



Slip line model for forces estimation in the radial-axial ring rolling process

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ABSTRACT

In the research presented in this paper, a slip line-based model is proposed for the estimation of both radial and axial force in the radial-axial ring rolling (RARR) process. Based on the shape of the contact arcs between ring and tools in the two deformation gaps present in the ring rolling process, a recursive algorithm for the calculation of the two slip line fields starting from the two pairs of opposite tools is derived and implemented in a commercial spreadsheet software (MS Excel). By considering the stress boundary conditions applied to the portion of material undergoing the deformation, both for the radial and axial deformation gaps, the pressure factors those make the two slip line fields starting from the two opposite tools to intersect are calculated and utilized for the estimation of radial and axial forces, for each round of the process. The developed model has been validated by cross-comparing its results with those of laboratory experiment and numerical simulation. For the validation study case, the average deviations, in comparison to the experimental results, are calculated in 1.86% and 4.55% for the slip line force model whereas in 6.86% and 0.88% for the numerical simulation, for the radial and axial forces respectively. The proposed slip line model has been also utilized for the estimation of radial and axial forming forces of nine different study cases of flat rings having the outer diameter ranging from 800 mm to 2000 mm, observing a maximum deviation, in comparison to the relevant FEM simulation, of 4.92% (radial force) and 5.88% (axial force). The developed slip line force model allows estimating almost in real time and with a reasonable accuracy the process forces and, for this reason, it may be of interest for both industrial and academic researchers dealing with the set-up and control of the radial-axial ring rolling process.

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1. Introduction

Radial-axial ring rolling (RARR) is a wide-spread forging process that allows manufacturing seamless rings and is utilized for the manufacturing of components for many different industrial sectors, among them machinery, metallurgy, aerospace and automotive [1]. In the last sixty years, several authors focused their attention on the development of both analytical and numerical models for the analysis of both the kinematics and the dynamics of the ring rolling process, aiming to develop algorithms and procedures for a quick and precise estimation of the process forces during the process.

One of the first contributions concerning the development of an analytical force model for the ring rolling process is due to Hawkyard et al. [2]. In their contribution, based on the Hill's slip line flat indenter theory [3], a model for the estimation of both load and torque is developed and validated by comparing the results with those of ring rolling laboratory experiments, carried out at cold process conditions. One of the main limitations of the Hill's solution (flat indenters) [3] is the assumption of a straight contact line between tool and workpiece. As demonstrated in

Quagliato and Berti [4], the fulfillment of this assumption is related to specific interactions between ring and tools diameters and fails to comply if the curvature of ring and tool, namely mandrel-main roll, become too similar. In this case, if the Hill's slip line solution for flat indenters is utilized, it results in an underestimation of the radial forming force up to almost 30%.

In a more recent work, Parvizi et al. [5] proposed a SLAB analysis for the determination of both load and torque in the ring rolling process but, based on the results presented in the paper, the model tends to underestimate both of them in case the mean radius of the ring is greater than 160 mm or in case of thickness reduction smaller than 40%. However, the thickness reduction and, especially, the ring diameter are normally not under the direct control of the process engineer and a variation of accuracy on their basis might be problematic

In fact, being the ring rolling an incremental forming process, the thickness reduction is not independent of the other process parameters and especially it is linked to the geometry of both ring and ring rolling mill, as initially suggested in the work of Lin and Zhi [6]. In addition to that, as proved in Berti et al. [7], the choice of the feasible ranges

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for both mandrel and upper axial roll feeding speeds should be oriented toward the stability of the process hence to thickness reduction between 10% and 20%, in order to reduce the spreading effect in the vertical direction of the ring.

Again concerning the development of analytical models for the force estimation in the RARR process, Parvizi and Abrinia [8] developed a two-dimensional upper bound method for the estimation of the radial forming force and compared the results with both previous upper bound literature models and literature experiments, namely the works of Ryoo and Yang [9] and Yang and Ryoo [10]. The results shown that, for most of the explored range of mean ring radii, a higher accuracy in comparison to previous literature upper bound models [9,10] is achieved but the comparison is carried out only at the first round of the rotation, which is normally not the round where the maximum force arises.

Concerning analytical models for the estimation of the axial forming force, only two recent contributions seem to be available. Wang and Wang [11] established a SLAB-based analytical model for the estimation of the forming force in the axial rolls deformation gap, presenting the results of its application on different study cases, where both inner and outer diameters of the rings have been varied. However, since no comparison with neither numerical nor experimental results is presented, the accuracy of the model is unknown. Same concerning the analytical estimation of the axial forming force, Kalyani et al. [12] presented a force equilibrium-based model for the estimation of both radial and axial forming force and compared the results with those of laboratory experiments. However, the comparison is carried out only at 60 s and 180 s and not for the whole rolling time, making it complicated to understand whether the maximum value of the axial forming force is well predicted or not. In addition to that, the maximum error in the estimation of the axial forming force, in comparison to the relevant experiment, is estimated in 16.9%, a deviation that may be considered too high for certain industrial or research applications.

In relation to the estimation of the forming force in the radial-axial ring rolling (RARR) process by means of numerical simulation, several authors developed different approaches and techniques to reduce the computational time while preserving or increasing the accuracy of the calculation. Yea et al. [13] developed a finite element code, namely SHAPE-RR™, specially designed for the ring rolling process, and utilized it for the calculation of spread, pressure distribution and roll force. The results have been compared with previous experimental works, proving the reliability of the developed software. In a subsequent work, Kim et al. [14] utilized the same SHAPE-RR™ code for the computation of the optimal mandrel and upper axial roll feeding speeds in order to minimize the load in the ring rolling process, validating the relevant results with industrial study-case experiments.

