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Abstract

In this paper, time-modulation is exploited for the synthesis of monopulse sub-arrayed antennas. The solution of the compromise sum-difference problem is obtained by setting the set of static excitations to an optimal sum set and synthesizing the "best compromise" difference pattern through a Continuous Partition Method (CPM) based approach. The array elements are aggregated into sub-arrays controlled by means of RF switches with optimized "on" time-durations. The switch-on instants of the pulse sequences are then computed by means of a particle swarm optimizer to reduce the interferences caused by the sideband radiations. A selected set of numerical results is reported to assess the potentialities of time-modulation in dealing with the synthesis problem at hand.

Key words: Sum and Difference Compromise Pattern Synthesis, Monopulse Antennas, Time-Modulated Arrays.

1 Introduction

Search-and-track radars based on monopulse principles require antenna systems generating sum and difference patterns. In the scientific literature, several approaches refer to the frequency domain and consider fixed antenna geometries as well as the exploitation of the degrees of freedom available in both the frequency domain and the spatial domain. Analytical procedures aimed at computing in an “optimal” way the excitation weights of the array elements belong to the former class. Patterns with either equi-ripple [1][2] or tapered [3][4] sidelobes have been efficiently obtained. Other strategies for the optimal synthesis of power patterns with arbitrary sidelobe bounds have been proposed [5][6][7], as well. Optimal patterns in the Dolph-Chebyshev sense have been determined. They realize an optimal trade-off between the sidelobe level (SLL) and the main lobe beamwidth (BW) or between the BW and the deepness of the slope along the boresight direction for a fixed SLL when dealing with sum patterns or difference patterns, respectively. Although the synthesis of optimal beams allows one to increase the resolution capability (i.e., a narrow BW and a deep boresight slope) and to enhance the reliability of the search and track system (i.e, a low SLL), it also requires the use of two independent feed networks.

In order to limit such a complexity constraint, additional degrees of freedom have been introduced by considering a partial sharing of the antenna circuitry between the two beams. In this framework, sub-arraying has been used [8] to approximate, in the least square sense, both sum and difference patterns starting from reference excitations. Towards this end, Taylor [9] and Bayliss [10] continuous distributions have been considered in [11] to optimize difference patterns by means of a Simulated Annealing (SA) algorithm. Moreover, following the guidelines originally presented by McNamara in [12], a growing attention has been also devoted to synthesize optimal compromise sum and difference patterns using sub-arrayed arrays. In such a case, the optimal sum pattern is usually generated through an independent beam-forming network, whereas the sub-optimal difference one is obtained spatially aggregating the elements into sub-arrays and assigning a suitable weight to each of them. Towards this purpose, analytical procedures [12][13], stochastic optimization algorithms [14][15][16][17], and hybrid methods [18][19] have been successfully applied.

Dealing with compromise solutions, this paper presents a new strategy aimed at exploiting time as an additional degree of freedom for the synthesis of difference patterns in sub-arrayed array antennas. Thanks to the use of RF switches, the approach enforces time-modulation to the static element excitations. Originally, time-modulation has been used for the synthesis of low and ultra-low sidelobe arrays for radar applications [20] and communication purposes [21]. More recently, some studies have been carried out to extend the application of time-modulation to other antenna synthesis problems. For instance, difference patterns have been synthesized by time-modulating a small number of elements of a two-section array generating a sum pattern [22]. However, even though pioneering works concerned with time-modulation date back to the end of 1950s [23], the potentialities of time-modulated arrays have been only partially investigated. This has been mainly due to the presence of undesired sideband radiations (SRs) which unavoidably affect the performance of time-modulated arrays. In order to minimize the SR power losses, different approaches based on evolutionary optimization algorithms have been proposed [24][25][26][27]. Otherwise, it has been demonstrated in [28] that the control of the sideband levels at the harmonic frequencies can be yielded by using suitable switching strategies providing effective pulse sequences.

In this paper, a time-modulation strategy is proposed as a suitable alternative to standard compromise methods, which neglect the time variable in the design process, to synthesize compromise arrays. Starting from a set of static excitations generating an optimal sum pattern at the carrier frequency, a compromise difference beam is synthesized through a sub-arraying pattern matching procedure [13] aimed at optimizing the pulse durations at the input ports of the sub-arrays. Successively, the SRs at the harmonic frequencies are minimized by performing a Particle Swarm Optimization (PSO) to set the switch-on instants of the time sequences.

The paper is organized as follows. The compromise problem is mathematically described in Sect. 2 where the pattern matching procedure as well as the strategy for the sideband level (SBL) minimization are also outlined. A selected set of numerical experiments are reported and discussed in Sect. 3 to point out advantages and limitations of the proposed technique. Finally, some conclusions are drawn (Sect. 4).

2 Mathematical Formulation

Let us consider a two-section linear array [29] of $N = 2 \times M$ elements equally-spaced (d being the inter-element distance) along the x-axis. According to the guidelines of the sub-arraying technique [12], the static real excitation coefficients $\mathbf{A} = \{\alpha_m = \alpha_{-m};\ m = 1, ..., M\}$ affording the sum pattern AF_Σ

$$AF_\Sigma\left(\theta;\ \mathbf{A}\right) = 2\sum_{m=1}^{M}\alpha_m \cos\left[\left(m - \frac{1}{2}\right)kd\sin\theta\right] \quad (1)$$

are computed using optimal techniques (e.g., [1][3][5]). Moreover, θ is the angular direction with respect to the array axis and $k = \frac{\omega_0}{c}$ is the wavenumber, ω_0 and c being the angular carrier frequency and the speed of light, respectively.

To generate the compromise difference patterns, the array elements are grouped into $R = 2 \times Q$ sub-arrays (i.e., Q for each half of the array). At each sub-array port, an RF switch is used to modulate the excitations of the elements assigned to the sub-array (Fig. 1). Mathematically, the process of enforcing a time-modulation to the sub-array signals can be described by defining a set of Q rectangular functions

$$U_q\left(t\right) = \begin{cases} 1 & t_q^{on} \leq t \leq t_q^{off} \\ 0 & otherwise \end{cases}, \quad q = 1, ..., Q \quad (2)$$

t_q^{on} and t_q^{off} being the sub-array *switch-on instant* and the *switch-off instant* of the q-th sub-array, respectively. The values of t_q^{on} and t_q^{off}, $q = 1, ..., Q$, are additional degrees of freedom to be determined for approximating the desired/reference difference pattern.

