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Questions About Quantifiers: Symbolic and Nonsymbolic Quantity Processing by the Brain

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Abstract

One approach to understanding how the human cognitive system stores and operates with quantifiers such as “some,” “many,” and “all” is to investigate their interaction with the cognitive mechanisms for estimating and comparing quantities from perceptual input (i.e., nonsymbolic quantities). While a potential link between quantifier processing and nonsymbolic quantity processing has been considered in the past, it has never been discussed extensively. Simultaneously, there is a long line of research within the field of numerical cognition on the relationship between processing exact number symbols (such as “3” or “three”) and nonsymbolic quantity. This accumulated knowledge can potentially be harvested for research on quantifiers since quantifiers and number symbols are two different ways of referring to quantity information symbolically. The goal of the present review is to survey the research on the relationship between quantifiers and nonsymbolic quantity processing mechanisms and provide a set of research directions and specific questions for the investigation of quantifier processing.

Keywords: Language processing; Natural language quantifiers; Number symbols; Symbolic representation; ANS; Nonsymbolic quantity

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1. Introduction

Humans can perceive, represent, and compare quantities perceptually, for example, extracted from visually presented arrays of objects or from aurally presented series of tones, as well as quantities presented using arbitrary symbols and natural language. In the former case, we can make an approximation of the quantity of elements (i.e., the cardinality or numerosity). In the latter case, we can learn a set of conventions representing the exact cardinality using number symbols (e.g., Arabic digits, number words, Roman numerals—7, "seven," VII). We also convey approximate cardinality and the relationship between cardinalities using natural language quantifiers (e.g., "some," "many," "most," etc.). While a lot of research has been devoted to how number symbols and nonsymbolic quantities[1] are linked in the human brain, substantially less is known about the link between quantifiers and nonsymbolic quantities.

The processing of number symbols has been the subject of extensive research within numerical cognition, given that number symbols referring to exact quantities play an essential role in everyday functioning in modern industrialized cultures and are used in mathematics (e.g., Eger, 2016; Nieder, 2016; Piazza & Eger, 2016; Sokolowski & Ansari, 2016). As the nonsymbolic quantity representation system is considered to be evolutionarily old and innate in humans, particular attention has been paid to the interaction of number symbols with, and their possible reliance on, the nonsymbolic quantity processing mechanisms.

Natural language quantifiers are pervasive in everyday communication to refer to quantity, even in individuals and cultures without extensive exact number symbol systems. Similar to number symbols, natural language quantifiers also potentially interact with and rely on neurocognitive systems for nonsymbolic processing of quantity. One reason to think so is that properties of natural language quantifiers have been shown to correspond to biases displayed by nonhuman primates in quantitative tasks (Chemla, Dautriche, Buccola, & Fagot, 2019), thus implying reliance on evolutionary older cognitive systems.

Suggestions about the existence of a potential link between quantifiers and nonsymbolic quantity processing are not new (e.g., Clark & Grossman, 2007; Coventry, Cangelosi, Newstead, Bacon, & Rajapakse, 2005; Holyoak & Glass, 1978; Pietroski, Lidz, Hunter, & Halberda, 2009), but the few studies that have investigated it have never been discussed extensively. In this paper, we offer an extensive review of published studies looking at this relationship to the backdrop of an up-to-date review of developmental, behavioral, and neuronal-level evidence accumulated in symbolic and nonsymbolic quantity processing research. We also suggest directions for future research in this line by formulating a set of questions. Our goal here is to use existing research questions into number symbols and paradigms used in this regard to help formulate new questions about quantifiers. We believe that enriching the research on quantifier processing by adopting the accumulated knowledge regarding number symbols is a fruitful way forward. The reader should bear in mind that the primary goal of this paper is not to defend any theoretical position but to survey the complementary literature in a way that would stimulate discussion and new research programs.

We consider the existing evidence and suggest future research questions for whether and how cognitive systems supporting nonsymbolic quantities are involved in processing natural language quantifiers. The main issue here is whether and to what extent the same representations and processing mechanisms are involved in quantifier and nonsymbolic quantity processing by the brain. As far as we know, we have reviewed all significant studies to date investigating this relation for quantifiers. Where possible, we draw parallels with evidence from research into number symbol processing. Importantly, quantifiers might be linked to nonsymbolic quantities to a more considerable extent than number symbols are, since they have a set of different properties sometimes better aligned with the features of nonsymbolic quantities (as discussed below).

The review is structured around three main issues: similar processing mechanisms for quantifiers and quantities, the possible codependence between quantifiers and nonsymbolic quantities in development, and the representation format of quantifiers. However, an initial note on how quantifiers differ from number symbols, as well as some relevant distinctions within the class of natural language quantifiers, is in order.

Our overarching goal in this paper is to stimulate new research on the numerical representations behind quantification, and the paper draws some theoretical and methodological lines along which such research could plausibly be conducted. We believe that the time is ripe for a more reciprocal relationship between the study of the human capacities for understanding language and magnitudes, since natural language semantics, especially quantification, and neurocognitive research on number symbols and nonsymbolic quantities are mature enough to make their integration a plausible next step on the research agenda. Such interdisciplinary research will not only bring semantics and cognitive science closer by identifying neurocognitive correlates of meaning, but will also enrich numerical cognition by showing how our brains encode quantities in language.

2. Preliminaries

2.1. *A note on quantifiers, number symbols, and nonsymbolic quantities*

We know that some languages have an upper limit to number words that exist to refer to exact cardinalities: some languages have number words only up to 3–5, some have a number higher than five as an upper limit, and a few are even reported to have an upper limit of "one" or "two" (e.g., Bowern & Zentz, 2012; Epps, Bowern, Hansen, Hill, & Zentz, 2012; see Carey & Barner, 2019; Núñez, 2017 for review and references). Thus, the symbolic number system (at least to the extent that Western cultures use it) does not spontaneously arise during human development, but instead requires specific training. In contrast, regardless of the scope of the numerical system of a language, all languages seem to have words to refer to approximate cardinalities employing quantifiers, analogous to, for example, "some," "several," "few," "many" in English (Bowern & Zentz, 2012). We also know that understanding and communicating using quantifiers does not require specific training because children can use them before they start math education (e.g., Barner, Chow, & Yang, 2009; Barner, Libenson, Cheung, & Takasaki, 2009; Dolscheid, Winter, Ostrowski, & Penke, 2017). Finally, in cultures with an extensive number symbol system, quantifiers are still used in communication even if the exact number of objects is known (e.g., someone saying that they "bought several books" even though they know that they bought exactly three books). Given these considerations, quantifiers can be seen as a more natural way to refer to nonsymbolic quantity information in human languages than number symbols (see also, e.g., Clark & Grossman, 2007; Coventry, Cangelosi, Newstead, & Bugmann, 2010, for this suggestion). Consistent with the possibility that quantifier processing is based on the nonsymbolic quantity processing mechanisms outlined above, speakers of all languages perform similarly well when it comes to nonsymbolic quantity perception and comparison (Ferrigno, Jara-Ettinger, Piantadosi, & Cantlon, 2017; Gibson, Jara-Ettinger, Levy, & Piantadosi, 2017; Pica, Lemer, Izard, & Dehaene, 2004).[2]

Another substantial difference between quantifiers and number symbols is the context-sensitivity of quantifiers (which is additional to the imprecise nature of the quantity to which they refer; see also Moxey & Sanford, 1993; Newstead & Coventry, 2000 for this point). While the number symbol “2” always refers to a cardinality two, there is no fixed cardinality or proportion for quantifiers. Possible exceptions to this are generalized universal quantifiers (“each,” “every,” “all”), because they refer to a fixed proportion, but even here, the exact quantity “all” represents may vary (i.e., “all” refers to a different quantity for a group of five objects than for a group of 10 objects). Rather, the quantity these quantifiers refer to depends on the expected quantity for a particular situation (e.g., “many,” when referring to “pandas” compared to “ants,” will mean a different quantity) (Heim et al., 2015; Ramotowska, Steinert-Threlkeld, Leendert, & Szymanik, 2023; Yildirim, Degen, Tanenhaus, & Jaeger, 2016) and possibly other factors.[3] The context-sensitivity of quantifiers makes them more compatible with nonsymbolic quantity representations than number symbols. We know, for example, that there are individual differences in performance with more difficult ratios in nonsymbolic quantity comparison tasks (what is typically referred to as *nonsymbolic number acuity*; e.g., Halberda & Feigenson, 2008). There are also individual differences in underestimation bias in estimation tasks (Crollen, Castronovo, & Seron, 2011). Furthermore, estimates of the cardinality of object arrays are influenced by how elements are clustered together and spatially organized within a visual scene (Im, Zhong, & Halberda, 2016). As far as we know, the connection between the context-sensitivity properties of quantifiers and the related specific features of nonsymbolic quantities has not yet been studied in the literature. Here, we do not attempt to relate the context-sensitivity properties of quantifiers to specific features of nonsymbolic quantity processing, as that would make the present effort unmanageable, but only note these properties and leave them for future research.

Related to the context sensitivity is the need to *choose* an appropriate quantifier to describe a certain quantity. This involves deciding whether, for example, the given proportion should be considered, for example, low, and which of a variety of similar-in-meaning quantifiers should be used (e.g., “few,” “several,” or “some”). Hence, decision-making processes will be involved in producing a quantifier, unlike number symbols, where there is only one corresponding symbol.

