A Simplified Model of the Ground Surface Vibration Arising from a Leaking Pipe

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Abstract. Acoustic techniques remain the bedrock of pipeline leak detection, particularly for the water industry. The correlation technique, in which leak noise measurements are made at accessible locations on the pipe, either side of the leak, is used world-wide. Unfortunately, especially in the case of plastic pipes, access points are often not spaced closely enough for effective leak detection to take place. An alternative to sensing on the pipe is to measure directly on the ground surface, using discrete sensors such as geophones or accelerometers. However, to do this, the vibrational field on the ground, produced by the leak, needs to be fully understood. The present author, alongside colleagues, has developed an analytical model to show how axisymmetric elastic waves propagating within the pipe radiate to the ground surface. The model, only valid directly above the pipe, shows that, dependent on the soil properties, both a conical shear wave and a conical compressional wave may radiate into the soil, and thence propagate to the ground surface. Moreover, the axial dependence of the ground surface response mirrors the axial dependence of the waves propagating within the pipe. Here, a simplified analytical model of the conical pipe-radiated waves, which encapsulates the essential phase-related features of the more complex development described previously, is presented. This then allows a relatively simple extension to predict the off-axis ground surface as well as that directly above the pipe. Numerical simulations and experimental investigations are also carried out to demonstrate the potentialities of the proposed model to reveal the underlying physics through a simple way.

Keywords: Leak Detection, Elastic Wave Propagation, Vibration, Buried Pipeline; Ground Surface Measurements.

1 Introduction

According to the World Bank, approximately 90 billion litres of water are lost due to leakage globally each day, this representing 30-50% of the world's pumped water [1].

The developed world is responsible for half this amount, with the UK's leakage alone accounting for approximately 3 billion litres per day, or over 3% of that lost globally [2]. Apart from the enormous loss of a treated resource, between 2% and 3% of the world's energy consumption is used to pump and treat water for urban residents and industry [3] so the energy wasted because of leakage corresponds to approximately 1% of the total global carbon footprint.

Whilst other methods are available, and have recently been extensively evaluated [3], acoustic techniques remain the bedrock of leak detection, particularly for the water industry [4,5]. The correlation technique, in which leak noise measurements are made at accessible locations on the pipe (for example, at hydrants) either side of the leak, is used world-wide. Unfortunately, in the case of plastic pipes, hydrants are often not spaced closely enough for effective leak detection to take place [6].

Acoustic alternatives to sensing on a hydrant or at a convenient access point include: measuring inside the pipe with hydrophones [7]; measuring directly at the ground surface, using discrete sensors such as geophones or accelerometers; measuring below the ground surface using a continuous or distributed sensor, such as a fibre optic cable; or sensing above the ground surface using a non-contact sensor such as a laser vibrometer. The present authors have developed a vibration-based method for locating buried pipes from the ground surface [8], supported by analytical modelling [9,10], but this has yet to be applied to the detection of leaks.

Leak noise from water (and, indeed, gas and oil) pipes tends to be concentrated at low frequencies [11,12]. At these frequencies, well below the pipe ring frequency, four wave types are responsible for most of the energy transfer in fluid-filled pipes [13, 14,15]: three axisymmetric waves (n=0) and the n=1 wave, related to beam bending. Of the n=0 waves, the first, termed s=1, is a predominantly fluid-borne wave; the second wave, s=2, is predominantly a compressional wave in the shell; the third wave, s=0, is a torsional wave uncoupled from the fluid. However, the authors' own work on buried plastic water pipes and others' work on a buried, gas-filled steel pipe [16] show that it is the fluid-dominated wave which dominates the radiated response from such structures. Previous work by the present authors, studying the radiation of the fluid-dominated wave into surrounding soil [9,10] has shown that, depending on the soil properties, both a conical shear wave and a conical compressional wave may radiate into the soil, and thence propagate to the ground surface. Moreover, directly above the pipe, the axial dependence of the ground surface response mirrors the axial dependence of the waves propagating within the pipe. Broadly, if the wavespeed of the wave in the pipe exceeds the respective wavespeed in the soil, the conical wave will radiate. For a typical sandy-type soil, for example, for which both the compressional and shear wavespeeds are low, both waves radiate. For a typical clay-type soil, only the compressional wave radiates. This behaviour has been successfully demonstrated at test sites in Brazil and the UK [17]. However, in addition to the waves radiated from the pipe, the source applied to the pipe (in this case a leak) will also excite body waves directly in the ground. The body waves will radiate spherically from the source location. A portion of the energy of these waves will also reach the measurement points, alongside the waves radiated from the pipe. When measuring at the ground surface, potentially both magnitude and phase information will be useful, although to visualize the wave fields described above, phase is likely to be the more important measure. Lines of constant phase represent wave fronts, encapsulating the relevant time delay information as waves travel from the input (via the pipe or otherwise) to the measurement location. Moreover, unwrapped phase is extremely robust in the presence of noise [8].