Since these rectangular pulses are periodic in time (with period T_p), each function $U_q\left(t\right)$, $q = 1, ..., Q$, is then expanded into its Fourier series and the condition $T_p \gg T_o = \frac{2\pi}{\omega_0}$ is assumed to hold true. It is then simple to show [20] that the arising expression of the array factor is composed by an infinite number of frequency components centered at ω_0 and separated by $h\omega_p = h\frac{2\pi}{T_p}$, h being the harmonic index. Let us choose to synthesize the difference pattern at the carrier frequency ($h = 0$). Accordingly, it results that

$$AF_\Delta^{(0)}\left(\theta;\ \mathbf{C},\ \mathbf{T}\right) = 2\sum_{m=1}^{M}\alpha_m\sum_{q=1}^{Q}\tau_q\delta_{c_m q}\sin\left[\left(m - \frac{1}{2}\right)kd\sin\theta\right] \quad (3)$$

where $\mathbf{T} = \{\tau_q;\ q = 1, ..., Q\}$ is the set of 0-th order Fourier coefficients (also called *normalized* *switch-on times*) given by

$$\tau_q = u_{hq}\rfloor_{h=0} \triangleq \frac{1}{T_p}\int_0^{T_p} U_n(t)\, e^{-jh\omega_p t} dt\Big\rfloor_{h=0} = \frac{t_q^{off} - t_q^{on}}{T_p}, \quad q = 1, ..., Q, \tag{4}$$

where $\delta_{c_m q}$ stands for the Kronecker delta function and $\mathbf{C} = \{c_m \in [0, Q]\,;\, m = 1, ..., M\}$ is the integer vector describing the sub-array configuration. As an example, $c_m = 0$ means that the excitation of the m-th element is not time-modulated.

In order to synthesize a compromise difference pattern close to a reference/optimal one, the definition of the two sets of unknowns $\mathbf{C}$ and $\mathbf{T}$ in (3) is then required. Towards this end, a suitable state-of-the-art sub-arraying procedure is used following the guidelines of the pattern matching procedure presented in [13]. More in detail, the following cost function

$$\Psi^{(0)}(\mathbf{C}, \mathbf{T}) = \frac{1}{M}\sum_{m=1}^{M}\left\|\alpha_m\left(\frac{\beta_m}{\alpha_m} - \sum_{q=1}^{Q}\delta_{c_m q}\tau_q\right)\right\|^2 \tag{5}$$

is minimized by means of the *contiguous partition method* (CPM) [13], where $\mathbf{B} = \{\beta_m = -\beta_{-m}$ $m = 1, ..., M\}$ is the set of reference/optimal excitation coefficients [2][4][6] that generate the reference difference pattern to match. As a matter of fact, a suitable customization of the CPM can be effectively used here starting from the key observation that the optimal and independent (when N RF switches are available) values of the switch-on times affording the desired pattern at ω_0 can be exactly computed by means of the techniques in [1][2][3][4][5][6][7]. Hence, the optimal excitation matching problem dealt with in [13] can be reformulated here as an optimal pulse matching problem. Accordingly, once the number of sub-arrays Q is given, the minimization of (5) allows to determine the number of elements within each group and the sub-array architecture where the cost function (5) is representative of a least square problem measuring the mismatch between the *optimal weights* $\frac{\beta_m}{\alpha_m}$, $m = 1, ..., M$, and the corresponding (unknown) *sub-array switch-on times* τ_q, $q = 1, ..., Q$. For the sake of clarity in the notation, let us indicate with τ_q^{CPM}, $q = 1, ..., Q$, and c_m^{CPM}, $m = 1, ..., M$, the values of the unknowns computed by minimizing (5) through the CPM.

It is worth noting that whether, on one hand, the "best compromise" difference pattern at ω_0 can be easily obtained by applying the CPM procedure, on the other hand, SRs are still present because of the commutation between the on and off state of RF switches that controls the time-modulation process. In order to reduce the interferences due to SRs, the optimization of $\mathbf{T}$ in uniform arrays [26] or the joint optimization of both $\mathbf{T}$ and $\mathbf{A}$ [24] has been performed in the literature. However, it should be pointed out [Eq. (3)] that a modification of the pulse durations τ_q^{CPM}, $q = 1, ..., Q$, causes the radiation of a different compromise difference pattern and no more the "best compromise" solution obtained through the CPM. Moreover, the static excitation vector $\mathbf{A}$ is *a-priori* fixed to generate the optimal sum pattern. Thus, neither $\mathbf{T}$ nor $\mathbf{A}$ can be now changed to address the SR minimization problem.

Towards this purpose, let us observe that the h-th Fourier coefficient ($h \neq 0$) is equal to

$$u_{hq} \triangleq \frac{1}{T_p} \int_0^{T_p} U_n(t)\, e^{-jh\omega_p t} dt = \frac{e^{-jh\omega_p t_q^{off}} - e^{-jh\omega_p t_q^{on}}}{2jh\pi} \tag{6}$$

and the corresponding harmonic pattern turns out to be

$$AF_{\Delta}^{(h)}(\theta; \mathbf{C}, \mathbf{U}_h) = 2\, e^{j(h\omega_p + \omega_0)t} \sum_{m=1}^{M} \alpha_m \sum_{q=1}^{Q} u_{hq} \delta_{c_m q} \sin\left[\left(m - \frac{1}{2}\right) kd \sin\theta\right], \quad |h| = 1, ..., \infty \tag{7}$$

where $\mathbf{U}_h = \{u_{hq};\, q = 1, ..., Q\} = \mathcal{F}\left(\mathbf{T}^{CPM}, \mathbf{T}^{on}\right)$ depends on the switch-on time $\mathbf{T}^{CPM} = \left\{\tau_q^{CPM};\, q = 1, ..., Q\right\}$ and the switch-on instants $\mathbf{T}^{on} = \left\{t_q^{on};\, q = 1, ..., Q\right\}$, since $t_q^{off} = \tau_q^{CPM} T_p + t_q^{on}$ [Eq. (4)]. Therefore, the set $\mathbf{T}^{on}$ can be profitably optimized to reduce the sideband level (SBL) of the harmonic radiations without modifying the pattern at the carrier frequency (i.e., $\mathbf{A}$ and $\mathbf{T}^{CPM}$). A strategy based on a Particle Swarm Optimizer (PSO) [30][31] is then applied to minimize the following cost function

$$\Psi\left(\mathbf{T}^{on}\right)\big|_{\mathbf{T}=\mathbf{T}^{CPM}} = \sum_{h=1}^{H} \left\{\aleph\left[SBL^{ref} - SBL^{(h)}\left(\mathbf{T}^{on}\right)\right] \left|\Delta_{SBL}^{(h)}\left(\mathbf{T}^{on}\right)\right|^2\right\} \tag{8}$$

where $\Delta_{SBL}^{(h)}\left(\mathbf{T}^{on}\right) = \frac{SBL^{ref} - SBL^{(h)}(\mathbf{T}^{on})}{SBL^{ref}}$ and $\aleph(\cdot)$ is the Heaviside function devoted to quantify the distance between the actual harmonic sideband levels, $SBL^{(h)} = SBL\left(\omega_0 + h\omega_p\right)$ [1], $h = 1, ..., H$ and the user-defined threshold SBL^{ref}.

