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ANALYSIS OF CREEP STRESSES AND STRAINS AROUND SHARP AND BLUNT V-NOTCHES

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Abstract

Geometrical discontinues, such as notches, need careful consideration due to stress concentration appearing in their proximity. Notches become even more dangerous in the case of mechanical and structural components subjected to high temperature and cyclic loading resulting often in localised creep deformations. It appears, based on the current state of art, that needed are numerical or analytical tools enabling the analysis and simulation of creep deformations near notches.

The knowledge of notch tip stresses and strains can be used as the input into stress-strain creep models used for strength and fracture evaluations of mechanical and structural components. The method discussed below is based on considerations of the Strain Energy Density variations near notches. Several examples concerning V-notches have been analysed to validate the concept.

Finally, the problem associated with the analysis of sharp notches and cracks was discussed in terms of the proposed methodology.

Keywords: creep, V-notches, non-localized creep, strain energy density, stress evaluation;

List of symbols

a	Notch depth
B	Creep constant
C_p	Plastic zone correction factor
d	Distance from the coordinate system origin at which the far field contribution is evaluated
E	Young's modulus
$H(2\alpha, \frac{R_0}{\rho})$	Function in the SED expression for blunt notches
I_1	Mode 1 function in the SED expression for sharp V-notches
K_{IC}	Material toughness
K_{Ω}	Strain energy concentration factor
K_t	Stress concentration factor
K_I	Mode I stress intensity factor
n	Creep exponent
q	Parameter linked to the V-notch opening angle
r	Radial coordinate
r_0	Distance within notch tip and coordinate system origin
R_0	Radius of the control volume
r_p	Plastic zone radius
SED	Strain Energy Density
t	Time
x(ρ)	Distance from the notch tip at which the far field contribution is evaluated
2α	Notch opening angle
$\Delta\varepsilon_{22}^{c_n}$	Creep strain increment at the notch tip at step n
$\Delta\varepsilon_{22}^{cf_n}$	Incremental far field creep strain
$\Delta\varepsilon_{22}^{t_n}$	Increment of total strain
ΔK_I	Mode I notch intensity factor ranke
ΔK_{th}	Threshold stress intensity factor range
Δr_p	Plastic zone increment
$\Delta\sigma_{22}^{t_n}$	Stress decrement at the notch tip at step n
Δt_n	Time increment
ε^{p0}	Plastic strain at time t=0
$\varepsilon_{22}^{c_f}$	Creep strain at the far field
$\varepsilon_{22}^{c_n}$	Creep strain at the notch tip
ε_{22}^{t}	Time dependent notch tip strain
ε_{ij}^{0}	Actual elastic-plastic strain
ε_{ij}^{e}	Hypothetical strain components obtained from linear elastic analysis
θ	Angular coordinate
λ_1	mode I eigenvalue

μ_1	mode I second order eigenvalue
ν	Poisson's ration
ρ	Notch tip radius
σ_{max}	Maximum stress at the notch tip
σ_{nom}	Applied nominal stress
σ_t	Ultimate tensile strength
σ_{ys}	Yield stress
σ_{22}^{f}	Far field stress
σ_{22}^{f0}	Far field stress, t=0
σ_{22}^{t}	Time dependent notch tip stress
σ_{ij}^{0}	Actual elastic-plastic stress
σ_{ij}^{e}	Hypothetical stress components obtained from the linear elastic analysis
χ_1	Mode I associated constant
Ω^e	Total strain energy density at the notch tip, obtained from linear elastic (LE) analysis
Ω^f	Total strain energy density at a pre-defined point in the far field, from LE analysis

1. Introduction

When dealing with high temperature applications and time dependent deformations resulting in a nonlinear stress-strain material response it is often necessary to account for creep (visco-plasticity) deformations. The creep phenomenon can be localized in a small region around the notch root tip (localized-creep) but it can also occur far away from the notch (non-localized or gross creep). The gross creep conditions refer to situations in which creep deformations occur also away from the crack tip and as a result it may intensify the creep process in the notch or crack tip region. Such a situation occurs in machine and structural components working at high temperatures such as power plants, gas turbines and nuclear pressure vessels. Gas turbine blades and disks are examples of components susceptible to creep effects. The material at the stress concentration point might be under steady strain during this period but the stress may relax due to creep [1].

The best to authors' knowledge only limited data sets concerning the evolution of creep stresses and strains around V-notches are available in literature when dealing with non-localized creep. It must be underlined the contribution by Kubo and Ohji [2] in which simple expressions are proposed for predicting the notch stress and strain, by applying the concept of the state of small-scale and large-scale creep conditions with the path-independence of the J-integral. Li at al. [3] have studied the dominance of asymptotic crack tip stress-strain fields using finite element solutions obtained for elastic power-law creeping solids. Both the small and large scale creep configurations were analysed. It was found that the HRR [4,5] theory could be applicable to a region extending over one fifth of the creep zone size in the case the small scale creep processes.

A differential form of the Neuber rule applicable to a general visco-plastic notch problem has been discussed in reference [6]. The results were in good agreement with the finite element data where both the creep of the nominal far stress field and the stress relaxation were taking place induced by remotely applied constant nominal strain. A very interesting observation was also made there that after sufficiently long period of steady loading both the stress and strain concentration factors were getting stabilized.

Zhu et al. [7] have recently obtained the plain strain singular stress-strain field near a sharp V-notch tip in a power law creep material. It has been mentioned earlier that the singularity exponent of the notch tip stress-strain field depends in the steady-state conditions on the creep law exponent and the V-notch angle. However the work, mentioned above, does not give any information concerning the evolution of stresses and strains as a function of time but it is focused on the time dependent variations of the stress-strain singularity order.

