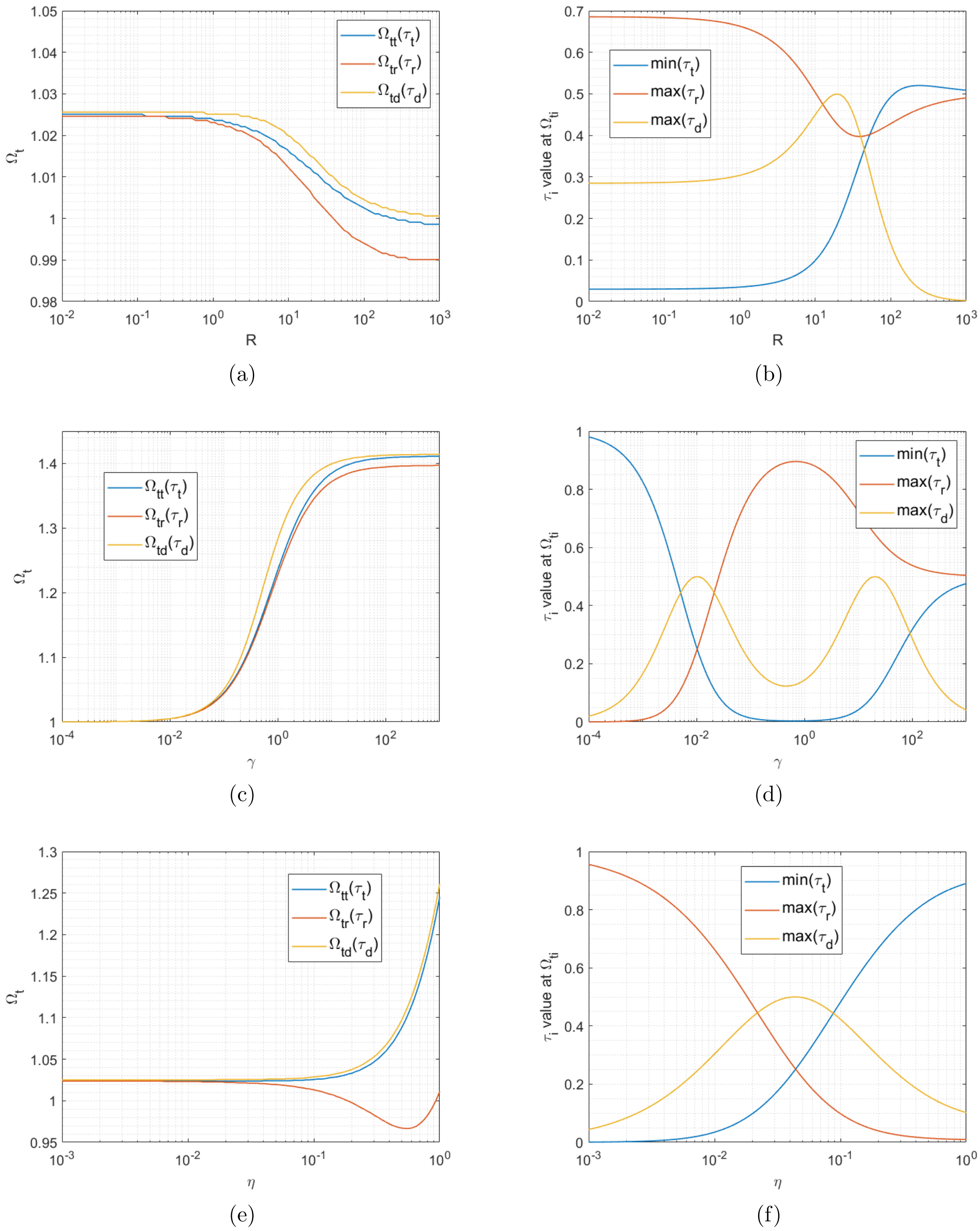

Figure 3: Parameter effects on the wave propagation. Optimal τ_t: blue line; optimal τ_r: red line; optimal τ_d: yellow line. (a) Tuned frequency ratio as function of R ($\gamma = 0.05$, $\eta = 0.01$); (b) Tuned values as function of R ($\gamma = 0.05$, $\eta = 0.01$); (c) Tuned frequency ratio as function of γ ($R = 1$, $\eta = 0.01$); (d) Tuned values as function of γ ($R = 1$, $\eta = 0.01$); (e) Tuned frequency ratio as function of η ($\gamma = 0.05$, $R = 1$); (f) Tuned values as function of η ($\gamma = 0.05$, $R = 1$).

