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# Parametric Real-Time System Feasibility Analysis Using Parametric Timed Automata

PhD Dissertation  
Yusi Ramadian

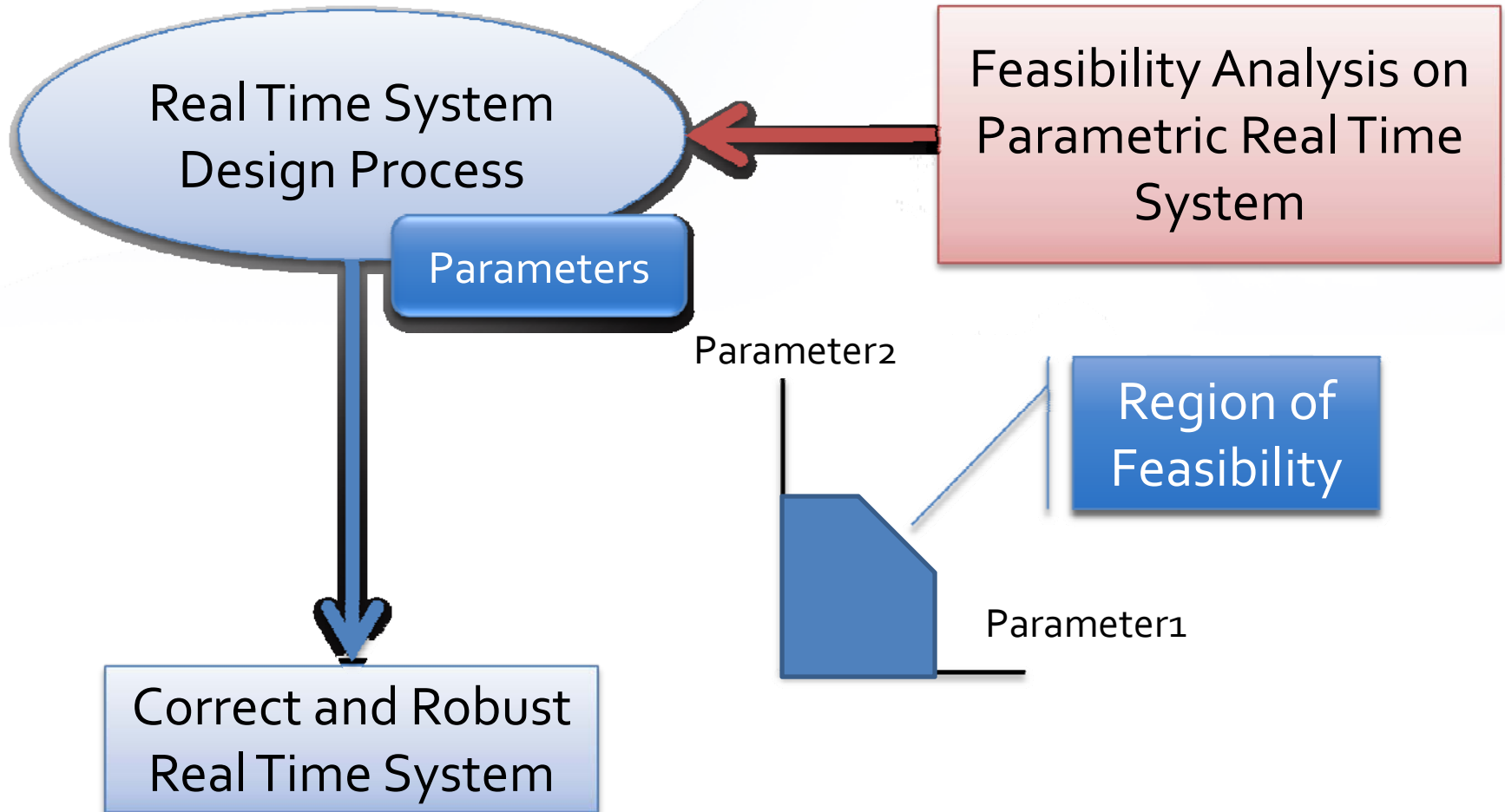
Advisor : Luigi Palopoli  
Co-advisor : Alessandro Cimatti

# Real-Time System Applications

- A computer based system, which produces results to inputs complying with some temporal constraints



# Main Research Idea





# Contribution of PhD research

- Parametric Timed Automata (PTA) definition and representation of Real Time System
- Parametric Verification of Temporal Properties (PVTP) method
- Implementation in tool Quinq
- Application in case problems :
  - periodic task system [RTSSo8],
  - heterogeneous system [ETFA10],
  - collaboration with Modular Performance Analysis Toolbox (MPA) [CASES11].

# Presentation Outline



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- Motivation
  - Real time system design
  - Example scenario
  - Problem Statement
- Solution
  - Parametric Timed Automata
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  - Demo
- State of the art
- Conclusion

# The Importance of CORRECT & ROBUST Real Time System



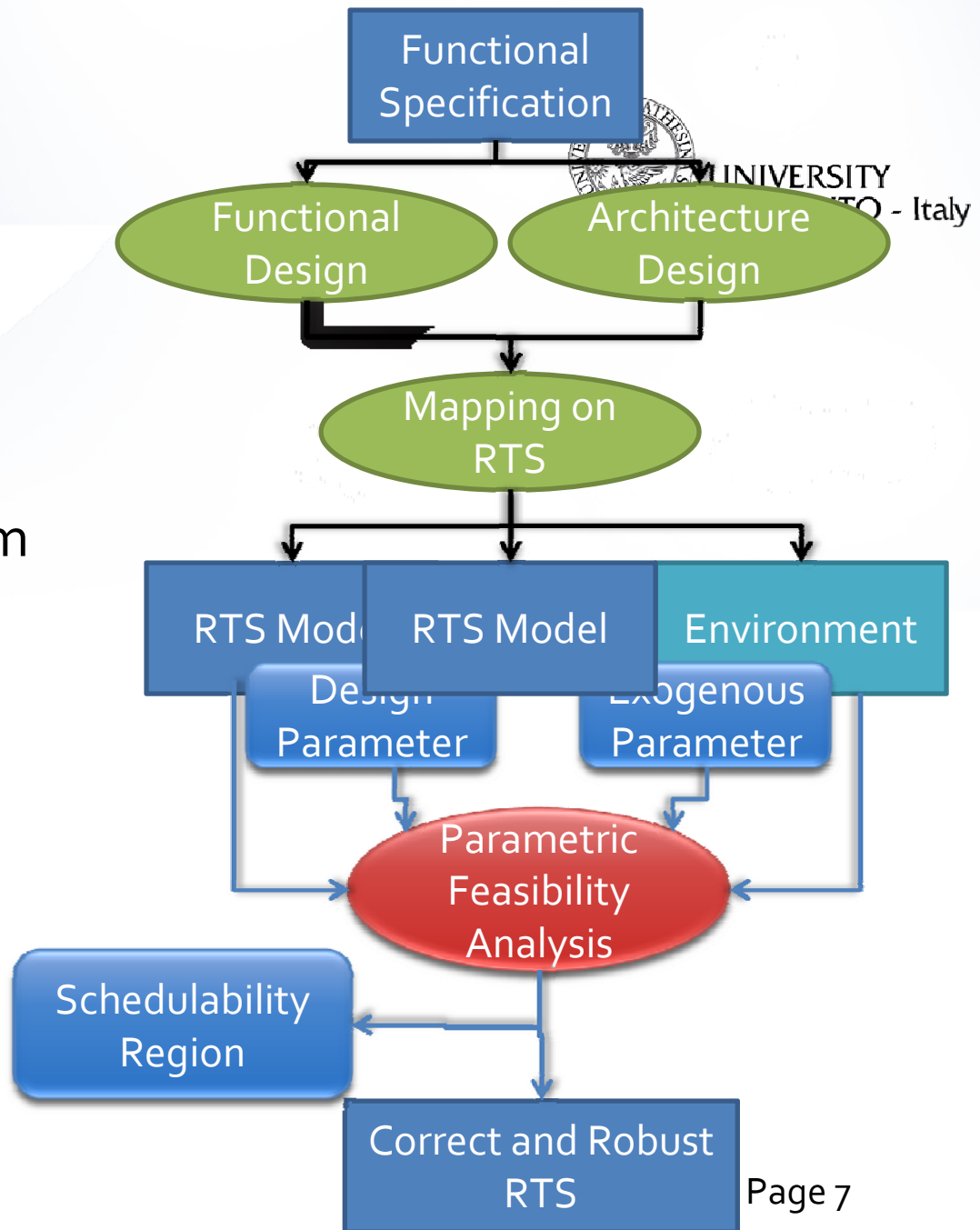
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- Safety consideration
- Manufacturing consideration
- Variability in environment and run-time

→ Need for formalization of  
design process

# Design Process

- Design & Modelling
  - Activation pattern
  - Timing properties
  - Scheduling Algorithm
- Robustness & Parameter Tuning
  - Assign values
  - Evaluate system robustness w.r.t to parameters





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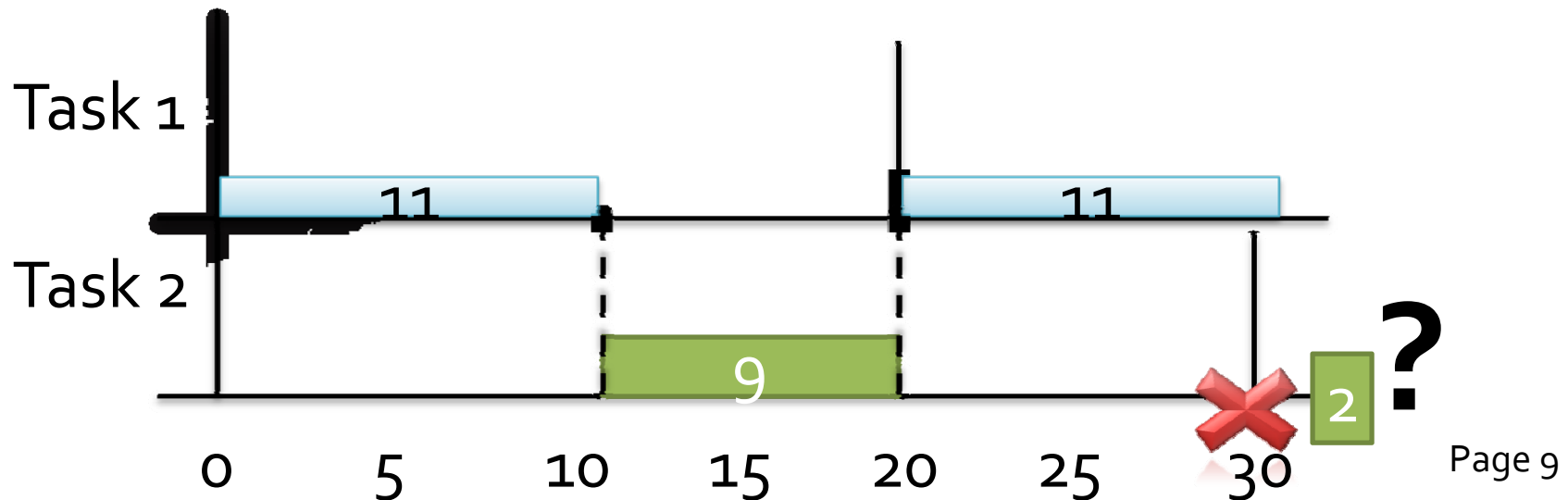
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# Sensitivity Analysis : Example Scenario



Parameters	Task 1	Task 2
Period	20	30
Deadline	20	30
Computation Time	11	12
Offset	0	0



