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THE ASYMPTOTIC LOSS DISTRIBUTION IN A FAT-TAILED FACTOR MODEL OF PORTFOLIO CREDIT RISK

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The Asymptotic Loss Distribution in a Fat-Tailed Factor Model of Portfolio Credit Risk

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Abstract

This paper extends the standard asymptotic results concerning the percentage loss distribution in the Vasicek uniform model to a setup where the systematic risk factor is non-normally distributed. We show that the asymptotic density in this new setup can still be obtained in closed form; in particular, we derive the return distributions, the densities and the quantile functions when the common factor follows two types of normal mixture distributions (a two-population scale mixture and a jump mixture) and the Student's t distribution. Finally, we present a real-data application of the technique to data of the Intesa - San Paolo credit portfolio. The numerical experiments show that the asymptotic loss density is highly flexible and provides the analyst with a VaR which takes into account the event risk incorporated in the fat-tailed distribution of the common factor.

Keywords: Factor model; asymptotic loss; Value at Risk.

1 Introduction

Quantitative risk management aims at building and estimating statistical models which describe the random behavior of portfolios; this is true both in market risk, where portfolios contain assets whose price is either known from the market or computed by means of mathematical models, and in

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credit risk. The latter can be divided in two main fields: "static" credit risk analyses loan and bond portfolios, "dynamic" credit risk deals with credit derivatives. Measuring the risk of credit derivatives is mainly based on the same tools employed in market risk, in that it uses option-theoretic tools; on the other hand, static credit risk management has to be tackled differently, both because its goals are not the same and because of limited data availability. This is especially true for retail portfolios, where: (i) the interest is not in predicting rating migrations or spread changes, but only default; (ii) the counterparties do not have debt and equity (or they are not exchange-traded), so that standard pricing techniques cannot be used; (iii) data is scarce.

In this paper, we consider the problem of finding the loss distribution, which is the basis for the computation of risk measures, in a typical static credit risk setup. One of the main challenges is the multivariate nature of the problem: a portfolio consists of many positions which are mostly correlated ([1], [10], [4]), so that taking into account this correlation is of crucial importance: a large literature (see [12], section 8.1.2, for an example concerning credit risk measurement) has shown that ignoring the correlation would indeed cause severe biases in the results.

Direct estimation of an unconstrained correlation matrix for the default indicators of all the positions of a typical portfolio is unfeasible. The reason is at least twofold. First, default is a "unique" event, so that it is impossible to estimate the default correlation between two counterparties by means of historical data, because, once a firm has defaulted, it no longer exists. The only way out would consist in estimating a default correlation coefficient for each pair of rating classes and assigning the same correlation to any pair of counterparties belonging to the same rating categories; however, even if we use all the defaults of two given rating classes, data are not enough, and statistical estimation of the correlation matrix would be likely to give poor estimates.

As a consequence, research has focused on identifying and modeling the factors which induce default correlation. This approach allows to build a correlation matrix on the basis of the knowledge of the correlation between the counterparties and some common factors, which influence in a different measure all the firms in the portfolio. The interpretation of these factors is clear, and common to other areas of finance and risk management: they incorporate the so-called systematic risk, which is typically related to the economic cycle, possibly at country and/or sector level.

In the setup of the so-called structural models (see, for example, [5], sect. 3.2), most commercial packages currently used by banks to measure portfolio credit risk (in particular, JP Morgan *CreditMetrics*© and Moody's|KMV *Credit Portfolio Manager*©) are based on a combination of two building blocks: the Merton model ([14]) is used for assessing the default probability and the Vasicek model ([16]) is a factor approach whose ultimate purpose consists in introducing some correlation structure. Vasicek contribution is particularly important because it provides us with a setup where many asymptotic results concerning the portfolio loss distribution can be derived; in particular, under the additional hypothesis of uniformity of the portfolio, the asymptotic distribution can be obtained in closed form.

In this paper we extend the basic Vasicek model for uniform portfolios to a setup where the common factor follows a non-normal distribution. From the risk management point of view, "interesting" distributions for the factor are fat-tailed, and we will examine in detail two distributional assumptions for the factor: (i) a finite mixture of normal distributions; (ii) a Student's t distribution. From the theoretical point of view, however, we will show that the density of the asymptotic percentage loss can be derived for general choices of the distribution of the factor.

This approach has two advantages. First, the factor distribution is extremely flexible, and for proper choices of the parameter(s) it becomes as leptokurtic as desired, a feature which is usually very important in models of the systematic risk component; second, the asymptotic loss distribution and its VaR are known in closed form. The latter issue marks a relevant difference with respect to approaches where the specific risk is also assumed to be non-normal (see, for example, the copula-based approaches outlined by [3], sect. 2.6), where the loss distribution cannot be obtained analytically.

The structure of the paper is as follows. In section 2 we will introduce the standard Vasicek model, its main implications and the notation used in the following. In section 3 we obtain analytically the asymptotic loss distribution and some of its features when we substitute to the normality assumption for the systematic risk factor a different distributional hypothesis. In section 4 we look more closely at the density when the factor is distributed as a finite normal mixture and as a Student's t distribution; several examples and an application to the default data of a specific sector of the portfolio of the Intesa - San Paolo banking group will also be provided. Section 5 concludes and outlines some directions for future research.