Here we present a simplified analytical model of the conical pipe-radiated waves which encapsulates the essential phase-related features of the more complex development described in [9,10]. This then allows a relatively simple extension to predict the off-axis ground surface as well as that directly above the pipe.

The paper is organized as follows. Followed by this introduction Section 2 presents the simplified model for the ground-surface response along with some theoretical and numerical investigations. Experimental investigations are then conducted in Section 3. Finally, the paper is closed with some conclusions and suggestions for future avenues of research in Section 4.

2 Simplified Model

A model of the expected ground surface phase response resulting from a leaking buried plastic water pipe is developed here. This model is based on the observed features of the response of analytical models developed previously [19-23]. We have established that there are body waves arising from excitation of the pipe due to a leak, which are radiated from the pipe towards the ground surface response as showed in Fig.1.

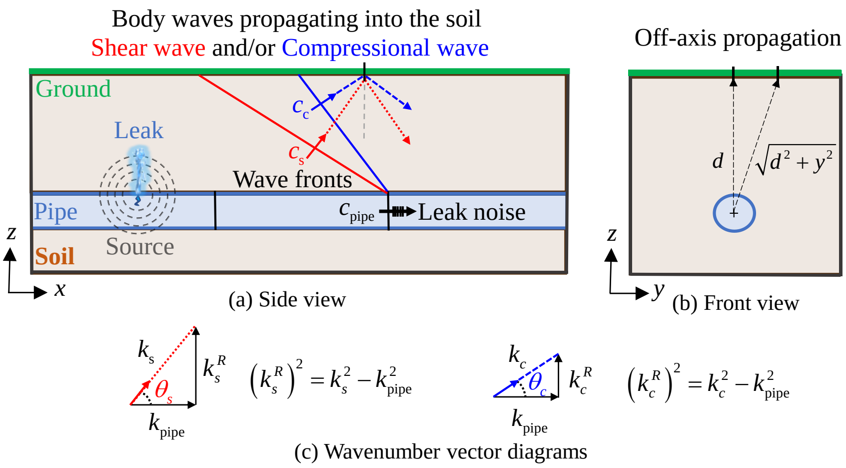

Fig. 1. Waves from a Leaking Pipe: (a) side view, (b) front view, (c) wavenumber diagrams. The terms c_s and c_c denote the wavespeed of shear and compressional wave respectively.

2.1 Radiated Conical Waves

In [9,10] and [23], expressions are derived for the ground surface vibration resulting from the axisymmetric, fluid-dominated (*n*=0, *s*=1) propagating in a buried pipe. The in-axis horizontal and vertical ground surface displacements, U_x and U_z, directly above the pipe are given as [10]:

$$\begin{Bmatrix} U_x \\ U_z \end{Bmatrix} = \sqrt{\frac{2}{\pi k_c^R d}} e^{\mathrm{i}\pi/4} \mathbf{u}_c A_m \mathrm{e}^{-\mathrm{i}k_c^R d} e^{-ik_{\text{pipe}} x} + \mathrm{i}\sqrt{\frac{2k_s^R}{\pi d}} e^{\mathrm{i}\pi/4} \mathbf{u}_s B_m \mathrm{e}^{-\mathrm{i}k_s^R d} e^{-ik_{\text{pipe}} x} \quad (1)$$