In order to characterize the creep cracking phenomenon some authors proposed energy based methods. In the pioneering work of Landes and Begley [8] the C* parameter was proposed for assessing the creep crack growth in a super alloy held at temperature of 920^o K. The parameter C*, derived analytically from the $\dot{J}$-integral, was proposed by other authors [9] as well. The crack growth rates were correlated with the C* parameter which can also be obtained from the line integral. However, it was found that the C* parameter was able to characterize the crack tip stress and strain rate field, provided that the elastic and transient creep effects were negligible. In spite of certain limitations the work of Landes and Begley [8] has enabled fracture mechanics based analysis of the crack creep phenomenon.

The strain energy-based fatigue and creep-fatigue damage parameter has been derived by Leis [10] stating that the material damage was dependent on the internal total octahedral strain energy density. The proposed general parameter was found applicable to cases of isothermal mechanical cyclic loading configurations.

A new strain energy-based fatigue damage parameter applicable in high temperature environments, was proposed by Lee [11] and co-workers. The method was based on the analysis of non-dimensionalized plastic strain energy density (PSED). The study was focused on the phenomenon of the low-cycle fatigue at high temperature (including creep and other high temperature effects) of 316L austenitic stainless and 429EM ferritic stainless steel material. The advantage of the method lies in the possibility of predicting low cycle fatigue lives at high temperatures based on only monotonic tensile material data. The high cycle fatigue was not considered there.

Another interesting approach to modelling the creep-fatigue phenomenon, involving the strain energy density parameter, has been presented in reference [12]. It was assumed that the density, on the macroscopic level, the dissipated strain energy can be used as the measure of the material creep damage. The assumption has subsequently led to the conclusion that the creep damage was proportional to the absorbed/dissipated amount of the internal strain energy density. Reasonably accurate fatigue-creep lives were predicted [12] based on the proposed methodology when compared with the experimental data. Zhu et. al. [13] have proposed another model applicable to fatigue-creep life predictions of turbine disks. The method was based on the analysis of the plastic strain energy density. It has resulted in the formulation of the strain energy density-based fatigue–creep damage parameter accounting for the creep, mean stress/strain and cyclic hardening effects. Results of their analysis indicate that the proposed model supplied better live estimations than the contemporary available methodologies.

In spite of the fact that a few strain energy density based methods, addressing problems of creep and creep-fatigue cracking, are available they predominantly apply to the low cycle fatigue situations.

A methodology for the non-localized notch creep stress-strain analysis has been recently presented in reference [14]. The method was derived by using the Neuber [15] rule concept. The proposed approach [14] yielded very good results when applied to U-notches ($2\alpha=0$ and $\rho\neq0$). The method made it possible to simulate the evolution of the time dependent (creep) notch tip stresses and strains based on the input data obtained from the linear elastic solution. Because the method was based on the Creager-Paris linear elastic notch-tip stress-strain field its application was limited to U-notches. The method proposed in reference [14] has been recently extended to a wider selection of geometrical configurations including blunt [16] V-notches by the present authors. The extension was based on the Lazzarin-Tovo equations [17] instead of using the original [18] notch tip Creager-Paris solution. It is worth to note that there are also other linear elastic notch stress field solutions [19,18,20,21] available in the literature. From the results presented in [16], it emerges a new limitation regarding the application of the method to sharp-notches, that justify the further development in terms of SED presented here. In fact, the analytically derived linear elastic stress and strain solutions are singular when dealing with sharp notches and cracks. Therefore strength and fatigue durability assessment methods based on the local stress or/and strain are no longer suitable and this problem includes also the creep phenomenon. This limitation can be easily overcome if an “averaged field” parameter, such as the averaged Strain Energy Density quantity, is considered.

The method and results presented below are concerned with the idea of applying the concept [22–24] of the average Strain Energy Density (SED) to the design and analysis of notched components experiencing non-localized creep deformations. The non-localized creep strains occur more often than expected in mechanical engineering components working in high temperature environments. Several investigations involving the extended SED approach, accounting for non-linear stress-strain effects, are discussed below. The method has characterized successfully the high temperature fatigue behaviour of various materials such as titanium and copper-beryllium alloys [24–29].

The paper is composed of three parts. The first part contains discussion and short presentation concerned with the behaviour of blunt V-notches in non-localized creep conditions, briefly recalling the fundamental results of Ref. [16]. This part is useful to give to the reader a better idea about the employed method, highlighting limitations that justify the further SED based improvement presented here. New geometries have been considered with respect to Ref. [16]. Secondly, discussed are numerical methods of estimating the strain energy density in the case of sharp V-notches. The third part is the discussion of results obtained and a few specific problems concerned with the application of the SED method to the creep analysis of sharp notches.

2. Evaluation of stresses and strains around blunt V-Notches in non-localized creep conditions – the extended Neuber rule

A method for the estimation of stresses and strains around U-notch tips, being in the state of non-localized creep, was presented in reference [14]. The method was based on the Neuber [15] concept extended to time dependent plane stress problems with the addition of the K_{Ω} parameter [30,31]. The K_{Ω} parameter enables to account for the effect of creeping of the entire gross cross section of a notched body, i.e. accounting for the effect of the non-localized creep. It was assumed that far field variations of the strain energy density magnify creep effects at the notch tip. The concentration factor K_{Ω} of the total strain energy density was introduced in order to appropriately scale the strain energy density level at the notch tip. Introduction of the K_{Ω} parameter into the time dependent Neuber rule is the main difference between the localized and non-localized formulation of the notch tip creep problem. Details of the original formulation can be found in references [14] and [16].