2 Setup: the standard single-factor Vasicek model

The so-called Vasicek model ([16]; [3], sect. 2.5.1; [8]) is the simplest factor model used in portfolio credit risk measurement. Formally, it assumes that the standardized return of the i-th counterparty is given by

$$r_i = \sqrt{\rho} Y + \sqrt{1-\rho} Z_i, \qquad i = 1, \dots, N, \tag{1}$$

where the single factor Y represents global economic conditions, ρ is the correlation between the returns, Z_i represents the idiosyncratic risk and N is the number of counterparties. The importance of this model is also related to the regulatory environment, as it is indeed the basis of the approach enforced to banks and financial institutions by the Basel II Accord for the computation of default probability and required capital.

Technically, (1) is based on the standard assumptions employed in factor analysis ([11], chap. 9): $Y \sim N(0,1)$, $Z_i \sim N(0,1)$, $\mathrm{cov}(Z_i, Z_j) = 0$, when $i \neq j$, $\mathrm{cov}(Z_i, Y) = 0$. It can be verified that $\boldsymbol{r} \sim N_p(\mathbf{0}, \boldsymbol{R})$, i.e., $r_i \sim N(0,1)$, $i = 1, \dots, p$ and $\mathrm{cov}(r_i, r_j) = \mathrm{corr}(r_i, r_j) = \rho$, $i \neq j$. In other words, each r_i is standard normal and any pair (r_i, r_j) has correlation ρ; in multivariate analysis, the matrix $\boldsymbol{R}$ is known as the *equicorrelation matrix*, and (1), also called *equicorrelation model*, is a repeated measurement model ([6], sect. 8.4).

The interpretation of these hypotheses is straightforward: standardized returns follow the standard normal distribution, systematic risk (Y) and specific risk (Z_i) are independent, as well as specific risks of different counterparties (Z_i, Z_j).

The model is based on a Merton-type approach; thus, there exists a threshold c_i such that $p_i = P(r_i < c_i)$, where p_i is the unconditional default probability. More formally, we can introduce a random variable $L_i = \mathbf{1}_{\{r_i<c_i\}}$ such that

$$L_i = \begin{cases} 1 & \text{with probability} \quad p_i, \\ 0 & \text{with probability} \quad 1 - p_i, \qquad i = 1, \ldots, N. \end{cases}$$

As $r_i \sim N(0, 1)$, we can also write $c_i = \Phi^{-1}(p_i)$. Finally, it is easy to prove that the conditional default probability is given by

$$p_i(Y) = \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right), \qquad i = 1, \ldots, N.$$

The results obtained so far are not specific to the Vasicek model, and analogous outcomes can be derived in multifactor settings. The significant advantage of the Vasicek model is that the asymptotic loss distribution can be derived without using simulation techniques, which are almost always necessary in more complicated models.

2.1 The asymptotic loss distribution in the general case

We now give a brief review of the limiting distribution theory concerning the standard Vasicek model, namely when the counterparties have different PD's; the reader interested in more details is again referred to [3] (sect. 2.5.1) and to the references therein. The case of a uniform portfolio (all the counterparties have the same PD) will be treated thoroughly in the next subsection.

The main theorem concerns the portfolio percentage loss $L^{(N)}$, which is defined as follows:

$$L^{(N)} = \sum_{i=1}^{N} w_i \eta_i L_i,$$

where $w_i = EAD_i / \sum_{j=1}^{N} EAD_j$ and EAD_i and η_i are respectively the *Exposure At Default* and the *Loss Given Default (LGD)* of the i-th counterparty. The following two hypotheses are required for the asymptotic results to hold, and in the following we will assume that they hold true:

$$\sum_{i=1}^{N} EAD_i \uparrow \infty \text{ as } N \to \infty; \tag{2}$$

$$\sum_{k=1}^{\infty} \left(\frac{EAD_k}{\sum_{i=1}^{k} EAD_i} \right)^2 < \infty. \tag{3}$$

Essentially, for $N \to \infty$, (3) defines an infinitely fine-grained portfolio, because the exposure share of each counterparty tends to zero as $N \to \infty$. These requirements are not very restrictive: it can be shown that a sufficient condition for (2) and (3) to hold is that $EAD_i \in (0, b]$, with $b \in \mathbb{R}^+$, for all $i = 1, \ldots, N$. On the other hand, (3) may not always be met in practice, because it is quite common for real bank portfolios to contain some "large" exposures.

We are now ready to give the following characterization of the limiting distribution of $L^{(N)}$.

Proposition 1 *If assumptions (2) and (3) hold, the percentage portfolio loss $L^{(N)}$ converges almost surely to the conditional expectation $E(L^{(N)}|Y)$:*

$$P\left(\lim_{N \to \infty} \left(L^{(N)} - E(L^{(N)}|Y) \right) = 0 \right) = 1.$$

Proof. See [3], pag. 88.

When applied to a portfolio where all the counterparties have the same default probability, proposition 1 allows to obtain the distribution and density functions of the asymptotic loss in closed form.

2.2 The asymptotic loss distribution of a uniform portfolio

In this subsection we focus on the uniform portfolio version of the Vasicek model; formally, this means that the random variable L_i is now defined as follows:

$$L_i = \begin{cases} 1 & \text{with probability} \quad p, \\ 0 & \text{with probability} \quad 1 - p, \qquad i = 1, \ldots, N. \end{cases}$$

In other words, in addition to the hypotheses listed after (1), we assume $p_i = p$ for all the counterparties.

