UNIVERSITY
OF TRENTO

DEPARTMENT OF INFORMATION AND COMMUNICATION TECHNOLOGY

38050 Povo – Trento (Italy), Via Sommarive 14
http://www.dit.unitn.it

REASONING WITH GOAL MODELS

Paolo Giorgini, Eleonora Nicchiarelli,
John Mylopoulous and Roberto Sebastiani

2002

Technical Report # DIT-02-0043

Also: In Proc. "Int. Conference of Conceptual Modeling -- ER2002"
Tampere, Finland, October 2002. LNCS series, © Springer Verlag.

Reasoning with Goal Models*

Paolo Giorgini[1], John Mylopoulos[2], Eleonora Nicchiarelli[1], and Roberto Sebastiani[1]

[1] Department of Information and Communication Technology
University of Trento - Italy
{pgiorgini,eleonora,rseba}@science.unitn.it
[2] Computer Science Department - University of Toronto - Canada
jm@cs.toronto.edu

Abstract. Over the past decade, goal models have been used in Computer Science in order to represent software requirements, business objectives and design qualities. Such models extend traditional AI planning techniques for representing goals by allowing for partially defined and possibly inconsistent goals. This paper presents a formal framework for reasoning with such goal models. In particular, the paper proposes a qualitative and a numerical axiomatization for goal modeling primitives and introduces label propagation algorithms that are shown to be sound and complete with respect to their respective axiomatizations. In addition, the paper reports on preliminary experimental results on the propagation algorithms applied to a goal model for a US car manufacturer.

1 Introduction

The concept of goal has been used in many areas of Computer Science for quite some time. In AI, goals have been used in planning to describe desirable states of the world since the 60s [5]. More recently, goals have been used in Software Engineering to model early requirements [2] and non-functional requirements [4] for a software system. For instance, an early requirement for a library information system might be "*Every book request will eventually be fulfilled*", while "*The new system will be highly reliable*" is an example of a non-functional requirement. Goals are also useful for knowledge management systems which focus on strategic knowledge, e.g., "*Increase profits*", or "*Become the largest auto manufacturer in North America*" [3].

Traditional goal analysis consists of decomposing goals into subgoals through an AND- or OR-decomposition. If goal G is AND-decomposed (respectively, OR-decomposed) into subgoals $G_1, G_2, \ldots, G_n$, then if all (at least one) of the subgoals are satisfied so is goal G. Given a goal model consisting of goals and AND/OR relationships among them, and a set of initial labels for some nodes

* We would like to thank Greg McArthur for sharing with us a version of his goal model for car manufacturing. We also thank the anonymous reviewers and Greg McArthur for helpful feedback on earlier drafts of this paper.

of the graph (S for Satisfied, D for Denied) there is a simple label propagation algorithm which can generate labels for all other nodes of the graph [6].

Unfortunately, this simple framework for modeling and analyzing goals won't work for many domains where goals are not formalizable and the relationships among them can't be captured by semantically well-defined relations such as AND/OR ones. For example, goals such as "*Highly reliable system*" have no formally defined predicate which prescribes their meaning, though one may want to define necessary conditions for such a goal to be satisfied. Moreover, such a goal may be related to other goals, such as "*Thoroughly debugged system*", "*Thoroughly tested system*" in the sense that the latter obviously contribute to the satisfaction of the former, but this contribution is partial and qualitative. In other words, if the latter goals are satisfied, they certainly contribute to the satisfaction of the former goal, but don't guarantee its satisfaction. The framework will also not work in situations where there are contradictory contributions to a goal. For instance, we may want to allow for multiple decompositions of a goal G into sets of subgoals, where some decompositions suggest satisfaction of G while others suggest denial.

We are interested in a modeling framework for goals which allows for other, more qualitative goal relationships and can accommodate contradictory situations. We accomplish this by introducing goal relationships labelled "+" and "–" which model respectively a situation where a goal contributes positively or negatively towards the satisfaction of another goal.

A major problem that arises from this extension is giving a precise semantics to the new goal relationships. This is accomplished in two different ways in this paper. In section 3 we offer a qualitative formalization and a label propagation algorithm which is shown to be sound and complete with respect to the formalization. In section 4, we offer a quantitative semantics for the new relationships which is based on a probabilistic model, and a label propagation algorithm that is also shown to be sound and complete. Section 5 presents preliminary experimental results on our label propagation algorithms, while section 6 summarizes the contributions of the paper and sketches directions for further research.

2 An Example

Suppose we are modeling the strategic objectives of a US car manufacturer, such as Ford or GM. Examples of such objectives are increase return on investment or increase customer loyalty. Objectives can be represented as goals, and can be analyzed using goal relationships such as AND, OR, "+" and "−". In addition, we will use "++" (respectively "−−") a binary goal relationship such that if ++(G,G') (−−(G,G')) then satisfaction of G implies satisfaction (denial) of G'.

For instance, increase return on investment may be AND-decomposed into increase sales volume and increase profit per vehicle. Likewise, increase sales volume might be OR-decomposed into increase consumer appeal and expand markets. This decomposition and refinement of goals can continue until we have goals that are tangible (i.e., someone can satisfy them through an appropriate course

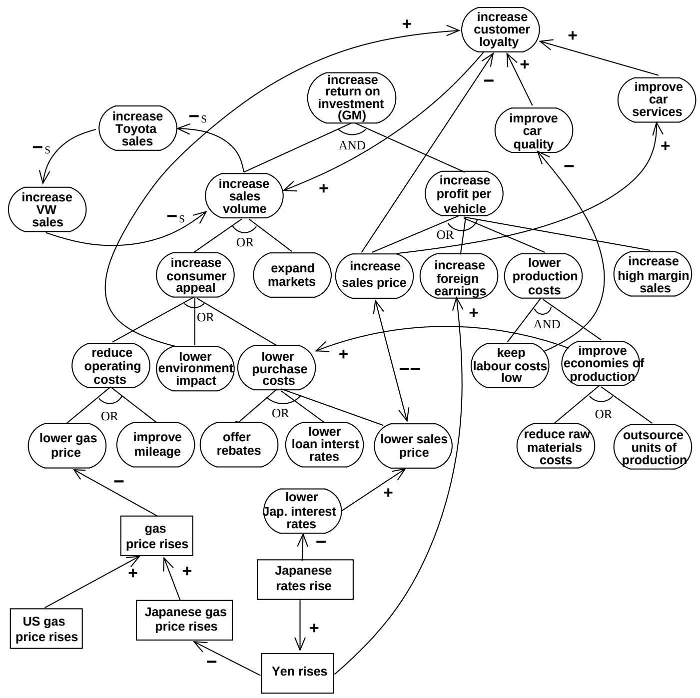

Fig. 1. A partial goal model for GM.

of action) and/or observable (i.e., they can be confirmed satisfied/denied by simply observing the application domain).