Finally, in contrast to number symbols, different quantifiers will lead to different inference patterns when interpreting them—for example, if “some people ate oranges” is true, then “some people ate” has to be true as well.[4] Downward monotone and upward monotone quantifiers are traditionally distinguished (Barwise & Cooper, 1981; this property is also referred to as *quantifier polarity*). This aspect is traditionally seen as purely linguistic (i.e., not involving quantity processing systems). While decision-making and inference licensing properties of quantifiers are essential, in this review, we do not fully cover them; they require a thorough consideration on their own. We consider these linguistic and decision-making processes additional to the quantity processing that takes place for quantifiers.

2.2. *Kinds of quantifiers*

While there are many ways to carve up the space of natural language quantifiers, for the purposes of this review, a fourfold partition will suffice. In particular, these four types of quantifiers differ in their relation to the magnitude system.

Existential quantifiers, like "some," "several," "a few," "a couple," "a dozen," and so on, are often considered to refer to imprecise/approximate cardinalities (Keenan, 2012).[5] The fact that they refer to imprecise cardinalities makes them potentially compatible with nonsymbolic approximate quantity representations in the brain—when someone refers to a quantity of objects as "several," we do not know what exact quantity they have in mind, just as we cannot perceive an exact quantity when presented with a set of objects and do not count them (Barth, Kanwisher, & Spelke, 2003; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Halberda & Feigenson, 2008). This makes such existential quantifiers more compatible with nonsymbolic quantity representations than number symbols are. Specifically, unlike number symbols, these quantifiers can sometimes directly refer to nonsymbolic cardinality representations.

Proportional quantifiers[6] like "many," "few," "most" are thought to refer to the ratio between two cardinalities—the cardinality of all objects in the context and the cardinality of objects that possess the relevant feature. We know that, when comparing two nonsymbolic cardinalities, behavioral performance and neuronal activation patterns are modulated by the ratio between two presented nonsymbolic cardinalities (Buckley & Gillman, 1974; Jacob & Nieder 2009a; Lyons & Beilock, 2018; Lyons, Ansari, & Beilock, 2015; Xu & Spelke, 2000). Moreover, we know that the brain represents ratio information along with cardinality information when we are presented with two nonsymbolic cardinalities (Jacob & Nieder 2009b; Jacob, Vallentin, & Nieder, 2012; Matthews & Lewis, 2017). Thus, the ratio is encoded and plays a crucial role in nonsymbolic quantity processing.

The link between the *universal quantifiers* like "all," "every," "each" and nonsymbolic quantities is less straightforward than in the case of other classes of quantifiers we consider here. On one view, the meaning of these quantifiers is evaluated using logical reasoning rather than the quantity system since knowledge of the quantity is not required to understand them. Instead, what is needed is the ability to find counterexamples (e.g., if at least one object of a set does not possess the property, "all" cannot be applied). Under such a view, these quantifiers can, in principle, be processed independently of quantity representations (argued by Halberda, Taing, & Lidz, 2008; Troiani, Peelle, Clark, & Grossman, 2009; using the same argument, these researchers suggest that "some" [which we here classify as generalized existential] does not involve quantity processing either). On the other hand, others have suggested that the ability to find at least one counterexample already entails that number processing is involved (Clark & Grossman, 2007; see also Olm, McMillan, Spotorno, Clark, & Grossman, 2014 for a similar argument). Relatedly, different generalized universal quantifiers have different semantic functions, such as distributivity. For instance, while "each" tends to refer to individuals and their properties, "all" and "every" usually refer to sets of objects. Recently acquired preliminary evidence suggests that this difference translates into variability in mental representations of the universal quantifiers (Knowlton, Pietroski, Halberda, & Lidz, 2022). Here, we do not take a position on whether generalized universal quantifiers should recruit nonsymbolic quantities but only highlight it. Multiple studies discussed below included an investigation of the processing of specifically the quantifier "all."

Finally, *modified numerals*, such as "more than two," "at least/at most five," and so on, are relevant for the present discussion because they include number symbols. These quantifiers require that a person has learned to operate with exact number symbols. When considering the involvement of brain mechanisms, those processing number symbols have to get involved in order for these quantifiers to be understood and produced. Since we know more about number symbol processing, we have specific predictions about the mechanisms that should be involved in their processing; for example, at the neuronal level, we expect to observe the involvement of neuronal populations in the intraparietal cortex (Arsalidou & Taylor, 2011; Goffin, Sokolowski, Slipenkyj, & Ansari, 2019; Holloway, Battista, Vogel, & Ansari, 2012; Notebaert, Nelis, & Reynvoet, 2010; Notebaert, Pesenti, & Reynvoet, 2010; Sokolowski & Ansari, 2016; Sokolowski, Fias, Mousa, & Ansari, 2017; Vogel et al., 2017). In this sense, this class of quantifiers may sometimes function as a good baseline for seeing the participation of quantity processing mechanisms in the case of other quantifiers.

3. Shared processing mechanisms

The existence of existential and proportional quantifiers, which, respectively, correspond to the numerosity and ratio information encoded by the human nonsymbolic quantity representation system, already points to an intriguing question about whether these are parallel ways to represent quantity and refer to quantity in natural language.

Open Question 1: *Can proportional quantifiers directly refer to the ratio information extracted by our nonsymbolic quantity processing system, while existential quantifiers can refer to the extracted approximate cardinality information, both computed and made available by our nonsymbolic quantity processing system?*

Several experiments have shown that people are able to give a number symbol estimate of a quantity of objects presented nonsymbolically. Because number symbols refer to exact cardinalities, it is possible to find one corresponding number symbol for any particular nonsymbolic quantity after counting. In contrast, due to their imprecise meaning, there are no unambiguous, objective nonsymbolic counterparts for generalized existential and proportional quantifiers. For an estimation task with quantifiers, the first question asked is, thus, which criteria people use to decide whether a quantifier is a good description of a specified cardinality. These studies ask whether a particular quantifier corresponds to a particular cardinality or ratio in a nonsymbolic quantity representation. In parallel with estimation tasks with number symbols, where participants were asked to give number symbols corresponding to the cardinality of an array of objects, here participants were asked to give a quantifier to describe the cardinality.

When evaluating the fit between the meaning of a quantifier (at least in the case of generalized existential and proportional quantifiers) and a particular visual scene consisting of an array of objects, at least two processes have to take place—retrieval of the meaning of the quantifier and assessment of the cardinality using the nonsymbolic quantity processing system. Several studies have looked into what kind of information about the nonsymbolic quantity is extracted and assessed with the proportional quantifier meaning. These studies aim to characterize *the interface* between quantifier meaning and nonsymbolic quantity representations.

3.1. The interface between quantifier meaning and quantity representations

Parallel to estimation tasks with number symbols, one can also look at estimation tasks with quantifiers where people are presented with a visual array of objects and asked to produce or choose a quantifier that best describes a target set of objects (e.g., red dots or red dots surrounded by dots of a different color). Such tasks have been used to determine a cardinality or a proportion to which each quantifier refers. Still, for the most part, they have only revealed the enormous context-sensitivity of generalized existential and proportional quantifiers with respect to the nouns with which they combine, the situational context, and individual speaker judgments (e.g., Coventry et al., 2010; Heim et al., 2015; Moxey & Sanford, 1993; Newstead & Coventry, 2000; Yildirim et al., 2016). These aspects make it difficult to pinpoint any particular reference in terms of proportions or approximate cardinalities for each quantifier.

A study from Pezzelle, Bernardi, and Piazza (2018, Experiment 1) presented participants with visual displays of two types of objects (e.g., five hedgehogs and 15 balls in one scene) and asked them to choose an appropriate quantifier from a range of alternatives to describe one of the sets. Pezzelle and colleagues wanted to determine the factors that influence which quantifier is picked as the best description. They ran a regression analysis with several potentially relevant variables (cardinality of targets, cardinality of nontargets, subitizing/nonsubitizing range, average size of targets, average size of nontargets) for each of the quantifiers they tested. Specifically, they tested some proportional quantifiers (“most,” “many”), some generalized existential quantifiers (“some,” “few,” “none,” “almost none”) as well as a generalized universal quantifier (“all”). For all quantifiers they tested, except for “almost none,” the proportion of the target items in the set of all items was the best predictor of the choice of the quantifier as the appropriate description.[7] Therefore, all these quantifiers seemed to have been interpreted as proportional. This result could be ascribed to the nature of the task—participants always saw displays of two sets of objects (which could have encouraged their comparison), and proportional quantifiers were intermixed with others (which could bias them to viewing all quantifiers as proportional). Nonetheless, the results support the view that generalized existential quantifiers can refer to proportions, at least in some contexts. Besides, it is surprising that the generalized universal "all" was dependent on the proportion; this supports the possibility that quantity processing plays a role in this class of quantifiers.

A number of studies investigate whether and how quantifiers recruit or interact with nonsymbolic quantity representations and processing mechanisms in sentence-picture verification tasks. In these studies, participants are required to understand the meaning of a sentence with a quantifier (e.g., "Many of the dots are blue") and, subsequently, decide whether the presented visual display matches the description. Therefore, it is assumed that participants in this task process the visual display with the particular goal of extracting specific information required by the given quantifier meaning.