where: *d* is the pipe depth; A_m and B_m are the compressional and shear wave potentials respectively; $\mathbf{u}_c$ and $\mathbf{u}_s$ are compressional and shear wave amplitude vectors incorporating the effects of reflection at the ground surface and functions of the axial and radial components of the shear and compressional wavenumbers in the ground only; and these surrounding medium compressional and shear radial wavenumbers k_c^R and k_s^R respectively are given by $\left(k_c^R\right)^2 = k_c^2 - k_{\text{pipe}}^2$ and $\left(k_s^R\right)^2 = k_s^2 - k_{\text{pipe}}^2$; k_c and k_s are the compressional (dilatational) and shear (rotational) wavenumbers in the surrounding soil, and k_{pipe} is the axial wavenumber in the pipe. Valid at frequencies for which $\left|k_c^R d\right| > 1$ and $\left|k_s^R d\right| > 1$, i.e. in the far field, a key observation regarding equation (1) may be made. Provided that the magnitude terms A_m and B_m, $\mathbf{u}_c$ and $\mathbf{u}_s$ vary slowly with frequency, the phase of the response at the ground surface is controlled by the terms $e^{-ik_c^R d}$ and $e^{-ik_s^R d}$, i.e. the phase at the surface is largely the same as had the ground surface not been present. Closer examination of the forms of the magnitude terms (equations (9), (18) and (20) in [9]), shows that they do indeed vary slowly with frequency compared with the phase terms above.

An important consequence of noting that the phase of the ground surface response can be determined on the basis of outgoing waves only is that the phase of the off-axis ground surface response can be estimated. This necessarily neglects the surface waves that can be generated at lateral distances from the pipe axis comparable or greater than the pipe depth [24,25].

However, it is anticipated that, due to the high attenuation in soils, the effect of these will be small. By incorporating the axial and time dependences, and with reference to Fig. 1(b), at a lateral distance from the pipe axis (off-axis), the horizontal, perpendicular and vertical ground-surface displacements can now be re-expressed as:

$$U_x\left(x, y, d\right) = \left(u_{xc} e^{-ik_c^R\sqrt{y^2+d^2}} + u_{xs} e^{-ik_s^R\sqrt{y^2+d^2}}\right)\sqrt{\frac{a}{\sqrt{y^2+d^2}}} e^{-ik_{\text{pipe}} x} \quad (2a)$$

$$U_y(x,y,d)=\left(u_{yc}e^{-ik_c^R\sqrt{y^2+d^2}}+u_{ys}e^{-ik_s^R\sqrt{y^2+d^2}}\right)\sqrt{\frac{y^2a}{\left(y^2+d^2\right)^{\frac{3}{2}}}}e^{-ik_{\text{pipe}}x} \quad (2b)$$

$$U_z(x,y,d)=\left(u_{zc}e^{-ik_c^R\sqrt{y^2+d^2}}+u_{zs}e^{-ik_s^R\sqrt{y^2+d^2}}\right)\sqrt{\frac{d^2a}{\left(y^2+d^2\right)^{\frac{3}{2}}}}e^{-ik_{\text{pipe}}x} \quad (2c)$$

where *a* is the pipe radius and the *u*'s are complex wave magnitudes (functions of the compressional and shear wavenumbers and the pipe radius). Wave attenuation is encapsulated in the exponential terms in which the wavenumbers comprise both real and imaginary components. This convenient form acknowledges both the cylindrically spreading nature of the waves and the dominant phase dependence. Moreover, it is a similar form to that previously derived in [15] when only one wave radiates (in this case a torsional wave). A 3D view of the conical waves is depicted in Fig. 2 by two cones describing the propagation of each wave type: shear wave (inner cone in red) and compressional wave (outer cone in blue). Depending on the soil properties, a conical shear wave and/or a conical compressional wave may propagate into the soil. If the speed of the leak noise wave in the pipe exceeds the respective wavespeed in the soil, the conical wave will then radiate towards the ground surface. The wave front of each conical wave has a certain angle of propagation which relates the wavespeed of the leak noise in the pipe with the wavespeed in the soil as depicted in Fig. 1(c) by the wavenumber diagrams. Attenuation, both in the pipe and in the soil, may be accounted for by introducing imaginary components into the wavenumbers k_c, k_s and k_{pipe}.