For vaguely stated goals, such as increase customer loyalty we may want to simply model other relevant goals, such as improve car quality, improve car services and relate them through "+" and "−" relationships, as shown in Figure 1. These goals may influence positively or negatively some of the goals that have already been introduced during the analysis of the goal increase return on investment. Such lateral goal relationships may introduce cycles in our goal models.

Examples of observable goals are Yen rises, gas prices rise etc. When such a goal is satisfied, we will call it an *event* (the kind of event you may read about in a news story) and represent it in our graphical notation as a rectangle (see lower portion of Figure 1).

Figure 1 shows a partial and fictitious goal model for GM focusing on the goal increase return of investment. In order to increase return of investment, GM has to satisfy both goals increase sales and increase profit per vehicle. In turn, increase sales volume is OR-decomposed into increase consumer appeal and expand markets, while the goal increase profit per vehicle is OR-decomposed into increase sales price, lower production costs, increase foreign earnings, and increase high margin sales. Additional decompositions are shown in the figure. For instance, the goal increase consumer appeal can be satisfied by satisfying lower environment impact, trying to lower purchase costs, or reducing the vehicle operating costs (reduce operating costs).

The graph shows also lateral relationships among goals. For example, the goal increase customer loyalty has positive (+) contributions from goals lower environment impact, improve car quality and improve car services, while it has a negative (−) contribution from increase sales price. The root goal increase return on investment (GM) is also related with goals concerning others auto manufacturer, such as Toyota and VW. In particular, if GM increases sales, then Toyota loses a share of the North American market; if Toyota increases sales increase Toyota sales), it does so at the expense of VW; finally, if VW increases sales (increase VW sales), it does so at the expense of GM.

So far, we have assumed that every goal relationship treats S and D in a dual fashion. For instance, if we have +(G,G'), then if G is satisfied, G' is partially satisfied, and (dually) if G is denied G' is partially denied. Note however, that sometimes a goal relationship only applies for S (or D). In particular, the – contribution from increase sales (GM) to increase sales (Toyota) only applies when increase sales (GM) is satisfied (if GM hasn't increased sales, this doesn't mean that Toyota has.) To capture this kind of relationship, we introduce $-_S$, $-_D$, $+_S$, $+_D$ (see also Figure 1). Details about the semantics of these are given in the next section.

3 Qualitative Reasoning with Goal Models

Formally, a *goal graph* is a pair $\langle \mathcal{G}, \mathcal{R} \rangle$ where $\mathcal{G}$ is a set of goals and $\mathcal{R}$ is a set of goal relations over $\mathcal{G}$. If $(G_1, ..., G_n) \stackrel{r}{\longmapsto} G$ is a goal relation in $\mathcal{R}$, we call $G_1 ... G_n$ *source goals* and G the *target goal* of r. To simplify the discussion, we consider only binary OR and AND goal relations. This is not restrictive, as all the operators we consider in this section and in Section 4 —i.e., $\wedge$, $\vee$, min, max, $\otimes$, $\oplus$— are associative and can be thus trivially used as n-ary operators.

3.1 Axiomatization of goal relationships

Let $G_1, G_2, ...$ denote goal labels. We introduce four distinct predicates over goals, $FS(G)$, $FD(G)$ and $PS(G)$, $PD(G)$, meaning respectively that there is (at least) *full* evidence that goal G is satisfied and that G is denied, and that there is at least *partial* evidence that G is satisfied and that G is denied. We also use the proposition $\top$ to represent the (trivially true) statement that there is at least null evidence that the goal G is satisfied (or denied). Notice that the predicates state that there is *at least* a given level of evidence, because in a goal graph there may be multiple sources of evidence for the satisfaction/denial of a goal. We introduce a total order $FS(G) \geq PS(G) \geq \top$ and $FD(G) \geq PD(G) \geq \top$, with the intended meaning that $x \geq y$ iff $x \rightarrow y$.

We want to allow the deduction of *positive* ground assertions of type $FS(G)$, $FD(G)$, $PS(G)$ and $PD(G)$ over the goal constants of a goal graph. We refer to externally provided assertions as *initial conditions*. To formalize the propagation of satisfiability and deniability evidence through a goal graph $\langle \mathcal{G}, \mathcal{R} \rangle$, we introduce the axioms described in Figure 2. (By "dual" we mean that we invert satisfiability with deniability.)

Goal	Invariant Axioms	
$G:$	$FS(G) \rightarrow PS(G)$	(1)
	$FD(G) \rightarrow PD(G)$	(2)
Goal relation	**Relation Axioms**	
$(G_2, G_3) \stackrel{and}{\longmapsto} G_1:$	$(FS(G_2) \wedge FS(G_3)) \rightarrow FS(G_1)$	(3)
	$(PS(G_2) \wedge PS(G_3)) \rightarrow PS(G_1)$	(4)
	$FD(G_2) \rightarrow FD(G_1), \quad FD(G_3) \rightarrow FD(G_1)$	(5)
	$PD(G_2) \rightarrow PD(G_1), \quad PD(G_3) \rightarrow PD(G_1)$	(6)
$G_2 \stackrel{+_S}{\longmapsto} G_1:$	$PS(G_2) \rightarrow PS(G_1)$	(7)
$G_2 \stackrel{-_S}{\longmapsto} G_1:$	$PS(G_2) \rightarrow PD(G_1)$	(8)
$G_2 \stackrel{++_S}{\longmapsto} G_1:$	$FS(G_2) \rightarrow FS(G_1),$	(9)
	$PS(G_2) \rightarrow PS(G_1)$	(10)
$G_2 \stackrel{--_S}{\longmapsto} G_1:$	$FS(G_2) \rightarrow FD(G_1),$	(11)
	$PS(G_2) \rightarrow PD(G_1)$	(12)

Fig. 2. Ground axioms for the invariants and the propagation rules in the qualitative reasoning framework. The (or), $(+_D)$, $(-_D)$, $(++_D)$, $(--_D)$ cases are dual w.r.t. (and), $(+_S)$, $(-_S)$, $(++_S)$, $(--_S)$ respectively.