# Sensitivity Analysis :

## Example Scenario Discussion #1

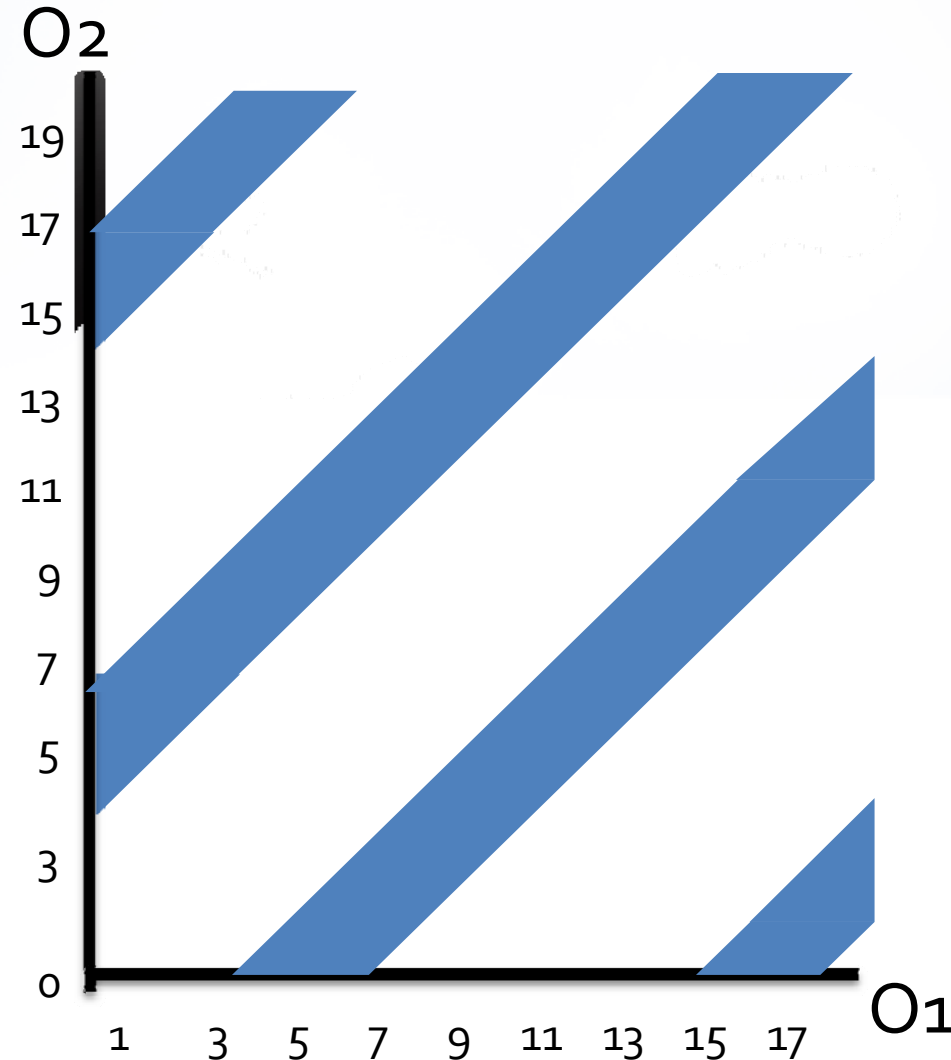


- Classical scheduling theory → **system failure**  
Task system is not schedulable
- Solution
  - Stronger machine (Hardware solution)
  - Tweaking offset..

# Sensitivity Analysis : Region of Schedulability on Offsets



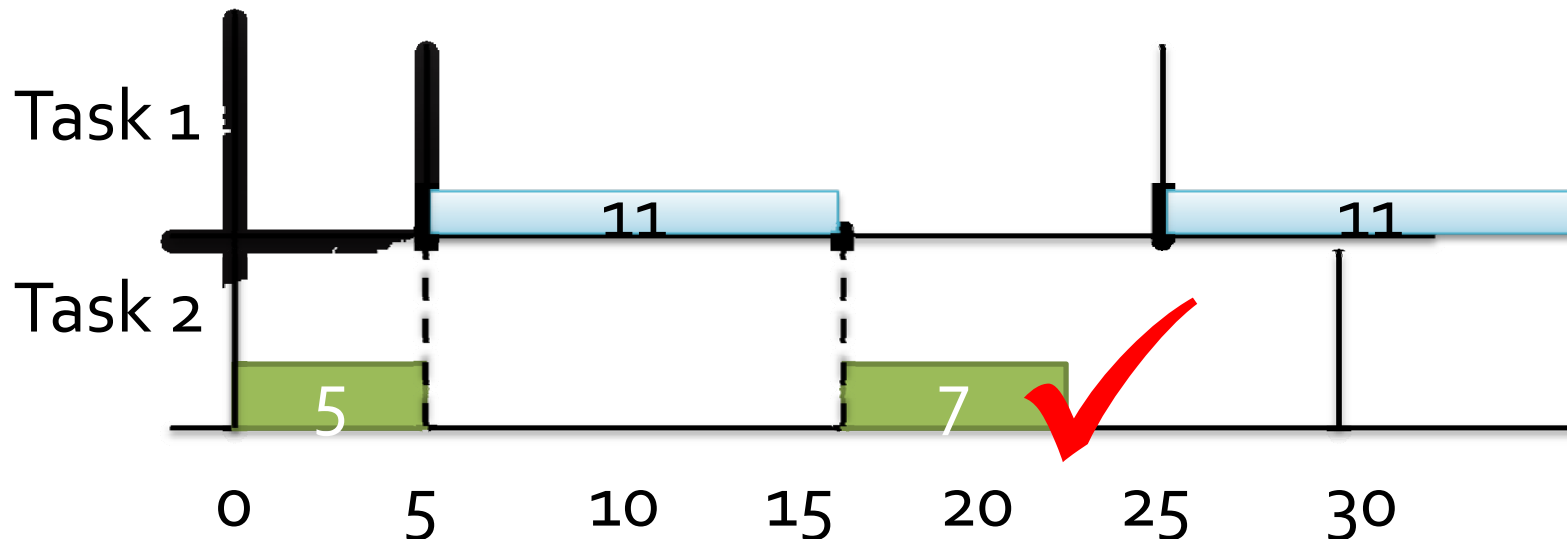
Parameters	Task 1	Task 2
Period	20	30
Deadline	20	30
Computation Time	11	12
Offset	?	?



# Sensitivity Analysis: Corrected Scenario



Parameters	Task 1	Task 2
Period	20	30
Deadline	20	30
Computation Time	11	12
<b>Offset</b>	<b>5</b>	<b>0</b>

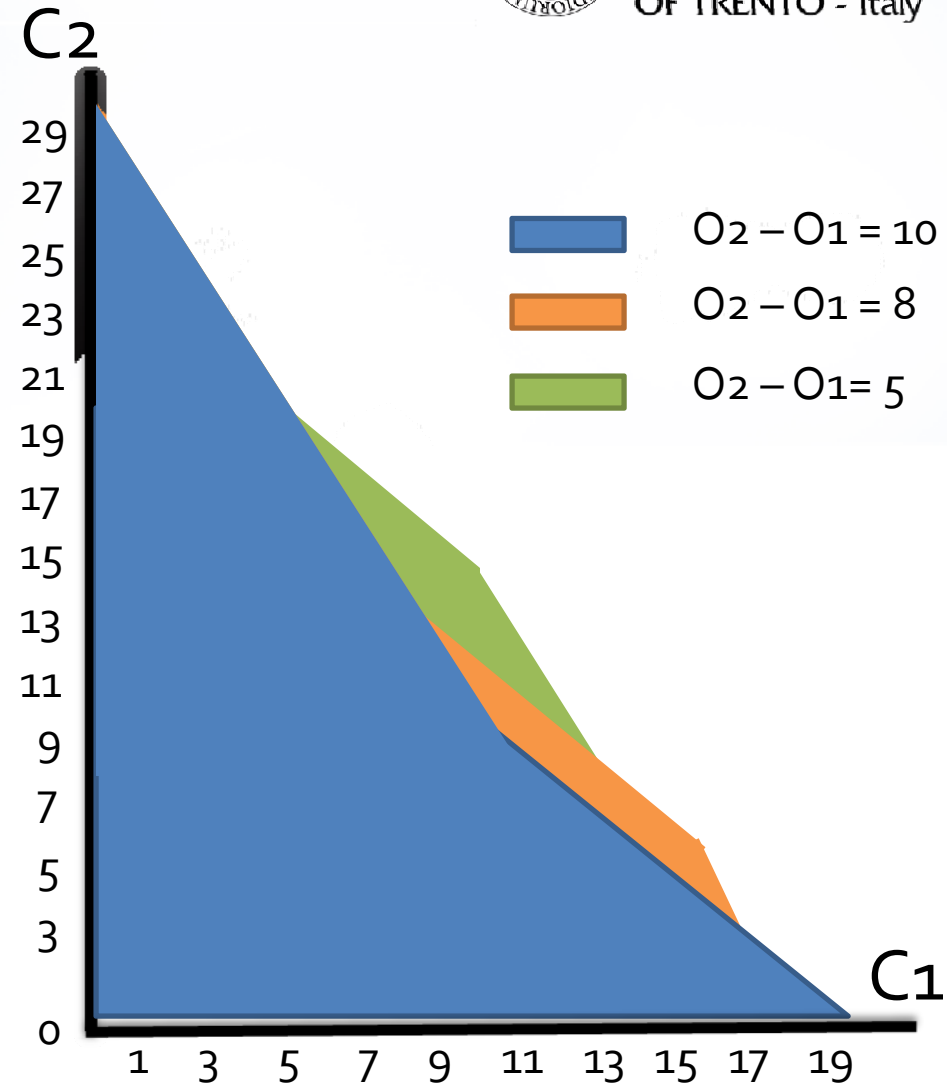


# Sensitivity Analysis: Example Scenario Discussion #2: Robustness



Parameters	Task 1	Task 2
Period	20	30
Deadline	20	30
Computation Time	?	?
Offset	5	?

System robustness = ?



# Sensitivity Analysis



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Requirement conclusion #1:

We want to find out :

the schedulability regions in the space of parameters

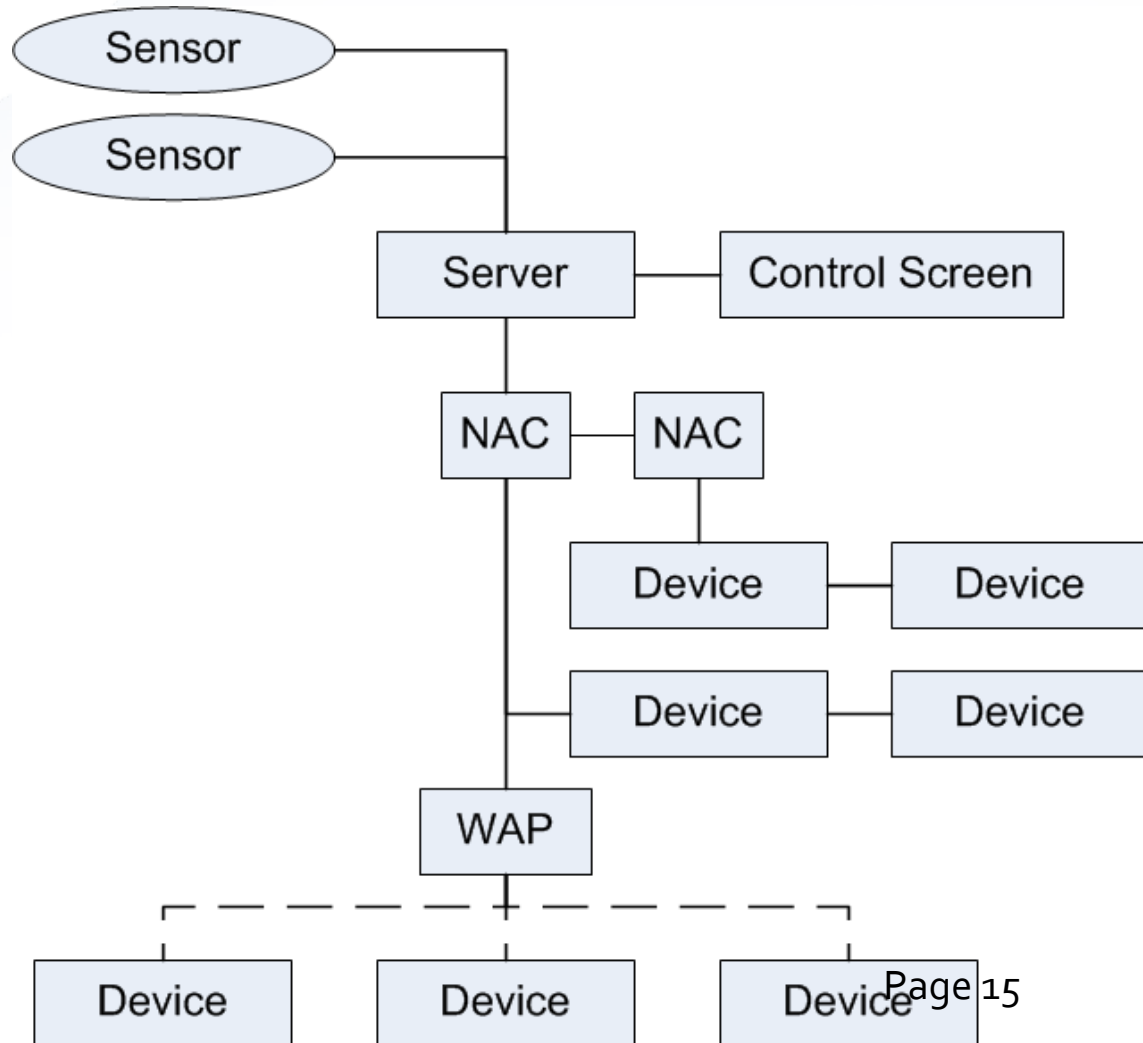
→ most robust design for our real-time systems

