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AN UNSUPERVISED STRATEGY BASED ON A FUZZY-LOGIC SYSTEM FOR THE EXPLOITATION OF THE INFORMATION CONTENT OF NOISY DATA IN INVERSE SCATTERING PROBLEMS

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An Unsupervised Strategy based on a Fuzzy-Logic System for the Exploitation of the Information Content of Noisy Data in Inverse Scattering Problems

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Inverse scattering data, although collected in a controlled-environment, are generally corrupted by an electromagnetic noise, which strongly affects the effectiveness of reconstruction strategies because of the intrinsic ill-conditioning of the problem. To limit the effects of the noise on the retrieval procedure, this paper presents an innovative fuzzy-logic-based approach, which allows one to take into account the corrupted nature of the data by fully exploiting all the available information content of the measurements. Selected numerical experiments in a noisy scenario are considered for assessing the potentialities of the approach also in comparison with a reference inverse scattering technique.

Introduction

Inverse scattering problems are intrinsically characterized by the ill-posedness and ill-conditioning due to the limited amount of information content available on collectable data [1]. To limit/overcome these drawbacks several approaches have been proposed. Some methods are aimed at quantifying the achievable spatial resolution in relation to the amount of data [2]. Others methodologies define suitable representations of the unknowns and customized strategies in order to (even though iteratively) enhance the reconstruction accuracy fully exploiting all the scattering information [3][4][5]. However, such techniques do not consider or partially address the problem of the reliability of the data. As a matter of fact, if quantifying the number of informative data is a key-issue in solving inverse scattering problems, on the other hand the effectiveness of a retrieval procedure strongly depends on the level of reliability of such data. Certainly, a direct evaluation of the data reliability would be really useful. In general, such a knowledge is available not as *objective knowledge* (i.e., in terms of a mathematical model or numerical values), but as *subjective knowledge*, which represents an information that is usually difficult or complex (and expensive) to quantify using traditional mathematics or experimental methods. Because of these problems, subjective information is usually ignored with a lost in the retrieval processing.

To exploit the subjective information by extracting the “clean” information-content available in the noisy data, this paper proposes an unsupervised approach, which takes into account the presence of the noise through a fuzzy-logic-based strategy [6]. Starting from the definition of a cost function composed of two terms, which depends on the scattered field collected in the observation domain and on the incident field measured in the investigation domain, the fuzzy-logic system (FLS) defines suitable regularization parameters able to weight more the one or the other term, depending on the uncertainty associated with each of them.

Mathematical Formulation

Let us consider a two-dimensional inverse scattering problem where a tomographic geometry is illuminated by a set of V plane waves characterized by radiated electric fields $E_{inc}^{v}(x,y)$, $v=1,...,V$. The investigation domain under test D_I is described with an unknown object function $\tau(x,y)$ to be determined by means of the reconstruction process. The relations between dielectric properties of the investigation domain and scattered fields are mathematically described through the well-known integral scattering equations [3], whose discretized counterparts [7] are

$$S_{scatt}^{v,m}=\sum_{n=1}^{N}\tau(x_n,y_n)E_{tot}^{v}(x_n,y_n)G_{ext}^{v}(x_m,y_m\,|\,x_n,y_n) \quad (x_m,y_m)\notin D_I \tag{1}$$

$$S_{inc}^{v,n}=E_{tot}^{v}(x_n,y_n)-\sum_{p=1}^{N}\tau(x_p,y_p)E_{tot}^{v}(x_p,y_p)G_{\mathrm{int}}^{v}(x_p,y_p\,|\,x_n,y_n) \quad (x_n,y_n)\in D_I \tag{2}$$