As indicated in Section 2, the propagation of goal satisfaction through a $++$, $--$, $+$, $-$ may or may not be symmetric w.r.t. that of denial. Thus, for every relation type $r \in \{++, --, +, -\}$, it makes sense to have three possible labels: "r_S", "r_S", and "r", meaning respectively that satisfaction is propagated, that denial is propagated, and that both satisfaction and denial are propagated. (We call the first two cases *asymmetric*, the latter *symmetric*.) For example, $G_2 \stackrel{-_S}{\longmapsto} G_1$ means that if G_2 is satisfied, then there is some evidence that G_1 is denied, but if G_2 is denied, then nothing is said about the satisfaction of G_1; $G_2 \stackrel{-_D}{\longmapsto} G_1$ means that if G_2 is denied, then there is some evidence that G_1 is satisfied, but if G_2 is satisfied, then nothing is said about the denial of G_1; $G_2 \stackrel{-}{\longmapsto} G_1$ means that, if G_2 is satisfied [denied], then there is some evidence that G_1 is denied [satisfied]. In other words, a symmetric relation $G_2 \stackrel{r}{\longmapsto} G_1$ is a shorthand for the combination of the two corresponding asymmetric relationships $G_2 \stackrel{r_S}{\longmapsto} G_1$ and $G_2 \stackrel{r_D}{\longmapsto} G_1$.

(1) and (2) state that full satisfiability and deniability imply partial satisfiability and deniability respectively. For an AND relation, (3) and (4) show that the full and partial satisfiability of the target node require respectively the full and partial satisfiability of all the source nodes; for a "$+_S$" relation, (7) show that only the partial satisfiability (but not the full satisfiability) propagates through a "$+_S$" relation. Combining (1) with (3), and (1) with (7), we have, respectively,

$$(G_2, G_3) \stackrel{and}{\longmapsto} G_1: \ (FS(G_2) \wedge PS(G_3)) \rightarrow PS(G_1) \tag{13}$$

$$G_2 \stackrel{+_S}{\longmapsto} G_1: \ FS(G_2) \rightarrow PS(G_1). \tag{14}$$

Thus, an AND relation propagates the minimum satisfiability value (and the maximum deniability one), while a "$+_S$" relation propagates at most a partial satisfiability value.

	$(G_2, G_3) \overset{and}{\longmapsto} G_1$	$G_2 \overset{+_S}{\longmapsto} G_1$	$G_2 \overset{-_S}{\longmapsto} G_1$	$G_2 \overset{++_S}{\longmapsto} G_1$	$G_2 \overset{--_S}{\longmapsto} G_1$
$Sat(G_1)$	$min \left\{ \begin{array}{l} Sat(G_2), \\ Sat(G_3) \end{array} \right\}$	$min \left\{ \begin{array}{l} Sat(G_2), \\ P \end{array} \right\}$	N	$Sat(G_2)$	N
$Den(G_1)$	$max \left\{ \begin{array}{l} Den(G_2), \\ Den(G_3) \end{array} \right\}$	N	$min \left\{ \begin{array}{l} Sat(G_2), \\ P \end{array} \right\}$	N	$Sat(G_2)$

Table 1. Propagation rules in the qualitative framework. The (or), ($+_D$), ($-_D$), ($++_D$), ($--_D$) cases are dual w.r.t. (and), ($+_S$), ($-_S$), ($++_S$), ($--_S$) respectively.

From now on, we implicitly assume that axioms (1) and (2) are always applied whenever possible. Thus, we say that $PS(G_1)$ is deduced from $FS(G_2)$ and $FS(G_3)$ by applying (3) —meaning "applying (3) and then (1)"— or that $PS(G_1)$ is deduced from $FS(G_2)$ and $PS(G_3)$ by applying (4) —meaning "applying (1) and then (4)".

We say that an atomic proposition of the form $FS(G)$, $FD(G)$, $PS(G)$ and $PD(G)$ *holds* if either it is an initial condition or it can be deduced via modus ponens from the initial conditions and the ground axioms of Figure 2. We assume conventionally that $\top$ always holds. Notice that all the formulas in our framework are propositional Horn clauses, so that deciding if a ground assertion holds not only is decidable, but also it can be decided in polynomial time.

We say that there is a *weak conflict* if either $PS(G)$ and $PD(G)$, $FS(G)$ and $PD(G)$, $PS(G)$ and $FD(G)$ hold for some goal G. We say that there is a *strong conflict* if $FS(G)$ and $FD(G)$ hold for some G.

3.2 The label propagation algorithm

Based on the logic framework of Section 3.1, we have developed an algorithm for propagating through a goal graph $\langle \mathcal{G}, \mathcal{R} \rangle$ labels representing evidence for the satisfiability and deniability of goals. To each node $G \in \mathcal{G}$ we associate two variables $Sat(G), Den(G)$ ranging in $\{F, P, N\}$ (full, partial, none) such that $F > P > N$, representing the current evidence of satisfiability and deniability of goal G. For example, $Sat(G_i) \geq P$ states that there is at least partial evidence that G_i is satisfiable. Starting from assigning an initial set of input values for $Sat(G_i)$, $Den(G_i)$ to (a subset of) the goals in $\mathcal{G}$, we propagate the values through the goal relations in $\mathcal{R}$ according to the propagation rules of Table 1.

The schema of the algorithm is described in Figure 3. $Initial$, $Current$ and Old are arrays of $|\mathcal{G}|$ pairs $\langle Sat(G_i), Den(G_i) \rangle$, one for each $G_i \in \mathcal{G}$, representing respectively the initial, current and previous labeling states of the graph. We call the pair $\langle Sat(G_i), Den(G_i) \rangle$ a *label* for G_i. Notationally, if W is an array of labels $\langle Sat(G_i), Den(G_i) \rangle$, by $W[i].sat$ and $W[i].den$ we denote the first and second field of the ith label of W.

The array $Current$ is first initialized to the initial values $Initial$ given as input by the user. At each step, for every goal G_i, $\langle Sat(G_i), Den(G_i) \rangle$ is updated by propagating the values of the previous step. This is done until a fixpoint is reached, in the sense that no further updating is possible ($Current == Old$).

