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A STUDY ON SEISMIC BEHAVIOUR OF MASONRY TOWERS

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SUMMARY

This study is dedicated to simplified vulnerability assessment of masonry towers, in particular to the definition of collapse mechanism geometry.

After a preliminary analysis on damages and collapse mechanisms caused to towers by the earthquakes and a review of analysis methods in literature, a model to determine the plane of fracture that defines the kinematic blocks of an overturning mechanism was proposed, based on simple equilibrium conditions.

According to the Italian codes, in fact, tower structures are classified as one of the churches macroelements, characterized by peculiar collapse mechanisms; respect to other macroelements, for towers a slight variation in mechanism geometry implies relevant variation in collapse multiplier values; this is mainly due to the importance of mass and height in these structures. Hence a correct definition of kinematism geometry results very important.

The proposed method was applied also including a limit on masonry compressive strength, despite traditional limit analysis method that usually assumes as infinite masonry compressive strength.

For the use in common practice, the curve of fracture was evaluated through parametric analyses for different geometrical configurations, to which many existing towers can be assimilated.

Finally, besides a comparison with real collapse mechanisms surveyed on towers after earthquakes, the proposed method was applied also in the vulnerability assessment of a medieval masonry tower, the Ghirlandina in Modena.

SOMMARIO

Il presente lavoro è dedicato all'analisi semplificata della vulnerabilità sismica delle torri in muratura, in particolare alla definizione della geometria del cinematismo di collasso.

Dopo un'analisi preliminare dei danni e dei meccanismi innescati dal sisma sulle torri, e una rassegna dei metodi di analisi presenti in letteratura, si è elaborato un metodo per determinare la geometria del piano di frattura che individua i blocchi di un meccanismo di ribaltamento globale, a partire da semplici considerazioni di equilibrio.

Secondo le Norme Tecniche Nazionali, infatti, le torri (campanarie), vengono classificate come uno dei macroelementi in cui vengono schematizzate le chiese, caratterizzato da propri meccanismi di collasso; a differenza tuttavia di altri macroelementi, per le torri, considerate le masse e le altezze in gioco, lievi variazioni nella geometria del meccanismo comportano sensibili modifiche nel moltiplicatore di collasso; è quindi importante una corretta definizione della geometria del cinematismo. Il metodo proposto è stato applicato anche rimuovendo l'ipotesi, tipica nell'analisi limite di strutture murarie, di resistenza a compressione infinita della muratura.

Al fine di rendere di immediato utilizzo pratico i risultati, l'andamento della frattura è stato determinato tramite analisi parametriche per diverse configurazioni geometriche a cui facilmente si possono ricondurre le strutture a torre esistenti.

Infine, oltre a un confronto con meccanismi reali rilevati a seguito di terremoti avvenuti in passato, si è applicato il metodo proposto alla analisi di vulnerabilità di una torre medievale, la Ghirlandina del Duomo di Modena.

CONTENTS

INTRODUCTION

This work presents a study on vulnerability assessment of masonry towers, defining a method to simply determine the plane of fracture that separates the overturning block of a tower collapsing in its typical kinematism, according to a macroelement classification.
The work is developed into four chapters:

In the first part the intrinsic characteristics and the properties of soil-structure interaction that influence the seismic behaviour of masonry towers are pointed out.
Typical collapse mechanisms, according to National Code are illustrated with some examples for each type, underlining vulnerability factors and interventions able to improve seismic capacity.

In the second chapter modeling strategies and analysis methods described in literature are presented, highlighting the presence of complex nonlinear methods and the shortcomings of simplified method considering mainly geometrical parameters; nevertheless the importance of limit analysis is clear both for vulnerability assessment, both as qualitative countercheck when running complex analyses.
Principles of limit analysis method are recalled and recent research developments, aimed to define the geometry of kinematic blocks, are described.

In the third chapter, following an approach defined to evaluate with limit analysis the safety of leaning towers, a method to calculate the curve of fracture and the corresponding collapse multiplier of an overturning kinematism is proposed.

Parametric analyses results, and comparisons with real collapses occurred during past earthquakes are described, a good correspondence is found between calculated curve of fracture and collapse occurred on real towers.

In the last chapter the method is applied to an existing medieval tower, the Ghirlandina in Modena.
After a brief historical report on construction phases and a description of in situ test results regarding masonry and foundation soil, a vulnerability assessment is performed exploring seismic capacity in six different conditions (depending on material properties assumptions and on the geometry of blocks). Seismic demand is determined in terms of spectral acceleration, according to hypothesis on soil-structure interaction.
Comparison shows a relevant influence of the curve of fracture calculated in defining the geometry of the kinematism and hence in the resulting collapse multiplier.

1. MASONRY TOWERS AND EARTHQUAKES

1.1 Masonry towers under earthquakes

Historical masonry towers (bell towers, civic towers, tower-houses, defense towers on the city walls…) are found throughout the entire Italian peninsula, where they represent a distinctive feature of many of its historical centers and its countryside. In roman and medieval times, some of them had a great strategic and military importance. The great variety of uses reserved to masonry towers is reflected in a considerable variety of constructive configurations. Their heights vary from the 60-70 meters of the 11^{th}-13^{th} century towers built with defensive functions (and also as a symbol of power and wealth of the owners) to the 20-30 meters of the tower houses, widely popular in central Italy in medieval times. Beside civic towers, a variety of bell towers is built next to almost every church; also bell towers present a variety of architectural styles and geometrical composition according to the historical period.
Evaluation of structural safety of historical masonry towers is an important issue in the maintenance of historical heritage of architectural monuments. An example of the interest arisen worlwide for these structures is given by the leaning tower of Pisa case, the bell tower of San Marco in Venice collapse, the Civic tower of Pavia, the bell tower of St. Magdalena in Goch.
Their vertical structure places towers at significant risk, not only due to the high stress level acting at their base but also because of their great susceptibility to dynamic actions consequent to events such as

earthquakes, bell motion, vibration produced by traffic or by the wind. In particular, the high vertical load value can cause crushing phenomena in the masonry or yielding of the foundation soil and therefore additional actions produced by the resulting leaning.

The extensive cracking revealed in many structures moreover testifies the action of thermal variations, and structural efforts experienced.

Figure 1 Bologna: examples of different typologies of tower.

Structural analysis on masonry tower is characterized by some specific aspects: these constructions usually are examples of great structural effort, sometimes extended for a long sequence of building phases and the result of their demanding design is that, in some cases, the materials are stressed until their limits even for simple dead load condition. Seismic events, considering the great masses involved and the height, on which they are distributed, often represents the most unfavorable load case condition.

In this chapter some qualitative aspects that determine towers seismic vulnerability are described.

1.1.1 Geometry

Dynamic behavior of masonry tower is heavily influenced by their particular geometry that defines a slender or a non-slender (massive) tower. Slenderness is a parameter with a wide variability for existing masonry towers: different examples are found from massive defensive towers for which a massive behavior (and shear failure) could be expected to slender bell towers from which a cantilever behavior as monodimensional element could be more representative.

In this latter case a good connection between adjacent walls is needed to guarantee a cantilever behavior with an associate stiffness corresponding to the entire cross section (assuming in plane deformation of sections). In general, slender towers, when able to exhibit a unitary behaviour, have natural modes of vibrations characterized by long period values and hence they should be protected by the frequency spectrum of the most seismic events; otherwise, when a good connection among external walls is not guaranteed, they exhibit a highly vulnerable behaviour.

Traditional techniques able to guarantee integrity of sections are rod ties and wooden deck well connected to the masonry walls; on the opposite, when in presence of vaults inside the tower, great care is needed to evaluate the effects of vault thrust because the unitary behavior could be locally prevented.

Figure 2, 3 Examples of isolated tower (San Marco bell tower) and connected to other buildings (Bell tower in Lucca)

Dynamic behavior is also influenced by the presence of adjacent structures able to produce some restraint to the tower. This is the common case of bell towers built in contact with the church façade, or the tower houses built in aggregate.

The presence of connections and restraints at different levels modifies natural frequencies of the structure and induces stress concentration on the stiffer parts.

Presence of slender elements on the top (spire, belfry, other architectural elements...) could modify structural vulnerability of the building and in general represents another very sensitive part of the tower respect earthquake; in fact the upper part of the structure could undergo to seismic motion amplification, whose structural effects could be aggravate by the reduced vertical load that cannot perform a stabilizing action toward the horizontal loads.

Also the presence of diffuse openings at certain levels heavily affects seismic vulnerability, introducing on the structure zones highly vulnerable respect to the horizontal actions.

Figure 4,5. Presence of openings: San Rocco bell tower in Frascati, San Gottardo bell tower in Milano

1.1.2 Existing damages

Vulnerability also depends on existing damages and deformations of the structure.

Damages include mechanical cracking, material decay (for chemical or physical effects) or any other phenomena influencing the original capacity of the material and the structures.

In masonry towers thermal variation is a common cause of typical vertical cracks mainly on the south façade, the presence of these cracks affects seismic response of the masonry becoming a quick path for cracks development; as thermal cracks also the presence of discontinuity (of material or geometry) due to different construction phases or repair interventions could modify collapse mechanisms.

For a complete analysis is hence very important to model existing damages, deformations and discontinuities of the structure.


:

Figure 6 Tormento tower in Vicenza: thermal load crack

1.1.3 Building history

Construction process, architectural alterations, additions or destructions of building parts and also events as earthquakes, fires, lightning, are essential for a realistic interpretation of structural behavior. In fact, for example, the performance shown during past seismic events must be evaluated to understand present seismic capacity.

Also architectural intervention aimed to modify the original structure must be evaluated carefully, for instance in the case of Pavia Tower the adjunction of the heavy granite belfry at the end of XVI sec certainly accelerated the crisis of the masonry for long term load (Binda 2008).

Figure 7 Pavia civic tower before collapse

1.2 Soil structure interaction and leaning phenomena

Beside intrinsic characteristics of the tower, another important aspect to determine seismic vulnerability is the restraint condition at soil level and hence the soil-structure interaction.
Foundation soil through its stratigraphy condition and its mechanical properties acts as a filter of seismic motion transferring it to the structure; hence it could determine seismic amplification respect to rigid soil condition.
Therefore, in every seismic analysis soil-structure interaction is a very important step to determine final results. Considering masonry towers this aspect assumes a major importance being towers modeled as cantilever beams fixed at the base by a spring with stiffness corresponding to soil properties: for such model the parameter that control dynamic properties is certainly the soil restraint (and elastic properties of the masonry).
Furthermore, foundation soil characteristics are very important not only to identify dynamic characteristics of the structure but also respect to leaning phenomenon, a very common effect of soil-structure interaction.
When earthquake occurs, seismic capacity of a leaning tower is "weakened" because a part of it is already absorbed by the additional effort in supporting bending moment due to eccentric load and stress concentration due to possible partialization of the lower sections.
Evidences of instability problems of towers, built on compressible ground, are shown in many different cases in the whole Italian territory; most famous cases are probably: Pisa tower in Campo dei Miracoli, Garisenda tower in Bologna, Santo Stefano tower in Venice.

Figure 8. Santo Stefano belfry in Venice

Leaning phenomena in masonry towers is due to instability caused by insufficient soil stiffness (excessive soil settlement under load). Being all foundation compressible to some extent, instability can occur also on a stiff stratum if the tower is tall enough and hence could reach critical conditions on the soil.

In seismic analyses of masonry towers, leaning represent an important factor to determine safety of the structure; being leaning basically a problem of equilibrium, the response of the system after the introduction of a perturbation (as earthquake could be intended) describes equilibrium stability condition – the more unstable as the soil stiffness decreases.

The reasons for which a slender structure cannot be built above a certain “critical” height on a compressible ground without introducing

a lean in the structure are extensively explained by the work of (Hambly 1984), here summarized.
A structure can fail either due to material failure or to instability; it could also happen that a structure fails for a combination of both causes, indeed material failure is generally preceded by inelastic phenomena which in general have a destabilizing effect on the structure. Hence, considering masonry towers, collapse in static conditions occurs for:
-buckling (foundation not stiff enough)
-bearing capacity failure (lack of strength of foundation or masonry)
Leaning phenomenon is due to stability problems (buckling); structural instability can occur also when stiffness of the soil is low and hence deformations are large. Being instability not caused by a lack of strength of the ground but by the insufficient stiffness and being every foundations compressible to some extent, instability problems can occur also in a tower on a stiff stratum if the building is very tall.
The height limits on structures built on compressible ground are explained by Hambly with a simple experiment.
Building a column of blocks on a springy foam pad, three different situation could be observed (figure 9):
a – the column is short and stable, an horizontal force is needed to give it lean and when the force is removed the column returns in the vertical position (*stable equilibrium*)
b – reached a certain critical height the column will not return to the vertical position after being perturbed by an horizontal force (*neutral equilibrium*).
c – the addition of any further weight will cause the column to lean over and an opposite horizontal force is needed to prevent toppling.

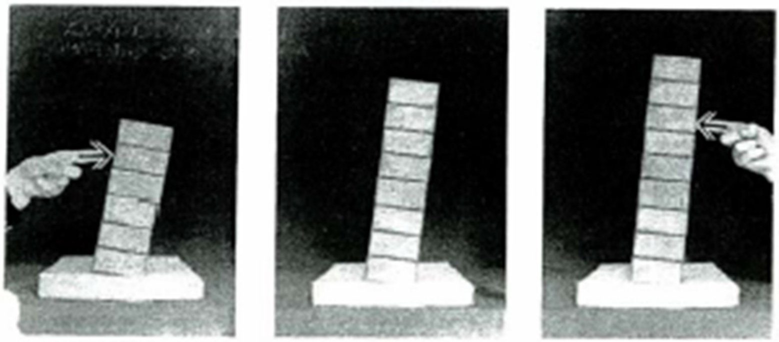

(a) (b) (c)

Figure 9 Column of blocks on a springy foam pad (Hambly 1984).

Even if the tower is built as vertical as possible it will become unstable and start to lean over when reached the critical height, also a column on a firm foundation will start to lean if the column is tall enough to reach the critical value.

Hambly then determine a critical height value depending on expression:

$$\frac{h_{cg} \cdot a_s}{\rho^2} \qquad (1)$$

being h_{cg} the height to center of gravity, a_s the average settlement and ρ^2 the radius of gyration. At this condition the tower starts to lean.

Seismic events can hence easily aggravate situations already near to collapse for simple static conditions. In particular, the vertical component of seismic action could make the structure reaching bearing capacity of the foundation soil; instead, the horizontal components of the seismic motion could make the tower reaching collapse for buckling crisis.

1.3 Damages survey in existing masonry towers

In a preliminary phase first descriptive data on masonry tower collapse mechanisms due to earthquakes were studied; data collected regard damaged towers in Italy only, from the 1976 Friuli earthquake to the most recent L'Aquila earthquake (2009); damages survey was done by earthquake and by collapse mechanism.
The aim of this survey was, besides reaching a more complete knowledge on towers collapse mechanisms, to collect geometrical data to compare the documented collapse mechanisms with the results of the analytical model proposed in the third chapter. According to (LL GG), the collapse mechanisms observed were divided into:
-*global mechanisms*, where the damage involves the whole structure, both with vertical or diagonal cracks on the façades of the building
-*belfry mechanisms*, where vulnerability of masonry walls is increased by multiple openings, usually arches, characterized by a low resistance to horizontal actions
-*overhanging element mechanisms (spire, steeple, statues*, *etc.)* involving architectural parts characterized by a weak inertia in one direction or the upper parts where a reduced axial load gives a minor stabilizing effect to the masonry
In the following tables, examples of the mentioned collapse mechanisms are reported divided by mechanism and earthquakes (considering all the major seismic events occurred on the italian territory: Friuli in 1976, Reggio-Emilia in 1996, Umbria-Marche in 1998, Molise in 2002, Salò in 2004 and L'Aquila in 2009).

In the following forms the damaged tower are compared with the undamaged state - when possible; the geographical localization is described and represented on a map (blue spot) with the epicentral area (red spot, in case of localization in the epicentral area only a red spot is drawn).

1.3.1 Global mechanisms

Global mechanisms are the most typical collapse modes of towers. They are divided in two main groups depending on the connection between adjacent walls; in fact, when in presence of a good connection or when tie rods guarantee the unitary behaviour, the tower presents a global overturning mechanism with a diagonal surface of fracture inclined on the façades.
Instead when the connection between walls is insufficient, or when in presence of existing damages (i.e. for thermal variation) that produce vertical discontinuity, the collapse mechanisms is represented by a general disaggregation phenomenon among the masonry walls; due to the opening of vertical cracks on the façades, the unitary behaviour is hence totally prevented.
Focusing the attention on the conservation aspects, it’s important to underline that most of towers damaged by a global mechanisms during the past earthquakes were completely demolished due to the difficulties in repairing such damages.
In the following tables examples of these mechanism are illustrated, in particular are described mechanisms of the first group, being these mechanisms the subject of the analytical model proposed in the third chapter to determine fracture surfaces.

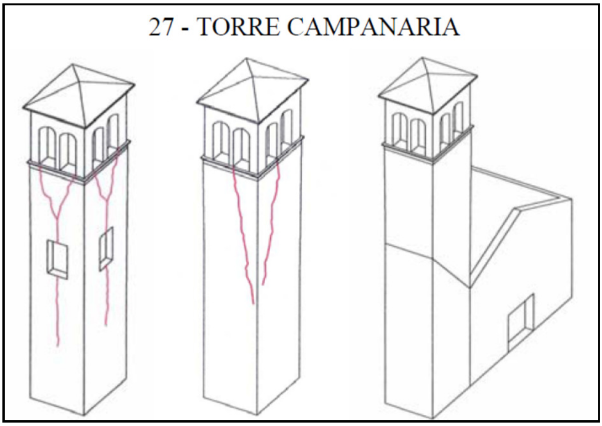
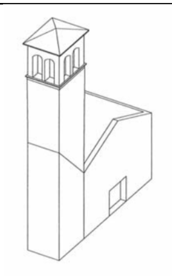


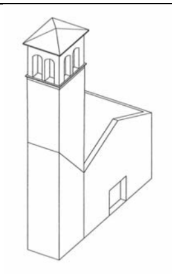
Figure 10 Global mechanisms according to (LL GG)

1.1 BELL TOWER OF *SAN PIETRO* - COPPITO (L'AQUILA)



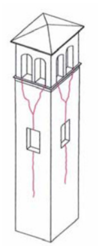
L'Aquila earthquake 2009

1.2 TOWER *MEDICEA* – SANTO STEFANO DI SESSANIO (L'AQUILA)



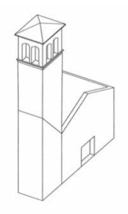
L'Aquila earthquake 2009

1.3 BELL TOWER OF *SAN SILVESTRO* - L'AQUILA



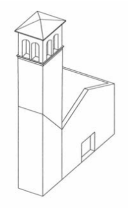
L'Aquila earthquake 2009

1.4 BELL TOWER OF *SAN GIULIANO DI PUGLIA* - CAMPOBASSO



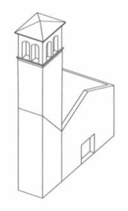
Molise earthquake 2002

1.5 BELL TOWER OF *SAN PIETRO IN VINCOLI* - CASTELLINO SUL BIFERNO (CAMPOBASSO)



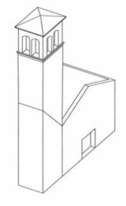
Molise earthquake 2002

1.6 *TORRAZZO* - BAGNOLO IN PIANO (RE)



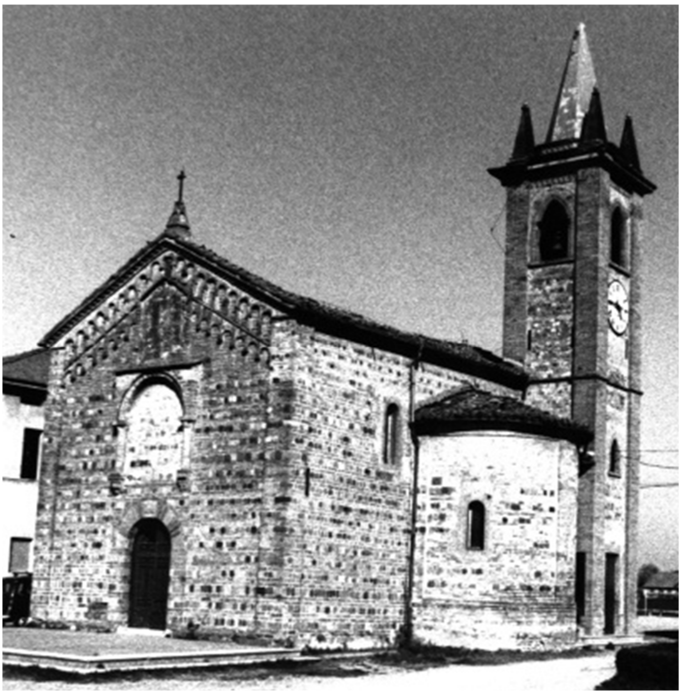
Reggio-Emilia
earthquake 1996

1.7 BELL TOWER OF BELL TOWER OF *SAN TOMMASO VESCOVO DI CANTERBURY* (REGGIO EMILIA)



Reggio-Emilia
earthquake 1996

-



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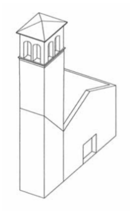
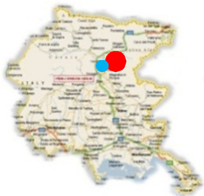
- BRAULINS (UD)</td></tr>
<tr><td></td><td>
Friuli earthquake 1976</td><td>-</td></tr>
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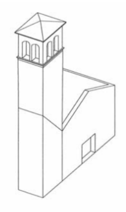
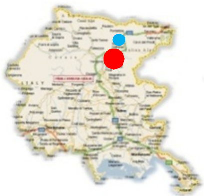
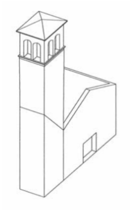
1.9 BELL TOWER OF *COLLE* (PORDENONE)

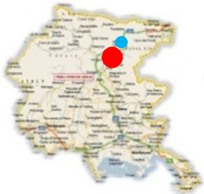




Friuli earthquake 1976

1.10 BELL TOWER OF *SAN MARTINO* - RESIUTTA (UDINE)





Friuli earthquake 1976

1.3.2 Belfry mechanisms

Belfry mechanisms are related to the presence of wide openings on the top of the bell tower, being the upper parts traditionally reserved to support and to protect the bells.
Often the openings of the belfry are constituted by an arched *loggia* or arched windows, hence the typical collapse mechanism usually coincides with mechanisms of in-plane loaded arches.
Also for these mechanisms the presence of tie rods could be determinant to the survival or not of the structure to the earthquake; in fact, by connecting the four walls of the belfry, they assure a major stiffness to the masonry structures, otherwise highly weakened by the presence of openings.