Concerning different integration approaches, Davey and Ward [15] developed an arbitrary Lagrangian–Eulerian strategy for the reduction of the computation time, in comparison to conventional Lagrangian approaches. The application of the proposed method allows reducing the nodes of the mesh to a range between 17.5% and 25% in comparison to the original full Lagrangian approach mesh system while granting a good agreement with experimental results. Lim et al. [16], by utilizing an implicit finite-strain updated-Lagrangian approach, developed a material mesh-computational hybrid numerical model for the reduction of the computational-time. The results showed a considerable computational-time reduction, estimated in 70%, in comparison to standard finite element technique but, although the considerable reduction, the computational time was still relatively high. In addition to that, in a recent work of Kim et al. [17], a dual-mesh approach by considering three different remeshing strategies in order to reduce the computational time, at some expense of the computational accuracy, has been proposed. By applying the proposed methodology, the ring is subdivided into different sectors meshed with different criteria in order to reduce the computational time related to the sections of the ring not directly under deformation. The results of the approach are promising but the continuous remeshing required by the algorithm reduces the ad-

vantages of utilizing a coarser mesh in the region of the ring not under direct deformation.

Although several authors focused their attention on the development of analytical models for the estimation of both the radial and the axial forming force in the radial-axial ring rolling process, a general approach, which can be applied to both deformation gaps, seems to be missing. In addition to that, as also stated in the relevant works, the utilization of the model available in the literature seems to be either limited in the range of application or present deviations, in comparison to experimental results, those might be unacceptable, especially if a reasonable estimation is required already in the early stages of the manufacturing and product design process. On the other hand, although the utilization of advanced and dedicated numerical approaches can allow a reduction in the computational-time, the intrinsic iterative nature of the finite element approach does not allow any almost-real-time calculation of the solution. For this reason, if many different combinations of process parameters and different study cases are meant to be analyzed, it might result in a huge computational burden.

Following these considerations, the research work presented in this paper presents a statically determined slip line force model for the estimation of both the radial and the axial forming force in the radial-axial ring rolling process of flat rings. Based on authors' previous work, ring preform and feasible ranges for the main process parameters as well as geometry, strain, temperature and strain rate evolution during the forming process can be analytically estimated [7,4,18] and, accordingly, the forming forces are estimated throughout the process, allowing to identify both their trend as well as their maximum value. In order to enhance the understanding of the paper contents as well as the paper flow, a summary of the main results of previous authors' works is presented in [Appendix A](#) whereas the governing equations at the basis of the slip line theory are reported in [Appendix B](#).

The proposed slip line algorithm and the developed numerical model, utilized for further investigations in the paper, have been validated by comparing their estimations with the results of laboratory experiments carried out on a laboratory-size ring rolling machine and by utilizing a Pb75–Sn25 alloy ring. In the comparison between analytical and experimental results, a maximum deviation of 1.86%, for the radial forming force, and 4.55%, for the axial forming force, has been observed. Concerning the numerical simulation, same in comparison to the relevant laboratory experiment, the maximum deviations have been identified in 6.86% and 0.88%, respectively. The validated analytical and numerical models have been applied to nine study cases where the outer diameter of the ring ranges from 800 mm and 2000 mm, resulting in maximum deviations of 4.92% and 5.88%, for the radial and axial forming forces respectively. Based on the positive results obtained both in the validation cases as well as in the wide range of study cases where the model has been applied, the proposed approach seems to be reliable for the application in both industrial and research environment, for a real-time and reasonable estimation of the forming forces.

2. Novelty of the research

The research presented in this paper provides the theoretical background and the full algorithm for the implementation of a slip line based algorithm for the estimation of the process forces in the radial-axial ring rolling process. As also highlighted in the introduction, the literature force models developed for the estimation of the radial forming force have a variable accuracy depending on the size of the ring or on the thickness reduction ratio, both of them not normally under the direct control of the process engineer. This fact has been proven in Quagliato and Berti [4] for the case of the application of the model proposed by Hawkyard et al. [2] and is included in the conclusions of the work of Parvizi et al. [5]. In addition to that, concerning the estimation of the axial forming force, only two literature contributions are available in the literature, as follows. In the work of Wang and Wang [11], no validation of the model is proposed thus the accuracy of the proposed formulation

is unknown. In the second available contribution, Kalyani et al. [12], a force equilibrium-based model is proposed but the comparison with the experimental results highlighted a deviation of 16.9%. Hence, the advantages of the model proposed in this paper, in comparison to those available in the literature, can be summarized in three different main aspects:

- The slip line based algorithm proposed in the research presented in this paper is a general model, which can be either applied to the mandrel-main roll deformation gap, for the calculation of the radial forming force, or to the axial rolls deformation gap, for the calculation of the axial forming force;
- As detailed in the results section of the paper, the size of the ring does not significantly influence the overall accuracy of the prediction, as it happens for the literature models [2,5];
- Although the maximum value for the radial and the axial forces are the most relevant ones, the proposed model also provides an indication of the trend of the forces during the rolling time.

In addition to that, the reliability of the proposed approach has been demonstrated by both experimental and numerical verifications, allowing to conclude that it can be applied to a large variety of rings, with different section profiles and diameters.

The force model algorithm presented in this paper, along with authors' previous works [7,18,4] allows having a full understanding of the influence of the process parameters on important process variables, such as the forming forces, those are normally utilized for the choice of the ring rolling mill. In addition to that, since the complete procedure for the construction of the slip line field is included in the paper, the proposed algorithm can be implemented also in already-existing codes and subroutines, widening its range of possible users. For these reasons, the developed approach is of interest of both industrial engineers and academic researchers dealing with the RARR process and can be potentially extended to different forging processes where similar tools/workpiece interactions can be identified.

3. Application of the slip line theory to the RARR process

The slip line theory is based on three main assumptions: (i) plane strain, (ii) perfect-plastic material and (iii) homogenous properties distribution. In this section of the paper, the admissibility of these three assumptions is discussed from the point of view of the RARR process aiming to show how they all can be reasonably applied. To this aim, authors' works results are utilized [7,18,4]. In addition to that, a summary of the main assumptions and results of the same previous works is detailed in Appendix A of the paper.

As proven in [18] and also summarized in Appendix A, in the cross-section of the ring the strain in the third direction, vertical one in the mandrel-main roll gap and radial in the axial rolls gap, is largely smaller than the other two. In other words, assuming a plane strain condition is not far from the reality of the process if the analysis is focused on the single deformation gap and not on the whole process. However, the third strain component has an influence on the estimation of the equivalent plastic strain thus, following the rationale detailed in [18], the equivalent plastic strain is estimated by accounting for all the three strain components along the three principal directions. Accordingly, the analytical estimation of the flow stress of the material, as proposed in [18,4], is accomplished by accounting for the 3D strain field in the ring section undergoing the deformation.