The updating of $\langle Sat(G_i), Den(G_i) \rangle$ works as follows. For each relation R_j incoming in G_i, the satisfiability and deniability values sat_{ij} and den_{ij} derived

```
1   label_array Label_Graph(graph ⟨𝒢,ℛ⟩, label_array Initial)
2       Current=Initial;
3       do
4           Old=Current;
5           for each G_i ∈ 𝒢 do
6               Current[i] = Update_label(i, ⟨𝒢,ℛ⟩, Old);
7       until not (Current==Old);
8       return Current;
9
10  label Update_label(int i, graph ⟨𝒢,ℛ⟩, label_array Old)
11      for each R_j ∈ ℛ s.t. target(R_j) == G_i do
12          sat_ij = Apply_Rules_Sat(G_i, R_j, Old);
13          den_ij = Apply_Rules_Den(G_i, R_j, Old);
14      return ⟨ max(max_j(sat_ij), Old[i].sat), max(max_j(den_ij), Old[i].den) ⟩
```

Fig. 3. Schema of the label propagation algorithm.

from the old values of the source goals are computed by applying the rules of Table 1. The result is compared with the old value, and the maximum is returned as new value for G_i.

3.3 Termination and complexity

Theorem 1. *Label_Graph(*$\langle \mathcal{G}, \mathcal{R} \rangle, Initial$*) terminates after at most* $6|\mathcal{G}|+1$ *loops.*

Proof. First, from lines 6 and 14 in Figure 3 we have that, for every goal G_i,

$$Current[i].sat = max(..., Old[i].sat),$$
$$Current[i].den = max(..., Old[i].den)$$

so that their values are monotonically non-decreasing. In order not to terminate, at least one value per step should monotonically increase. Each of the $2|\mathcal{G}|$ variables $Sat(G_i)$ and $Den(G_i)$ admits 3 possible values and at each non-final loop at least one value increases. Thus the procedure must terminate after at most $6|\mathcal{G}| + 1$ loops. □

Notice that the upper bound is very pessimistic, as many value updates are done in parallel. In Section 5, we report on experiments we have conducted which suggest that the algorithm generally converges after a few loops.

3.4 Soundness and completeness

We call a *value statement* an expression of the form $(v \geq c)$, $v \in \{Sat(G_i), Den(G_i)\}$ for some goal G_i and $c \in \{F, P, N\}$, with the intuitive meaning "there is at least evidence c for v". Thus from now on we rewrite the assertion $FS(G)$, $PS(G)$, $\top$ as $(Sat(G) \geq F)$, $(Sat(G) \geq P)$, $(Sat(G) \geq N)$ and $FD(G)$, $PD(G)$, $\top$ as $(Den(G) \geq F)$, $(Den(G) \geq P)$, $(Sat(G) \geq N)$ respectively.

We say that $(v_1 \geq c_1)$ *is deduced* from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by a relation axiom meaning that the corresponding assertions are deduced. For instance, $(Sat(G_1) \geq P)$ is deduced from $(Sat(G_2) \geq F)$ by axiom (7) as $PS(G_1)$ is deduced from $FS(G_2)$ by axiom (7).

We say that $(v_1 \geq c_1)$ *derives* from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by means of one propagation rule $x = f(y, z)$ [$x = f(y)$] of Table 1 if $c_1 = f(c_2, c_3)$ [$c_1 = f(c_2)$]. For instance, $(Sat(G_1) \geq P)$ derives from $(Sat(G_2) \geq P)$ and $(Sat(G_3) \geq F)$ by means of the first propagation rule. Notice that this is possible because the operators *min* and *max* are *monotonic*, that is, e.g., $max(v_1, v_2) \geq max(v_1', v_2')$ iff $v_1 \geq v_1'$ and $v_2 \geq v_2'$.

To this extent, the rules in Table 1 are a straightforward translation of the axioms (1)-(12), as stated by the following result.

Lemma 1. *$(v_1 \geq c_1)$ derives from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by means of the propagation rules of Table 1 if and only if $(v_1 \geq c_1)$ is deduced from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] with the application of one of the relation axioms (3)-(12).*

Proof. For short, we consider only the AND and $+_S$ cases for $Sat(G_1)$, as the other cases are either analogous or trivial and can be verified by the reader.

$(G_2, G_3) \overset{and}{\longmapsto} G_1$**:** If either $(Sat(G_2) \geq N)$ or $(Sat(G_3) \geq N)$, then $(Sat(G_1) \geq N)$ is derived. This matches one-to-one the fact that from $\top$ nothing else is deduced;
otherwise, if either $(Sat(G_2) \geq P)$ or $(Sat(G_3) \geq P)$, then $(Sat(G_1) \geq P)$ is derived. This matches one-to-one the fact that from $(Sat(G_2) \geq P)$ and $(Sat(G_3) \geq P)$ axiom (4) is applied, so that $(Sat(G_1) \geq P)$ is deduced;
finally, if both $(Sat(G_2) \geq F)$ and $(Sat(G_3) \geq F)$, then $(Sat(G_1) \geq F)$. This matches one-to-one the fact that from $(Sat(G_2) \geq F)$ and $(Sat(G_3) \geq F)$, axiom (3) is applied, so that $(Sat(G_1) \geq F)$ is deduced.

$G_2 \overset{+_S}{\longmapsto} G_1$**:** If $(Sat(G_2) \geq N)$, then $(Sat(G_1) \geq N)$. Again, this matches one-to-one the fact that from $\top$ nothing else is deduced;
otherwise, if either $(Sat(G_2) \geq P)$ or $(Sat(G_2) \geq F)$, then we have that $Sat(G_1) \geq min(Sat(G_2), P) \geq P$. This matches one-to-one the fact that from either $(Sat(G_2) \geq P)$ or $(Sat(G_2) \geq F)$ axiom (7) is applied and $(Sat(G_1) \geq P)$ is deduced. □

Given an array of labels W, we say that $(Sat(G_i) \geq c)$ [resp. $(Den(G_i) \geq c)$] *is true in* W if and only if $W[i].sat \geq c$ [resp. $W[i].den \geq c$]. This allows us to state the correctness and completeness theorem for *Label_Graph()*.