[1] $SBL^{(h)} \triangleq max_\theta \left\{AF_{\Delta}^{(h)}(\theta)\right\}$

3 Numerical Results

In order to discuss the potentialities and current limitations of the proposed approach, the results from two representative experiments are analyzed. More specifically, the same array geometry is considered in both cases, but different static (sum) excitations as well as different numbers of sub-arrays have been used. Since this is the first (to the best of the authors' knowledge) application of the time-modulation to the synthesis of monopulse sub-arrayed antenna where the sum and the difference patterns are simultaneously generated, no comparisons with other methods are possible. However, since the independent generation of difference patterns by modulating a limited number of static excitations that afford a Villeneuve sum pattern has been described in [22], similar scenarios have been considered as reference geometries. Accordingly, let us refer to a $N = 30$ element array with inter-element spacing $d = 0.7\lambda$ [22]. In the first experiment (*Experiment 1*), the set of static sum excitations $\mathbf{A}$ has been chosen to synthesize a Villeneuve sum pattern with $SLL = -20\,dB$, $\overline{n} = 3$ and $\nu = 0$ [32]. To generate the compromise difference pattern, $R = 8$ sub-arrays have been used as in [22] (Tab. 4 - Case *B*). The CPM has been run by setting the reference difference excitations to those of a Modified Zolotarev pattern [4] with $SLL = -30\,dB$ and $\overline{n} = 5$. The "best compromise" solution, obtained after 16 iterations in $1.7 \times 10^{-5}\,[sec]$ (on a $3\,GHz$ PC with $1\,GB$ of RAM), is shown in Fig. 2(*a*) together with the reference difference pattern. The corresponding element switch-on times, $\mathbf{T}^{CPM}$, and the sub-array configuration $\mathbf{C}^{CPM}$ computed through the minimization of (5) are shown in Fig. 2(*b*) and reported in Tab. I, respectively. For completeness, the plot of the reference excitations is displayed in Fig. 2(*b*) (dotted line). From Fig. 2(*a*), it can be seen that there is a good matching between the main lobes of the reference and compromise difference patterns. As a matter of fact, the $-3\,dB$ beamwidth (BW) is equal to $BW^{ref} = 2.57^o\,[deg]$ and $BW^{CPM} = 2.58^o\,[deg]$, respectively. Therefore, the resolution capability of the monopulse tracking systems (i.e., the deepness of the main lobe along the boresight direction [33]) is kept almost unaltered. Secondly, although the envelope of the secondary lobes is no more decaying as $\frac{1}{sin\theta}$ as for the reference pattern, the SLL of the compromise pattern is close to the optimal one ($SLL^{CPM} = -26.9\,dB$ vs. $SLL^{ref} = -30.0\,dB$) with still a satisfactory ability to suppress interferences and clutters [34].

As far as the CPM solution is concerned, $N_{TM} = 20$ elements over $N = 30$ are time-modulated, while the others are kept time-constant and set to the corresponding static sum excitations (Tab. I). Concerning SRs, Figure 3 shows the patterns radiated at $|h| = 1, 2$. As it can be observed, the highest lobes principally lie in the angular region close to that of the main difference lobes and the values of the SBLs turn out to be $SBL_{CPM}^{(1)} = -14.9\,dB$ and $SBL_{CPM}^{(2)} = -22.4\,dB$, respectively. In order to minimize the SBL, the PSO strategy has been successively applied by setting $H = 1$, as in [22] [(2)], and $SBL^{ref} = -20\,dB$. Moreover, the following PSO setup has been chosen according to the guidelines in [35]: $S = 10$ particles, $w = 0.4$ (*inertial weight*), and $C_1 = C_2 = 2$ (*cognitive/social acceleration coefficient*).

At the convergence, after 500 iterations and $63.5\,[sec]$, the optimized values of the switch-on instants t_q^{on}, $q = 1, ..., Q$, are those given in Tab. II ($Q = 4$). Moreover, the plot of the pulse sequence is shown in Fig. 4(*a*), while the corresponding patterns are displayed in Fig. 4(*b*). It is worth noticing that, without additional hardware, but simply adjusting the on-off sequence of the RF switches, the $SBL_{CPM}^{(1)}$ value is lowered of more than $4\,dB$ (i.e., $SBL_{CPM-PSO}^{(1)} = -19.2\,dB$ vs. $SBL_{CPM}^{(1)} = -14.9\,dB$). It is worth noting that neglecting the small "on-time interval" at the beginning of the period T_p for elements 5, 11, 20 and 26 [Fig. 4(*a*)] the features of both the main pattern at central frequency and the harmonic patterns slightly modify (e.g., the SLL and the $SBL^{(1)}$ increase of $0.3\,dB$ and $0.5\,dB$, respectively). This fact would avoid these small intervals to be the bottleneck of the time-modulation system, allowing the RF switches to have less restrictions about their switch-on-to-switch-off speed.

For completeness, although the comparison is not completely fair since different synthesis problem are at hand, the solutions obtained with the $CPM - PSO$ and those shown in [22] are then analyzed by comparing the corresponding patterns at both the carrier frequency [Fig. 5(*a*)] and when $|h| = 1,\, 2$ [Fig. 5(*b*)]. The power losses due to SRs, quantified through the close form relationship in [36], amounts to $P_{SR} = 21.3\%$ of the total radiated power in correspondence with the $CPM - PSO$. Otherwise ([22] - Tab. 4, Case *B*), the wasted power is only $P_{SR}^{SA} = 3\%$ and the SBL is much smaller [Fig. 5(*b*)] since only $N_{TM}^{SA} = 8$ elements are time-modulated (instead of $N_{TM}^{CPM} = 20$). On the other hand, the efficiency of the $PSO - CPM$ approach

[(2)] Only the first harmonic mode has been optimized since the power loss reduces when the order of the harmonic mode increases.

in minimizing the SLL of the compromise difference patterns ($h = 0$) is non-negligible [Fig. 5(*a*)] ($SLL^{SA} = -14.9\,dB$ vs. $SLL^{CPM} = -26.9\,dB$).