Applying proposition 1 to this new framework is straightforward. Consider the asymptotic loss $\lim_{N\to\infty} L^{(N)} =: L^{(\infty)}$. As for the conditional expectation $E(L^{(N)}|Y)$, it is easy to see that

$$E(L^{(N)}|Y) = \sum_{i=1}^{N} w_i E(L_i|Y) = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right),$$

where $\Phi(\cdot)$ is the standard normal distribution function. Proposition 1 implies that

$$L^{(\infty)} = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right) \qquad \text{a.s.} \tag{4}$$

As pointed out by [3] (pag. 89), this result does not depend on the distribution of the factor Y; however, when $Y \sim N(0,1)$, the distribution and density functions can be obtained in closed form. Assuming, without loss of generality, $EAD_i = \eta_i = 1$ ($i = 1, \dots, N$), the distribution function is given by:

$$\begin{aligned} F_{L^{(\infty)}}(x) = P(L^{(\infty)} \leq x) = P\left(\Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right) \leq x\right) = \\ = P\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}} \leq \Phi^{-1}(x)\right) = \\ = P\left(-Y \leq \frac{1}{\sqrt{\rho}}(\Phi^{-1}(x)\sqrt{1-\rho} - \Phi^{-1}(p))\right) = \\ = \Phi\left(\frac{1}{\sqrt{\rho}}(\Phi^{-1}(x)\sqrt{1-\rho} - \Phi^{-1}(p))\right). \end{aligned} \tag{5}$$

In order to find the density we have to differentiate $F_{L^{(\infty)}}(x)$ with respect to x. Putting $s(x) = \frac{1}{\sqrt{\rho}}(\Phi^{-1}(x)\sqrt{1-\rho} - \Phi^{-1}(p))$ and applying the chain rule we get:

$$f_{L^{(\infty)}}(x) = \frac{\partial F_{L^{(\infty)}}(x)}{\partial x} = \frac{\partial \Phi(s(x))}{\partial x} = \phi(s(x))\frac{\partial s(x)}{\partial x}, \tag{6}$$

where $\phi(\cdot)$ is the density of the standard normal distribution. As for the partial derivative of $s(x)$, using the inverse function differentiation theorem we immediately get

$$\frac{\partial s(x)}{\partial x} = \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{\partial \Phi^{-1}(x)}{\partial x} = \\ = \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}. \tag{7}$$

It follows that

$$f_{L^{(\infty)}}(x) = \phi(s(x))\sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}. \tag{8}$$

Finally, some straightforward algebraic manipulations of the two standard normal densities in (8) give the density function of $L^{(\infty)}$:

$$f(L^{(\infty)}) = \sqrt{\frac{1-\rho}{\rho}} \cdot \\ \cdot \exp\left(-\frac{1}{2\rho}\left((1-2\rho)(\Phi^{-1}(x))^2 - 2\sqrt{1-\rho}\Phi^{-1}(x)\Phi^{-1}(p) + (\Phi^{-1}(p))^2\right)\right).$$

The density depends on ρ and p and exhibits different shapes as the numerical values of the parameters change; moreover, its quantile function and moments can be obtained analytically ([3], pagg. 91-95). In the next section we extend these results to the case where the factor Y follows a non-normal distribution.

3 The asymptotic loss under non-normal factor distributions

Consider now a setup where all the hypotheses of section 2.2 remain unchanged except the distribution of the factor $\tilde{Y}$, which is assumed to be given by a non-normal, and for now unspecified, distribution (in the following, all the quantities whose distribution is different from the standard case will be denoted by the symbol "~"). We are now going to derive the density of $\tilde{L}^{(\infty)}$ under this more general assumption.

Before proving the main result, it is worth stressing that, although proposition 1 does not rely on any specific hypothesis concerning the distribution of the common factor, when $\tilde{Y}$ is non-normal the distribution of $\tilde{r}_i = \sqrt{\rho}\tilde{Y} + \sqrt{1-\rho}Z_i$ is no longer normal: it is indeed the sum of a random variable $\tilde{Y}$ and of a standard normal random variable Z_i. Usually the distribution of $\tilde{r}$ is not known analytically but can be easily simulated, so that $c = F_{\tilde{Y}}^{-1}(p)$ has to be computed by Monte Carlo simulation. In section 4 we will give more details about the distributions obtained with specific choices of $\tilde{Y}$.

Proposition 2 *Suppose that all the hypotheses of proposition 1 hold true, with the exception of the distribution of the common factor $\tilde{Y}$, which is now assumed to follow an unspecified distribution $F_{\tilde{Y}}$. Then the density of the asymptotic percentage loss $\tilde{L}^{(\infty)}$ is given by*

$$f_{\tilde{L}^{(\infty)}}(x) = \sqrt{\frac{1-\rho}{\rho}} \cdot f_{\tilde{Y}}(\tilde{s}(x)) \cdot \frac{1}{\phi(\Phi^{-1}(x))}.$$

Proof. We can rewrite (4) as

$$\tilde{L}^{(\infty)} = \Phi\left(\frac{F_{\tilde{Y}}^{-1}(p) - \sqrt{\rho}\tilde{Y}}{\sqrt{1-\rho}}\right) \qquad \text{a.s.}$$