# Sensitivity Analysis:

## Example Scenario Discussion #3:



- System model not in classical RTS
- Examples:
  - System with buffers
  - Complex activation pattern
  - Heterogeneous, distributed system
  - Flexible deadline (e.g. Firm Deadline)





# Sensitivity Analysis



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- Requirement conclusion #2

Sensitivity analysis for **general** real-time system



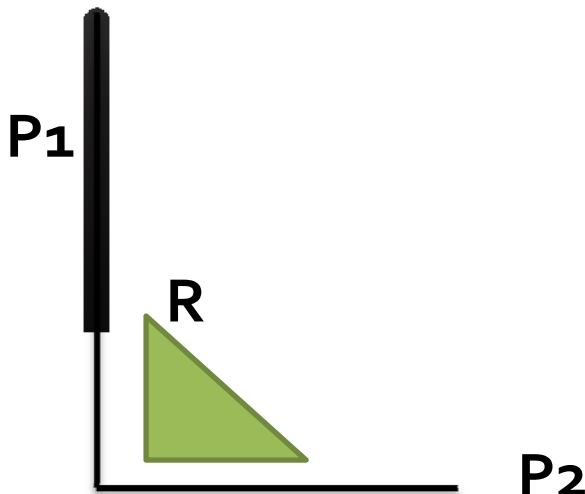
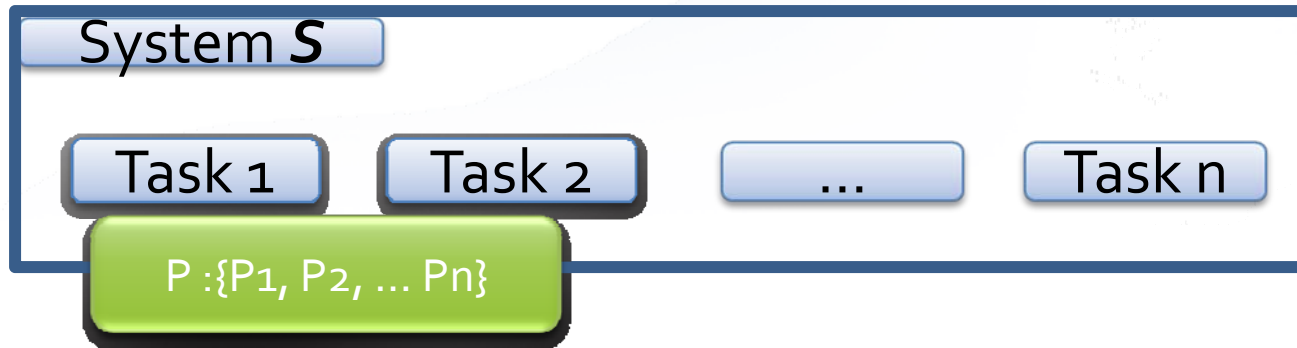
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# Sensitivity Analysis: Formal problem definition



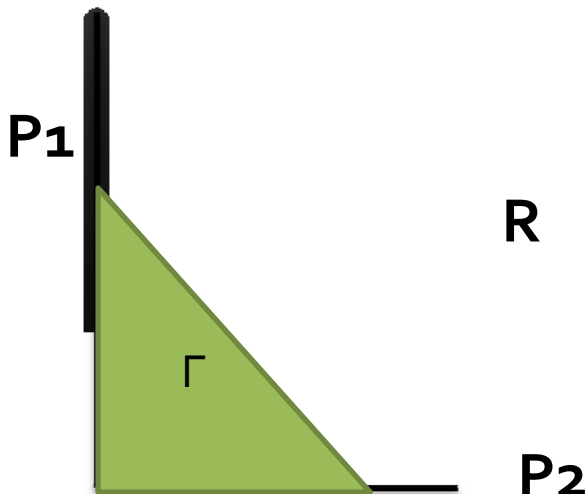
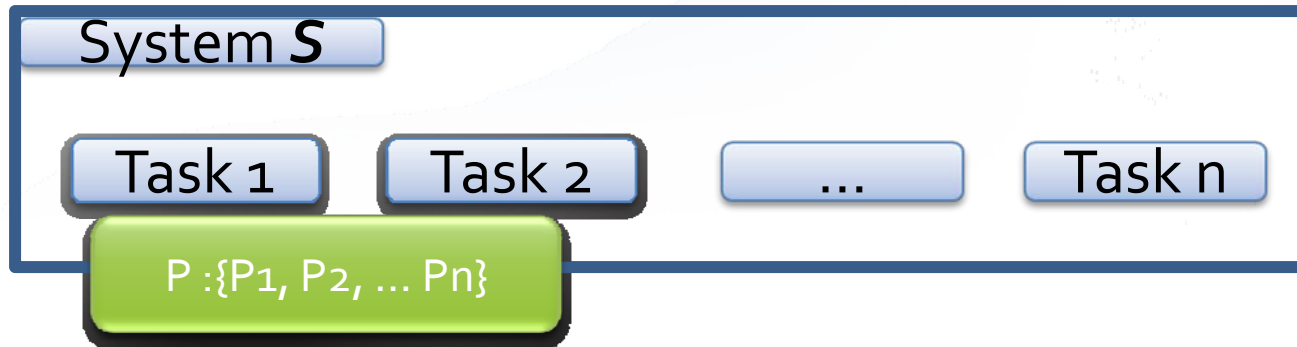
- **Problem 1:**



# Sensitivity Analysis: Formal problem definition



- Problem 2:



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# Parametric Timed Automata



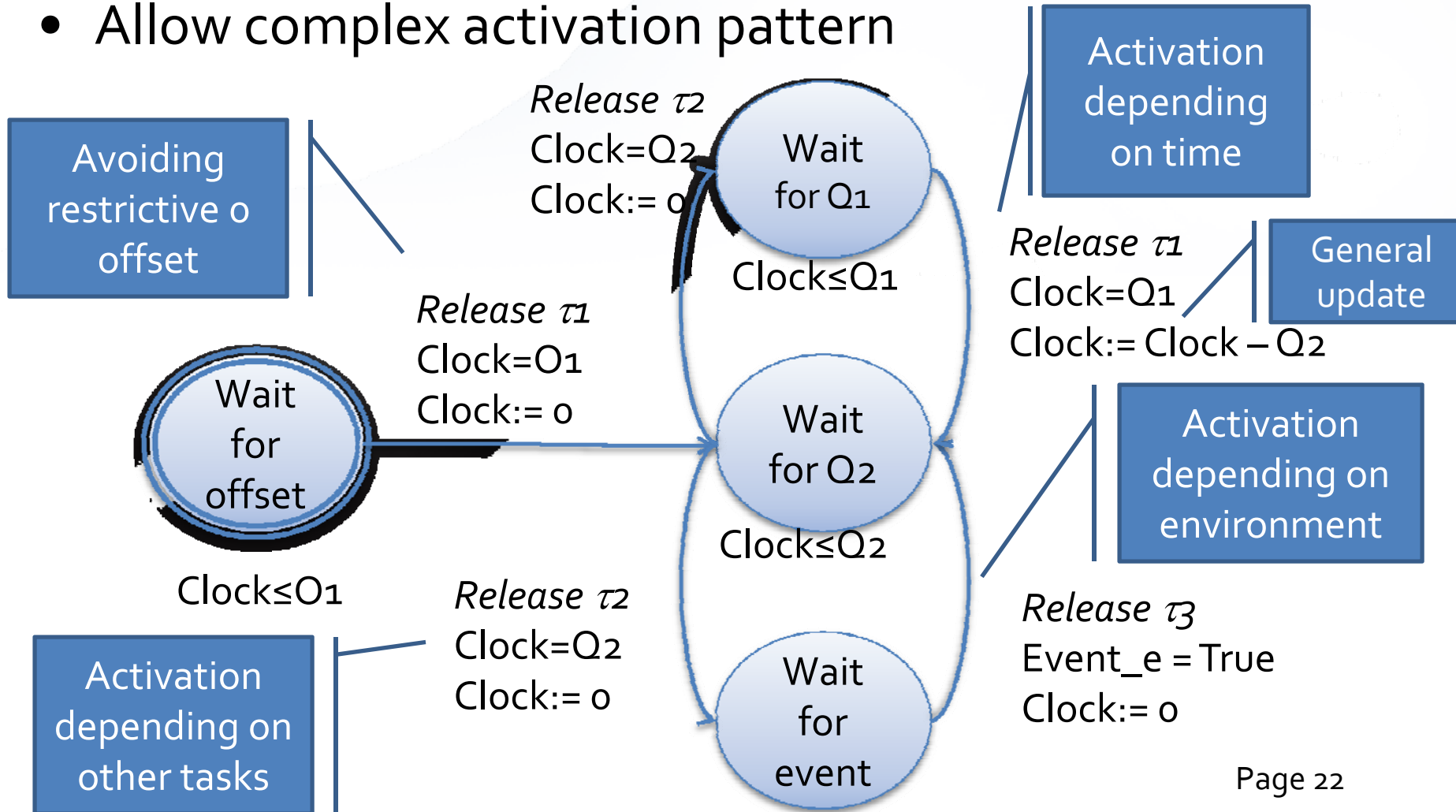
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- Timed automata with parameters extension
- Main differences:
  - Parameters
  - Auxiliary variables
  - General update statement