The plane strain assumption is introduced only in the slip line model on the basis of the fact that the material plastic flow along the third-direction, vertical one in the mandrel-main roll deformation gap and radial one in the axial rolls gap, is largely lower than that along the other two directions. The radial-circumferential (mandrel-main roll gap) and vertical-circumferential (axial rolls gap) directions are those along which the slip line fields are constructed and are those relevant for the major strain contributions. Therefore, the orientation of the plane strain

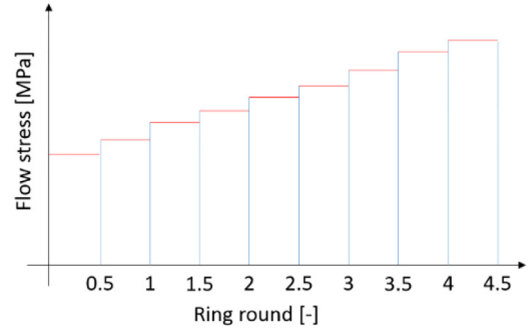


Fig. 1. Material flow stress update concept.

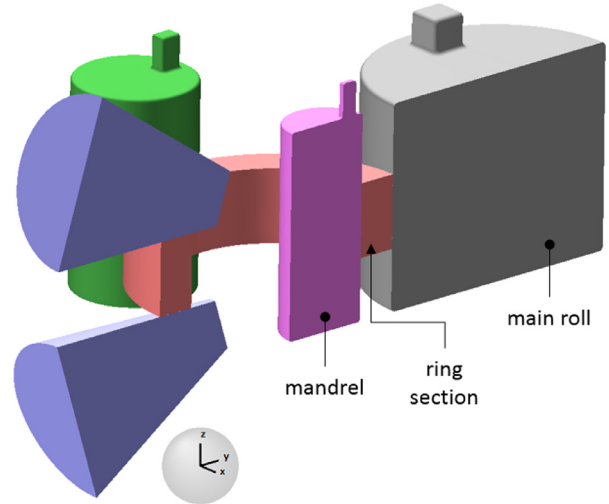


Fig. 2. Section of ring and tools at the symmetry axis (Z-Y plane).

(plane) changes according to the considered deformation gap and according to the directions along which the plastic flows are predominant. This allows fulfilling the plane strain assumption, required by the slip line theory while granting good accuracy of the prediction. The reliability of the last statement will also be proven in the results section of the paper, where good agreement has been observed among experimental, numerical and slip line model-based values for both the radial and the axial forming forces.

Considering the perfect plasticity assumption, required by the slip line theory, the values of strain, strain rate and temperature are estimated according to previous authors' models [18,4] at the end of the deformations when the considered portion of the ring (slice, see Appendix A) has exited the deformation gap. Then, by inputting the estimated strain, strain rate and temperature values in the flow stress equation of the material, the final peak of the material hardening, at its exit from the deformation gap, is calculated. According to the slice concept [7,18,4], explained in Appendix A, the material flow stress is calculated for each portion in which the ring has been subdivided (viz. slice) and for each round of the process, allowing to estimate the increase of the flow stress of the material, as a consequence of strain, strain rate, and temperature variation, as shown in Fig. 1. The flow stress is updated at each half-round of the process or, in other words, at each deformation gap. By applying this rationale, the perfect-plastic material behavior assumption, required from the slip line field theory, is correctly fulfilled and, at the same time, the variation of the material flow stress along the process can be taken into account as well.

Considering the homogeneity of process variables and the relevant influence on the material properties, i.e. the influence of the temperature gradient in the ring thickness on the material flow stress, an impor-