In the second experiment (*Experiment 2*), the number of control elements is reduced by considering $R = 4$ RF switches ([22] - Tab. 4, Case *C*). The sum pattern is a Villeneuve pattern with $SLL = -20\,dB$, $\overline{n} = 3$, and $\nu = 1$ [32]. Moreover, the reference difference set $\mathbf{B}$ has been selected to generate a Modified Zolotarev difference pattern [4] with $SLL = -20\,dB$ and $\overline{n} = 4$. Figure 6(*a*) shows the approximated pattern synthesized at the convergence of the CPM-based matching procedure by applying the pulse sequence $\mathbf{T}^{CPM}$ in Fig. 6(*b*). The corresponding sub-array configuration is given in Tab. I, as well. As for the first experiment, the secondary lobes do not decrease when θ grows [Fig. 6(*a*)], but the SLL value of the compromise pattern turns out to be lower than that of the Zolotarev one ($SLL^{CPM} = -23.3\,dB$ vs. $SLL^{ref} = -21.0\,dB$). Moreover, the same beamwidth has been achieved ($BW^{ref} = 2.36^o\,[deg]$ and $BW^{CPM} = 2.37^o\,[deg]$). Concerning the computational burden, 5 CPM iterations and $\sim 10^{-6}\,[sec]$ are enough to find the final solution.

Successively, the $SBL^{(1)}$ has been minimized by optimizing $\mathbf{T}^{on}$ with a PSO swarm of $S = 5$ particles. For comparison purposes, Figure 7 shows the patterns at $|h| = 0,\,1,\,2$ synthesized with the CPM and after the PSO optimization. Despite the reduced number of sub-arrays ($Q = 2$), the value of $SBL^{(1)}_{CPM} = -17.3\,dB$ has been reduced to $SBL^{(1)}_{CPM-PSO} = -19.3\,dB$ in $7.25\,[sec]$ after 100 iterations by defining the values of the final switch-on instants reported in Tab. II.

For completeness, the $CPM - PSO$ patterns and those in [22] with four switches are shown in Fig. 8(*a*) ($h = 0$) and Fig. 8(*b*) ($|h| = 1,\,2$). As regards to the number of time-modulated elements, it results that $N^{CPM}_{TM} = 10$ and $N^{SA}_{TM} = 4$. Consequently, $P^{CPM}_{SR} = 16.9\%$ and $P^{SA}_{SR} = 2.1\%$, while $SLL^{CPM} = -23.3\,dB$ and $SLL^{SA} = -15.2\,dB$.

4 Conclusions

In this paper, the potentialities of time-modulation when dealing with the synthesis of monopulse sub-arrayed antennas have been investigated. Starting from a set of static excitations affording an optimal sum pattern, the signals at the sub-arrayed feed network have been time-modulated to generate a compromise difference pattern. Both the sub-array configuration and the duration of the time-pulse at each sub-array have been optimized solving a pattern matching problem by means of the CPM. A particle swarm optimization has been successively performed to minimize the SBL of the sideband radiations.

The obtained numerical results seem to indicate the proposed approach as a suitable alternative for the synthesis of compromise sum and difference patterns. As a matter of fact, the main advantages of the proposed approach are the possibilities on one hand of using simple RF devices (i.e., switches) in the feed network reducing the complexity of the antenna system and on the other hand of shaping the beam pattern by only changing the pulse sequence at the sub-array ports.

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FIGURE CAPTIONS

- **Figure 1.** Sketch of the antenna feed network.

- **Figure 2.** *Experiment 1* ($Q = 4$) - Plots of (*a*) the reference (Modified Zolotarev [4], $SLL = -30\,dB$, $\overline{n} = 5$) and CPM-synthesized power patterns at the carrier frequency ω_0 ($h = 0$) and (*b*) the corresponding switch-on times.

- **Figure 3.** *Experiment 1* ($Q = 4$) - Normalized power patterns generated at ω_0 ($h = 0$) and $|h| = 1, 2$ by means of the CPM.

- **Figure 4.** *Experiment 1* ($Q = 4$) - PSO-optimization: (*a*) switch-on times and (*b*) power patterns at $|h| = 1, 2$.

- **Figure 5.** *Experiment 1* ($Q = 4$) - (*a*) Normalized difference power patterns at ω_0 ($h = 0$) synthesized through the SA [22] and the $CPM - PSO$. (*b*) Polar plots of the corresponding sideband radiations at $|h| = 1, 2$.

- **Figure 6.** *Experiment 2* ($Q = 2$) - Plots of (*a*) the reference (Modified Zolotarev [4], $SLL = -20\,dB$, $\overline{n} = 5$) and CPM-synthesized power patterns at the carrier frequency ω_0 ($h = 0$) and (*b*) the corresponding switch-on times.

- **Figure 7.** *Experiment 2* ($Q = 2$) - Normalized power patterns at ω_0 ($h = 0$) and $|h| = 1, 2$ synthesized by means of the CPM and the $CPM - PSO$ approach.

- **Figure 8.** *Experiment 2* ($Q = 2$) - (*a*) Normalized difference power patterns at ω_0 ($h = 0$) synthesized through the SA [22] and the $CPM - PSO$. (*b*) Polar plots of the corresponding sideband radiations at $|h| = 1, 2$.

TABLE CAPTIONS

- **Table I.** Sub-array configurations for the compromise difference patterns when $Q = 4$ and $Q = 2$.

- **Table II.** PSO-optimized switch-on instants for the compromise difference patterns when $Q = 4$ and $Q = 2$.