In two studies with the proportional quantifier "most," participants were asked to answer the question "Are most of the dots yellow?" (or "blue" in the second study; Lidz, Pietroski, Halberda, & Hunter, 2011; Pietroski et al., 2009). Participants saw visual displays with dots of two or more colors for 150–200 ms. Given the restriction in the time for which the visual arrays were displayed, participants were prevented from counting. In each trial, they answered the same question by pressing "yes" or "no." Within the visual displays, the ratio of dots of the target color and nontarget colors was varied; presented ratios were 1:2; 2:3, 3:4, 4:5, . ., 9:10. In these studies, the accuracy of the participants' responses varied according to the ratio, mirroring the performance expected in a nonsymbolic cardinality comparison task. Such results suggest that ratio information from the nonsymbolic quantity representation system is indeed extracted to evaluate fit against the meaning of the quantifier "most." The authors of these studies interpret the results as showing that the canonical verification strategy for "most" is based on cardinality judgments and those judgments may be well modeled with reference to Approximate Number Sense (ANS). One point of criticism is that participants saw 350–400 trials always to verify the same sentence. Thus, it is not necessarily the case that participants were retrieving the meaning of "most" with every trial. Participants might have been instructed to compare the cardinality of the dots of two different colors in a purely perceptual experiment. The second important point is that participants in these studies had only 150 or 200 ms to view the visual displays. Ideally, we would like to know whether the quantity-processing system is involved for longer viewing times as well or whether it was simply an artifact of this particular setup.[8]

A different sentence-picture verification study with quantifiers, by Deschamps, Agmon, Loewenstein, and Grodzinsky (2015) (see also Heim et al., 2012 for a similar setup and results), avoided the issues of using a single sentence across the whole experiment as well as a short duration of visual display presentation. In this study, participants were presented with the quantifiers "more/less than half," "many," "few," "more/less […] than […]" in sentences (e.g., "Many of the circles are yellow"). Each trial started with an auditory presentation of the sentence to be judged and followed with a visual display. The visual displays contained circles of two colors, with the ratio between the cardinalities of the circles of each color being manipulated. The visual displays stayed on the screen for 1100 ms. The performance of participants in terms of accuracy as well as reaction time was modulated by ratio for all quantifiers. Thus, here the authors obtain the same effects, while the instructions differed in each trial and the duration of the visual display was longer.[9]

Another similar study, by Shikhare, Heim, Klein, Huber, and Willmes (2015), also manipulates the sentence with the quantifier that is to be evaluated between trials and presented visual stimuli for 1000 ms. Shikhare and colleagues asked participants to verify statements with the proportional quantifiers "many" and "few" (e.g., "many/few of the circles are yellow") and modified numerals "at least n" and "at most n" (e.g., "at least/at most seven of the circles are yellow"). For the proportional quantifiers, they also observed slower responses and more errors with smaller ratios. Of special interest are the conditions with the modified numerals. Here, participants compared number symbols and nonsymbolic quantities, but with an additional direction of comparison/instruction given by the "at least/at most" quantifier. When the actual quantity of the dots of the corresponding color on display was closer to the reference number (e.g., 8 as opposed to 12 circles displayed for the sentence "at least seven of the circles are yellow"), the reaction times were longer, and accuracy was lower. Therefore, the ratio effect was also preserved here. We discuss further findings in these conditions in Section 5.2, in relation to whether quantifiers bias the nonsymbolic quantity processing mechanism.

Finally, yet another series of sentence-picture verification studies compare processing times for proportional and other quantifier classes. In these studies, the duration of the visual display was long enough to allow counting if participants so wished. The studies find that participants are fastest for the quantifiers "all" and "some," followed by modified numerals ("less than eight," "more than seven") and, finally, with the proportional quantifiers "more than half" and "less than half" taking the most time (Szymanik & Zajenkowski, 2010). Recent Electroencephalography (EEG) results confirm that this distinction is also associated with differences in the evoked potential (Bremnes, Szymanik, & Baggio, 2022). Moreover, schizophrenic patients fell behind control subjects, in terms of accuracy, only on proportional quantifiers (Zajenkowski, Styła, & Szymanik, 2011). Furthermore, the numerical distance between the two cardinalities to be compared in the case of proportional quantifiers influences verification time and accuracy (Zajenkowski, Szymanik, & Garraffa, 2014). Szymanik and colleagues suggest that proportional quantifiers take longer to evaluate because they involve comparing the cardinalities of two sets, requiring the involvement of working memory and executive processes, whereas "all" and "some" do not require such comparison (see Szymanik, 2016 for an overview).

Looking at the interface more directly, several Functional magnetic resonance imaging (fMRI) studies used a sentence-picture verification task. In each trial, participants first saw a sentence containing a quantifier. Subsequently, they saw the same sentence, accompanied by a picture depicting a certain number of objects. The participants' task was to indicate whether the statement was a suitable description of the image. McMillan, Clark, Moore, Devita, and Grossman (2005) compared the BOLD response that was present during the display of a sentence together with a picture with the BOLD response that was present when just the sentence was presented. This point in time was thought to reflect the process of verification of the quantifier rather than the reading of the sentence. In this comparison, McMillan and colleagues reported more neural activity in the right inferior parietal cortex for verification than for reading the sentence. The quantifiers they used were "all," "some," as well as modified numerals. All were analyzed together as one group. Furthermore, McMillan and colleagues compared activity at this point for different quantifiers. In all the different quantifier conditions, some verification process (and visual array processing) was taking place, meaning that any differences between quantifiers can be attributed to a difference in the verification processes specific to the quantifiers. They did not find more activity in this brain region for modified numerals than for "some" and "all," which speaks to the parietal areas being at least equally active for "all" and "some" as for modified numerals.[10] In a different study using a similarly structured sentence-picture verification task, Olm et al. (2014) compared the neural activity for verifying sentences with the quantifiers "some," "at least half,"[11] and modified numerals. All quantifiers were analyzed together as a group, compared with the neural activity for verifying number words (e.g., "three").[12] They observed more activity in the bilateral inferior and superior parietal cortices for quantifiers relative to number words. However, both of these studies analyzed the modified numerals and other quantifiers together as a group, meaning that the modified numeral alone could be driving the observed effect.

3.2. The mechanisms for processing quantifiers and nonsymbolic quantities

When we consider the potential of shared mechanisms underlying quantifier and nonsymbolic quantity processing, it is fruitful to frame the question in neuronal terms: we are interested in whether processing quantifiers requires the involvement of populations of neurons that also process nonsymbolic quantities. The areas within the prefrontal and parietal cortices, and especially the intraparietal sulcus and area around it, are thought to play a crucial role in both symbolic and nonsymbolic quantity processing (Arsalidou & Taylor, 2011; Sokolowski & Ansari, 2016; Sokolowski et al., 2017). If quantifiers interact with or are references to nonsymbolic quantity representations, as we propose, we would expect these same neuronal populations to be crucial for processing quantifier meaning as well. Alternatively, quantifiers might be represented as a separate, independent network, for example, in the left temporal lobe similar to other semantic categories (e.g., Binder & Desai, 2011; Matchin & Hickok, 2020; Ralph, Jefferies, Patterson, & Rogers, 2017).

Given that there are certain differences between quantifiers and nonsymbolic quantities, we do not expect the processing mechanisms to be identical, still, it would be interesting to know to what extent the two mechanisms are related to each other. Specifically, for example, for deciding which of the possible quantifiers best suits a cardinality description or a proportion, we would expect decision-making processes to be involved. Given the context-sensitivity of quantifiers, the fact that they give rise to pragmatic inferences, and that they need to be read as words before their meaning is understood, we would expect to see some involvement of general language processing areas. For example, a recent fMRI study showed that simply reading quantified sentences did not recruit the same parietal areas as reading mathematical statements (Amalric & Dehaene, 2019; see also Liu et al., 2017), indicating that simply reading a quantifier is not sufficient to engage the numerical processing system. However, in this review, we only briefly mention studies relevant to these additional processes as discussing them in detail is beyond the scope of the present paper. Instead, we focus on the involvement of the quantity processing system, and ask:

Open Question 2 *Do the neuronal populations involved in processing various classes of quantifiers overlap with the neuronal populations involved in nonsymbolic quantity estimation and comparison?*

To date, only one study that we are aware of, compares fMRI BOLD signal during quantifier comprehension, quantity processing, and comprehension of other words to directly test whether the parietal areas crucial for quantity processing get involved in quantifier processing more than in semantic processing of words in general (Wei, Chen, Yang, Zhang, & Zhou, 2014). In different trials within this study, participants saw pairs of quantifiers, Arabic digits, number words, dot arrays, frequency adverbs, or animal names and were asked to choose the one that was most similar in meaning to a third stimulus of the same type. Wei and colleagues reasoned that if an overlapping neuronal population subserves the processing of nonsymbolic quantity as well as symbolic quantity in the form of number words, digits, quantifiers (specifically, generalized existential and proportional quantifiers were included), and frequency adverbs (such as “always,” “often,” “never”), this population should be more involved in reasoning about all of these materials when compared to reasoning about animal names (specifically, they expected to find at least one overlapping area when looking at the BOLD signal for each of them as compared to animal names). They did not find any such common area. In contrast, when looking solely at an area that was involved in processing number words, digits, and dot arrays together (i.e., excluding quantifiers and quantity adverbs), they did find a region that seemed to participate in processing all of these but not animal names; this area was in the right intraparietal sulcus. This result speaks against quantifier meaning being represented by a neuronal population overlapping with that representing Arabic digits, number words, and dot array representations. However, the inclusion of frequency adverbs in the analysis, together with quantifiers, precludes us from drawing reliable conclusions since we are not confident that frequency adverbs also involve quantity processing mechanisms.[13] Unfortunately, no analysis excluding frequency adverbs is reported by Wei and colleagues, so further research is needed to make confident conclusions. An fMRI study on Chinese classifiers by Her, Chen, and Yen (2018), using a similar paradigm to Wei and colleagues, supports this reservation: compared to tool nouns, classifiers—that are arguably more linguistic than quantifiers—did show increased activation in the bilateral inferior parietal lobule, including the intraparietal sulcus.