# Real Time System in PTA: Activation Pattern

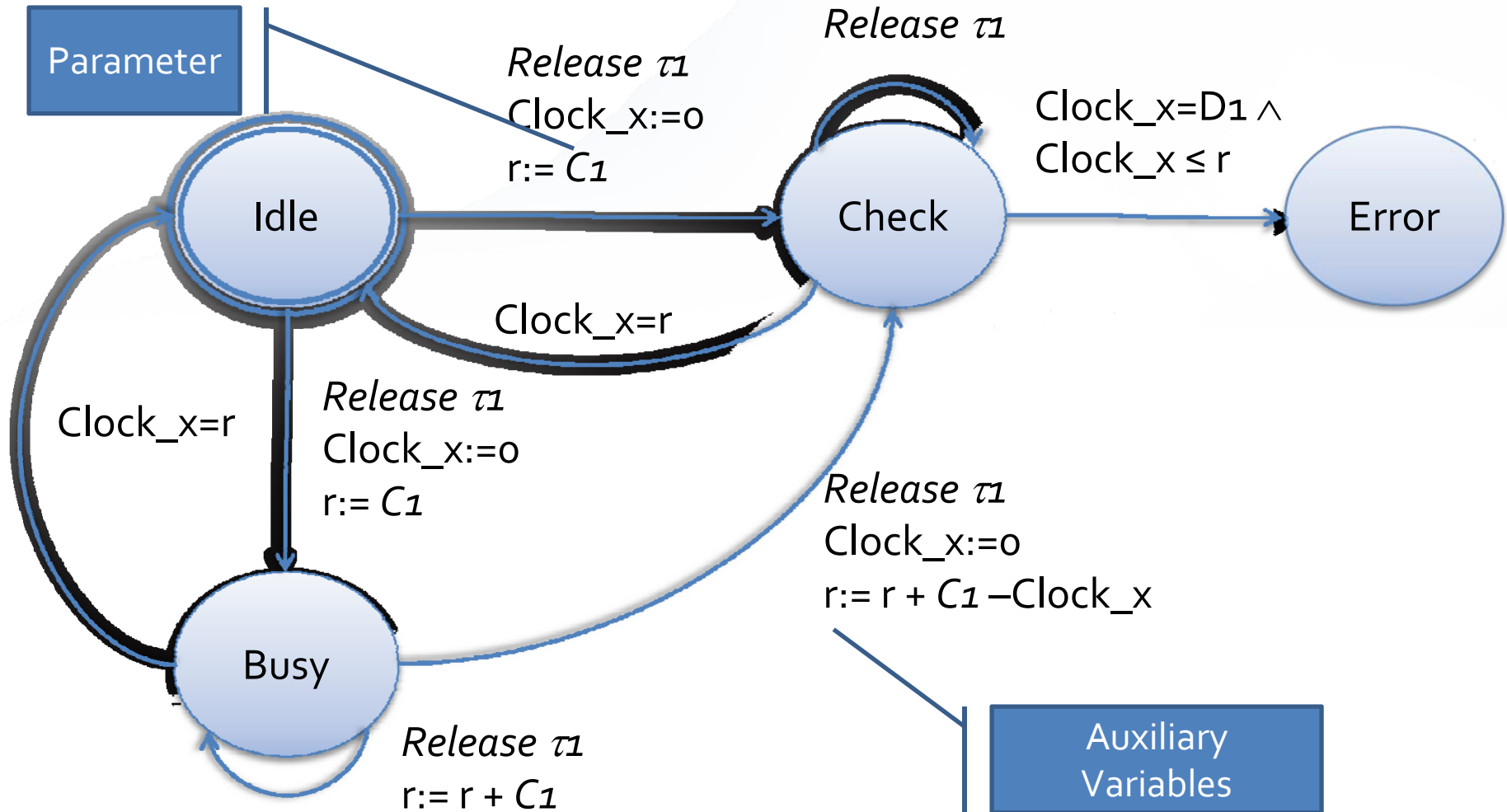


- Allow complex activation pattern





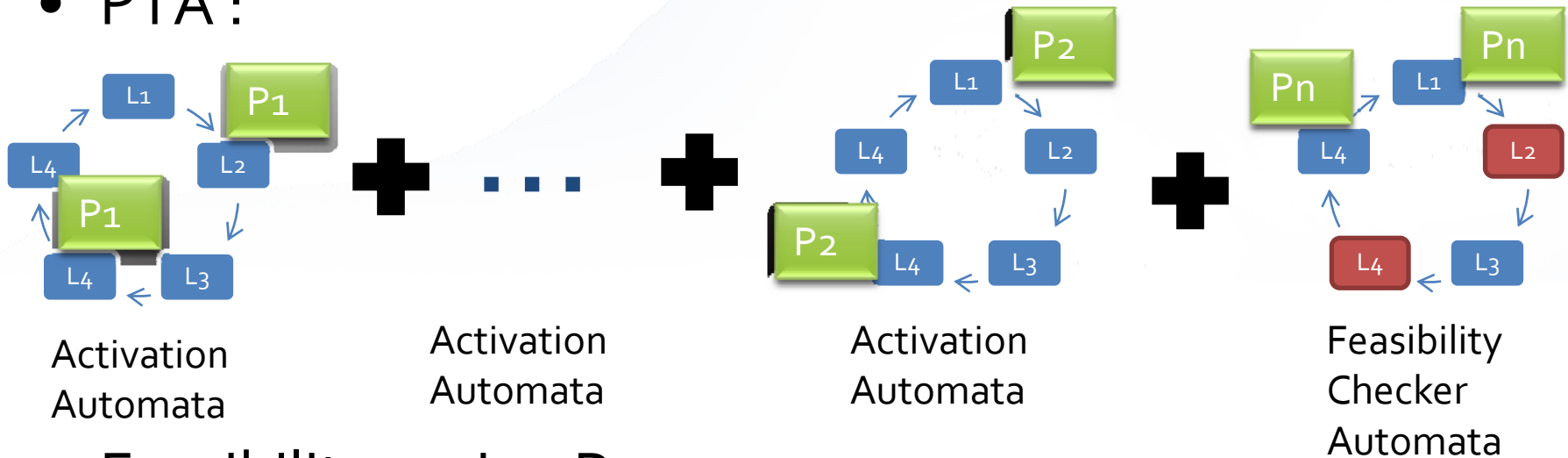
# Real Time System in PTA: Feasibility Checker



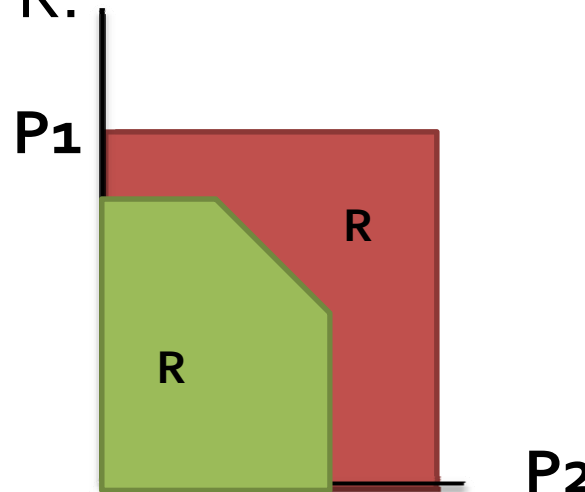


# Sensitivity Analysis via PTA

- PTA :



- Feasibility region R:



# Symbolic representation of PTA

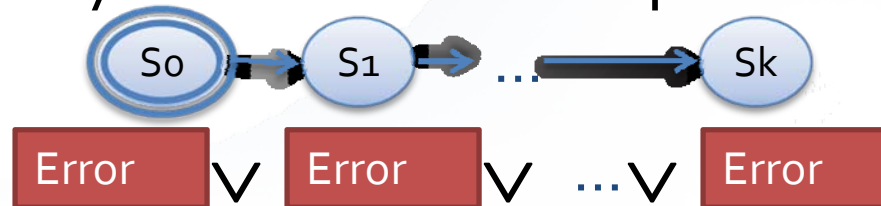


- Current state variables  $V$ :  
Discrete vars  $D$ : Location, transitions as *boolean*  
Continuous vars  $X$ : Clocks and other variables as *real*
- Symbolic model of PTA : set of constraints on boolean and real variables
- Examples:  
 $Loc_i \rightarrow x-y \leq O1$   
 $Trans_i \rightarrow (x \geq C1) \wedge (x' = C1+x) \wedge (y' = y)$

# Bounded Model Checking (BMC)



- Look only for counterexample made of  $k$ -states



- $BMC(k)$ :
- $I(V^0) \wedge R(V^0, V^1) \wedge \dots \wedge R(V^{k-1}, V^k) \wedge Error(V^k)$
- Completeness of the solution is not guaranteed
- Complementing method : inductive reasoning

# Presentation Outline

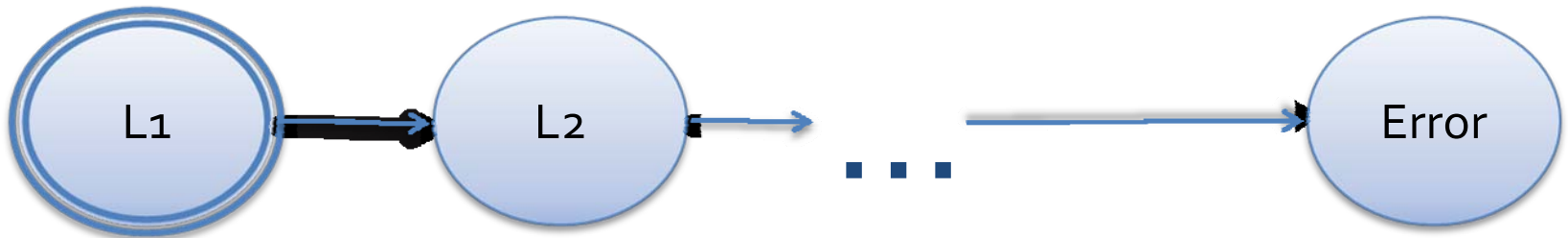


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# PTVP algorithm intuition: Search for an error trace



- Verification on reachability problem using BMC:  
An error trace for every found counterexample
- Alternatively, error trace can be searched via non-parametric model checker
- An error trace  $\pi$  :

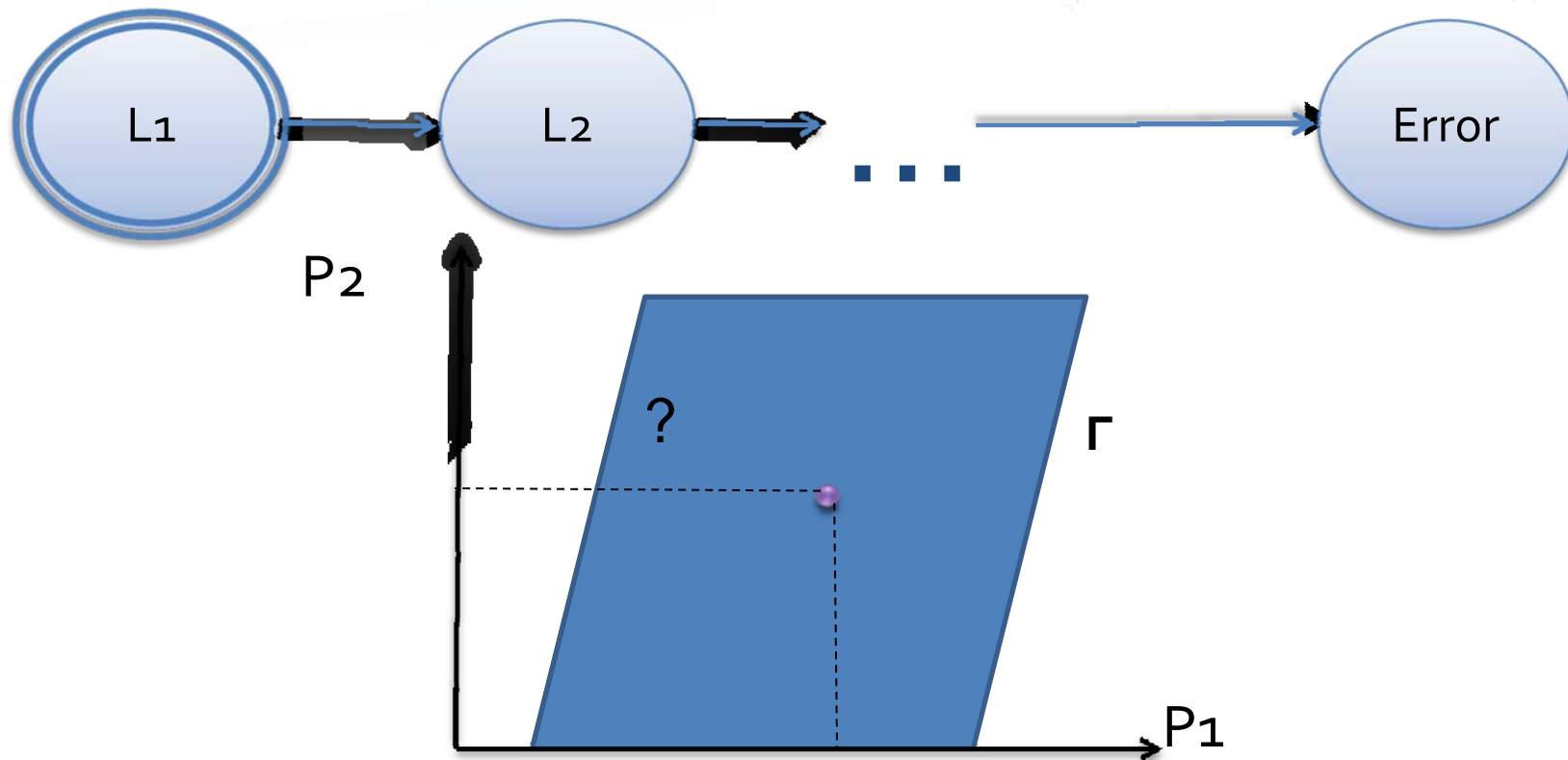


# PTVP algorithm intuition:

## An error trace $\pi$



- Along with the trace, an assignment for the parameters that validate the trace is produced



