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Uncertain interval TOPSIS and potentially regrettable decisions within ICT evaluation environments

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A B S T R A C T

It is generally assumed that the rankings provided by Multi-Attribute Decision-Making (MADM) techniques are definitive. Once the ranking is delivered, decision makers (DMs) are expected to choose the first alternative and dismiss the remaining ones, concluding the application of the corresponding model. The MADM literature has incorporated fuzziness and imprecision to its models to deal with evaluation uncertainties but has not accounted for its consequences defined in terms of regrettable choices. That is, MADM models do not consider the possible consequences of having chosen an alternative whose actual characteristics do not correspond to those expected by the DM. This paper aims at designing an integrated MADM framework with interval variables where the DM is allowed to modify the initial alternative chosen after observing the realizations of its characteristics. In order to do so, sequences of alternatives including the initial choice as well as subsequent alternate choices should be ranked in place of single alternatives. We analyze the combinatorial decision environment that arises from defining and evaluating sequences of choices by accounting for the whole set of potential realizations and any subsequent change in the alternatives selected. The TOPSIS method is used to design the integrate evaluation framework producing the final ranking. A case study analyzing the entry decision of a firm within a group of European countries based on their levels of ICT development is presented. We illustrate how the countries selected and their order may differ substantially when accounting for the complementarities existing among them. Moreover, the selection process and any subsequent decision vary with the number of modifications considered relative to the initial country selected. The results obtained are of interest not only to firms facing a similar problem, but also to DMs or managers dealing with strategic selection processes where the wrong choice of alternatives may lead to increasingly complex sequential disruptions.

1. Introduction

1.1. Motivation

When applying Multi-Attribute Decision-Making (MADM) techniques, rankings are delivered to decision makers (DMs) who are expected to choose the first alternative and dismiss the remaining ones.

Fuzziness and imprecision have been widely introduced in MADM models to deal with evaluation uncertainties, with the most common assumption being the definition of the values of the different characteristics of the alternatives as interval variables (i.e., uncertain intervals).

However, to the best of our knowledge, no MADM model has considered the possible consequences of having chosen an alternative whose actual characteristics do not correspond to those expected by the DM or analyzed these consequences in terms of regrettable choices. No MADM framework with interval variables has accounted for the very concrete possibility that after observing realizations of the characteristics from an initially selected alternative different from those expected, the DM may want to modify the initial choice.

In this regard, the MADM literature related to the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and its extensions to uncertain interval settings focuses on the capacity of models to rank alternatives but does not consider the actual consequences from the decisions made when the realizations are observed (Dymova et al. [1]; Niroomand et al. [2]). In other words, the uncertainty inherent to the evaluations may lead DMs to modify their initial choices after observing a realization from the alternative selected and such a possibility must be incorporated in the corresponding model.

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1.2. ICT evaluation environments: Problem description

The problem of uncertainty in Information and Communication Technology (ICT) environments goes well beyond the introduction of fuzzy variables to account for the consequences of approximate reasoning (Alshahrani et al. [3]; Ruiz et al. [4]; Stawowy et al. [5]). This is particularly the case when considering the actual implementation of the models to real life scenarios (Pashutan et al. [6]; Trzaska et al. [7]; Wachnik et al. [8]). For example, Dzemydienė et al. [9] compared the Simple Additive Weighting (SAW) method and TOPSIS when evaluating the access and usage of ICT in businesses for a group of European countries throughout the period 2013–2017. These authors focused on the access and usage of ICT in business enterprises as the unique decision variable.

Consider now the problem faced by a firm that must enter a country and interact with the local firms, workers, and institutions. Following the above example, one of the standard variables that may be used by the firm when making an entry decision is the percentage of local firms giving portable devices for a mobile connection to the internet to their employees. These percentages can be directly applied to a MADM technique to obtain a ranking among countries. The intuition is clear, higher values of the variable imply that a larger percentage of firms and workers are aware of the existence and capable of using ICT devices. The percentage is a crisp value and, as such, its interpretation is intuitive and direct.

This information, while important, is highly approximate. Higher values imply a higher probability of interacting with local firms more developed than others located in countries exhibiting lower values. However, the firm entering must consider the possibility of obtaining a suboptimal performance from interacting within the higher valued country. Note that fuzzy variables and possibility theory do not suffice to incorporate this type of uncertainty into the analysis. Thus, the scenarios derived from the subsequent interactions, which may force the firm to leave the country if the performance does not satisfy some minimum requirements, are not generally analyzed. However, market frictions are extremely important when firms select a country to interact with in the international business literature (Nguyen et al. [10]; Sanna-Randaccio and Veugelers, [11]).

1.3. Challenges

Consider a ranking produced within a MADM framework characterized by uncertain intervals evaluations. Suppose that the DM is given the possibility to modify the alternatives initially chosen when the actual characteristics observed do not correspond to the expected ones.

This possibility implies that the ranking delivered by the MADM technique should consider the initial choice as well as any subsequent modification in case the alternatives ranked in the initial positions perform suboptimally. That is, sequences of alternatives including the initial choice as well as subsequent alternate choices should be ranked in place of single alternatives.

Hence, incorporating uncertainty about the potential realizations of the characteristics requires designing a decision path determined by the information available per alternative, the type of uncertainty faced by the DM, and the set of potential realizations.

More precisely, the choice made initially by a DM must consider all the potential combinatorial scenarios that derive from the number of modifications that the DM is willing to consider relative to the alternative initially selected whenever the ranking obtained leads to regrettable decisions. Moreover, the sequential evaluations and choice paths necessary to formalize the analysis within each scenario should be designed as part of the decision process resulting from the MADM technique applied.

1.4. Objective and contribution

The main objective of the current paper is to define optimal decision paths within a MADM uncertain setting where the DM is allowed to modify the initial alternative chosen after observing the realizations of its characteristics.

We illustrate and analyze the ranking modifications that arise when considering potential interactions among the realizations of the alternatives and regrettable choices. This analysis yields a combinatorial decision structure within which optimal decision paths are defined and shown to vary depending on the number of modifications allowed to be introduced. Each of these paths represents a decision strategy describing the optimal sequence of choices that should be followed by the DM if, at some point, he decides to modify a given selection and proceed with a different alternative.

We show that the complexity of the combinatorial process increases as we introduce additional regrettable decisions based on the set of available alternatives. For instance, when considering pairs of potential realizations, we must categorize the resulting combinations within two sets of ordered alternatives. The number of categories increases to six when accounting for triples of potential realizations and so forth.

We incorporate the combinatorial decision process within an interval TOPSIS setting. The resulting integrated evaluation framework is used to produce a final ranking of optimal decision paths.

Incorporating sequential evaluations and sequences of alternate choices into a MADM framework guarantees that the potential modifications are formalized before an initial decision is made. This is a particularly important feature of the proposed approach considering the fact that, in real-life applications, information may be costly to retrieve and update and the selection of new alternatives is generally constrained by the existence of structural pecuniary costs (Álvarez et al. [12]; Arikan et al. [13]). From a more general viewpoint, the incorporation of strategic elements into a MCDM setting allows for a substantial number of extensions into a completely unstudied area of research.

The proposed integrated model could have been defined using different MADM models such as, for instance, SAW or VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje). However, the combinatorial process does not modify the features of the MADM technique with which it is integrated. Hence, there are no specific advantages or limitations in implementing one method in place of another, if not those already considered in the MADM literature for the aforementioned techniques.

Finally, a case study analyzing the entry decision of a firm within a group of European countries based on their levels of ICT development is presented. We incorporate to the MADM framework the effects derived from the resolution of uncertainty and the capacity of firms to enter different countries after making suboptimal decisions. We illustrate how the countries selected and their order may differ substantially when accounting for the complementarities existing among them. Moreover, the selection process and any subsequent decision vary with the number of modifications considered relative to the initial country selected. The increase in complexity that follows from incorporating additional potentially regrettable alternatives to the choice process of the DM is also discussed.

1.5. Why using TOPSIS

The reason behind the decision of using TOPSIS to develop the proposed extended MADM approach is threefold. First, TOPSIS constitutes one of the main MADM techniques applied to deal with sustainable scenarios within a wide spectrum of research applications. This is the case, for instance, when considering sustainable transportation problems. Broniewicz and Ogrodnik [14] reviewed this branch of the literature and concluded that AHP and TOPSIS were the methods implemented more frequently, both in crisp and fuzzy environments. Within this area of research, Shen et al. [15] analyzed different green traffic scenarios using TOPSIS while Aljohani and Thompson [16] applied this technique to locate consolidation facilities.

An additional advantage of TOPSIS is its malleability and subsequent capacity to generate hybrid MADM models with other decision techniques, particularly the Analytic Network Process (ANP), in sustainability assessment research involving, for instance, cities (Ozkaya and Erdin, [17]) and big data centers (Zhang and Yang, [18]).

Finally, TOPSIS is one of the main MADM techniques used to evaluate the sustainability of ICT projects. For instance, TOPSIS has been applied to analyze scenarios involving the use of ICTs in European firms (Vasilić et al. [19]), energy policies within the European Union (Andreopoulou et al. [20]), the implementation of circular economy strategies (Husain et al. [21]), sustainable industrial relations (Galik et al. [22]), sustainable smart waste (Seker, [23]) and sustainable business management settings (Singh et al. [24]).

The importance of TOPSIS has consistently increased in the analysis of sustainable supply chains (Rajesh, [25]), particularly in situations dealing with risk and uncertainty (Prakash et al. [26]; Zhang and Song, [27]). Paul et al. [28] provide a comprehensive review of the recent literature on sustainable supply chain management and MADM models.

The use of TOPSIS to analyze the sustainability of Industry 4.0 environments represents one of its most recent areas of application (Nara et al. [29]; Samadhiya et al. [30]).

The rest of the paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the basic TOPSIS environment. Section 4 defines the assumptions required to develop the evaluation framework introduced in Section 5, which is extended through Sections 6 and 7. Section 8 analyzes the case study. Section 9 discusses the policy implications that can be drawn from of the results obtained paying particular attention to strategic sustainable development environments. Section 10 concludes and suggests potential extensions.