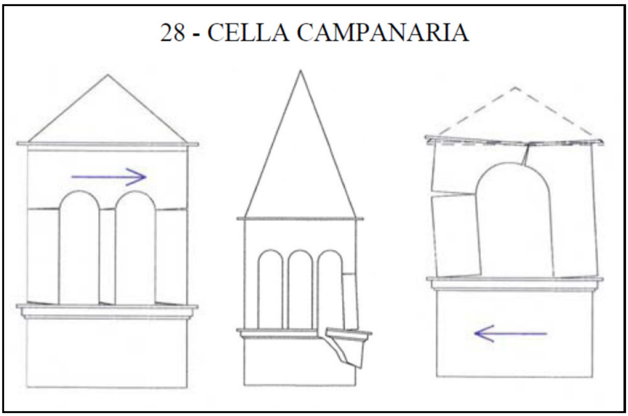


Figure 11 Belfry mechanism according to (LLGG)

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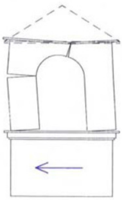
<tr><td></td><td>
Salò earthquake 2004</td><td>-</td></tr>
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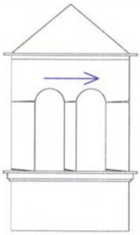
2.2 BELL TOWER OF *SAN BERNARDINO* - L'AQUILA



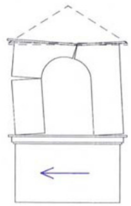
L'Aquila earthquake 2009

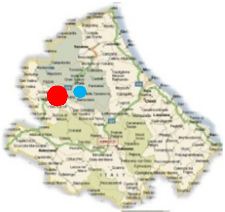
2.3 BELL TOWER OF *SAN DOMENICO* - L'AQUILA		
	 *L'Aquila earthquake 2009*	-
		

2.4 BELL TOWER OF *SAN FRANCESCO* - CASTELVECCHIO SUBEQUO (L'AQUILA)

L'Aquila earthquake 2009

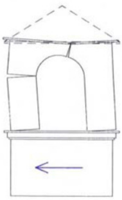
2.5 BELL TOWER OF *SAN FELICE* - POGGIO PICENZE (L'AQUILA)



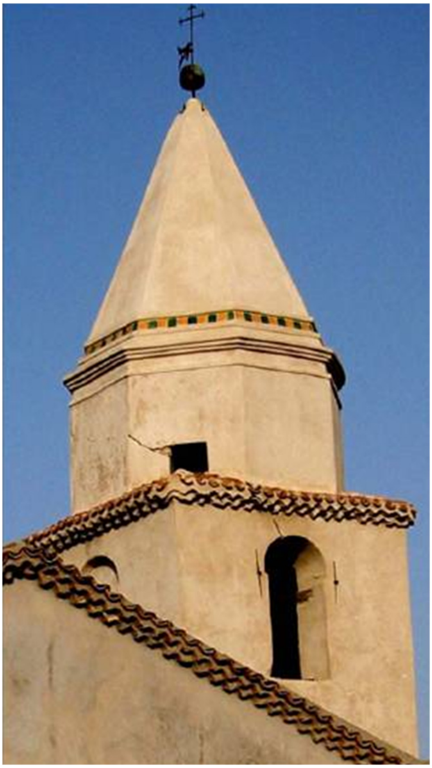


L'Aquila earthquake 2009

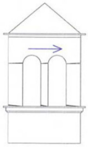
2.6 BELL TOWER OF *SANT' ALFONSO DEI LIGUORI* - CAMPOBASSO



Molise earthquake 2002



2.7 BELL TOWER OF *ANDUINS* - PORDENONE



Friuli earthquake 1976

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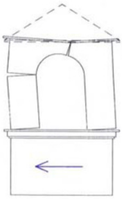
<tr><td></td><td>
Friuli earthquake 1976</td><td>-</td></tr>
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</table>

1.3.3 Overhanging parts mechanisms

These mechanisms are the most frequent for towers having slender elements built on the top; the upper parts in fact are more vulnerable due to the reduced axial load that gives a minor stabilizing effect respect to lower parts of the tower; other mechanisms of this group are those involving architectural parts characterized by a weak inertia in one direction as *vela* belfry that is characterized by a high vulnerability in the out of plane direction both for the weak inertia plane both for the presence of the bells.

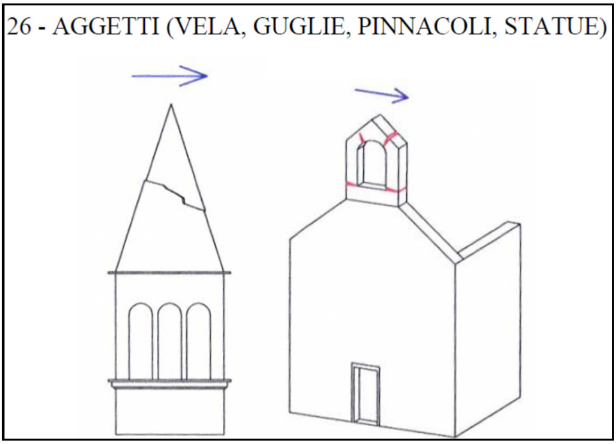
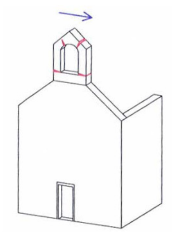


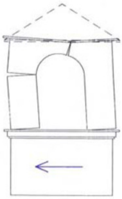
Figure 12 Overhanging parts mechanisms according to (LL GG)

3.1 BELL TOWER OF *CHIESA MATRICE DI SAN MARCO* - CASTELDELMONTE (L'AQUILA)

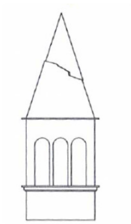


L'Aquila earthquake 2009

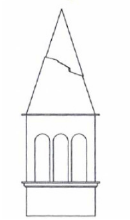
3.2 BELL TOWER IN SALÒ (BRESCIA)

	 Salò earthquake 2004	-

3.3 BELL TOWER IN SALÒ (BRESCIA)

	 Salò earthquake 2004	-

3.4 BELL TOWER OF *SANTA CROCE DI MAGLIANO* - CAMPOBASSO



Molise earthquake 2002

1.3.4 Notes on most common damages

Observing damages caused by past seismic events, most vulnerable and critical zones result represented by:

- foundation and base section where high dead load stress values could be aggravate by horizontal seismic loads, determining global mechanisms;
- parts connected to other buildings, as churches and bell towers, where different stiffness could produce a stress concentration due to effects of concentrated loads transferred by the connecting element (Church of *San Giuliano di Puglia* in Campobasso 1.4, Church of *S.Michele Arcangelo* in Braulins 1.8).
- lanterna or other geometrical discontinuities on the upper part where a reduced axial load gives a minor stabilizing effect. (church of *Santa Maria Matrice* in Casteldelmonte 3.1)

Others important vulnerability factors are the absence of tie rods connecting opposite walls, (church of *San Bernardino* in L'Aquila 2.2), disconnection in the masonry due to different building phases or discontinuity of materials. These situations in fact represent zones di of high vulnerability, where cracks can develop and trigger a collapse mechanism.

It's also important in order to determine seismic vulnerability to identify the presence of rigid diaphragms and r.c slabs, which in some cases are built as strengthening intervention (as *Santo Stefano* tower in Sessanio 1.2).

2 MODELING AND ANALYSIS OF MASONRY TOWERS

2.1. Modeling

The structural problem is generally concerned with geometrical relations dealing with displacements and deformations, with static and dynamic relations dealing with equilibrium conditions, and with the constitutive laws of material which can be seen as a link between the two aspects.

Specifically, seismic behavior analysis of masonry structures is a challenging topic due to the incomplete experimental characterization of the mechanical properties, to difficulties in numerical modeling when nonlinear behavior of the material is taken into account, and in some cases to the complexity of geometrical configuration.

A general view of the different modeling strategies and analysis methods adopted in the masonry research field is described.

2.1.1 Material modeling

Traditional and historical materials, as brick or stone masonry, are characterized by complex mechanical and strength phenomena, due to the fact that their characteristics depend from the properties of their components and from the construction geometry and the block placing. As traditionally observed, masonry has a composite character, a brittle behavior in tension with almost null tensile strength, a frictional response in shear and a response highly sensitive to load orientation (anisotropy).

A complete material modeling should consider the following aspects:

-masonry is a discrete material (composed by blocks and mortar) in which the dimension of the single constituting element is large compared to the dimensions of the structural element

-geometry and blocks placing can vary considerably

-blocks are generally stiffer than mortar

-stiffness of the vertical joints is remarkably smaller than stiffness of the horizontal joints

-mortar thickness is limited compared to block dimensions

In general hence, interaction between masonry components depends on properties of the mortar, properties of the blocks and construction scheme. The need of characterizing masonry with a suitable constitutive model led to different modeling strategies (Lourenço 2002), (Roca et al. 2010); according to the level of accuracy expected, these methods can be grouped as:

Detailed micro-modeling: the different components (units, mortar and unit-mortar interface) are distinctly described; this is the most accurate tool to simulate masonry behavior, in particular for the local response of the material.

Blocks and mortar are modeled with continuum finite elements, while the unit-mortar interface is represented by discontinuous elements accounting for potential crack or slip planes. Elastic and inelastic properties of the components can be taken into account.

The main drawback of this accurate modeling is certainly the intensive computational effort needed. Micro-modeling is hence

suitable only for small structural elements of particular interest in strongly heterogeneous states of stress and strain.
Some difficulties are partially solved by simplified models (Lofti and Shing 1994) where expanded units represented by continuum elements are used to model both units and mortar, while the behavior of the mortar joints and unit-mortar interface is lumped to the discontinuous elements; masonry is hence considered as a set of elastic blocks bonded by potential fracture/slip lines at the joints (Lourenco and Rots 1997), (Gambarotta and Lagomarsino1997).
Detailed micro-modeling deals at the same time with constitutive law of materials and structural modeling (see 2.1.2), since the microscopic approach allows a lack of any further kinematic model.
Simplified micro-modeling: is represented by the homogenized modeling.
If the structure is composed by a finite repetition of an elementary cell, masonry is considered as a continuum whose constitutive relations are derived from the characteristics of its individual components and from the geometry of the elementary cell. Most of the methods of homogenization simplify the geometry of the basic unit with a two-step introduction of vertical and horizontal joints and thus without taking into account the regular offset of vertical mortar joints. This approach could produce significant errors in nonlinear-analyses. To overcome this approximation micromechanical homogenization, based on the detailed finite element analysis of the elementary cell, was derived by (Van der Pluijm1999), (Lopez et al. 1999), (Zucchini & Lourenco 2002).
A micro-mechanical model for the homogenized limit analysis of in-plane loaded masonry has been proposed by (Milani et al. 2006 I - II), (Milani et al. 2007). It's developed to obtain the homogenized failure surfaces for masonry. The strength domains are implemented

in finite element limit analysis codes and numerically treated both with lower and an upper bound approach.

Macro-modeling: is the most common approach, it does not make any distinction between units and mortar and considers the material as a fictitious homogeneous orthotropic continuum. In fact, in practice-oriented analysis on full structures a detailed description of the interaction between units and mortar may not be necessary.

The macro models can be related to plasticity or damage constitutive laws, an example is given in (Lourenco 1996, 1998) where a non-linear constitutive model, for in plane loaded walls, based on plasticity theory is presented. The main drawback is that these continuum mechanics (finite element) models would describe damage as a smeared property spreading over a large part of the structure; in real masonry damage instead is normally localized in concentrated large cracks.

2.1.2 Structural modeling

Once defined the material modeling strategy, another complex issue in historical masonry structures analysis is the choice of a suitable structural model representing the structure.

In the social sciences, the concept of structure refers to the organizing principle of a lexicon (R. Barthes). Similarly, in construction science structural modeling deals with the correlation of displacements and deformations. Very often, thus, structural theories have their rationale in kinematic laws simplifying the underlying continuum formulation, according to the geometry of the problem.

In the hypothesis of homogeneous or homogeneized material, different models can be identified:

- Models with structural components among which can be distinguished:

Models with beams and columns: this model defines in detail the behaviour of the system for a façade for instance and makes possible to determine nonlinearly the collapse state both statically and dynamically.

Strut and tie models: these models give the possibility of using simple equilibrium models to estimate the ultimate capacity of masonry shear-walls. These models are based on load-path or strut-and-tie schemes representing the combination of the compression or tension stress fields which are mobilised at the ultimate condition. General rules for the construction of the models and specific solutions are presented for elementary solid walls subjected to different load conditions in literature (Roca 2006).

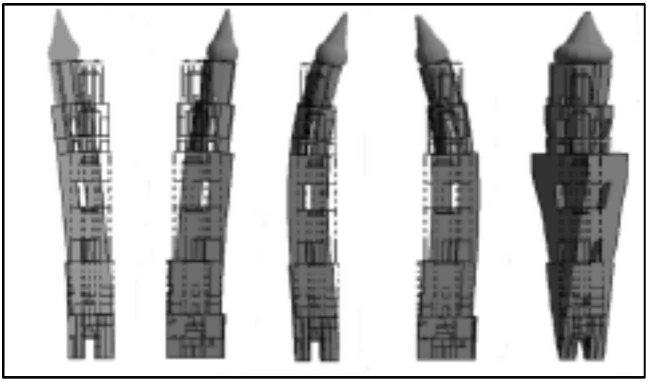
Macroelements models: following this model the structures is divided into a whole of so-called macroelements which are studied independently trough limit analysis method. The macroelement model, once identified the rigid panels or blocks, can be studied by advanced computer developments based on limit analysis, (Lourenço 2002), (Lagomarsino & Podestà 2004), (Orduna & Lourenço 2005), (Lagomarsino 2006).

- Finite element method: according to this method, the main geometrical approximation is a space discretization allowing to solve the structural problem ODEs by means of simple linear systems. Models here can be either in plane or in 3d space and can be composed by monodimensional elements (beams) bidimensional elements (plates) or three dimensional (bricks) elements. Plate elements in general give faster and more controllable models because of the presence of a smaller number of nodes if compared with a corresponding brick model. On the contrary a brick model

allows the visualization of the stresses evolution inside the structure.
- Discrete element method: is characterized by modeling the structure as an assemblage of distinct blocks interacting along boundaries. According to (Cundall and Hart 1971) the name discrete element applies to a computer approach only if it allows finite displacement and rotations of discrete bodies, including the complete detachment and it can recognize new contacts between blocks automatically as the calculation progresses.
Interesting application of this method to historical masonry structures are described in (Lemos 2007), (De lorenzis et al. 2007).
- On the other hand, material non homogeneization implies the already mentioned detailed micro-modeling approach, for which geometrical relations are just obvious.

2.2 Analysis methods for masonry towers

A general overview of most common analysis methods used to determine seismic behavior of masonry towers is described in the following.
A well assessed procedure includes FEM models associated to a dynamic identification through in situ test: FE model, first designed according to geometrical survey, is hence updated (in terms of mechanical properties of masonry) in order to give natural frequencies results in agreement with in situ measurements (Ivorra & Pallares 2006). Once the FE model is judged reliable on dynamic aspects, a spectral linear analysis could be run. An example of this procedure is illustrated in (Ceroni et al. 2010).



Fiugure 13 Natural frequency analysis of the bell Tower of Santa Maria del Carmine (Ceroni et al. 2010).

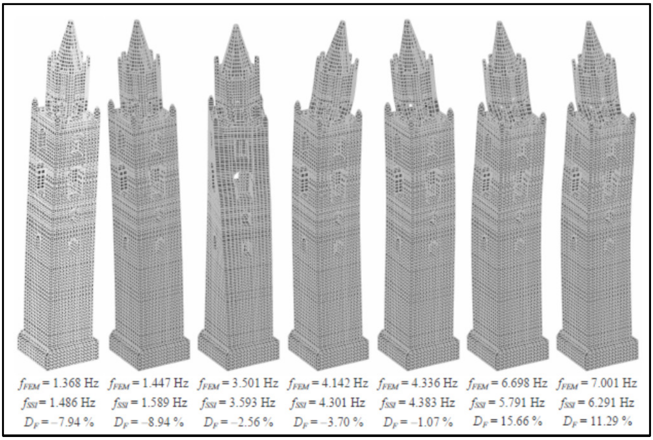


Figure 14 Comparison between vibration modes of the Fem model and experimental data for a bell tower in Teramo (Gentile & Benedettini 2007).

It's important to underline that not often, analyses dealing with FEM models include also a model for soil-structure interaction (Abruzzese & Vari 2003), (Fanelli 1993), while many analyses consider the tower with a fixed restraint at the base.

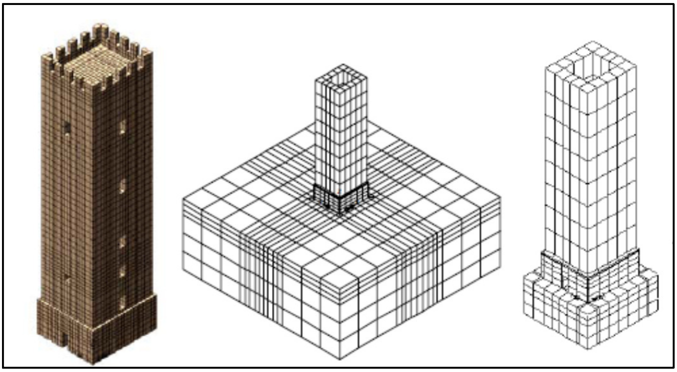


Figure 15 Torre dei Capocci, example of FEM analysis including the influence of soil modeling (Abruzzese & Vari 2003).

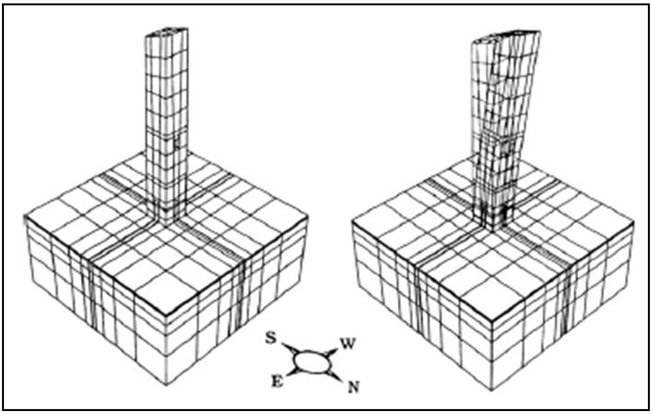
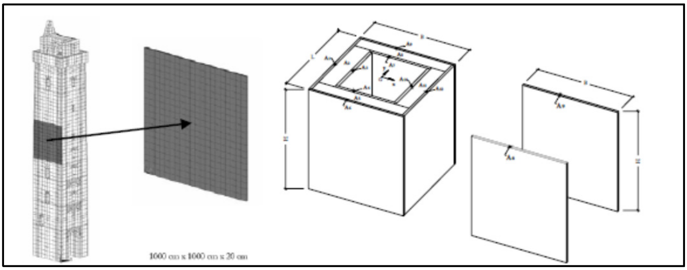


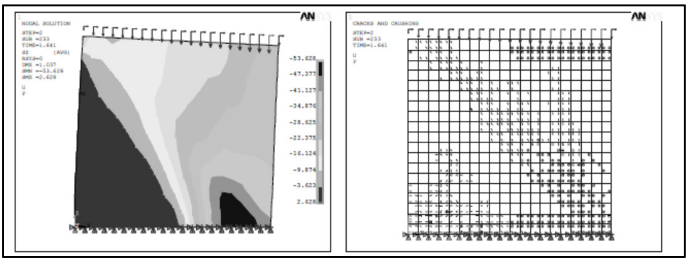
Figure 16 Discretized geometry including soil foundation and first vibration mode of a masonry tower (Fanelli 1993).

It's important to point out that while fixed restraint assumption can be accepted for new buildings as it generally implies an increase in seismic demand, it's not equally acceptable to verify existing structures as it would state an unrealistic failure condition.

Another analysis method using a FEM model to run a global linear analysis followed by nonlinear analysis of some masonry panels is described in (Bartoli et al. 2006): evaluation on seismic reliability of an ancient tower is done by a preliminary static and dynamic characterization of an elastic FE model performed with respect to a series of in situ measurements. Identification model is lately used to evaluate time history of the global force acting on each section due to seismic load.

After the evaluation of the time-history of each internal action, for some sections of the tower, the evaluation of seismic reliability was carried out analyzing two limit state (tower overturning and mechanical collapse of masonry panel).





Figures 17, 18 Identification of the panels; vertical stress diagram and crack pattern of a single panel (Bartoli et al. 2006).

Main advantage of this method is that since the whole model is a linear one computational effort needed for analysis is not heavy and nonlinear analysis are developed only on a reduced model of an elementary panel.

(Pena et al. 2010) instead proposed a method including a combination of different FE models: complex tridimensional models dedicated to dynamic identification and for calibration of simplified models, beam models for nonlinear analyses and rigid models as comparison. The use of different models allows overcoming the complexity on the study of seismic behavior of masonry structures; in fact combining the results it's possible to obtain a better and more comprehensive interpretation of seismic behavior. In particular results obtained from nonlinear static analysis and dynamic analysis indicates a different response to the earthquake of a slender minar tower. Nonlinear static analysis shows that the lowest part of the structure exhibits a diffuse cracking and a base overturning mechanism could be detected. Instead, the nonlinear dynamic analysis carried out indicates that the part more susceptible to seismic damages coincides with the upper levels where the highest accelerations and drifts are found. The difference in results is due to high influence of the higher modes in the seismic behavior of the tower; in fact, the nonlinear static analysis does not take into account the participation of different modes. The modal pushover analysis, which considers influence of higher modes, cannot reproduce the appendix-like behavior of the last levels satisfactory and this is due to the change of dynamic properties during the damage process. The results of nonlinear analysis are considered more representative of real seismic behavior since historical damage by earthquake is concentrated in the upper levels.

Considering hence the importance of higher modes in tower seismic behaviour, whose effects are appreciable only in a nonlinear dynamic analysis, some works are dedicated to identify a model simple enough to perform a wide number of nonlinear dynamic analyses avoiding a part of computational effort.

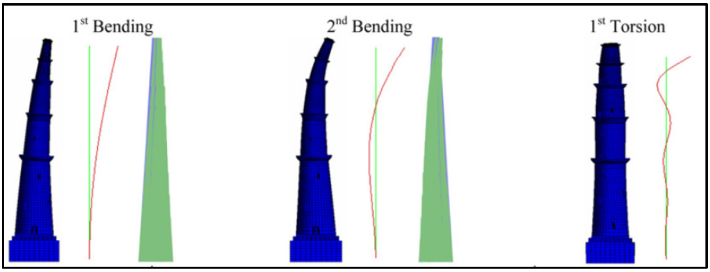


Figure 19 Solid model, beam model and rigid model for Qutb Minar (Pena et al. 2010).

Another example of these procedures is proposed for Asinelli tower in Bologna (Riva et al. 1998) and uses a simplified beam model to perform a nonlinear dynamic adopting as input earthquake time-histories recorded during events in nearby area. The advantage of using a simplified model permitted to run the analysis for a significant number of seismic events obtaining a more complete picture of the seismic behaviour of the tower.

Due to the geometrical simplicity of masonry towers, different fiber models were developed to study with a reduced computational effort towers behavior.

(Casolo 1998) proposed a fiber model for hollow squared section to describe global dynamic response of slender masonry towers to be used in deterministic vulnerability analyses. The model accounts for the tridimensional response of the structure and the relations between coupling effects and masonry characteristics; a parametric

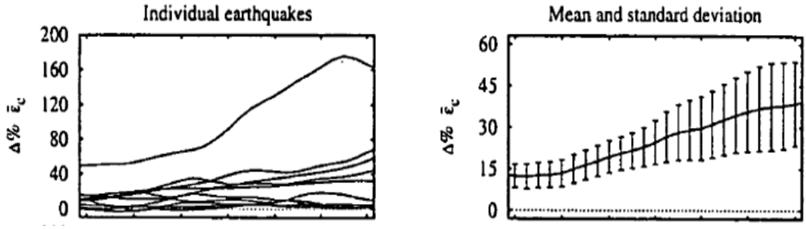


Figure 20 Influence of tower height on percentage variation in mean deformation indices, percentage variation are determined comparing results of analysis which considers the three components of earthquakes with those considering only horizontal components (Casolo 1998).

study indicates that compression strength and height are the most important parameters determining global response to seismic events and that the response is often very sensitive to vertical component of the ground motion.

A numerical model is proposed in (Lucchesi & Pintucchi 2007) to enable performing nonlinear dynamic analysis of slender masonry structures, such as towers and columns. Such structures are represented by a continuous one dimensional model and the main mechanical characteristics of the material in all cross-sections along the height are taken into account by means of a nonlinear elastic constitutive law formulated in terms of generalized stress and strain, under the assumption that the material has no resistance to tension and limited compressive strength.

Fiber models applied to beam elements are a computationally efficient mean for the frequency characterization of structures as masonry towers, for which the material non-linearities (e.g. NRT material) result non neglectable in predicting their dynamical properties.

Finally, an important classical method for seismic assessment of masonry structures is represented by limit analysis. Limit analysis is used both as an independent seismic assessment method for simplified vulnerability analysis (D'Ayala & Speranza 2003), (Speranza 2003), both as comparison and qualitative countercheck when running complex numerical analyses - an example for tower analysis is given in (Salvatore et al. 2003).