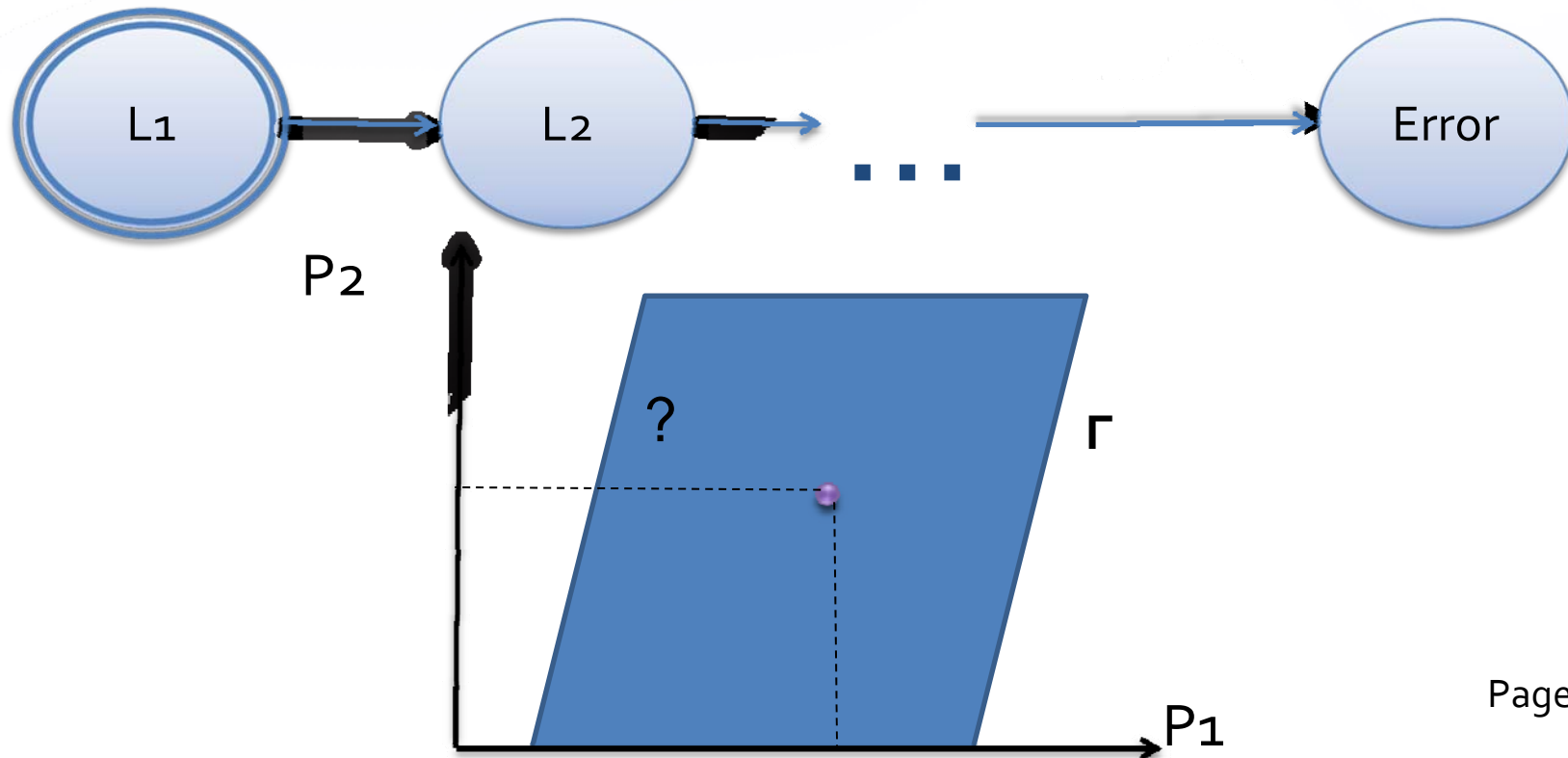


# PTVP algorithm intuition:

## Sensitivity analysis to an error trace



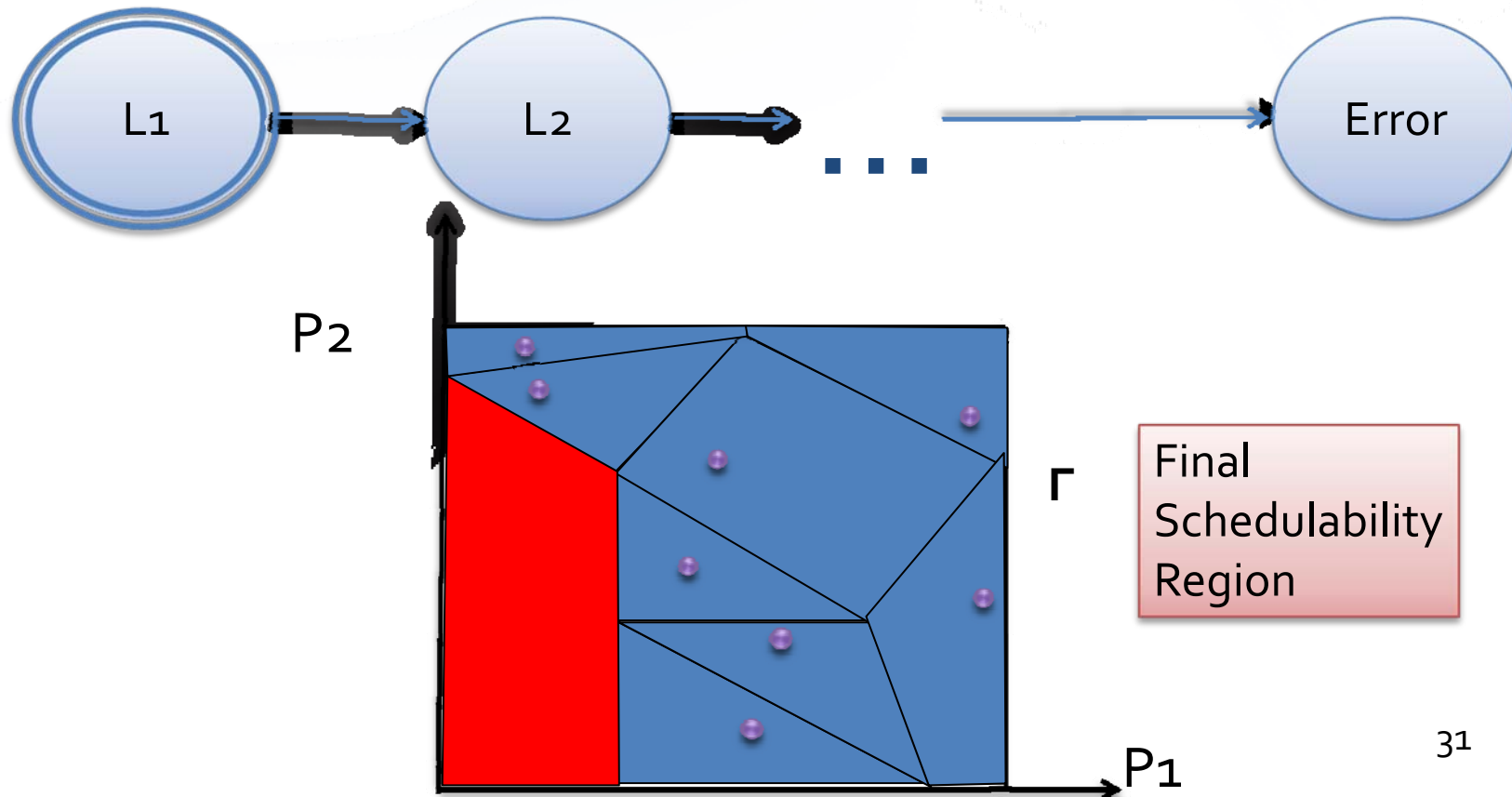
- By processing the trace, the surrounding region of parameters that make the trace true is identified
- ...And we rule this region out from the next search



# PTVP algorithm intuition: Schedulability region



- Feasibility region : found by iteratively bounding the parameter space from the unschedulability regions



# Parametric Verification of Temporal Properties (PVTP) Algorithm



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**Require:** PTA describing activations and scheduling of  $n$  tasks

**Ensure:** Schedulability Region

```
1: for  $i = 1$  to  $n$  do  
2:   PTA.init(ParamSchedProblemForTask( $i$ ))  
3:    $j = 0$   
4:   while PTA.reachable(Error) do  
5:     trace = PTA.get_trace()  
6:     Unfeasible[ $j$ ] = PTA.get_parameter(trace)  
7:     PTA.add_constraints( negate( Unfeasible[ $j$ ]))  
8:      $j++$   
9:   Feasible[ $i$ ] = not(big_or(0,  $j$ , Unfeasible))  
10: Return big_and(0,  $n$ , Feasible)
```

Error Trace Search

Sensitivity Analysis

Region Exclusion

Collection of regions



# Sensitivity Analysis

- Given Polyhedron in the space of clocks and parameters  $\text{Poly}\{P, X\}$
- Obtain  $\text{Poly}\{P\} \leftrightarrow \exists X, \text{Poly}\{P, X\}$

**→ Existential Quantifier  
Elimination**

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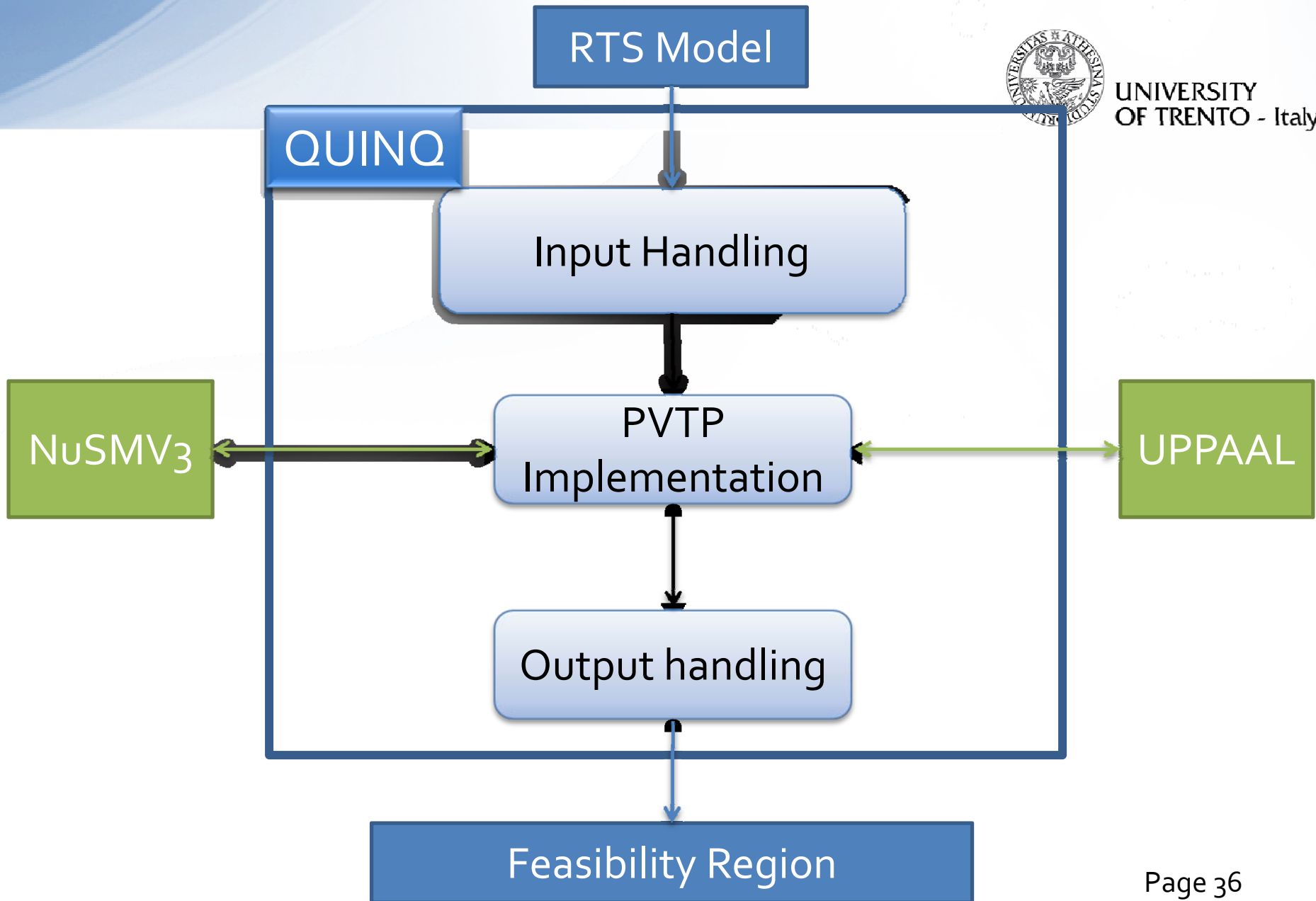
# Implementation in Quinq