The way of reasoning followed in the standard case remains unchanged until (5) because it does not require any specific assumption about $\tilde{Y}$. Thus we have

$$\begin{aligned} F_{\tilde{L}^{(\infty)}}(x) &= P(\tilde{L}^{(\infty)} \le x) = P\left(\Phi\left(\frac{F_{\tilde{Y}}^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right) \le x\right) = \\ &= P\left(\frac{F_{\tilde{Y}}^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}} \le \Phi^{-1}(x)\right) = \\ &= P\left(-\tilde{Y} \le \frac{1}{\sqrt{\rho}}(\Phi^{-1}(x)\sqrt{1-\rho} - F_{\tilde{Y}}^{-1}(p))\right) = \\ &= F_{\tilde{Y}}(\tilde{s}(x)), \qquad x \in [0,1]. \end{aligned} \tag{9}$$

To find the density, we have to differentiate (9) with respect to x. We have:

$$\frac{\partial F_{\tilde{Y}}(\tilde{s}(x))}{\partial x} = f_{\tilde{Y}}(\tilde{s}(x))\frac{\partial \tilde{s}(x)}{\partial x}.$$

Now, $\tilde{s}(x)$ differs from $s(x)$ only because $F_{\tilde{Y}}^{-1}(p)$ replaces $\Phi^{-1}(p)$; thus, from (7) we conclude that the partial derivative of $\tilde{s}(x)$ is unchanged with respect to the standard case:

$$\frac{\partial \tilde{s}(x)}{\partial x} = \frac{\partial s(x)}{\partial x} = \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{\partial \Phi^{-1}(x)}{\partial x} = \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}.$$

To conclude the proof we only have to combine (7) and (9) in order to obtain the density $f_{\tilde{L}^{(\infty)}}(x)$. It is given by

$$\frac{\partial F_{\tilde{L}}^{(\infty)}(x)}{\partial x} = f_{\tilde{L}}^{(\infty)}(x) = f_{\tilde{Y}}(\tilde{s}(x)) \cdot \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}, \quad x \in [0,1]. \quad (10)$$

■

The density (10) is completely defined only when the distribution of the factor $\tilde{Y}$ is specified. We now work out the details when the factor is distributed respectively as a finite normal mixture and as a Student's t distribution.

Example 1 (*Finite Normal Mixture*). We assume that $\tilde{Y}$ follows a finite normal mixture, whose density is given by $f_{\tilde{Y}}(y) = \sum_{i=1}^{k} \pi_i \phi_{\mu_i, \sigma_i^2}(y)$. According to proposition 2, the asymptotic loss density is given by:

$$f_{\tilde{L}^{(\infty)}}(x) = \left(\sum_{i=1}^{k} \pi_i \phi_{\mu_i, \sigma_i^2}(\tilde{s}(x)) \right) \cdot \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}, \qquad x \in [0,1].$$

■

Example 2 (*Student's t*). Let $\tilde{Y}$ follow a Student's t distribution with ν degrees of freedom; then (10) becomes

$$f_{\tilde{L}^{(\infty)}}(x) = f_{t_\nu}(\tilde{s}(x)) \cdot \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{1}{\phi(\Phi^{-1}(x))}, \qquad x \in [0,1],$$

where $f_{t_\nu}(\tilde{s}(x))$ is the t_ν density. ■

This approach has another desirable feature: quantiles can be computed in closed form. We summarize the result in the next proposition (see also [8] for more details on risk measures in a factor model setup).

Proposition 3 *The quantile function of $\tilde{L}^{(\infty)}$ is given by*

$$q_\alpha(\tilde{L}) = p(-q_\alpha(Y)) = \Phi\left(\frac{F_{\tilde{Y}}^{-1}(p) + \sqrt{\rho} q_\alpha(\tilde{Y})}{\sqrt{1-\rho}} \right),$$

where $q_\alpha(\tilde{Y})$ is the α quantile of $\tilde{Y}$.

Proof. The proof is identical to the one given by [3] (pag. 94), because the only relevant condition ($p(\cdot)$ strictly decreasing) is obviously verified for any choice of the distribution of $\tilde{Y}$.

Finally, as concerns the expectation of $\tilde{L}^{(\infty)}$, it is easy to see that it is equal to p, as in the standard Vasicek setup. We have:

$$E(\tilde{L}^{(\infty)}) = E(p(Y)) = \sum_{i=1}^{N} w_i E(L|Y) = E(L|Y) = p.$$

As for the variance, the result valid in the standard case, namely $\mathrm{var}(L^{(\infty)}) = \Phi_2(\Phi^{-1}(p), \Phi^{-1}(p); \rho)$ (see [3], pag. 94), does not extend to the setup of the present paper. We ran some simulation experiments (not shown here to save space) from which it turned out that the variance of $\tilde{L}^{(\infty)}$ is larger than the variance of $L^{(\infty)}$, and the difference increases as the tails of $\tilde{Y}$ get heavier.

4 Applications

4.1 Choosing a distribution for $\tilde{Y}$

Two types of mixtures are common in finance and risk management: the scale mixture of normals and the so-called "jump" mixture.

The two-component scale mixture of normals has density

$$f_S(x) = \pi_1 \phi_{\mu_{11},\sigma_{11}^2}(x) + (1-\pi_1)\phi_{\mu_{12},\sigma_{12}^2}(x), \qquad x \in I\!R,$$

where π_1 is "large" ($\pi_1 \in [0.9, 1]$, say), $\mu_{11} = \mu_{12} = 0$ and σ_{12} is larger than σ_{11} ($\sigma_{12} \geq 4\sigma_{11}$, say). The density is unimodal and symmetric and its kurtosis increases as σ_{12} increases.