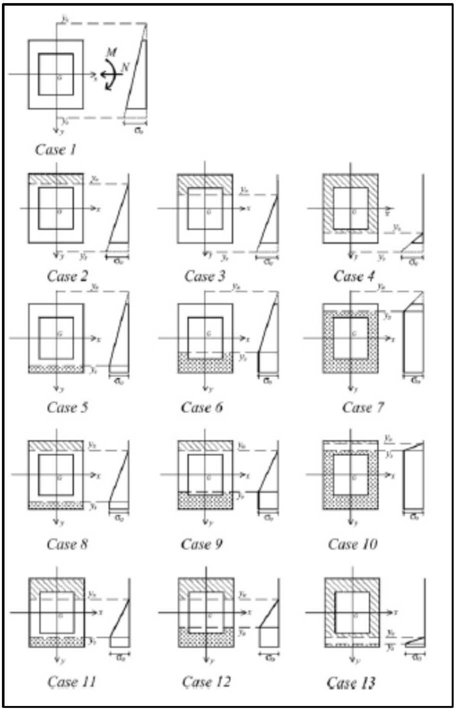


Figure 21 Cross-section cases for continuous one dimensional model of (Lucchesi & Pintucchi 2007)

An important contribute in masonry tower limit analysis is given by (Heyman 1992) but the work is dedicated to leaning analysis and it does not concern directly the seismic behaviour. In the following paragraph an extensive discussion on limit analysis and its application on historical architecture and masonry towers will be presented.

2.3 Limit analysis

The general method of limit analysis is aimed to determine the collapse load of a structure.
The static and kinematic theorems of the limit analysis Godzev (1938) and Drucker, Prager and Greenberg (1952), are:

Static theorem:
The plastic collapse load multiplier γ_p is the largest of all the multipliers γ_σ correspondent to the statically admissible set ($\gamma_p > \gamma_\sigma$).
For a statically admissible set, a stress distribution in equilibrium with the external forces that in no point violates the plastic conditions is intended.
Kinematic theorem
The plastic collapse load multiplier γ_p is the smallest of all the multipliers γ_σ correspondent to possible collapse mechanisms ($\gamma_p > \gamma_\sigma$).
For kinematically admissible set, a kinematism or a distribution of velocity of plastic deformations, related to the distribution of plastic

hinges, which satisfies the condition of kinematic compatibility is intended.

From these theorems two calculus methods are derived:

Static method

This method consists in assuming a distribution of statically admissible stresses dependent by a certain numbers of parameters and searches them so that the correspondent load multiplier is maximum.

Kinematic method

This method consists in assuming a collapse mechanism dependent on some geometrical parameters and in the following minimization of the correspondent multiplier to the considered mechanism.

According to the uniqueness theorem, a multiplier that is statically and kinematically admissible coincides necessarily to the collapse multiplier.

2.3.1 Limit analysis of masonry structures

When applying limit analysis method to masonry structures analysis it is necessary to take into account that: masonry constitutive model is of fragile type with a high value of collapse in compression, compared to tension; ultimate tensile stress is not only small but is characterized also by a high uncertainty of values because of a great scattering of the experimental results. In limit analysis, hence, a simplified diagram of infinitive compressive strength and no tensile strength is in general adopted.

The application of limit analysis to masonry structures was firstly studied by (Coulomb 1977) for determining their collapse behaviour, Coulomb proposed the use of a theory of “maxima and minima” to determine the position of the most unfavorable hinges position. In recent times (Koorian 1953) demonstrated how stone masonry can be studied through plasticity theorems, and lately a wide contribution on the subject was done by (Heyman 1966, 1969, 1995) who indicates some hypothesis on the mechanical behaviour of masonry, the basis of modern limit analysis.

Following assumptions regarding material properties are made:

1- Masonry has no tensile strength; this statement corresponds not only to the effective masonry tensile strength experimental values but also to the case where forces are transferred trough joints without mortar (*a secco*)

2- Infinite compressive strength of the blocks, considering the fact that usually masonry structures reach collapse for a mechanism state before than compression failure

3- Sliding inside the masonry and between parts of the structure cannot occur, considering that generally the angle between the thrust line and the sliding surface is greater than the friction angle.

Under these assumptions, unique collapse mode is a mechanism one, involving the rotation of a rigid block relatively to another about a common hinge point and masonry behaves as an assemblage of rigid bodies held up by compressive contact forces. The collapse is characterized by the formation of internal hinges.

Unique and safe theorems can be expresses as follows

“*If a thrust line representing an equilibrium condition for the structure under certain loads lies fully within the masonry, and allows the formation of sufficient hinges to transform the structure into a*

mechanism, then the structure is about to collapse. Further, in case of proportional loads, the loads multiplier at collapse is unique"
"*If a thrust line, in equilibrium with the external loads and lying wholly within the structure, can be found, then the structure is safe*"
In spite of its ancient origin, limit analysis is regarded today as a powerful tool realistically describing the safety and collapse of structures composed by blocks; however it must be remarked that this analysis can hardly be used to describe the response and predict damage for moderate or service load levels not leading to a limit condition.
In engineering common practice, when dealing with structures under dynamic excitation (as seismic load), deformable continuum behavior is assumed. Under this assumption, in fact, the main code-prescribed analysis (linear static analysis, modal dynamic analysis, push over analysis and nonlinear dynamic analysis) are developed. There exist, however, a number of structural types for which rigid body motion may represent a significant structural behavior; in fact phenomenon of separation or lift off has been observed to occur between structural parts in numerous earthquakes.
In particular for masonry buildings, the experimental observation of collapse mechanism consequent to hinges formation on the wall section led to the bases of masonry limit analysis centuries ago.
Both advanced continuous models, anisotropic based models, and discrete (micro-) models for masonry structures have been developed in the last decades. Nevertheless, the drawback of using nonlinear finite element analysis in practice includes: requirement of adequate knowledge of sophisticated nonlinear process and advanced solution techniques by the engineer; comprehensive mechanical characterization of the materials and large time requirements for modeling, for performing the analysis with a

significant number of combinations, and for reaching proper understanding of the result significance. Of course for special cases, as complex, important structures, nonlinear analysis should not be ignored as an analysis tool.
Linear elastic analysis can be assumed more practical, even if the time requirements of modeling are similar. Nevertheless these analyses fail to give an idea of the structural behavior beyond the beginning of cracking. Due to the low tensile strength of the masonry, linear elastic analyses seem to be unable to represent adequately the behavior of historical constructions.
Limit analysis combines, on one side, sufficient insight into the collapse mechanisms, ultimate stress distributions (at least on critical sections) and load capacities and, on the other, simplicity in practical computational tool. Another important feature of limit analysis is the reduced number of necessary material parameters, given the difficulties in obtaining reliable data for historical masonry.

2.3.2 Macroelement analysis method

Limit analysis principles, combined with survey and recognition of frequent collapse modes of certain typology of structures, led to the macroelement analysis: according to most common damages, observed during earthquakes, the buildings are subdivided into a certain number of macroelements depending of their typology. Each macroelement is characterized by a sort of independent behavior expressed in some classified collapse mechanisms.

This structural interpretation allows defining the global behavior as a sum of single macroelement mechanisms and a sum of disaggregation phenomena between adjacent macroelements.
Aggregation lines are geometrical surfaces that connect adjacent macroelement, therefore being zones of forces transmission, these parts are very important from a structural point of view and the global behavior of the building depends from their connection. The presence of tie rods, the presence of rigid decks connected to the masonry walls or the presence of vaults could strongly affect global structural behavior preventing or encouraging detachments and relative movements.
In fact, although damage survey and catalogs of damages due to past earthquakes allows determining the most probable behavior of a macroelement, the activation of a certain mechanism depends on many boundary conditions depending on aggregation lines.
Collapse mechanism of the single masonry macroelement is generated by *fractures lines* that separate the macroelement in rigid blocks transforming the structural part in a labile system.
Blocks are considered usually as bidimensional solids (in plane or out of plane surfaces with a finite thickness) and they can assume a kinematic configuration that produces the collapse.
Fracture line represents an acquired discontinuity of the masonry wall.
Dynamic properties of the structure change for the presence of line of fracture; hence interaction with seismic motion is modified.
The damaged structure, in dynamic phase dissipates a lot of energy along the fracture line where relative sliding and rotations can occur.
When line of fracture presents mainly a detachment motion perpendicular to the fracture, fracture line is defined as activated in

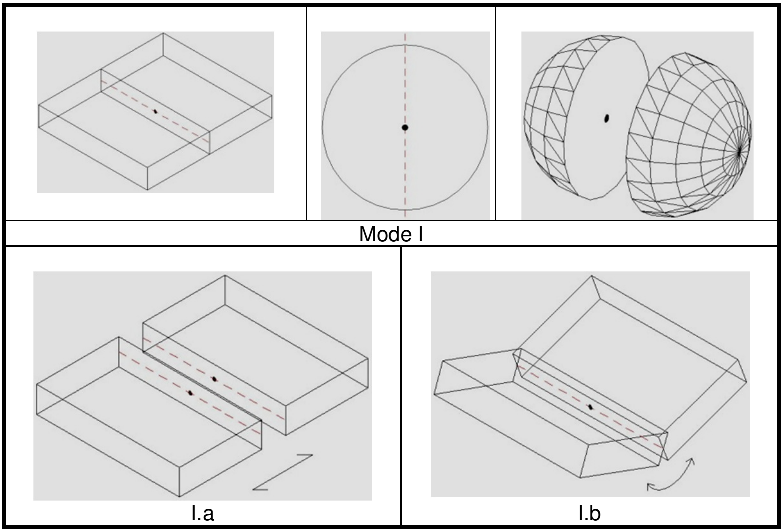


Figure 22 Fracture in mode I

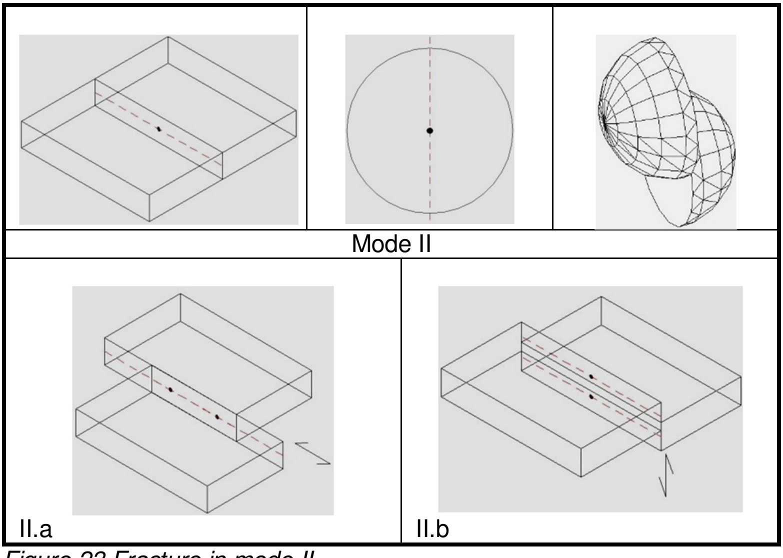


Figure 23 Fracture in mode II

mode I, when the fracture line presents sliding is defined as activated in mode II.

Fracture lines can be divided also into: I.a when corresponding to a relative translation of blocks and I.b when corresponding to a relative rotation of blocks with center of rotation along the fracture line. Moreover fracture lines are identified as II.a type if translation remains on the middle plane of the original element (in plane movement), they are identified as II.b if the translation of blocks occurs with a motion perpendicular to middle plane.

In the described models masonry is assumed as isotropic material with homogenized properties, idealization particularly functional to study collapse of macroelements due to seismic action.

In fact inertia forces due to relative motion could determine a lack of equilibrium for the system; through this model it’s possible to appreciate the failure mode in most cases.

As every model this one applies better to some cases and worse in other situation: for instance is suitable for brick masonry structures but less suitable to describe behavior of masonry composed by large stone blocks (for which hypothesis of homogeneous solid is not correct and where fracture lines are heavily influenced by joints positions). This model is even less suitable in case of poor masonry quality; in fact poor masonry structures reach collapse by disaggregation of masonry panels.

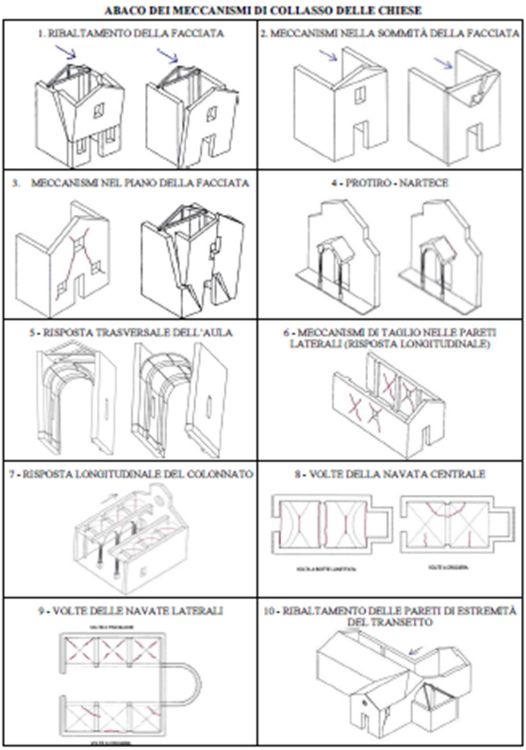


Figure 24 Collapse mechanisms for churches

The macroelement analysis method was introduced to study damages on historical buildings, churches in particular, after the Friuli earthquake (Doglioni et al. 1994); nevertheless some recurrent collapse mechanisms were already been identified by (Rondelet 1802) and proposed by (Giuffrè 1991) for seismic analysis of masonry buildings, by decomposing them into rigid blocks. In the latest decade it became a common analysis method for masonry structures also thanks to the possibility to combine blocks analysis with the capacity spectrum method (Fajfar 1999), for the seismic assessment of masonry structures. The method is applied to churches, buildings and towers; the verification methodology has been adopted by the seismic Italian code since the OPCM 3274.

Advanced computer developments based on limit analysis can be found in (Orduna & Lourenco 2001), (Lourenço & Rots 1997), Lourenço & Rots 1998), (Lourenço 1996).

Examples of application of macroelement limit analysis coupled to graphic static on real historic churches can be found in (Roca P et al. 1998) and in (Huerta S 2001).

Description of the analysis method

Once identified the mechanisms, the seismic force, activating them and causing the collapse of the structure, must be determined: the analysis is aimed to quantify the factor λ, multiplier of horizontal loads that activate the kinematic mechanism.

Local collapse mechanisms analysis is developed through equilibrium limit analysis following a kinematic approach that is based on the choice of mechanism and the evaluation of the horizontal action that cause its activation.

The comparison of λ values obtained for different kinematic mechanisms allows to identify the one causing the failure of the structure as the mechanism identified by the minor multiplier among all the possible kinematic mechanisms. To this ultimate multiplier value a correspondent seismic acceleration can be related; this analysis permit also to determine most critical zones of the structure for the presence of possible collapse mechanisms with a low multiplier of activation.

For each potential collapse method the procedure requires to transform a part of the building in a labile system identifying the rigid blocks through possible surface of fracture; lately for each mechanism collapse multiplier λ is determined.

To calculate the collapse multiplier it's necessary to apply to the rigid blocks system, forming the kinematic chain, all the actions active on the system:
- Dead load of the blocks applied on the center of mass of each block
- Vertical loads supported by the blocks
- An horizontal forces system proportional to the vertical loads supported
- Others eventual external forces (as tie rods)
Multiplier is then obtained by applying virtual work principle, in terms of displacements, imposing the equality from total work made by external forces applied to the system in a virtual motion condition to the work of eventual internal forces:

$$\lambda \cdot \left[\sum_{i=1}^{n} P_i \cdot \delta_{ix} + \sum_{j=n+1}^{n+m} P_j \cdot \delta_{jx}\right] - \sum_{i=1}^{n} P_i \cdot \delta_{iy} - \sum_{h=1}^{O} F_h \cdot \delta_h = L_{fi} \quad (2)$$

being:
- *n* the number of all the self-weight forces applied to various blocks of the cinematic chain
- *m* the number of forces not directly acting on the blocks, whose masses, as consequence of seismic action, determine horizontal forces on kinematic chain element (when not transferred to other parts of the building)
- *o* is the number of external forces, not associated with the masses, applied on the blocks
- P_i is a generic self-weight force applied on the block
- P_j is a generic self-weight force acting not directly on the block, whose mass, as consequence of seismic action, determines horizontal forces on kinematic chain element (when not transferred to other parts of the building)

- δ_{ix} is the horizontal virtual displacement of the application point of the i-th force P_i, assuming as positive the direction associated to that where seismic force activating the mechanism is acting.
- δ_{jx} is the horizontal virtual displacement of the application point of the j-th force P_j, assuming as positive the direction associated to that where seismic force activating the mechanism is acting
- δ_{iy} is the vertical virtual displacement of the application point of the i-th force Pi, positive if upward
-F_h is the absolute value of a generic external force applied to a block
- δ_h is the virtual displacement of the application point of h-th external force, in the direction of the force, positive if in the opposite direction
- L_{fh} is the work of eventual internal forces

The displacements of the forces application points are calculated considering geometry of the structure and assigning a virtual rotation at the generic block.

In recent years some effort was addressed to make this simplified macroelement analysis more accurate, as including a limit on masonry compressive strength or trying to define analytically the correct geometry of rigid blocks.

To determine the shape of fracture surfaces that divide the structure in rigid blocks, beside qualitative methods using recurrent collapse mechanisms, there exist in literature methods based on micro-mechanical models for the homogenised limit analysis of in-plane loaded masonry. (Milani et al. 2006 I - II); assuming brickwork under plane stress condition and adopting a polynomial expansion for the 2D stress field, a linear optimisation problem is derived on the elementary cell in order to recover the homogenised failure surface of the brickwork.

(De felice & De Buhan 1997) proposed a closed-form solution obtained through a kinematic approach where the homogenized

material derived is infinitely resistant in the compression-compression region, while is orthotropic at failure in the tension–tension field.
Some methods uses discrete element method to define surface of fracture considering the external geometry of units constituing masonry walls as geometry of discrete elements (de Felice & Mauro 2010); other methods consider friction effects on fracture joints (D'ayala e Casapulla 2003), methods including an explicit evaluation of seismic resistance to changes in the geometry and in the masonry fabrics that can be used for practical design (De Felice 2001).
The method proposed in (Ochsendorf et al.2004) defines a stress free surface of fracture from the assumption of unilateral behaviour of masonry that induces, at the limit of overturning of a block, that part of the block will separate from the rest if not held in compression.
In particular the identification of blocks geometry represent a very important issue because, being the calculation of collapse multiplier, essentially a problem of equilibrium, geometry of the kinematic chain highly affects the results.
As discussed in the previous chapter the Italian code includes in the description of local mechanisms - to be studied by kinematic analysis - all the mechanism of bell towers (considering bell towers one of the macroelement of churches). Nevertheless the code, while encouraging limit analysis for masonry towers, does not give any formulation to determine the shape of rigid blocks forming the mechanism.

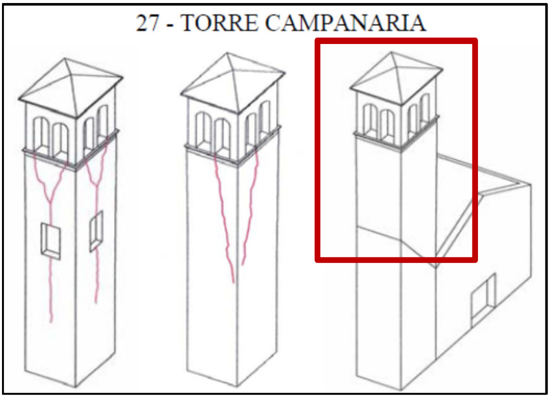


Figure 25 Towers collapse mechanisms, global overturning (LLGG)

In the following chapter a simple analytical model able to identify the geometry of collapse mechanism is described.

Considering the specific topic of masonry towers in the limit analysis method some aspects will be taken into account:

- due to the high value of compression stresses at the base of masonry towers some considerations on material properties are needed, in particular a limit on compressive strength must be considered

- considering the dimensions of the element a very simplified global collapse mechanism which does not consider that during overturning the masonry volume not subjected to compression will separate from the rest, and hence will not give any weight contribution to stabilizing moment, would result very unsafe.

2.4 Seismic analyses of masonry structures in National Codes and Eurocodes

Eurocode 08 (EC08), Italian national codes for building construction and seismic risk (NTC 2008 and OPCM 3431), and the italian document *Linee guida per la valutazione e la riduzione del rischio sismico del patrimonio culturale* (LLGG in the following) dedicated to seismic assessment of architectural heritage have been considered.
The latter document was thought to adapt requirements stated by the building code for new constructions to the different situations that can be found in ancient architecture.
Guidelines (LLGG) are written to specify the knowledge process, to evaluate seismic assessment and to define a design suitable to cultural heritage requirements; the aim is to evaluate safety while guaranteeing conservation through a structural upgrading. The document refers only to masonry structures.
Since the situations found in architectural heritage could be very different the code gives only general guide lines and it is let to the engineer the task to define a suitable model for safety assessment that must be justified according to the specific situation.
In the guide lines given by the code, high importance is given to historical-critical analysis, aimed to identify the building process and the intervention on the structure; fundamental is also the geometrical and structural survey that must include crack patterns and structural damages.
The code underlines also the importance of mechanical properties identification of materials through in situ analysis whose number and type must be justified by their employment in the structural assessment.

According to the knowledge achieved in the preliminary phases are defined three different level of knowledge (*livelli di conoscenza*) to which correspond different confidence factors (*fattori di confidenza*); these factors must be used as partial safety factors that consider the incomplete description of model parameters.
Structural demands are those calculated for new construction, but comparing demand and structural response national code states that for architectural heritage safety factors could be decided according to the specific case, furthermore the interventions on historical architectures could be devoted to achieve only a seismic upgrading.

Rilievo geometrico	**Rilievo materico e dei dettagli costruttivi**	**Proprietà meccaniche dei materiali**	**Terreno e fondazioni**
rilievo geometrico completo $F_{C1} = 0.05$	limitato rilievo materico e degli elementi costruttivi $F_{C2} = 0.12$	parametri meccanici desunti da dati già disponibili $F_{C3} = 0.12$	limitate indagini sul terreno e le fondazioni, in assenza di dati geologici e disponibilità d'informazioni sulle fondazioni $F_{C4} = 0.06$
rilievo geometrico completo, con restituzione grafica dei quadri fessurativi e deformativi $F_{C1} = 0$	esteso rilievo materico e degli elementi costruttivi $F_{C2} = 0.06$	limitate indagini sui parametri meccanici dei materiali $F_{C3} = 0.06$	disponibilità di dati geologici e sulle strutture fondazionali; limitate indagini sul terreno e le fondazioni $F_{C4} = 0.03$
	esaustivo rilievo materico e degli elementi costruttivi $F_{C2} = 0$	estese indagini sui parametri meccanici dei materiali $F_{C3} = 0$	estese o esaustive indagini sul terreno e le fondazioni $F_{C4} = 0$

Figure 26 Confidence factor for historical masonry structures (LLGG)

2.4.1 Horizontal actions

Being the ground acceleration function of the seismic code, the difficulty in considering a suitable horizontal action applicable on masonry structures is here enlightened; in (Meli & Sanchez-Ramirez

1996) the effects of different types of ground motion on monuments and the qualification of the seismic action are discussed.