2. Literature review

Fernández-Portillo et al. [31] found a positive relationship between ICT development and economic growth when analyzing a sample of European Union countries. The intuition supporting this result builds on the research linking GDP and ICT (Ho et al. [32]; Vu, [33]; Warr and Ayres, [34]). In this regard, the relationship between ICT developments and economic growth has been validated at the country (Venturini, [35]) and company levels (Albiman and Sulong [36]; Gërguri-Rashiti et al. [37]). At the same time, the relation existing between ICT and technological change implies productivity increments (Jorgenson and Vu, [38]) and externalities derived from the propagation of knowledge and innovations (Fossen and Sorgner, [39]; Vu, [33]). This last quality is important when considering the entry behavior of firms in countries where they aim at merging or competing with local companies (Álvarez et al. [40]; Sopha et al. [41]).

That is, as highlighted by the international business literature, the interactions and decisions determining the mode of entry into a country are strategic and made under uncertainty (Kim et al. [42]; Klimas et al. [43]). However, much information a firm has, the capacity to enter a market, interact with local firms – which are generally less developed technologically –, and evolve is limited and subject to strategic considerations (Barnard, [44]; Findlay et al. [45]). That is, when receiving and analyzing information regarding the technological development level of a country and its ICT infrastructure, firms do not know to what extent their subsequent interactions will be successful.

The set of frictions that may be triggered by the entry decision is not limited to the risks involved in the interactions with local firms and the potential loss of strategic information, but extends to incompatibilities, losses to competitors who select better partners, institutional problems, and many others. The literature on this topic is quite extensive and wide-ranging (Baier-Fuentes et al. [46]; Guimarães et al. [47]; O'Connor et al. [48]; Popli et al. [49]; Ragmoun, [50]; Strange et al. [51]).

The operations research and management literature has consistently analyzed the uncertainty existing within business environments and its strategic consequences. For instance, Bahli and Rivard [52] analyzed different measures of the risk factors existing in information technology outsourcing environments. Rodríguez et al. [53] designed a fuzzy hybrid model to analyze and evaluate risks in information technology projects. Zare et al. [54] reviewed the Multiple Criteria Decision Making (MCDM) literature on E-learning, while Mardani et al. [55] focused on fuzzy developments of the Stepwise Weight Assessment Ratio Analysis (SWARA) and Weighted Aggregated Sum Product Assessment (WASPAS) techniques. Bolukbas and Guneri [56] clustered firms within categories determined by their technology management competencies. Chen and Ming [57] integrated the best–worst method and data envelopment analysis within a rough environment to select the service module of smart products. Sotoudeh-Anvari [58] concluded that TOPSIS was one of the main MADM methods applied to analyze the consequences of the COVID-19 pandemic.

Fuzzy TOPSIS models may incorporate strategic considerations but do not consider the probabilistic consequences derived from the set of potential realizations observed after making an initial choice (Santos Arteaga et al. [59]). For instance, Karabašević et al. [60] extended TOPSIS to incorporate neutrosophic sets and applied the resulting model to the selection of strategies for e-commerce. Ocampo et al. [61] extended the TOPSIS-Sort model within an intuitionistic fuzzy environment to categorize the degree of exposure of customers to COVID-19 in the Philippine restaurant industry. Li et al. [62] used a fuzzy TOPSIS model to implement an information fusion approach when dealing with expert reliability problems.

Similarly, the literature on interval TOPSIS – a model designed specifically to deal with interval evaluation uncertainties – does not consider the set of sequential interactions that arise after an initial choice is made and the subsequent realizations observed. This literature focuses on the capacity of TOPSIS to incorporate interval evaluations and the resulting applications (Jahanshahloo et al. [63]; Dymova et al. [1]). The same problem arises when considering alternative formulations such as fuzzy (Wang and Elhag, [64]) and Bag-Based TOPSIS (Rebai, [65]). A recent extension into the regret domain is provided by Zhu et al. [66], who combined regret theory with the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II) to cluster heart disease patients within a fuzzy environment.

Given these premises and considering the fact that firms may modify their initial decisions if interactions do not proceed as expected, we incorporate to our MADM framework the effects derived from the resolution of uncertainty and the capacity of firms to enter different countries after making suboptimal decisions. We will analyze the behavior of different combinatorial environments determined by the set of potential realizations of the interval evaluations received by DMs. These scenarios will allow us to illustrate the increase in complexity that follows from incorporating additional potentially regrettable alternatives to the choice process of DMs.

Table 1
The current literature on TOPSIS extensions vs. the current study.

TOPSIS extensions			
Fuzzy strategic	Manipulation	Fuzzy (HFN) manipulation	Combinatorial regret fuzzy (HFN) interval manipulation
Karabašević et al. [60]		Santos Arteaga et al. [59]	
Ocampo et al. [61]	Dong et al. [67]		Future research
Li et al. [37]			
Interval		**Combinatorial regret interval**	
Jahanshahloo et al. [63]		Current paper	
Dymova et al. [1]			

Table 1 summarizes the current literature dealing with extensions of TOPSIS to fuzzy, fuzzy interval and/or manipulation environments. This table also shows where the contribution of the current study stands within the existing literature. The future research lines which are naturally implied by the published works and the approach proposed in this study are also displayed. The next research steps are further described in the policy implication section (Section 9).

3. TOPSIS environment

We summarize the steps composing the TOPSIS technique and illustrate how it is not well suited to handle uncertain information using the expectations operator. In a nutshell, TOPSIS allows to rank several alternatives based on different criteria, whose values are either observed by the DM or received from one or a series of experts. TOPSIS computes positive and negative ideal reference points for each criterion and calculates the relative distance between each of the characteristics defining the different alternatives and these reference values.

Denote by $A_1, A_2, \ldots, A_m$ the m alternatives available to the DM and by $C_1, C_2, \ldots, C_n$ the n evaluation criteria. We represent via x_{ij} the performance of alternative A_i, $i = 1, \ldots, m$, in terms of criterion C_j, $j = 1, \ldots, n$. These performances compose the following decision matrix

	C_1	C_2	$\ldots$	C_n
A_1	x_{11}	x_{12}	$\ldots$	x_{1n}
A_2	x_{21}	x_{22}	$\ldots$	x_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\ldots$	x_{mn}
	$W = [w_1, w_2, \ldots, w_n]$			

The relative weight or importance assign to each criterion j is given by the terms $w_j, j = 1, \ldots, n$. Criteria can be categorized as positive or negative depending on whether higher or lower values are preferred by the DM, respectively.

TOPSIS implements the following set of steps to rank the alternatives.

Step 1. Normalize the decision matrix so that criteria can be directly compared:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_i x_{ij}^2}}, i = 1, \ldots, m,\ j = 1, \ldots, n; \tag{1}$$

Step 2. Multiply each column of the decision matrix by the corresponding weight:

$$v_{ij} = w_j r_{ij},\ j = 1, \ldots, n, \tag{2}$$

to obtain the weighted normalized decision matrix.

Step 3. Compute the ideal positive, v_i^+, and negative, v_i^-, reference values per criterion. The ideal criteria values are summarized in the following vectors:

$$A^+ = \left(v_1^+, \ldots, v_n^+\right) \tag{3}$$

with

$$v_i^+ = \max\left\{(v_{ij}) | j \in \textit{positive criteria}\right\} \tag{4}$$

$$v_i^+ = \min\left\{(v_{ij}) | j \in \textit{negative criteria}\right\} \tag{5}$$

when accounting for the best positive values, and

$$A^- = \left(v_1^-, \ldots, v_n^-\right) \tag{6}$$

where

$$v_i^- = \min\left\{(v_{ij}) | j \in \textit{positive criteria}\right\} \tag{7}$$

$$v_i^- = \max\left\{(v_{ij}) | j \in \textit{negative criteria}\right\} \tag{8}$$

when considering the worst negative values.

Step 4. Compute the distances between the value of each positive and negative characteristic per alternative and the ideal ones:

$$d_i^+ = \left[\sum_{j=1}^{n} \left(v_j^+ - v_{ij}\right)^2\right]^{1/2}, \quad i = 1, \ldots, m \tag{9}$$

$$d_i^- = \left[\sum_{j=1}^{n} \left(v_j^- - v_{ij}\right)^2\right]^{1/2}, \quad i = 1, \ldots, m \tag{10}$$

Step 5. Compute the relative distance of each alternative A_i from the negative ideal solution using the d_i^+ and d_i^- values. This distance determines the position of an alternative within the ranking.

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, \ldots, m \tag{11}$$

The scores assigned to each alternative range between the highest value of $R_i = 1$, and the lowest one given by $R_i = 0$.

3.1. Basic numerical example

We illustrate through a basic numerical example the consequences from incorporating uncertainty in the evaluations and making decisions via the expectations operator. Consider a decision environment defined by three alternatives and two criteria summarized through the following decision matrix:

	C_1	C_2
A_1	10	0
A_2	5	5
A_3	0	10
	$W = [w_1, w_2] = [0.5, 0.5]$	

For expositional simplicity, assume that both criteria correspond to positive characteristics. It is intuitively clear that implementing TOPSIS will lead to an identical evaluation and rank of the three alternatives. The weighted normalized decision matrix is given by:

	C_1	C_2
A_1	0.4471	0
A_2	0.22361	0.22361
A_3	0	0.44721

leading to a value of $R_i = \frac{1}{2}$, $i = 1, 2, 3$. As a result, DMs should be indifferent between the three alternatives.

Consider now an uncertain setting where the DM receives evaluation intervals defining the set of potential realizations of each characteristic per alternative. Assume that the evaluations provided correspond to the best potential realization that may be observed. That is, alternatives displaying higher valued are expected to perform better than those with lower values to the extent reported in the evaluations.

If DMs were to compute the expected values from each evaluation interval using a uniform distribution to account for the uncertainty, we would obtain the following decision matrix:

	C_1	C_2
A_1	5	0
A_2	2.5	2.5
A_3	0	5

The entries of this matrix are given by the expected value of the uncertain realizations defined over the different evaluation intervals, that is, for $i = 1, 2, 3$ and $j = 1, 2$, the ijth entry is the value of $\int_0^{x_{ij}^r} \frac{1}{x_{ij}^r} x_{ij} dx_{ij}$, with x_{ij}^r corresponding to the upper limit of the evaluation domains. For example, the first entry is given by $\int_0^{x_{11}^r} \frac{1}{x_{11}^r} x_{11} dx_{11} = \int_0^{10} \frac{1}{10} x_{11} dx_{11}$. Once again, TOPSIS delivers identical values of $R_i = \frac{1}{2}$, $i = 1, 2, 3$, for all the alternatives.