The sparsity of direct comparisons forces us to consider the question more indirectly: Studies investigating brain regions that subserve quantifier processing in healthy adults point to a network of right (or in some studies bilateral) parietal (specifically, the intraparietal sulcus and areas close to it in the inferior and superior parietal cortices) and prefrontal areas, parallel to the network of quantity processing for number symbols and nonsymbolic quantity (Heim et al., 2012, 2016; McMillan et al., 2005; Olm et al., 2014; Troiani et al., 2009). However, there have been only a few studies to date, even considering a less precise sample. Additionally, they each have relatively few participants, so while we mention the regions in which an increased BOLD signal was observed for quantifiers, these regions are not, in fact, very specific. For this line of research, it is only possible to make rough generalizations about the involved regions, compared to the line of inquiry into number symbols, where substantially more evidence has been accumulated. The more informative aspect of each of these studies is the comparative involvement of different brain regions in different conditions within the same study. A multi-voxel pattern analysis approach could be applied to improve on the situation, as this technique has been utilized to make fine-grained distinctions between brain areas involved in symbolic and nonsymbolic processing (Bulthé, De Smedt, & Op de Beeck, 2014; Eger et al., 2009).

A different approach to data analysis has been adopted by Heim et al. (2012) who investigated brain activity in response to verifying phrases with proportional quantifiers (precisely, "many," "few," "most," "very few," "more than half," "less than half"), also against a visual scene. Instead of comparing brain activity at different points in time, they systematically manipulated the number of target items (blue circles in the case of "many of the circles are blue") and the ratio between target and comparison items (e.g., the number of blue circles relative to yellow circles). To tap into the semantic processing of quantifiers, they looked for the regions in which activity correlated with the change of ratio between target and comparison items and which were also involved during comprehension of the sentence with the quantifier before that. In the study, sentences were presented auditorily before visual scene presentation. The result of this analysis was thought to specifically reflect an evaluation of the meaning of the quantifier and whether it fit the picture. The bilateral intraparietal sulcus and inferior parietal cortex were identified among the regions correlating with the semantic processing of quantifiers. Given that there were no modified numerals among the materials used in this study, these results cannot be attributed to their presence.[14,15] Thus, for proportional quantifiers, we have rather strong evidence for the involvement of the brain regions critical for quantity processing.

One additional source of evidence comes from looking at patients with damage in the parietal cortex.[16] Several studies have been conducted with participants with Corticobasal syndrome (CBS) who are known to have impaired processing of both number symbols and nonsymbolic quantities. Participants were typically asked to make judgments about whether a particular statement or sentence with a quantifier correctly described a picture. For modified numerals, several studies have reported impaired knowledge in CBS patients, both relative to healthy age-matched controls and relative to patients with damage to other parts of the brain (McMillan, Clark, Moore, & Grossman, 2006; Morgan et al., 2011; Troiani, Clark, & Grossman, 2011; Troiani et al., 2009). Performance with a limited selection of quantifiers has been investigated in CBS patients. Three studies have looked at their performance with the quantifiers "some" and "all." Troiani and colleagues (Troiani et al., 2009) observed worse performance in CBS patients for modified numerals than these quantifiers and interpreted this as evidence for "some" and "all" not recruiting parietal areas. However, the CBS patients, in fact, performed worse for both modified numerals and "some" and "all" when compared to the control group of Parkinson's disease patients; thus, it seems like "some" and "all" also rely on parietal areas, just to a lesser extent than modified numerals. Supporting this possibility, McMillan et al. (2006) report worse performance for "some," "all," and modified numerals combined (this, however, means that only modified numerals might have been responsible for the effect as a group) by CBS patients relative to age-matched controls, patients with Alzheimer's disease, and frontotemporal dementia (which does not typically involve parietal lobe damage). On the other hand, contrary to the results of these two studies, Morgan and colleagues (Morgan et al., 2011) found comparable performance for "some" and "all" in CBS patients relative to age-matched controls and frontotemporal dementia patients.

Other quantifiers that have been investigated with CBS patients are "at least/more/less than half," again with mixed results. Troiani et al. (2011) report impaired performance with these quantifiers compared to healthy seniors and a brain-damaged control group, but they were analyzed together with modified numerals so that the latter may have been driving the effect. On the other hand, Morgan et al. (2011) do not find impaired performance with these quantifiers.

Finally, one recent study investigated the performance of CBS patients in a production task. They were asked to describe a picture; the authors observed fewer uttered quantifiers by these patients but did not provide a comparison between different classes of quantifiers (Ash et al., 2016).

Overall, so far, it has been consistently observed that CBS patients are impaired in the processing of modified numerals, whereas the results with other quantifier classes remain mixed. For the quantifiers "at least/more/less than half," one study observed impaired performance in CBS patients, whereas another did not. For "some" and "all," two out of three studies to date suggest that CBS patients are impaired for these quantifiers.

Open Question 3 *Are patients with damage to the parietal lobe impaired in their knowledge of quantifiers other than modified numerals?*

Another line of research with patients has investigated the performance of patients with semantic dementia. This neurodegenerative disorder mostly affects the left temporal lobe and results in a gradual loss of semantic memory (of semantic concepts such as knowledge about different animals, tools, etc.). Because of this behavioral manifestation of the atrophy, it is thought that the left temporal lobe plays a crucial role in the storage of semantic information. Studies of quantifier knowledge with these patients, therefore, can help us understand whether quantifiers are stored together with other semantic concepts (in which case, we expect to see a deterioration in the knowledge of quantifiers as well) or separately, for example, relying more heavily on parietal areas (in this case, we expect to see the knowledge of quantifiers mostly preserved). In this line of research, only two studies with just three patients have been reported. Two out of these three patients had retained knowledge of the meaning of quantifiers (as well as unimpaired performance in purely quantity-related tasks), while at the same time, they had a severely damaged understanding of the meaning of other words (Cappelletti, Butterworth, & Kopelman, 2006. In the study of Cheng et al. (2013), one patient with mild semantic dementia did not have impaired quantifier processing, whereas another patient with severe semantic dementia *was* found to be impaired on quantifier comprehension. However, this could be attributed to a more fundamental deterioration of language skills. These studies tested knowledge of generalized existential and proportional quantifiers, but not generalized universal quantifiers or modified numerals. Therefore, we cannot exclude the possibility that unimpaired processing was due to the participants being good at mostly or only modified numerals. Also, we cannot say whether participants were perhaps impaired in terms of their knowledge of generalized universal quantifiers.

3.3. Differences between kinds of quantifiers

As mentioned in Section 2.2, not all quantifiers are expected to behave the same. In particular, there are universal quantifiers—that are expected to rely primarily on the detection of examples or counterexamples—and a subset of the existential quantifiers that may be more related to signal detection or logical reasoning than quantity processing.

Open Question 4: *Are the generalized universal/existential quantifiers processed independently of quantity representations?*

If generalized universal quantifiers rely on logical reasoning, we would expect corresponding mechanisms to be involved (see, e.g., McMillan et al., 2005; Szymanik, 2007; Troiani et al., 2009 for discussions of the implications of these differences for the brain regions involved in processing quantifiers).

Overall, the results described in the preceding sections are compatible with the possibility that the verification of sentences with the quantifiers "some" and "all" recruit roughly those brain regions known to be involved in quantity processing. In contrast to these studies, Troiani et al. (2009) observed more BOLD signal for modified numerals than for "some" and "all" in the bilateral intraparietal sulcus, and argue based on this that "some" and "all" do not recruit these areas.[17] One way to explain these results would be to recall that Aristotelian quantifiers may be verified by searching for (counter-)examples without any reference to number processing. However, note that there are some methodological difficulties with interpreting the above result. First of all, the authors do not compare brain activity during processing "some" and "all" to processing other words. Hence, the parietal areas that they identified may be involved in processing "some" and "all" as well, just to a smaller extent than in processing modified numerals. Second of all, modified numerals are harder to verify than "some" and "all" (Szymanik & Zajenkowski, 2010) so in order to get a more robust comparison of the activations, one would have to normalize trial difficulty.[18] In any event, it is safe to conclude that question 4 remains an open question.

3.4. Open questions

Overall, when considering the accumulated evidence, we see that it is consistent with the possibility that the right or bilateral intraparietal sulcus and surrounding areas are involved in processing modified numerals and various proportional quantifiers. For the generalized existential "some" and generalized universal "all," the evidence so far is mixed. The present review points out that only a limited set of quantifiers has been investigated in work with patients and neuroimaging research. Specifically, various generalized existential quantifiers besides "some" should be investigated[19] (e.g., "several," "a few," "a couple"; and the value judgment "many" and "few," if that can be made explicit by the task) as well as generalized universal quantifiers besides "all" (e.g., "every," "each"). The modified numerals or bare number words could function as a good baseline if the processing of other quantifiers is compared to their processing in the same context. At the same time, these studies should aim to first identify the neuronal populations responsible for nonsymbolic quantity processing to allow for drawing conclusions about potential overlap directly within the same study rather than based on previous studies (given that differences in, e.g., data processing procedures or a specific set of participants make it difficult to draw such conclusions based on data across different studies).

An important consideration for future studies is to distinguish quantifier representations and decision-making, or accompanying logical reasoning processes within tasks. The analyses conducted by Heim et al. (2012) address these issues by looking for regions that are involved in *both* processing the sentence with a quantifier without any visual display and processing the visual presentation itself. A different solution to this issue is to not include a visual quantity comparison task at all, as in the study by Wei et al. (2014); though in their case, an additional issue is that the neural populations involved in *reasoning about* quantifiers might not be the same as those representing the meaning of the quantifier.