2.4.2 Equivalent seismic forces

In linear static analysis, loads equivalent to the seismic action are applied on the structure through the introduction of proportional weight forces.
In the (EC08), (OPCM3431) and (NTC 2008) the force is evaluated as:

$$F_i = F_h \cdot z_i \cdot W_i / \sum z_j \cdot W_j \tag{3}$$

Where:
$F_h = S_d(T_1) \cdot m \cdot \lambda$
$S_d(T_1)$ is the ordinate in the design spectra assumed by the building in the considered direction
W is the total weight of the construction
λ a reductive coefficient equal to 0.85 if the building is composed by at least three levels and if $T_1<2T_c$, equal to 1 in any other case.
g is the gravity acceleration
z_i and z_j are the distance form the foundation level of masses i and j
W_i and W_j are the weights of masses i and j

2.4.3 *Elastic spectra*

The earthquake motion in a given point of the structure is represented by an elastic ground acceleration response spectrum "elastic response spectrum"; the shape of the elastic response spectrum is the same for the Ultimate Limit State and for the damage limitation requirement (Damage Limit State).
According to EC08 and OPCM 3431the elastic spectrum (of vertical component) is defined as:

$0 < T < T_B \qquad S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2.5 - 1)\right]$

$T_B < T < T_C \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5$

$T_C < T < T_D \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5 \cdot \left(\frac{T_C}{T}\right)$

$T_D < T < 4s \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5 \cdot \left(\frac{T_C \cdot T_D}{T^2}\right) \qquad (4)$

Where a_g is the design ground acceletration, *S* is the soil factor, *T* is the vibration period of a single-degree-of freedom system, η is the damping correction factor with reference value of 1 for 5% viscous damping ξ, T_B-T_C are the limits of the constant spectral acceleration branch, T_D is the value defining the beginning of the constant displacement response range of spectrum.
The value of a_g varies in function of the seismic zones and the values of S, T_B, T_C and T_D are function of the soil type (with slight differences between OPCM 3431 and EC08).
It's important to point out that the material and the type construction do not play any role in the elastic spectra definition, so that they are valid for any structure.

The main difference of NTC 2008 method respect to previous codes is the definition of a_g that is determined according to seismic microzonazione (with factors F_0 and T_{C^*}) instead of being classified approximately in 4 different values corresponding to different seismic zones of the country.
Elastic spectrum (of the horizontal component) is hence calculated as:

$$0 < T < T_B \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \left[\frac{T}{T_B} + \frac{1}{\eta \cdot F_0} \cdot \left(1 - \frac{T}{T_B}\right)\right]$$

$$T_B < T < T_C \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot F_0$$

$$T_C < T < T_D \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \cdot \left(\frac{T_C}{T}\right)$$

$$T_D < T < 4s \qquad S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \cdot \left(\frac{T_C.T_D}{T^2}\right) \qquad (5)$$

Where T and Se are respectively the period and the corresponding spectral acceleration and:
S is a coefficient taking into account soil type and topography
η is a factor that modifies the spectrum for viscous damping ratio of the structure different from conventional ξ = 5%
F_0 is a factor that quantifies maximum spectral amplification, depending on site
T_C is the value defining the beginning of the constant velocity branch of the spectrum (defined from a soil coefficient depending on the site)
T_B is the value defining the beginning of the constant acceleration branch of the spectrum, $T_B=T_C/3$
T_D is the value defining the beginning of the constant displacement response range of spectrum
In figure 27 the three elastic spectra are compared.

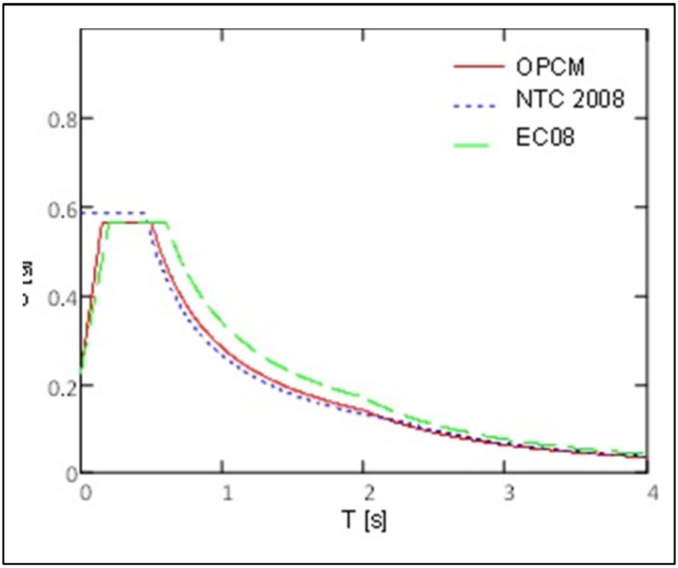


Figure 27 Elastic spectra calculated for the same structure according to the different codes prescriptions.

Respect to OPCM 3431 and EC8 it must be underlined that NTC 2008 can be said more performance design oriented as a proper lifetime, a set of four limit states and four utilization classes can be chosen in the seismic demand definition of a building

2.4.4 Design Spectra

The capacity of structural systems to resist seismic actions in the nonlinear range generally permits their design for smaller forces than those corresponding to a linear elastic response. To avoid explicit inelastic structural analysis in design, the capacity of the structure to dissipate energy, mainly through ductile behavior of its elements, is taken into account by performing an elastic analysis based on a reduced response spectrum with respect to the elastic one, called “design spectrum”. This reduction is accomplished by introducing the behavior factor *q*. The factor *q* is often recalled as the ratio of the seismic forces that the structure would experience if its response was completely elastic to the minimum seismic forces that may be used in design still ensuring a satisfactory response of the structure.

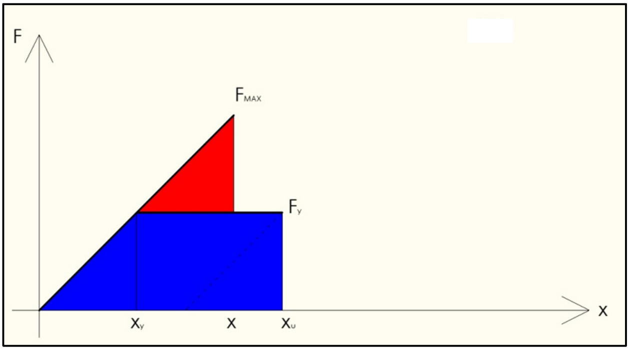


Figure 28 Forces and displacements in the elastic and elasto-plastic behaviour: the definition of behaviour factor.

It should be not forget that ductility and behavior factor, in the regard of high frequencies/low periods are connected by the relation:

$$q = \sqrt{2\mu - 1}$$

where μ is the ductility factor that is the ratio between ultimate and elastic displacements X_u/X_y. As the figure 28 shows, in fact, it is allowed to reduce seismic forces from the elastic analysis F_{max} by using the behavior factor, only if the same amount of energy is absorbed by the structure in the plastic domain when displacing at $X_u>x>X_y$ under a minor force F_y.

The values of *q* are given by the code provisions for the various materials and structural systems.

According to EC08 values for unreinforced masonry vary from 1.5 to 2.5.

In the Italian code OPCM 3431 the factor q is defined according to building technique and if the structure is a new construction or an existing building. For existing building it's equal to the product of a number (function of the regularity in height) and a coefficient α_u/α_1, defined as:

$-\alpha_1$ is the multiplier of the horizontal seismic action for which, keeping constant the other actions, the first masonry panel reaches the ultimate strength (for shear or compression and bending)

$-\alpha_u$ is the 90% of the seismic horizontal action for which, keeping constant the other actions, the building reaches the maximum resistant force.

The value of this ratio can be calculated through a nonlinear static analysis and cannot be larger than 2.5, or values given by the code (varying from 1.3 to 1.8) can be used.

To obtain value of q factor previous coefficient must be multiplied for 2 in case of regular buildings, for 1.5 in the other cases.

In the TU the q factor is calculated as:

$q = q_0 \cdot K_R$

Where q_0 for unreinforced masonry is calculated as

$q_0 = 2.0 \cdot \alpha_u/\alpha_{u1}$

The values of ratio α_u/α_1 given by the code are the same than those of the previous OPCM 3431; K_R is a reductive factor depending on regularity in height of the structure, its value can be 1 for regular building and 0.8 for the other cases.

The design spectrum indicated in the EC8 is:

$0 < T < T_B$ $\quad S_e(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$

$T_B < T < T_C$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q}$

$T_C < T < T_D$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left(\frac{T_C}{T}\right)$

$T_D < T < 4s$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left(\frac{T_C \cdot T_D}{T^2}\right)$ (6)

Where a_g is the design ground acceleration, *S* is the soil factor, *T* is the vibration period of a single-degree-of freedom system, *q* is the behavior factor, T_B-T_C are the limits of the constant spectral acceleration branch, T_D is the value defining the beginning of the constant displacement response range of spectrum.
In the OPCM 3431 the ultimate limit state design spectrum is:

$0 < T < T_B$ $\quad S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - 1\right)\right]$

$T_B < T < T_C$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q}$

$T_C < T < T_D$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left(\frac{T_C}{T}\right)$

$T_D < T < 4s$ $\quad S_e(T) = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left(\frac{T_C \cdot T_D}{T^2}\right)$ (7)

Where a_g is the design ground acceleration, *S* is the soil factor, *T* is the vibration period of a single-degree-of freedom system, *q* is the behavior factor, T_B-T_C are the limits of the constant spectral acceleration branch, T_D is the value defining the beginning of the constant displacement response range of spectrum.
The corresponding design spectrum according to the recent Italian code NTC 2008 is described as:

$0 < T < T_B$ $\quad S_e(T) = a_g \cdot S \cdot \frac{1}{q} \cdot F_0 \left[\frac{T}{T_B} + \frac{1}{\eta \cdot F_0} \cdot \left(1 - \frac{T}{T_B}\right)\right]$

$T_B < T < T_C$ $\quad S_e(T) = a_g \cdot S \cdot \frac{1}{q} \cdot F_0$

$T_C < T < T_D$ $\quad S_e(T) = a_g \cdot S \cdot \frac{1}{q} \cdot F_0 \cdot \left(\frac{T_C}{T}\right)$

$T_D < T < 4s$ $\quad S_e(T) = a_g \cdot S \cdot \frac{1}{q} \cdot F_0 \cdot \left(\frac{T_C \cdot T_D}{T^2}\right)$ (8)

Where T and Se are respectively the period and the corresponding spectral acceleration and:

S is a coefficient taking into account soil type and topography

q is the behaviour factor

F_0 is a factor that quantifies maximum spectral amplification, depending on site

T_C is the value defining the beginning of the constant velocity branch of the spectrum (defined from a soil coefficient depending on the site)

T_B is the value defining the beginning of the constant acceleration branch of the spectrum, $T_B=T_C/3$

TD is the value defining the beginning of the constant displacement response range of spectrum

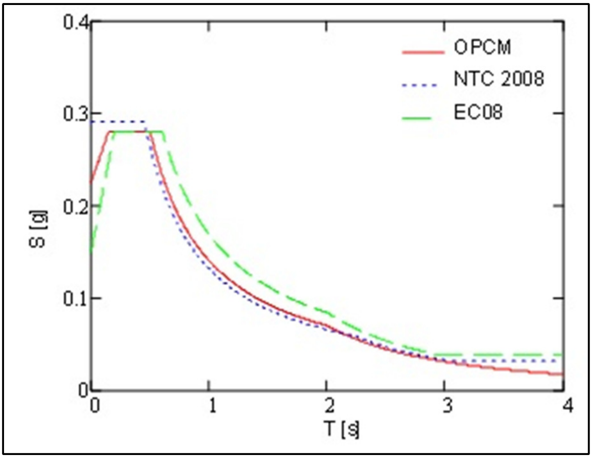


Figure 29 Design spectra calculated for the same structure according to the different codes prescriptions.

2.4.5 Analyses methods

According to Italian codes and Eurocode, seismic assessment of historical masonry buildings can be evaluated in the following methods:

- linear static analysis (equivalent seismic forces)
- linear dynamic analysis (considerate medoto lineare di riferimento)
- nonlinear static analysis (push over)
- nonlinear dynamic analysis

These methods are common to other typologies of structures; for masonry structures, in particular, is admitted also the limit analysis method, intended as:

- linear cinematic analysis
- nonlinear cinematic analysis

2.4.6 Linear static analysis

Linear static analysis method consists in the application of a force system distributed along the height of the building, in the assumption of a linear distribution of the displacements. For buildings made of several floors, the forces are applied at each slab where it's assumed that the forces are concentrated; otherwise a distributed load proportional to the masses can be adopted.

Nevertheless this method should be avoided in all the cases where the contribution of superior modes is relevant, being this the case of masonry tower (according to NTC 2008).

Guidelines (LLGG) states that the development of simplified models, able to analyze towers collapse mechanisms (depending on their slenderness and on the geometrical variety found) is not possible, hence the document suggest to perform “specific analysis even if simplified”. Finally, for a quantitative evaluation on simplified models the document suggest a sectional check under compression and bending conditions considering masonry as a NRT material.

2.4.7 Linear dynamic analysis (modal dynamic analysis)

This method is considered the reference method for existing building according to the latest Italian code.
The modal analysis, associated with the design response spectrum, can be performed on bi or three dimensional models in order to obtain the stresses values in the elements. In this analysis, all the vibration modes with a participant mass bigger than 5% must be considered and summed up so that the total participating mass result bigger than 85%. Lately a SRSS or CQC combination method must be employed to have final results in terms of stresses and displacements.
Italian guidelines for seismic vulnerability reduction of architectural heritage although usually discourage linear dynamic analysis, judge this kind of analysis more feasible to masonry towers considering the geometrical simplicity that allows to model them as cantilever with a fixed restraint at the base, recalling the fact that stress redistribution in a isostatic structure is modest (LLGG).

2.4.8 Nonlinear static analysis

This method is represented by the evaluation of the seismic behaviour of structure (generalized relation force-displacement), in particular in the capacity displacement at ultimate limit state that must be compared to the displacement demand of the seismic motion evaluated in spectral terms.
This analysis can be run on global models representing the behaviour of the whole structure or on local models (macroelement models).
The nonlinear static analysis consists in the application on the structure of the vertical loads and a horizontal forces system monotonously increasing until the reaching of the limit conditions.
Capacity curve of the structure can be determined from general relation force-displacement obtained through an incremental analysis via finite element method using nonlinear material law and eventually considering also a geometrical nonlinearity.
As alternative at the finite element method a nonlinear cinematic analysis can be done, according to document 11.C (in OPCM 3431); assigning incrementally finite displacements to the cinematic mechanism to be analyzed.
The method is introduced in OPCM 3431 seismic code and it is present also in the latest national code (NTC 2008).
In the case of architectural heritage the variety of geometries and structural systems makes impossible to determine a general force distribution corresponding to seismic motion. Analysis can be run considering two different forces distributions: proportional to the masses and proportional to the first natural mode.

2.4.9 Nonlinear dynamic analysis

Nonlinear dynamic analysis con be run on finite elements nonlinear models if the material laws can simulate the decay in stiffness and resistance at local level and also the damping properties due to hysteresis.

The analysis needs different groups of acceleration input (at least three) chosen in accordance to the response spectrum.

Nonlinear dynamic analysis, due to high computational effort requested, does not represent the most common analysis method in engineering practice and is dedicated only to very complex structural systems where the contribution of superior modes is not neglectable.

3. A SIMPLIFIED MODEL PROPOSED FOR LIMIT ANALYSIS OF MASONRY TOWERS

3.1 Masonry towers limit analysis

For a common masonry building, simplified seismic analysis can be performed through an exhaustive sum of local mechanisms analysis; instead, for masonry towers, seismic analysis must include, beside local mechanisms, also a global overturning check. In common practice this latest analysis, in lack of an alternative well-defined procedure, is often represented by an elastic analysis followed by a simple bending and compression section check. To extend limit analysis method to masonry towers a simplified model is proposed in this chapter.

When analyzing masonry towers through limit analysis, the material and geometrical properties introduced in the previous chapter must be taken into account.

Hence, in the following paragraphs it will be described a limit analysis method to evaluate safety, respect to a global overturning mechanism. The procedure presented should maintain the advantages of a traditional limit analysis (to remain distinguished from more complex nonlinear analyses, since their purpose is different and here a simplified analysis is considered) improving it by considering some specific aspects not neglectable for a safe assessment.

3.1.1 Relevance of finite masonry compressive strength

Limit analysis of masonry structures is frequently performed under the assumptions of masonry without tensile strength (*Non Resistant to Tension material or No-Tension material*) and infinite compressive strength (Heyman, 1977). As a consequence of these assumptions, cylindrical hinges are placed at one edge of cross sections (considering the case of beam-columns elements) and thrust line in collapse conditions lies at one edge of hinged cross sections. The assumption of masonry infinite compressive strength is suitable for most cases. Nevertheless, in cases of very poor masonry compressive strength and/or high compressive normal force, collapse loads resulting from these assumptions would be over evaluated.

This is in fact the situation of masonry towers analysis: the stress values at the base are in general very high, in some case near to the ultimate value yet in dead load condition (some values in table 1), hence in this case a limit analysis not including an evaluation on compressive stress is surely unsafe.

Tower	Medium compressive stress at base section
Torrazzo Cremona	1.5 MPa
Torre Duomo Monza	2.2 MPa (max)
Torre Pavia	2 MPa
Torre Ghirlandina	1.2 MPa (max)
Campanile San Marco	2.8 MPa

Table 1 Medium and maximum stress values at the base section calculated for some important masonry towers in Italy (underlined the collapsed ones),(Binda 2008).

Indeed, in cases of towers, the assumption of infinite masonry compressive strength (which, in more detail, assumes that masonry compressive stresses are small compared to strength), is not always reliable. This is because the weight of the structure produces a high axial load and high compressive stress at the base sections. In such cases, it has to be taken into account that in the lower cross sections (near the base) the application point of normal force (and thus, the hinge) cannot be placed at the cross section edge, but, at a certain distance from it, depending on axial load and masonry compressive strength (figure 30). Compared to the above case, this fact reduces the structure capacity, due to the reduction of the activation multiplier (for the diminishing of the weight moment arm).

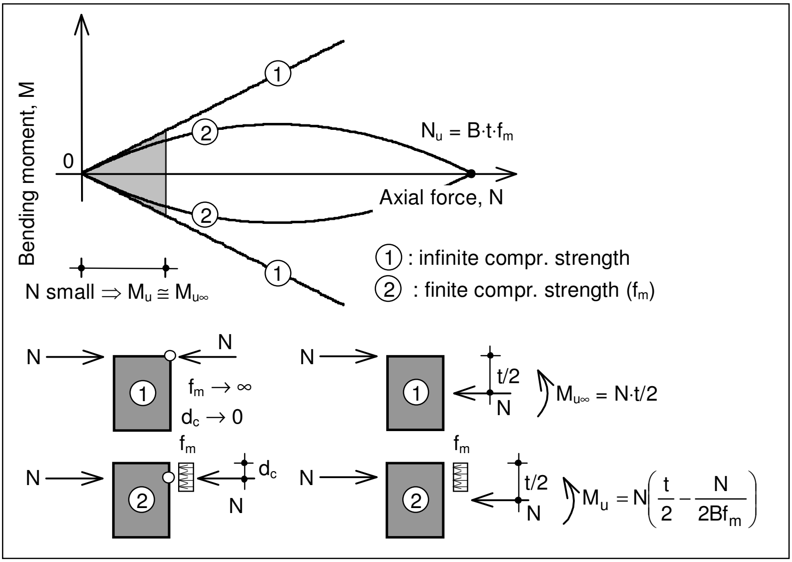


Figure 30 Axial load effects on hinges position

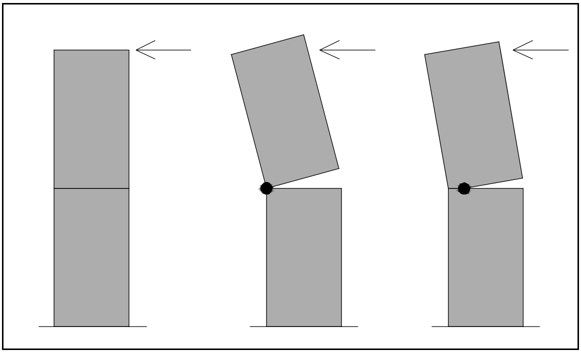


Figure 31 : Hinges positions considering infinite compressive strength or finite compressive strength

This principle applies also when dealing with soil compressive strength – for example when considering a global overturning on a base hinge point on soil foundation level, and soil compressive strength must be taken into account.

3.1.2 Relevance of fracture shape

In masonry limit analysis, the structures at collapse condition are considered subdivided into a number of monolithical blocks that form the failure mechanism; the geometry of blocks, determining their weight value and their centroid position, has a great influence on the collapse multiplier result.
Of course, considering a real masonry structure, the geometry of blocks forming the kinematic mechanism should correspond to bricks position due to the fact that the weakest interface is usually the joint; hence the crack pattern, at collapse limit state, will follow the joints position.

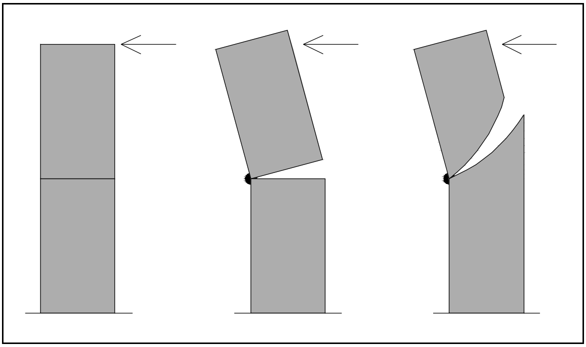


Figure 32 (a,b,c) Mechanism considering monolithical block or cracked block

For simple cases, as overturning of a wall, sections are small enough comparing to brick or stone blocks, hence an analysis on simplified geometry (figure 32 b) could result reasonable.

But analyzing big structures or global mechanisms, as evaluating the ultimate load factor for towers overturning, the geometry of the overturning mechanism should take into account that masonry is a unilateral material able to resist high compressive stresses but with feeble tensile strength.

As a consequence of this masonry characteristic, at the limit of overturning, a part of the masonry will remain attached to the base and a stress-free surface of fracture will form (Heyman 1992).

According to simple elastic theory, when the line of thrust falls outside the section kern a stress-free zone will develop (figure 33), defined by the condition, in a solid rectangular section i.e., that the distance of line of thrust from the section edge result equal to *0.333L*, being *L* the length of uncracked region (under the assumption that the compressive stress distribution is linear in the fractured region).

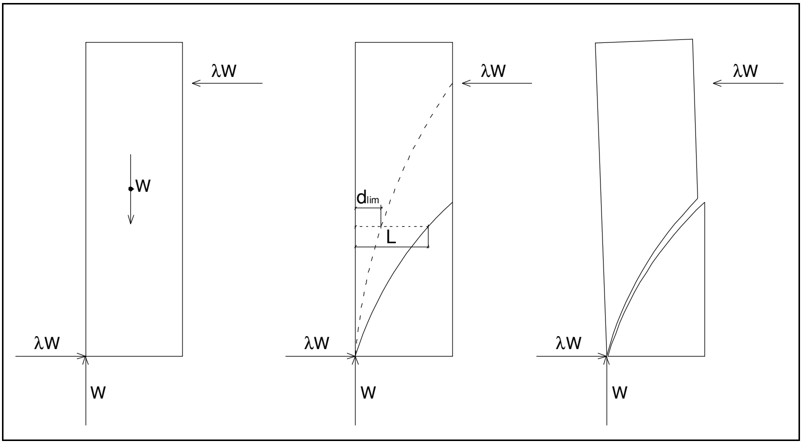


Figure 33 Position of line of thrust respect to section kern and fracture developing

In general the limit distance value d_{lim} must be calculated for each case depending on section geometry.
Geometry of the block involved in the kinematic mechanism is hence modified compared to simplified general analysis that, once defined the blocks constituting the mechanism, does not verify the exclusive compressive state (figure 32).

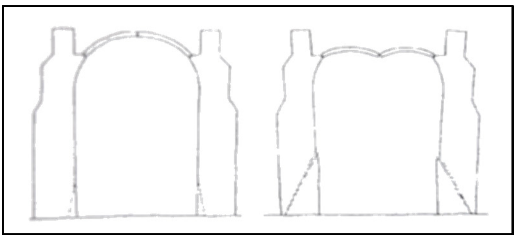


Figure 34 Example for masonry buttress (Ochsendorf et al. 2004)

An example of this procedure is given in (Ochsendorf at al. 2004) where the method is applied to buttresses supporting arches or vaults (figure 34).
In the following paragraphs, through an analytical model, is therefore determined a line of fracture. The fracture excludes a part of masonry that does not give any contribution in terms of dead load or stabilizing moment, not participating to the mechanism.

