RECONSTRUCTION OF DIELECTRIC OBJECTS FROM AMPLITUDE-ONLY DATA – ADVANTAGES AND OPEN PROBLEMS OF A TWO-STEP MULTI-RESOLUTION STRATEGY

G. Franceschini, M. Donelli, D. Franceschini, M. Benedetti, P. Rocca, and A. Massa

January 2011

Technical Report # DISI-11-240

Reconstruction of Dielectric Objects from Amplitude-Only Data - Advantages and Open Problems of a Two-Step Multi-resolution Strategy

G. Franceschini, M. Donelli, D. Franceschini, M. Benedetti, P. Rocca and A. Massa

Department of Information and Communication Technology, University of Trento
Via Sommarive 14, 38050 Trento, Italy,
Email: gabriele.franceschini@dit.unitn.it, massimo.donelli@dit.unitn.it, davide.franceschini@dit.unitn.it,
manuel.benedetti@dit.unitn.it, paolo.rocca@dit.unitn.it, andrea.massa@ing.unitn.it

Abstract

In the following contribution an innovative strategy for the inversion of amplitude-only data in microwave imaging applications is presented. The method consists of two steps. At the first step the source is synthesized in order to compute the incident field in the investigation domain. In the second step the profile of the object is reconstructed thanks to the iterative multi-scaling approach combined to the Particle Swarm Optimiser, an innovative and effective evolutionary minimization technique. The effectiveness of the algorithm is preliminary assessed through the inversion of experimental data concerning an inhomogeneous dielectric scatterer.

I. INTRODUCTION

The reconstruction of the geometrical and physical characteristics of an unknown object is a topic of great interest in many different fields, such as biomedical and industrial diagnostic. The microwave imaging techniques are potentially suitable for these problems, but they have some drawbacks related to the nature of the mathematical model and to the hardware setup required to collect the necessary field measurements. As a matter of fact, the inverse scattering problems are ill-posed, highly non-linear and the amount of collectable information is limited also if multi-illumination, multi-view and multi-frequency systems are considered. In [1] and [2] a criterion related to the geometrical and physical characteristics of the system is provided in order to evaluate the upper-bound to the achievable information and to chose the optimal number of unknowns. Moreover, the data acquisition requires complex and expensive hardware setups. In particular, the measurement of the phase distribution turns out to be critical when high frequencies are considered. As a matter of fact, holographic and interferometric techniques (generally used in optical application [3][4]) allow to retrieve the phase information starting from amplitude-only data, but they require undesired additional post-processing. In order to realize a reliable and cost-effective imaging apparatus, some different strategies based on phaseless data have been developed in the past. Two main paths of research seem to be usually taken into account: (*a*) the direct application of a reconstruction algorithm for the processing of phaseless field data (Single-Step Strategy) (see for example [5]-[6]); (*b*) the splitting of the phaseless-data reconstruction into a two-step process (Two-Step Strategy) where the first step deals with a phase-retrieval problem for completing the amplitude-only inversion data and the latter is concerned with a standard reconstruction from complete field data (see for example [7][8]).

In this contribution an innovative two-step strategy belonging to the second class is proposed and presented in Sect. II. In Sub-Sect. II.A an inverse source problem is presented and solved through the modeling of the electric field according to the *Distributed-Cylindrical-Waves Model* (*DCW*-Model). In Sub-Sect. II.B, a multi-resolution cost functional [9] is defined and minimized using the Particle Swarm Optimizer [10], one of the most effective evolutionary iterative procedures. In section III, some experimental results are presented in order to draw some preliminary conclusions (Sect. IV) on the effectiveness of the proposed methodology.