Theorem 2. *Let $Final$ be the array returned by Label_Graph($\langle \mathcal{G}, \mathcal{R} \rangle, Initial$). $(v \geq c)$ is true in $Final$ if and only if $(v \geq c)$ can be deduced from $Initial$ by applying the relation axioms (3)-(12).*

Proof. First we define inductively the notion of "deduced from $Initial$ in k steps": (i) an assertion in $Initial$ can be deduced from $Initial$ in 0 steps; (ii) an assertion can be deduced from $Initial$ in up to $k+1$ steps if either it can be deduced from *Initial* in up to k steps or it can be deduced by applying a relation axiom to some assertions, all of which can be deduced from *Initial* in up to k steps.

Let $Current_k$ be the value of *Current* after k loops. We show that $(v \geq c)$ is true in $Current_k$ if and only if $(v \geq c)$ can be deduced from *Initial* in up to k steps. The thesis follows from the fact that $Final = Current_k$ for some k.

We reason by induction on k. The base case $k = 0$ is obvious as $Current_0 = Initial$. By inductive hypothesis, we assume the thesis for k and we prove it for $k + 1$.

If. Consider $(v \geq c)$ such that $(v \geq c)$ is deduced from *Initial* in up to $k + 1$ steps. Thus, $(v \geq c)$ is obtained by applying some relation axiom AX to some assertion(s) $(v_1 \geq c_1)$ [and $(v_2 \geq c_2)$], which can be both deduced from *Initial* in up to k steps. Thus, by inductive hypothesis, $(v_1 \geq c_1)$ [and $(v_2 \geq c_2)$] occur in $Current_k$. Then, by Lemma 1, $(v \geq c)$ derives from $(v_1 \geq c_1)$ [and $(v_2 \geq c_2)$] by means of the propagation rules of Table 1. Thus, if v is $Sat(G_i)$ [resp $Den(G_i)$], then c is one of the values sat_{ij} [resp. den_{ij}] of lines 14, 15 in Figure 3, so that $Current_{k+1}[i].sat \geq c$ [$Current_{k+1}[i].den \geq c$]. Thus $(v \geq c)$ is true in $Current_{k+1}$.

Only if. Consider a statement $(v \geq c)$ true in $Current_{k+1}$. If $(v \geq c)$ is true also in $Current_k$, then by inductive hypothesis $(v \geq c)$ can be deduced from *Initial* in up to k steps, and hence in $k+1$ steps. Otherwise, let R_j be the rule and let $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] the statements(s) true in $Current_k$ from which v has been derived. By inductive hypothesis $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] can be deduced from *Initial* in up to k steps. By Lemma 1 $(v \geq c)$ can be deduced from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by the application of one relation axiom, and thus can be deduced from *Initial* in up to $k + 1$ steps. □

Thus, from Theorem 2, the values returned by *Label_Graph*$(\langle \mathcal{G}, \mathcal{R} \rangle, Initial)$ are the maximum evidence values which can be deduced from *Initial*.

4 Quantitative Reasoning with Goal Models

The qualitative approach of Section 3 allows for setting and propagating partial evidence about the satisfiability and deniability of goals and the discovery of conflicts.

We may want to provide a more fine-grained evaluation of such partial evidence. For instance, when we have $G_2 \overset{+_S}{\longmapsto} G_1$, from $PS(G_2)$ we can deduce $PS(G_1)$, whilst one may argue that the satisfiability of G_1 is in some way less evident than that of G_2. For example, in the goal graph of Figure 1, the satisfaction of the goal Lower environment impact may not necessarily imply satisfaction of Increase customer loyalty, so it may be reasonable to assume "less evidence" for the satisfaction of the latter compared to the former. Moreover, the different relations which mean partial support – i.e., $+_S$, $-_S$, $+_D$, $-_D$ – may have different strengths. For instance, in the goal graph of Figure 1, `US Gas price rises`

	$(G_2, G_3) \overset{and}{\longmapsto} G_1$	$G_2 \overset{w+_S}{\longmapsto} G_1$	$G_2 \overset{w-_S}{\longmapsto} G_1$	$G_2 \overset{++_S}{\longmapsto} G_1$	$G_2 \overset{--_S}{\longmapsto} G_1$
$Sat(G_1)$	$Sat(G_2) \otimes Sat(G_3)$	$Sat(G_2) \otimes w$		$Sat(G_2)$	
$Den(G_1)$	$Den(G_2) \oplus Den(G_3)$		$Sat(G_2) \otimes w$		$Sat(G_2)$

Table 2. Propagation rules in the quantitative framework. The (or), $(+_D)$, $(-_D)$, $(++_D)$, $(--_D)$ cases are dual w.r.t. (and), $(+_S)$, $(-_S)$, $(++_S)$, $(--_S)$ respectively.

may have a bigger impact on `Gas price rises` than Japanese gas price rises if the manufacturer's market is mainly in the US.

To cope with these facts, we need a way for representing different *numerical* values of partial evidence for satisfiability/deniability and for attributing different weights to the $+_S$, $-_S$, $+_D$, $-_D$ relations. And, of course, we also need a formal framework to reason with such quantitative information.

4.1 An extended framework

In our second attempt, we introduce a *quantitative* framework, inspired by [1]. We introduce two real constants inf and sup such that $0 \leq inf < sup$. For each node $G \in \mathcal{G}$ we introduce two *real* variables $Sat(G), Den(G)$ ranging in the interval $\mathcal{D} =_{def} [inf, sup]$, representing the current evidence of satisfiability and deniability of the goal G. The intended meaning is that inf represents no evidence, sup represents full evidence, and different values in $]inf, sup[$ represent different levels of partial evidence.

To handle the goal relations we introduce two operators $\otimes, \oplus : \mathcal{D} \times \mathcal{D} \longmapsto \mathcal{D}$ representing respectively the evidence of satisfiability of the conjunction and that of the disjunction [deniability of the disjunction and that of the conjunction] of two goals. $\otimes$ and $\oplus$ are associative, commutative and monotonic, and such that $x \otimes y \leq x, y \leq x \oplus y$; there is also an implicit unary operator $inv()$, representing negation, such that $inf = inv(sup)$, $sup = inv(inf)$, $inv(x \oplus y) = inv(x) \otimes inv(y)$ and $inv(x \otimes y) = inv(x) \oplus inv(y)$.

