

FRONTLINE • OPEN ACCESS

From optimal engines to minimal refrigerators: the Carnot case

To cite this article: Stefano Oss 2026 *Phys. Educ.* **61** 033004

View the [article online](#) for updates and enhancements.

You may also like

- [Upper Limits to Long-term Variability of Solar-type Stars from Observations of the Open Cluster M67](#)
Steven R. Spangler
- [GASP: A Corrected, Cross-matched Catalog of 19,190 Gaia DR3 Asteroid Reflectance Spectra](#)
Werner Scheibenpflug
- [Monolithic integration for self-contained skin electronics](#)
Daegun Kim, Giwon Lee and Sungjun Park

From optimal engines to minimal refrigerators: the Carnot case

Stefano Oss* 

Physics Department—University of Trento, 38123 Povo (Trento), Italy

E-mail: stefano.oss@unitn.it



Abstract

The Carnot limit defines the maximum efficiency for reversible heat engines. However, when applied to reversible refrigerators operating within a fixed temperature range, the Carnot cycle corresponds—counterintuitively—to the minimum possible performance. While this fact is established in formal thermodynamics, it is frequently overlooked or misunderstood by students. This paper presents a physical explanation for this asymmetry without explicitly relying on entropy formalism. A direct, simple mathematical demonstration of this result is provided, along with a concrete illustrative example.

Keywords: Carnot cycles, thermodynamics, efficiency and performance

1. Introduction

A first-year physics student of mine asked for help because, after calculating the performance of a reversible refrigerator, he had obtained a value larger than the one predicted by the corollary of Carnot's theorem, which, according to the student, set an upper limit on the efficiency when operating between only two ideal reservoirs. He struggled to find the mistake, but there was none. Students in introductory thermodynamics quickly learn a powerful slogan: Carnot sets an ultimate limit.

For heat engines, this message is clear and undisputable: a reversible machine operating between the hottest and coldest reservoirs provides a strict upper bound on its efficiency when compared with engines exchanging heat with more than two reservoirs still reversibly. This is rarely discussed explicitly in text books, so the same slogan is often implicitly transferred to refrigerators and heat pumps, leading to the natural expectation (like that of my student) which turns out to be incorrect: that the Carnot coefficient of performance (COP), built from the extreme temperatures, also represents the best, maximum value. While indeed it sets a limit, when a reversible refrigerator exchanges heat with more than two reservoirs, the Carnot value based on the extremes becomes a minimum! While comprehensive analyses of thermodynamic cycles confirm that Carnot efficiency represents indeed a lower bound for COP

* Author to whom any correspondence should be addressed.



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

under fixed temperature extremes [1], this result often strikes students as paradoxical due to the emphasis usually placed on Carnot engines [2]. The aim of this paper is not to re-derive general and known inequalities, but to frame a physically intuitive explanation—based on the concept of entropy as ‘thermal weight’—that clarifies why the ‘best’ engine becomes the ‘worst’ refrigerator.

2. Temperature jumps and the ‘weight’ of heat

An explanation of this asymmetry is based on the fact that large temperature jumps are advantageous for producing work, but they are not for cooling. This offers a robust pedagogical reinforcement of an appropriate interpretation of thermodynamics principles. Everything starts in recognizing and stressing that heat and work are not equivalent modes of energy transfer. Work is an ordered transfer: in principle it can be converted entirely into other forms without leaving any thermodynamic trace. Heat, on the contrary, is a transfer of energy whose foundation is tied to microscopic degrees of freedom and to the temperature at which it occurs: when energy is transferred as heat, it is not only the amount that matters, but also the temperature at which the transfer is done. We could rephrase this by saying that the same amount of heat has a different thermodynamic impact depending on the temperature. A given amount of energy exchanged with a very hot reservoir produces only a small change in its thermal state, whereas the same amount of energy exchanged with a cold reservoir produces a much larger change. In this sense, heat exchanged at low temperature is thermodynamically ‘heavy’ (one might say ‘strong’), while heat exchanged at high temperature is comparatively ‘light’ (or ‘weak’). While the core of this argument does not require the formal introduction of entropy, it is worth noting for clarity that this macroscopic metaphor perfectly mirrors its behaviour: exchanging a given amount of heat at a lower temperature inherently results in a larger, or ‘heavier’ (i.e. ‘stronger’), thermodynamic impact, exactly as a smaller denominator dictates in the

relation $\Delta S = Q/T$. A reversible thermal machine cannot eliminate this difference; it can only redistribute it among reservoirs. Of course, these statements find their natural confirmation and formal justification once entropy is introduced as a state function that quantitatively accounts for the temperature dependence of thermal energy transfer. In fact, the perspective of seeing entropy more like a ‘thermal force’—either as an alternative to or alongside the original Clausius definition—is not new and has been already framed as a useful pedagogical approach [3, 4].