- Based on NuSMV3 symbolic model checker with underlying MathSAT SMT solver
- Main functionalities :
  - Input handling
  - PVTP algorithm implementation
  - Completion check
  - Output handling
- Components
  - Sensitivity add-on
  - High level periodic system analysis
  - Search optimization
  - Model checker drivers
  - Graph generator
- Blackbox components : UPPAAL, JUNT, existelim

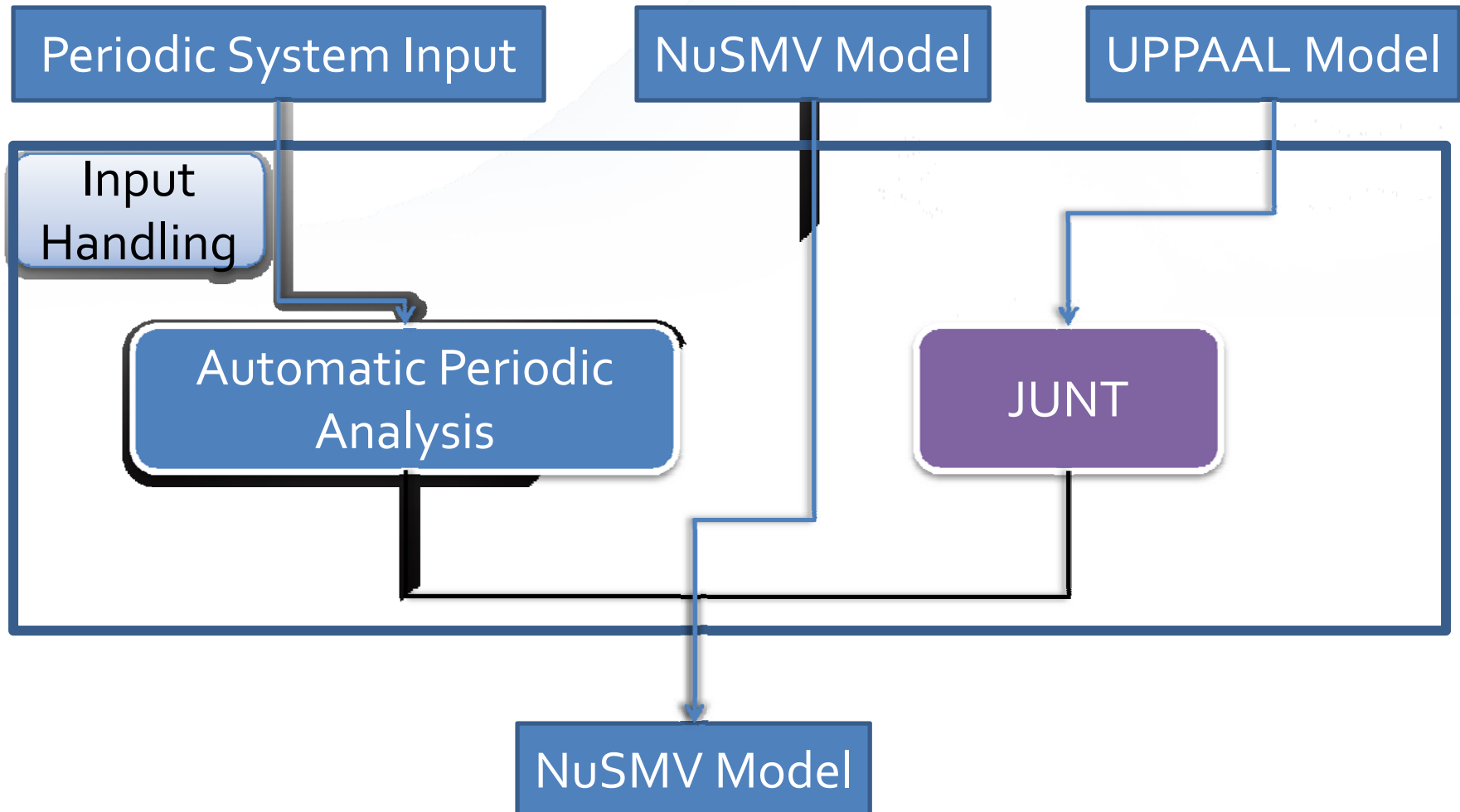
# Architecture



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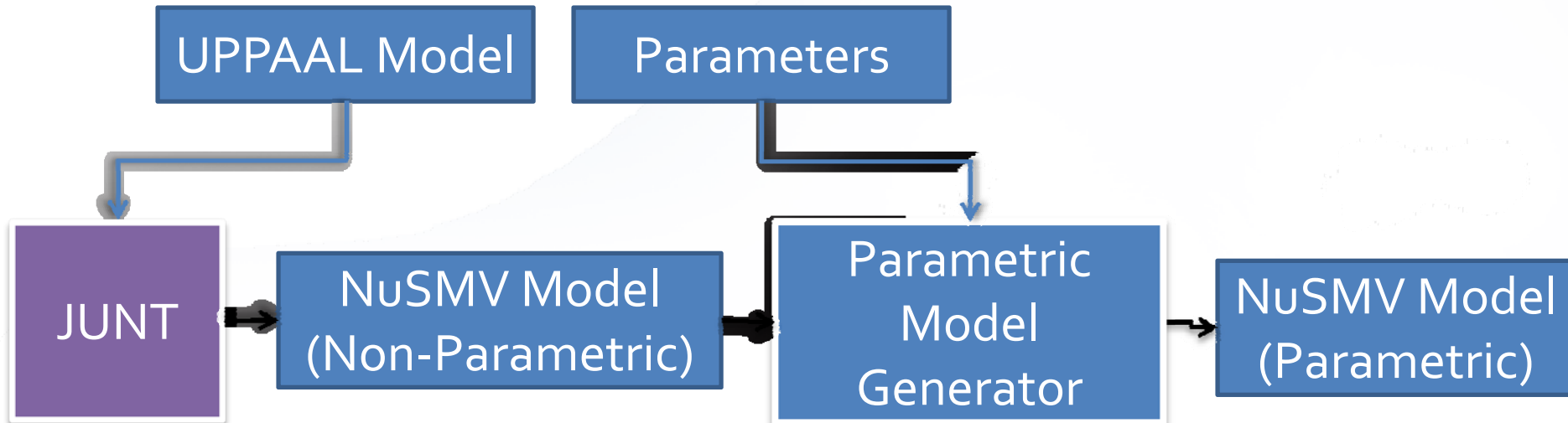
# Input Handling







# Input via UPPAAL Model

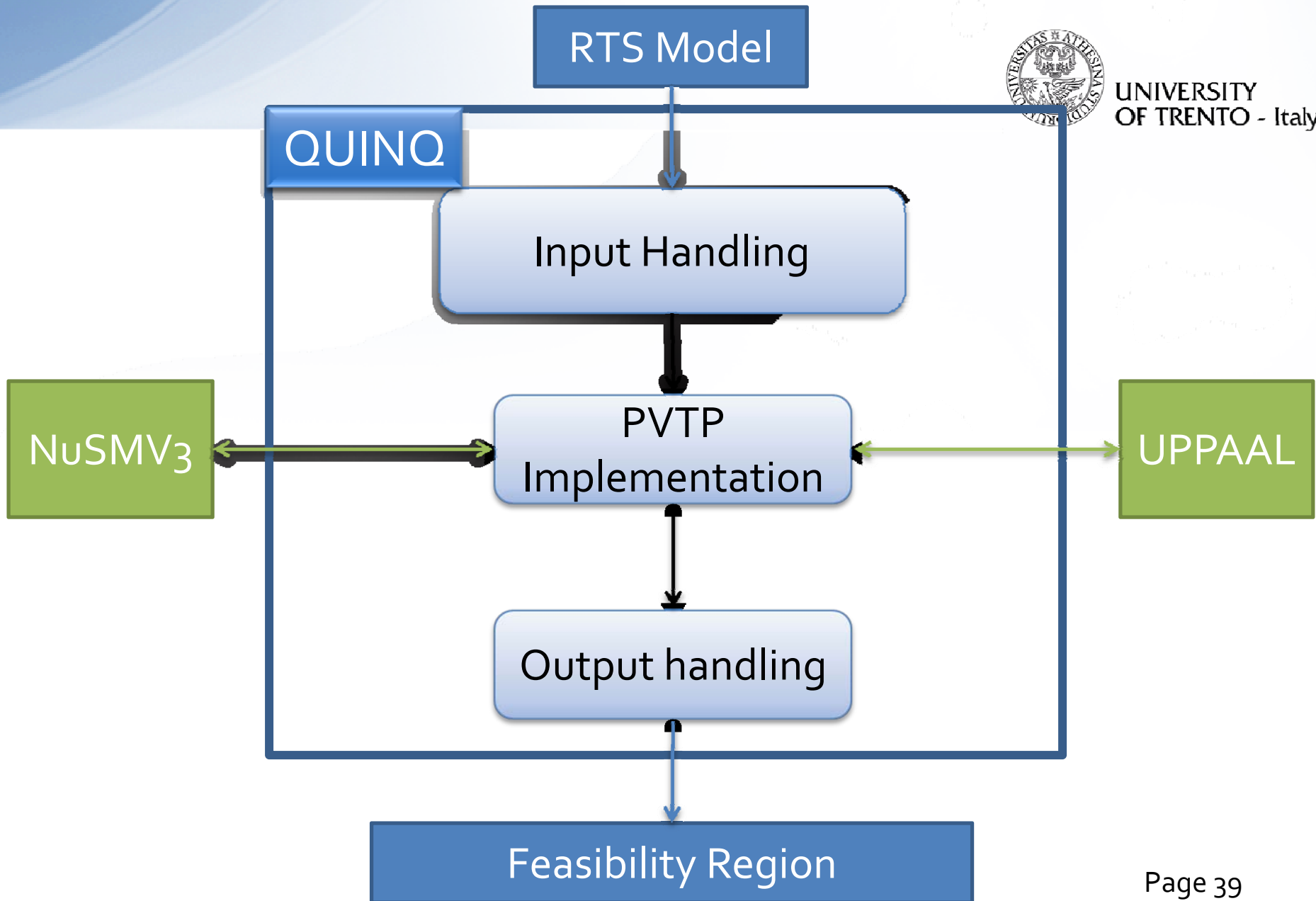


- Point of Considerations:
  - Integer vs Real domain
  - Array data structure
  - Clocks
  - Transition synchronization