We also attribute to each goal relation $+_S$, $-_S$, $+_D$, $-_D$ a weight $w \in]inf, sup[$ stating the strength by which the satisfiability/deniability of the source goal influences the satisfiability/deniability of the target goal. The propagation rules are described in Table 2. As in the qualitative approach, a symmetric relation —such as, $G_2 \overset{w+}{\longmapsto} G_1$— is a shorthand for the combination of the two corresponding asymmetric relationships sharing the same weight w —e.g., $G_2 \overset{w+_S}{\longmapsto} G_1$ and $G_2 \overset{w+_D}{\longmapsto} G_1$.

There are a few possible models following the schema described above. In particular, here we adopt a *probabilistic* model, where the evidence of satisfiability $Sat(G)$ [resp. deniability $Den(G)$] of G is represented as the probability that G is satisfied (respectively denied). As usual, we adopt the simplifying hypothesis that the different sources of evidence are independent. Thus, we fix $inf = 0$, $sup = 1$, and we define $\otimes, \oplus, inv()$ as:

$$p_1 \otimes p_2 =_{def} p_1 \cdot p_2, \quad p_1 \oplus p_2 =_{def} p_1 + p_2 - p_1 \cdot p_2, \quad inv(p_1) = 1 - p_1$$

Goal relation	Axioms	
$(G_2, G_3) \stackrel{and}{\longmapsto} G_1$:	$(Sat(G_2) \geq x \wedge Sat(G_3) \geq y) \rightarrow Sat(G_1) \geq (x \otimes y)$	(15)
	$(Den(G_2) \geq x \wedge Den(G_3) \geq y) \rightarrow Den(G_1) \geq (x \oplus y)$	(16)
$G_2 \stackrel{w+_S}{\longmapsto} G_1$:	$Sat(G_2) \geq x \rightarrow Sat(G_1) \geq (x \otimes w)$	(17)
$G_2 \stackrel{w-_S}{\longmapsto} G_1$:	$Sat(G_2) \geq x \rightarrow Den(G_1) \geq (x \otimes w)$	(18)
$G_2 \stackrel{++_S}{\longmapsto} G_1$:	$Sat(G_2) \geq x \rightarrow Sat(G_1) \geq x$	(19)
$G_2 \stackrel{--_S}{\longmapsto} G_1$:	$Sat(G_2) \geq x \rightarrow Den(G_1) \geq x$	(20)

Fig. 4. Axioms for the propagation rules in the quantitative reasoning framework. The x, y variables are implicitly quantified universally. The (or), $(+_D)$, $(-_D)$, $(++_D)$, $(--_D)$ cases are dual w.r.t. (and), $(+_S)$, $(-_S)$, $(++_S)$, $(--_S)$ respectively.

that is, respectively the probability of the conjunction and disjunction of two independent events of probability p_1 and p_2, and that of the negation of the first event. To this extent, the propagation rules in Table 2 are those of a Bayesian network, where, e.g., in $G_2 \stackrel{w+_S}{\longmapsto} G_1$ w has to be interpreted as the conditional probability $P[G_1\ is\ satisfied \mid G_2\ is\ satisfied]$. Notice that the qualitative framework of Section 3 can be seen as another such model with $\mathcal{D} = \{F, P, N\}$, $\otimes = min()$ and $\oplus = max()$. [3]

4.2 Axiomatization

As with the qualitative case, we call a *value statement* an expression of the form $(v \geq c)$, $v \in \{Sat(G_i), Den(G_i)\}$ for some G_i and $c \in [0, 1]$, with the intuitive meaning "there is at least evidence c of v". We want to allow the user to state and deduce non-negated value statements of the kind $(Sat(G) \geq c)$ and $(Den(G) \geq c)$ over the goal constants of the graph. We call externally provided assertions about the satisfaction/denial of goals *initial conditions*.

To formalize the propagation of satisfiability and deniability evidence values through a goal graph, for every goal and goal relation in $\langle \mathcal{G}, \mathcal{R} \rangle$, we introduce the axioms (15)-(20) in Figure 4. Unlike those of Figure 2, the relation axioms in Figure 4 are not ground Horn clauses —thus, propositional—but rather first-order closed Horn formulas, so that they require a first-order deduction engine.

We say that a statement $(v \geq c)$ holds if either it is an initial condition or it can be deduced from the initial conditions and the axioms of Figure 4. We implicitly assume that $(Sat(G_i) \geq 0)$ and $(Den(G_i) \geq 0)$ hold for every G_i, and that the deduction engine —either human or machine— can semantically evaluate $\otimes$ and $\oplus$ and perform deductions deriving from the values of the evaluated terms and the semantics of $\geq$. For instance, we assume that, if $(G_2, G_3) \stackrel{and}{\longmapsto} G_1$ is in $\mathcal{R}$, then $(Sat(G_1) \geq 0.1)$ can be deduced from $(Sat(G_2) \geq 0.5)$, $(Sat(G_3) \geq 0.4)$, as from (15) it is deduced $(Sat(G_1) \geq 0.5 \otimes 0.4)$, which is evaluated into $(Sat(G_1) \geq 0.2)$, from which it can be deduced $(Sat(G_1) \geq 0.1)$.

[3] Another model of interest is that based on a serial/parallel resistance model, in which $inf = 0$, $sup = +\infty$, $x \oplus y = x + y$, $x \otimes y = \frac{x \cdot y}{x+y}$ and $inv(x) = \frac{1}{x}$ [1].

We say that there is a *weak conflict* if both $(Sat(G) \geq c_1)$ and $(Den(G) \geq c_2)$ hold for some goal G and constants $c_1, c_2 > 0$. We say that there is a *strong conflict* if there is a weak conflict s.t. $c_1 = c_2 = 1$.

4.3 The label propagation algorithm

Starting from the new numeric framework, we have adapted the label propagation algorithm of Figure 3 to work with numeric values. The new version of the algorithm differs from that of Section 3 in that: the elements in $Initial$, $Current$ and Old range in $[0, 1]$; the input graph contain also weights to the $+_S$, $-_S$, $+_D$, $-_D$ goal relations; and, the propagation rules applied are those of Table 2.