2 DoF neutraliser can be obtained from the following equations:

$$m_1 = \frac{m_t}{1 + m_t\Phi^2(L/2)} \tag{10a}$$

$$m_2 = m_t\left(1 + \frac{1}{m_t\Phi^2(L/2)}\right)^{-1} \tag{10b}$$

$$k = \frac{\omega_1^2}{1 + m_t\Phi^2(L/2)}\left(\frac{m_t}{1 + 1/m_t\Phi^2(L/2)}\right) \tag{10c}$$

The values of ω_1, m_t and $\Phi^2(L/2)$ can be obtained from an analytical model as discussed or from experimental measurements.

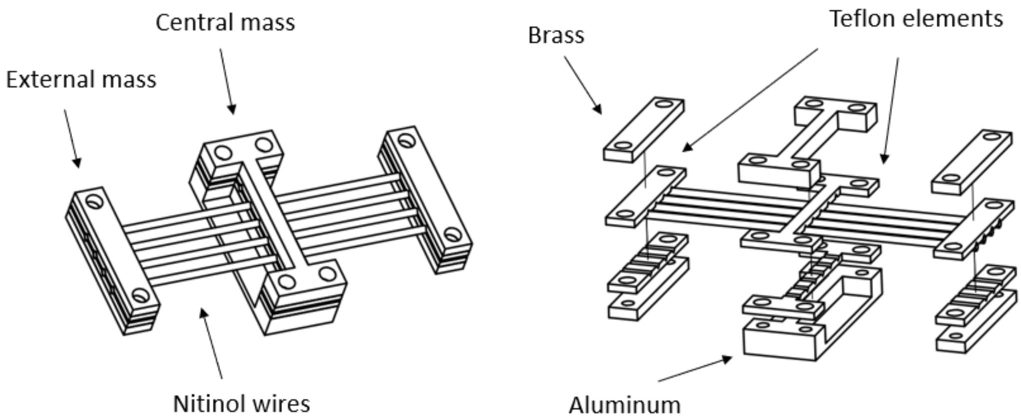

Figure 4: Neutraliser design.

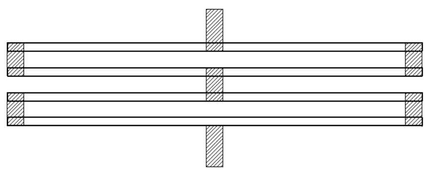

Figure 5: Parallel electrical configuration of the wires.

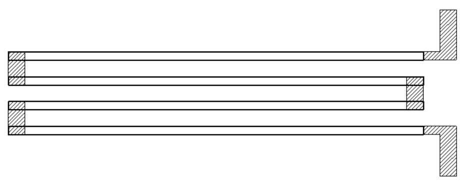

Figure 6: Series electrical configuration of the wires.

The prototype designed is shown in figure 4. Four Nitinol wires are held by a central mass which is used as the clamping point to the host structure. The Nitinol wires were supplied by Nanografi Nano Technology (Turkey). They had a 2 mm diameter. The manufacturer data gave an austenite finish temperature at $80-85\,^\circ$C. The base material is aluminium ($\rho \approx 2700$ kg/m^3) to limit R, and hence m_1. The external masses are made of brass elements ($\rho \approx 8500$ kg/m^3) in an attempt to increase γ for the reasons described above. To insulate the system electrically, Teflon elements are used. Channels were machined into the insulating element to position the Nitinol wires at a predetermined distance. The conductive path of the Nitinol wires is closed by brass sheet elements positioned between the insulators. The various parts are kept in place by the pressure obtained with threaded joints. Figures 5 and 6 show the two electrical configurations used to supply power, therefore heating the system.