3.1.3 Importance of considering material and fracture properties

Finally, as stated at the beginning of this third chapter, the proposed method must consider both the limit on masonry compressive strength, both the fact that masonry is a unilateral material so at the collapse state an inclined line of fracture will form.
These conditions will affect the geometry of blocks and also the hinges position, determining a lower collapse multiplier compared to the one calculated under traditional assumptions.

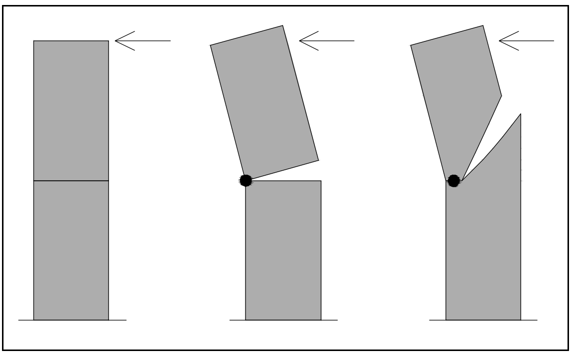


Figure 35 Mechanism considering monolithical block or cracked block with a finite masonry strength

The scheme of the modified mechanism is illustrated in figure 35, compared to the geometry of a traditional limit analysis

In the following the curve of fracture will be determined under the assumption of masonry elastic behaviour.

In simple elastic behaviour the fracture will form when the line of thrust is at limit position from the edge and the corresponding hinge point on the section should be at the same distance d_{lim} from the edge (figure 36 b).

Nevertheless, in the model proposed in the following, when considering a finite value of masonry compressive strength the compressed area has been determined assuming masonry strength as uniformly distributed on the compressed area (figure 36 c, red diagram), hence in this case the hinge should lie in the centroid of the section (x_G in figure 36).

This assumption on stress distribution on the lowest fractured section implies that in the adjacent cross-sections stress peaks values higher than masonry compressive strength are accepted.

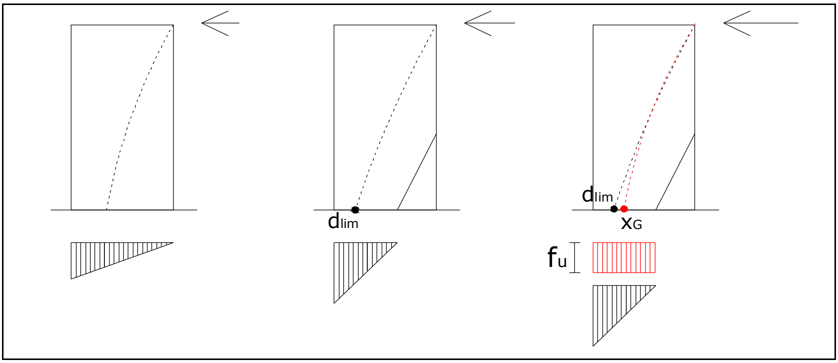


Figure 36 (a,b,c,) Hinge positions in case of different stress distribution assumptions

In order to avoid stress peaks in the lower sections, the curve of fracture should change accordingly to the modified position of line of thrust.
In the algorithm proposed in the following, assuming neglectable the differences in curve of fracture evaluation, the kinematic mechanism will be calculated assuming the hinge point in the center of mass and neglecting the calculation of the new fracture geometry.

3.2 Horizontal slice equilibrium model

Simplifying the problem into a plane problem, with reference to figure 39 a differential equation is searched whose solution is the curve of fracture $z = z(l)$
A tower of height h_t, having a constant cross-section is considered; on the tower are applied the dead load and an horizontal load with a known distribution proportional to the mass high enough to determine section partialization between $z=0$ and $z=h_{fp}$; the following assumptions are made:
-null masonry tensile strength (no tension material)
-elastic behaviour of masonry in compression
-at mechanism condition, only the masonry in compression is involved
-cross-section is constant in the volume where the fracture develops
Hence, the fracture will form in each cross-section when the line of thrust reaches the edge of the section kern.
The distance of section kern from the external edge, in case of squared cross-section of side L_e, is:

$$d_{lim}(l) = \frac{L_e - l}{3}$$

(9)

In case of hollow squared cross-sections the function must be preliminarily calculated as:

$$d_{lim}(l) = L_g(l) - \frac{\rho^2(l)}{L_e - l - L_g(l)}$$

(10)

where:

$$\rho(l) = \sqrt{\frac{J(l)}{A(l)}}$$

(11)

being $\rho(l)$ the radius of gyration of the uncracked section, $L_g(l)$ is the distance of the section centroid to the edge in compression; $A(l)$ and $J(l)$ are respectively the section area in compression, and its moment of inertia, being L_i and L_e as in figure 39.

Distance from the edge, normalized respect the uncracked length of the section, results:

$$d(l) = \frac{d_{lim}(l)}{L_e - l}$$

(12)

An examples is shown in figure 37 for two different values of ratio L_e/L_i

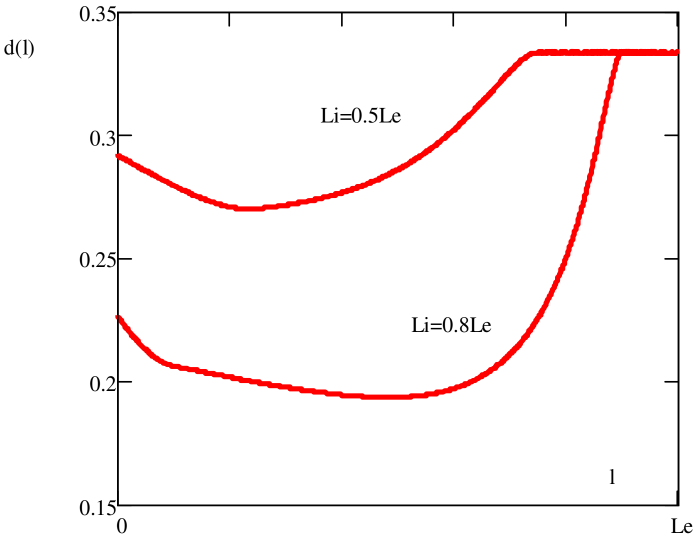


Figure 37 Values of equation d(l) for different values of ratio L_e/L_i

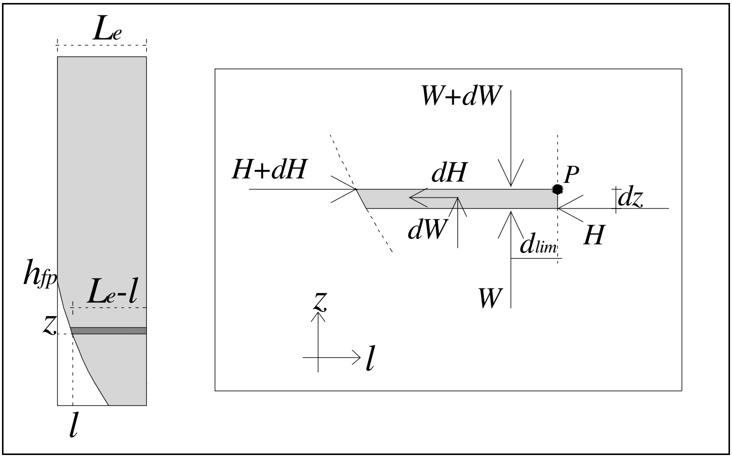


Figure 38 Elementary slice of the tower in the fractured zone

Equation of fracture is determined from equilibrium conditions of an elementary slice of tower in the fractured zone (figure 38).
W(l,z(l)) is the weight of the part in compression above height *z*; *H(l,z(l))* is the resultant of the horizontal load above height *z*, activating the mechanism.
A parameter h_{fp} is defined as the starting point of the fracture along the *z* axis, hence the condition $z(0)=h_{fp}$. is imposed. The criterion followed to choose the h_{fp} value is explained in section 3.2.1.
Expressing moment equilibrium at point *P* (figure 38) on an elementary horizontal slice of width (L_e-l), it can be obtained:

$$W(l,z(l))\cdot d_{lim}(l)+dW\big(l,z(l)\big)\cdot L_g(l)=W\big(l,z(l)\big)\cdot d_{lim}(l)+ d\big[W\big(l,z(l)\big)\cdot d_{lim}(l)\big]-H\big(l,z(l)\big)\cdot dz-dH(l,z(l))\cdot\frac{dz}{2} \tag{13}$$

Simplifying and neglecting second order terms as infinitesimal quantities:

$$dW(l,z(l))\cdot L_g(l)=d\big[W\big(l,z(l)\big)\cdot d_{lim}(l)\big]-H(l,z(l))\cdot dz \tag{14}$$

Developing differential of $dW[(l,z(l))d_{lim}(l)]$:

$$dW(l,z(l))\cdot l_g(l)=-H\big(l,z(l)\big)\cdot dz+dW\big(l,z(l)\big)\cdot d_{lim}(l)+ W(l,z(l))\cdot d(d_{\mathrm{lim}}(l)) \tag{15}$$

That is

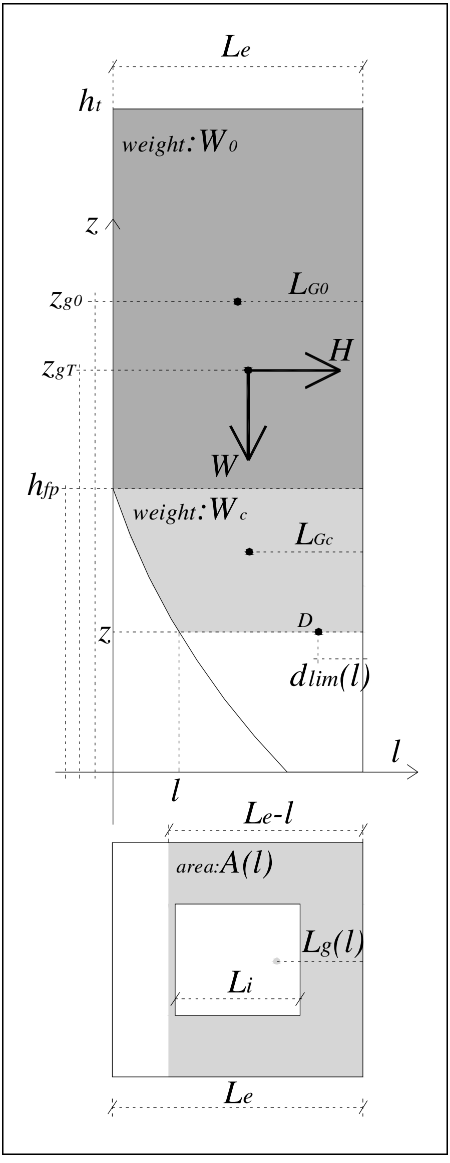


Figure 39 Geometrical model of the tower

$$.dW(l,z(l)) \cdot [l_g(l) - d_{lim}(l)] = -H\big(l,z(l)\big) \cdot dz + W(l,z(l)) \cdot d(d_{\mathrm{lim}}(l)) \tag{16}$$

Substituing:

$$dW(l,z) = -\gamma_m . A(l). dz \tag{17}$$

where γ_m is the density value of masonry,

$$dz \cdot \big\{\big[d_{lim}(l) - l_g(l)\big] \cdot A(l) \cdot \gamma_m + H(l,z)\big\} = W(l,z(l)) \cdot d(d_{\mathrm{lim}}(l)) \tag{18}$$

and dividing both terms by dl, finally, equation (18) can be written as:

$$\frac{dz}{dl} = W(l,z(l)) \cdot \frac{d(d_{\mathrm{lim}}(l))}{\big\{\big[d_{lim}(l) - l_g(l)\big] \cdot A(l) \cdot \gamma_m + H(l,z)\big\}} \tag{19}$$

That represents the differential equation of the fracture curve.
H(l,z), is expressed from moment equilibrium at a distance d_{lim} from the section edge, point D in figure 39, (where, in the fractured zone, the line of thrust lies for assumption):

$$H \cdot z_{Gt}(l,z(l)) - W(x,z(l)) \cdot [L_{Gt}(l,z(l)) - d_{lim}(l) \cdot (L_e - l)] = 0 \tag{20}$$

Hence *H(l, z)* can be defined as:

$$H(l,z(l)) = \frac{W(l,z(l))\cdot[L_{Gt}(l,z(l)) - d_{lim}(l)\cdot(L_e - l)]}{z_{Gt}(l,z(l))} \tag{21}$$

3.2.1 Implementation notes

The differential equation of the curve, having boundary condition *z(0)=h_{fp}*, was solved via a numerical *ODE solver* that uses the *Runge-Kutta* method in the fourth order increment approximation, obtaining a family of fracture curves *z(l)* varying with parameter h_{fp}.
In a first solution step, the algorithm performs a do-loop on the h_{fp} parameter until the fracture curve reaches the external edge of the section that corresponds to assume infinite masonry compressive strength.
In a second step the curve of fracture has been determined by imposing to reach the ultimate resisting moment at the base section considering a finite value of masonry compressive strength, which defines the final h_{fp} in the iterative scheme.
Once determined h_{fp} value and the corresponding curve of fracture according to assumptions made on material properties, the collapse multiplier λ can easily be obtained as the ratio between horizontal force and dead load of the overturning part.

3.3 Global equilibrium model

Finally, is observed that (19) could be obtained also from a different method, under the same assumptions considering applied to the

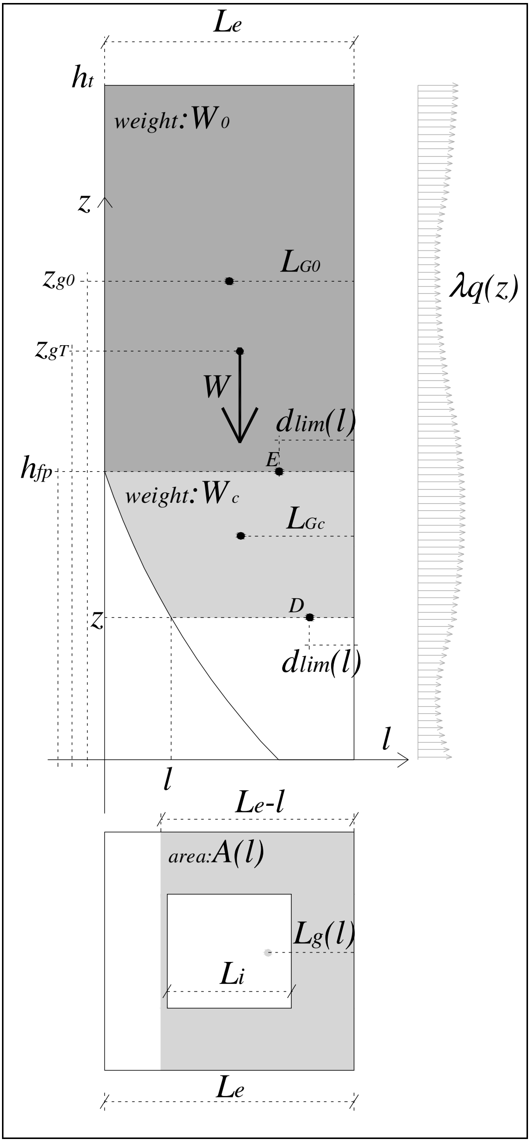


Figure 40 Geometrical model of the tower

tower the dead load and an horizontal load with a known distribution *q(z)*, being $\lambda q(z)$ the horizontal load at height *z* with λ a real multiplier and being the tower cross section constant in the range $0<z<h_t$.

Considering a tower cross-section at height $h_{fp}<h_t$ and being λ value high enough to determine section partialization between $z = 0$ and $z=h_{fp}$ but enough small not to induce the collapse of tower under dead load and horizontal distribution a $\lambda q(z)$.

For this λ value, the sections included between $z = 0$ and $z = h_{fp}$ are partialized, that is, in these sections the neutral axis divides the section in a compression zone and a stress-free zone; *l(z)* is defined as the locus of neutral axis positions between $z = 0$ e $z = h_{fp}$.

Referring to figure 40, rotational equilibrium at point D of the uncracked tower, above a generic height z included between 0 and h_{fp} gives:

$$\lambda\int_z^{h_t} q(\zeta)\cdot(\zeta - z)d\zeta - W(z,l(z))\cdot\left[L_e - L_g(z,l(z)) - d_{\lim}(l(z))\right] = 0 \qquad (22)$$

That is

$$\lambda\int_z^{h_t} q(\zeta)\cdot\zeta dz - \lambda z\int_z^{h_t} q(\zeta)d\zeta - W(z,l(z))\cdot\left[L_e - L_g(z,l(z)) - d_{\lim}(l(z))\right] = 0 \qquad (23)$$

Being *W(z,l)* the weight of the part in compression above height *z*, and $L_g(z,l(z))$ the abscissa of *W(z,l(z))* centroid, calculated as:

$$W(z,l(z)) = \gamma \cdot \int_z^{h_f} A(l(\zeta))d\zeta + W_0(h_f) \tag{24}$$

$$L_g(z,l(z)) = \frac{1}{W(z,l(z))} \cdot \left\{ \gamma \cdot \int_z^{h_t} A(l(\zeta)) \cdot [L_e - L_g(l(\zeta))]dz + W_0(h_f) \cdot L_{G0}(h_{fp}) \right\} \tag{25}$$

being (figure 40) $W_0(h_{fp})$ e $L_{G0}(h_{fp})$ the weight and the centroid abscissa of the uncracked tower (above the height h_{fp})
Differentiating (23) respect to z, it can be obtained:

$$\begin{aligned} &\lambda \frac{d}{dz} \int_z^{h_t} q(\zeta) \cdot \zeta dz - \lambda \frac{d}{dz} \left[z \int_z^{h_t} q(\zeta) d\zeta \right] + \\ &- L_e \frac{d}{dz} W(z,l(z)) + \frac{d}{dz} [W(z,l(z)) \cdot L_g(z,l(z))] + \\ &+ \frac{d}{dz} [W(z,l(z)) \cdot d_{\lim}(l(z))] = 0 \end{aligned} \tag{26}$$

Differentiating (24) e (25):

$$\frac{d}{dz} W(z,l(z)) = -\gamma \cdot A(l(z)) \tag{27}$$

$$\frac{d}{dz} [W(z,l(z)) \cdot L_g(z,l(z))] = -\gamma \cdot A(l(z)) \cdot [L_e - L_g(l(z))] \tag{28}$$

Substituting (27) and (28) in (26) it can be obtained

$$
\begin{aligned}
& -\lambda q(z)\cdot z-\lambda\left[\int_z^{h_t} q(\zeta)\,d\zeta - zq(z)\right]+ \\
& + L_e\gamma A(l(z))-\gamma\left[A(l(z))\cdot\left(L_e - L_g(l(z))\right)\right]+ \\
& +\left[-\gamma A(l(z))\cdot d_{\lim}(l(z))+W(z,l(z))\cdot\frac{d}{dz}d_{\lim}(l(z))\right]=0
\end{aligned}
\tag{29}
$$

that can be written as:

$$
\begin{aligned}
& -\lambda H(z)+\gamma A(l(z))\left[L_g(l(z))-d_{\lim}(l(z))\right]+ \\
& W(z,l(z))\cdot\frac{d}{dl}d_{\lim}(l)\cdot\frac{d}{dz}l(z)=0
\end{aligned}
\tag{30}
$$

being *H(z)* the resultant of horizontal load between height *z* and the top of the structure.

Finally:

$$
\frac{d}{dz}l(z)=\frac{\lambda H(z)-\gamma A(l(z))\left[L_g(l(z))-d_{\lim}(l(z))\right]}{W(z,l(z))\cdot\frac{d}{dl}d_{\lim}(l)}
\tag{31}
$$

is the differential equation that together with the boundary condition:

$$
z(0)=h_{fp}
\tag{32}
$$

allows to determine the curve of fracture *l=l(z)*, once determined the load distribution *q(z)* and the multiplier λ.

In the present case an horizontal load distribution proportional to the mass is assigned, hence:

$$q(z) = \gamma \cdot A(l(z)) \tag{33}$$

$$H(z) = \gamma \int_{z}^{h_{fp}} A(l(\zeta))d\zeta + W_0 = W(z, l(z)) \tag{34}$$

Being W_0 the weight of the uncracked part (above height h_{fp}) and L_{G0}, z_{g0} the coordinate of its centroid; hence (31) and (32) become

$$\begin{cases} \dfrac{d}{dz} l(z) = \dfrac{\lambda W(z, l(z)) - \gamma A(l(z))\lfloor L_g(l(z)) - d_{\lim}(l(z))\rfloor}{W(z, l(z)) \cdot \dfrac{d}{dl} d_{\lim}(l)} \\ l(0) = h_f \end{cases} \tag{35}$$

For each height h_{fp} a multiplier λ is associated, imposing that the line of thrust at height h_{fp} lies on the edge of section kern, point E in figure 40.

$$\lambda W_0(h_f) \cdot (z_{G0}(h_f) - h_f) = W_0(h_f) \cdot \lfloor L_e - L_{G0}(h_f) - d_{\lim}(0)\rfloor \tag{36}$$

Hence

$$\lambda = \frac{L_e - L_{G0}(h_{fp}) - d_{\lim}(0)}{z_{G0}(h_{fp}) - h_{fp}} \tag{37}$$

Therefore, chosen arbitrarily a value h_{fp}, a multiplier λ and its corresponding curve of fracture can be determined. Lately, among the infinitive curves of fracture calculated, the one corresponding to a defined collapse condition on base section is determined (depending on masonry compressive strength assumptions).
Both methods allow to determine the same curve of fracture; in fact multiplying the (31) by dz/dl and rearranging terms the (19) can easily be obtained.

3.4 Parametric analyses and results

As example of this method the equation is solved for an ideal tower with a common geometry.
The curve of fracture calculated for a tower of height *60 m*, with a squared cross section defined by $L_e = 10\ m$ and $L_i = 7\ m$, of desity $\gamma_m = 1800\ kg/m^3$ is plotted in figure 41.
The curve of fracture obtained presents a curvature with convexity downward; in the lower part, where the fracture is developed in the full thickness of masonry, the curve is almost a straight line.
Of course, the shape of real fracture will depend on masonry blocks position and will be influenced by the presence of discontinuities such as openings or other geometrical discontinuities on the structure (figure 43).

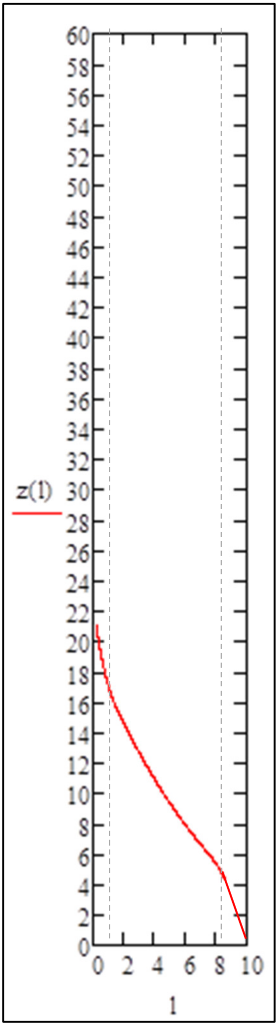
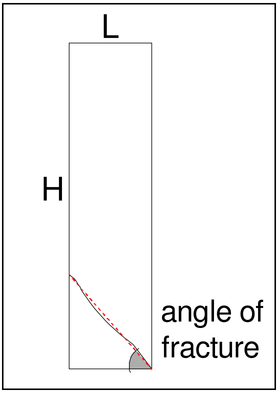


Figure 41 Curve of fracture



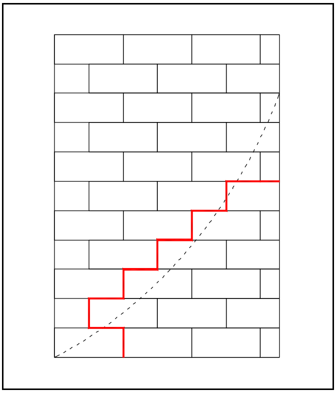


Fig 42,43 Angle of fracture definition and real crack shape

The angle of fracture (defined as in figure 42) was calculated for different tower height values and for different dimensions of square hollow sections.