G_{ext}^{v} and G_{int}^{v} being the external and internal free-space Green's functions, respectively; $\tau(x_n,y_n)$ and $E_{tot}^{v}(x_n,y_n)$ are the object function and the electric field at the *n*th discretization cell of the investigation domain ($n=1,...,N$). Through a multi-view/multi-illumination measurement system, the data of the inverse scattering problem are acquired. The scattered electric field $E_{scatt}^{v}(x_m,y_m)$, $v=1,...,V$ is collected at M measurement points ($m=1,...,M$) external to D_I and equally-spaced along a circular observation domain. Moreover, the scattering data are completed by measuring the incident field $E_{inc}^{v}(x,y)$; $n=1,...,N$; $v=1,...,V$ in D_I. After the acquisition, the retrieval problem is then solved by processing the data through the minimization of the cost function

$$\Phi(\underline{u})=\frac{\sum_{v=1}^{V}\sum_{m=1}^{M}\left\{\alpha^{v,m}\left|E_{scatt}^{v}(x_m,y_m)-S_{scatt}^{v,m}\right|^2\right\}}{\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{scatt}^{v}(x_m,y_m)\right|^2}+\frac{\sum_{v=1}^{V}\sum_{n=1}^{N}\left\{\beta^{v,n}\left|E_{inc}^{v}(x_n,y_n)-S_{inc}^{v,n}\right|^2\right\}}{\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{v}(x_n,y_n)\right|^2} \tag{3}$$

whose reliability coefficients $\alpha^{v,m}$ and $\beta^{v,n}$ are defined according to the fuzzy-logic strategy described in the following Section and where $\underline{u}=\left\{\tau(x_n,y_n),E_{tot}^{v}(x_n,y_n);n=1,...,N;v=1,...,V\right\}$.

Fuzzy-Logic-Based Reconstruction Strategy

In order to define the weighting coefficients, by taking into account the presence of the noise on the measured data, the fuzzy-logic strategy is implemented according to the following step-by-step process:

- The samples of inverse scattering data are normalized and the following coefficients are defined

$$\eta^{v,m}=\frac{\left|\dfrac{E_{scatt}^{v}(x_m,y_m)}{E_{tot}^{v}(x_m,y_m)}\right|}{\max\limits_{v}\left\{\max\limits_{m}\left|\dfrac{E_{scatt}^{v}(x_m,y_m)}{E_{tot}^{v}(x_m,y_m)}\right|\right\}},\quad \zeta^{v,n}=\frac{\left|E_{inc}^{v}(x_n,y_n)\right|}{\max\limits_{v}\left\{\max\limits_{n}\left|E_{inc}^{v}(x_n,y_n)\right|\right\}},\quad \begin{array}{c}v=1,...,V\\ m=1,...,M\\ n=1,...,N\end{array} \tag{4}$$

  which are used as inputs of the "*fuzzifier*" block.

- The fuzzifier determines the fuzzy counterparts of the input coefficients $\varphi(\eta^{v,m})$ (or $\varphi(\zeta^{v,n})$) through a Gaussian function φ centered in $\eta^{v,m}$ (or $\zeta^{v,n}$) and characterized by an assigned variance value.
- The "*inference*" block, which operates according to a specific set of heuristically defined "*if-then-else*" rules composed by a set of *antecedents* $\Gamma = \{\Gamma_r; r = 1,..., R\}$ and relative *consequences* $C = \{C_r; r = 1,..., R\}$, starting from the activation values $\mu_r(\eta^{v,m}) = \varphi(\eta^{v,m}) \cap \Gamma_r$, $r = 1,..., R$ (or $\mu_r(\zeta^{v,n}) = \varphi(\zeta^{v,n}) \cap \Gamma_r$, $r = 1,..., R$) determines the degree of truth of every consequence $\mu_r(\eta^{v,m}) \cap C_r$ (or $\mu_r(\zeta^{v,n}) \cap C_r$).
- The "*defuzzifier*" block computes $\alpha^{v,m}$ and $\beta^{v,n}$ as the centroid of the area obtained through the superposition of the activated consequences

After the computation of the reliability indexes, the inversion process is successively completed by determining an estimate of the unknown object function, through the iterative minimization of (3) (where each measured data contributes according to own reliability index).