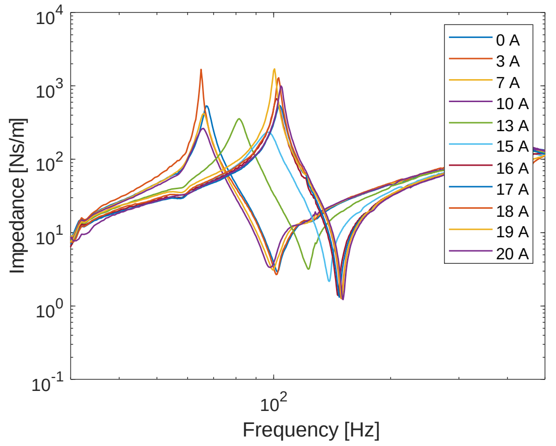

Figure 7: Base impedance of the neutraliser as a function of the heating current in parallel configuration.

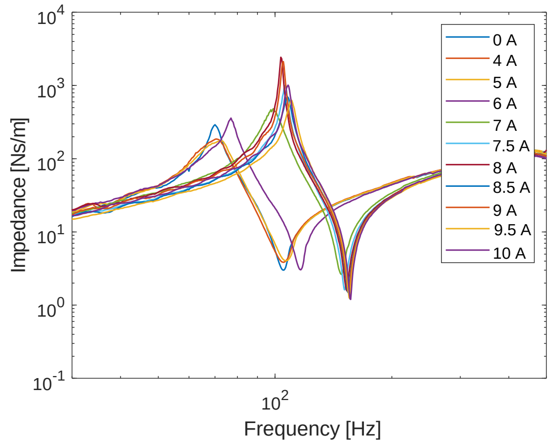

Figure 8: Base impedance of the neutraliser as a function of the heating current in series configuration.

3.2. Experimental characterisation of the SMA adaptive vibration neutraliser prototypes

Modal analysis was performed on the neutraliser in both series and parallel configurations. The measured impedance for both cases is shown in figures 7 and 8. It is possible to see that the frequency response of the system increases by about 40 Hz when transitioning from martensite (low temperature lattice configuration) to austenite (high temperature lattice state). The neutraliser frequency f_a for both the configurations increases from about $60 - 65$ Hz to about $105 - 110$ Hz, which corresponds to an increase of 66% in the tuned frequency.

Figures 9 and 10 show the variation of $f_a = \omega_a/2\pi$ with respect to the current supplied to the system. The offset in the current between parallel and series configurations is caused by the different resistances of the circuits. Experimental transition temperatures for parallel and series setups, in particular A_s (austenite start) and A_f (austenite finish) [7], can be estimated respectively as about $40 - 45\,°$C and about $80 - 85\,°$C from thermocouple measurements. From the peak frequency position the elastic modulus of the beam-like part can also be calculated. Since ω_a is proportional to $\sqrt{E}$, it is found that E varies approximately from 40 GPa to 100 GPa, which corresponds to an increase in Young's modulus of about 150%.

4. Performance prediction for flexural wave control

Although the effect that the neutraliser has on wave propagation has not yet been found experimentally, a prediction is here reported for the series configuration mounted on an infinite beam. The steel beam model has a 50×6.4 mm cross section, a Young's modulus of 210×10^9 Pa and a density of 7900 kg/m^3. For the series configuration three impedance measurements at three different temperatures are taken as a reference. From the experimental data the stiffness, damping and masses of the 2 DoF system are calculated. The parameters found are then inserted in the wave propagation model to find an estimate of the effect that the neutraliser has on the vibration of the host structure. The results obtained are visible in figures 11 and 12. The values used for the numerical calculations are presented in table 1. It is clear the effect that temperature has on both stiffness and loss factor.

The curves of the optima, either minimum or maximum, presents a shift of about 40 Hz. Since damping is strongly affected by the lattice structure, the optimal transmitted power is around 40% at low temperature/martensitic phase (0 A) and 10% in the high temperature/austenitic phase (10 A). Optimal dissipation instead remains constant at about $45 - 50$%.

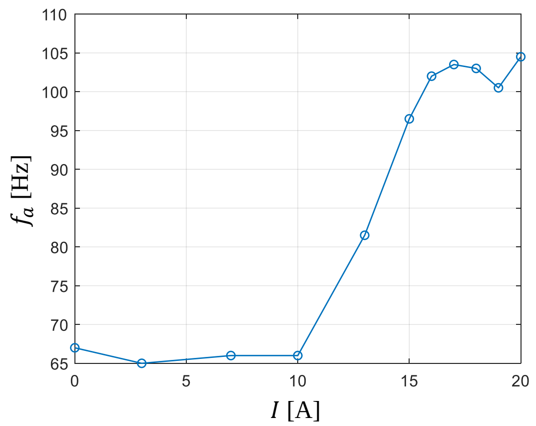

Figure 9: Variation of $f_a = \omega_a/2\pi$ with respect to the heating current in parallel configuration.