Accordingly, the Carnot result for heat engines becomes almost self-evident. A reversible engine absorbs heat from hot reservoirs, rejects heat to cold ones, and produces work as the difference. To maximize the work output, it is advantageous to absorb heat where it is ‘lightest’, i.e. from the hottest available reservoir and reject heat where it is ‘heaviest’, i.e. to the coldest available reservoir. For engines working reversibly with more reservoirs, the spreading of heat absorption over cooler reservoirs makes the incoming energy ‘heavier’ and therefore harder to compensate; any spreading of heat release over warmer sinks makes the rejected energy ‘lighter’ and forces more energy to be discarded. In this precise physical sense, the Carnot engine is optimal because it exploits the largest possible temperature difference.

The situation is completely different for refrigerators, even though the same reversible constraint applies. A refrigerator must remove heat from cold reservoirs and deliver it to warmer ones, using work as a resource. Removing heat from a very cold reservoir is thermodynamically expensive: each unit of extracted energy is extremely ‘heavy’. Delivering heat to a hot reservoir, by contrast, is relatively inexpensive per unit energy. Seen in this light, the Carnot refrigerator represents the least efficient possible way of cooling. It forces the entire heat extraction to occur at the lowest available temperature and the entire heat delivery at the highest available temperature, thereby imposing the maximum temperature jump on the cooling process. Every joule extracted is as heavy as it can possibly be, and the work required to compensate

this effect is therefore maximal. For a refrigerator operating reversibly among more reservoirs, so extracting part of the heat from reservoirs at higher temperatures (still below the hot extreme), the extracted energy becomes thermodynamically less expensive. The same reversible balance can then be achieved with less work per unit of total extracted heat, and the performance increases. From a physical standpoint, cooling is easier when it is performed through smaller temperature differences.

3. A simple demonstration

These statements of course admit a formal derivation, which we propose here as a minimal version of the more general and articulated approach of [1]. The argument can be adapted also for use at the high school level and is therefore more accessible to undergraduate students, while remaining instructive even at advanced levels. It makes explicit use of the two principles of thermodynamics in the form of energy conservation, $\Delta U = Q - W = 0$ and the Clausius relation for reversible processes, $\sum_k (Q_k/T_k) = 0$.

One starts from a reversible machine that exchanges heat with ideal reservoirs at fixed temperatures. The heat engine extracts positive (for the machine) amount of heat $Q_k^{(H)}$ from hot reservoirs at temperatures $T_k^{(H)}$, with a maximum value $T_{\max}$, and releases negative amounts of heat $Q_k^{(C)}$ to cold reservoirs at temperature $T_k^{(C)}$, with a minimum value $T_{\min}$. The total extracted heat is $Q_H = \sum_k Q_k^{(H)}$, while the total released heat is $Q_C = \sum_k |Q_k^{(C)}|$, where the absolute value allows the energy balance of the complete cycle to be written as $W = Q = Q_H - Q_C > 0$. The Clausius relation then leads to

$$\begin{aligned} 0 &= \sum_k \frac{Q_k^{(H)}}{T_k^{(H)}} - \sum_k \frac{|Q_k^{(C)}|}{T_k^{(C)}} \\ &\geq \frac{\sum_k Q_k^{(H)}}{T_{\max}} - \frac{\sum_k |Q_k^{(C)}|}{T_{\min}} = \frac{Q_H}{T_{\max}} - \frac{Q_C}{T_{\min}}. \end{aligned} \quad (1)$$

The total work per cycle is therefore

$$W = Q_H - Q_C \leq Q_H \left(1 - \frac{T_{\min}}{T_{\max}}\right). \quad (2)$$

Which implies, from the definition of the efficiency $\eta = W/Q_H$, that

$$\eta \leq 1 - \frac{T_{\min}}{T_{\max}} = \eta_{\text{CARNOT}}. \quad (3)$$

This is the well-known result stating that the reversible Carnot cycle is the most productive among all reversible cycles exchanging heat at intermediate temperatures.

The same reasoning can be applied to a reversible refrigerator operating between ideal reservoirs with temperatures ranging again from $T_{\min}$ to $T_{\max}$. Careful attention must be paid to the signs: in this case the ‘hot’ reservoirs receive heat (negative for the refrigerator), while the ‘cold’ ones release heat (positive). The first law then reads $W = Q = -Q_H + Q_C < 0$ where now $Q_H = \sum_k |Q_k^{(H)}|$ and $Q_C = \sum_k Q_k^{(C)}$. One thus obtains

$$\begin{aligned} 0 &= \sum_k \frac{Q_k^{(C)}}{T_k^{(C)}} - \sum_k \frac{|Q_k^{(H)}|}{T_k^{(H)}} \\ &\leq \frac{\sum_k Q_k^{(C)}}{T_{\min}} - \frac{\sum_k |Q_k^{(H)}|}{T_{\max}} = \frac{Q_C}{T_{\min}} - \frac{Q_H}{T_{\max}}. \end{aligned} \quad (4)$$

The work required per cycle is therefore

$$\begin{aligned} W &= Q_C - Q_H \geq Q_C \left(1 - \frac{T_{\max}}{T_{\min}}\right) \Leftrightarrow \\ |W| &= Q_H - Q_C \leq Q_C \left(\frac{T_{\max}}{T_{\min}} - 1\right). \end{aligned} \quad (5)$$

Which implies, from the definition of the coefficient of performance $\text{COP} = Q_C/|W|$, that

$$\text{COP} \geq \frac{T_{\min}}{T_{\max} - T_{\min}} = \text{COP}_{\text{CARNOT}}. \quad (6)$$

Which is the expected result.