Numerical Assessment

In this Section, a selected set of numerical results will be shown to give some indications on the improvement over a standard approach when pre-processing the input data with the fuzzy-system. Such results will be concerned with two different scattering scenarios characterized by a signal-to-noise-ratio (SNR) equal to $10\,dB$ and $5\,dB$, respectively. The two-scatterers geometry under test [Fig. 1(*a*)] is located in a square investigation domain λ-sided and it has been probed by a set of $V = 8$ monochromatic plane waves. The scattering data have been numerically computed in $M = 8$ sampling points by adding a random Gaussian noise to simulate realistic environmental conditions. As far as the minimization procedure is concerned, since the focus is on the comparison between the fuzzy-logic-based technique and the reference one (the "standard" approach where $\alpha^{v,m} = \beta^{v,n} = 1.0$), a simple deterministic iterative conjugate-gradient optimizer has been used. As expected, the fuzzy-logic data processing, acting before the minimization process, significantly impacts when the measured data are seriously corrupted by the noise. Such an event can be pictorially noticed by comparing the images of the reconstructed contrasts when $SNR = 10\,dB$ [Fig. 1(*b*) vs. Fig. 1(*c*)] and $SNR = 5\,dB$ [Fig. 1(*d*) vs. Fig. 1(*e*)].

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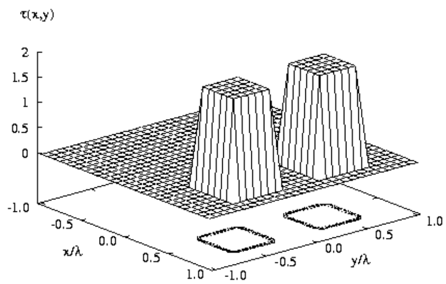
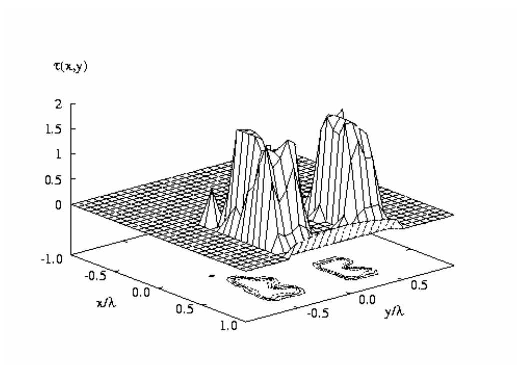
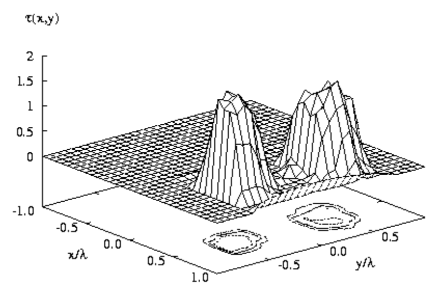


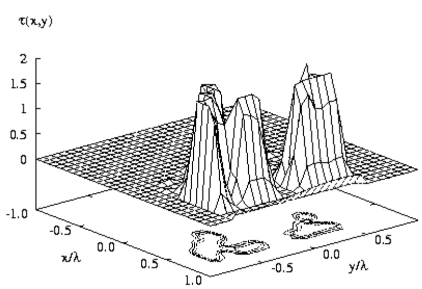
(*a*)



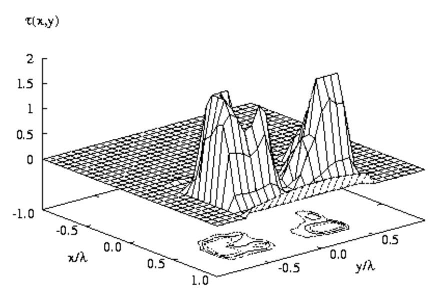
(*b*)



(*c*)



(*d*)



(*e*)

Figure 1. Two-scatterers geometry – Actual profile (*a*) and dielectric distributions reconstructed with the “standard” approach (*b*)-(*d*) and the “fuzzy-logic-based” strategy (*c*)-(*e*) when $SNR = 10\,dB$ (*b*)-(*c*) and $SNR = 5\,dB$ (*d*)-(*e*).