Another mixture often used in applications ([7], [2]) is a three-population distribution with $\mu_{21} < 0$, $\mu_{22} = 0$ and $\mu_{23} > 0$, and usually $\sigma_{21} = \sigma_{22} = \sigma_{23} =: \sigma_2$. The random variable in this case can be defined as

$$X = X_2 + X_J,$$

where X_2 is allowed to follow any distribution and the jump component has a trinomial distribution:

$$X_J = \begin{cases} 0 \text{ with probability } (1 - \pi_{21} - \pi_{23}); \\ D \text{ with probability } \pi_{21}; \\ U \text{ with probability } \pi_{23}, \end{cases}$$

where $\pi_{21} \in [0, 1]$, $\pi_{23} \in [0, 1]$, $\pi_{21} + \pi_{23} < 1$, $D \in I\!R^-$ and $U \in I\!R^+$. Thus, X is a mixture of three random variables X_D, X_2 and X_U, whose density is given by:

$$f_J(x) = \pi_{21} f_D(x) + (1 - \pi_{21} - \pi_{23}) f_2(x) + \pi_{23} f_U(x), \qquad x \in I\!R. \tag{11}$$

Assuming $f_2 \sim N(\mu_{22}, \sigma_2^2)$, X is a mixture of three normal densities with the same variance σ_2^2 and means respectively equal to μ_{21}, μ_{22} and μ_{23}, with $\mu_{21} = \mu_{22} + D$ and $\mu_{23} = \mu_{22} + U$; the observations coming from f_D and f_U are sometimes called jumps. If (11) is used as a model for $\tilde{Y}$, jumps can be interpreted as the quantification of the impact of events taking place with small probability and affecting the whole economic system (for example, unexpected good or bad news concerning the economy of major countries, terroristic attacks, catastrophic weahter events and so on).

Finite mixture disributions have been employed in several fields of finance and risk management: see [9], [18], [17], [15], [7] and [2]. Efficient estimation algorithms have also been developed: see [13] for a review.

In our opinion, mixture distributions are preferable to the Student's t distribution as a model for $\tilde{Y}$. The reason is essentially the larger flexibility that can be obtained by varying the number of components k and the values of the parameters π, μ_i and σ_i $(i = 1, \ldots, k)$: as will be seen in section 4.2, the Student's t is actually very similar to the scale mixture of two normals. However, the latter provides us with the possibility of modeling more precisely the "degree of heaviness" of the tails, because when using the Student's t we can only vary the numerical value of one parameter, namely the number of degrees of freedom.

4.2 The distribution of returns

Consider first the distribution of the returns $\tilde{r}_i = \sqrt{\rho}\tilde{Y} + \sqrt{1-\rho} Z_i$. Figures 1 and 2 show the Q-Q plots of the returns simulated respectively when $\tilde{Y}$ is a scale mixture of two normals and a Student's t, for various values of the parameters.

From graph 1 it can be seen that the distribution gets more leptokurtic as the variance of the second mixture component and/or the numerical value of

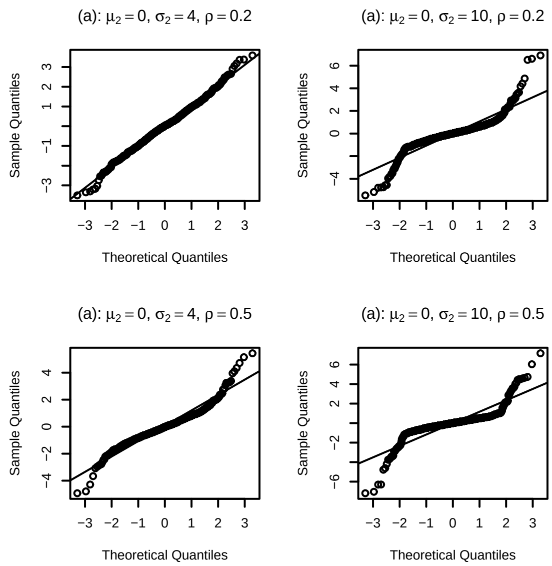

Figure 1: Q-Q plot of the returns when $\tilde{Y}$ is a scale mixture.

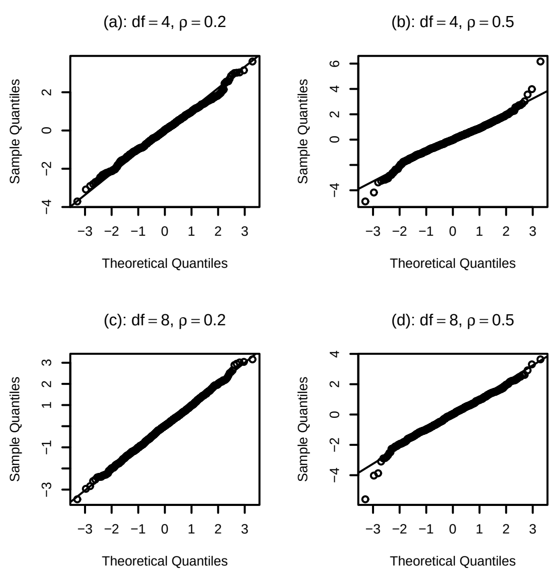

Figure 2: Q-Q plot of the returns when $\tilde{Y}$ Student's t.