4.4 Soundness and completeness

We say that $(v_1 \geq c_1)$ *derives* from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by means of one propagation rule $x = f(y, z)$ [$x = f(y)$] of Table 2 if $c_1 = f(c_2, c_3)$ [$c_1 = f(c_2)$]. For instance, $(Sat(G_1) \geq 0.4)$ derives from $(Sat(G_2) \geq 0.8)$ and $(Sat(G_3) \geq 0.5)$ by means of the first and propagation rule. Again, this is possible because the the operators $\otimes$ and $\oplus$ are monotonic.

Lemma 2. *$(v_1 \geq c_1)$ derives from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] by means of the propagation rules of Table 1 if and only if $(v_1 \geq c_1)$ is deduced from $(v_2 \geq c_2)$ [and $(v_3 \geq c_3)$] with the application of one of the relation axioms (15)-(20).*

Proof. Trivial observation that the axioms (15)-(20) are straightforward translation of the propagation rules in Table 1. □

Given an array of labels W, we say that $(Sat(G_i) \geq c)$ [resp. $(Den(G_i) \geq c)$] *is true in* W if and only if $W[i].sat \geq c$ [resp. $W[i].den \geq c$]. This allows for stating the correctness and completeness theorem for *Label_Graph()*.

Theorem 3. *Let $Final$ be the array returned by Label_Graph($\langle \mathcal{G}, \mathcal{R} \rangle, Initial$). $(v \geq c)$ is true in $Final$ if and only if $(v \geq c)$ can be deduced from $Initial$ by applying the relation axioms (15)-(20).*

Proof. Identical to that of Theorem 2, substituting the axioms (15)-(20) for the axioms (3)-(12), the rules in Table 2 for those in Table 1 and Lemma 2 for Lemma 1. □

Again, from Theorem 3, the values returned by *Label_Graph*($\langle \mathcal{G}, \mathcal{R} \rangle, Initial$) are the maximum evidence values which can be deduced from $Initial$.

4.5 Termination

To guarantee termination, the condition $(Current == Old)$ of line 7 in Figure 3 is implemented as:

$$\begin{array}{l} max_i\,(|Current[i].sat - Old[i].sat|) < \epsilon \textbf{ and} \\ max_i\,(|Current[i].den - Old[i].den|) < \epsilon, \end{array} \tag{21}$$

ϵ being a sufficiently small real constant. (This is a standard practice to avoid numeric errors when doing computations with real numbers.) Thus, the algorithm loops until all satisfiability or deniability value variations have become negligible.

Theorem 4. *Label_Graph*($\langle \mathcal{G}, \mathcal{R} \rangle, Initial$) *terminates in a finite number of loops.*

Proof. For the same reason as in Theorem 1, the values $Current[i]_k.sat$ and $Current[i]_k.den$ are monotonically non-decreasing with k. As every monotonically non-decreasing upper-bounded sequence is convergent, both $Current[i]_k.sat$ and $Current[i]_k.den$ are convergent. Thus they are also Cauchy-convergent. [4] It follows that condition (21) becomes true after a certain number of loops. □

Remark 1. In the proof of Theorem 3, we have assumed the terminating condition ($Current == Old$), whilst in our implementation we use condition (21). Thus, whilst *Label_Graph()* stops when (21) is verified, the corresponding deductive process might go on, and thus deduce a value slightly bigger than the one returned by the algorithm. Since we can reduce ϵ at will, this "incompleteness" has no practical consequences.

5 Experimental Results

Both qualitative and quantitative algorithms have been implemented in Java and a series of tests were conducted on a Dell Inspiron 8100 laptop with a Pentium III CPU and 64 MB RAM (OS: GNU/Linux, kernel 2.4.7-10). The tests were intended to demonstrate the label propagation algorithms, also to collect some experimental results on how fast the algorithms converge.

The first set of experiments was carried out in order to demonstrate the qualitative label propagation algorithm. For each experiment, we assigned a set of labels to some of the goals and events of the auto manufacturer graph (Figure 1) and see their consequences for other nodes. The label propagation algorithm reached a steady state after at most five iterations.

In a second set of experiments we assigned numerical weights to "+", "−" and "$-_S$" lateral relationships as reported in Table 3. For instance, the goal increase sales volume contributes negatively to the goal increase Toyota sales with a weight 0.6, while the goal increase car quality contributes positively to the goal increase customer loyalty with weight 0.8. Table 4 reports the results of four different experiments. For each goal/event, the table shows the initial (Init) label assignment to the variables S and D, also their final (Fin) value after label propagation. For instance, in the first experiment, the initial assignment for the goal expand markets is S=.3 and D=0, while the final values for increase return on investment (GM) are S=0 and D=.4.

The results of these experiments confirm those obtained with the qualitative algorithm. However, the numeric approach allows us to draw more precise conclusions about the final value of goals. This is particularly helpful for evaluating contradictions. For instance, in the second experiment (Exp 2), even though we

[4] An infinite sequence a_n is Cauchy-convergent if and only if, for every $\epsilon > 0$, there exist an integer N s.t. $|a_{n+1} - a_n| < \epsilon$ for every $n \geq N$. a_n is convergent if and only if it is Cauchy-convergent.

Goal/Event	Relationship	Goal/Event
increase sales volume	$\overset{0.6-}{\longmapsto}_S$	increase Toyota sales
increase Toyota sales	$\overset{0.6-}{\longmapsto}_S$	increase VW sales
increase VW sales	$\overset{0.6-}{\longmapsto}_S$	increase sales volume
increase customer loyalty	$\overset{0.4+}{\longmapsto}$	increase sales volume
increase sales prices	$\overset{0.5-}{\longmapsto}$	increase customer loyalty
increase car quality	$\overset{0.8+}{\longmapsto}$	increase customer loyalty
improve car services	$\overset{0.7+}{\longmapsto}$	increase customer loyalty
lower environment impact	$\overset{0.4+}{\longmapsto}$	increase customer loyalty
increase sales prices	$\overset{0.3+}{\longmapsto}$	improve car services
keep labour costs low	$\overset{0.7-}{\longmapsto}$	increase car quality
improve economies of production	$\overset{0.8+}{\longmapsto}$	lower purchase costs
Yen rises	$\overset{0.8+}{\longmapsto}$	increase foreign earnings
lower Japanese interest rates	$\overset{0.4+}{\longmapsto}$	lower sales price
Japanese rates rises	$\overset{0.8-}{\longmapsto}$	lower Japanese interest rates
Japanese rates rises	$\overset{0.6+}{\longmapsto}$	Yen rises
Yen rises	$\overset{0.4-}{\longmapsto}$	Japanese gas price rises
Japanese gas price rises	$\overset{0.6+}{\longmapsto}$	gas price rises
US gas price rises	$\overset{0.6+}{\longmapsto}$	gas price rises
gas price rises	$\overset{0.8-}{\longmapsto}$	lower gas price

Table 3. Quantitative relationships for the auto manufacturer example of Figure 2

have a contradiction for the final values of the root goal increase return on investment (GM) (i.e., S=0.8 and D=0.4), we have more evidence for its satisfaction than its denial. With the qualitative approach we had S=P and D=P, and there was nothing else to say about this contradiction. Analogous comments apply for the fourth experiment.