II. TWO STEP ALGORITHM

Let us consider the classical tomographic imaging configuration in which an unknown cylindrical dielectric object is located in an inaccessible investigation domain D_I and is illuminated by a set of V TM-polarized incident electromagnetic waves. The aim of the proposed algorithm is the reconstruction of the contrast function $\tau(\mathbf{r})$ defined in (1), where ε_r is the relative dielectric permittivity.

$$\tau(\mathbf{r})=\varepsilon_r(\mathbf{r})-1 \qquad (\mathbf{r})\in D_I \tag{1}$$

For the proposed methodology, only the knowledge of the amplitude of the total field, $\left|E_{tot}^{v}(\mathbf{r}_{m(v)})\right|$ is assumed, together with the amplitude and phase of the incident electric field, $E_{inc}^{v}(\mathbf{r}_{m(v)})$, in $M^{(v)}$ points, $(\mathbf{r}_{m(v)}) \in D_M$, being D_M the observation domain external to D_I. Assuming the knowledge of the phase, we do not limit the phaseless nature of the algorithm because the measurements can be executed only once and off-line for each hardware setup and they are not so-expensive being limited to a reduced number of points in the observation domain.

The relation between unknowns ($\tau(\mathbf{r})$ and $E_{tot}^{v}(\mathbf{r})$) and data is expressed by the following equations

$$\left|E_{tot}^{v}(\mathbf{r}_{m(v)})\right| = \left|E_{inc}^{v}(\mathbf{r}_{m(v)}) + j\omega\mu_0 \int_{D_I} \tau(\mathbf{r}')E_{tot}^{v}(\mathbf{r}')G(\mathbf{r}_{m(v)}/\mathbf{r}')d\mathbf{r}'\right| \tag{2}$$

$$\left|E_{inc}^{v}(\mathbf{r})\right| = \left|E_{tot}^{v}(\mathbf{r}) - j\omega\mu_0 \int_{D_I} \tau(\mathbf{r}')E_{tot}^{v}(\mathbf{r}')G(\mathbf{r}/\mathbf{r}')d\mathbf{r}'\right| \tag{3}$$

where $G(\mathbf{r}/\mathbf{r}')$ is the free space green function.

It can be notice that in (3) the measurements of the amplitude of the incident field in the investigation domain are necessary. From a practical point of view, it is a critical issue because the measurements have to be performed in a large number of points if a satisfactory resolution level is desired. Moreover, the experimental system (and in particular the electromagnetic sensors) is moved by means of a mechanical apparatus with some tolerances in the positioning. Therefore, a reduced sampling distance between adjacent positions in D_I would result in an inaccurate measure of the field and, consequently, each field sample would be corrupted by a non-negligible error. For avoiding such a drawback, a suitable model of the radiating source will be defined in the following.

A. SOURCE SYNTHESIS

Because of the complexity and of the difficulties in collecting reliable and independent measures in a dense grid of points, let us assume that the incident field, $E_{inc}^{v}(\mathbf{r}_{m(v)})$, is only available at the measurement points belonging to the observation domain. Therefore, in order to apply the constraints stated through (3) and before facing with the data inversion, it is mandatory to develop a suitable model able to predict the amplitude of the incident field radiated by the actual electromagnetic source in the investigation domain D_I. In the *DCW* Model, the antenna is represented by means of a linear array of W equally spaced line-sources and therefore the electric field can be expressed as

$$\varsigma^{v}(\mathbf{r}) = -\frac{k_0^2}{8\pi f \varepsilon_0}\sum_{w=1}^{W} A_w H_0^{(2)}(k_0 d_w) \tag{4}$$

where d_w is the Euclidean distance between the position of *w-th* element of the array and **r**, k_0 is the free-space wavenumber and $H_0^{(2)}$ is the *0-th* order second-kind Hankel function. The optimal configuration of the unknown coefficients, A_w, is determined minimizing the differences between the measures of the incident field and the synthesized values in the observation domain D_M

$$\overline{\mathbf{A}}_{opt} = \arg\left\{ \min_{\overline{\mathbf{A}}} \left(\frac{\sum_{v=1}^{V}\sum_{m(v)=1}^{M^{(v)}} \left\| E_{inc}^{v,meas}(\mathbf{r}_{m(v)}) - \varsigma^{v}(\mathbf{r}_{m(v)}) \right\|^2}{\sum_{v=1}^{V}\sum_{m(v)=1}^{M^{(v)}} \left\| E_{inc}^{v,meas}(\mathbf{r}_{m(v)}) \right\|^2} \right) \right\} \tag{5}$$

Such a problem is solved using the well-known *Singular-Value-Decomposition* algorithm and once the parameters are tuned, the electric field can be evaluated in every point of the investigation domain according to (4).