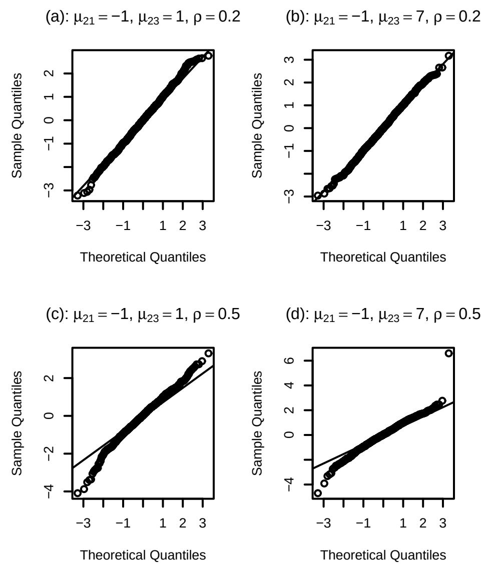

Figure 3: Q-Q plot of the returns when $\tilde{Y}$ is a jump mixture.

ρ increase. A similar message comes from graph 2: the tails become heavier as the number of degrees of freedom decreases and/or the numerical value of ρ increases.

Returns simulated from a jump mixture are shown in figure 3. Tails are generally heavier than in the normal case; notice in particular (see figure 3d) that the distribution becomes asymmetric when $|\mu_{23}| > |\mu_{21}|$; this implies the possibility of giving more weight to a negative event than to a positive event.

4.3 The asymptotic loss density

We concentrate now on the asymptotic loss density (10). In this and in the next section, results obtained with the Student's t distribution are essentially identical to those obtained with the scale mixture of two normals, so that, to save space, we do not show them. Figures 4 and 5 display the density obtained using respectively the scale mixture and the jump mixture for the factor $\tilde{Y}$.

In both cases the density is, for some parameter configurations, bimodal, with a mode at zero (m_1, say) and another mode farther away from zero

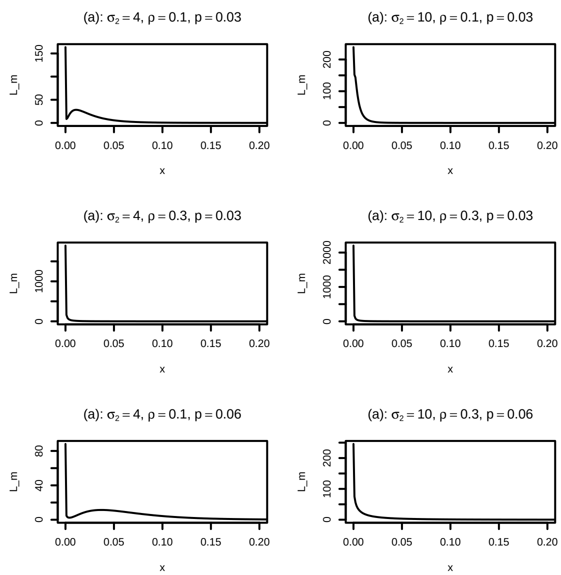

Figure 4: The asymptotic loss density when $\tilde{Y}$ follows a scale mixture.

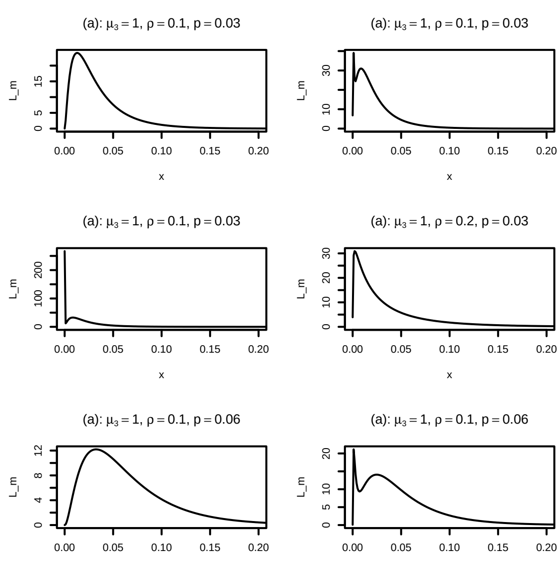

Figure 5: The asymptotic loss density when $\tilde{Y}$ follows a jump mixture.

(m_2). This feature is very interesting because of the following interpretation: when the economy expands, the probability of observing a percentage loss close to zero (m_1) is particularly high; when the economy is in recession, there is a larger probability of observing a larger percentage loss (m_2); in other words, the heaviness of the tails of $\tilde{Y}$ seems to determine the presence of the mode m_2.

Finally, we compute VaR for the standard Vasicek, the scale-mixture and the jump mixture distributions. Results are shown in tables 1, 2 and 3.

Table 1. VaR_α in the standard Vasicek case

	$\alpha = 0.95$	$\alpha = 0.99$	$\alpha = 0.999$
$p = 0.01,\ \rho = 0.1$	0.0285	0.0468	0.0775
$p = 0.03,\ \rho = 0.1$	0.0758	0.1137	0.1704
$p = 0.06,\ \rho = 0.1$	0.1377	0.1940	0.2713
$p = 0.1,\ \rho = 0.1$	0.2111	0.2825	0.3742
$p = 0.01,\ \rho = 0.3$	0.0442	0.1043	0.2244
$p = 0.03,\ \rho = 0.3$	0.1208	0.2342	0.4110
$p = 0.06,\ \rho = 0.3$	0.2173	0.3687	0.5654
$p = 0.1,\ \rho = 0.3$	0.3246	0.4965	0.6883

Table 2. VaR_α in the scale-mixture case (in all cases we set $\pi_1 = 0.9$ and $\sigma_{11} = 1$)