The extension of Neuber's concept to time dependent stress-strain notch problems led to a set of analytically derived complex differential equations. In order to numerically solve those equations Moftakhar et al. [30] have proposed a special time integration routine. Namely, the integration time period was divided into a finite number of discrete time steps, Δt_n, and then the solution was generated for each subsequent time step. Such an approach resulted in the formulation of a set of incremental equations linking the applied load, geometry, time step and other necessary material properties. The set of equations [14] to be solved for each time increment is as follows:

- Equation involving the creep strain increment at the notch tip occurring during the time step n:

$$\Delta\varepsilon_{22}^{c_n} = \Delta t_n \cdot \dot{\varepsilon}_{22}^{c}(\sigma; t) \quad (1)$$

This is the incremental form of the creep law, where $\dot{\varepsilon}_{22}^{c}(\sigma;t) = A\sigma^{\alpha}t^{\beta}$ is the Norton type [32] creep power law.

- The stress decrement at the notch tip induced by the creep process during the time step *n*:

$$\Delta\sigma_{22}^{t_n} = \frac{(K_\Omega C_p)\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cfn} - \sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{cn}}{\frac{2}{E}\sigma_{22}^{t_{n-1}} + \varepsilon^{p0} + \varepsilon_{22}^{cn}}, \tag{2}$$

where: C_p is the [33] plastic zone correction factor.

The correction factor C_p depends on the first approximation of the plastic zone size r_p and the plastic zone size increment Δr_p resulting from the notch tip stress redistribution. It is to emphasize that the time-dependent strain consists of the elastic, initial plastic and the creep strain contribution. The effect of the non-localized creep, resulting from the material creeping in the far field, is represented by the term $(K_\Omega C_p)\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cf_n}$. Neglecting this term leads to the localized notch tip creep formulation.

- Increment of the total strain:

$$\Delta\varepsilon_{22}^{t_n} = \Delta\varepsilon_{22}^{c_n} - \frac{\Delta\sigma_{22}^{t_n}}{E} \tag{3}$$

The total strain increment is obtained as the difference between the creep strain increment at the step n and the elastic strain decrement (replaced by the stress decrement according to the Hooke law) at the time t.

The key to the extension of the Nuñez-Glinka method to blunt V-notches is the introduction of the Lazzarin-Tovo equations [17] into the original formulation involving the plastic zone correction factor C_p. Main steps of the proposed methodology can be summarised as follows:

- Replace the Creager-Paris solution by the Lazzarin-Tovo equations describing the stress distribution around the notch tip;
- Determine, according to reference [17], the origin r_0 of the coordinate system being the function of the opening angle and the notch tip radius;
- Re-define the plastic zone correction factor C_p by using the actual plastic zone size r_p and the plastic zone size increment Δr_p;

Definitions of parameters C_p, r_p and Δr_p are not much different from those reported in reference [33] except those equations describing the notch tip stress distribution. By considering, in polar coordinates (Fig. 1), the von Mises Plastic Yielding Criterion [34] in the form of Eq. 4 and by expressing stress components in terms of the Lazzarin-Tovo solution [17] it is possible to obtain the first numerical approximation of the plastic zone size r_p.

$$\sigma_{ys} = \sqrt{\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2} \tag{4}$$

Once the plastic zone size r_p is known, the F_1 parameter shown in Fig. 1-b can be evaluated:

$$\begin{aligned} F_1 &= \int_{r_0}^{r_p} \sigma_\theta dr - \sigma_\theta(r_p) \cdot (r_p - r_0) = \\ &= \frac{K_t\sigma_{nom}}{4}\Bigg\{ (r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1 - 1}\Bigg[(\lambda_1 - 1) \\ &+ \chi_1(1-\lambda_1)\left[1 - \left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right] \\ &+ (3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\Bigg] \\ &- \frac{\left[(\lambda_1+1) + \chi_1(1-\lambda_1)\right]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1} \\ &+ \frac{\left[\chi_1(1-\lambda_1) - (3-\lambda_1)\right]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\Bigg\} \end{aligned} \tag{5}$$

Where:

$$\begin{aligned} \sigma_\theta(r_p) = \frac{K_t\sigma_{nom}}{4}\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\Bigg[&(1+\lambda_1) + \chi_1(1-\lambda_1) \\ &+ \left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\left[(3-\lambda_1) - \chi_1(1-\lambda_1)\right]\Bigg] \end{aligned} \tag{6}$$

The stress $\sigma_y(r)$ within the plastic zone i.e. for $r_0 < r < r_p$, needed for estimating the C_p correction factor, was assumed constant $\sigma_y(r)=\sigma_y(r_p)$. Therefore the correction factor C_p may slightly overestimate the effect of the stress redistribution resulting from the yielding in the notch tip region. It is to note that the lower integration limit is r_0 and it depends on the notch opening angle and the notch tip radius. In order to satisfy equilibrium conditions, the F_1 force shown in (Fig. 1-b) has to be carried through by the material outside of the initially estimated plastic zone size r_p. This is the main reason of the stress redistribution and subsequent increase of the plastic zone size by the increment Δr_p. Since forces F_1 and F_2 in (Fig. 1-b) must satisfy the force equilibrium condition then:

$$F_1 = F_2 = \sigma_\theta(r_p) \cdot \Delta r_p. \tag{7}$$

Then the plastic zone increment Δr_p can be determined as:

$$\Delta r_p = \frac{F_1}{\sigma_\theta(r_p)} \tag{8}$$

Substitution of expression (5) and (6) into (8) makes it possible to determine the plastic zone size increment Δr_p:

$$\begin{aligned}\Delta r_p = &\left\{\left(\frac{r_p}{r_0}\right)^{1-\lambda_1}\left[(r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]\right.\right.\\ &-\frac{[(\lambda_1+1)+\chi_1(1-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1}\\ &\left.\left.+\frac{[\chi_1(1-\lambda_1)-(3-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\right]\right\}\\ &/\left\{(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right\}\end{aligned} \tag{9}$$