3.1. Carnot and Stirling cycles: an example

Although this demonstration is conclusive in itself, it may be useful to provide students with a concrete example. For this purpose, of course any reversible cycle is suitable. As a possible choice, one may consider the well-known and easily tractable Stirling sequence (an ideal gas undergoing two isothermal and two isochoric reversible transformations without heat regeneration along them). To maintain strict reversibility during these isochoric processes, it is explicitly assumed that the system exchanges heat with an infinite succession of reservoirs, each at a temperature that differs infinitesimally from that of the gas. For this cycle, convenient expressions for the thermal efficiency η_{STIRLING} and the $\text{COP}_{\text{STIRLING}}$ are straightforwardly computed and given by

$$\eta_{\text{STIRLING}} = \frac{R(r_T - 1) \ln r_V}{c_V(r_T - 1) + r_T R \ln r_V},$$

$$\text{COP}_{\text{STIRLING}} = \frac{c_V(r_T - 1) + R \ln r_V}{R(r_T - 1) \ln r_V} \quad (7)$$

where $r_T = T_{\text{max}}/T_{\text{min}}$ and $r_V = V_{\text{max}}/V_{\text{min}}$ denote, respectively, the ratios of highest to lowest temperatures and of the largest to smallest volumes of gas in the cycle. It is straightforward to show that $\eta_{\text{CARNOT}} > \eta_{\text{STIRLING}}$ and $\text{COP}_{\text{CARNOT}} < \text{COP}_{\text{STIRLING}}$ for any values of r_T and r_V , in agreement with equations (3) and (6). This result is even more clearly illustrated using temperature-entropy diagrams. Even if ‘entropy’ is used here only as a formal coordinate, its interpretation as the ‘thermal weight’ of energy—as hinted at above—is preferable for intuitive understanding. Figure 1 shows the T–S diagrams for Carnot and Stirling cycles operating in both directions with the same areas (work done/required). These graphs can also be used for quantitative analysis: as a useful practical exercise, one can compute the ratios of efficiencies and COPs by accounting for the areas under the isochoric transformations characteristic of the Stirling cycle, which are simply given, per mole of substance, by $Q_{D'A}^{(+)} = Q_{C'A}^{(-)} = c_V(T_H - T_C)$.

4. Conclusion

To support and reinforce the deep connections between heat and work and the first and second

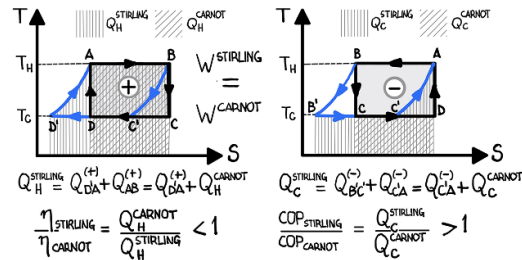


Figure 1. Comparison of Carnot and Stirling cycles using Gibbs (T-S) diagrams. The areas (work done/required) are equal by construction.

laws of thermodynamics, a simple analysis of reversible cycles is suggested. Although this work does not introduce any new physical results, it is intended as a pedagogical contribution aimed at improving the teaching of reversible thermal machines and of the different thermodynamic roles played by heat and work. Here, we have emphasized that relatively large temperature differences are beneficial for engines, as they maximize the potential to convert heat into work, whereas the same differences are unfavourable for refrigerators, because they make heat extraction maximally expensive in terms of work. This asymmetric behaviour, which makes engines and refrigerators so different, is not rooted in the definitions of efficiency or COP. Instead, it is attributed to the intrinsic non-equivalence of heat and work as carriers of energy. Once students recognize that heat exchanged at different temperatures has a different thermodynamic ‘weight’ (which will eventually be clarified in terms of entropy), it becomes natural that the Carnot cycle is a maximum for engines and a minimum for refrigerators.

Data availability statement

No new data were created or analysed in this study.

ORCID iD

Stefano Oss  0000-0001-9427-2151

Received 21 January 2026, in final form 26 March 2026

Accepted for publication 13 April 2026

<https://doi.org/10.1088/1361-6552/ae5eb5>

References

- [1] Leff H S 2018 Reversible and irreversible heat engine and refrigerator cycles *Am. J. Phys.* **86** 344–53
- [2] Cochran M J and Heron P R L 2006 Development and assessment of research-based tutorials on heat engines and the second law of thermodynamics *Am. J. Phys.* **74** 734–41
- [3] Falk G, Herrmann F and Schmid G B 1983 Energy forms or energy carriers? *Am. J. Phys.* **51** 1074–7
- [4] Fuchs H U 1987 Entropy in the teaching of introductory thermodynamics *Am. J. Phys.* **55** 215–9