	$\alpha = 0.95$	$\alpha = 0.99$	$\alpha = 0.999$
$\sigma_{12} = 4,\ p = 0.01,\ \rho = 0.1$	0.0235	0.0454	0.0869
$\sigma_{12} = 10,\ p = 0.01,\ \rho = 0.1$	0.0003	0.0015	0.0064
$\sigma_{12} = 4,\ p = 0.03,\ \rho = 0.1$	0.0790	0.1322	0.2162
$\sigma_{12} = 10,\ p = 0.03,\ \rho = 0.1$	0.0664	0.1420	0.2785
$\sigma_{12} = 4,\ p = 0.06,\ \rho = 0.1$	0.1554	0.2363	0.3493
$\sigma_{12} = 10,\ p = 0.06,\ \rho = 0.1$	0.1905	0.3284	0.5157
$\sigma_{12} = 4,\ p = 0.1,\ \rho = 0.1$	0.2399	0.3405	0.4681
$\sigma_{12} = 10,\ p = 0.1,\ \rho = 0.1$	0.3171	0.4824	0.6699
$\sigma_{12} = 4,\ p = 0.01,\ \rho = 0.3$	0.0097	0.0396	0.1344
$\sigma_{12} = 10,\ p = 0.01,\ \rho = 0.3$	< 0.0001	< 0.0001	< 0.0001
$\sigma_{12} = 4,\ p = 0.03,\ \rho = 0.3$	0.1117	0.2618	0.5049
$\sigma_{12} = 10,\ p = 0.03,\ \rho = 0.3$	0.0509	0.2151	0.5641
$\sigma_{12} = 4,\ p = 0.06,\ \rho = 0.3$	0.2487	0.4607	0.7094
$\sigma_{12} = 10,\ p = 0.06,\ \rho = 0.3$	0.3943	0.7189	0.9370

From table 2 we see that the scale mixture VaR tends to increase as σ_{12} increases, that is, as the tails of the factor distribution get heavy; moreover, in general, the scale-mixture VaR is larger than the standard VaR. There is, however, a significant exception: when $p = 0.01$, the distribution tends to collapse to δ_0 (a Dirac delta centered at zero) more quickly than in the standard case; this explains the VaR figures obtained in the 2-nd and 10-th row of table 2.

Table 3. VaR_α in the jump-mixture case
(in all cases we set $\pi_{21} = 0.1$, $\pi_{23} = 0.1$, $\mu_{22} = 0$ and $\sigma_2 = 1$)

	$\alpha = 0.95$	$\alpha = 0.99$	$\alpha = 0.999$
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.01$, $\rho = 0.1$	0.0323	0.0525	0.0859
$\mu_{21} = -3$, $\mu_{23} = 3$, $p = 0.01$, $\rho = 0.1$	0.0187	0.0319	0.0549
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.01$, $\rho = 0.1$	0.0493	0.0772	0.1210
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.03$, $\rho = 0.1$	0.0828	0.1232	0.1829
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.03$, $\rho = 0.1$	0.1159	0.1664	0.2376
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.06$, $\rho = 0.1$	0.1447	0.2026	0.2817
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.06$, $\rho = 0.1$	0.1942	0.2626	0.3517
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.1$, $\rho = 0.1$	0.2172	0.2896	0.3821
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.1$, $\rho = 0.1$	0.2808	0.3619	0.4606
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.01$, $\rho = 0.3$	0.0678	0.1477	0.2924
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.01$, $\rho = 0.3$	0.1355	0.2563	0.4385
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.03$, $\rho = 0.3$	0.1649	0.2986	0.4886
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.03$, $\rho = 0.3$	0.2801	0.4457	0.6419
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.06$, $\rho = 0.3$	0.2682	0.4317	0.6286
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.06$, $\rho = 0.3$	0.4114	0.5879	0.7649
$\mu_{21} = -1$, $\mu_{23} = 1$, $p = 0.1$, $\rho = 0.3$	0.3749	0.5506	0.7348
$\mu_{21} = -1$, $\mu_{23} = 7$, $p = 0.1$, $\rho = 0.3$	0.5311	0.6999	0.8471

The interpretation of table 3 is similar to table 2: VaR increases as the probability of observing a large positive value of the factor increases (when $\mu_{23} = 7$). As the parameters p and ρ vary, the VaR figures show a behavior similar to the standard case; in other words, jump mixture VaR, while being larger than standard VaR (in a measure quantified by the tail heaviness of the distribution of $\tilde{Y}$), exhibits a dependence on p and ρ similar to the one of standard VaR. This feature is very desirable because it allows to quantify directly (in term of risk measures) how the portfolio is affected by a fat-tailed factor distribution. Moreover, the possibility of choosing a skewed distribution for the factor makes the approach even more flexible.

4.4 A real-data application

In this section we apply the methodology developed so far to the counterparties of the retail sector of the credit portfolio of the Intesa - San Paolo banking group. These counterparties are small, and do not have exchange-traded equity and/or debt, so that a Merton-based approach is unfeasible; although we do not have any information concerning the exposures, this should also imply that the portfolio share of each counterparty is negligible. The sector contains approximately 550,000 counterparties, so that it makes sense to resort to asymptotic theory.

In absence of exchange-traded equity and debt, the popular commercial versions of the structural models, like Creditmetrics© and MKMV©, cannot be used. Essentially, what is known of a counterparty is the default probability; with this limited information, however, we can still implement the methodology proposed here.