The plastic zone correction factor C_p [33] can be finally determined as:

$$\begin{aligned}C_p = 1+\frac{\Delta r_p}{r_p} = 1+&\left\{\left(\frac{r_p}{r_0}\right)^{1-\lambda_1}\left[(r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\right.\right.\right.\right.\\ &\left.\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\Bigg]-\frac{[(\lambda_1+1)+\chi_1(1-\lambda_1)]\left[r_0-r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1}+\end{aligned} \tag{10}$$

$$\left.\left.\frac{[\chi_1(1-\lambda_1)-(3-\lambda_1)]\left[r_0-r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\right]\right\}\Big/\left\{r_p\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]\right]\right\}$$

It is note that the procedure for the determination of the plastic zone adjustment Δr_p is analogous to that one proposed by Irwin [35] for cracks.

The subsequent general stepwise procedure for the stress-strain creep analysis at notches is as follows [14,30,31]:

1. Determine the 'pseudo-elastic' notch tip stress, σ_{22}^{e}, and strain, ε_{22}^{e}, by using the linear-elastic stress analysis.
2. Determine the notch tip stress, σ_{22}^{0}, and corresponding elastic-plastic strain, ε_{22}^{0}, by using the Neuber rule [15], the ESED [36] method or the finite element analysis.
3. Begin the creep stress-strain analysis by calculating, from Eq. (1), the creep strain increment, $\Delta\varepsilon_{22}^{c_n}$, accumulated during the time increment Δt_n. One must obey the assumed creep hardening rule.
4. Determine, from Eq. (2), the stress decrement, $\Delta\sigma_{22}^{t_n}$, associated with the previously determined increment of the creep strain, $\Delta\varepsilon_{22}^{c_n}$.
5. Determine, from Eq. (3), the increment of the total strain at the notch tip, $\Delta\varepsilon_{22}^{t_n}$, occurring during the time increment Δt_n.
6. Repeat steps from 3 to 5 over the required period of time t.

The method can be easily programmed and yields reasonable results for a variety of material constitutive creep curves and creep hardening models when applied to notched components subjected to steady loads. The method can also be extended to cyclic loading histories.

2.1 Creep stress-strain analysis around blunt V-notches

The method presented above and discussed in reference [16] has been use for the creep analysis of plates (Fig. 2) weakened by two V-edge notches. The plates were subjected to axial uniformly distributed tensile nominal stress of σ_{nom}=345 MPa. Dimensions of the plate were (Fig. 2): the height H=192 mm and the width W=100 mm. Other dimensions were: the opening angle 2α=135°, the notch depth a=10 mm, the notch tip radii ρ=0.5, 1.0 and 6 mm respectively. The numerical results have been obtained with the help of the MATLAB® software.

It is fundamental at this point to define the "far stress field" characterized by the distance *d* measured from the origin of the system of coordinates (see Fig. 1-a). Following the definition proposed in reference [14] the far field stress level was assumed to be equal to the elastic stress found at the point located away from the notch tip at the distance 'd', being equal to three notch tip radii 3ρ. However, this assumption generated satisfactory results only when the notch tip radius was larger than 2 mm. Since the proposed *d* parameter was too small for smaller notch tip radii then the distance at which the elastic far field contribution was finally evaluated had to be larger than the three times of the notch tip radius Therefore appropriate *d* values have been determined with help of the finite element method and they are listed in Table 1 . Parameter *d* is the distance measured from the origin of the system of coordinates while parameter *x*(ρ) is the distance measured from the notch tip (see Fig. 1-a)

All finite element analyses were carried out with the help of the ANSYS® Finite Element code. The plate has been modelled using the two-dimensional finite element SOLID183 8 node and assuming the plane stress state. The material elastic (E, ν, σ_{ys}) and creep (n, B) properties respectively are given in Table 2. The assumed creep law was:

$$\dot{\varepsilon_{22}} = \mathrm{B} \cdot \sigma^{\mathrm{n}} \cdot t^{\beta} \tag{11}$$

Where β=0. Equation (11) is very similar to that proposed by Nuñez–Glinka [14] by assuming parameter β equal to 0.

A uniformly distributed stress σ_{nom}= 345 MPa was applied (Fig. 2) at the end of the specimen. In order to simulate the creep relaxation process the load was applied in two steps. First, the almost instantaneous load step was applied over relatively short time increment of $t = 10^{-5}$ h. Then the load was maintained constant over the desired period of time *t*. All stress and strain components were periodically saved at predefined time intervals.

The analysis results are presented in Figs. 3, 4 and 5. The data illustrate the stress and strain evolution at various notches determined by the finite element analysis and those obtained from the proposed method denoted as the 'theoretical solution'. The theoretical results are in good agreement with the finite element data. The largest discrepancy concerning strain predictions was less than 20%.

This error was most likely due to different approximation procedures introduced into the theoretical formulation, such as the assumption of the elastic-perfectly plastic behaviour of material in the plastic zone and the application of the Irwin method to estimate the extent of the plastic zone.

Similar level of discrepancies between finite element results and theoretical predictions were found by other authors [37] as well.