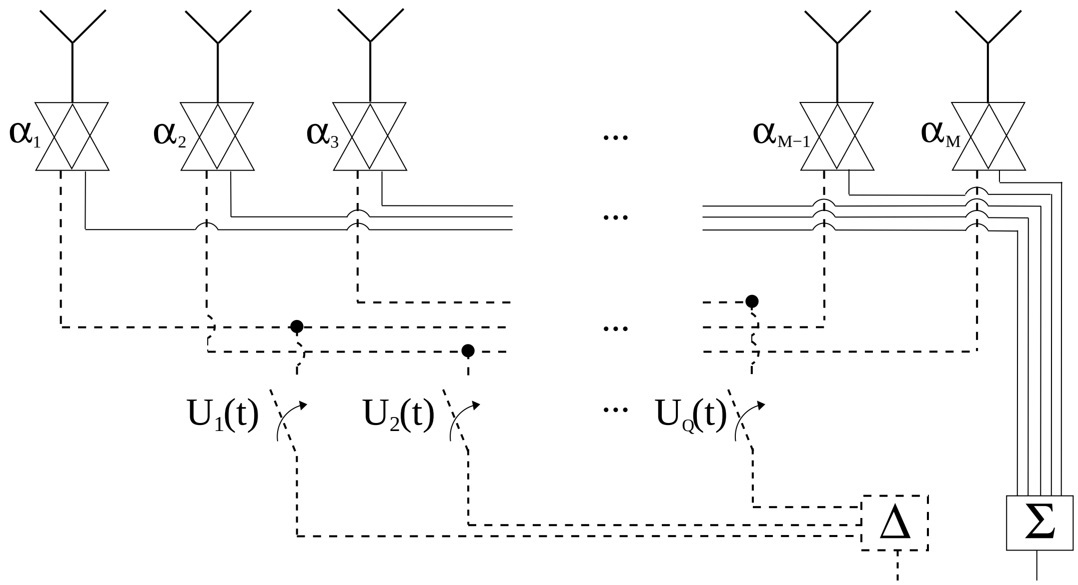
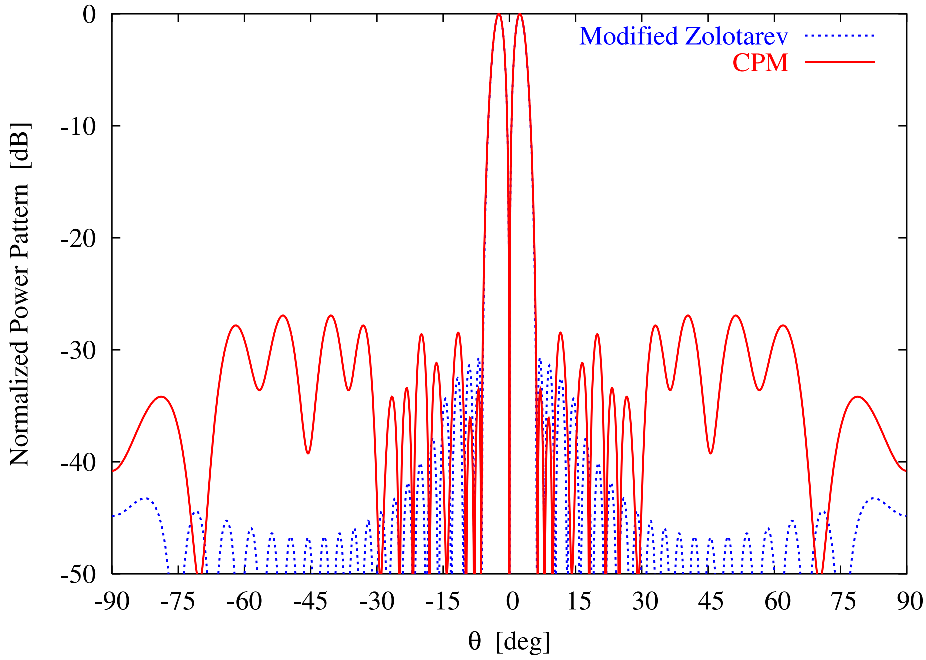
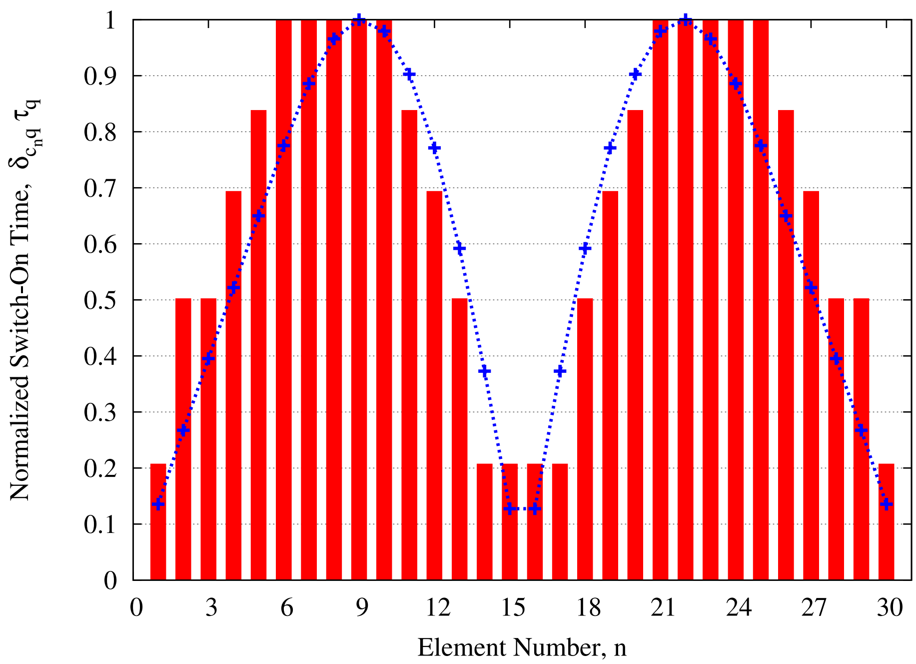


Fig. 1 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."



(a)



(b)