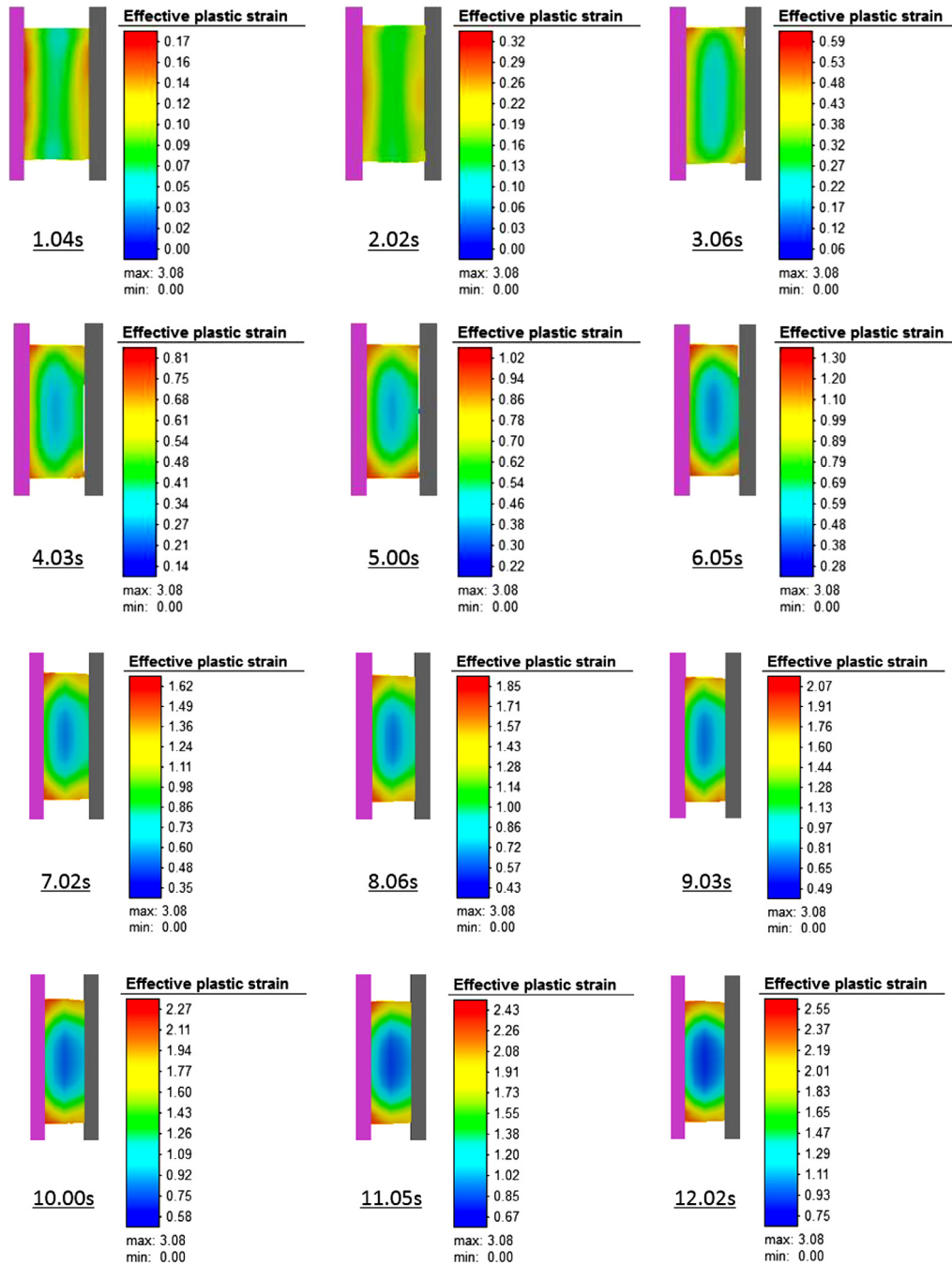


Fig. 3. Evolution of the equivalent plastic strain in the cross-section of the ring for increasing process time.

tant issue, which partially anticipates the results section of the paper, should be addressed.

Although the process variables, such as temperature, strains, and stress, are not homogeneous along the thickness, or the height, of the ring, only a small portion of the thickness, or of the height, are actually interested by a strong gradient whereas the remaining parts show a more uniform behavior. In addition to that, the strain and stress field in the central region of the ring cross-section is largely lower than that close to the surface as well as the temperature is higher. The validity of these last two statements can be assessed, for instance, by analyzing the variation of the strain field in the cross-section of the ring while it is being deformed, Fig. 2, for increasing time steps during the deformation process. To this aim, a ring with initial dimensions equal to 600 mm, 821 mm and 214.7 mm and final dimensions equal to 1270 mm, 1400 mm and

195 mm, for the inner radius, outer radius and height respectively, has been taken into account. The ring object of this preliminary analysis is part of a larger research presented in a previous authors' work [4]. By analyzing the evolution of the equivalent plastic strain in the cross-section of the portion of the ring undergoing the deformation in the mandrel-main roll deformation gap, Fig. 3, it is possible to infer how the high deformation is located in a small portion of the thickness of the ring whereas, in the central area, its value is greatly lower and with an almost homogenous distribution.

As previously highlighted, in the research proposed in this paper the flow stress is estimated on the basis of the values of strain, strain rate and temperature at the final stage of the deformation or, in other words, considering the maximum (for strain and strain rate), or minimum (for the temperature), values of these parameters when the considered portion

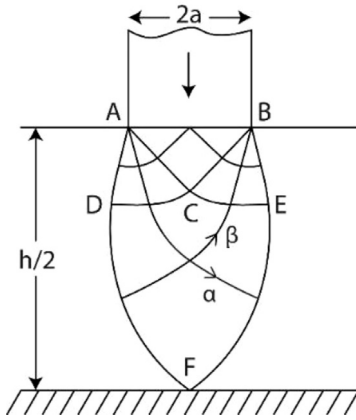


Fig. 4. Hill's flat indenter slip line solution.

of material is exiting the deformation gap. Accordingly, the flow stress is considered to be uniform in the cross-section of the ring but this fact makes the proposed algorithm to substantially overestimate the state of stress in the cross-section, allowing a calculation in favor of safety.

According to the considerations highlighted in this section of the paper, the utilization of the slip line field theory for the estimation of the process forces in the radial-axial ring rolling process is reasonable and all its assumptions are taken into account and correctly fulfilled. In addition to that, as shown in the result section of the paper, the proposed slip line based algorithm is able to predict both radial and axial forces with a reasonable deviation in comparison to both numerical and experimental results, proving the reliability of the application of the slip line theory to the RARR process.

4. Slip line network construction algorithm

The slip line theory has been utilized for the calculation of the pressure distribution in the plastically deformed region of the material in several different metal forming and forging processes [2,19–25]. Especially, in [2] the Hill's flat indenter slip line solution [3] has been utilized for the estimation of load and torque in the radial ring rolling process. In order to demonstrate the improvement of the proposed approach, in comparison to that available in the literature, a summary of the algorithm proposed in [2] is hereafter reported. In the Hill's flat indenter slip line solution [3], utilized by Hawkyard et al. [2], the slip line network is calculated for the case of a flat punch indenting on a flat-smooth surface, Fig. 4. Considering the stress boundary conditions on A and B both the magnitude of the hydrostatic pressure $\bar{\sigma}$ as well as the angle ϑ can be calculated, allowing to draw the first two slip lines starting from A and B, those cross in the point C (see Eqs. (B.1)–(B.3), Appendix B). In the flat indenter slip line solution, being AB a straight line aligned with the x-direction and having neglected the friction contribution in the contact between the flat punch and the flat surface, the AC (α -line) and BC (β -line) slip lines are straight lines and their inclinations with respect to the x-direction are $\pi/4$ and $-\pi/4$, respectively. Starting from the ABC triangular zone, the remaining two fans areas ACD and BCE can be constructed and have the shape of circular sectors. Finally, based on the solution calculated on DC and CE curves, the last portion of the slip line network, DCEF, can be constructed, allowing to calculate the point F, ending point of the plastically deformed region of the material in the thickness of the workpiece. However, as previously anticipated and also detailed in [4], the approximation of the ring/tools contact surface as a straight line is not always a good representation of the reality of the RARR process, Fig. 5a. For this reason, the real curved contact geometry between ring and tools, as shown in Fig. 5b and c, has been taken into account for the construction of the slip line field.

Although the contact geometry of the mandrel-main roll deformation gap differs, in principle, from that of the axial rolls gap, a common feature is present in both of them, namely a curved contact arc, starting with a certain angle and ending with a horizontal tangent, where the point A represents the onset the deformation gap whereas the point B its end. The slip lines starting from point A and point B will have different inclinations, depending on the relevant stress boundary conditions, geometrical boundary conditions and on the friction law.

