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ANALYSIS OF PERTURBED FRACTALS FOR ANTENNA  
SYNTHESIS

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## **Analysis of Perturbed Fractals for Antenna Synthesis**

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### **Introduction**

In the last years there has been an increasing demand for compact and multi-service systems driven by the great expansion of telecommunications sectors such as mobile telephony or personal communications. The development of these systems involves the synthesis of innovative antennas characterized by small dimensions and able to manage simultaneously different communication standards [1]. In this framework, fractal-shaped antennas are promising solutions thanks to their properties. Among them, the self-similarity property and the possibility to shrink an infinite-length curve in a finite area make fractal geometries excellent candidates for both multi-band [2][3] and miniaturization [4] purposes. However, standard fractal-shaped antennas, as the Koch monopole [5], usually present an harmonic behavior rather than a real multi-band behavior. Possible solutions to such a limitation are based on the perturbation of the fractal shape [6][7] in order to increase the degrees of freedom during the synthesis process for better tuning and controlling the working frequencies of the arising structure.

In this paper, the effects of some variations of the geometry of a Koch-like fractal antenna are carefully analyzed to define some a-priori rules for the synthesis of efficient antennas. Towards this end, a set of descriptive parameters which “control” the antenna structure is defined and the achievable antenna performances are studied by means of numerical simulations to define some analytic relationships for the behavior of resonant frequencies. However, since only an exhaustive search of the whole set of possible combinations of the antenna parameters would provide an analytic synthesis tool, the analysis is devoted to provide a suitable initialization for global optimization procedures aimed at finding the best set of antenna parameters without excessive computational costs.

### **Fractal Antenna Analysis and Synthesis**

Let us consider the Koch-like geometry derived from the Koch curve described in [8]. Following the notation in [8], the antenna is generated from the Koch curve by iteratively (let  $k$  be the fractal iteration index) applying the Hutchinson

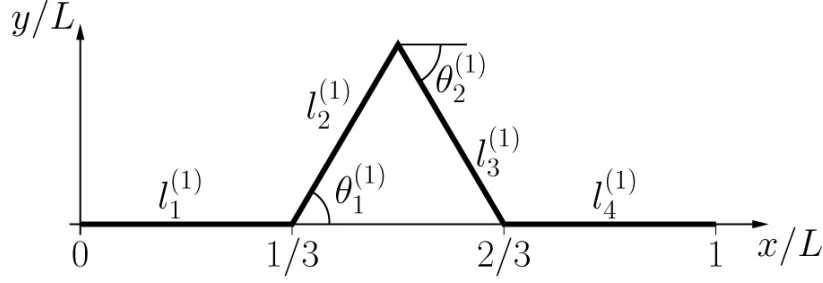


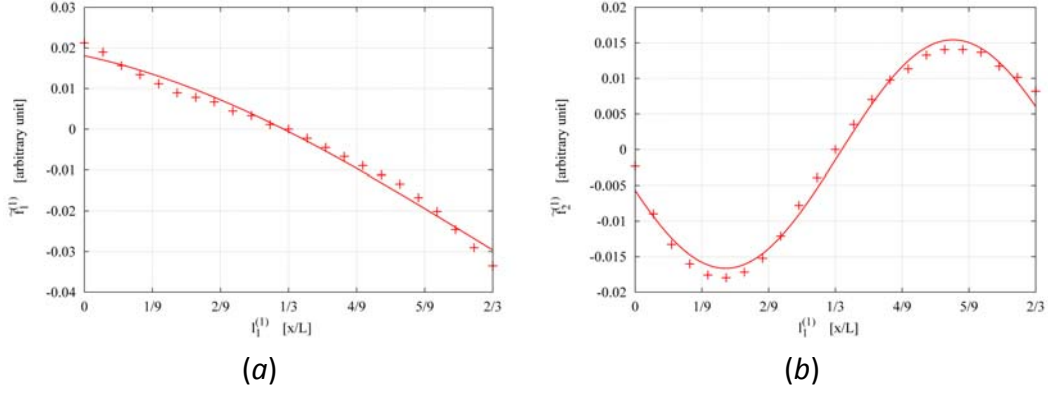
Fig. 1 – Koch-like antenna geometry at  $k = 1$ .

operator. At the  $k$ -th stage the antenna structure is uniquely determined by the descriptive vector  $\mathbf{w}_k$