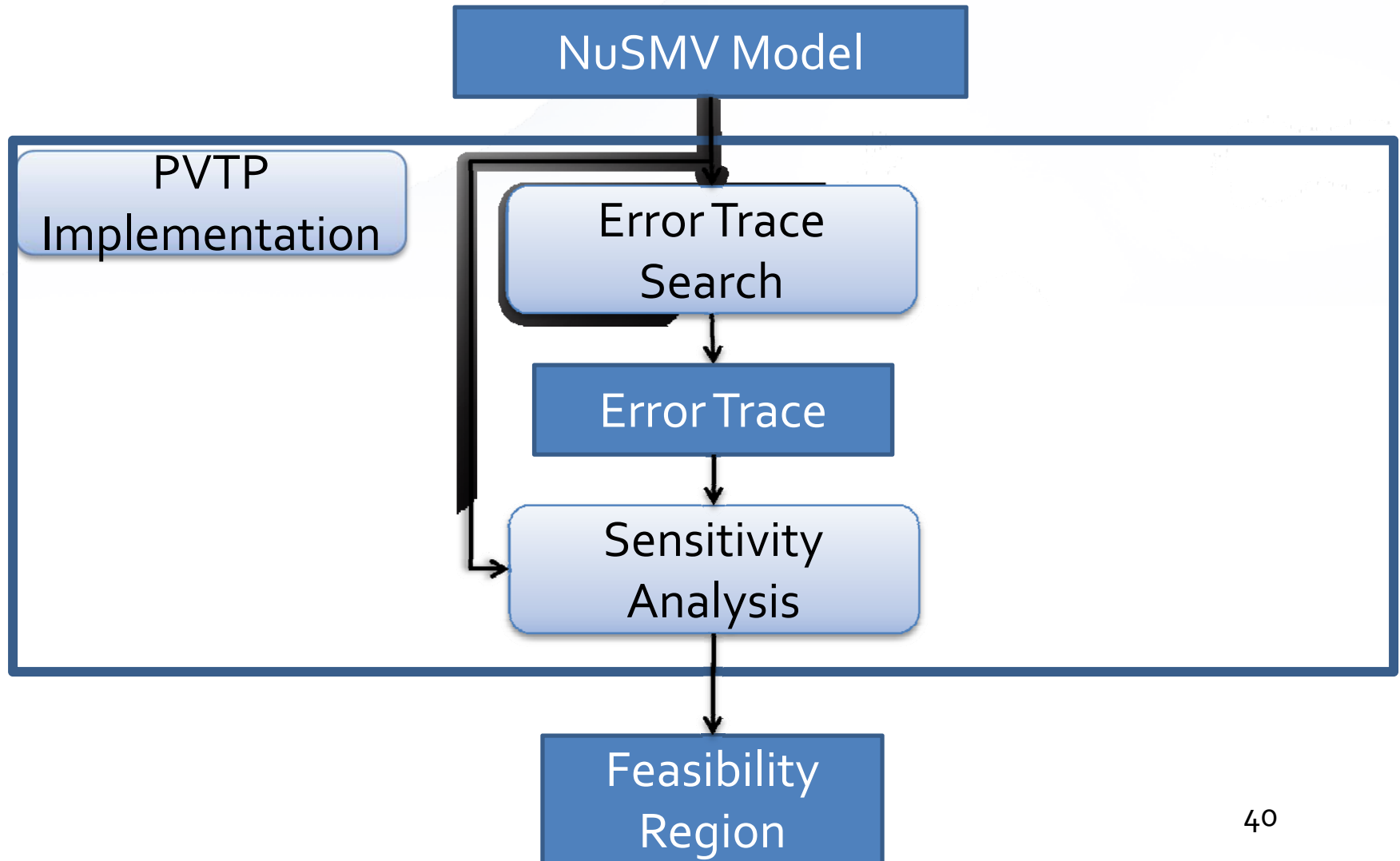
# Architecture



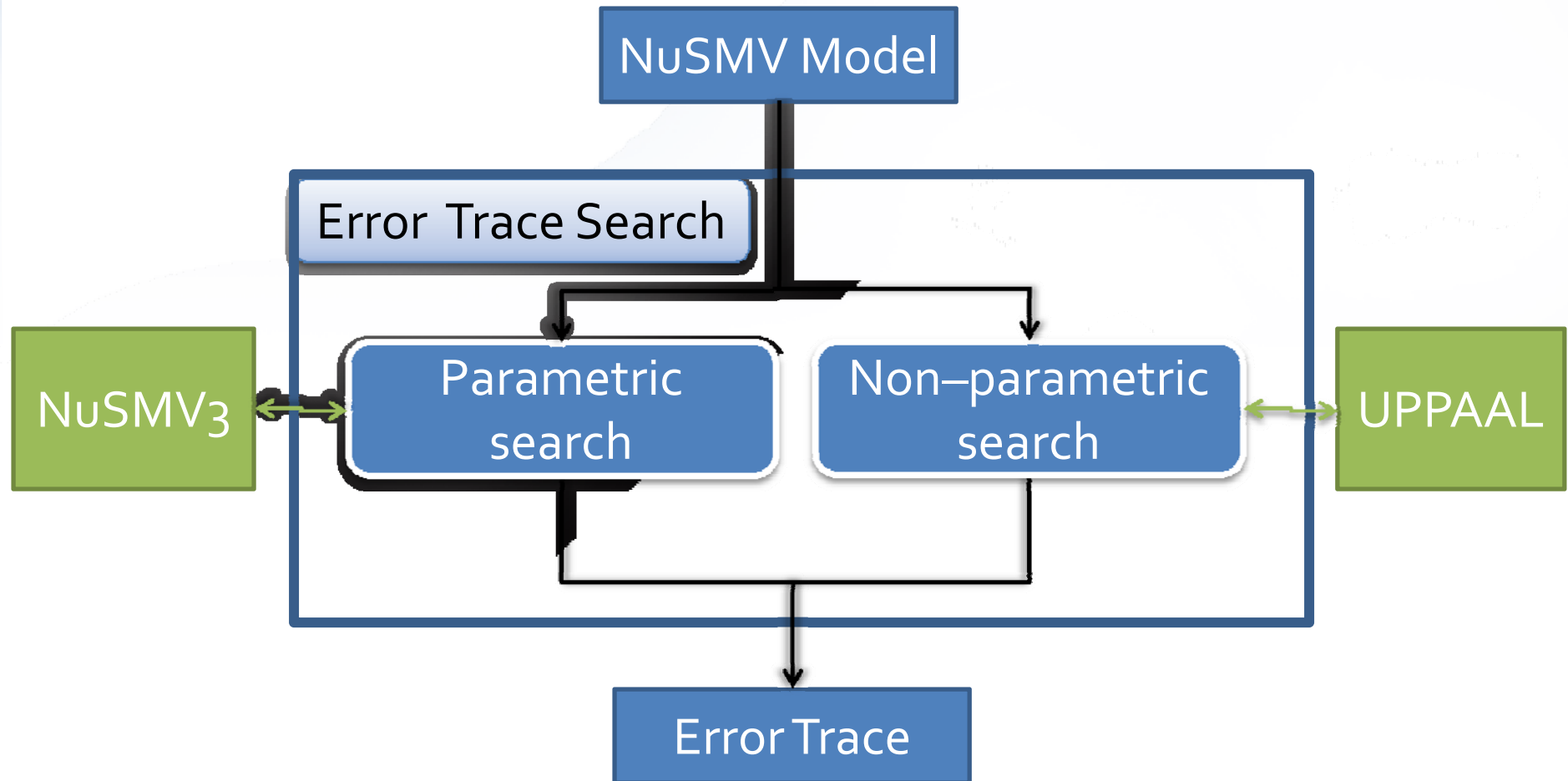
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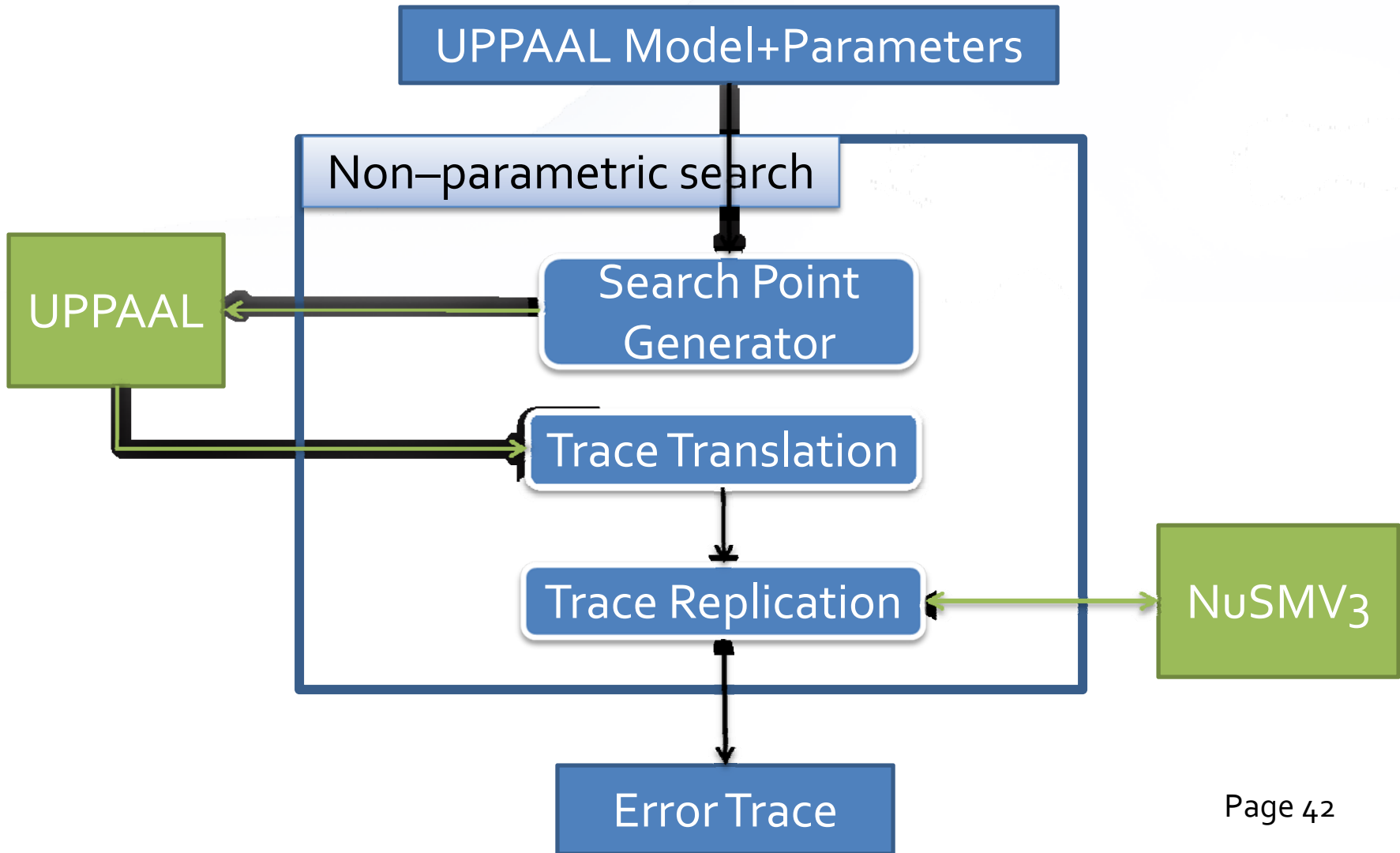
# Parametric Verification of Temporal Property Implementation