In the hypothesis of a squared base tower, cross section is expressed in terms of percentage of area respect to full section (100% means a full section), the lower limit was taken as 9,75%, that for a squared section of side 10 m corresponds to a thickness of one brick.

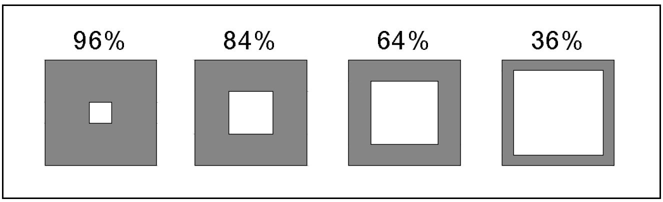


Figure 44 Examples of different percentage of hollow section

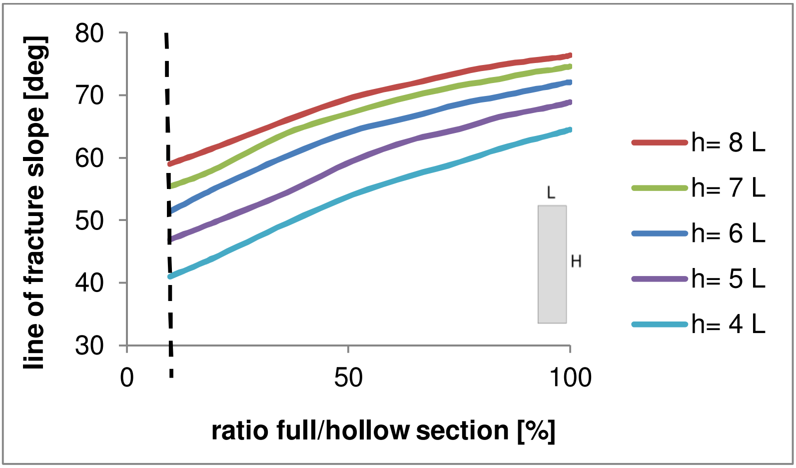


Figure 45 Line of fracture tilt for different geometry of the tower

The height of the towers is expressed as a multiple of the base length; the geometrical proportions taken into account (according to existing masonry towers surveyed) range from 4 to 8, intended as values of the ratio: height of the tower / base width.

Results are illustrated in figure 45; the fracture slope can hence be compared to real cases and used for a global mechanism analysis without calculating the equation of line of fracture.

As shown in the plot, the angle that the fracture forms with the horizontal axe is wider for full section and smaller when the hollow part of cross-section becomes not neglectable. For the geometries considered, the values of fracture angle vary from 41 deg to 76 deg.

A comparison between the collapse multipliers of uncracked and fractured tower is plotted in figure 46 for a varying geometry.

The collapse multiplier of cracked tower was calculated assuming a straight fracture line (the secant of the curve of fracture) beginning at the edge of the base section with a slope according to values

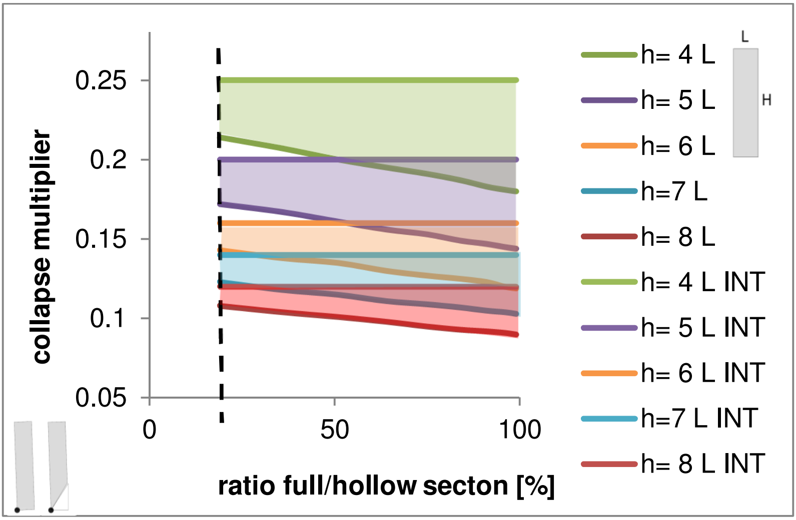


Figure 46. Collapse multipliers for different geometry of the tower (calculating assuming as line of fracture the secant line) compared with those horizontal calculated for uncracked tower (INT)

calculated in the plot of figure 45; hinges for both uncracked and fractured tower lies on the external edge of the base section. In case of uncracked tower the multiplier value is indifferent to the ratio of full/hollow section, hence in the plot they are represented by an horizontal line.

The difference from the multiplier calculated on the uncracked tower is lower for very slender tower and more accentuated for short tower. The difference from multipliers of uncracked towers rises as the percentage of full section increases; in fact being bigger the slope of the line of fracture, the contribution of a wider part of tower is excluded from the stabilizing moment value.

Once determined the collapse multipliers for a fracture line crossing the edge of the section, the influence of masonry compressive strength has been evaluated, calculating new fracture lines. The procedure to determine a correct fracture line taking into account a ultimate compressive strength of masonry (or foundation soil) needs first to determine this ultimate value.

Masonry strength values were chosen in consideration of examples in table 1 where the medium stress values of some important tower are described. As shown in this table medium compressive stresses on a base section of a tower could be quite high, hence, assuming in this analysis a very low value, the tower could result unsafe yet in

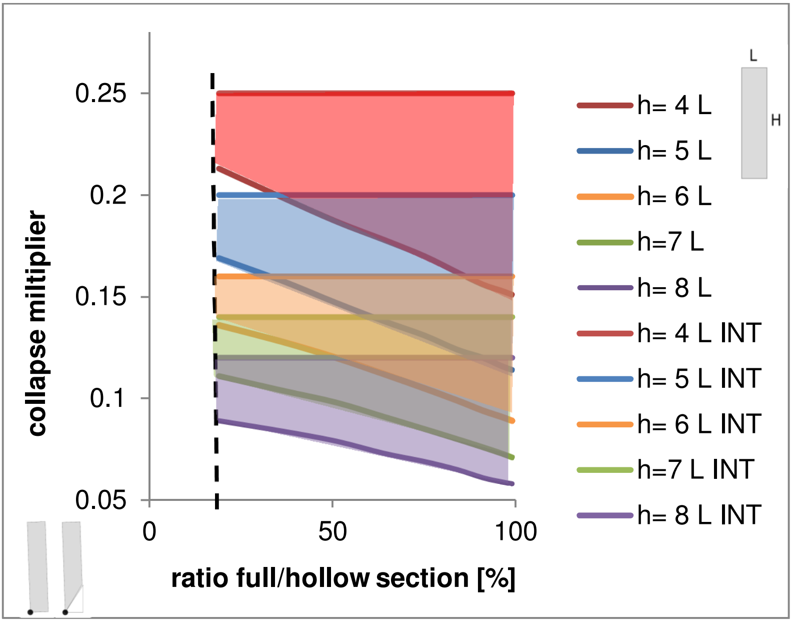


Figure 47 Collapse multipliers for different geometry of the tower compared with those calculated for uncracked tower (INT, constant values); respect to plot of figure 46, here curves of fractures were determined under the assumption of masonry compressive strength equal to 3 MPa

simple dead load condition. For this reason the example value chosen was *3 MPa*.

The fracture line determining this stress value at the base will correspond to a lower horizontal force (respect to the case of fracture on the edge of section) since the line of fracture, in order to respect the condition on compressive stress, must define an uncracked base section whose area is N/f_{ult} , being N the axial load and f_{ult} the ultimate masonry compressive strength.

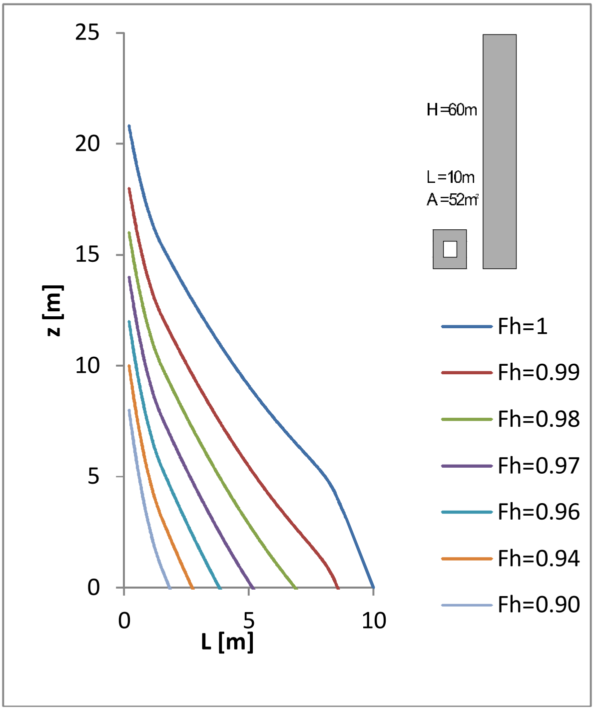


Figure 48 For a given geometry, different curves of fracture are plotted for variable horizontal force values (being the unitary value the one producing a fracture crossing the section edge).

The comparison between collapse multiplier of uncracked and fractured tower taking into account the ultimate compressive strength is plotted in figure 47 for a varying geometry.

As shown in the plot, the difference from the uncracked condition is wider than for the case of fracture crossing the section edge (figure 46). Also the influence of section geometry is more relevant.

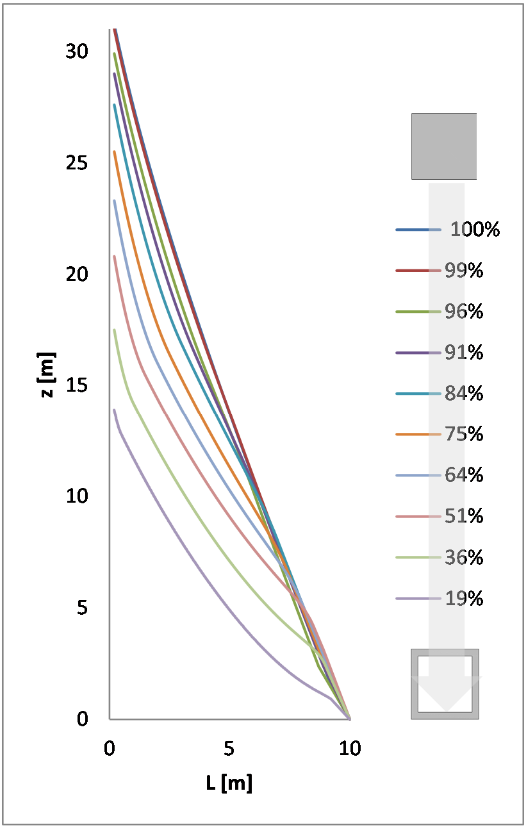


Figure 49 In figure are plotted different curves of fracture corresponding to different cross-section of a tower b=10m L=60.

In figure 48, for a given tower of known geometry, different curves of fracture are compared corresponding to different values of horizontal forces.

In the plot is assumed as unitary value the horizontal force determining a fracture that crosses the edge of the section and the lower forces are expresses as ratio of this unitary force.

Figure 49 illustrates how the curve of fracture changes for different values of ratio full/hollow section; the curve is almost a straight line in case of full section and more curved for decreasing value of the mentioned ratio.

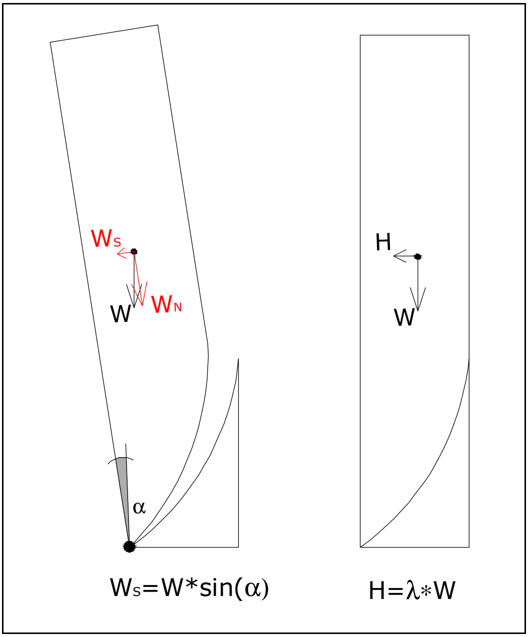


Figure 50 Comparison between angle of tilt causing the collapse (Heyman model) and ultimate multiplier of horizontal loads (proposed model)

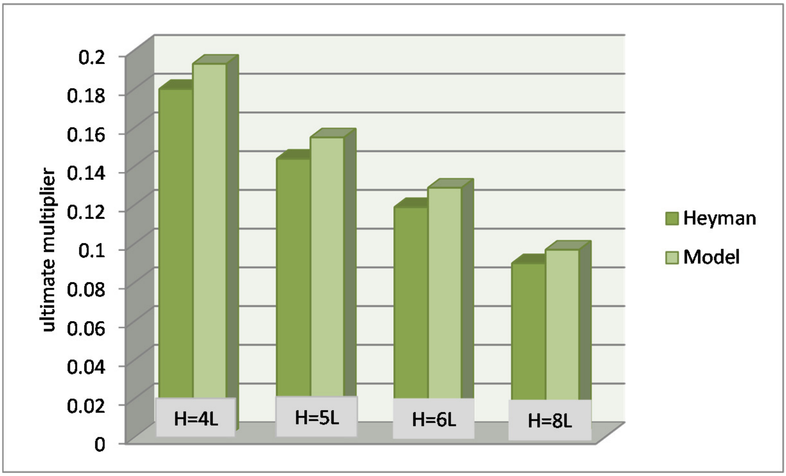


Figure 51 Comparison between ultimate multipliers calculated by Heyman and results from the analytical model proposed

Finally, collapse multipliers evaluated with this method were compared to collapse tilt values calculated with a similar approach for leaning towers by (Heyman 1992) and given by the author for different values of the ratio tower height/base width.
The comparison between the two groups of values – angles of leaning and collapse multiplier of a horizontal force proportional to the masses - was made possible in consideration of figure 50. Both groups of values are referred to a squared full section.
As shown in figure 51 a good correspondence is found, the proposed model gives multiplier values 6 % higher respect to Heyman model.

3.5 Comparison with real cases

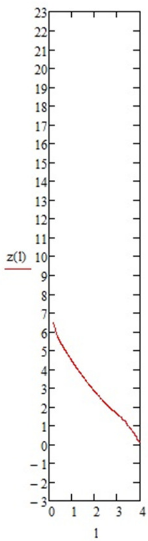
A comparison with real collapse mechanisms due to earthquakes, determining a global overturning of a tower, was performed to validate the simplified analytical model proposed.
Four cases found in literature were studied, Bell Tower of *San Martino* church in *Resiutta*, Bell Tower of *San Michele Arcangelo* in *Braulins*, Bell Tower of *Colle* in *Arba* (near *Udine*) damaged by the Friuli earthquake in 1976 and Bell Tower of *San Tommaso vescovo di Canterbury* (near *Reggio Emilia*), damaged by *Emilia Romagna* earthquake.
From geometrical data found in literature (Doglioni 1994), (survey from Reggio Emilia Municipality) the main dimensions for each tower were determined. The height was calculated starting from the lower fractured section (*z=0* where the fracture reaches the external edge of the tower) and curve of fracture was calculated in the hypothesis of infinitive masonry compressive strength.
Curves obtained were then compared with the crack pattern documented by pictures and drawings, in terms of angle of fracture (defined as in figure 42).
As illustrated in the following tables a good correspondence was found both in terms of slope and shape of fracture; the curve of fracture calculated is plotted next to the damage survey and then directly compared.

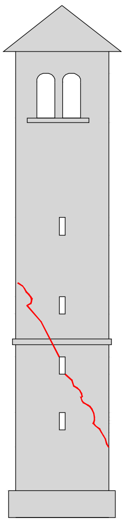
Bell Tower of San Martino church in Resiutta (Udine)

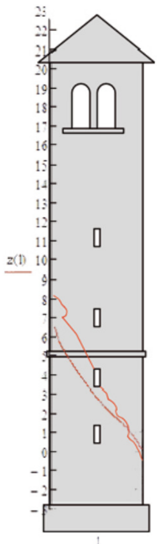
Damaged by Friuli Earthquake in 1976

b=4 m
h=23 m
percentage of full section: 64%

Angle of fracture measured **59 deg**
Angle of fracture calculated **60 deg**





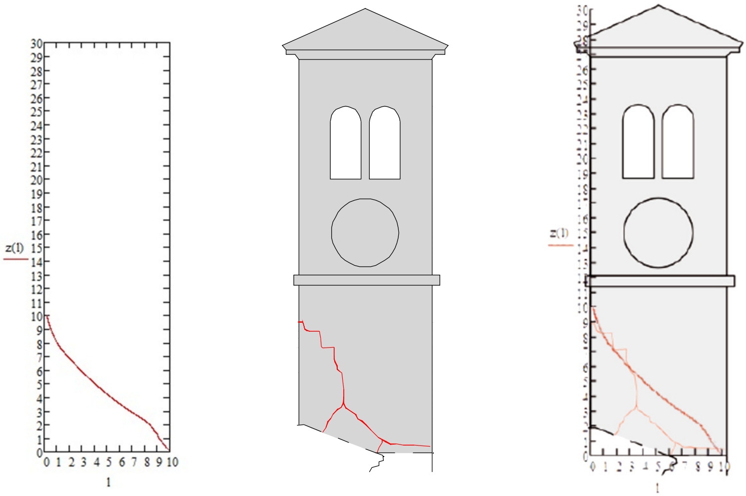


Bell Tower of San Michele arcangelo in Braulins (Udine)

Damaged by Friuli earthquake in 1976

b/h=1/3
percentage of full section: 50%

Angle of fracture measured **47 deg**
Angle of fracture calculated **44 deg**

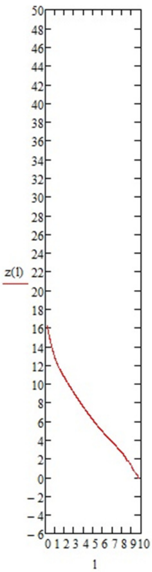


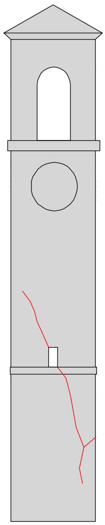
Bell Tower of Colle in Arba (Udine)

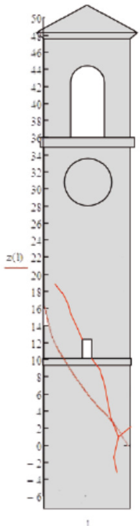
Damaged by Friuli earthquake in 1976

b/h=1/5.6
percentage of full section: 50/60%

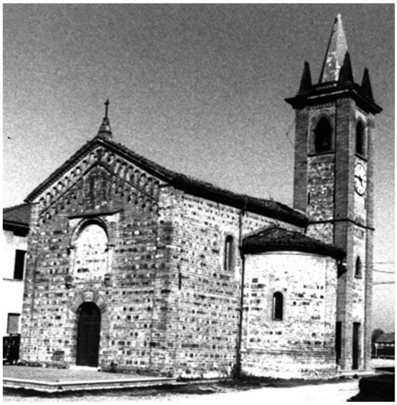
Angle of fracture measured 6**7 deg**
Angle of fracture calculated **63 deg**







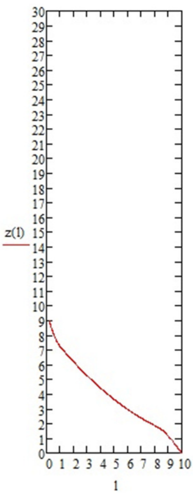
Bell Tower of San Tommaso vescovo di Canterbury (Reggio Emilia)

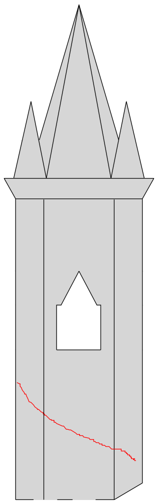


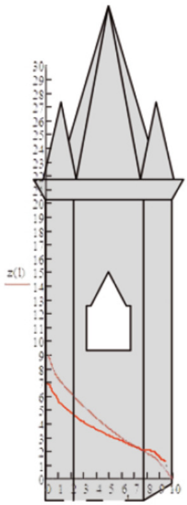
Damaged by Reggio Emila earthquake in 1996

b/h=1/3
percentage of full section: 50%

Angle of fracture measured **40 deg**
Angle of fracture calculated **45 deg**







4. CASE STUDY: GHIRLANDINA TOWER IN MODENA

The analysis method described in the previous chapter is here applied for the analysis of a slender masonry tower, evaluating the collapse multiplier of a global overturning mechanism by taking into account the formation of a fracture surface according to the model described in chapter 3.

4.1 Geometrical and structural description

The Ghirlandina tower is the ancient bell tower of the Cathedral of Modena, both included in the UNESCO site of Piazza Grande. Ghirlandina tower is a squared based (side: 10,8 m) structure 87 m high; the structure has a regular outer section from the base up to 48 m, with an inner hollow section, thicker on the corner for the presence of four masonry pillars; in the inner part an open stair run along the structure from the base up to the upper part where the belfry and the spire roof complete the architectural composition.
The tower is characterized by a tall and slender spire built on its top and preciously decorated that defines the slender architectural appearance.
The masonry diaphragms built in the tower are: the vault on the first floor, the floor of the *Torresani* cell and the vault above the belfry (the

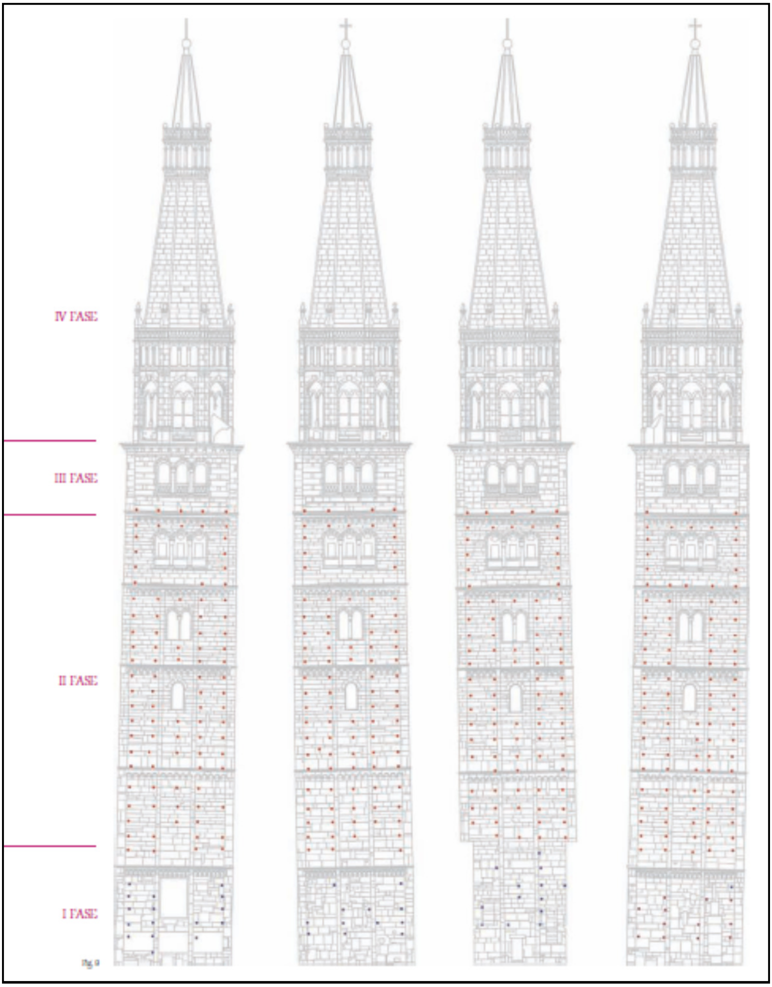


Figure 52 Construction phases of Ghirlandina Tower (Labate 2009)

deck instead is a timber structure). At the base of the tower, two masonry arches connect the structure with the cathedral.