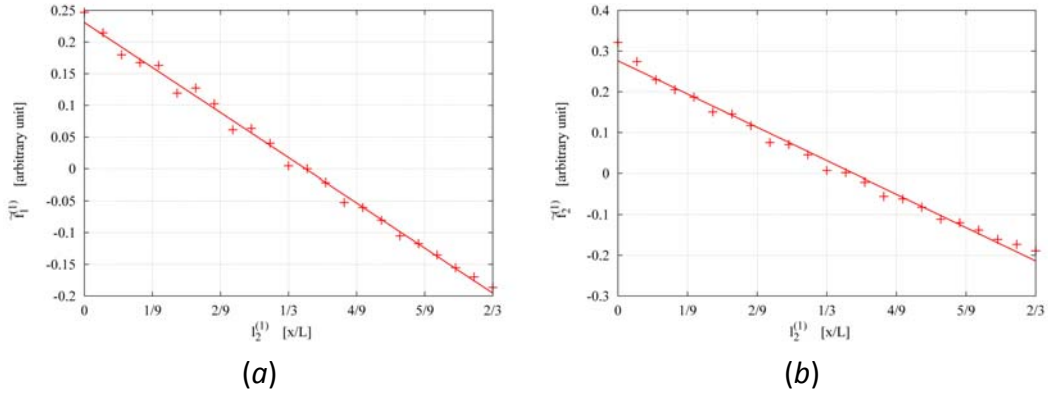
$$\mathbf{w}_k = \{l_i^{(k)}, \theta_j^{(k)}; i = 1, \dots, 4^k; j = 1, \dots, 2 \cdot 4^{k-1}\} \quad (1)$$

where  $l_i^{(k)}$  is the length of the  $i$ -th antenna segment at the  $k$ -th iteration, whereas  $\theta_j^{(k)}$  is the  $j$ -th angle at the same stage. As an example, the normalized (with respect to the antenna length  $L$ ) antenna geometry at  $k = 1$  is shown in Fig. 1. For the sake of simplicity, let us focus on the Koch geometry at the first iteration to study the impact of the geometrical perturbations on the resonant frequency behavior of the fractal antenna. More specifically, the location of the first  $M$  resonant frequencies  $f_m^{(1)}; m = 1, \dots, M$  is analyzed. Towards this purpose, the following constraints are imposed on the antenna structure:  $l_3^{(1)} = l_2^{(1)}$ ,  $\theta_2^{(1)} = \theta_1^{(1)}$ , and  $l_4^{(1)} = 1 - l_1^{(1)} - 2l_2^{(1)} \cos(\theta_1^{(1)})$ . The last requirement forces the antenna to always occupy the same area. Moreover, in order to avoid any reliance on the working frequency, the resonant frequencies have been normalized such that  $\tilde{f}_m^{(1)} = \frac{f_m^{(1)} - f_m^{(std)}}{f_m^{(std)}}; m = 1, \dots, M$ , where  $f_m^{(std)}; m = 1, \dots, M$  denote the first  $M$  resonant frequencies of the standard Koch-like monopole antenna ( $l_1^{(1)} = l_2^{(1)} = \frac{1}{3}$  and  $\theta_1^{(1)} = \frac{\pi}{3}$ ).

To illustrate some representative results from the analysis, two different test cases are reported in the following. The first one is aimed at verifying the effects of varying  $l_1^{(1)}$  while keeping the others set to the reference monopolar structure ( $l_2^{(1)} = \frac{1}{3}$  and  $\theta_1^{(1)} = \frac{\pi}{3}$ ). Figure 2 shows the first and the second resonant frequencies in correspondence with the admissible values of  $l_1^{(1)}$ . A sinusoidal behavior, characterized by the following relations



**Fig. 2** – Behavior of (a)  $\tilde{f}_1^{(1)}$  and (b)  $\tilde{f}_2^{(1)}$  varying the  $l_1^{(1)}$  parameter.



**Fig. 3** – Behavior of (a)  $\tilde{f}_1^{(1)}$  and (b)  $\tilde{f}_2^{(1)}$  varying the  $l_2^{(1)}$  parameter.

$$\begin{aligned}\tilde{f}_1^{(1)} &= -0.028 + 0.05 \cos(1.875l_1^{(1)} + 0.357) \\ \tilde{f}_2^{(1)} &= -0.001 + 0.016 \cos(8.382l_1^{(1)} + 1.894)\end{aligned}\quad (2)$$

is obtained whatever the resonant frequency. In the second test case,  $l_2^{(1)}$  is varied in the range  $\left[0, \frac{2}{3}\right]$  while  $l_1^{(1)} = \frac{1}{3}$  and  $\theta_1^{(1)} = \frac{\pi}{3}$ . As it can be noticed (Fig. 3), the resonant frequencies vary with a linear trend according to the following

$$\begin{aligned}\tilde{f}_1^{(1)} &= 0.231 - 0.646l_2^{(1)} \\ \tilde{f}_2^{(1)} &= 0.276 - 0.742l_2^{(1)}\end{aligned}\quad (3)$$

From an engineering perspective, (2) and (3), as well as the others obtained varying the other descriptors, give a set of an *a-priori* information to be fully exploited as a suitable initialization for a successive optimization process for

refining the choice of the best solution (i.e., the optimal antenna parameters). As a matter of fact, a good initialization usually results in an efficient method for minimizing the computational time as well as the required resources.

### Conclusions

In this paper, the effects of geometrical perturbations on the geometry of a reference fractal antenna have been carefully analyzed to provide a set of analytic rules for describing the resonant behavior of the antenna. As regards to the antenna synthesis, the obtained results can be profitably used to initialize an optimization process for identifying an optimal solution thus allowing non-negligible saving of the computational resources.

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