As pointed out before, in order to derive explicitly the loss density we have to assume that the portfolio is uniform. Considering that the sector is organized in rating classes, we decided to compute, for each rating category, the average default probability $\bar{p}$ of the individual PD's, and assigned to each counterparty of the class a PD equal to $\bar{p}$; this hypothesis seems reasonable because the PD's of firms with the same rating are similar.

The internal rating system of the bank consists of 14 categories, including the D (Default) class, labelled as $A1, A2, \ldots, A14$; as an illustration, we compute VaR for classes $A1$, $A7$ and $A13$. The bank has developed several internal models for each sector of its portfolio; the PD's used here are actually obtained, for confidentiality reasons, by means of a slightly modified version of one of these models. The average PD values for the three classes are $\bar{p}_{A1} = 0.00057$, $\bar{p}_{A7} = 0.0125$ and $\bar{p}_{A13} = 0.33291$. Tables 4, 5 and 6 show selected results.

Table 4. 99% VaR for class $A1$ ($\bar{p}_{A1} = 0.00057$)
(in all cases we set $\sigma_{11} = 1$, $\pi_{21} = 0.1$, $\pi_{23} = 0.1$, $\mu_{22} = 0$ and $\sigma_2 = 1$)

	$L_m^{(V)}$	$L_m^{(S)}$	$L_m^{(J)}$
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.1$	0.0039	0.0006	0.0049
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.1$	0.0039	< 0.0001	0.0087
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.3$	0.0090	< 0.0001	0.0140
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.3$	0.0090	< 0.0001	0.0338

Table 5. 99% VaR for class $A7$ ($\bar{p}_{A7} = 0.0125$)
(in all cases we set $\sigma_{11} = 1$, $\pi_{21} = 0.1$, $\pi_{23} = 0.1$, $\mu_{22} = 0$ and $\sigma_2 = 1$)

	$L_m^{(V)}$	$L_m^{(S)}$	$L_m^{(J)}$
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.1$	0.0562	0.0576	0.0620
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.1$	0.0562	0.0058	0.0912
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.3$	0.1238	0.0666	0.1738
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.3$	0.1238	< 0.0001	0.2887

Table 6. 99% VaR for class $A13$ ($\bar{p}_{A13} = 0.33291$)
(in all cases we set $\sigma_{11} = 1$, $\pi_{21} = 0.1$, $\pi_{23} = 0.1$, $\mu_{22} = 0$ and $\sigma_2 = 1$)

	$L_m^{(V)}$	$L_m^{(S)}$	$L_m^{(J)}$
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.1$	0.6256	0.7046	0.6209
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.1$	0.6256	0.8367	0.6940
$\sigma_{12} = 4$; $\mu_{21} = -1$, $\mu_{23} = 1$; $\rho = 0.3$	0.8430	0.9240	0.8472
$\sigma_{12} = 10$; $\mu_{21} = -1$, $\mu_{23} = 7$; $\rho = 0.3$	0.8430	0.9901	0.9217

The tables show that, at the 99% level, the VaR obtained with $L_m^{(J)}$ is larger than with $L_m^{(V)}$ and $L_m^{(S)}$; as for the latter, both for class $A1$ and for class $A7$ we observe the same behavior noted in section 4.3, namely a very small VaR, due to the fact that the distribution tends to concentrate near zero; on the other hand, for class $A13$ the scale mixture provides us with the largest VaR figures.

According to these results, in our opinion, if one seeks a model where the factor has a fat-tailed distribution (that is, in other words, event risk is appropriately accounted for), and this distributional property impacts the percentage portfolio loss, the jump mixture model seems to be the most approriate one, because it produces larger risk measures while remaining easily interpretable and feasible to different setups.

Finally, it is worth pointing out that the only purpose of the examples shown so far consists in investigating the mathematical and probabilistic features of the asymptotic distribution in this new framework. A complete implementation would entail an empirical investigation of the distribution of $\tilde{Y}$, possibly based on historical data, in order to give a sound statistical foundation to the choice of the distribution and of the numerical values of the parameters.

5 Conclusions

The aim of this paper was to generalize the standard asymptotic results concerning the percentage loss distribution in the Vasicek uniform model. We showed that the asymptotic density in the uniform portfolio can still be obtained in closed form under different distributional hypotheses for the common factor. Moreover, we derived the return distributions, the densities and the quantile functions obtained with two types of mixture distributions (a two-population scale mixture and a jump mixture) and with the Student's t distribution. Finally, we applied the methodology to some real data from the Intesa - San Paolo credit portfolio.

The approach constitutes an extension of the standard Vasicek model in that it allows to obtain a wide range of densities, which can accommodate many different features required by the investigator; in particular, we stress the importance of bimodal asymptotic loss densities, which can be well-suited to incorporate event risk. Its implementation is straightforward: the only quantity that cannot be computed analytically is the default threshold

c. However, its estimation by means of Monte Carlo simulation is easy.

As concerns the application of the model, the main issues to be addressed in the future seem to the be choice of the distribution of the common factor and the estimation of its parameters; a thorough statistical analysis is necessary in order to avoid a misspecification of the distribution that would lead to substantial errors in the estimation of risk measures. Estimation of ρ is also of crucial importance.

From a theoretical point of view, the extension of the analytical results to the standard Vasicek setup (where with "standard" we mean the non-uniform portfolio case) does not appear immediate. Thus, it is quite likely that future research shall concentrate on trying to improve copula-based models, which currently constitute the most general approach to credit loss distributions.

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