The reviewed studies that looked at the interface between quantifiers and nonsymbolic quantity system (Deschamps et al., 2015; Heim et al., 2012; Lidz et al., 2011; Pietroski et al., 2009; Shikhare et al., 2015), as well as a study with children by Odic and colleagues (Odic, Pietroski, Hunter, Lidz, & Halberda, 2013) to be discussed below, all report the same ratio-dependent performance even though they investigate different proportional quantifiers—"most," "more/less than," "many," "few," "more." The interesting question here is whether, by interfacing with quantity processing mechanisms, these quantifiers simply extract the ratio between two sets, which is then used to decide whether the quantifier is applicable. If this is the case, as the evidence seems to say, any differences between their meanings should not be due to quantity processing but to specific extracted ratio values (e.g., for the difference between "many" and "few") and possibly some other properties (e.g., inference patterns, pragmatic aspects, etc.; e.g., in the case of "many" and "most," where we intuitively believe there is a difference). Alternatively, however, it could be said that the setup of these tasks was such that participants did not, in fact, evaluate quantifier meanings but performed a perceptual judgment—simply chose the larger/smaller quantity set. In this case, a better task would be required to allow us to observe the differences between the kind of information that is extracted from quantity representation mechanisms. One possibility, for example, instead of an experiment with many trials where participants may develop and adjust strategies, is to administer few items but with many participants (this has been done, e.g., by Register, Mollica, & Piantadosi, 2020). Another possibility would be to study the extracted ratio in a purely linguistic setup, as has been first proposed by Ramotowska et al. (2023), and compare performance in the setup with a visual scene and a setup without a visual scene (e.g., Schlotterbeck, Ramotowska, van Maanen, & Szymanik, 2020).

Open Question 5 *Do the proportional quantifiers extract ratio information from nonsymbolic quantity representations in tasks other than sentence-picture verification?*

Furthermore, in terms of the interface between quantifiers and nonsymbolic quantity, it remains to be seen whether generalized existential quantifiers, such as "some," "several," "a few," "enough," "a couple," "a dozen," and so on, as well as generalized universal quantifiers, such as "all," "every," "each," also interface with quantity processing systems, and if they do, what kind of information they extract. For the generalized existential quantifiers, we would expect cardinality information to be extracted, whereas for the generalized universal quantifiers, it is more difficult to make predictions. Researchers are starting to look at the interface between generalized universal quantifiers and nonsymbolic quantities. Knowlton et al. (2022) present experimental results suggesting that verifying sentences with the quantifiers "all" and "every" against visual displays triggers a representation of the cardinality of a set, whereas "each" does not.

Open Question 6 *Do generalized existential and universal quantifiers also interface with quantity processing systems, as has been shown for proportional quantifiers? If yes, what kind of information do they extract?*

4. Codependence between quantifiers and nonsymbolic quantity in development

Provided that the adult system for quantifier processing interfaces with the system for nonsymbolic quantity, and that these two systems partially share neurobiologically grounded processing mechanisms, it is natural to ask whether the two systems develop in parallel or whether the development of one system is parasitic on the other. We know that already prelinguistic infants can distinguish nonsymbolically presented quantities to some extent and that their ability to discriminate improves with development, allowing increasingly smaller ratios to be distinguished (e.g., Izard, Dehaene-Lambertz, & Dehaene, 2008; Spelke, 2011; Wynn, 1998; Xu & Spelke, 2000). Children are also able to understand some quantifiers from approximately the age of 2 (see Barner & Chow et al., 2009; Barner & Libenson et al., 2009). Parallel to the hypotheses about number symbol learning (Carey, 2001; Carey, 2009), one theory about quantifier learning would be that since children have the nonsymbolic quantity processing system available, they simply associate or map the quantifier meanings onto these nonsymbolic quantity representations. An alternative hypothesis is that quantifier comprehension and production develop as a separate system, not relying on nonsymbolic quantities.[20]

4.1. Learning quantifiers by mapping them to nonsymbolic quantities

To look at the interface between quantifiers and nonsymbolic quantities in young children, Odic et al. (2013) used a sentence-picture verification task with the comparative quantifier "more." Eighty children aged 2–4 years were asked to verify the statement "are more of these dots blue or yellow?".[21] They reasoned that if children use their nonsymbolic quantity processing system to evaluate whether the quantifier fits the description, they should find the typical psychophysical pattern of ratio-based performance for nonsymbolic quantity comparison seen in adults, albeit given more noisy representations. Children performed above chance in this task at approximately age 3.3. The children who succeeded indeed showed a pattern of performance consistent with nonsymbolic quantity processing. Odic and colleagues interpret their results as suggesting that "more" interfaces with perceptual quantity processing mechanisms and that children have access to this interface as soon as they understand the meaning of the comparative "more."

If quantifiers indeed rely on nonsymbolic quantity processing, one could expect children who perform nonsymbolic quantity comparison at a higher level (i.e., who can distinguish smaller ratios) to understand quantifiers better. The only study to date we are aware of investigating this question looked at the correlation between the two abilities. Dolscheid et al. (2017) asked 39 children aged between 3 and 6 years old to give several objects corresponding to one of eight German quantifiers ("Can you put all/a/none/both/most/many/some of the bananas into the bowl?"). The children's performance in this task (assessed based on whether they gave a quantity in the range matching that of adult control participants) was overall correlated with the ratio they were able to discriminate in a nonsymbolic comparison task. This correlation was significant when controlling for age, IQ, and the children's level of knowledge of number symbols. However, when investigated more closely based on performance with individual quantifiers, only the quantifiers "both" and "most" were related to performance on the nonsymbolic quantity comparison task. The fact that only two quantifiers were related to nonsymbolic quantity performance is unexpected as among the quantifiers presented to the children, at least "many" and "some" can be thought of as quantifiers that should be related to nonsymbolic quantities. A potential explanation may lie in the fact that the average age of children who participated in this task was 4.5 years old. These children have likely already mastered other quantifiers rather well (surpassing the initial reliance on purely nonsymbolic quantities) and perhaps showed ceiling performance that did not allow for correlations. Indeed, when examining the performance for each quantifier, it becomes apparent that they perform at the ceiling for all quantifiers except for "most," "both," and "some."[22] This explains why, for other quantifiers, there was no relationship. Though it still does not answer why there was no relationship with "some."

4.2. *Order of acquisition of quantifiers*

We know that the meaning of number symbols is acquired in a particular order—"one" through "four" sequentially, followed by an understanding of the cardinality principle. Multiple factors play a role here, including the way the number symbols are used (lexical frequency, contextual diversity, word co-occurrence, or distributional similarity) (Willits, Jones, & Landy, 2016). A parallel question for quantifiers would be whether there is any particular universal order of acquisition of quantifiers by children learning different languages. Katsos et al. (2016) suggest that if quantifiers, like number symbols, are acquired in order of increasing cardinality, it follows that "a few" and "some" should be learned earlier in development, whereas "most" and "all" should be acquired later. This prediction is not borne out given the observation that children as early as 2 years old understand "all," but even some 7-year-old children have not yet fully acquired the meaning of "most" (see, e.g., Barner et al., 2009). Instead, given that quantifiers are richer in meaning (due to the inference patterns they give rise to), Katsos and colleagues suggest that there are constraints in quantifier learning absent in learning number symbols. They present four such constraints (given the monotonicity, totality, complexity, and informativeness properties of quantifiers) based on which they make predictions for quantifiers corresponding to the English "all," "none," "some," "some…not," and "most." Katsos and colleagues collected data from children learning 31 different languages.[23] Children learning most of these languages conformed in their performance to predictions based on each of their proposed constraints. Katsos and colleagues, therefore, suggest that the order of acquisition of quantifiers is driven by properties that can be characterized as something like "semantic complexity"[24] rather than the cardinalities to which they refer.

One of the central problems in number acquisition literature is to explain how children go from understanding the meaning of a few numbers to having access to a powerful concept of a number line. Piantadosi, Tenenbaum, and Goodman (2012) show, via computational modeling, how such an inductive leap can be in principle accomplished by rational Bayesian agents as a statistical inference over a sufficiently powerful representational system (the language of thought). Interestingly, for our considerations, a similar model can be used to explain properties of natural language quantifiers (Piantadosi, Tenenbaum, & Goodman, 2013), that is, to explain why certain quantifier meanings are learnable/lexicalized, while others are not. Essentially, the authors try to explain learnability by inducing bias toward quantifiers that have shorter descriptions in the Language of Thought (LoT). Similarly, in a recent paper, van de Pol, Lodder, van Maanen, Steinert-Threlkeld, and Szymanik (2023) show that quantifiers with shorter minimal description lengths are more likely to be lexicalized and Steinert-Threlkeld and Szymanik (2020) show that artificial neural network models have a similar learning bias.

4.3. Open questions

One set of questions regarding the acquisition of quantifiers concerns the (availability of the) interface between quantifier comprehension and perceptual systems of nonsymbolic quantities in sentence-picture verification. The only such study with children was conducted by Odic et al. (2013). This study suggests that children make use of nonsymbolic quantity representations to evaluate "more" as soon as they understand the comparative meaning of "more." This observation needs to be confirmed in replications. Also, follow-up research should investigate whether this generalizes to other quantifiers, such as "some," "several," "many," and so on. If it does, what kind of information do children then extract from nonsymbolic quantity representations using this interface for each of the quantifiers, and do they change throughout development?