The uncertainty inherent to the evaluations received leads to a natural question: what would happen if the DM could account for the realization observed after the uncertainty is resolved when selecting the initial alternative? Furthermore, how should the DM behave if the alternative chosen initially performs worse than expected and a different alternative had to be selected?

The answer delivered by TOPSIS is simple, given the identical ranking of all the alternatives, any choice sequence is as good as the others. We illustrate how this is not the case if the DM incorporates the set of potential consequences derived from any decision made when selecting a sequence of alternatives.

4. Combinatorial regret and dynamic behavior

In order to simplify notation, we focus on a unique characteristic per alternative. Let x^i, x^s and x^k be the evaluation values assigned to alternatives i, s and k, with $i \neq s \neq k$. Intuitively, the evaluations received assign a relative position to each alternative in terms of DM preferences. The potential realizations of alternative i will be distributed within the interval $[0, y^{iM}]$, with $y^i \in [0, y^{iM}]$ referring to one of these realizations and the superscript M to the upper limit value of the interval. The same notation and intuition apply to all remaining alternatives.

Note that, while the upper limits of the sets of potential realizations of the different variables are generally different (i.e., $y^{iM} \neq y^{sM}$), the lower limits of the intervals have been all unified at the value of zero. That is, for i and s with $i \neq s$, we have $y^i \in [0, y^{iM}]$ and $y^s \in [0, y^{sM}]$. This assumption has been introduced to simplify both notations and computations, without leading to the generality of the results. Relaxing this assumption by assigning different positive lower limit values would complicate the presentation without modifying the qualitative results obtained.

We assume that the relative position of the potential realizations reflects the uncertainty inherent to the evaluations, that is, $x^i > x^s > x^k$ implies $y^{iM} > y^{sM} > y^{kM}$ for any i, s and k, with $i \neq s \neq k$. We simplify the presentation and subsequent computations by assuming that $y^{iM} = x^i$ for every evaluation i. This notation has been introduced to differentiate the set of potential realizations y^i from the categorization derived from the initial evaluations, represented by x^i.

Finally, the numerical illustration of the evaluation and regret processes requires the introduction of probability functions $f_i : [0, y^{iM}] \to [0, 1]$, with $i = 1, \ldots, m$, to describe the beliefs of the DM regarding the distribution of potential realizations derived from the choice of a given alternative.

4.1. Uncertain interval evaluations

Given the categorization of the ith alternative through x^i, the DM considers the set of potential realizations derived from $[0, y^{iM}]$ after the alternative is selected. The uncertainty inherent to the characteristics defining the alternatives implies that the DM faces a set of potential realizations that must be accounted for in the initial decision. Information entropy is maximized through a uniform density defined on the set of potential realizations $[0, y^{iM}]$ as follows:

$$f_i(y^i) = \begin{cases} \frac{1}{x^i} & \text{if } y^i \in [0, y^{iM}] = [0, x^i] \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The main results derived from the model are independent of the density function defined for $[0, y^{iM}]$ and remain valid when introducing different probability functions, such as a normal.

4.2. Negative criteria

Note that the descriptions provided so far correspond to positive characteristics of the alternatives. The intuition regarding negative characteristics is similar. In this case, the intervals of potential realizations would be given by $[y^{im}, M]$, with y^{im} referring to the lowest potential realization and M to the highest value of the characteristic. The combinatorial structure of the model would however differ depending on the type of characteristic considered, as will be illustrated through the next sections.

Note that in this case, the lower limits of the sets of potential realizations are the values allowed to be generally different, while the upper limits are assumed to take all the same value M. That is, for i and s with $i \neq s$, $y^i \in [y^{im}, M]$ and $y^s \in [y^{sm}, M]$. As for positive characteristics, the introduction of this assumption serves the purpose of simplifying both notations and computations. Assigning different negative upper limit values would imply an adjustment of the formulas and a more complex presentation but will not modify the qualitative results obtained.

5. Combinatorial value functions with two alternatives

The combinatorial structure of the value functions is determined by the potential realizations of the characteristics defining the alternatives and the order in which they are selected by the DM. We will assume that the i, s and k sequence defines the order in which alternatives are selected by DMs.

5.1. $x^i \leq x^s$ setting

Consider the case with one regrettable decision and two alternatives. Two potential evaluation scenarios can be defined depending on the relative position of the categorization variables. We start by focusing on the $x^i \leq x^s$ setting, where the first alternative selected is ranked within a lower category than the second. The set of potential realizations is calculated through a

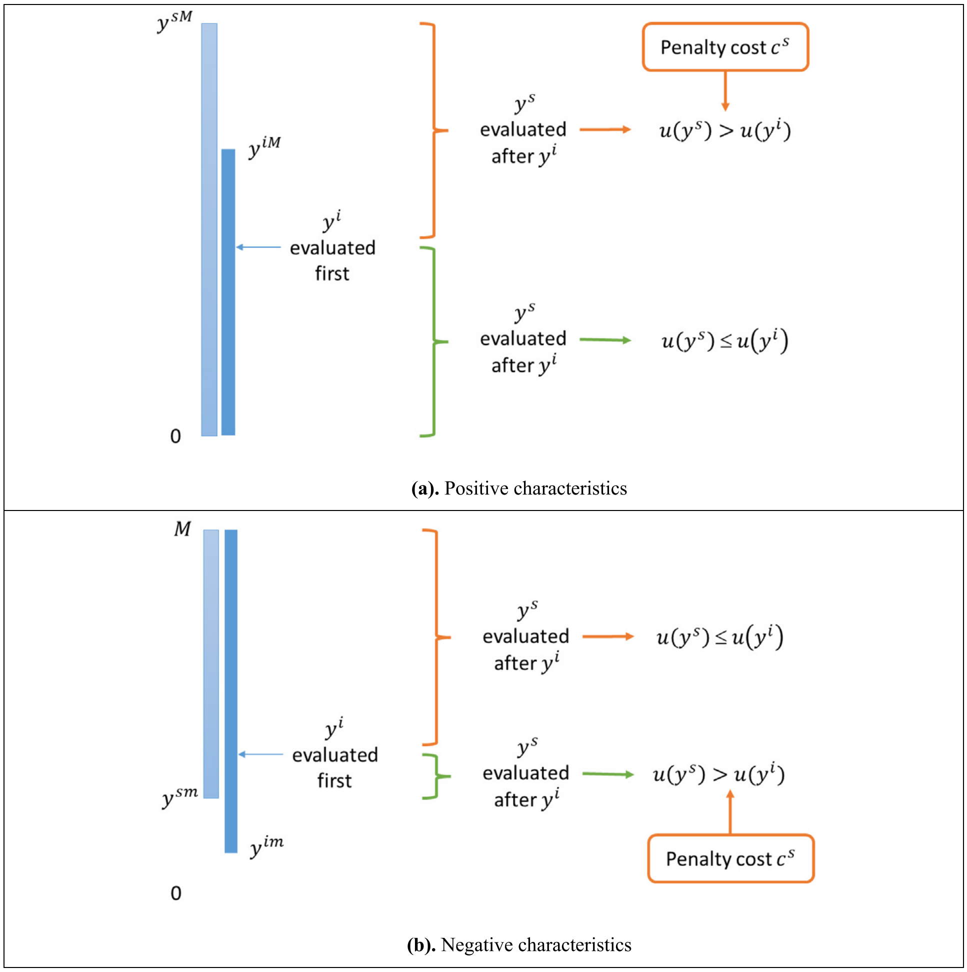


Fig. 1. Relative positions of the potential realizations y^i and y^s in the $x^i \leq x^s$ setting.

value function defined for every possible pair (y^i, y^s) within their respective domains

$$V(x^i, x^s, y^{iM}, y^{sM}) = \int_0^{y^{iM}} \frac{1}{x^i} \left[\int_0^{y^i} \frac{1}{x^s} \left(y^i\right) dy^s + \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s \right] dy^i, \quad y^{iM} \leq y^{sM} \tag{13}$$

where the densities associated with each set of potential realizations, y^i and y^s, are given by $\frac{1}{x^i}$ and $\frac{1}{x^s}$, respectively.

The categorization of alternatives i and s is reflected in the upper realization limits $y^{iM} \leq y^{sM}$. The highly ranked alternative selected in second place may improve upon the first one ranked lower. A penalty cost c^s is introduced to reflect the potentially suboptimal initial decision made. That is, after selecting the alternative i and observing y^i, the potential realization of the next alternative s, y^s, may be located within either $[0, y^i]$ or $[y^i, y^{sM}]$. These potential realizations are formalized by the integrals $\int_0^{y^i} \frac{1}{x^s} \left(y^i\right) dy^s$ and $\int_{y^i}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s$ for the cases $y^s \in [0, y^i]$ and $y^s \in [y^i, y^{sM}]$, respectively.

Fig. 1(a) provides a graphical illustration of the potential realizations y^i and y^s along their respective domains within the $x^i \leq x^s$ setting. The figure describes the different realizations y^s that may be observed relative to an initial realization y^i, imposing a penalty cost to the DMs if modifying their initial choices.

The same intuition applies to the negative criteria case described below.

5.1.1. $x^i \leq x^s$ setting: negative criteria

A different set of value functions must be defined when considering negative criteria. The intuition is similar to the positive case, except for the fact that the evaluations received should be considered as the minimum potential realizations that may be observed.

The required modifications are implemented within the value function described in Eq. (14), where the intervals of potential realizations determine the limits of the corresponding densities. Note that, as in the positive case, the value function is determined by the order in which the alternatives are evaluated as well as their relative values.

$$V_{[-]}(x^i, x^s, y^{im}, y^{sm}) = \int_{y^{im}}^{y^{sm}} \frac{1}{M - x^i} (y^i) dy^i + \int_{y^{sm}}^{M} \frac{1}{M - x^i} \left[\int_{y^{sm}}^{y^i} \frac{1}{M - x^s} \left(y^s + c^s\right) dy^s + \int_{y^i}^{M} \frac{1}{M - x^s} \left(y^i\right) dy^s \right] dy^i, \quad y^{im} \leq y^{sm} \tag{14}$$

Fig. 1(b) describes a scenario with two potential realizations, y^i and y^j, within the $x^i \leq x^j$ negative criteria setting. Note how, in this case, a lower domain constitutes a potential advantage for the alternative evaluated initially.