Since the inclinations of the slip lines are normally measured as counterclockwise angle from the x-direction, in A the angle of the slip line trajectory must be summed to that given by the relevant geometrical boundary conditions, namely inclination of the tangent to the roll at the onset of the deformation gap (α in Fig. 6). In the point B, being the trajectory of the contact curve aligned to the x-direction, no additional geometrical boundary conditions are present but only the relevant stress boundary conditions and the friction angle should be taken into account.

In the slip line field, friction acts as an additional angle that has the effect of changing the initial directions of the slip lines starting from A and B, as shown in Fig. 6. The β -angle represents the inclination of the β -line starting from B and is initially calculated on the basis of its stress boundary conditions but must be reduced of the friction angle contribution estimated as $\beta' = \alpha \tan(\mu)$, where μ is the friction coefficient. Accordingly, if no friction is considered in the calculation, Fig. 6a, the two segments AB and BC result in having the same length. However, if friction is accounted for, as in Fig. 6b, the effect is a sort of rotation of the slip line field towards the onset of the deformation gap. This issue has the effect of increasing the stress boundary conditions on A and B, which results in a higher stress state in the plastically deformed region of the material, thus in a higher process force. The relationship between α and β also affect the two angles of the two adjacent fans DAC and CBE according to the following Eq. (1), derived from the first Hencky's theorem (Appendix A).

$$\theta - \psi = \pi/4 + \alpha - \beta \quad (1)$$

Following the same nomenclature for the points of the slip line field shown in Fig. 4, but changing the shape of the contact zone according to the real one between tools and ring in the ring rolling process, in the following three sections of the paper the algorithms through which to build the slip line network are detailed.

4.1. Construction of the ABC region (dead metal zone)

In the present research work, as in the previous literature works [21,23,24], the triangular region ABC is defined as dead metal zone, a portion of the material that is deformed and afterward rigidly moves toward the exit of the deformation gap. In addition to that, on AB, the friction stress will vanish in the neutral point, which is not known a priori, thus no boundary can be defined on the AB curve, except for its two extremity points A and B. In order to simplify the calculation, no slip-page is assumed between the tool and ring in both deformation gaps, allowing defining AB as a sticking region, where the relative velocity between ring and tool vanishes.

In the radial-axial ring rolling process, the contact curve between tools and ring, in both deformation gaps, can be considered as that shown in Fig. 7, where the trajectory of the contact curve in A (entry point of the gap) forms an β_1 angle with the horizontal plane whereas, in B (exit point of the gap), the angle $\beta_2 = 0$, being the trajectory aligned with the horizontal plane. In addition to that, based on the stress boundary conditions defined on A and B, the initial angles of the slip lines starting from these two points, referred to the tangent to the trajectory in those points, can be defined as $\vartheta_{A,1}$ and $\vartheta_{B,1}$, respectively.

In order to create the triangular-like region ABC, the α -line starts from A and the β -line from B have to intersect themselves in the point C forming a $\pi/2$ angle. For this reason, each one of them can be considered as an arc of circumference starting at point A, for the α -line, and in B for β -line. The radii of these arcs are orthogonal to the line inclined,

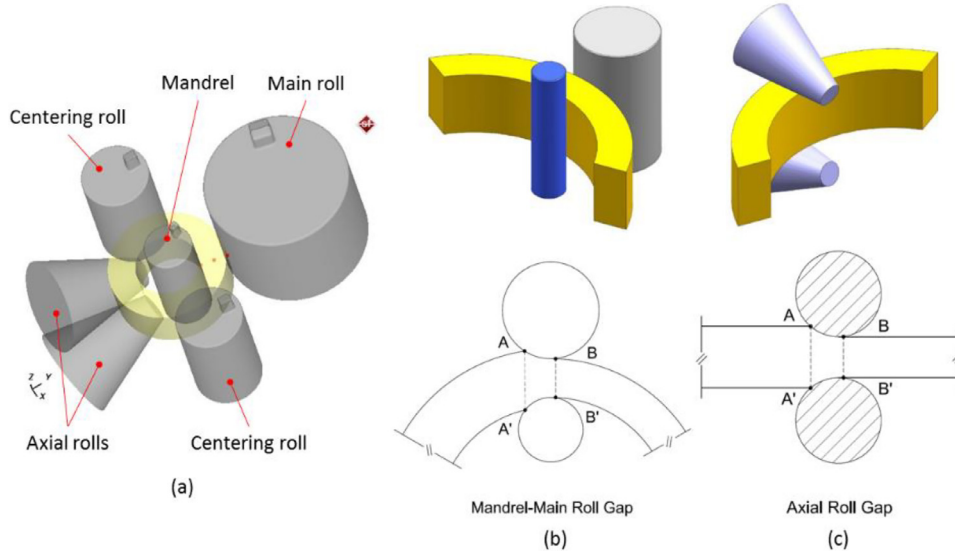


Fig. 5. Radial-axial ring rolling schematic representation (a) and contact geometry between ring and tools in the (b) mandrel-main roll gap and (c) in the axial rolls gap.

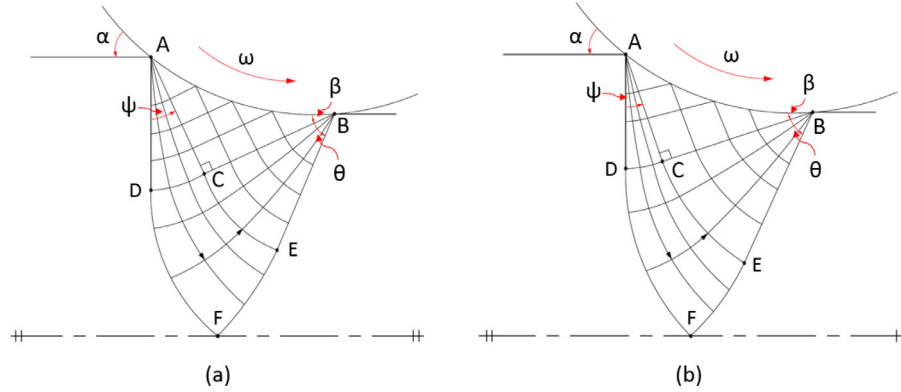


Fig. 6. Influence of the friction on the shape of the slip line field. (a) Without and (b) with friction fields.