3 The averaged strain energy density (SED) method

The averaged Strain Energy Density (SED) concept, developed originally for sharp and blunt notches by Lazzarin and co-workers [38] has been derived from Neuber's concept of the elementary volume [39] and the local mode I idea proposed by Erdogan and Sih [40]. Fundamentals of the SED methodology and its recent developments have been discussed in references [22,41]. This method has also shown some potentials [25,27,29,42] being applicable to the stress-strain analysis of notched components working in elevated temperatures and at different scales [43].

The main hypothesis of the method is that the failure of a solid body occurs when the averaged strain energy density within a given control volume, $\overline{W}$, reaches the critical energy level W_c, being the material property.

In the case of two dimensional problems the control volume becomes a circle or a circular sector with the radius R_c as shown in Fig. 6. The control volume can be easily evaluated [44] for metallic materials from the fracture toughness and the ultimate tensile strength (in case of a static loading) or from the smooth specimen fatigue limit and the threshold stress intensity factor (in the case of cyclic loading). The critical strain energy value W_c can be can be evaluated [26, 28] from the material monotonic tensile strength σ_t in the case of quasi-static load or the fatigue strength $\Delta\sigma_0$ in the case of cyclic loading.

The SED approach requires appropriate definition of the control volume and validity of the assumption that the critical strain energy does not depend on the notch tip radius. The method was first derived and applied to sharp (zero tip radius) V-notches and subsequently extended to blunt U- and V-notches under mode I loading mode. Its extension to pure mode II and III loading mode and later to the mixed mode loading case has been discussed in references [24,38,45,46]. It is to note that in the case of mixed mode loading configurations the region of the control volume is not symmetrically spread out around the symmetry plane of the notch but it is shifted off along the notch tip contour towards the point of the maximum tensile principal stress.

The SED approach was successfully applied [47,48] to assess the strength of notched and welded components subjected to either static or fatigue subjected to single or mixed loading modes.

Other important advantages of the SED method are as follows:

- The method makes it possible to account for the scale effect fully included into the Notch Stress Intensity Factor NSIF approach;
- The method is capable of accounting for the effect of various loading modes;

- It allows for the evaluation of the effect of the cyclic loading stress ratio;
- It makes it possible to compare notches with various opening angles;
- It takes into account the T-stress effect in the case of thin wall structures;
- It can be combined with the FE method and relatively coarse finite element mesh models;
- It makes it possible to quantify the influence of residual stresses in fatigue strength in case of notches [49] and butt-welded joints [50].

4 Evaluation of the Strain Energy Density in the presence of creep deformations

The analytically derived linear elastic stress and strain solutions are singular when dealing with sharp notches and cracks, i.e. they tend to infinity in the close neighbourhood of a notch or crack tip. Therefore strength and fatigue durability assessment methods based on the local stress or/and strain are no longer suitable and this problem includes also the creep phenomenon. As a consequence, the method proposed earlier [16] and briefly discussed above cannot be directly applied to sharp V-notches. However, it should be noted that the singularity of the stress and strain field at a sharp notch\crack tip does not mean that the averaged over a finite area/volume strain energy is singular and therefore it can be used as a potential fracture parameter. For these reasons, such an approach is also applicable when dealing with the creep analysis around sharp notches or cracks. In order to verify such a possibility and obtain numerical creep stress-strain data a series of finite element non-linear analyses were carried out for sharp symmetric edge V-notches. The geometry of analysed notched specimens is presented in Fig. 7. The notch opening angles 2α were 60°, 90°, 120°, 135° and 160°. All analyses were carried out using the Solid183 8 node finite elements available in the ANSYS® finite element software package. In order to simulate the creep relaxation phenomenon the load was applied in two steps. The first step was a quick increase of the load from zero to the required level over relatively short time interval $t=10^{-5}$ h (instantaneous load). Then the load was kept constant over the assigned time period t. Then the Strain Energy Density (SED) variations in the notch tip region (the control volume) was determined for subsequent uniformly spaced time intervals. Necessary material properties of studied specimens are given in Table 3 and they are the same as those listed in reference [7].
The Strain Energy Density (SED) plotted as a function of the normalized time τ [7] is shown in Figs. 8-10.

$$\tau = \frac{B\sigma_g^n t}{\sigma_g / E} \tag{12}$$

The averaged SED showed certain time dependent trend for all analysed geometrical configurations and creep law exponents. The averaged SED showed noticeable time dependency in the case of all considered geometries and creep law/exponents. The initial magnitude of the Strain Energy Density at the time τ=0 was obtained from the linear elastic stress-strain solution. Then the SED was decreasing, as reported in reference [3], until it had stabilised reaching steady levels of the stress and strain concentration factor. The time necessary to reach the steady state depended on the creep exponent. The higher was the creep exponent 'n' the shorter was the time period 'τ 'necessary for reaching the steady stress-strain state (the strain-time plateau). It is also worth noting that the average SED was decreasing with the increase of the opening angle 2α.

4.1 The effect of finite element mesh refinements on the evaluated creep SED

It is well known that numerical determination of NSIFs (Notch Stress Intensity Factors) requires using very refined finite element meshes. However, the strain energy Density (SED) can be determined from a relatively coarse Finite Element mesh model [51] providing that the control volume is appropriately defined. The SED can be determined directly from nodal displacements and forces which are sufficiently accurate even in the case of coarse FE meshes. In order to analyze the effect of the FE mesh refinements on the accuracy of the SED assessment several geometrical configurations with sharp V-notches have been analyzed in the presence of the localized creep deformations. The geometry of analyzed notched configurations is shown in Fig. 7. The analyses were carried out for three different values of the creep exponent n and for a variety of notch opening angles 2α. For the sake of brevity only results obtained for the creep exponent n=5 and the notch opening angles 2α=60° and 120° have been reported in Tables 4 and 5. The number of FE elements in the critical volume, the time t, the averaged level of the SED and the size of the control volume are reported in those tables as well. It appears, based on the results under discussion, that the averaged SED is only slightly influenced by the mesh refinement and it resulted in differences less than 12%. It is worth noting that all SED data sets were overestimated resulting in safe/conservative estimations of the final notch tip creep results. Similar results were obtained for other creep laws and notch opening angles to be reported later.
The non-linear-creep analyses discussed above indicate that the SED estimations remained unaffected be the FE mesh refinements. Therefore, the methodology permits using coarse FE meshes for analyzing creep stresses and strains in notched components made of non-linear (power law) materials and resulting in only slightly overestimated notch tip stresses and strains.