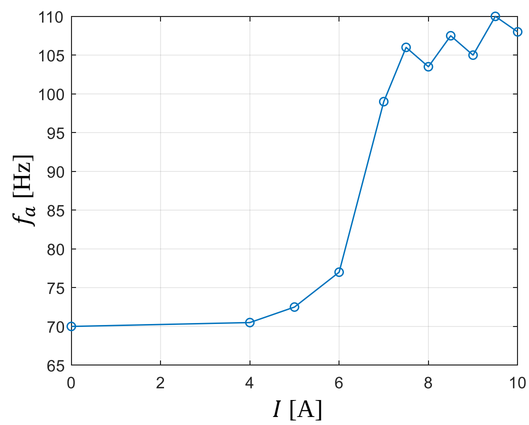

Figure 10: Variation of $f_a = \omega_a/2\pi$ with respect to the heating current in series configuration.

Table 1: Parameters of the two DoF neutraliser obtained from the experimental data of the series configuration.

I	f_a	m_1	m_2	k	η
0 A	70 Hz	0.03952 kg	0.04777 kg	9240.83 Nm	0.055
7 A	99 Hz	0.03952 kg	0.04777 kg	18483.56 Nm	0.04
10 A	108 Hz	0.03952 kg	0.04777 kg	21996.96 Nm	0.02

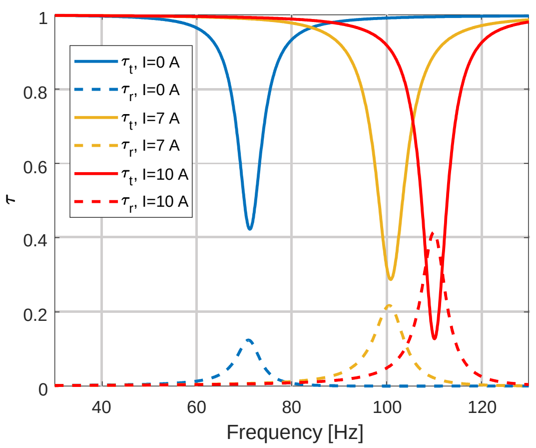

Figure 11: Estimate of the transmission and reflection ratios with the prototype parameters.

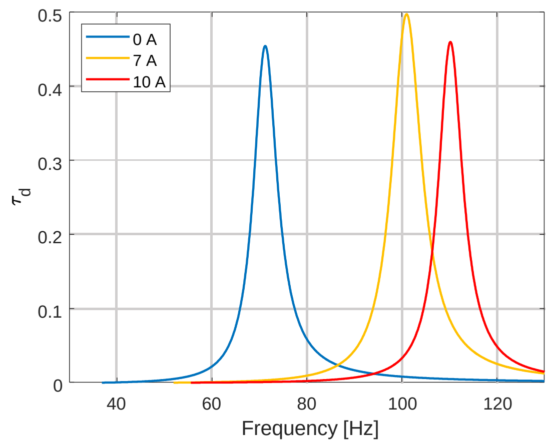

Figure 12: Estimate of the dissipation ratio with the prototype parameters.

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5. Conclusions

In this work modelling, design and experimental validation of a neutraliser for the control of flexural waves on an infinite beam has been described. The inclusion of Nitinol wires acts as a tuning element to cope with possible variations in the incident wave frequency. A beam-like neutraliser carrying masses was modelled to accurately describe the expected frequency response functions of the prototype along with its 2 DoF approximation. The experimental results on the prototype shows a significant shift of about 40 Hz (approximately +66%) in the impedance curve. This effect is related to the martensite-austenite phase transition and the relative increase in the elastic modulus (approximately +150%). From the experimental data the parameters of the 2 DoF system were extracted and an estimate of the neutraliser effect on the infinite beam given. The models developed are well-suited to describe the prototype implemented and the approach used is capable of modelling the behaviour of the system. The feasibility of an SMA beam-like neutraliser was verified. Future work will be focused on the experimental effect that the neutraliser has on the host structure.

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