Fig. 2 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

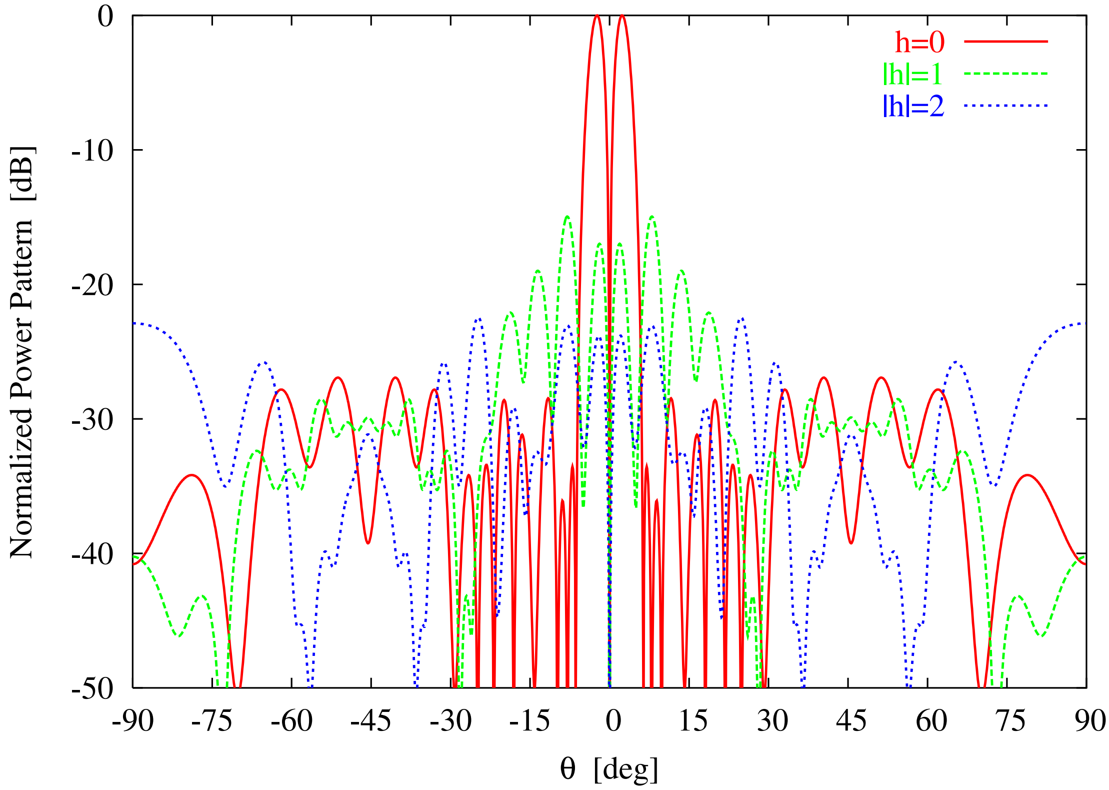


Fig. 3 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

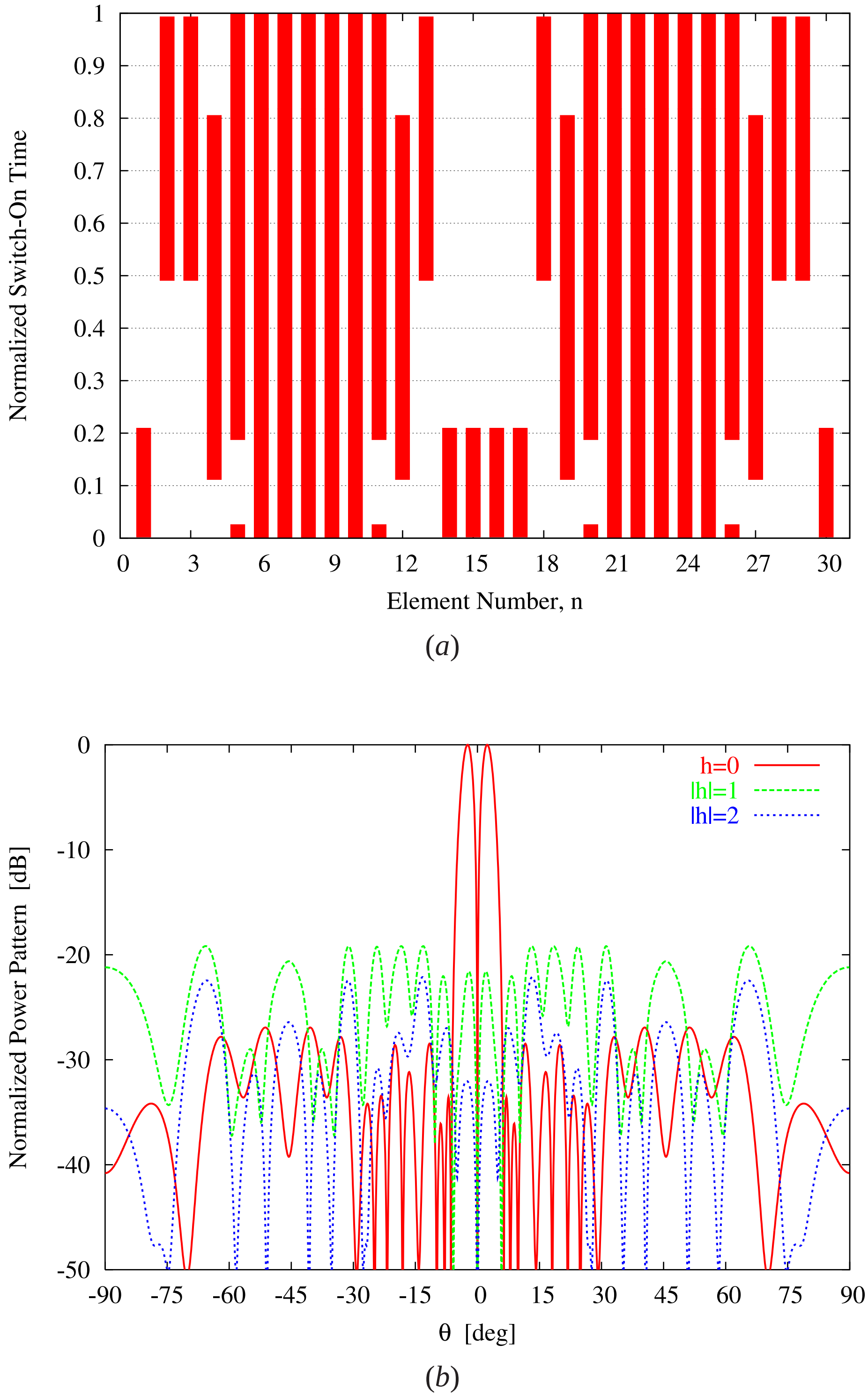
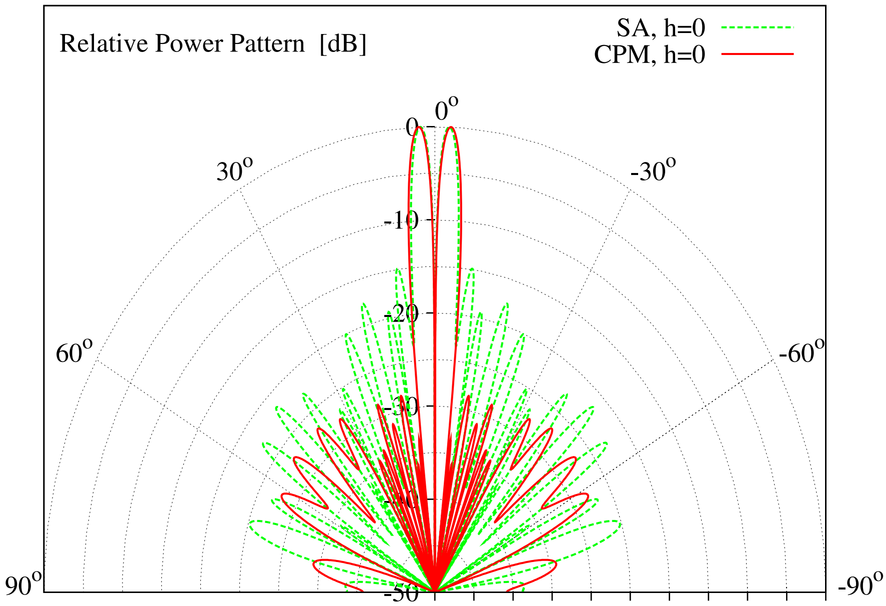
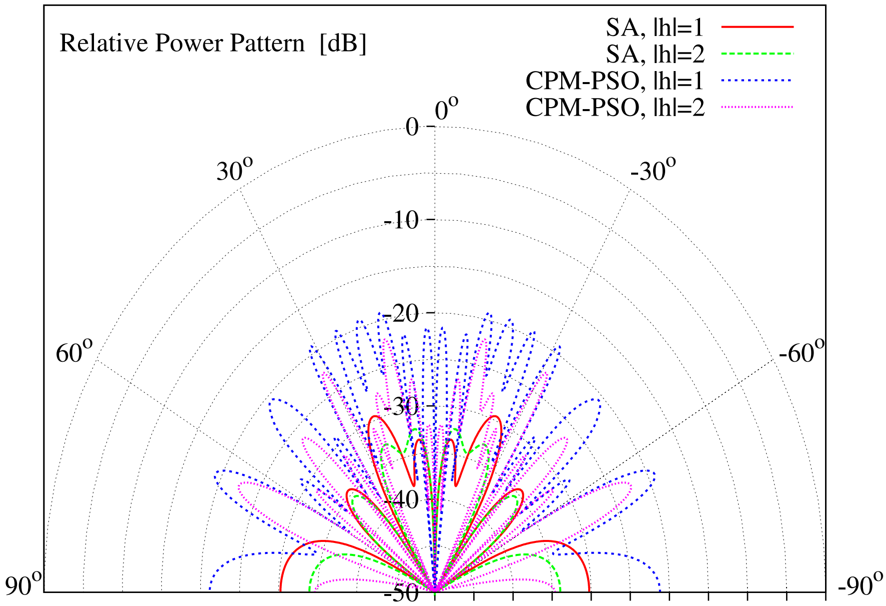