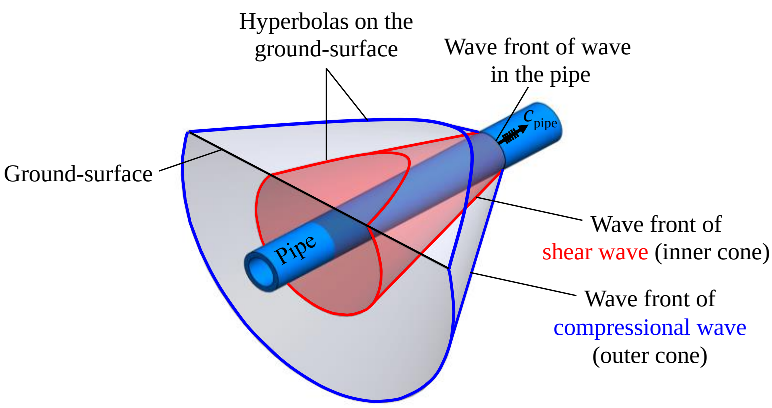

Fig. 2. Typical conical wave intersections with the surface of the ground.

2.2 Investigations using theoretical model and finite element model

In order to estimate the ground surface responses using both theoretical and numerical models, a rectangular grid of points was considered. The investigations were performed by simulating only responses in the vertical direction, given by Eq. 2(c), at various position over a grid of 32x7 points 0.5 m apart (axial and lateral distances) on the layer above a plastic water pipe which is buried at a depth of 1 m. Effects from reflected waves were neglected for simplification purposes

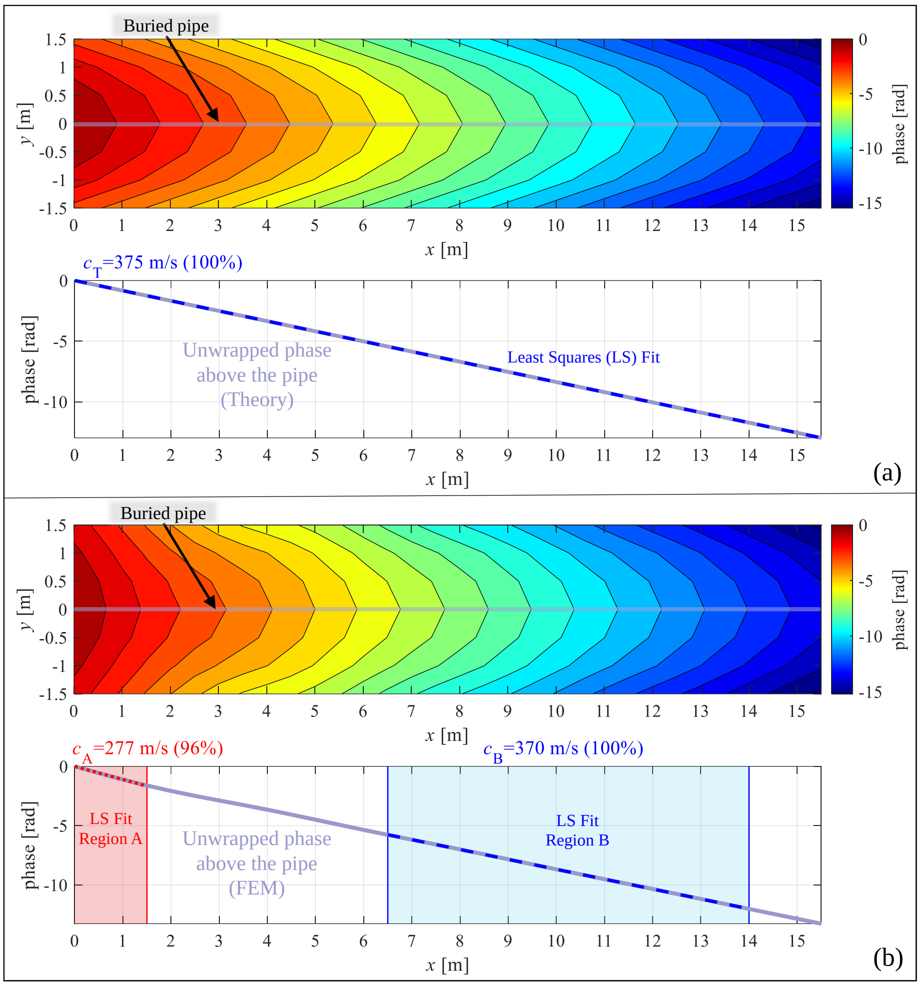

Fig. 3. Contour plots of spatially unwrapped phase for ground-surface response (top), phase gradients right above a plastic pipe (bottom) at 56 Hz: (a) Theoretical model; (b) FEM. The term c_T is the wavespeed predicted above the pipe using least squares fit.