B. OBJECT FUNCTION RECONSTRUCTION

The amount of information in phaseless data is extremely limited. In fact, beyond the typical limitation of the inverse scattering problem, when amplitude-only data are considered the collectable information is reduced further on. Therefore, the iterative multi-scaling approach [9] has been customized for amplitude-only data in order to control the dimension of the search space and to improve the quality of the reconstructed profiles. Such an iterative procedure is initialised assuming a uniform distribution of the unknowns which are chosen according to [1][2]. Moreover, the value of the incident field in each sub-domain of D_I is evaluated and the system (2)-(3) numerically solved through the minimization of a suitable cost function.

Then, at each step the resolution is adaptively improved in the *Regions of Interest* (*RoIs*) where the object is supposed to be located [9]. Accordingly, a multi-resolution grid is obtained and a multi-resolution cost function is defined

$$\Phi_{IMSA-PD}=\frac{\sum_{v=1}^{V}\sum_{r=1}^{s}\sum_{n(r)=1}^{N(r)}\left\|\varsigma^{v}\left(\mathbf{r}_{n(r)}\right)\right|-\left|\zeta^{v}\left(\mathbf{r}_{n(r)}\right)\right\|^{2}}{\sum_{v=1}^{V}\sum_{r=1}^{s}\sum_{n(r)=1}^{N(r)}\left|\varsigma^{v}\left(\mathbf{r}_{n(r)}\right)\right|^{2}}+\frac{\sum_{v=1}^{V}\sum_{m(v)=1}^{M^{(v)}}\left\|E_{tot}^{v,meas}\left(\mathbf{r}_{m(v)}\right)\right|-\left|\xi^{v}\left(\mathbf{r}_{m(v)}\right)\right\|^{2}}{\sum_{v=1}^{V}\sum_{m(v)=1}^{M(v)}\left|E_{tot}^{v,meas}\left(\mathbf{r}_{m(v)}\right)\right|^{2}} \tag{6}$$

$$\left|\xi^{v}(\mathbf{r}_{m(v)})\right|=\left|E_{inc}^{v,meas}(\mathbf{r}_{m(v)})+\sum_{t=0}^{s-1}\sum_{q(t)=1}^{N(t)}\left\{\omega_{q(t)}^{(s)}\left[\tau(\mathbf{r}_{q(t)})\xi^{v}(\mathbf{r}_{q(t)})G(\mathbf{r}_{m(v)}/\mathbf{r}_{q(t)})\right]\right\}\right| \tag{7}$$

$$\left|\zeta^{v}(\mathbf{r}_{n(r)})\right|=\left|\xi^{v}(\mathbf{r}_{n(r)})-\sum_{t=0}^{s-1}\sum_{q(t)=1}^{N(t)}\left\{\omega_{q(t)}^{(s)}\left[\tau(\mathbf{r}_{q(t)})\xi^{v}(\mathbf{r}_{q(t)})G(\mathbf{r}_{n(r)}/\mathbf{r}_{q(t)})\right]\right\}\right| \tag{8}$$

where the weighting function $\omega_{q(t)}^{(s)}$ can assume 0 or 1 value [9]. In order to completely exploit all the achieved information, each intermediate reconstruction is used as initial solution of the successive minimization process.

However, the cost function (6) is still highly non-linear and suffers of local minima problem. Therefore it is minimized using the Particle Swarm Optimiser (for a detailed description see [10]-[12]), one of the most effective recent evolutionary techniques based on the observation of the movement of swarms of insects looking for food. Finally, the multi-resolution procedure is iterated until a stationary condition is reached [9].