Open Question 7 *Is children's understanding of quantifiers correlated with their nonsymbolic number acuity? Does improvement in nonsymbolic number acuity result in an improved understanding of quantifiers?*

Katsos and colleagues (Katsos et al., 2016) argue that cardinality does not play a role in the order of acquisition because children do not master quantifiers in the order of the cardinality or proportion to which they refer. However, there is an alternative hypothesis about the order of the acquisition of quantifiers that can be derived from what we know about the development of nonsymbolic quantity processing. We know that children improve in their ability to distinguish between two nonsymbolic quantities in the course of development—their estimates become more accurate, and they learn to distinguish increasingly smaller ratios (e.g., Feigenson, 2007; Halberda & Feigenson, 2008). Perhaps predictions about the order of acquisition should be related to how well children can distinguish between pairs of quantifiers (or even more broadly, between words with a magnitude semantic component) at a given developmental stage rather than to the specific cardinalities to which each quantifier refers. The further apart two ranges of cardinalities or proportions, associated with the quantifiers, are from one another, the sooner children would be able to successfully distinguish them perceptually and, therefore, the earlier they will master the difference between the corresponding quantifier pairs. This proposal predicts, for example, that children will successfully distinguish between "few" and "many" at an earlier point in development than they successfully distinguish between "few" and "several" or between "many" and "most." While Katsos and colleagues present convincing evidence that semantic complexity plays a role in the acquisition process, the development of nonsymbolic quantity representations may play a role in the order of acquisition alongside these factors. Note also that whether order-of-acquisition accounts can predict the order of learning of "all" depends on whether we consider generalized universal quantifiers to also rely on the nonsymbolic quantity system.

Open Question 8 *Is the order of quantifier acquisition linked to the development of nonsymbolic number acuity?*

Only one study has examined whether there is a potential correlation between nonsymbolic quantity discrimination performance and quantifier knowledge (Dolscheid et al., 2017), in one language and a sample of 39 children. Studies with a larger sample and age range of children, as well as with different languages, are needed to see if this relationship exists. Moreover, as we have observed, it would also be essential to break down the relationships by specific quantifiers or quantifier classes. In addition, in analogy to studies on the relationship between number symbols and nonsymbolic quantities, one could also investigate whether training participants to discriminate nonsymbolic quantities improves their performance with quantifiers. While such training results in little or no improvement in number symbol tasks, if natural language quantifiers rely on nonsymbolic quantity representations, this may result in improved performance with quantifiers.

5. The nature of quantifier representations

For number symbols, one prominent research direction has been investigating whether their neural representations are similar to that of nonsymbolic quantity representations. However, note that from the fact that some neurons are responding both to, say, quantifiers and nonsymbolic number representations, it is not easy to draw any cognitive conclusions as we still do not know how to theoretically interpret such facts. Another complicating factor is that quantifiers, unlike number symbols, display no strict linear order in terms of the cardinality or proportion to which they refer. Several studies look into the underlying dimensions behind quantifiers, and we briefly touch upon these.

5.1. The representation format of quantifiers

It has been suggested that the representation format for number symbols in the human brain parallels that for nonsymbolic quantities. Analogously, we may ask about the quantifier representation format. Specifically, quantifiers may be organized in a network of ordered and noisy units. Those referring to larger approximate cardinalities or proportions (high-magnitude quantifiers) have more intersection with each other than those referring to smaller approximate cardinalities or proportions (low-magnitude quantifiers). Evidence for such a format has been reported not only for approximate cardinalities but also for ratio information (Jacob & Nieder, 2009a; Jacob et al., 2012). Therefore, such a representation format is at least possible for generalized existential (which we think refers to cardinalities or proportions) and proportional quantifiers (which we think refer to proportions). The representation format of generalized universal quantifiers is more tricky because, as discussed, it is not transparent to what extent they are related to quantity processing rather than logical reasoning. On the other hand, possibly even these quantifiers rely on quantity information. There is some empirical evidence to suggest that they are also understood as referring to proportions: The study by Pezzelle et al. (2018), discussed in Section 3.1, did observe empirical results to that effect, and also analyzed the particular proportions that participants associated with each quantifier. There was a substantial intersection between the proportions to which quantifiers referred (e.g., when the target objects constituted 20% of all objects on the screen, participants chose “few,” “the smaller part,” or “almost none” to describe their cardinality). It was nonetheless possible to order quantifiers in terms of their preferred proportion ranges or most preferred proportion. The resulting order was: “none,” “almost none,” “few,” “the smaller part,” “some,” “many,” “most,” “almost all,” “all.” Interestingly, the range of preferred proportions was smaller, and there was less intersection for low-magnitude quantifiers (i.e., quantifiers referring to smaller proportions) than for high-magnitude quantifiers. This means that low-magnitude quantifiers had relatively more specific meanings.

As an alternative to the hypothesis that the representation format of quantifiers mirrors that of nonsymbolic quantity, quantifiers may be organized as discrete entities, not in linear order and without any intersection in meaning representations. In such a network, each quantifier representation would be separate from others, not competing for activation due to intersection in meaning. In that case, quantifiers could possibly still compete for activation for other reasons. For instance, positive and negative quantifiers could be organized into separate networks. Schlotterbeck et al. (2020) try to explain the difference between pairs of antonymous quantifiers (“more than half” vs. “fewer than half”) by modeling reaction times and accuracy from two verification experiments (a sentence-picture and a purely linguistic verification task), using the diffusion decision model. The authors’ results do not allow us to distinguish between two competing explanations: a two-step processing model, that is, negative quantifiers are harder because they involve a hidden negation (“fewer than half” is represented mentally as “it’s not the case that more than half”), or a pragmatic model, that is, negative sentences are harder due to pragmatic factors like frequency.[25]

Importantly, unlike for number symbols, quantifiers have prominent features in addition to their reference to quantity—they are context-sensitive, give rise to different pragmatic inference patterns, and some are in a special antonym relation to each other (e.g., “many” vs. “few”). Thus, their representations should contain more information than simply referring to quantity and might be organized along more than one dimension (Pezzelle et al., 2018; Routh, 1994), forming multiple different networks.

To investigate the features that comprise quantifier representations in the human cognitive system, another experiment in the study mentioned above by Pezzelle et al. (2018, Experiment 2) asked participants to evaluate the semantic similarity of pairs of quantifiers on a scale from 1 to 7. The authors then used multidimensional scaling to look for underlying dimensions that would explain the judgments of similarity. The results indicated that just two aspects presented a rather good fit for their data ($R^2 = .988$). One dimension seemed to correspond to a separation between the low- and high-magnitude quantifiers (“none,” “almost none,” “few,” “the smaller part” vs. “many,” “most,” “almost all,” “all”). The second dimension distinguished between the low-magnitude quantifiers themselves while not distinguishing between the high-magnitude quantifiers. This suggests that the intersection between low-magnitude quantifiers’ representations is substantially lower than the intersection between representations of high-magnitude quantifiers, as would be expected from an organization format similar to that of nonsymbolic quantity representations. What we mean by that is that, in model-theoretic terms, the classes of models corresponding to low-magnitude quantifiers intersect less than the classes of models corresponding to high-magnitude quantifiers. The results of their other experiment showed less intersection in distributions of proportions that were judged to correspond to lower-magnitude quantifiers. The data from Pezzelle and colleagues support the hypothesis about the existence of ordered representations of quantifiers with more intersection for quantifiers denoting larger proportions.

5.2. *Quantifiers biasing the nonsymbolic quantity processing mechanism*

Because quantifiers have additional pragmatic/linguistic features compared to number symbols, we can ask whether these properties can influence nonsymbolic quantity processing. Assuming that there exists an interface between quantifiers and the nonsymbolic quantity representation system, one possibility is that certain quantifiers influence the comparison process in the quantity processing system when extracting information.

Specifically, does the potential top-down influence of specific quantifiers on nonsymbolic quantity perception force us to use different mechanisms/strategies for quantity comparison when different quantifiers extract quantity information? Shikhare et al. (2015) suggest that quantifier semantics does bias quantity processing mechanisms indeed. Let us take the example they give of comparing an array of five dots against a modified numeral—“at least seven” where the key will be “at least.” They argue that to perform this comparison, we need to activate a quantity distribution corresponding to the reference quantity “seven” and compare it to the observed quantity 5. However, because “at least” typically focuses our attention on larger quantities than the reference (e.g., “at least seven” is typically used to mean “seven or more”), the quantity distribution of “seven” will be skewed toward larger quantities; if we imagine the quantity representations in a left-to-right direction, it will have a right skew.[26] We are, therefore, comparing a normal distribution around 5 to a right-skewed distribution around

7. Because the distribution for 7 is right-skewed, there will be less intersection with the distribution for 5 than if both distributions were normal. This should result in faster reaction times and higher accuracy for “at least seven” than if we were to simply compare the quantities 5 and 7. Thus, the ratio effect will be different from the case where two quantities are compared without the quantifier biasing the comparison process. The opposite should be the case for “at most seven,” since “at most” focuses our attention on smaller quantities than the reference (e.g., “at most seven” is typically used to mean “a maximum of seven, or less”).[27]

Another example of the potential influence of quantifier semantics on the comparison process is the contrast between “most” and “more than half.” It has been suggested in the literature and supported by experiments that the two quantifiers have roughly the same extension, that is, both mean “more than half.” Hackl (2009) has observed that these quantifiers are potentially associated with different information being extracted from the quantity processing system. While “more than half As are B” involves dividing the total number of A’s in half, verifying “most As are B” requires comparing the total number of A’s that are B’s with the number of A’s that are not B’s. However, others have failed to replicate these results and instead suggest that the different roles that the working memory plays in the verification of each of these quantifiers, as well as individual differences in the use of various cognitive strategies, are a better explanation for the difference that Hackl observes (Steinert-Threlkeld, Munneke, & Szymanik, 2015; Talmina, Kochari, & Szymanik, 2017). Independently, Solt (2016), using corpus data, suggests that whereas “most” can be used when only approximate cardinality information is available, “more than half” can only refer to the result of a precise comparison. Hence, it possibly relies on symbolic number processing. Solt also suggests that the difference in meaning between the two quantifiers should be accounted for in terms of pragmatic strengthening. Carcassi and Szymanik (2021), following the pragmatic route, propose a computational model of usage in the Rational Speech Act framework. They argue that the two quantifiers may be truth-conditionally equivalent and that the difference in typical proportions associated with the two expressions can be explained with previous, independently motivated semantic and pragmatic mechanisms of distance-minimization among listeners and the structural account of alternatives. On the other hand, Ramotowska et al. (2023) have applied new modeling techniques to the verification data and discovered that mental representations of “most” (operationalized as thresholds separating true and false instances) vary across subjects and affect the verification process. However, these effects are not present for “more than half.” Building on that work, Denić and Szymanik (2022) present experimental work showing that most preserves the “significantly more than half” interpretation in downward-entailing environments. This finding speaks against the pragmatic strengthening hypothesis and in favor of there being a difference between the two quantifiers at the level of truth conditions. Summing up, this debate leaves us with two possibilities: either “most” and “more than half” have the same meaning but interact differently with the nonsymbolic quantity system, or they subtly differ in meaning.