5. The SED as the creep characterizing parameter

The main advantage of the SED method [16] is the possibility of relatively easy and quick determination of creep stresses and strains at notches without the necessity of using complex and time-consuming FE non-linear analyses. The method has been validated for U-notches and blunt V-notches. Moreover, the proposed notch creep formulation can be easily simplified for cases of only localized creep, i.e. without the far field creep effects. Its general formulation enabling wide range of applications is mainly due to the employment of Lazzarin-Tovo equations [17] valid for a wide variety of geometrical notch configurations.
Although Lazzarin and Tovo equations [17] are valid for sharp V-notches the near the notch tip stress and strain do not represent any meaningful parameters for material failure analyses. Various methods are available in the literature dealing with this matter like the theory of critical distances [52–54] or the approach based on the Strain Energy Density method. These methods, being applied mostly to linear-elastic cases, have also shown some potential when analysing non-linear problems.
However, several issues remain still open such as:
- the time dependence of the singularity order of the stress-strain field near the tip of a sharp notch or crack: when considering creep conditions, the singularity order does not stay constant but it varies with time. This phenomenon has been observed by various authors [7] and the phenomenon is still under investigation.
- the time transition from the elastic to the elastic-plastic or fully plastic state of a notched body, especially when dealing with high temperature environments; concepts accounting for those transitions are needed for adequate modifications of the model proposed.

Due to promising results obtained from preliminary analyses it was decided to combine the proposed model with new ideas enabling prediction of stresses and strain based on the averaged SED concept. The aim is to develop a general tool for dealing with notches subjected to creep regardless of their geometrical features.

6. Conclusions

The analysis and data presented above was devoted to the extension of the Strain Energy Density (SED) method to mechanical and structural components experiencing non-localized creep deformations. Estimations of the SED were carried out by implementing a special numerical technique enabling to simulate the creep stress-strain evolution at the notch tip as a function of time. The method [14] formulated initially for a specific notch configuration has been extended [16] to a wider variety of notch geometries by applying the approach [17] of Lazzarin and Tovo. The notch tip stresses and strains can be subsequently used as input parameters necessary for creep stress-

strain analysis models addressing localized creep phenomena. However, when dealing with sharp notches and cracks the local near the tip strains and/or stresses tend theoretically to infinity and therefore they are not suitable quantities for any fracture or strength analyses. On the other hand the Strain Energy Density averaged over specified control volume remains finite. Therefore the method can be applied to both blunt and sharp notches and cracks.
The data presented above have shown good agreement of finite element simulations and theoretical results obtained from the SED method.

The final conclusions can be formulated as follows:

- A reliable and robust analytical/numerical tools are needed when dealing with creep phenomena at notches.
- The method proposed addresses the problem of the localized creep stress-strain analysis at notches and it has been validated for edge V-notches.
- The main advantage of the averaged SED method is its mesh independence when using the FE stress-strain analysis method for creating the input data for subsequent creep analyses.

APPENDIX A

Fundamental equations for the extended Neuber method.

Application of the Neuber rule to creep stress-strain analysis.

The original formulation of the Neuber rule [15] is

$$K_t^2 = K_\sigma K_\varepsilon \ . \qquad \text{(A.1)}$$

Equation (A.1) can also be written in the form of Eq. (A.2) relating the hypothetical stress and strain components at the notch tip, obtained from the linear elastic analysis, to the actual elastic-plastic stress and strain components:

$$\sigma_{ij}^e \varepsilon_{ij}^e = \sigma_{ij}^0 \varepsilon_{ij}^0 \qquad \text{(A.2)}$$

In the case plane stress state Eq. (A.1) reduces to:

$$\sigma_{22}^e \varepsilon_{22}^e = \sigma_{22}^0 \varepsilon_{22}^0 \ . \qquad \text{(A.3)}$$

Assuming that plastic deformations remain localized (also under creeping conditions), the total strain energy density at the notch tip remains almost constant and the Neuber rule [15] can be directly extended [31]to time-dependent problems:

$$\Omega = \sigma_{22}^{e}\varepsilon_{22}^{e} = \sigma_{22}^{0}\varepsilon_{22}^{0} = \sigma_{22}^{t}\varepsilon_{22}^{t} \tag{A.4}$$

Localized creep formulation [14,30,31]**.**

Assuming that the creep deformation is localized, it can be stated that:

$$\sigma_{22}^{0}\varepsilon_{22}^{0} = \sigma_{22}^{t}\varepsilon_{22}^{t} \ . \tag{A.5}$$

The time-dependent strain can be decomposed into its elastic, mechanically induced plastic and creep strain contributions:

$$\varepsilon_{22}^{t} = \varepsilon_{22}^{et} + \varepsilon_{22}^{p0} + \varepsilon_{22}^{ct} \ . \tag{A.6}$$