The finite element model (FEM), implemented in COMSOL Multiphysics commercial software, describes a pipe system modelled as a four-part system comprising water, the pipe, the surrounding soil and a layer that corresponds to the ground. A monopole pulsating source was placed 1 m away from the first row of sensor to excite the fluid-dominated wave (s=1) in the pipe. A fine mesh of triangular elements was used in the discretisation of the model along with a perfectly matched layer (PML) applied on the boundary of the computational domain to simulate an infinite medium.

The phase analysis was performed by using a two-dimensional unwrapping algorithm applied over the entire grid for both theoretical and numerical ground surface responses. Overall, the contour plots represent wave fronts. Fig. 3 shows the results for a particular frequency of 56 Hz. The wavespeed predicted is showed below the contour plot together the metric Value of Fit (VoF) which measures in % the accuracy of the results. For the FEM results, the analysis is divided into two regions: region A that has influence from the monopole source and region B where the prediction matches with the wavespeed in the pipe.

3 Experimental results using a test rig located in the UK

A typical problem of mapping and locating a buried plastic pipe using ground vibration measurements is depicted in Fig. 4. The test site is located near Blithfield reservoir in Staffordshire in the UK and consists of a 120 m long medium-density polyethylene (MDPE) pipe buried at a depth of about 1 m. A leak was generated by opening the valve and the vibration signals were measured by accelerometers 1 and 2 to predict the wavespeed of s=1 wave in the pipe, which was found to be around 375 m/s.

The ground surface is grass and the soil in which the pipe is buried, typically found in this region, is a mixture of gravel, sand and clay. Only main features of the test rig were included here, and more details about the test rig can be found in [18]. The cross power spectral density (CPSD) between the velocity measured by each geophone right above the pipe and the acceleration measured at the hydrant by the source was calculated for the entire grid of points as depicted in Fig. 4. The spatially unwrapped phases using the radial responses were then estimated. Results using the geophones that are positioned directly above the pipe (when y=0) are shown below each contour plot as a function of axial distance along the pipe.

At a first glance, the run of the pipe can be detected and interaction between different wave types radiating into the surrounding medium are evident for distances up to 5 metres away from the source as showed by the phase contour plots. In order to estimate the wavespeed using the ground responses, a least-squares fit was applied based on the linear interpolation of the measured phase gradients for region A (up to 1.5 m) as well as for the region B (from 6.5 m up to 14 m). The jumps observed in the phase plots arise due to several reasons such as external noise, discontinuities from wave interactions and effects of reflections. Moreover, some uncertainties present in the soil also affected the quality of the measured data. The results provided evidence that after some distance from the source, approximately 7 m, only the pipe wave dominates as verified in both theoretical and numerical examples. For region A (up to 2.5 m away from the source) there is a combination of waves from the source and body waves.

Overall, it has been found that, when calculating the CPSD between the vibrational velocity on the ground surface and the vibration at the source, the phase information is important and the contour plots of the spatially unwrapped phase on the ground surface can be an effective tool to reveal the location of the pipe. As can be observed in the phase contour plots, the run of the pipe using the experimental data is less evident and noisier compared to the theoretical and numerical results, since some effects mentioned earlier were neglected in both models.