The threshold ϵ (equation 21) used in the experiments has been chosen as the smallest positive value of type float (i.e., `Float.MIN_VALUE`). With this threshold, the algorithm converged in five iterations for all four experiments.

6 Conclusions

We have presented a formal framework for reasoning with goal models. Our goal models use AND/OR goal relationships, but also allow more qualitative relationships, as well as contradictory situations. A precise semantics has been given for all goal relationships which comes in a qualitative and a numerical form. Moreover, we have presented label propagation algorithms for both the qualitative and the numerical case that are shown to be sound and complete with respect to their respective axiomatization. Finally, the paper reports some preliminary experimental results on the label propagation algorithms applied to a goal model for a US car manufacturer.

Future research directions include applying different techniques, such as Dempster Shafer theory [7], to take into consideration the sources of information in the propagation algorithms. This will allow us to consider, for instance, the reliability and/or the competence of a source of evidence . We also propose to apply our framework to more complex real cases to confirm its validity.

Goals/Events	Exp 1				Exp 2				Exp 3				Exp 4			
	Init		Fin		Init		Fin		Init		Fin		Init		Fin	
	S	D	S	D	S	D	S	D	S	D	S	D	S	D	S	D
increase return on investment (GM)	0	0	0	**.4**	0	0	**.8**	**.4**	0	0	**.9**	**.2**	0	0	**.9**	**.6**
increase sales volume	0	0	**1**	**.1**	0	0	**1**	**.1**	0	0	**1**	**.2**	0	0	**1**	**.6**
increase profit per vehicle	0	0	0	**.4**	0	0	**.8**	**.4**	0	0	**.9**	0	0	0	**.9**	0
increase customer appeal	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0
expand markets	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0
increase sales price	0	**.5**	0	**.8**	0	**.5**	0	**.8**	0	**.5**	0	**.8**	0	**.5**	0	**.8**
increase foreign earnings	0	**.9**	0	**.9**	0	**.9**	**.8**	**.9**	0	**.9**	**.8**	**.9**	0	**.9**	**.8**	**.9**
lower production costs	0	0	0	**.9**	0	0	0	**.9**	0	0	**.6**	0	0	0	**.6**	0
increase high margin sales	0	**.6**	0	**.6**	0	**.6**	0	**.6**	0	**.6**	0	**.6**	0	**.6**	0	**.6**
reduce operating costs	0	0	**.8**	0	0	0	**.8**	0	0	0	**.8**	0	0	0	**.8**	0
lower environmental impact	**.9**	0	**.9**	0	**.9**	0	**.9**	0	**.9**	0	**.9**	0	**.9**	0	**.9**	0
lower purchase costs	0	0	**.9**	0	0	0	**.9**	0	0	0	**.9**	0	0	0	**.9**	0
keep labour costs low	0	**.9**	0	**.9**	0	**.9**	0	**.9**	**.9**	0	**.9**	0	**.9**	0	**.9**	0
improve economies of production	0	0	0	0	0	0	0	0	0	0	**.7**	0	0	0	**.7**	0
improve mileage	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lower gas price	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0
offer rebates	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0	**.3**	0
lower loan interest rates	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lower sales price	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0	**.8**	0
reduce raw materials costs	0	0	0	0	0	0	0	0	**.7**	0	**.7**	0	**.7**	0	**.7**	0
outsource units of production	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
gas price rises	0	0	0	0	0	0	0	**.2**	0	0	0	**.2**	0	0	0	**.2**
lower Japanese interest rates	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
US gas price rises	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Japanese gas price rises	0	0	0	0	0	0	0	**.2**	0	0	0	**.4**	0	0	0	**.4**
Yen rises	0	0	0	0	**1**	0	**1**	0	**1**	0	**1**	0	**1**	0	**1**	0
Japanese rates rise	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
improve car quality	0	0	**.6**	0	0	0	**.6**	0	0	0	0	**.6**	0	0	0	**.6**
improve car services	0	0	0	**.2**	0	0	0	**.2**	0	0	0	**.2**	0	0	0	**.2**
improve customer loyalty	0	0	**.5**	**.2**	0	0	**.5**	**.2**	0	0	**.5**	**.2**	0	0	**.4**	**.5**
increase Toyota sales	0	0	0	**.6**	0	0	0	**.6**	0	0	0	**.6**	**1**	0	**1**	**.6**
increase VW sales	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0	**1**	**.6**

Table 4. Results with the quantitative approach

References

1. A. Bundy, F. Giunchiglia, R. Sebastiani, and T. Walsh. Calculating Criticalities. *Artificial Intelligence*, 88(1-2), December 1996.
2. A. Dardenne, A. van Lamsweerde, and S. Fickas. Goal-directed requirements acquisition. *Science of Computer Programming*, 20(1–2):3–50, 1993.
3. R. Jarvis, G. McArthur, J. Mylopoulos, P. Rodriguez-Gianolli, and S. Zhou. Semantic Models for Knowledge Management. In *Proc. of the Second International Conference on Web Information Systems Engineering (WISE'01)*, 2001.
4. J. Mylopoulos, L. Chung, and B. Nixon. Representing and Using Non-Functional Requirements: A Process-Oriented Approach. *IEEE Transactions on Software Engineering*, 6(18):483–497, June 1992.
5. A. Newell and H. Simon. GPS: A Program that Simulates Human Thought. In E. Feigenbaum and J. Feldman, editors, *Computers and Thought*. McGraw Hill.
6. N. Nilsson. *Problem Solving Methods in Artificial Intelligence*. McGraw Hill, 1971.
7. G. Shafer. *A Mathematical Theory of Evidence*. Princeton University Press., 1976.