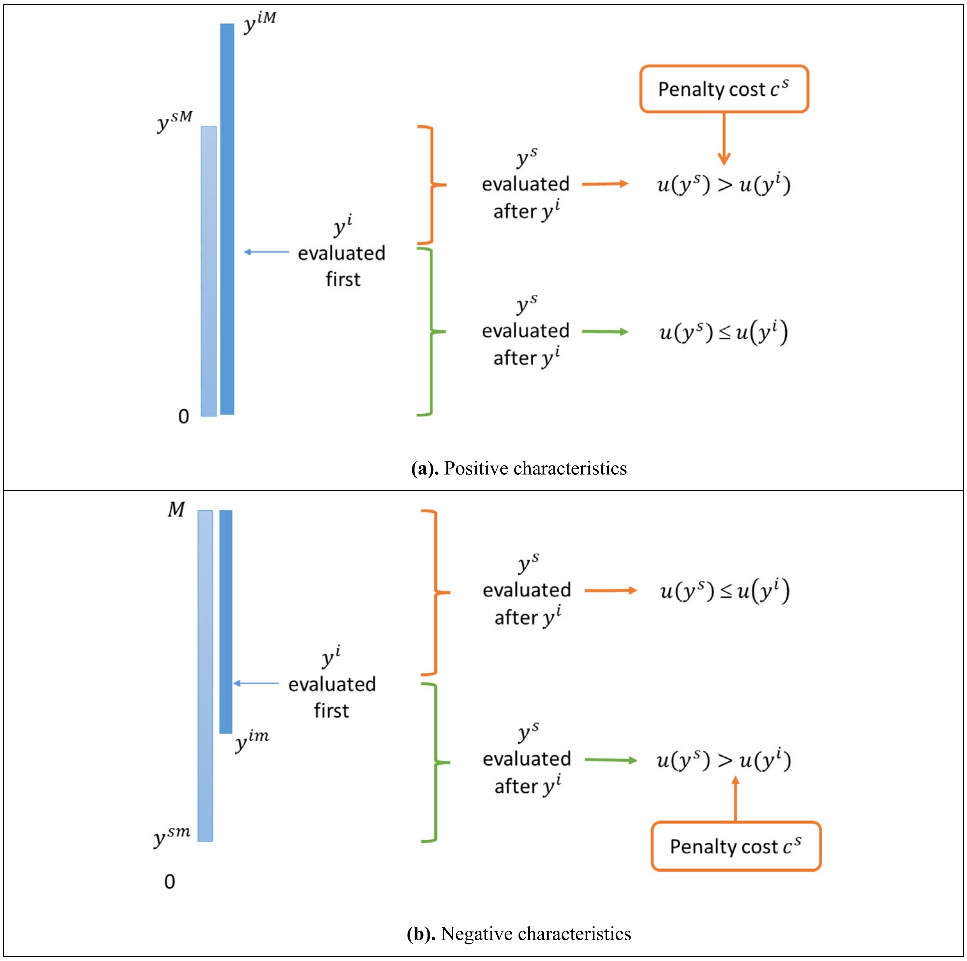


Fig. 2. Relative positions of the potential realizations y^i and y^s in the $x^i > x^s$ setting.

5.2. $x^i > x^s$ setting

We analyze now the scenario where the first alternative selected is the one ranked higher, that is, $x^i > x^s$. The value function must be adapted to the respective domains categorizing the alternatives as follows

$$V(x^i, x^s, y^{iM}, y^{sM}) = \int_{y^{sM}}^{y^{iM}} \frac{1}{x^i}\left[y^i\right] dy^i + \int_0^{y^{sM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^s}\left(y^i\right) dy^s + \int_{y^i}^{y^{sM}} \frac{1}{x^s}\left(y^s - c^s\right) dy^s\right] dy^i, \quad y^{iM} > y^{sM} \tag{15}$$

The realization of y^i may now be higher than that of y^{sM}, as can be inferred from the first term composing the value function. y^{sM} corresponds to the upper limit of the domain on which y^s is defined. The second realization can either improve upon the first one, incurring a cost of c^s, or provide a lower value. The expressions within the second term composing the value function describe the outcome from the realizations y^s located within $[0, y^i]$ and $[y^i, y^{sM}]$, respectively.

Note that alternatives belonging to categories with higher limit values may underperform relative to those in lower ones. Fig. 2(a) illustrates the domains describing the potential realizations y^s for a given initial value of y^i within the $x^i > x^s$ scenario.

5.2.1. $x^i > x^s$ setting: negative criteria

When accounting for a negative criterion within the $x^i > x^s$ setting, we must once again adapt the value function to the respective domains categorizing the alternatives.

$$V_{[-]}(x^i, x^s, y^{im}, y^{sm}) = \int_{y^{im}}^{M} \frac{1}{M - x^i}\left[\int_{y^{sm}}^{y^i} \frac{1}{M - x^s}\left(y^s + c^s\right) dy^s + \int_{y^i}^{M} \frac{1}{M - x^s}\left(y^i\right) dy^s\right] dy^i, \quad y^{im} > y^{sm} \tag{16}$$

Fig. 2(b) presents the domains describing the potential realizations y^s for a given initial value of y^i within the $x^i > x^s$ setting. In this case, a higher initial evaluation may constitute a drawback for the initial alternative, an effect reflected in the first term of Eq. (16), while the second term represents the potential realizations of the initial alternative that outperform those of the second.

6. Incorporating a third alternative

The complexity of the value function increases with every new alternative incorporated to the analysis. Note that whenever a new alternative is added, we must account for two different sets of combinations, namely, those following from the alternatives selected and the order of selection. This implies that the subsequent uncertainty inherent to the potential realizations must be adapted to the domain limits described by the order in which the alternatives are selected and evaluated. In addition, the value received after proceeding through a first and second round of regret must incorporate an increasing penalty that will be assumed to be linear and given by c^s and $2c^s$, respectively.

Consider three different potential realizations denoted by $y^i, y^s, y^k \in Y$ for alternatives i, s, and k, with $i \neq s \neq k$. A total of six permutations will be required to define the value function $V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM})$, which, as stated above, will be conditioned by the order in which the alternatives are selected and evaluated.

6.1. $x^i \leq x^s \leq x^k$ setting

As an illustrative example, we consider the $x^i \leq x^s \leq x^k$ scenario. In this case, x^s can improve upon x^i through the section of the y^s domain located above y^{iM}. The same intuition applies to the potential realizations y^k relative to y^{iM} and y^{sM}. The required modifications have been added to the value function described in Eq. (17), where the different intervals of potential realizations determine the limits of the corresponding densities.

$$V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \int_0^{y^{iM}} \frac{1}{x^i} \left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^s} \left[\int_0^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k \right. \\ \left. + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s + \\ \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left[\int_0^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k \right. \\ \left. + \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s \end{array} \right] dy^i \tag{17}$$

Four terms compose this value function. The first two are located within the domain where the realizations of the second alternative are below those of the first and the third. The first term describes the case where the initial selection delivers the highest realization. The second term corresponds to the case where the final selection provides the highest realization at a cost of $2c^s$. The complementary interval – which includes the realizations of the second evaluation higher than those of the first one – contains the third and fourth terms of the value function. In this case, the realizations of the third alternative are respectively lower and higher than those of the second, as intuitively illustrated by the corresponding costs introduced within each term.

6.1.1. $x^i \leq x^s \leq x^k$ setting: negative criteria

As in the positive case, six permutations are required to define the value function $V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km})$, conditioned by the order in which the alternatives are selected and evaluated.

As an illustrative comparative example, consider the $x^i \leq x^s \leq x^k$ scenario. In this case, x^i is better positioned than x^s since a section of the y^i domain is located below y^{sm}. The same intuition applies to the potential realizations y^k relative to y^{im} and y^{sm}, given the lower negative evaluations received.

The i, s, k sequence conditions the evaluation process, which must account for the different permutations that can be defined based on the relative value of the alternatives. As a result, each value function will differ significantly from the others, though they all follow the same intuition.

$$V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \int_{y^{im}}^{y^{sm}} \frac{1}{M - x^i} \left[y^i\right] dy^i + \int_{y^{sm}}^{y^{km}} \frac{1}{M - x^i} \left[\int_{y^{sm}}^{y^i} \frac{1}{M - x^s} \left(y^s + c^s\right) dy^s + \int_{y^i}^{M} \frac{1}{M - x^s} \left(y^i\right) dy^s \right] dy^i + \int_{y^{km}}^{M} \frac{1}{M - x^i} \left[\begin{array}{l} \int_{y^{sm}}^{y^{km}} \frac{1}{M - x^s} \left[y^s + c^s\right] dy^s + \\ \int_{y^{km}}^{y^i} \frac{1}{M - x^s} \left[\int_{y^{km}}^{y^s} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \right. \\ \left. + \int_{y^s}^{M} \frac{1}{M - x^k} \left(y^s + c^s\right) dy^k \right] dy^s + \\ \int_{y^i}^{M} \frac{1}{M - x^s} \left[\int_{y^{km}}^{y^i} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \right. \\ \left. + \int_{y^i}^{M} \frac{1}{M - x^k} \left(y^i\right) dy^k \right] dy^s \end{array} \right] dy^i \tag{18}$$

Five terms compose this value function. The first two are located within the domain where the realizations of the first alternative are located below those of the second and the third. The first term describes the case where the initial selection delivers the lowest negative realization. That is, the choice of the i alternative as the first one composing the ranking was the correct one and no penalty must be added to the evaluation. The second term correspond to the case where the second and the initial alternative exhibit the lowest realizations, respectively. Note that, if the second alternative displays the lowest realization, a cost of c^s must be added to the resulting expression.

The complementary interval includes realizations from all the alternatives, where each of them can be lower than the others, imposing the corresponding costs if selected in second or third place. The first subcase describes the second evaluation being lower than the initial and third one. The second term contains the cases where the first alternative displays a higher evaluation, while the third and second alternatives exhibit the lowest ones, respectively. The final subcase is given by the third and the initial alternative having the lower evaluations. A penalty of $2c^s$ is applied to the former term.