Substituting Eq. (A.6) into Eq. (A.5) results in the strain energy density equation involving the creep strain:

$$\sigma_{22}^{0}\varepsilon_{22}^{0} = \sigma_{22}^{t}\varepsilon_{22}^{et} + \sigma_{22}^{t}\varepsilon_{22}^{p0} + \sigma_{22}^{t}\varepsilon_{22}^{ct} \ . \tag{A.7}$$

Differentiation of Eq. (A.7) with respect to time leads to the rate equation:

$$0.0 = \sigma_{22}^{t}\dot{\varepsilon}_{22}^{et} + \dot{\sigma}_{22}^{t}\varepsilon_{22}^{et} + \dot{\sigma}_{22}^{t}\dot{\varepsilon}_{22}^{ct} + \dot{\sigma}_{22}^{t}\dot{\varepsilon}_{22}^{ct} \ . \tag{A.8}$$

Considering that the elastic strain rate is defined according to Eq. (A.9)

$$\dot{\varepsilon}_{22}^{et} = \dot{\sigma}_{22}^{t} / E \ , \tag{A.9}$$

Eq. (A.8) can be transformed to

$$0.0 = \frac{2}{E}\sigma_{22}^{t}\dot{\sigma}_{22}^{t} + \dot{\sigma}_{22}^{t}\varepsilon_{22}^{p0} + \sigma_{22}^{t}\dot{\varepsilon}_{22}^{ct} + \dot{\sigma}_{22}^{t}\varepsilon_{22}^{ct} \ . \tag{A.10}$$

In the case of finite increments one can obtain the following expression for the stress and strain rate:

$$\dot{\sigma}_{22}^{t} = \frac{\mathrm{d}\sigma_{22}^{t}}{\mathrm{d}t} \cong \frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n} = \frac{\sigma_{22}^{t_n} - \sigma_{22}^{t_n - 1}}{t_n - t_{n-1}} \ , \tag{A.11}$$

and

$$\dot{\varepsilon}_{22}^{ct} = \frac{\mathrm{d}\varepsilon_{22}^{ct}}{\mathrm{d}t} \cong \frac{\Delta\varepsilon_{22}^{c_n}}{\Delta t_n} = \frac{\varepsilon_{22}^{c_n} - \varepsilon_{22}^{c_{n-1}}}{t_n - t_{n-1}} \ . \tag{A.12}$$

Substitution of Eqns. (A.10) and (A.11) into Eq. (9) results in Eq. (A13).

$$0.0 = \frac{2}{E}\frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\sigma_{22}^{t_{n-1}} + \frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\varepsilon_{22}^{p0} + \frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\varepsilon_{22}^{c_n} + \sigma_{22}^{t_{n-1}}\frac{\Delta\varepsilon_{22}^{c_n}}{\Delta t_n} \quad . \tag{A.13}$$

Equation (A13) can subsequently be solved for the stress increment, $\Delta\sigma_{22}^{t_n}$ occurring during the time step Δt_n.

$$\Delta\sigma_{22}^{t_n} = \frac{-\sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{c_n}}{\frac{2}{E}\sigma_{22}^{t_{n-1}} + \varepsilon^{p0} + \varepsilon_{22}^{c_n}} \tag{A.14}$$

The increment of the creep strain can be directly obtained from appropriate creep law.

$$\Delta\varepsilon_{22}^{t_n} = f\left(\sigma, \varepsilon, \Delta t_n\right) \tag{A.15}$$

Non-localized creep formulation [14,30,31]**.**

The strain energy density concentration factor proposed by Moftakhar et al. [30] is

$$K_{\Omega} = \frac{\Omega^e}{\Omega^f} = \frac{\sigma_{22}^e\varepsilon_{22}^e}{\sigma_{22}^f\varepsilon_{22}^f} \quad . \tag{A.16}$$

The effect of the far field creep under constant load on the notch tip behavior can be interpreted as an additional input of the strain energy density that can be inputted into Eq. (A.5) resulting in Eq, (A17).

$$\sigma_{22}^0\varepsilon_{22}^0 + K_{\Omega}\Omega^{cf} = \sigma_{22}^t\varepsilon_{22}^t \ , \tag{A.17}$$

where

$$\Omega^{cf} = \sigma_{22}^f\varepsilon_{22}^{cf} \ . \tag{A.18}$$

Decomposition of the time-dependent strain into its elastic, initial plastic and creep contribution, enables writing down the energy density equation at the notch tip as:

$$\sigma_{22}^0\varepsilon_{22}^0 + K_{\Omega}\sigma_{22}^f\varepsilon_{22}^{cf} = \sigma_{22}^t\varepsilon_{22}^{et} + \sigma_{22}^t\varepsilon_{22}^{p0} + \sigma_{22}^t\varepsilon_{22}^{ct} \ . \tag{A.19}$$

Differentiation of Eq. (A.19) with respect to time leads to Eq. (A.20).

$$K_{\Omega}(\sigma_{22}^f\dot{\varepsilon}_{22}^{cf} + \dot{\sigma}_{22}^f\varepsilon_{22}^{cf}) = \sigma_{22}^t\dot{\varepsilon}_{22}^{et} + \dot{\sigma}_{22}^t\dot{\varepsilon}_{22}^{et} + \dot{\sigma}_{22}^t\varepsilon_{22}^{p0} + \sigma_{22}^t\dot{\varepsilon}_{22}^{ct} + \dot{\sigma}_{22}^t\varepsilon_{22}^{ct} \quad . \tag{A.20}$$

In Eq. (A.21), it is assumed that that the far field stress remains constant during the hold time, and it takes the value of the elastic stress in the far field calculated at time t=0:

$$\sigma_{22}^{f} = \sigma_{22}^{f}\,|_{t=0.0} = \sigma_{22}^{f0}\ . \tag{A.21}$$