The verticality has been corrected several times during the different phases of construction; it is in fact possible to observe, along the façades, segments of variable leaning as corrections of settlement problems. The tower presents a visible leaning, in particular on the S-W corner where two masonry arches are built to connect the tower to the cathedral.

4.2 History of the structure

Evaluation on structural behavior of historical monuments must necessarily begin from knowledge of the constructive history; analyzing transformations occurred during its life, damages undergone and retrofit interventions completed on the building. From historical analysis is also possible to discover which material was employed and which building techniques were used.
According to archeological survey (Labate 2009) the tower construction can be divided into four different construction phases:
I) to this first phase corresponds the construction of the foundation and of the base up to the first cornice; reusing brick fragments from the roman era are used combined with mortar, the four stonework pillars are made exclusively of 60 cm roman bricks; the external part is covered with stone blocks (mainly *Pietra d'Istria* and *Pietra di Vicenza* stone, also reused elements). The reason for which the tower construction was interrupted was probably due to the ground yielding and the structure settlement.
II) to this second phase corresponds the construction of the shaft and the first loggia, also in this phase reused roman bricks are

employed, in the loggia are also found first-use medieval bricks. The exterior walling is clad with reused stone blocks

III) the construction of the second loggia is attributed to this phase, ammonite stone is used for the cladding while the plastered interiors do not allow to specify which bricks are used

IV) this is the completing phase of the construction that includes the cusp; cladding material are the same than those used for the loggia, while for the wall system a new type of brick was used, slightly smaller than those present in the lower parts.

1099	*Beginning of Cathedral construction.*
1319	*The complex of Cathedral and Ghirlandina is completed*
1481	*A lightening hit and burn the upper part of the steeple*
1483	*Restoration*
1488	*After repairing parts of the building the restoration work continues with an almost complete substitution of the outer stone skin.*
1501	*Earthquake*
1504	*Beginning of tower restoration*
1505	*Earthquake*
1510	*Restoration of the upper Ghirlandina and substitution of wooden deck of bell dome.*
1554-1590	*Restoration works of the tower*
1600	*New wooden stairs are built for the steeple*
1609	*Repair of the foundation structure and strengthening intervention of the base of the cusp*
1820	*Repairs of the upper parts in order to prevent water seepage*
1890-1897	*Strengthening and restoration (also due to damages done by a lightning and an earthquake)*
1901	*Stability analyses and foundation inspection*
1972-1973	*Restoration intervention on the stone cladding*

Table 2. Construction history (Dieghi 2009)

Definition of the most important events in the building history was hence considered essential to complete the constructive knowledge on the tower. A synthetic historical review was done to remark the main events of structural importance for the building and the main construction and restoration phases occurred during centuries
In table 2, fundamental chronology of the building is pointed out, based on *cronistoria* collected by (Dieghi 2009).

4.3 Crack pattern and in situ test results (sonic test)

The tower presents a crack pattern both due to some intrinsic characteristics/weakness of the tower structure, both due to leaning phenomena.
The walls presenting major cracks in facts are those under leaning, the western and southern façades; other vertical cracks distributed also on the other walls are probably related to a general “opening” phenomenon, common to other masonry tall building when not tightened in the upper parts.
These vertical cracks have probably been increased also by thermal variation (in particular those on the southern façades where their effect is accentuated) and also by the presence of the inner staircase built on untightened arches.
Other very sensitive zones are the lower sections on masonry pillars, built on the corner of the inner section; in fact, due to the presence of the staircase, in some section the pillars are hollow thus determining a high stress concentration on the surrounding masonry structures.

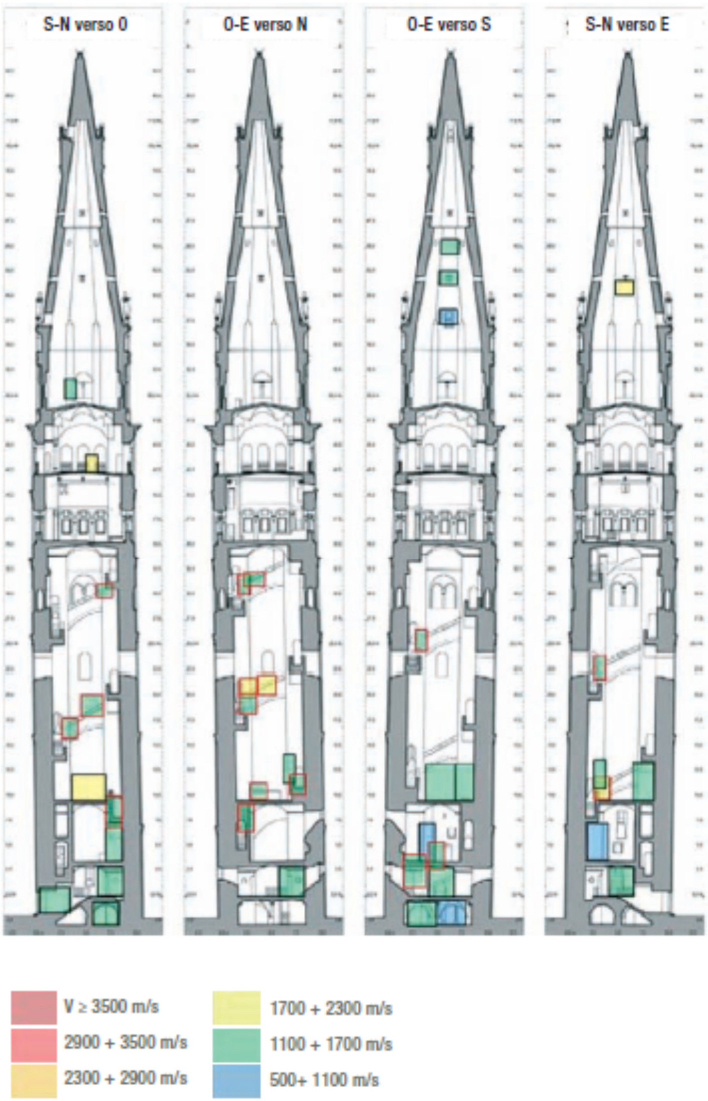


Figure 53 Sonic test velocities on the structure

In order to have a complete structural model, in situ testing was planned and executed using sonic technique and taking masonry specimens for a mechanical characterization. Sonic test velocities are illustrated in figure 53 (Colla & Pascale 2009).

4.4 Seismic demand

Modena is an area of ordinary seismic risk, where a number of average intensity earthquakes occurred in history. In figure 54 seismic activities, recorded form XIII century until now, are illustrated comparing magnitude at the epicenter and on Modena site for different seismic actions.

The effects of some of these earthquakes are described also in historical documents, as in *Cronaca Modenese* of *Tommasino de Bianchi* where the tower is described during earthquake moving as a tree in the wind *("la tore del domo fu veduta dondolare come una pioppa agitata dal vento"),*(Dieghi 2009).

To evaluate seismic vulnerability of the tower, as first step the seismic demand, in terms of spectral acceleration was determined.

Seismic action to be considered at a specific site is usually described in terms of peak ground acceleration a_g, this latter being associated to a rigid soil formation and to free-field conditions, and to the elastic response spectrum $S_e(T)$.

Being the demand defined also by soil characteristics, some preliminary evaluations on foundation soil are needed.

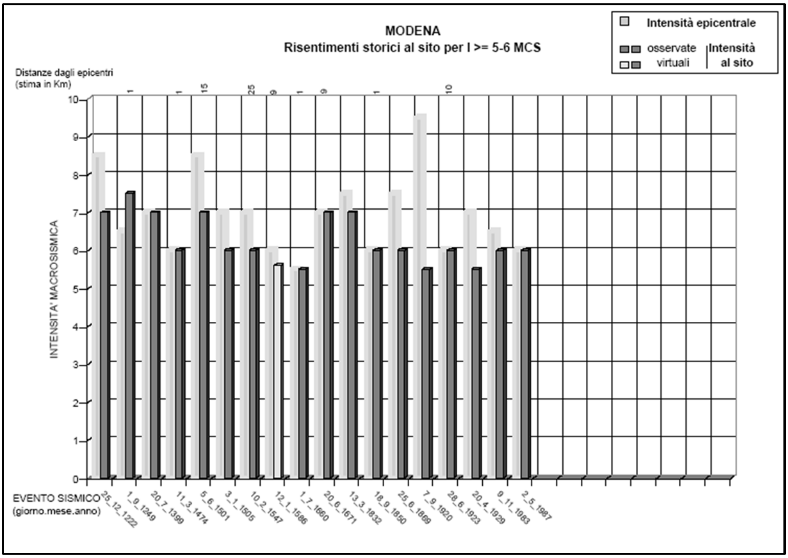


Figure 54 Macroseismic intensity in Modena for historical earthquakes

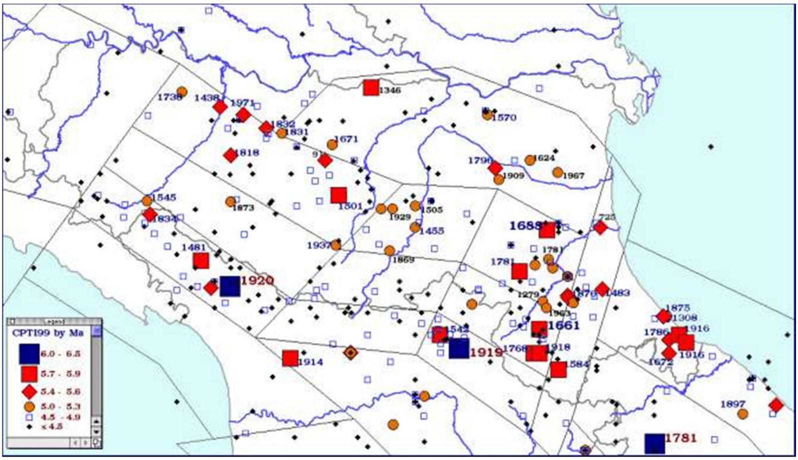


Figure 55 Epicenters of historical earthquakes and their intensity in the region

4.4.1 Soil parameters

Geotechnical analysis results on Ghirlandina soil are described in (Lancellotta 2009).
According to theoretical and experimental evidence, earthquake waves are affected by soil condition and topography, so that the size of seismic waves may be modified (increased) as they pass from the rigid basement to the soil surface. This phenomenon, known as soil amplification, requires specific site studies, or may be based on lumped parameters. One of these parameters is the shear waves velocity V_s 30, characterizing the upper 30 m thick horizon. For this reason the geotechnical survey included the execution of cross-hole tests, shallow seismic exploration tests of soils represent an important class of field tests, because of their noninvasive character. This allows to preserve the initial structure of soil deposits as well as the influence of all diagenetic phenomena contributing to a stiffer mechanical response. Therefore, the cross-hole test represents one of the most reliable methods of determining the shear modulus at small strain amplitude. Based on the results referred in figure 1, a relevant shear wave velocity V_s 30 equal to 192 m/s was deduced, that allows to classify the subsoil into the class C, according to the Eurocode and the National Standard Code.

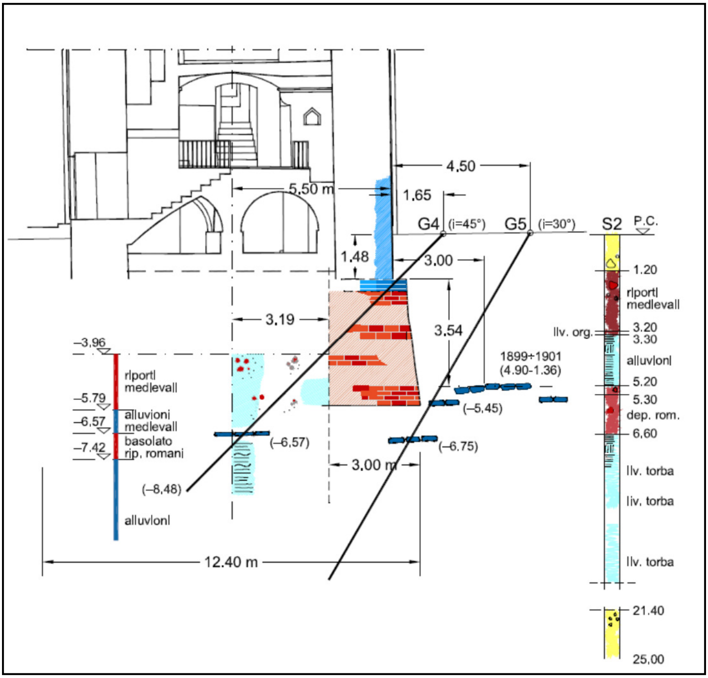


Figure 56. Details of soil profile and foundation of Ghirlandina Tower (Lancellotta 2009)

4.4.2 Soil-structure interaction

The seismic analysis of a tower is not an easy task, because of the interaction of structural and geotechnical aspects, mainly in presence of high values of slenderness. During the first stage of construction the tower could have been not so far from a soil *bearing capacity* collapse, due to *lack of strength* of the soil, and safely survived thanks to some delay or interruption of the building process. This

analysis is strongly dependent on the soil response, and in order to model soil response most of the approaches are based on the so called macro-element approach (Hambly 1985), (Heyman 1992), (Lancellotta 1993) (Desideri and Viggiani 1994); (Marchi 2008). This approach is aimed at representing soil response in terms of generalised forces and related displacement components, i.e. a formulation suitable for soil-structure interaction, moving from advanced hardening plasticity, in order to account for the irreversible and nonlinear soil behaviour.
According to geotechnical analysis (Lancellotta 2009) in the present analysis two assumptions are used, as far as the rotational stiffness is corcerned.
(*a*) Moving from the shear wave velocity equal to v_s*=125 m/s*, a small-strain shear modulus has been deduced. This value refers to free field conditions, so that it has been corrected in order to account for the stress level induced by the tower, by taking into account the strain level and was further increased in order to account for the foundation depth (Gazetas, 1991) giving a corrected stiffness (Di Tommaso el al. 2010) equal to:

$$K_{\alpha \min} = 3.97 \cdot 10^5 \quad kN \cdot m$$

(*b*) An upper bound value was estimated by using the elastic shear modulus, moving from the assumption that soil behaviour could still be dominated by an elastic response due to creep hardening (Di Tommaso el al. 2010):

$$K_{\alpha \max} = 2.4 \cdot 10^6 \quad kN \cdot m$$

4.4.3 Natural frequencies of the tower

From a dynamic point of view, tower-like structures, intended as slender and tall buildings, present in general some common behaviour under dynamic excitation.
However, the basic geometry could show distinct structural components jutting out, or other substructure that can significantly influence the dynamic behaviour of the tower.
They can be incorporated as substructures into the total structural system but, nevertheless, still exhibit their own local behavior.
Slender towers vibrate relatively slowly in their fundamental mode; the calculation of the bending frequency is best carried out by Rayleigh's method:

$$f_e = \frac{1}{2\pi}\sqrt{\frac{\sum m_j \,.\, g \,.\, y_j}{\sum m_j \,.\, y_j^2}} \quad [Hz] \tag{38}$$

Where m_j is the mass of *j-th* discretized section of the tower, y_j is the deflection caused by the applied horizontal inertia force $m_j\, g$.
In the calculation of deflection it is important to consider the deformability of of the tower foundation; the displacements due to the deformations of the tower structure have to be added to the displacements due to the rotation of the foundation in the plane of bending. The influence of flexible supports can be considerable.
Another important issue in the dynamical properties of towers is their double symmetry, generally associated to a uniformly distributed seismic mass per unit height: this implies that higher modes are more important with respect to common structures, where in-plan

stiffness distribution very often determines the major role of fundamental frequency.

While Rayleigh's method is sufficient for the calculation of the fundamental bending frequency, for determination of higher frequencies finite element method or classical approach (matrix iteration) may be required (Bachmann 1995).

Natural frequencies of the tower were hence determined by FEM analysis on a simplified elastic model made of beam elements (figure 57).

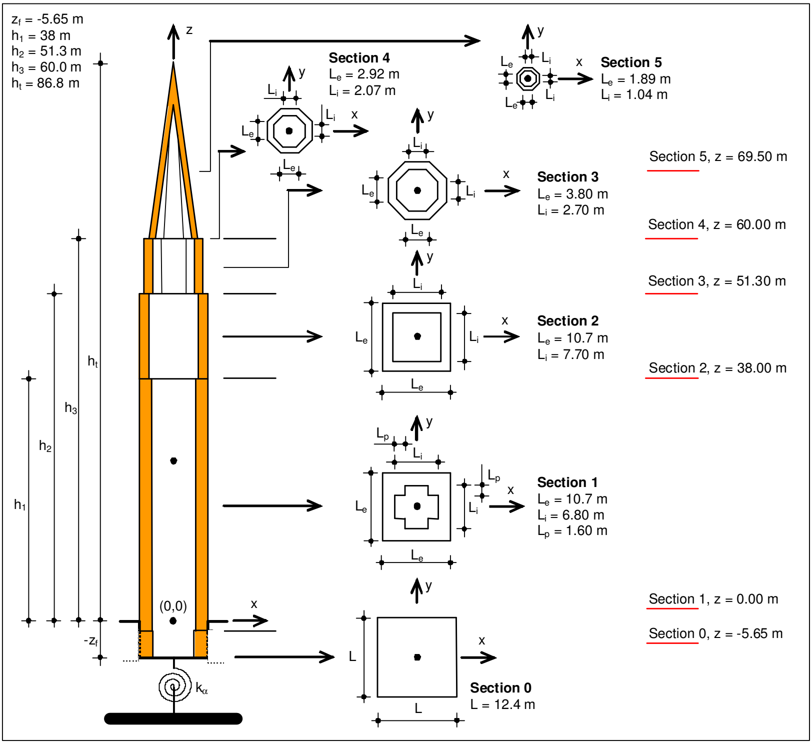


Figure 57 Geometrical model of the tower

4.4.4 Parametric analysis on natural frequencies

Referring to figure 58 where the tower is considered made of elastic material and soil-structure interaction is modeled as an elastic spring with rotational stiffness K_α it's clear that the elastic properties of the system (masonry Young modulus and spring stiffness K_α), determining natural frequencies of the system, modify the seismic action in terms of expected spectral acceleration.
Geotechnical analysis gives two different stiffness values for the elastic spring, corresponding to different assumption about soil behavior. To evaluate the effect of the two values on seismic action, a simple parametric analysis was performed considering also the influence of the masonry Young modulus.

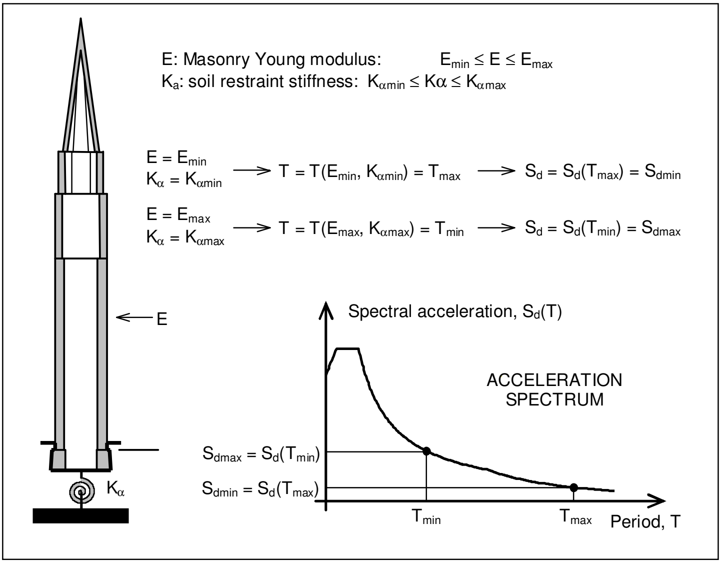


Figure 58 Expected spectral acceleration depending on elastic properties of the system

This parameter was varied ranging between 2000 MPa and 4000 MPa; the tower natural frequencies were then calculated for five different stiffness values (between 0.5 10^6 kNm and 2.5 10^6 kNm) of the spring representing the soil restraint.
The results of the analysis, performed on a simplified elastic model of the tower, are shown in figure 59, 60, 61.
Only the first three modal shapes were considered, being the first and the second flexural shapes and the third a torsional one.
From the obtained results, a different effect of Young modulus depending on stiffness assigned to base restraint can be observed.
In particular, when the spring stiffness is low a variation in elastic properties of the tower has a low influence on the natural frequency of the first mode. On the opposite, when the spring stiffness is higher the Young modulus variation effects results more evident.

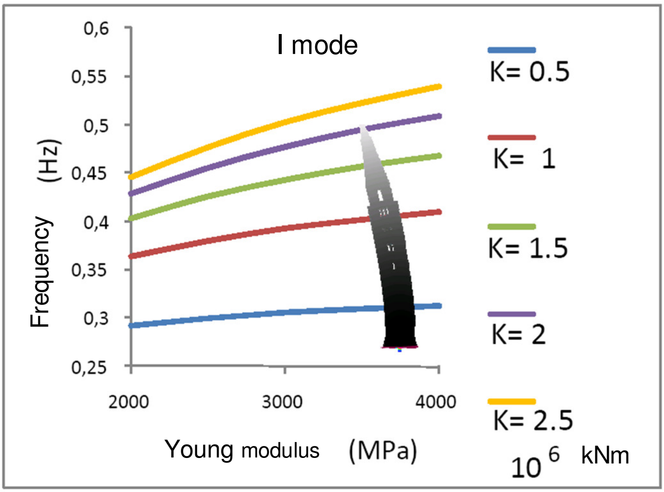


Figure 59 Natural frequencies of I mode depending on elastic properties of the system.

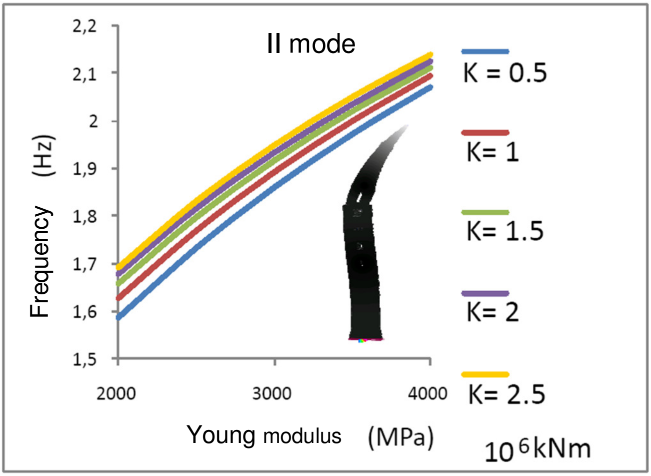


Figure 60 Natural frequencies of II mode depending on elastic properties of the system.

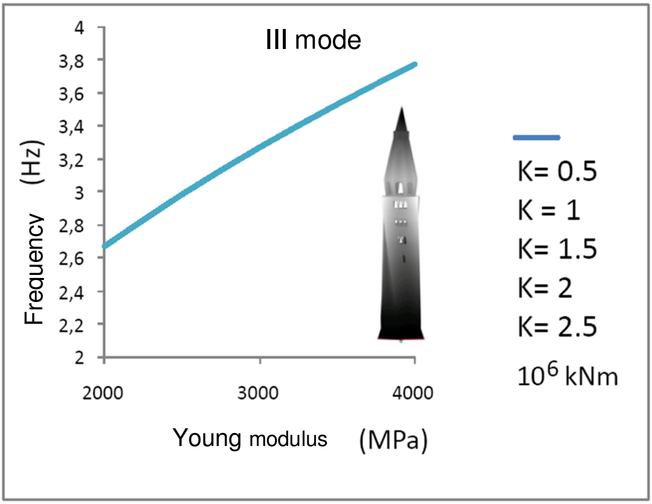


Figure 61. Natural frequencies of III mode(torsional) depending on elastic properties of the system.