The remaining set of combinatorial scenarios that must be defined by the DM when adding a third variable to the value function is described in the Appendix section.

7. Further intuition: $x^i \leq x^s \leq x^k \leq x^g$ setting

Adding a fourth alternative increases the complexity of the combinatorial problem. As an illustrative example, we define the initial setting out of a total of 24 composing the evaluation scenarios. We focus on this setting due to its relative simplicity among those that must be generated, an intuition that follows from those presented when considering three alternatives.

The simplicity of the evaluation process stems from the selection of alternatives, which are ordered following an increasing pattern, allowing for an intuitive understanding of the interactions among the different domains defining the integration limits.

$$
\begin{aligned}
&V(x^i, x^s, x^k, x^g, y^{iM}, y^{sM}, y^{kM}, y^{gM}) = \\
&\int_0^{y^{iM}} \frac{1}{x^i} \Bigg[\int_0^{y^i} \frac{1}{x^s} \Bigg[\int_0^{y^i} \frac{1}{x^k} \Bigg[\int_0^{y^i} \frac{1}{x^g} \left(y^i\right) dy^g + \int_{y^i}^{y^{gM}} \frac{1}{x^g} \left(y^g - 3c^s\right) dy^g \Bigg] dy^k + \\
&\qquad \int_{y^i}^{y^{kM}} \frac{1}{x^k} \Bigg[\int_0^{y^k} \frac{1}{x^g} \left(y^k - 2c^s\right) dy^g + \int_{y^k}^{y^{gM}} \frac{1}{x^g} \left(y^g - 3c^s\right) dy^g \Bigg] dy^k \Bigg] dy^s + \\
&\quad \int_{y^i}^{y^{sM}} \frac{1}{x^s} \Bigg[\int_0^{y^s} \frac{1}{x^k} \Bigg[\int_0^{y^s} \frac{1}{x^g} \left(y^s - c^s\right) dy^g + \int_{y^s}^{y^{gM}} \frac{1}{x^k} \left(y^g - 3c^s\right) dy^g \Bigg] dy^k + \\
&\qquad \int_{y^s}^{y^{kM}} \frac{1}{x^k} \Bigg[\int_0^{y^k} \frac{1}{x^g} \left(y^k - 2c^s\right) dy^g + \int_{y^k}^{y^{gM}} \frac{1}{x^g} \left(y^g - 3c^s\right) dy^g \Bigg] dy^k \Bigg] dy^s \Bigg] dy^i
\end{aligned}
\tag{19}
$$

The value function is relatively intuitive and can be described following a sequential evaluation structure:

1. The realization of the first variable, y^i, endowed with the lowest upper limit of the domain, may either not be improved upon by the next alternative, $y^s \in \left[0, y^i\right]$, defining the upper section of Eq. (19), or it may be improved upon by the next alternative, $y^s \in \left[y^i, y^{sM}\right]$, which constitutes the lower section of Eq. (19).
2. The next variable, y^k, may (when $y^k \in \left[y^i, y^{kM}\right]$), or may not (when $y^k \in \left[0, y^i\right]$), improve relatively to y^i, whenever y^s does not improve upon y^i (i.e., $y^s \in \left[0, y^i\right]$). These possibilities define the next set of potential realizations within the expression of the upper section. At the same time, y^k may (when $y^k \in \left[y^s, y^{kM}\right]$), or may not (when $y^k \in [0, y^s]$), improve relatively to y^s, whenever y^s improves upon y^i (i.e., $y^s \in \left[y^i, y^{sM}\right]$) defining the next set of potential realizations within the expression of the lower section.
3. We conclude the analysis considering the realizations y^g described within the corresponding sets of brackets, which may be located

    - below or above y^i when all the previous realizations are below y^i;
    - below or above y^k whenever this last variable is above y^i and y^s but this latter variable is below y^i.

    Both possibilities define the upper section of the value function. At the same time, the realizations y^g may be located

    - below or above y^s when the previous realizations are above y^i but below y^s;
    - below or above y^k whenever this last variable is above y^i and y^s.

Note how the evaluation patterns provide a consistent description of the potential realizations that may be observed depending on those of the previous variables. At the same time, the sequential evaluation structure highlights the need for a heuristic mechanism to be defined and implemented whenever additional variables are added to the analysis. This last feature depends on the number of regrettable choices that the DM is willing to consider, a quality limited by real-life constraints regarding the subsequent costs implied. We discuss this possibility more in detail within the conclusion section.

7.1. Back to the basic interval TOPSIS

We illustrate the consequences from incorporating sequential regrettable decisions by applying our formal framework to the basic numerical example presented in Section 3.1. The table below describes the combinations that may be defined based on the set of potential pairs of realizations derived from the three alternatives composing the decision problem. The expected evaluations per pair of alternatives are determined by the different potential realizations that may be observed when incorporating a penalty cost equal to $c^s = 0.1$.

Combinatorial scenarios and expected evaluations

Alternatives	C_1 pairs	C_2 pairs	$V(C_1, c^s)$	$V(C_2, c^s)$
A_1A_2	([0,10], [0,5])	([0], [0,5])	5.3917	2.4
A_1A_3	([0,10], [0])	([0], [0,10])	5	4.9
A_2A_1	([0,5], [0,10])	([0,5], [0])	5.3417	2.5
A_2A_3	([0,5], [0])	([0,5], [0,10])	2.5	5.3417
A_3A_1	([0], [0,10])	([0,10], [0])	4.9	5
A_3A_2	([0], [0,5])	([0,10], [0,5])	2.4	5.3917

The weighted decision matrix and subsequent R_i values are given by:

	C_1	C_2	R_i
A_1A_2	0.24752	0.11018	0.5
A_1A_3	0.22954	0.22495	0.85159
A_2A_1	0.24522	0.11477	0.50439
A_2A_3	0.11477	0.24522	0.50439
A_3A_1	0.22495	0.22954	0.85159
A_3A_2	0.11018	0.24752	0.5

The resulting ranking of alternatives equals $A_1A_3 \sim A_3A_1 \succ A_2A_1 \sim A_2A_3 \succ A_1A_2 \sim A_3A_2$. It can be observed how the first and third alternatives are preferred over any combination that includes the second alternative. We must note that the introduction of penalty costs conditions the expected values obtained, allowing the order in which alternatives are selected to determine the resulting values.

8. Numerical evaluations and frequency distributions

Tables 2 and 3 describe the main characteristics of the countries analyzed within the classification problems considered. These tables differ due to the lack of data regarding the [TIN00085] and A1 variables described below. These variables are not available for all countries and have therefore been omitted from the analysis in Table 3. This drawback is compensated by an increment in the sample of countries. The results will obviously differ, but the objective of the paper is to illustrate the combinatorial evaluation and choice paths arising as potentially regrettable decisions are incorporated into the analysis.

We have chosen 2017 as the year to analyze the ICT status of these countries since it is the last year for which the ICT Development Index is available from the International Telecommunication Union (ITU) of the United Nations. It is also one of the most complete years in terms of data availability for the countries

Table 2
Value of the variables used in the initial classification problem.

		Eurostat variables							OECD variables		ITU variable
		[TIN00115]	[TIN00110]	[TIN00125]	[TIN00116]	[TIN00090]	[TIN00111]	[TIN00085]	A1	C5B	IDI relative
1	Bulgaria	17	5	51	13	89	7	2.71	30.16	63.41	76.39
2	Czechia	12	31	79	16	98	24	3.07	48.83	84.64	79.73
3	Estonia	15	16	74	15	95	16	4.09	48.47	88.10	90.65
4	Greece	10	4	52	15	85	11	1.44	43.71	69.89	80.51
5	Croatia	19	11	83	12	95	18	2.49	50.24	67.10	80.62
6	Lithuania	28	13	88	24	100	22	2.57	45.95	77.62	80.07
7	Hungary	9	20	70	9	91	13	3.56	47.17	76.75	77.17
8	Poland	21	15	70	16	95	10	2.47	45.61	75.99	76.73
9	Romania	7	8	50	13	82	8	2.36	35.56	63.75	72.16
10	Slovenia	15	16	81	13	99	18	2.66	56.63	78.89	82.18
11	Slovakia	15	22	82	17	95	15	3.18	50.42	81.63	78.62

Table 3
Value of the variables used in the second classification problem.

		Eurostat variables						OECD variable	ITU variable
		[TIN00115]	[TIN00110]	[TIN00125]	[TIN00116]	[TIN00090]	[TIN00111]	C5B	IDI relative
1	Bulgaria	17	5	51	13	89	7	63.41	76.39
2	Czechia	12	31	79	16	98	24	84.64	79.73
3	Estonia	15	16	74	15	95	16	88.10	90.65
4	Greece	10	4	52	15	85	11	69.89	80.51
5	Croatia	19	11	83	12	95	18	67.10	80.62
6	Latvia	6	9	73	13	99	11	81.32	80.85
7	Lithuania	28	13	88	24	100	22	77.62	80.07
8	Hungary	9	20	70	9	91	13	76.75	77.17
9	Poland	21	15	70	16	95	10	75.99	76.73
10	Portugal	17	16	71	18	98	18	73.79	79.40
11	Romania	7	8	50	13	82	8	63.75	72.16
12	Slovenia	15	16	81	13	99	18	78.89	82.18
13	Slovakia	15	22	82	17	95	15	81.63	78.62

considered, which concentrate on the eastern members of the European Union together with Portugal and Greece.

The variables used to evaluate the development of the ICT environment and the potential interaction capacities of local firms within each country are taken from three different sources. The variables have been selected based on their availability, avoiding concept overlaps and focusing on the specific qualities of each agency. All variables are positive and considered equally important. The main data sources are Eurostat, the Organisation for Economic Co-operation and Development (OECD), and the ITU. All data is publicly available in the websites of the corresponding agencies.