Consistent with particular quantifiers biasing the quantity comparison mechanisms, Deschamps et al. (2015) found a difference in performance between evaluating a phrase with a quantifier as opposed to the same meaning being conveyed using a mathematical symbol. For example, “Many of the circles are blue” as opposed to instructions given as a depiction of a blue square followed by a sign “ >” and followed by a yellow square (the alternative color of circles in the visual display). Whereas error rates and reaction times were different for pairs of antonymous quantifiers despite the only difference being in the direction that they referred to (e.g., “many” vs. “few”)[28]; no such difference was observed for the two opposite mathematical symbols. The fact that simply giving instructions using a quantifier resulted in a different performance speaks to the idea that the quantifier did somehow influence or bias the comparison process.

5.3. Open questions

When thinking about quantifiers in terms of the questions that have been asked for number symbols, one consideration is whether we can expect to see the same format of representations for quantifiers and nonsymbolic quantities. Do quantifiers similar in meaning overlap in their neuronal representations more than quantifiers that are far apart in meaning? Neuroimaging methods can also be used to tap into this question. A ratio-dependent similarity has been observed for nonsymbolic quantity representations in the adaptation paradigm and representational similarity analysis (RSA) analyses of fMRI BOLD data (Lyons & Beilock 2018). Using such an approach for quantifiers, one would expect to see more similarity in neural activation patterns for quantifiers whose meanings intersect more. For example, we would expect to see more similar activation patterns for high-magnitude quantifiers than for low-magnitude quantifiers (as discussed above) based on the results of Pezzelle and colleagues.

In parallel to these studies on number symbols, questions about whether representations of quantifiers and nonsymbolic quantities overlap can be investigated using an adaptation paradigm (possibly with a change in notation—adaptation to a dot array representing a certain proportion followed by a deviant quantifier that refers to a similar or dissimilar proportion; e.g., 3/10 followed by “few” or “many”) and RSA analyses (similarly to the study and analysis conducted by Lyons and colleagues, 2018, one could look at the similarity of activation patterns in response to quantifiers and dot arrays). In a similar vein, one could investigate the similarity between “most” and “more than half” by looking at the similarity in corresponding neural activations between these quantifiers.

Open Question 9 *How is the meaning of quantifiers represented in the brain? Are quantifiers stored together with other semantic concepts (such as animals, tools, etc.) or do they rely on, for example, parietal lobe areas? Is there more overlap in representations of quantifiers that intersect more in terms of the cardinality or ratio to which they refer? Is there a neural overlap between specific ratio representations and specific quantifier representations?*

We suggested two possibilities for the representation format of quantifiers—linearly ordered intersecting representations and increasing quantities or proportions (parallel to nonsymbolic quantities) or a network of discrete items. The evidence presented by Pezzelle and colleagues (Pezzelle et al., 2018) supports the former, but these results are based on explicitly requested similarity judgments for which participants used their intuition. To gather further evidence, some of the paradigms used to investigate the number symbol representation format can also be used with quantifiers. Specifically, matching and priming tasks could be used, where different pairs of quantifiers instead of pairs of number symbols would be presented (see, e.g., Koechlin, Naccache, Block, & Dehaene, 1999; Verguts & Van Opstal, 2005). If their representations intersected, quantifiers more similar to each other in meaning would be more difficult to distinguish, resulting in longer Reaction Times (RTs) and lower accuracy, and would prime each other more. As mentioned, however, quantifier representations potentially contain features other than quantity information. These aspects should be taken into account in designing experiments and interpreting results.

A related question is why quantifier meanings/representations should intersect more for increasing quantities or proportions. We have a suggestion for this that could be explored in future work. Low-magnitude quantifiers refer to larger ratios between target objects and the total number of relevant objects (e.g., "few" referring to 3 out of 10 items, ratio 3:10). In contrast, high-magnitude quantifiers refer to smaller ratios between target objects and the total number of relevant objects (e.g., "many" referring to 7 out of 10 items, ratio 7:10). Since our nonsymbolic quantity-representation system is more accurate with larger proportions, it is also more capable of supporting quantifiers referring to larger proportions. Larger ratios intersect less and remain sharp, so they result in less confusion and fewer errors. On the other hand, the meaning of quantifiers referring to smaller ratios is blurry/imprecise. Our nonsymbolic quantity representation system is not capable of perceiving these differences to the same extent. In the long run, such constraints may affect the repertoire of words. If language users share these limitations in their ratio experiences, then words may have settled, over time, to cover the space to match discriminability across words. In this way, language could optimize the informativeness by reducing the probability of potential miscommunication (cf. Gibson et al., 2019).

One could use computational modeling to investigate such questions. A learning model could start with equal intersections for lexical items referring to cardinalities and ratios across the whole range, and with a system where quantity representations have properties of nonsymbolic analog quantities that humans have. We predict that such a model would allow blurrier or imprecise high-magnitude quantifiers. One could build a model parallel to the one by Verguts and Fias (2004), which would learn quantifiers (instead of number symbols) along with nonsymbolic quantities.

Open Question 10 *What is the representational format of quantifiers and how does it relate to symbolic and nonsymbolic quantity representations?*

Tying in with Section 4, it is also natural to ask whether this results from the development of these three systems.

Open Question 11 *Do children make use of nonsymbolic quantity representations to interpret quantifiers against a visual scene? If yes, is the information that children extract from nonsymbolic quantity representations different for each of these quantities? Does the extracted information change throughout development?*

An exciting new line of research is the one looking at whether and how quantifiers potentially bias quantity comparison mechanisms. As discussed, there are suggestions and some empirical support that they do (Deschamps et al., 2015; Shikhare et al., 2015). Follow-up research could gather more empirical support[29] and compare a wider range of quantifiers.[30]

Open Question 12 *Can and do quantifiers bias (i.e., assert top-down influence on) nonsymbolic quantity comparison mechanisms?*

6. Conclusion

This paper presents an attempt to connect what we know about the relationship between number symbols and nonsymbolic quantity processing to research into the semantics of natural language quantifiers. Both number symbols and natural language quantifiers can be seen as symbolic references to perceptually perceived quantity information. We reviewed past studies relevant to the relationship between natural language quantifiers and nonsymbolic quantity processing. Importantly, we presented many new research directions and specific questions regarding the processing of quantifiers, which we hope will inspire follow-up research and further theoretical considerations.

We believe that understanding the regularities and limitations in the way the linguistic expressions get mapped on cognitive representation, which in turn get mapped into language-driven behavior, like verification, is crucial for every cognitively committed theory of meaning. However, in this paper, we are not only taking the cognitive perspective, but we also review the neuroscientific literature on how those meanings and behaviors are implemented in the brain. This perspective may not be easily translatable into semantic or even cognitive theories of quantifiers. Still, we find it a crucial ingredient of the interdisciplinary study of quantification. Throughout the text, we are trying to distinguish between those various perspectives carefully, but we admit that the borders are not always easy to identify and may require further interpretation. Triggering the discussion about the reciprocal relevance of these various perspectives is yet another rationale for this paper.

While there are many open questions, it might still be beneficial to highlight what we can know with a higher degree of certainty. First, the processing of quantifiers—at the very least proportional quantifiers—does rely on brain areas associated with nonsymbolic processing to a certain extent. This has been demonstrated in a growing number of neuroimaging and patient studies, and can also be inferred from the properties shared between nonsymbolic processing and the processing of quantifiers, ratio-dependence being the foremost example. From this observation, it is natural to assume that the acquisition of quantifiers consists of mapping certain meanings onto corresponding representations of nonsymbolic quantities, and there is some evidence to that effect. The main problem with drawing this inference, however, is that we do not have adequate theoretically and empirically motivated predictions about precisely what this correspondence is. It is our belief that the contradictory findings from the developmental literature derive from this deficiency. Relatedly, while there is evidence to suggest that the representations of lower magnitude quantifiers (“none,” “few,” etc.) intersect less than higher magnitude quantifiers (“most,” “almost all,” etc.) and that quantifiers can bias our representations of nonsymbolic quantities, there is no unified concept of quantity representation that can bring these cognitive systems together.