The elastic strain can be subsequently replaced by the stress based on the Hooke.

$$K_{\Omega}\sigma_{22}^{f0}\dot{\varepsilon}_{22}^{cf} = \frac{2}{E}\sigma_{22}^{t}\dot{\sigma}_{22}^{t} + \dot{\sigma}_{22}^{t}\varepsilon_{22}^{p0} + \sigma_{22}^{t}\dot{\varepsilon}_{22}^{ct} + \dot{\sigma}_{22}^{t}\varepsilon_{22}^{ct} \tag{A.22}$$

Substitution of Eqns (A.11) and (A.12) into Eq. (A.22) leads to Eq. (A23).

$$K_{\Omega}\sigma_{22}^{f0}\frac{\Delta\varepsilon_{22}^{cf_n}}{\Delta t_n} = \frac{2}{E}\frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\sigma_{22}^{t_{n-1}} + \frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\varepsilon_{22}^{p0} + \frac{\Delta\sigma_{22}^{t_n}}{\Delta t_n}\varepsilon_{22}^{c_n} + \sigma_{22}^{t_{n-1}}\frac{\Delta\varepsilon_{22}^{c_n}}{\Delta t_n}\ . \tag{A.23}$$

The actual stress increment can be subsequently obtained as:

$$\Delta\sigma_{22}^{t_n} = \frac{K_{\Omega}\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cf_n} - \sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{c_n}}{\frac{2}{E}\sigma_{22}^{t_{n-1}} + \varepsilon^{p0} + \varepsilon_{22}^{c_n}}\ , \tag{A.24}$$

Where:

$$\left|K_{\Omega}\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cf_n}\right| < \left|\sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{c_n}\right|\ . \tag{A.25}$$

The plastic zone adjustment factor [14,20,30,31]

The plastic zone correction/adjustment factor has been defined as

$$C_{\mathrm{p}} = 1 + \frac{\Delta r_{\mathrm{p}}}{r_{\mathrm{p}}}\ , \tag{A.26}$$

Therefore Eq. (A.19) can be written as:

$$\sigma_{22}^{0}\varepsilon_{22}^{0} + (K_{\Omega}C_{p})\sigma_{22}^{f}\varepsilon_{22}^{cf} = \sigma_{22}^{t}\varepsilon_{22}^{et} + \sigma_{22}^{t}\varepsilon_{22}^{p0} + \sigma_{22}^{t}\varepsilon_{22}^{ct}\ . \tag{A.27}$$

Final expressions for calculating the notch tip stress increment and the actual strain increment have been derived as:

$$\Delta\sigma_{22}^{t_n} = \frac{(K_{\Omega}C_{p})\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cf_n} - \sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{c_n}}{\frac{2}{E}\sigma_{22}^{t_{n-1}} + \varepsilon^{p0} + \varepsilon_{22}^{c_n}} \tag{A.28}$$

$$\Delta\varepsilon_{22}^{t_n} = \Delta\varepsilon_{22}^{c_n} - \frac{\Delta\sigma_{22}^{t_n}}{E}\ . \tag{A.29}$$

APPENDIX B

Fundamental equations and tables of parameters for the averaged SED method.

Static loading (for further details refer to Ref. [22,38,41]).

The critical strain energy density can be derived in the case of static loading in the form of Eq. (B.1):

$$W_c = \frac{\sigma_t^2}{2E} \tag{B.1}$$

The control volume, over which the strain energy density is being averaged, is a function of certain mechanical properties:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \qquad \text{plane strain} \tag{B.2}$$

$$R_c = \frac{(5-3\nu)}{4\pi}\left(\frac{K_C}{\sigma_t}\right)^2 \qquad \text{plane stress} \tag{B.3}$$

In case of sharp and blunt V-notches, the averaged strain energy density can be obtained from Eqs. (B.4) and (B.5). Necessary material properties are given in Tables B.1, B.2 and B.3, and they can be also found in Ref. [22,41].

$$\overline{W}_1 = \frac{I_1}{4\,E\,\lambda_1\,(\pi-\alpha)}\left(\frac{K_1}{R_0^{1-\lambda_1}}\right)^2 \qquad \text{sharp V-notches} \tag{B.4}$$

$$\overline{W}_1 = F(2\alpha)\times H(2\alpha,\frac{R_0}{\rho})\times\frac{K_{t,n}^2\sigma_{nom}^2}{E}, \qquad \text{blunt V-notches} \tag{B.5}$$

Cyclic loading (for further details refer to Ref. [22,38,41]).

The critical strain energy density range, in case of cyclic loading, can be written down as

$$\Delta W_c = \frac{\Delta\sigma_0^2}{2E} \quad . \tag{B.6}$$

The control volume can be evaluated from Eq. (B.7) and (B.8):

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \quad \text{plane strain} \tag{B.7}$$

$$R_c = \frac{(5-3\nu)}{4\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \quad \text{plane stress.} \tag{B.8}$$

In the case of sharp and blunt V-notches, the averaged strain energy density can be evaluated from Eqs. (B.9) and (B.10). Those parameters are listed in Tables B.1, B.2 and B.3 and given in Ref. [22,41].

$$\overline{W}_1 = \frac{I_1}{4\,E\,\lambda_1\,(\pi-\alpha)}\left(\frac{\Delta K_1}{R_0^{1-\lambda_1}}\right)^2 \quad \text{sharp V-notches} \tag{B.9}$$

$$\Delta\overline{W} = F(2\alpha)\times H(2\alpha,\frac{R_0}{\rho})\times\frac{K_{t,n}^2\Delta\sigma_{nom}^2}{E} \quad \text{blunt notches} \tag{B.10}$$

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