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Kinetic energy of rolling motion from the contact point

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Abstract

We make use of the fact that the instantaneous velocity field of a body in a planar, pure rolling motion is identical to that of a rigid rotation about the contact point. As a consequence, the total kinetic energy can be written with respect to that point. For bodies with circular symmetry, the usual Koenig decomposition into translational and rotational contributions is immediately recovered, and remains valid for any radially symmetric mass distribution.

Keywords: pure rolling, kinetic energy, Koenig decomposition

1. Introduction

The kinetic energy of an in-plane rolling, non-slipping rigid body is often written, according to the well-known Koenig decomposition, as

$$K = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2. \quad (1)$$

The distinctive kinematic feature (and definition) of the pure rolling motion is that the point of contact between the body and the surface is instantaneously at rest. In this note we

exploit this property directly: once the structure of the instantaneous velocity field is identified, the kinetic energy follows immediately. An alternative derivation based on summing kinetic energies of elementary portions of the rolling body has been recently presented [1]; here, by contrast, we emphasize the role of the contact point and obtain the same result in a more direct way. From a teaching perspective, this viewpoint makes explicit the connection between three ideas that are often introduced separately: the vanishing velocity of

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the contact point, the notion of instantaneous centre of rotation, and the standard expression for the kinetic energy of rolling motion. Difficulties in connecting these ideas have been documented in the literature [2, 3]. The argument presented here may help clarify the connection between these aspects.

2. Instantaneous velocity field and kinetic energy

As schematically depicted in figure 1, at a given instant, the rolling body, with radius R , centre O and instantaneous contact point Q , has velocity $\vec{v}_O$ at its centre (the centre of mass) and angular velocity with magnitude $\omega = \frac{v_O}{R}$. We can write, for a planar rigid-body motion, the velocity of an arbitrary point P of the body at distance r from O in the laboratory frame as

$$\vec{v}_P = \vec{v}_O + \vec{\omega} \times \vec{OP} = \vec{v}_O + \vec{\omega} \times \vec{r} \quad (2)$$

Applying this same relation to the contact point Q , i.e. taking $\vec{r} = -\vec{R}$, and using $\vec{v}_Q = 0$ because of the non-slipping condition, we obtain that $\vec{v}_O = -\vec{\omega} \times \vec{OQ} = \vec{\omega} \times \vec{R}$. As a consequence,

$$\vec{v}_P = \vec{\omega} \times (\vec{R} + \vec{r}) = \vec{\omega} \times \vec{\rho}. \quad (3)$$

Thus the velocity of every point is perpendicular to the vector $\vec{\rho} = (\vec{R} + \vec{r}) = \vec{QP}$ which joins point P to the contact point Q . The speed is then $v_P = \rho\omega$. This means that the instantaneous velocity field is identical to that of a rigid rotation about the contact point Q . The circles centred at Q should be interpreted as instantaneous kinematic circles with radius ρ which describe the velocity field at that instant. Of course, they are not the actual trajectories of the points of the body in the laboratory frame as the contact point keeps changing during the motion and so does the centre of the instantaneous circles. It is now immediate to obtain the kinetic energy of the body in this general form:

$$K = \frac{1}{2} \int v_P^2 dm = \frac{1}{2} \omega^2 \int \rho^2 dm = \frac{1}{2} I_Q \omega^2 \quad (4)$$

in which we introduce the moment of inertia of the body about the instantaneous centre of rotation Q .

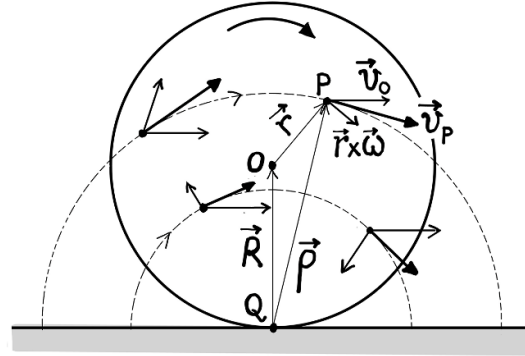


Figure 1. A rolling body with two instantaneous kinematic circles (dashed). For a generic point P , the distance from the contact point Q is $\rho = QP$, while $r = OP$ is the distance from the centre O .

The standard expression for the kinetic energy of equation (1) is then recovered immediately from the parallel-axis theorem $I_Q = I_O + MR^2$. For a body with circular symmetry, that same result can also be obtained directly from $\rho^2 = (\vec{R} + \vec{r})^2 = R^2 + r^2 + 2\vec{r} \cdot \vec{R}$, since the integral of the inner product $\vec{r} \cdot \vec{R}$ over the whole body vanishes by circular symmetry. Equation (4) shows that the kinetic energy of a rolling body can be interpreted as the rotational kinetic energy about the instantaneous contact point. This follows directly from the instantaneous structure of the velocity field. In this way, the familiar decomposition of the kinetic energy of rolling motion appears as a direct consequence of the instantaneous rotational structure of the velocity field.

3. Conclusions

We have recovered the well-known result that, in a planar, pure rolling motion, the instantaneous rest of the contact point Q allows the velocity field to be described as that of a pure rotation about that point. The kinetic energy can therefore be written as $K = \frac{1}{2} I_Q \omega^2$ and the usual Koenig decomposition follows immediately from the parallel-axis theorem. Besides providing a compact derivation, this viewpoint may be pedagogically useful because it directly connects the kinematics of the contact point, the instantaneous centre of rotation, and the standard expression for the kinetic energy.

Data availability statement

No new data were created or analysed in this study.

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