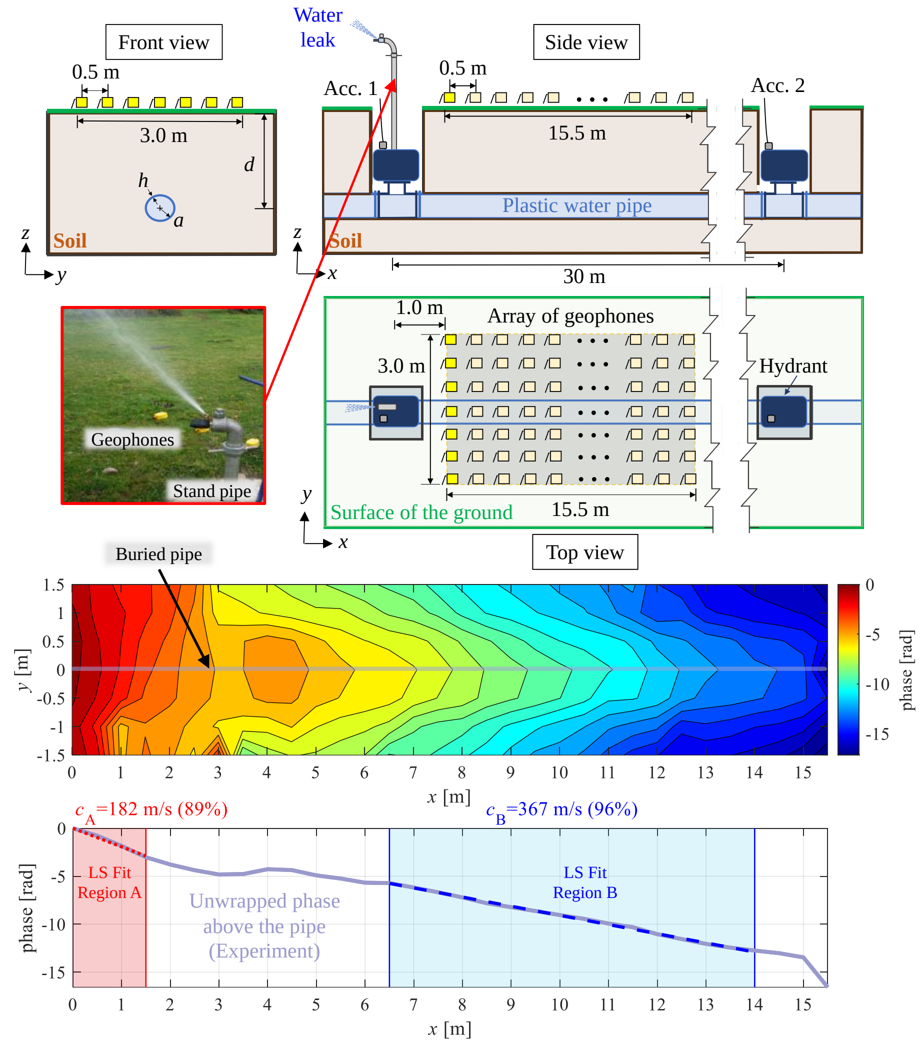

Fig. 4. Schematic diagram of the test rig in Blithfield-UK (not to scale) and the contour plots of spatially unwrapped phase for the grid of geophones at a frequency of 56 Hz together with the phase gradients using the geophones positioned right above the buried plastic pipe.

Another marginal difference is the excitation mechanism adopted in each case. In Blithfield test rig, a standpipe was attached to a hydrant to reproduce a leak as depicted in Fig. 4, and their dynamical effects have not been included in both theoretical and numerical simulations. Furthermore, by examining the gradients of the spatially unwrapped phase it is possible to estimate the wavespeed on the ground-surface which is mostly dominated by the *s*=1 wave, allowing to identify which type of wave propagating from the pipe within the soil is responsible for the ground vibration.

4 Conclusions

In this paper, a simplified approach for understanding the interaction of waves propagating from a plastic buried pipe towards the surface of the ground has been presented. This allows to exploit features from the leak noise wave radiation using ground vibration response phase data. Physical insight on the interaction of waves is given for a better understanding about how the soil properties are acting on the propagation characteristics of a leak noise wave travelling along the pipe-wall and propagating into the surrounding soil towards the ground surface. In addressing such a problem, there is always likely to be a trade-off between the need to provide a mathematical model which is sophisticated enough to encapsulate the key elements of the observed behaviour, yet simple enough to be of general applicability and amenable to straightforward interpretation. Moreover, it is important to limit both the number of unknown parameters and the overall size of the parameter space to reduce overall computational costs, but perhaps more crucially, to increase the likelihood of identifying globally optimal solutions, rather than local minima.

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