This highlights some important limitations in our interpretation of the results presented in the paper. In particular, we view quantifiers from various perspectives or levels of analysis. Semantic theories of quantification are our starting point to think more about the way people are processing quantifier meaning. We talk about meaning representations, abstract cognitive or computational objects embedding quantifiers' meaning in a given context, for instance, the range of proportions that quantifiers refer to, the threshold that a given numerosity should reach for people to declare a quantifier sentence correct, prototypes, and so on. Importantly, there is growing evidence that those different approaches may often be indistinguishable from the empirical perspective (van Tiel, Franke, & Sauerland, 2021). However, we remain neutral to what extent such representations should be used directly as building blocks in semantic theories. One could easily imagine that the meaning, whatever it is, gets mapped into cognitive representation, but the mapping is not necessarily unique.

To complicate the issue even more, almost all experiments discussed in the paper have a form of verification task. Given a certain quantifier sentence, the subject needs to decide whether it is a true or accurate description of some state of the world. Hence, in almost all of these studies, we face the situation when the underlying meaning needs to be inferred from the associated verification strategy. This is where context plays a crucial role. For instance, while verifying sentences with the quantifier "most" against the large set of randomly distributed colorful objects, the cognitive system may map "most" to a specific range of proportions and then use the perceptual system to decide whether the observed situation falls within the prescribed ranges. However, when verifying the same sentence against an ordered set of objects, the meaning may get mapped into a different representation, allowing easy input to the perceptual system to check whether the representation satisfies most-condition.

Let us, therefore, close the paper with a more general question lurking behind all the research that we have summarized. In the quantifier literature, different ways to decide what should be considered a quantifier have been proposed, as have different classifications of quantifiers. Quantifiers can be divided with respect to their linguistic properties (A- and D-quantifiers), logical definability (first-order, e.g., "all" or "some" vs. higher-order quantifiers, e.g., "most"), computational complexity (e.g., recognizable by finite-automata, like "all," or not recognizable by finite-automata, like "an even number of"), historical reasons (e.g., distinguishing Aristotelian quantifiers "all," "some," "not all," "some not"), or even combinations of these various criteria (e.g., Barwise & Cooper, 1981; Keenan, 2012; Partee, 1995; Szymanik, 2016). All these classifications turned out to be very useful in understanding the formal properties of quantifiers, formulating quantifier theories, and empirical predictions about quantifier distribution or processing. However, as those classifications were not designed explicitly with cognitive science in mind, they seem to be insufficient for capturing and categorizing the varieties of quantifier cognition. We believe that our survey is a witness to this difficulty. Moreover, even though we support the idea of considering all quantifiers separately, in practice, it is only possible to make progress by trying to draw some semantic generalizations, also given the cross-linguistic variation in quantifier repertoires. We therefore believe that one of the central goals for the future of generalized quantifier theory would be to propose cognitively plausible distinctions among quantifiers. Again, we could start by trying to explicitly think of connecting quantification with symbolic and nonsymbolic quantity processing.

Question 13 *What are natural and robust quantifier classes with respect to symbolic and nonsymbolic quantity processing?*

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Notes

1 Throughout the paper, we refer to perceptually extracted quantities as nonsymbolic.

2 A related point is that while it is, in principle, possible to have a one-to-one correspondence in terms of quantity between quantities represented nonsymbolically and by number symbols (e.g., six dots and the digit "6"), it is not possible to find correspondence between quantifiers and nonsymbolic quantities in this way. Therefore, it might seem like quantifier meanings are less comparable to nonsymbolic quantities than number symbol meanings are. However, considering that nonsymbolic quantities beyond the subitizing range are not exactly represented anyway, this correspondence is not useful from the perspective of questions about shared or distinct brain processing and representations.

3 For proportional quantifiers, part of context-sensitivity can be potentially explained by the fact that they refer to a proportion that is invariant to absolute quantities. However, this still does not explain context-sensitivity in terms of speaker differences—different people have different internal criteria for what proportion should be considered "many ants" (perhaps for a person with an insect phobia, just three ants would be sufficient; moreover, we know that the people's internal thresholds can also change in the course of a conversation) (Heim et al., 2015; Ramotowska, Steinert-Threlkeld, Leendert, & Szymanik, 2023; Yildirim, Degen, Tanenhaus, & Jaeger, 2016).

4 In contrast, if "five people ate oranges" is true, it is not then the case that "five people ate" has to be true (there could be additional people present in the context who ate things other than oranges). There is a debate (see, e.g., Hurewitz, Papafragou, Gleitman, & Gelman, 2006, for a short review) about whether the number symbols in this context are interpreted as "at least five (people ate oranges)," in which case the number symbol meaning would be upward monotone. We will leave this debate aside, however.

5 Alternatively, some semanticists claim (see, e.g., Partee, 2004), that these quantifiers refer to proportional information or are ambiguous between the approximate cardinality and proportional readings. This is because their meaning does not refer to any particular approximate quantity (it is not the case that, e.g., "some" always refers to 2–5 objects),

but instead, at least in some cases, they seem to be dependent on the total number of objects available in the relevant context (for supporting empirical evidence see, e.g., Pezzelle, Bernardi, & Piazza, 2018, Experiment 1; Newstead & Coventry, 2000).

6 And sometimes potentially also existential quantifiers, see the footnote above.

7 Note that when the analysis was restricted to trials with target object quantities within the subitizing range, for "few," "none," and "almost none," the best predictor was the number of target objects.

8 Cf. with experiments in Knowlton et al. (2021), some of which did not involve time pressure, and hence strengthen the original interpretation of the results.

9 While the authors do not explicitly mention that the order of presentation of trials with different quantifiers was randomized, this is implicit in the arguments that they make in the paper. We thus infer that the trials with different quantifiers were intermixed.

10 Of course, it should be kept in mind that this is a null finding—not seeing a difference is not enough to claim that there was no difference, it only supports such a possibility.

11 The article discusses "logical quantifiers" as a category, but gives only "some" as an example (with no full stimuli list available), so we are not sure whether there were any other quantifiers included in this category.

12 Verifying sentences with number words also involves numerical knowledge, so it is an exceptionally strong control ensuring that they capture quantifiers' meaning processing rather than a pure assessment of the quantity of objects on display. We already know that parietal lobe areas are involved in estimating and comparing quantity information, so any task that requires this will be bound to result in parietal area activations. The tricky part is finding out what part of such activation is due to quantity processing itself and what part of such activation is due to perhaps the retrieval of and maintenance of the meaning of the quantifier.

13 Actually, lots of the literature argues that space, time, and number all reside in the parietal lobe (Bonn & Cantlon, 2017; Cantlon, Platt, & Brannon, 2009; Casasanto & Boroditsky, 2008; Lourenco, 2015; Walsh, 2003). Hence, even if frequency adverbs do not connect to numerical quantity, they are likely to connect to temporal quantity—which also resides in the parietal lobe. We thank one of the reviewers for pointing this out.

14 One caveat in this study is that the number of tested participants is not reported in the article, meaning that we cannot be sure about how robust/trustworthy these results are.

15 Interestingly, follow-up fMRI studies pointed explicitly to the left inferior frontal gyrus as subserving, specifically the semantic/linguistic aspect of quantifier processing/polarity processing (Heim et al., 2012, 2016; see also Wei, Chen, Yang, Zhang, & Zhou, 2014 for corroborating evidence). This is expected given that neuronal populations in this area are considered to be some of the crucial ones for language processing (Friederici, 2002; Hagoort, 2017; Matchin & Hickok, 2020).

16 Given that different quantifiers may require working memory, logical reasoning, and lexical retrieval, patients with atrophy or damage to other parts of the brain that result in deficits in these capacities have also been impaired regarding some types of quantifiers. Still, here we refer the reader to the specific studies for more information (see Ash et al.,

2016; Morgan et al., 2011, for recent reviews of quantifier processing impairments for other damaged brain areas).

17 This study had a similar design to the two studies described above, except that they presented visual objects whose quantity was to be evaluated against quantifier sentences serially rather than simultaneously as McMillan and colleagues did, and without the statement with the quantifier present on display at the same time.

18 We are grateful to one of our reviewers for pointing out this possibility to us.

19 Note that the above-described study by Wei et al. (2014) did include other generalized existential quantifiers, but we believe their analyses are not sufficiently informative.

20 In the case of specifically generalized universal quantifiers, recalling that logical reasoning might be especially important, children would need to develop this first to understand and use these quantifiers correctly. For example, children understand the meaning of "some" at an adult level (i.e., interpret it as adults do) at a later point in development than they understand "all" (Barner et al., 2009; Barner et al., 2009; Dolscheid, Winter, Ostrowski, & Penke, 2017). We will not discuss the development of logical reasoning in detail here, focusing solely on the question of the relation to the nonsymbolic quantity processing system.

21 As well as the statement "is more of the goo blue or yellow?" in another condition for which they observed the same result.

22 Interestingly, children learning English show a parallel pattern, with "both" and "most" being the most difficult quantifiers to acquire (Barner et al., 2009); see also (Sullivan, Bale, & Barner, 2018) for evidence that "most" might not be fully acquired until later in childhood.

23 All languages were those of industrialized societies with complete number symbol systems.

24 See also Szymanik and Thorne (2017).

25 Both nondecision times and drift rates, two of the free model parameters of the DDM, were affected by the monotonicity manipulation.

26 As a reviewer pointed out, one should also ask here about the potential role of implicit negation, as for instance, "at least seven" might be represented as "not less than seven."

27 See Carcassi and Szymanik (2022) for formalizing the use of modified numerals with a computational model based on a similar idea.

28 Possibly due to bias introduced by each quantifier, as suggested by Shikhare and colleagues; see Deschamps, Agmon, Loewenstein, and Grodzinsky (2015) for an extensive discussion of other possible explanations.

29 In the case of Shikhare and colleagues, this was a post-hoc suggestion based on an asymmetrical pattern they observed in their data, which means that a replication is necessary.

30 Shikhare and colleagues themselves attempt to do this for "many" and "few."

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