The main set of variables is taken from Eurostat (https://ec.europa.eu/eurostat/web/main/data/database). These variables describe the ICT capacities of local firms and the relative importance of the ICT sector among the employed population

1. Enterprises whose business processes are automatically linked to those of their suppliers and/or customers [TIN00115]
2. Share of enterprises' turnover on e-commerce - % [TIN00110]
3. Enterprises giving portable devices for a mobile connection to the internet to their employees [TIN00125]
4. Enterprises using software solutions, like CRM to analyze information about clients for marketing purposes [TIN00116]
5. Enterprises with broadband access [TIN00090]
6. Enterprises having received orders online (at least 1%) - % of enterprises [TIN00111]
7. Percentage of the ICT personnel on total employment [TIN00085]

In order to avoid concept overlaps when retrieving data from the OECD database (https://stats.oecd.org/Index.aspx?DataSetCode=ICT_BUS), we focus on the use of technology at work and by the general population

8. A1: Persons employed regularly using a computer in their work (%)
9. C5B: Individuals using the Internet — last 3 m (%) All (individuals aged 16–74)

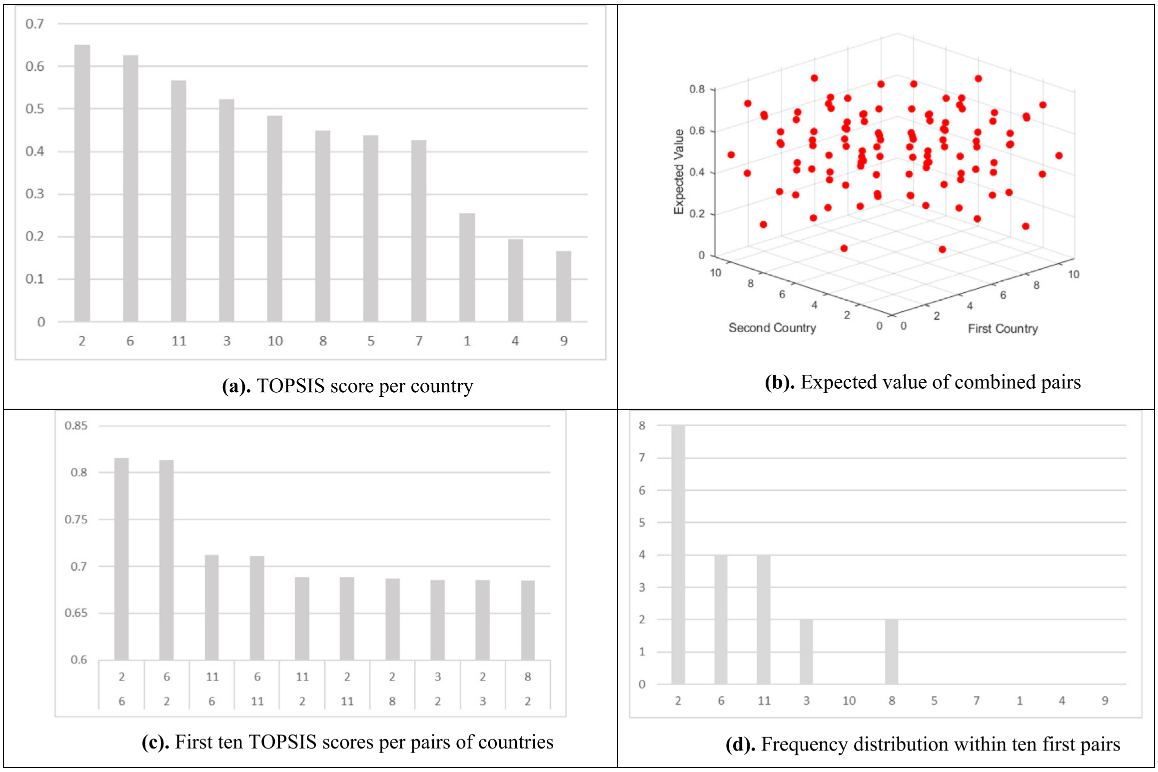
Finally, we have retrieved the value of the last available ICT Development Index from the ITU database (https://www.itu.int/en/ITU-D/Statistics/Pages/IDI/default.aspx). The last index available corresponds to 2017 and has been normalized with respect to the value obtained by the most developed country.

10. IDI Value Relative to that of Iceland

Consider the data provided in Tables 2 and 3. Clearly, these percentages constitute an approximate indicator of how well the interactions arising between local and foreign companies may proceed. However, they are not guarantees of a given outcome. Selecting an alternative endowed with a higher set of potential realizations than another may result in a lower realization. This event happens with lower probability than the opposite one but remains a possibility that must be accounted for when selecting a country. Thus, while the evaluations provide a basic guideline to firms, the latter must also consider the fact that realizations may differ from the expected values, forcing them to change the initial country selected. This is the main feature motivating the implementation of the model introduced in this manuscript.

8.1. Analysis of the results

Figs. 3 and 4 illustrate the expected evaluations obtained when considering combinations of pairs and triples within the initial classification problem, respectively. Fig. 3(a) describes the ranking values R_i, $i = 1, \ldots, 11$, derived from a direct implementation of TOPSIS to the data presented in Table 2. The numerical identification of the countries defined in Table 2 implies that



(a). TOPSIS score per country

(b). Expected value of combined pairs

(c). First ten TOPSIS scores per pairs of countries

(d). Frequency distribution within ten first pairs

Fig. 3. First scenario: Initial results and paired evaluations. Horizontal axes represent countries in all figures.

the first three composing the ranking are Czechia, Lithuania, and Slovakia. Thus, if a firm is deciding which country to enter and interact with from a technological and commercial perspective, TOPSIS provides a direct and clear answer. If the interactions with Czech firms are suboptimal, the DM should proceed with Lithuania, even though this possibility is not explicitly considered.

When accounting for the possibility of making a regrettable decision, we get a total of 110 pairs of combined alternatives. The expected values obtained are presented in Fig. 3(b), where the symmetry in the evaluations can be observed. As already stated, the value differences derived from the order of evaluation are the result of including a cost variable, which, in this case, is assumed equal to $c^s = 0.1$.

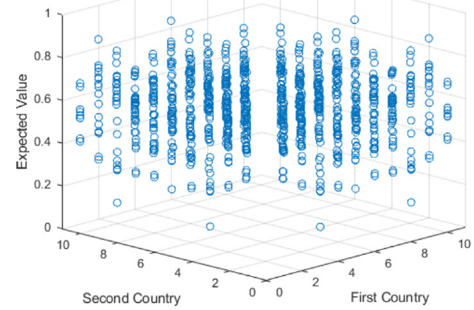
Fig. 3(c) represents the evaluations of the first ten paired alternatives obtained from TOPSIS while Fig. 3(d) illustrates the frequency distribution of the different countries within these first ten pairs. Clearly, highlighting the consistency of the framework implemented, the best potential combinations are the ones following from the two alternatives receiving the highest individual scores. The frequency distribution illustrates how these two alternatives – together with the third one – are the main ones considered by the DM. However, as can be observed in Fig. 3(c), Slovakia combines with Lithuania and not with Czechia within the third and fourth ranking positions, illustrating how the set of combinations may vary from the ones that may be initially expected.

This intuition is validated when analyzing the 990 triples of alternatives described in Fig. 4. As in the previous scenario, Fig. 4(a) presents the evaluation of the first ten triples of alternatives obtained from TOPSIS while Fig. 4(b) represents the expected values of the whole set of combinations excluding the third country from the triple. Fig. 4(c) illustrates the frequency distribution of the different countries within the first ten triples of alternatives. The most interesting feature is the position of Estonia within the first four triples. Note that Estonia was ranked fourth individually but becomes a fundamental alternative when accounting for two potentially regrettable choices. Its frequency overtakes that of Slovakia, which was ranked third individually and relatively more important when considering one regrettable choice. The frequency distribution also illustrates how Poland is eliminated from the group of ten preferred combinations.

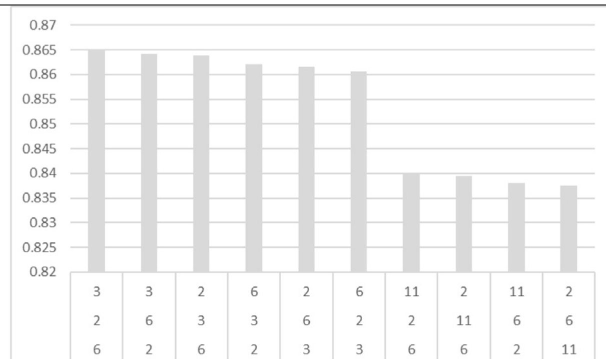
The second classification problem extends the set countries considered while accounting for a lower number of criteria. The number of pairs and triples generated is therefore higher than in the previous problem, resulting in a total of 156 pairs and 1716 triples. Figs. 5 and 6 illustrate the expected evaluations obtained when considering combinations of pairs and triples, respectively. The first three alternatives composing the extended ranking are given by Lithuania, Czechia, and Slovakia.

The conclusions obtained are similar across scenarios. Note, however, that in the latter problem the results follow a more intuitive pattern with combinations of the first three alternatives ranked individually composing most of the preferred pairs and triples. Poland and Croatia, ranked seventh and eighth individually, are included within the first ten pairs. Similarly, Poland arises as a potential option within the triples, displaying an even higher frequency than Portugal, which was ranked fourth individually.

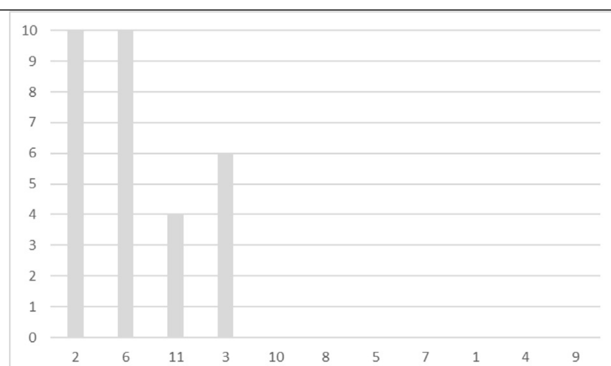
The rankings obtained illustrate how alternatives not necessarily considered from an individual standpoint become important when incorporating potentially regrettable choices and complementarities among their characteristics. Once again, rankings will vary depending on the number of choices that the DM is expected or allowed to regret. Computing the sets of potential combinations may impose a considerable burden on the DM in terms of information acquisition or computational costs. At the same time, considering further regrettable combinations implies allowing for an increasing number of suboptimal decisions, limiting the applicability of the corresponding framework in real-life settings.



(a). Expected value of combined triples absent the third country



(b). First ten TOPSIS scores per triples of countries



(c). Frequency distribution within ten first triples

Fig. 4. First scenario: Evaluation of triples.