Fig. 4 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

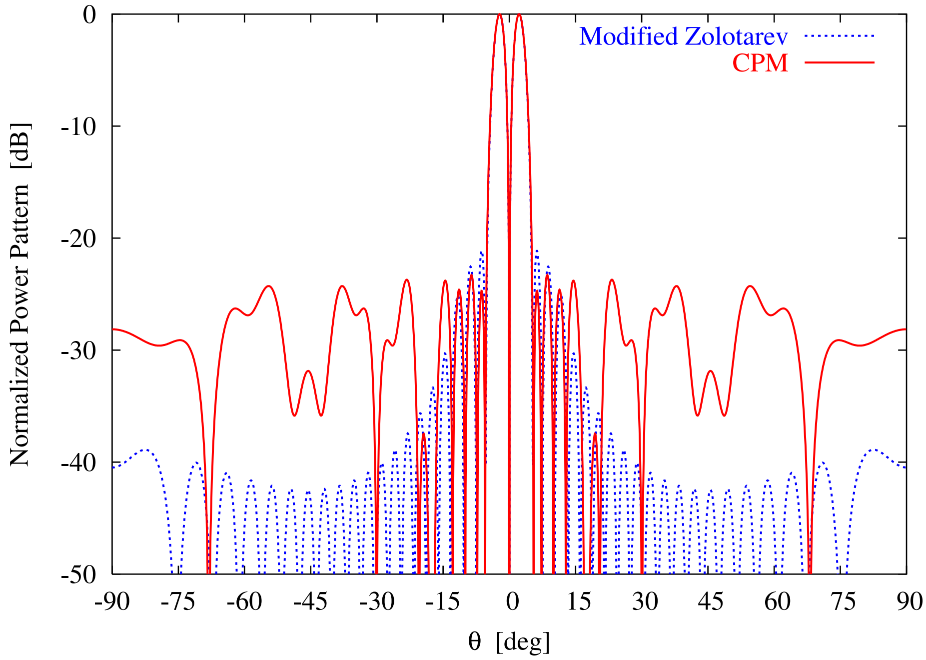


(a)

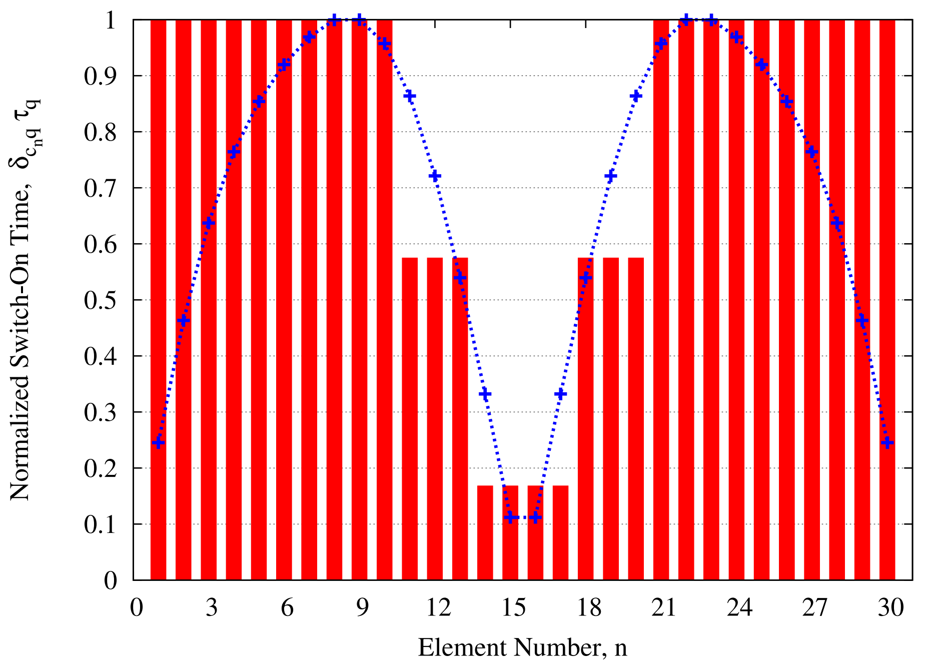


(b)

Fig. 5 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."