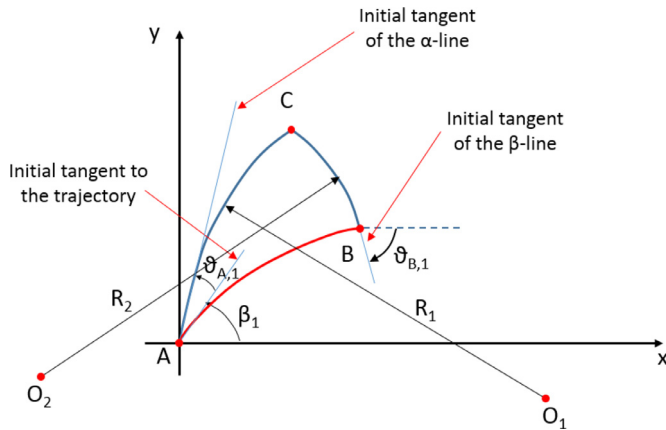


Fig. 7. Initial slip line trajectory from A and B points.

in their relevant starting point, of an $\beta_i + \vartheta_{j,i}$ angle, therefore the line where the center of arc O_i lays is determined. The subscript “ i ” refers to the counter for the lines starting from one point of the slip line domain, namely $i = 1 \dots n$, whereas “ j ” stands for the point name counter, namely $j = A$ or B .

The radii of these arcs, R_1 and R_2 in Fig. 7a, are unknown but they can be related to the slip lines angles $\vartheta_{A,1}$ and $\vartheta_{B,1}$ by an inverse pro-

portionally, the wider the angle the smaller the relevant radius. In the point C, these two arcs form a $\pi/2$ angle, which means that the triangle $O_1 O_2 C$ is rectangular in $O_1 C O_2$, with the two cathouses having lengths equal to R_1 and R_2 . In order to trace these two arcs, their relevant centers, O_1 and O_2 , have to be determined considering that, by construction, $R_1^2 + R_2^2 = O_1 O_2^2$, and that, the following correlation is considered between radii and inclination angles, namely $R_1/R_2 = \vartheta_{B,1}/\vartheta_{A,1}$. The coordinates of the two centers, O_1 and O_2 , can be defined based on the coordinates of A and B and on the β_i and $\vartheta_{j,i}$ angles, as shown in Eqs. (2) and (3), respectively, where $\vartheta_{B,1} < 0$ angle.

$$O_1 = \begin{cases} x : A_x + R_1 \sin(\vartheta_{A,1} + \beta_1) \\ y : A_y - R_1 \cos(\vartheta_{A,1} + \beta_1) \end{cases} \quad (2)$$

$$O_2 = \begin{cases} x : B_x - R_2 \sin(\vartheta_{B,1}) \\ y : B_y - R_2 \cos(\vartheta_{B,1}) \end{cases} \quad (3)$$

Considering the relation between R_1 , R_2 and $O_1 O_2$, and Eqs. (2) and (3), the equation system, in the two unknown variables R_1 and R_2 shown in Eq. (4), can be written. Being a second-order polynomial equation, the solution which satisfies $R_1 + R_2 > O_1 O_2$ is that which should be considered since the other one does not have physical meaning. The parameters P, Q, M and N in (13) are defined as $P = A_y - B_y$, $Q = A_x - B_x$, $M = (\vartheta_{B,1}/\vartheta_{A,1}) \cos(\beta_1 + \vartheta_{A,1}) - \cos \vartheta_{B,1}$ and $N = (\vartheta_{B,1}/\vartheta_{A,1}) \sin(\beta_1 + \vartheta_{A,1}) + \sin \vartheta_{B,1}$, respectively.

In C, the triangular shape formed by R_1 , R_2 and $O_1 O_2$ has the configuration shown in Fig. 8 thus, once the coordinates of the two centers

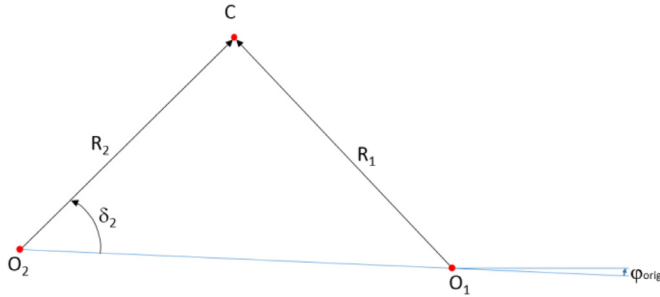


Fig. 8. Geometry of the dead metal zone.

O_1 and O_2 are known, also ϕ_{orig} can be calculated as in Eq. (5).

$$\begin{cases} R_2 = \left[2PM - 2QN \pm \sqrt{(2PM - 2QN)^2 - 4(N^2 + M^2)(Q^2 + P^2)} \right] \frac{1}{2(N^2 + M^2)} \\ R_1 = R_2 \frac{|\theta_{B,1}|}{|\theta_{A,1}|} \end{cases} \quad (4)$$

$$\phi_{orig} = \arctan \left(\frac{O_{1,y} - O_{2,y}}{O_{1,x} - O_{2,x}} \right) \quad (5)$$

Accordingly, also the angle δ_2 and the coordinates of the point C can be calculated according to Eqs. (6) and (7), respectively. At this point of the calculation, the position of the point C has been calculated based on the intersection of the two slip lines starting from A and B, allowing to determine the first portion of the slip line field.

$$\delta_2 = \arccos \left(\frac{R_2}{|O_2O_1|} \right) \quad (6)$$

$$\begin{aligned} C_x &= R_2 \cos(\delta_2 + \phi_{orig}) + O_{2,x} \\ C_y &= R_2 \sin(\delta_2 + \phi_{orig}) + O_{2,y} \end{aligned} \quad (7)$$

4.2. Construction of the ADC and BCE fans regions

The regions of the left and on the right of the dead metal zone are defined, in the present research work, as fans. For the construction of these two portions of the slip line domain, an additional consideration regarding the hydrostatic pressure $\bar{\sigma}$ should be given. In the case of compressive deformation of materials, the boundary of the slip line network is defined as the border, in the plain strain-section of the ring, where the hydrostatic pressure becomes zero and afterward positive. Although the mathematical solution, according to the governing equation, can be calculated, it does not have physical meaning, since the material is undergoing a compression deformation thus no tension stresses should arise. The width of the two regions on the left and on the right of the ABC dead metal zone depends on the pressure boundary conditions between tool and workpiece which are not known a priori.