III. EXPERIMENTAL VALIDATION

In order to assess the robustness and the effectiveness of the algorithm the real dataset of measurements kindly provided by M. Saillard and K. Belkebir (for details see [13]) has been used. The considered test case the so-called "*FoamDielIntTM*" scattering configuration characterized by the following quantities: $\tau_{obj}^{1}=2.0\pm0.3$, $R_{obj}^{1}=1.5\times10^{-2}m$, $\tau_{obj}^{2}=0.45\pm0.15$, $R_{obj}^{2}=4.0\times10^{-2}m$. The object is located in a square investigation domain of side $L_I=3.0\times10^{-1}m$ and $V=8$ different views and $M^{(v)}=241$ measurement points have been taken into account. Single frequency data ($f=2GHz$) have been inverted. According to the studies in [10]-[12], the following configuration of PSO parameters is selected: constant inertial weight $\omega=0.4$, acceleration coefficients $C_1=C_2=2.0$, swarm dimension $I=\frac{5}{100}U$, being U the number of unknowns.

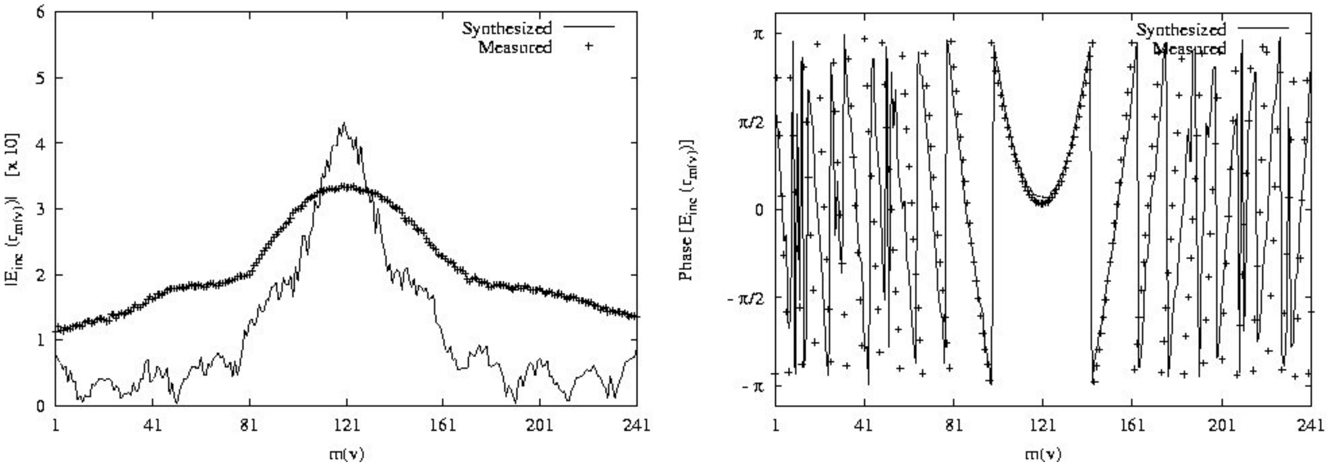

Figure 2. Matching between synthesized and measured amplitude of the incident field in D_M

Firstly, let us consider the synthesis of the source. It has provided a good agreement between measured and synthesized values of the phase of the electric incident field as shown in Fig. 2. On the other hand, the amplitude turns out to be more critical, but it is still an acceptable approximation to our aims. As a matter of fact the reconstruction of Fig. 3 points out the presence of two different levels of contrast. The scatterer is well located and dimensioned also if the reconstructed shape is not so accurate. From a quantitative point of view, the dielectric properties are satisfactorily estimated.