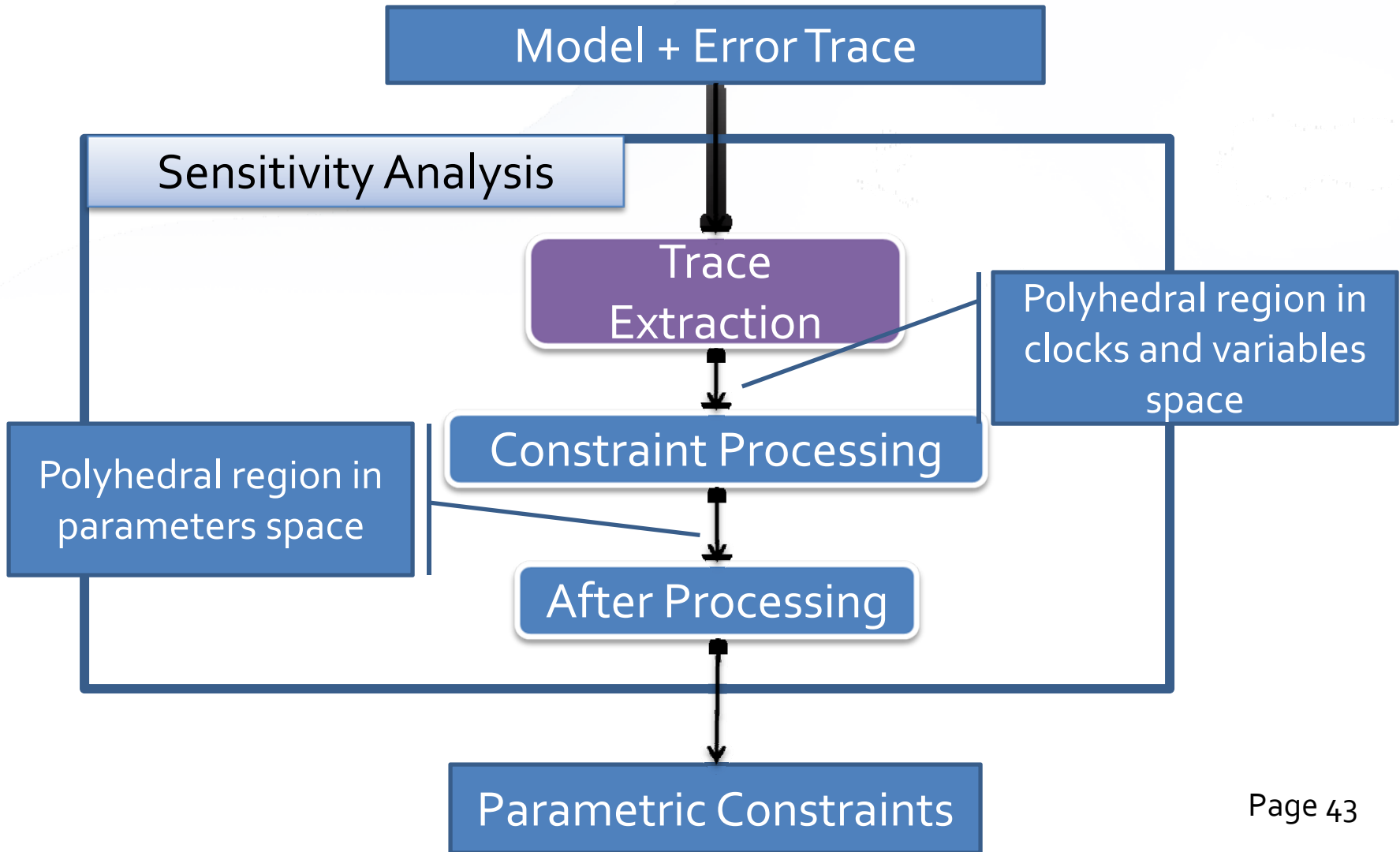
# PTVP Algorithm Implementation



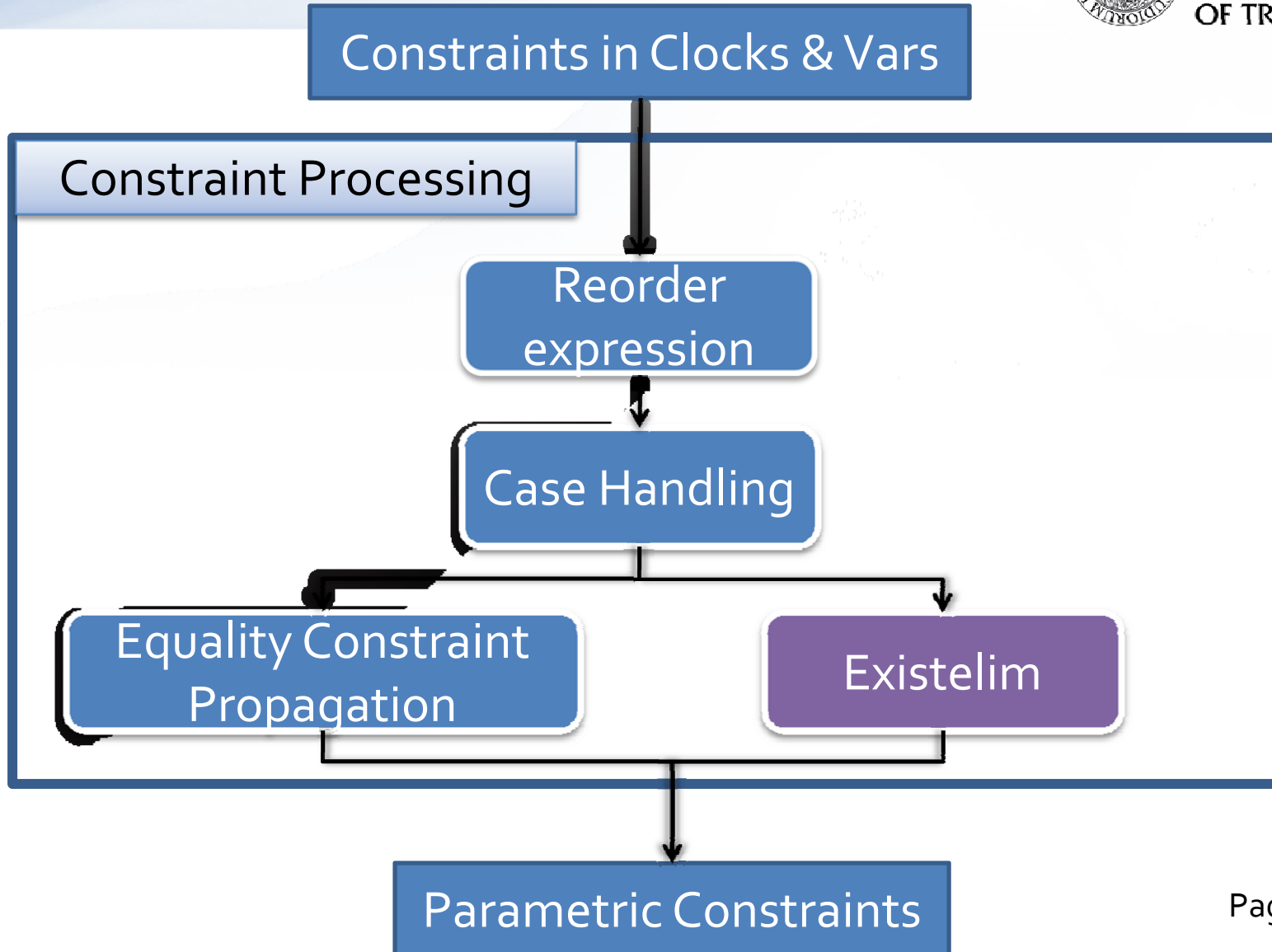
# Non-Parametric Search



# Sensitivity Analysis : Implementation



# Constraint Processing



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# DEMO

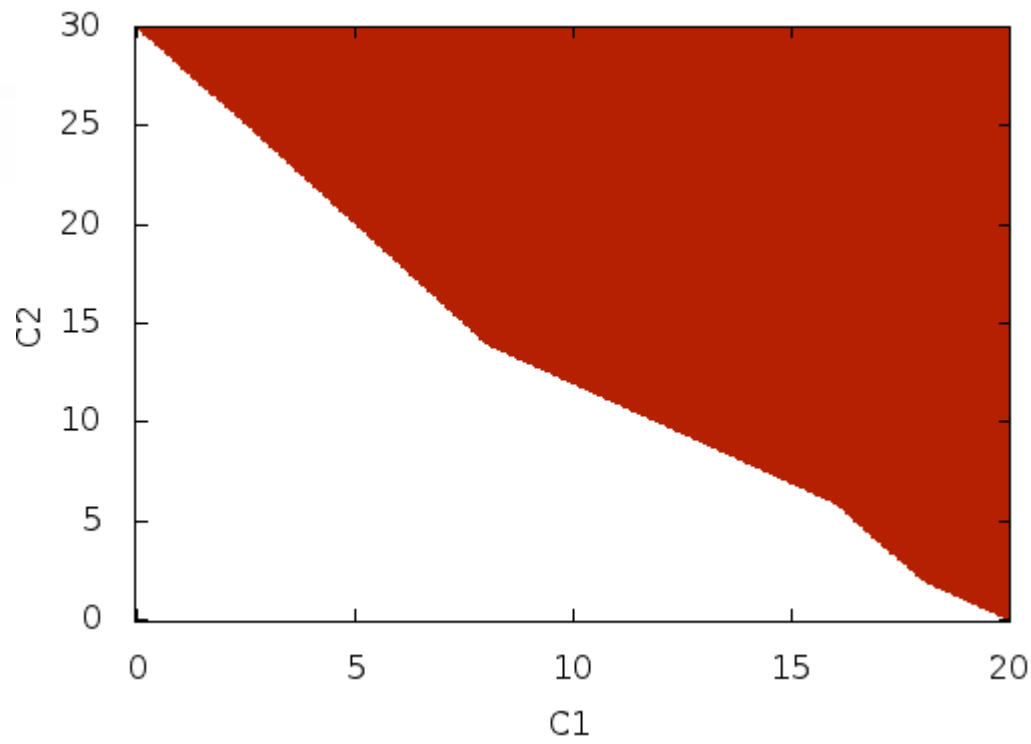


- Previously Illustrated example:  
2 periodic tasks system with 3 parameters
- $S = \{\text{task}_1, \text{task}_2\}$
- Periodic tasks :  $T_1=D_1=20, T_2=D_2=30$
- Offset :  $O_1 = 0$
- Parameters :
  - Computation time:  $C_1, C_2$
  - Offset :  $O_2$

# Demo:result



- $\text{Offset}_2 = 8$





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# Sensitivity Analysis Tools



Tool	Flexible RTS	Expression	System known apriori	Inference Point	Number of Parameters	Feasibility Region
Bini	-	-	-	-	No limit	✓
MAST	-	-	-	✓	-	-
SMART	✓	*	✓	-	No limit*	✓
IMITATOR	✓	*	-	✓	2	✓
QUINQ	✓	✓	-	-	No limit*	✓

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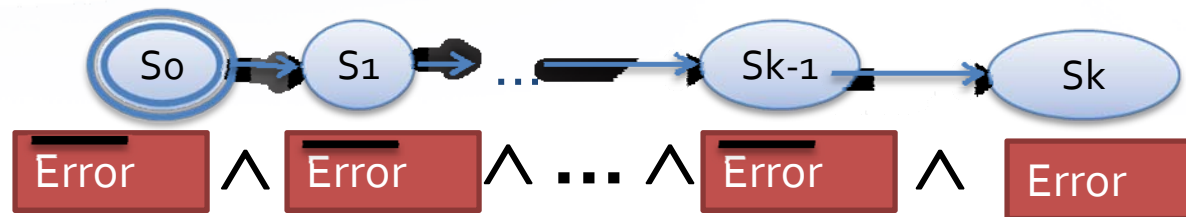
# Conclusion

- PTA representation → flexible activation pattern ,  
general RTS presentation
- PVTP algorithm → General method to obtain  
feasibility region
- Implemented in Quinq with applications on some  
example cases
- Edge on comparison with other tools:
  - Flexible RTS representation
  - No reference point input needed
  - Whole region of schedulability result



# K-Induction

- Does there exist  $k$  such that the following formula is unsatisfiable



- if *unsatisfiable* and  $\text{BMC}(k)$  *unsatisfiable* then error state **unreachable**