(*a*)



(*b*)

Fig. 6 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

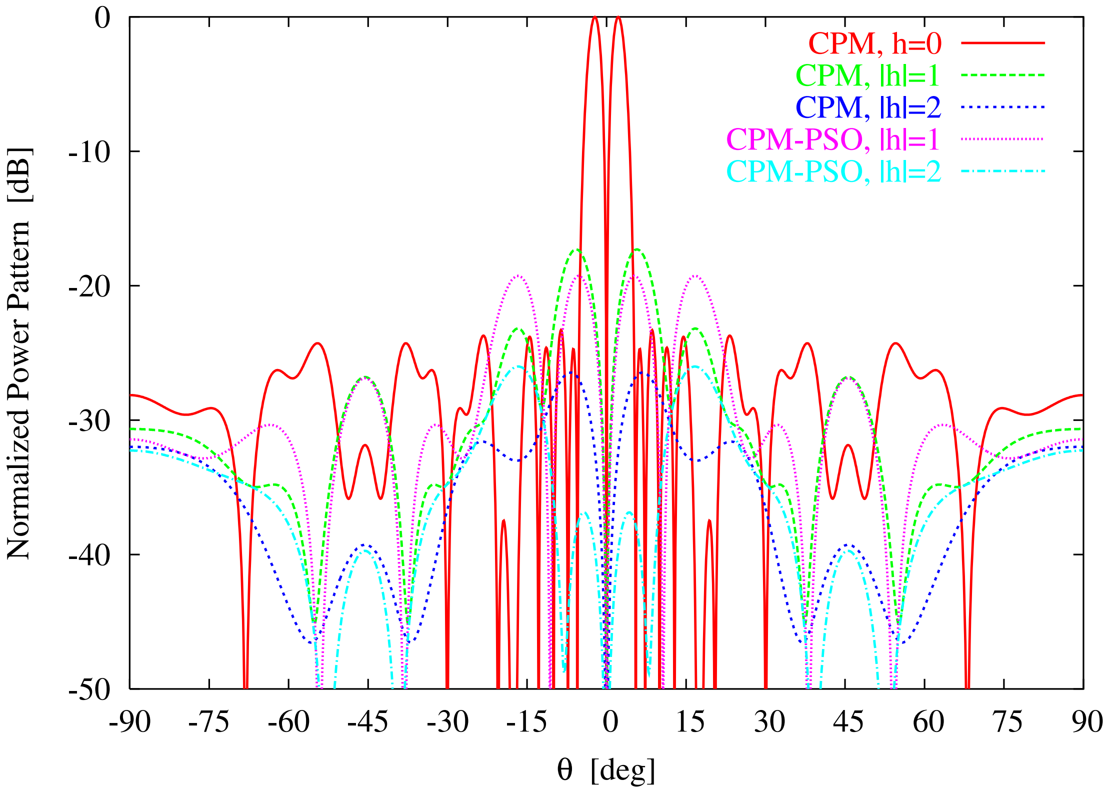
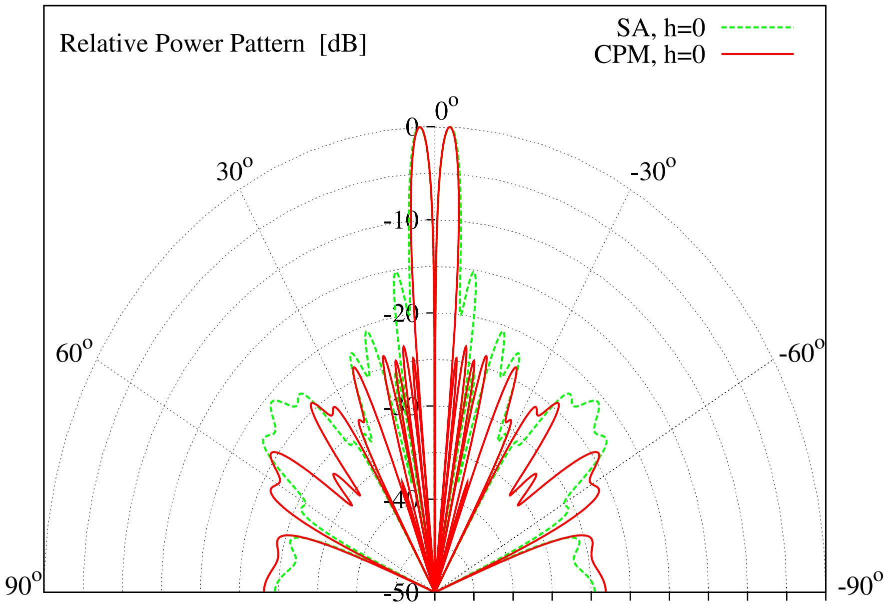
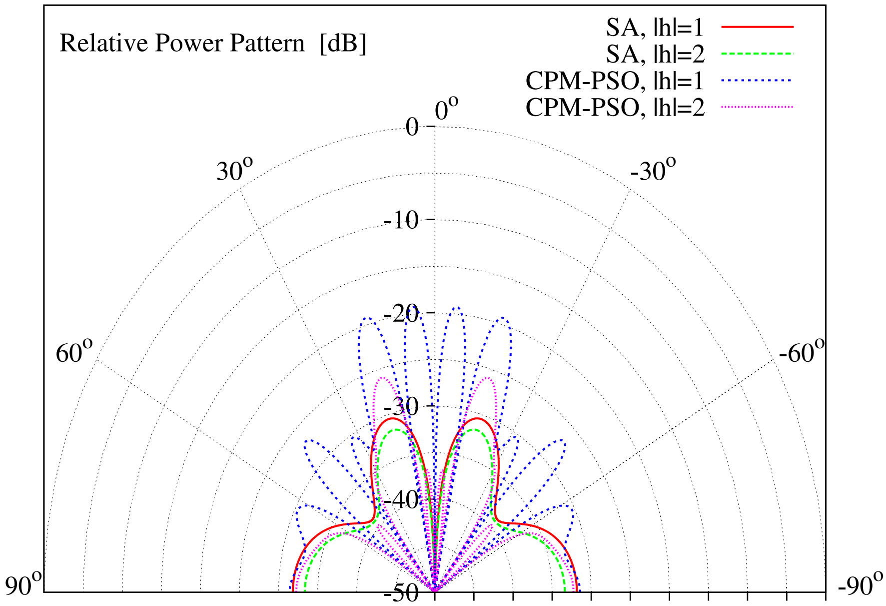


Fig. 7 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."



(*a*)



(*b*)

Fig. 8 - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

	$\mathbf{C} = \{c_m;\, m = 1, ..., M\}$
$M = 15,\, Q = 4$	1 1 2 3 4 0 0 0 0 0 4 3 2 2 1
$M = 15,\, Q = 2$	1 1 2 2 2 0 0 0 0 0 0 0 0 0 0

Tab. I - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."

	t_q^{on} [sec]			
q	1	2	3	4
$Q = 4$	0.00	0.49	0.11	0.19
$Q = 2$	0.89	0.18	$-$	$-$

Tab. II - P. Rocca *et al.*, "Synthesis of Compromise Sum-Difference Arrays ..."