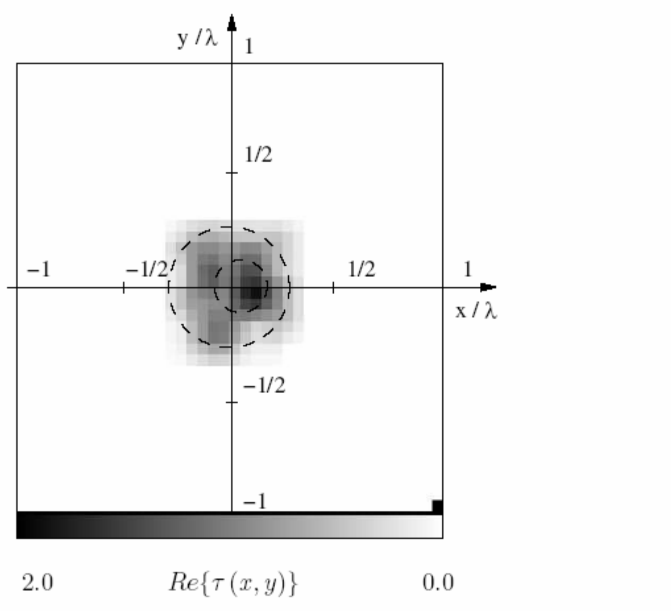

Figure 3. Reconstructed profile of the real part of the object function

IV. CONCLUSIONS

In this contribution an innovative two-step strategy has been presented and its performances analysed considering experimental data. The results point out the effectiveness of the approach and the feasibility of the inversion of amplitude-only data without the need of expensive post processing of the data or phase retrieval algorithms. The *DCW*-Model allows us to avoid the critical measurements in the investigation domain and the iterative multi-scaling approach integrated with the Particle Swarm Optimiser has shown a good effectiveness also in dealing with phaseless real data.

REFERENCES

[1] O. M. Bucci, and G. Franceschetti, ”On the degrees of freedom of scattered fields”, *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 918-926,1989.

[2] O. M. Bucci and T. Isernia, “Electromagnetic inverse scattering: retrievable information and measurements strategies,” *Radio Science*, pp. 2123-2138, 1997

[3] E. Wolf, “Determination of the amplitude and the phase of the scattered fields by holography,” *J. Opt. Soc. Am. A,* vol. 60, pp. 18-20, 1970.

[4] G. W. Faris and H. M. Hertz, “Tunable differential interferometer for optical tomography,” *Appl. Opt.,* vol. 28, pp. 4662-4667, 1989.

[5] S. Caorsi, A. Massa, M. Pastorino, and A. Randazzo, "Electromagnetic detection of dielectric scatterers using phaseless synthetic and real data and the memetic algorithm," *IEEE Trans. Geosci. Remote Sensing*, vol. 41, pp. 2745-2753, Dec. 2003.

[6] T. Takenaka, D. J. N. Wall, H. Harada, and M. Tanaka, "Reconstruction algorithm of the refractive index of a cylindrical object from the intensity measurements of the total field," *Microwave Optical Technol. Lett.*, vol. 14, pp. 182-188, Feb. 1997.

[7] M. H. Maleki, A. J. Devaney, and A. Schatzberg, "Phase retrieval and intensity-only reconstruction algorithms from optical diffraction tomography," *J. Opt. Soc. Am. A*, vol. 10, pp. 1086-1092, 1993.

[8] L. Crocco, M. D'Urso, and T. Isernia, "Inverse scattering from phaseless measurements of the total field on a closet curve," *J. Opt. Soc. Am. A*, vol. 21, Apr. 2004.

[9] S. Caorsi, M. Donelli, D. Franceschini, and A. Massa, "A new methodology based on an iterative multiscaling for microwave imaging," *IEEE Trans. on Microwave Theory Tech.*, vol.51, pp. 1162-1173, Apr. 2003.

[10] J. Robinson and Y. Rahmat-Sami, "Particle swarm optimization in electromagnetics," *IEEE Trans. on Antennas and Propagation*, vol.52, pp. 771-778, Mar. 2004.

[11] J. Kennedy, R. C. Eberhart, and Y. Shi, *Swarm Intelligence*, San Francisco, Morgan Kaufmann Publishers, 2001.

[12] M. Donelli and A. Massa, "Computational approach based on a particle swarm optimizer for microwave imaging of two-dimensional dielectric scatterers," *IEEE Trans. on Microwave Theory Tech.*, vol.53, pp. 1761-1776, May 2005.

[13] K. Belkebir and M. Saillard, Special issue on "Testing inversion algorithms against experimental data: inhomogeneous targets," *Inverse Problems*, vol.21, pp. 1-3, Dec. 2005.