For this reason, the developed algorithm is designed in order to determine the increment of the ϑ angle that makes the relevant hydrostatic pressure to become zero and, accordingly, to subdivide into equi-spaced sub-sectors the so-determined portion of the domain, allowing to construct the two fans with a chosen discretization angle. By definition, each slip line of the domain must satisfy the Hencky's equations for both the α -line and the β -line (Appendix B), meaning that the sum, with a sign, of hydrostatic pressure and the $2k\vartheta$ factor is constant along a slip line. Considering the β -line exiting from point C and reaching a generic point D where the hydrostatic pressure vanished ($\bar{\sigma}_D = 0$), this point represents the boundary between compression and tension stress solutions and only compressive solutions are considered to be physically acceptable. Applying the Hencky's equation for the β -line to point C and point D, the absolute angle in point D is determined as in ((8)), limiting the extension of the left fan.

$$\vartheta_D = \vartheta_C + \frac{\bar{\sigma}_C}{2k} \quad (8)$$

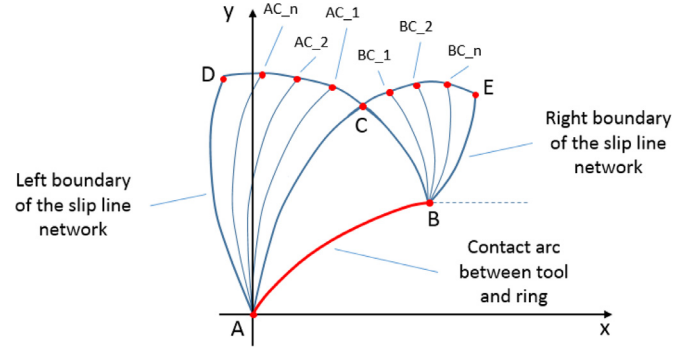


Fig. 9. Final configuration of the slip line field after the creation of right and left fans.

The same rationale can be applied to the α -line exiting from point C and reaching a generic point E, where the hydrostatic pressure vanishes as well ($\bar{\sigma}_E = 0$). Applying the Hencky's equation for the β -line to point C and point E, the absolute angle in point E is determined as in (9), limiting the extension of the right fan. The boundary of the two fans, after the relevant construction, is shown in Fig. 9.

$$\vartheta_E = \vartheta_C - \frac{\bar{\sigma}_C}{2k} \quad (9)$$

The difference between ϑ_C and ϑ_D , as well as that between ϑ_C and ϑ_E , allow determining the angular extension of both left and right fans and, according to a chosen number of subdivisions n_{subdiv} , the discretized slip line network can be constructed. The slip line network, for each fan, is represented by arcs centered in fan center (point A for left fan, point B for right fan) whose shape is obtained by rigid rotation around the fan center of an incremental angle calculated as in (10), for the left fan and for the right fan, respectively, as shown in Fig. 9.

$$\Delta\vartheta_1 = \frac{\vartheta_D - \vartheta_C}{n_{subdiv}}; \quad \Delta\vartheta_2 = \frac{\vartheta_E - \vartheta_C}{n_{subdiv}} \quad (10)$$

The coordinates of the crossing points between α -lines and β -lines, moving from C to D (or from C to E), are determined as the final point of the vector $\hat{L}_A$ (or $\hat{L}_B$) from the center of the fan to point C, rotated around the center of the fan by the incremental angle calculated in (10).

Based on the Hencky's equations (Appendix B), also the ϑ angle and the hydrostatic pressure $\bar{\sigma}$, for all the points on the β -line from C to D and on the α -line from C to E, can be calculated.

The set of equations for the left fan, related to the β -line passing through BCD, is given in (11) whereas that for the right fan, related to the α -line passing through ACE, are reported in (12). In (11) and (12), the subscript "j" stands for the point counter and starts from $j=0$ in the point C to $j=n$ in the point D and to $j=-n$ to the point E. Finally, λ_{CD} and λ_{CE} , define the angle between the $\hat{L}_A$ and $\hat{L}_B$ vectors with respect to the horizontal axis.

$$\begin{aligned} \hat{L}_A &= \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}; \quad \lambda_{AC} = \arctan \left(\frac{y_C - y_A}{x_C - x_A} \right) \\ x_{AC_j} &= \hat{L}_A \cos(\lambda_{AC} + j\Delta\vartheta_1) + x_A \\ y_{AC_j} &= \hat{L}_A \sin(\lambda_{AC} + j\Delta\vartheta_1) + y_A \end{aligned} \quad (11)$$

$$\begin{aligned} \vartheta(\alpha)_{AC_j} &= \vartheta_j + j\Delta\vartheta_1 \\ \bar{\sigma}_{AC_j} &= \bar{\sigma}_j - j\Delta\vartheta_1 2k \\ \hat{L}_B &= \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}; \quad \lambda_{BC} = \arctan \left(\frac{y_C - y_B}{x_C - x_B} \right) \\ x_{BC_j} &= x_B - \hat{L}_B \cos(\lambda_{BC} + j\Delta\vartheta_2) \\ y_{BC_j} &= y_B - \hat{L}_B \sin(\lambda_{BC} + j\Delta\vartheta_2) \\ \vartheta(\alpha)_{BC_j} &= \vartheta_j + j\Delta\vartheta_2 \\ \bar{\sigma}_{BC_j} &= \bar{\sigma}_j - j\Delta\vartheta_2 2k \end{aligned} \quad (12)$$

4.3. Construction of the DCEF edge region

Based on the solutions, in terms of hydrostatic pressure $\bar{\sigma}$ and ϑ angle calculated on the CD and CE curves, the remaining portion of the slip

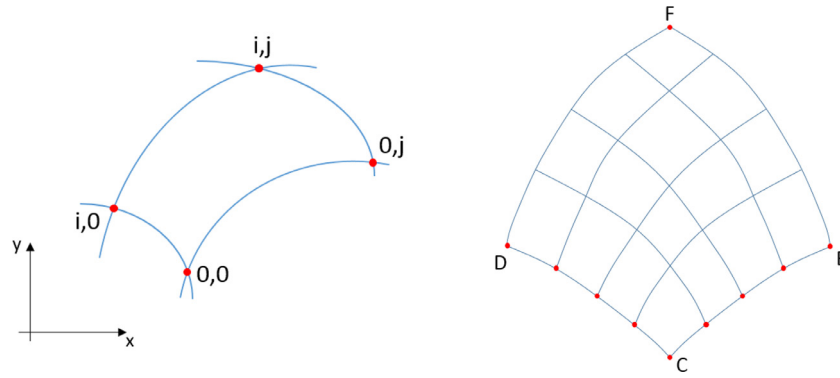


Fig. 10. (a) Generic quadrilateral element in the slip line network and (b) determination of the end-point F from the solution.

line network can be constructed following the procedure detailed in this paragraph.