The frequency identification of the torsional mode is obviously independent by the flexural stiffness of the base spring.
Spectral acceleration values obtained for hypothesis a) and b) (paragraph 4.4.2) assuming a Young modulus value equal to 4000 MPa are illustrated on the design spectrum (NTC 2008) in figure 62.

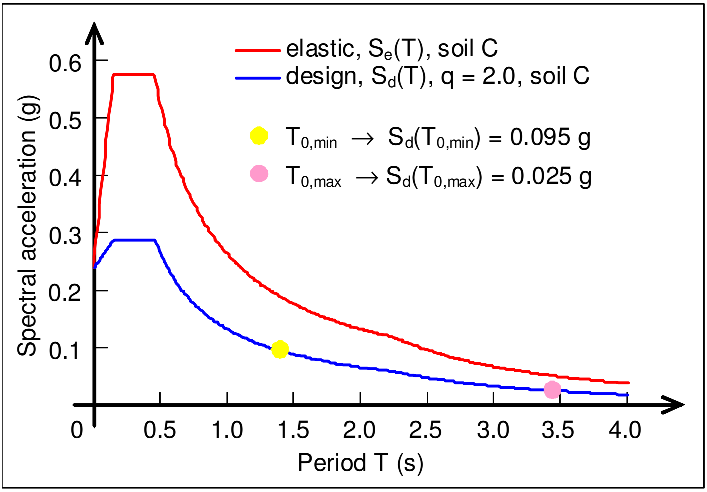


Figure 62. Spectral accelerations corresponding to assumptions a) and b) on soil-structure interaction.

4.5 Seismic capacity

Seismic capacity has been calculated, according to Italian code prescription (NTC2008) for a global overturning collapse mechanism. Mechanisms involving an "opening" of the structure along the existing vertical cracks were not included because, in order to guarantee a unitary behaviour of the tower, a preventive intervention

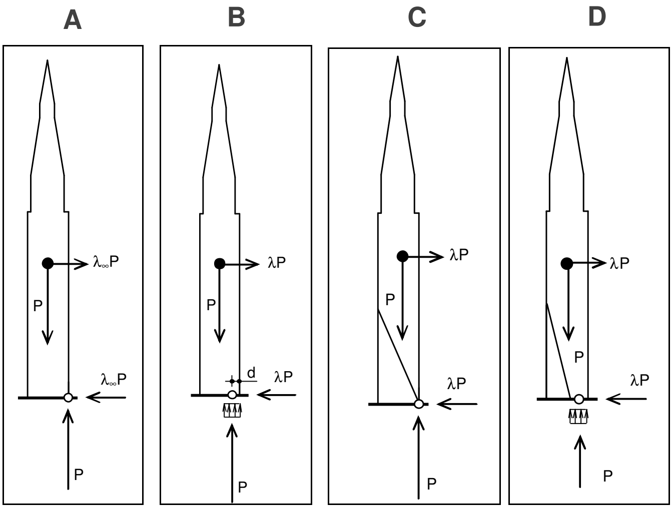


Figure 63.Collapse mechanisms evaluated

with tie-rods able to hold opposite walls together and interventions to repair the masonry are necessarily needed.

Hence, the collapse for global overturning has been evaluated (as an *in plane* problem) in case of different assumptions on system properties, to determine the effects of hinge position and surface of fracture (figure 63).

Results are then compared and a reliability assessment is performed. The considered simplified geometrical model of the tower is shown in figure 64, where the main variations in cross section are considered, windows and openings are ignored and a hollow base section is assumed. In particular at foundation level where the real tower has a discontinuity in cross-sections, the model assumes a constant cross-section equal to the cross-section at the base level;

the foundation area is assumed as a squared full section of width 12 m.

In fact, beside a collapse mechanism for overturning at base level (meaning at level of the ground, at height $z=0$ in figure 65), also a conservative evaluation for overturning at foundation level (meaning at foundation soil level, $z=-5\ m$) was performed, taking into account soil properties.

When considering short term perturbations (earthquakes or wind effects), failure mechanisms are explored with reference to undrained conditions. For this reason, the bearing capacity has been evaluated in terms of total stress and assumed as: $q_{lim}=0.714MPa$ (Lancellotta 2009).

Masonry compressive strength, considering the results of sonic test and the heterogeneity of calculated velocities, is assumed as *3MPa*.

Leaning of the tower (1 deg) has been taken into account considering the effective position of the centroid (figure 65).

4.5.1 Uncracked tower overturning

Base level

When assuming a global overturning at base level ($z = 0$), neglecting masonry compressive strength, the hinge will form at the external edge of the cross-section (figure 63 A); the corresponding collapse multiplier is:

$\lambda=0.196$

Considering, instead, a finite value of masonry compressive strength (figure 63 B), in this case assumed as *f_m=3MPa,* the hinge moves to the centroid of the uncracked base section (compressed zone), and the corresponding multiplier results:

$\lambda = 0.153$

Foundation level

Evaluating the overturning at foundation level (-5 m from the base level) the bearing capacity of soil must be taken into account, considering that at the overturning limit condition the normal stress on foundation level is uniform and equal to the strength of soil-foundation system.

Equilibrium between the self-weight of the tower and soil reaction resultant gives the extension of compression area (figure 65):

$$x_n = \frac{W_{tot}}{L_e . q_{lim}} = 9.7\ m \tag{39}$$

Being W_{tot} = *85546 kN* the tower weight, L_e = *12.4 m* the side of the squared foundation area and q_{lim}= *0.714 MPa*.

Considering the tower overturning around the centroid of the foundation compression area (figure 63 B), rotational equilibrium gives the ultimate multiplier:

λ=*0.038*

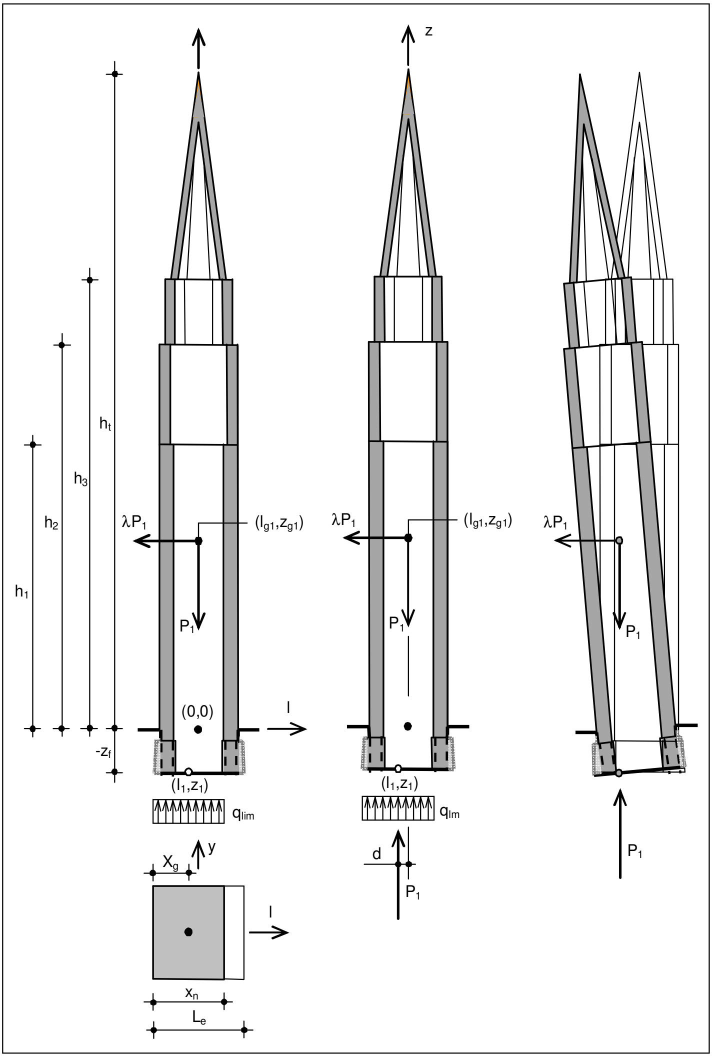


Figure 64. Geometrical model for limit analysis: overturning mechanism at foundation level.

4.5.2 Cracked tower overturning

The same overturning collapse mechanism has been evaluated including the hypothesis that at the limit of overturning a fracture in the masonry will form due to nonresistance to tension of masonry.
The fracture line is defined by the equation (19) and has been evaluated in case of:
-rotation at base level and masonry infinite compressive strength
-rotation at base level and masonry finite compressive strength
-rotation at soil foundation level and soil finite compressive strength

Base level
In the first case, among the fracture lines defined by (19), the one intercepting the edge of the base cross section is determined, assuming masonry infinite compressive strength (figure 63 C). This line is shown in figure 67, a; the corresponding collapse mechanism has the multiplier:

$\lambda=0.143$

When instead a finite value of masonry compressive strength is assumed (figure 63 D), the area of the uncracked part in compression at base level is determined considering $f_m=3MPa$ and the corresponding collapse mechanism has a multiplier:

$\lambda=0.127$

The curve of fracture is drawn in figure 67, b.

Foundation level

Considering, finally, an overturning at foundation level and assuming the soil compressive strength $q_{lim}=0.714MPa$ (figure 63 D), among the fracture lines defined by h_{fp} parameter (figure 39), the one intercepting the edge of the compression part at foundation level is determined. This line is shown in figure 67, c; the corresponding collapse multiplier is:

$\lambda=0.022$

According to (NTC 2008) to each ultimate multiplier value the corresponding spectral acceleration activating the mechanism can be associated:

$$a_0^* = \frac{\lambda \cdot g}{e^* \cdot FC} \tag{40}$$

where *FC* (assumed 1.35 in this case) is a factor taking into account the level of knowledge of the structure (*fattore di confidenza*), and e* the ratio of participating mass, defined as:

$$e^* = \frac{M^* \cdot g}{P_{TOT}} \tag{41}$$

Where P_{TOT} is the total weight of the involved masse and M* is the participating mass, defined as:

$$M^* = \frac{1}{g \cdot \sum_{j=1}^{n} P_j \cdot \delta_j^2} \cdot \left(\sum_{i=1}^{n} P_i \cdot \delta_i \right)^2 \tag{42}$$

Being P_j and δ_j respectively the weight and the displacement of the j-th block constituting the mechanism.

In figure 66, the collapse multipliers calculated and the corresponding accelerations are compared for different cases.
Seismic capacity for uncracked and fractured tower was compared to the seismic demand, defined at paragraph 4.4 (figure 66).
Overturning at foundation level results the mechanism with the lowest collapse multiplier, due to the small dimensions of foundation area and the increase of the global centroid height; in this case almost the whole foundation area is needed to respect condition on soil bearing capacity, hence the line of fracture separates just a small part of masonry; nevertheless a relevant variation in multiplier values can be observed.

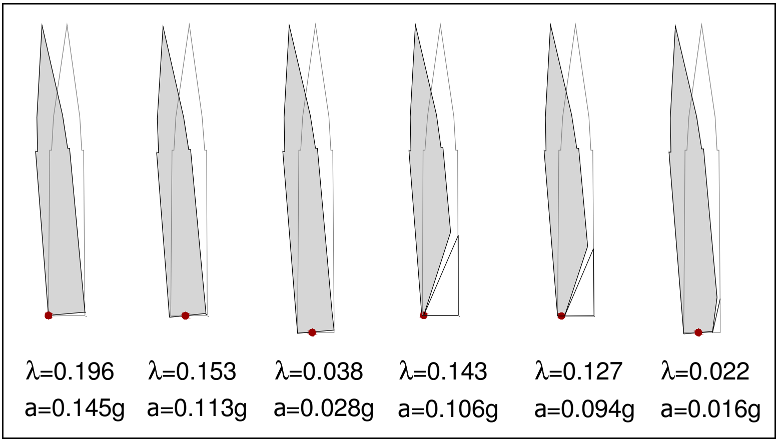


Figure 65 Collapse multiplier and acceleration values for different configurations of global overturning

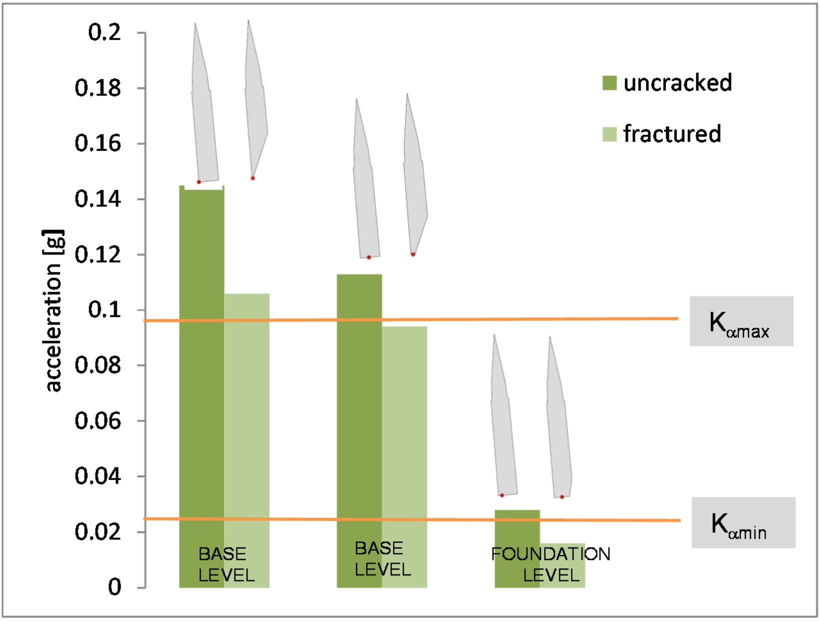


Figure 66 Comparison between seismic demand and seismic capacity.

For the other cases, where the condition on materials strength determines a fracture that propagates higher in the tower the effect of considering the inclined line of fracture reduces the resistance of the tower to overturning of 36% (neglecting masonry compressive strength) and 20% (considering masonry compressive strength), indicating that for a safe simplified assessment these condition must be evaluated.

The reduction in capacity varies depending on the geometry of the tower but this result justifies the present study and the importance of considering fractured geometry.

The value of collapse multipliers calculated on the model of figure 57 in case of finite/infinite masonry compressive strength and inclined

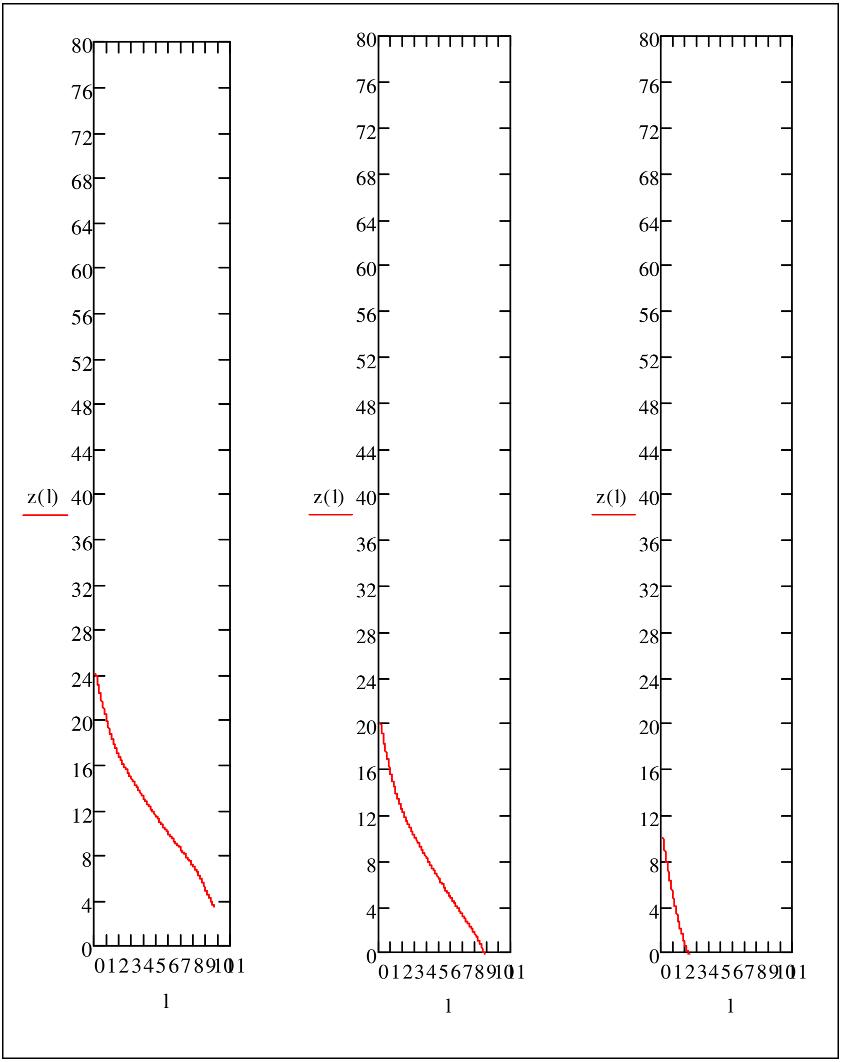


Figure 67 Curves of fracture (a), (b), (c) – z and l axes are expressed in m.

line of fracture (λ=0.127 and λ=0.143 respectively) can be compared with the values of figure 47, corresponding to a tower of simplified geometry.

Assuming geometrical parameters as: B/H=1/5.8 (including in total height only half of the spire) and the percentage of full section as 66%, the values in diagram of figure 47 would result:

- for masonry infinite compressive strength: $0.130<\lambda<0.156$
- for masonry finite compressive strength (3MPa): $0.112< \lambda <0.138$

The values determined from a detailed geometrical model are hence included in the previous ranges.

4.5.3 Overturning collapse under different assumptions

In the present case a difference, whose relevance depends on material properties assumptions, is observed analyzing tower overturning with an inclined line of fracture, respect to the case where the tower is considered uncracked, defining as more conservative the method presented.

In the case of Ghirlandina, the overturning at foundation level, according to assumption on soil behavior, results the most dangerous because the situation of Ghirlandina tower is characterized by a narrow widening of foundation area, respect to tower cross-section at the base, and the tower is built on medium/high plasticity inorganic clays.

In case of lower masonry strength and foundation soil made of gravel or rock an inverted hierarchy of collapse mechanisms can be reached.

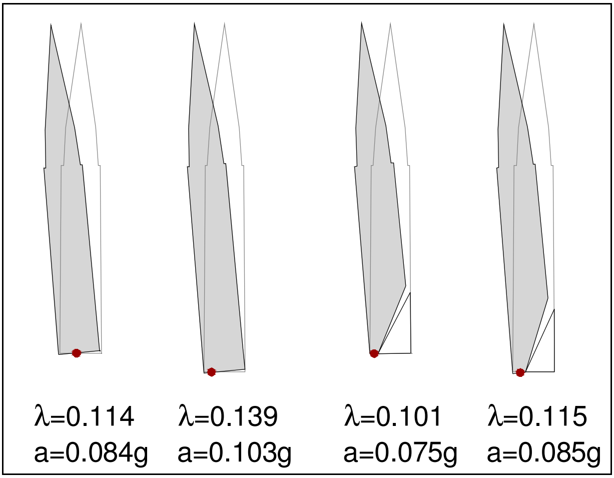


Figure 68 Collapse multipliers and spectral accelerations that activate the mechanisms, for different assumptions on material properties.

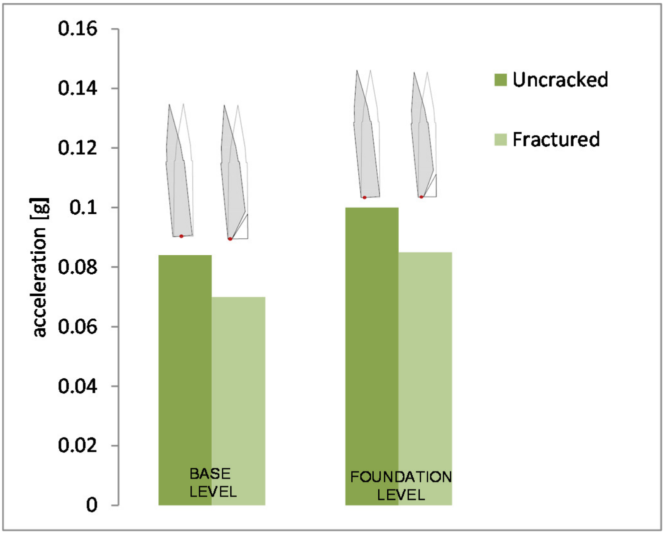


Figure 69 Seismic capacity in case of different assumptions

Hence for a safe simplified evaluation both curves of fracture and soil conditions are very important.

As example the ultimate multipliers of the same tower were calculated in the assumption of soil compressive strength equal to $q_{lim}=1.4\ MPa$, a foundation area *1 m* wider respect to base cross-section and masonry compressive strength equal to *2 MPa*.

Results are illustrated in figure 68 as collapse multipliers and spectral accelerations and compared to the cases where the line of fracture is neglected in figure 69.

Comparing results with those in case of masonry compressive strength 3 MPa and $q_{lim}=0.71\ MPa$ bearing capacity of the soil (figure 67) it can be seen that the more the material compressive strength is high the more relevant is the effect of the fracture line in the ultimate multiplier evaluation, in fact for low material strength almost the whole cross section is needed to respect condition on compressive strength, hence the fracture can develop just in a reduced part of the tower.

CONCLUSIONS

In the present study a simplified method for seismic assessment of masonry tower is presented.
The survey of seismic damages in real towers, besides defining the characteristics that influence their behavior under earthquakes, denounces the global overturning mechanism as the most dangerous also in terms of maintenance of architectural heritage; most of towers damaged by this mechanism were in fact lately demolished for the difficulties in repairing similar damages.
An overview of modeling and analysis method, without neglecting the National and European codes prescriptions, was done, thus demonstrating that respect to the many complex nonlinear methods developed for towers, in the last decades, less simplified strategies of analysis corresponds. In particular, considering the macroelement method, the codes specify that towers vulnerability, being the towers one of the macroelements composing the churches, can be studied through a kinematic analysis but does not give any formulation able to determine the shape of the kinematic block.
Respect to other mechanisms, in the global overturning of towers, considering the height and the total mass of the structure a slight change in the geometry of the kinematism can determine a sensible variation in ultimate multiplier results.
For this reason a method based only on equilibrium considerations was developed to determine the shape of overturning block assuming masonry as a NRT material; under this assumption the equation of the curve of fracture was calculated varying geometrical properties of towers. Results were then compared with those corresponding to an overturning mechanism calculated in the

assumption of an uncracked tower (considered as a monolitical element), hence ignoring the non-resistance to tension of masonry.
The comparison shows that, according to the proposed method, the reduction in ultimate multiplier is relevant, thus determining as unsafe the assessment neglecting the fracture.
Curves obtained were compared also to crack patterns surveyed in damages analysis on monuments after earthquakes; the comparison highlighted a good correspondence to real crack patterns, both in terms of slope and shape.
Finally the analysis method described was applied to the analysis of a slender masonry tower (Ghirlandina in Modena), evaluating the collapse multiplier of a global overturning mechanism by taking into account the formation of a fracture surface according to the presented model.
Collapse multipliers has been evaluated according to different assumptions on material properties and on geometry of blocks, demonstrating the effects of considering the non-resistance to tension of masonry also for the definition of blocks geometry.
When the material strength is low compared to axial stress in static dead load condition, the effect of the fracture is feeble because, almost the entire section is in compression state, hence the fracture can develop just in a small portion of the structure.
In this case the multipliers result very low both in case of considering the curve of fracture both ignoring it.
But, as described with a quantitative example, going far from the ultimate resistance, that is when the foundation area is bigger or the normal stresses in static analysis are far enough from compressive strength, the line of fracture can develop in a wider zone of the tower thus determining an important difference respect to the case of uncracked tower.

Further developments of this simplified method of analysis should include:

- the possibility of calculating the curve of fracture also in presence of cross-section discontinuities along the height of tower (discontinuities can be determined by the presence of openings in the façades or due to widening of the wall sections)
- the evaluation of curve of fracture for a nonlinear behaviour of masonry.

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