9. Policy implications

TOPSIS is often used in sustainability environments where initial decisions can be modified, particularly in such strategic sectors as ICT. In this regard, the role played by uncertainty and interval evaluations in group decision-making settings within MADM scenarios such as TOPSIS has become increasingly relevant in the later years (Ramakrishnan and Chakraborty, [68]). For instance, Pamučar et al. [69] implemented rough interval numbers in MADM environments exploiting the interval-type information associated to the data observed. Similarly, Narayanamoorthy et al. [70], used interval-valued intuitionistic hesitant entropy to assign weights within a VIKOR framework.

MADM models do not generally consider the strategic incentives of experts, which constitutes a substantial problem when dealing with uncertain settings. For instance, climate skeptics use fake experts as a common communication strategy (Schmid-Petri and Bürger, [71]), producing a considerable amount of misinformation on environmental and sustainability topics (Treen et al. [72]; Johansson et al. [73]).

The uncertainty inherent to the evaluations provided by the experts differs from a reporting strategy (Di Caprio and Santos Arteaga, [74]). Fuzzy MADM models have been introduced to deal with the former (Awasthi et al. [75]), while the strategic case remains mainly unstudied in the literature. There are however models that warn about the capacity of experts to manipulate the rankings of MADM techniques under asymmetric information (Dong et al. [67]). The existence of strategic reports and the subsequent games have been generally analyzed through real-life settings focused on sustainable production environments (Agi et al. [76]). Despite this fact, the literature on MADM has not incorporated strategic interactions to its analysis.

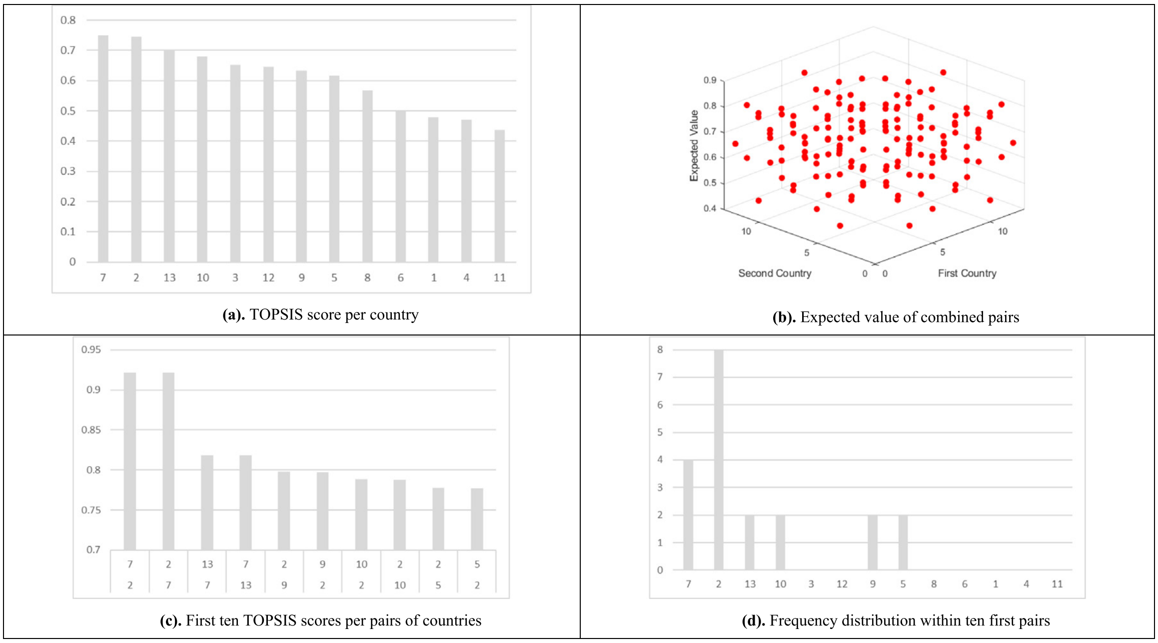


Fig. 5. Second scenario: Initial results and paired evaluations.

Real-life cases such as that of Madrid Central (Salas et al. [77]) demonstrate the necessity of incorporating a formal strategic framework into MADM environments. In this case, the opinions and strategies of experts differed to the point of implementing substantial modifications to an initially established sustainable environmental project. Sustainability problems consider many indicators and policy criteria, the latter being highly strategic and requiring specific models to account for the potential consequences.

The model introduced in the current paper constitutes a first step in this direction, allowing DMs to consider the possibility of modifying an initial choice after observing realizations from the alternatives ranked in the first positions. The interval evaluations determining the initial ranking are the immediate consequence from accounting explicitly for the uncertainty inherent to the data, with standard indicators providing approximations to the realizations that may be observed. The introduction of strategic reports formalized as intervals of potential realizations together with a credibility assigned to each of the experts or sources consulted define the next step.

10. Conclusion

The MADM literature generally assumes that once DMs are provided with a ranking generated by implementing any of its techniques, they choose the first alternative and dismiss the remaining ones. However, the information retrieved regarding the capacity of an alternative to perform according to a set of predetermined standards is generally imprecise. There is a concrete possibility that the DM observes realizations from an initially selected alternative that differ from those expected and may want to modify the initial choice. Clearly, the same reasoning applies to any of the alternatives classified as second, third, and so on.

The aim of the current paper has been to define optimal decision paths within a MADM uncertain setting where the DM is allowed to modify the initial alternative chosen after observing the realizations of its characteristics. Each of these paths represents a decision strategy describing the optimal sequence of choices that should be followed by the DM if, at some point, he decides to modify a given selection and proceed with a different alternative.

We have achieved the stated aim by analyzing the consequences from making potentially regrettable choices triggered by the uncertainty inherent to MADM models dealing with interval data. The value functions introduced in this paper define both the alternatives that should be considered by the DM as well as the order in which they should be chosen. The domains of the potential realizations within which the different alternatives are defined determine the value taken by the corresponding functions. We have also highlighted the increasing complexity of the combinatorial process required to generate all potential evaluation paths as the number of regrettable alternatives incorporated into the analysis increases.

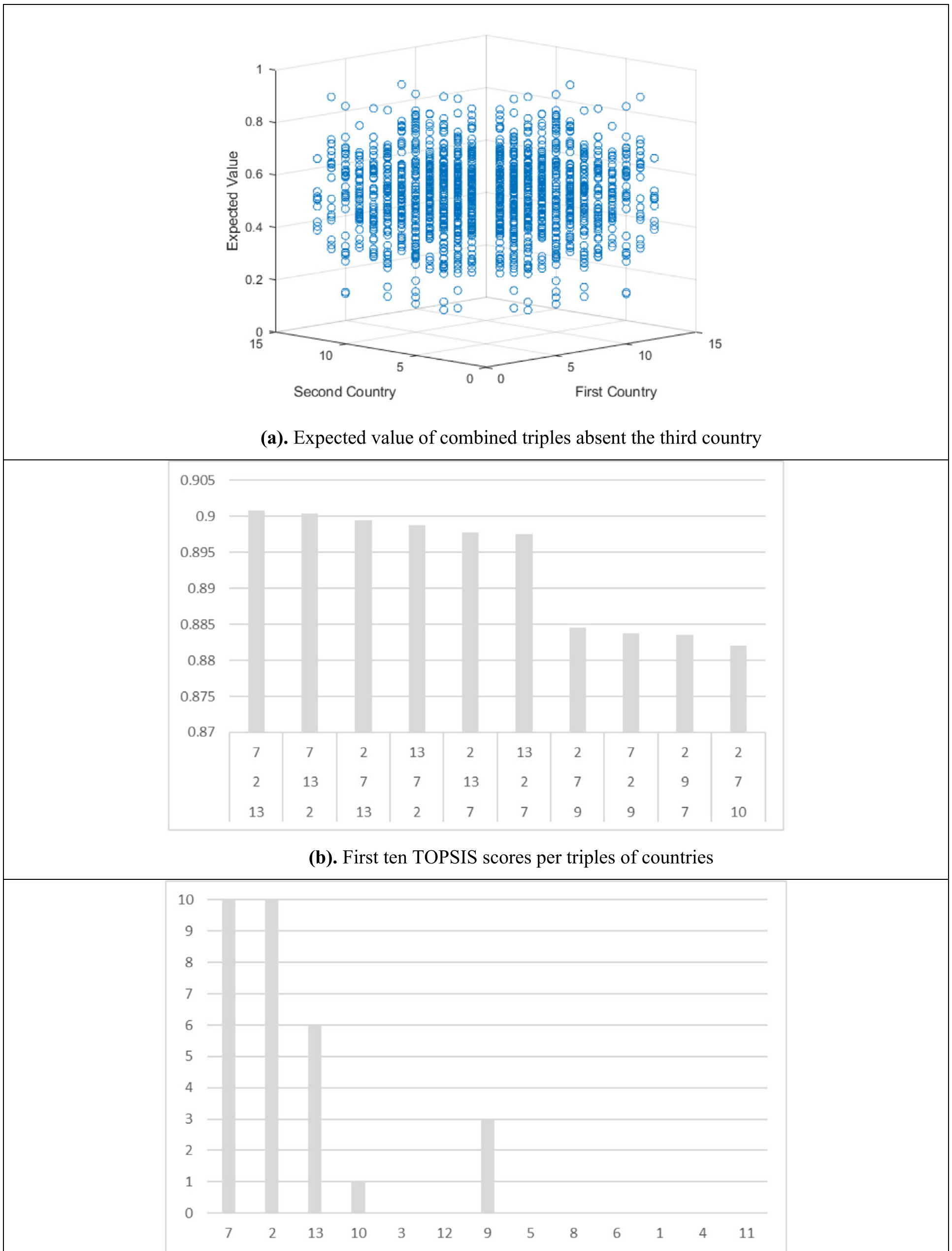
We have incorporated the combinatorial decision process within an interval TOPSIS setting. A numerical example has been provided to show how the resulting integrated evaluation framework is implemented to produce a final ranking of optimal decision paths.

A case study analyzing the entry decision of a firm within a group of European countries based on their levels of ICT development has been presented. We have incorporated to the MADM framework the effects derived from the resolution of uncertainty and the capacity of firms to enter different countries after making suboptimal decisions.

