LOOKING FOR STOKES THEOREM IN AN ELEMENTARY TRAPEZOID.

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Abstract. The aim of this very short note it to show that even in a totally elementary framework, it is possible to glimpse the Stokes theorem.

The Note

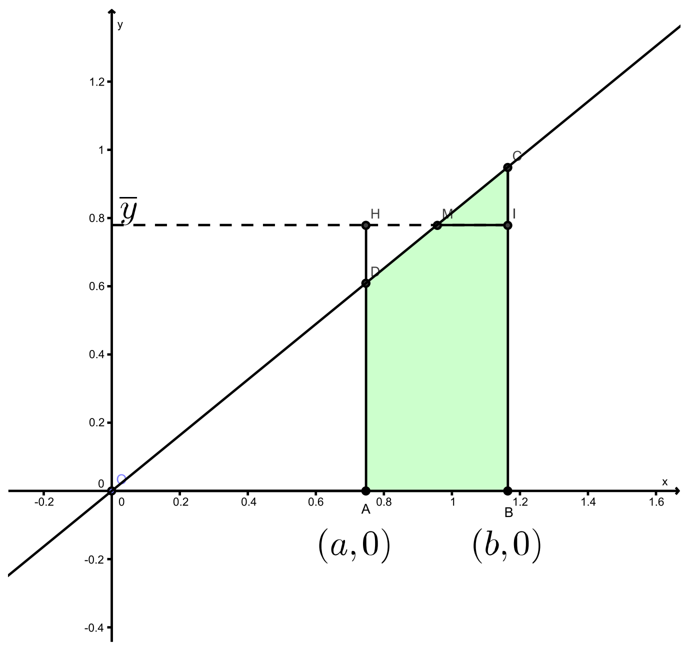

Figure 1

The Stokes theorem tell us that

$$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega. \tag{1}$$

where Ω is a suitable domain and $\partial\Omega$ its boundary. Even in the calculation of the area of an elementary trapezoid, we use, almost certainly in a non-conscious way, either the LHS or the RHS of equation (1). Let

Date: September 21, 2012.
1991 *Mathematics Subject Classification.* 97A80,97A99.
Key words and phrases. elementary mathematics, Stokes Theorem,
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us consider a rectangular trapezoid $ABCD$ as shown in Figure 1 and let be $y = mx$ the straight line through O and C. Of course we can calculate the area of the trapezoid in two different ways:

(1) We can think to the trapezoid as equivalent to the rectangle $ABIH$.
(2) We can think to the trapezoid as the difference between the triangles COB and DOA.

If we think to the trapezoid as equivalent to the rectangle $ABIH$ then, since its area is $\mathcal{A} = \overline{y}(b-a)$, we can imagine this as theas the LHS of (1) where Ω is the interval $[a, b]$ and $d\omega = (mx)dx$. If we calculate the area as difference between the area of the triangle OCB and the area of the triangle DOA then we have

$$A = \frac{b\,(mb)}{2} - \frac{a\,(ma)}{2} = \frac{1}{2}mb^2 - \frac{1}{2}ma^2 = \omega\,(b) - \omega\,(a) = \int\limits_{\partial\Omega} \omega$$

namely, the RHS of (1) because the boundary of Ω, trivially is given by $\{b, a\}$.

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