The policy implications that can be drawn from of the results obtained have been discussed paying particular attention to strategic sustainable development environments.

Rankings vary depending on the number of modifications of the initial choice that the DM is willing to consider. In this regard, future research could analyze the heuristic mechanisms that should be implemented if we were to introduce further combinatorial settings in the analysis. It must be noted that the number of regrettable choices should be bounded by the DM, given the information and structural costs resulting from a suboptimal selection.

Finally, introducing strategic reports on the side of a group of experts assigned to evaluate different alternatives constitutes



(a). Expected value of combined triples absent the third country

(b). First ten TOPSIS scores per triples of countries

(c). Frequency distribution within ten first triples

Fig. 6. Second scenario: Evaluation of triples.

an extension of the current framework into the game-theoretical domain that seems worth investigating. The intuition provided implies that an expert with strategic interests will report high potential positive values and low negative ones for his preferred alternatives. The resulting model should account for this feature when aggregating the reports of the experts and weighting their relative importance.

CRediT authorship contribution statement

Debora Di Caprio: Formal analysis, Investigation, Methodology, Writing – review & editing. **Francisco J. Santos-Arteaga:** Formal analysis, Investigation, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request

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Appendix. Triple combinations for positive and negative criteria

We introduce now the remaining combinations required to define the complete set of potential triples determined by the relative limits of the evaluation intervals and the order of choice selected by the DM.

A.1. $x^i \le x^k \le x^s$ setting

$$
\begin{aligned}
&V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \\
&\int_0^{y^{iM}} \frac{1}{x^i} \left[\begin{array}{l}
\int_0^{y^i} \frac{1}{x^s} \Big[\int_0^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k \\
+ \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s + \\
\int_{y^i}^{y^{kM}} \frac{1}{x^s} \Big[\int_0^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k \\
+ \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s \\
+ \int_{y^{kM}}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s
\end{array} \right] dy^i
\end{aligned}
\tag{A.1}
$$

A.1.1. $x^i \le x^k \le x^s$ setting: negative criteria

$$
\begin{aligned}
&V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \\
&\int_{y^{im}}^{y^{km}} \frac{1}{M - x^i} \left[y^i\right] dy^i + \\
&\int_{y^{km}}^{y^{sm}} \frac{1}{M - x^i} \Big[\int_{y^{sm}}^{M} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^i} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
&+ \int_{y^i}^{M} \frac{1}{M - x^k} \left(y^i\right) dy^k \Big] dy^s \Big] dy^i + \\
&\int_{y^{sm}}^{M} \frac{1}{M - x^i} \left[\begin{array}{l}
\int_{y^{sm}}^{y^i} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^s} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
+ \int_{y^s}^{M} \frac{1}{M - x^k} \left(y^s + c^s\right) dy^k \Big] dy^s + \\
\int_{y^i}^{M} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^i} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
+ \int_{y^i}^{M} \frac{1}{M - x^k} \left(y^i\right) dy^k \Big] dy^s
\end{array} \right] dy^i
\end{aligned}
\tag{A.1.1}
$$

A.2. $x^k \le x^i \le x^s$ setting

$$
\begin{aligned}
&V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \\
&\int_{y^{kM}}^{y^{iM}} \frac{1}{x^i} \left[\int_0^{y^i} \frac{1}{x^s} \left(y^i\right) dy^s + \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s \right] dy^i + \\
&\int_0^{y^{kM}} \frac{1}{x^i} \left[\begin{array}{l}
\int_0^{y^i} \frac{1}{x^s} \Big[\int_0^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k \\
+ \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s + \\
\int_{y^i}^{y^{kM}} \frac{1}{x^s} \Big[\int_0^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k \\
+ \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s \\
+ \int_{y^{kM}}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s
\end{array} \right] dy^i
\end{aligned}
\tag{A.2}
$$

A.2.1. $x^k \le x^i \le x^s$ setting: negative criteria

$$
\begin{aligned}
&V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \\
&\int_{y^{im}}^{y^{sm}} \frac{1}{M - x^i} \Big[\int_{y^{sm}}^{M} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^i} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
&+ \int_{y^i}^{M} \frac{1}{M - x^k} \left(y^i\right) dy^k \Big] dy^s \Big] dy^i + \\
&\int_{y^{sm}}^{M} \frac{1}{M - x^i} \left[\begin{array}{l}
\int_{y^{sm}}^{y^i} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^s} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
+ \int_{y^s}^{M} \frac{1}{M - x^k} \left(y^s + c^s\right) dy^k \Big] dy^s + \\
\int_{y^i}^{M} \frac{1}{M - x^s} \Big[\int_{y^{km}}^{y^i} \frac{1}{M - x^k} \left(y^k + 2c^s\right) dy^k \\
+ \int_{y^i}^{M} \frac{1}{M - x^k} \left(y^i\right) dy^k \Big] dy^s
\end{array} \right] dy^i
\end{aligned}
\tag{A.2.1}
$$

A.3. $x^k \le x^s \le x^i$ setting

$$
\begin{aligned}
&V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \\
&\int_{y^{sM}}^{y^{iM}} \frac{1}{x^i} \left[y^i\right] dy^i + \int_{y^{kM}}^{y^{sM}} \frac{1}{x^i} \Big[\int_0^{y^i} \frac{1}{x^s} \left(y^i\right) dy^s \\
&+ \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left(y^s - c^s\right) dy^s \Big] dy^i + \\
&\int_0^{y^{kM}} \frac{1}{x^i} \left[\begin{array}{l}
\int_0^{y^i} \frac{1}{x^s} \Big[\int_0^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k \\
+ \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s + \\
\int_{y^i}^{y^{kM}} \frac{1}{x^s} \Big[\int_0^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k \\
+ \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \Big] dy^s \\
+ \int_{y^{kM}}^{y^{sM}} \frac{1}{x^j} \left(y^s - c^s\right) dy^s
\end{array} \right] dy^i
\end{aligned}
\tag{A.3}
$$

A.3.1. $x^k \leq x^s \leq x^i$ setting: negative criteria

$$V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \int_{y^{im}}^{M} \frac{1}{M-x^i} \left[\begin{aligned} & \int_{y^{sm}}^{y^i} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^s} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^s}^{M} \frac{1}{M-x^k} \left(y^s + c^s\right) dy^k \right] dy^s + \\ & \int_{y^i}^{M} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^i} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^i}^{M} \frac{1}{M-x^k} \left(y^i\right) dy^k \right] dy^s \end{aligned} \right] dy^i \quad \text{(A.3.1)}$$

A.4. $x^s \leq x^i \leq x^k$ setting

$$\begin{aligned} & V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \\ & \int_{y^{sM}}^{y^{iM}} \frac{1}{x^i} \left[\int_{0}^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^i + \\ & \int_{0}^{y^{sM}} \frac{1}{x^i} \left[\begin{aligned} & \int_{0}^{y^i} \frac{1}{x^s} \left[\int_{0}^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s + \\ & \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left[\int_{0}^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k + \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s \end{aligned} \right] dy^i \end{aligned} \quad \text{(A.4)}$$

A.4.1. $x^s \leq x^i \leq x^k$ setting: negative criteria

$$\begin{aligned} & V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \\ & \int_{y^{im}}^{y^{km}} \frac{1}{M-x^i} \left[\int_{y^{sm}}^{y^i} \frac{1}{M-x^s} \left(y^s + c^s\right) dy^s + \int_{y^i}^{M} \frac{1}{M-x^s} \left(y^i\right) dy^s \right] dy^i + \\ & \int_{y^{km}}^{M} \frac{1}{M-x^i} \left[\begin{aligned} & \int_{y^{sm}}^{y^{km}} \frac{1}{M-x^s} \left(y^s + c^s\right) dy^s + \\ & \int_{y^{km}}^{y^i} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^s} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^s}^{M} \frac{1}{M-x^k} \left(y^s + c^s\right) dy^k \right] dy^s + \\ & \int_{y^i}^{M} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^i} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^i}^{M} \frac{1}{M-x^k} \left(y^i\right) dy^k \right] dy^s \end{aligned} \right] dy^i \end{aligned} \quad \text{(A.4.1)}$$

A.5. $x^s \leq x^k \leq x^i$ setting

$$\begin{aligned} & V(x^i, x^s, x^k, y^{iM}, y^{sM}, y^{kM}) = \\ & \int_{y^{kM}}^{y^{iM}} \frac{1}{x^i} \left[y^i\right] dy^i + \int_{y^{sM}}^{y^{kM}} \frac{1}{x^i} \left[\int_{0}^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^i + \\ & \int_{0}^{y^{sM}} \frac{1}{x^i} \left[\begin{aligned} & \int_{0}^{y^i} \frac{1}{x^s} \left[\int_{0}^{y^i} \frac{1}{x^k} \left(y^i\right) dy^k + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s + \\ & \int_{y^i}^{y^{sM}} \frac{1}{x^s} \left[\int_{0}^{y^s} \frac{1}{x^k} \left(y^s - c^s\right) dy^k + \int_{y^s}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^s\right) dy^k \right] dy^s \end{aligned} \right] dy^i \end{aligned} \quad \text{(A.5)}$$

A.5.1. $x^s \leq x^k \leq x^i$ setting: negative criteria

$$\begin{aligned} & V_{[-]}(x^i, x^s, x^k, y^{im}, y^{sm}, y^{km}) = \\ & \int_{y^{im}}^{M} \frac{1}{M-x^i} \left[\begin{aligned} & \int_{y^{sm}}^{y^{km}} \frac{1}{M-x^s} \left(y^s + c^s\right) dy^s + \\ & \int_{y^{km}}^{y^{im}} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^s} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^s}^{M} \frac{1}{M-x^k} \left(y^s + c^s\right) dy^k \right] dy^s + \\ & \int_{y^{im}}^{y^i} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^s} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^s}^{M} \frac{1}{M-x^k} \left(y^s + c^s\right) dy^k \right] dy^s + \\ & \int_{y^i}^{M} \frac{1}{M-x^s} \left[\int_{y^{km}}^{y^i} \frac{1}{M-x^k} \left(y^k + 2c^s\right) dy^k + \int_{y^i}^{M} \frac{1}{M-x^k} \left(y^i\right) dy^k \right] dy^s \end{aligned} \right] dy^i \end{aligned} \quad \text{(A.5.1)}$$

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