The importance of the final region of the slip line field is mainly represented by the position of the final point F, which is the control point for the convergence criteria of the method, as it will be explained in the following section of the paper. By applying the Hencky's equations (Appendix B) to a generic quadrilateral element bounded by two pairs of slip lines, as shown in Fig. 10a, it is possible to determine the correlation between the hydrostatic pressure on these four points, ((13) and (14), as well as that between the relevant ϑ angles, as shown in Eq. (15).

$$\bar{\sigma}_{i,j} - \bar{\sigma}_{0,0} = (\bar{\sigma}_{i,j} - \bar{\sigma}_{i,0}) + (\bar{\sigma}_{i,0} - \bar{\sigma}_{0,0}) = 2k(2\vartheta_{i,0} - \vartheta_{i,j} - \vartheta_{0,0}) \quad (13)$$

$$\bar{\sigma}_{i,j} - \bar{\sigma}_{0,0} = (\bar{\sigma}_{i,j} - \bar{\sigma}_{0,j}) + (\bar{\sigma}_{0,j} - \bar{\sigma}_{0,0}) = 2k(\vartheta_{i,j} + \vartheta_{0,0} - 2\vartheta_{0,j}) \quad (14)$$

$$\vartheta_{i,j} = \vartheta_{i,0} - \vartheta_{0,0} + \vartheta_{0,j} \quad (15)$$

If the ϑ angle is known for three out of four points on two pairs of slip lines enclosing a portion of material, for instance, point $(0,0)$, $(i,0)$ and $(0,j)$, that of point (i,j) can be calculate according to (15) as well as the radii, to trace the relevant α -line from $i,0$ and the β -line from $0,j$, are determined according to Eq. (16). In (16), $R_{\alpha-i,j}$ and $R_{\beta-i,j}$ represent the radii of the arcs, starting from $i,0$ and from $0,j$, whose intersection allows determining point i,j . The coordinates of the generic point i,j of the domain are calculated starting from those of the adjacent ones, as shown in Eq. (17).

$$R_{\alpha-i,j} = \frac{1}{\tan[(\vartheta_{i,j} + \vartheta_{i,0})/2]} \quad (16)$$

$$R_{\beta-i,j} = \frac{1}{\tan[(\vartheta_{i,j} + \vartheta_{0,j})/2]}$$

$$x_{i,j} = -(y_{i,0} - y_{0,j} - R_{\alpha-i,j}x_{0,j} - R_{\beta-i,j}x_{i,0}) \frac{1}{(R_{\alpha-i,j} + R_{\beta-i,j})} \quad (17)$$

$$y_{i,j} = y_{i,0} + (x_{i,j} - x_{i,0}) \tan\left(\frac{\vartheta_{i,j} + \vartheta_{i,0}}{2}\right)$$

Finally, based on (13) and (14), also the hydrostatic pressure for the generic point i,j of the domain, namely $\bar{\sigma}_{i,j}$, can be calculated. By iteratively applying this procedure, section by section, the DCE curve, the final part of the slip line network can be build, allowing to determine the position of the point F, as shown in Fig. 10b.

5. Slip line algorithm utilization

The developed algorithm, as presented in the previous section of the paper, has been implemented in an MS Excel spreadsheet and has been utilized for the calculation of both radial and axial forming force in the ring rolling process. However, it can easily be implemented also in already-existing subroutines or in different spreadsheets o programs and, it can potentially be extended to different metal forming and forging processes where similar interactions between tools and workpiece can be identified.

For the calculation of the process force, an additional parameter is defined as the ratio between the stress boundary conditions of the points A and B, assumed equal, and two times the yield shear stress under plane strain assumption, namely $2k$. This ratio is defined as pressure factor, identified with the letter γ , and it is considered to be the solution of the algorithm, to be inputted in the force model.

In the developed algorithm, starting from $\gamma = 1$, the pressure boundary conditions applied to A and B are gradually increased until the two stress field, one starting from the upper roll and one from the lower roll, intersect in one point. Since the two slip line networks, representing the two stress fields starting from the contact arcs between tools and workpiece touch in one point, the whole thickness of the workpiece is interested by plastic deformation, allowing a uniform expansion of the ring. This progressive application of increasing pressure boundary conditions on the two sides of the workpiece is shown in Fig. 11, from (a) to (d). In Fig. 11a, $\gamma = 1$, represents the initial state with the initial boundary conditions whereas in Fig. 11d, $\gamma = 2.39$, identifies the point where the two slip line networks touch at one point and is identified as the solution for the considered case, taken as an example. The two intermediate steps represent examples of how the algorithm progressively increases the stress boundary conditions of both top and bottom, onset and exit, contact points, in order to determine the solution.

For the computation of the process force, the pressure factor is inputted in the force model Eq. (18), for the mandrel-main roll deformation gap, and (19), for the axial rolls gap. The difference between these two equations is given by the calculation of the contact area between tools and ring. In the first case, the projection of the contact arc is calculated according to [26] formulation, as reported in (20) whereas the height of the ring based on authors' previous model [7]. For the latter one, the contact arc is calculated according to authors' algorithm presented in [4] whereas the thickness of the ring same according to authors' previous work [7].

$$F = 2k\gamma\bar{\sigma}L_{C,rad} \quad (18)$$

$$F = 2k\gamma\bar{h}L_{C,axi} \quad (19)$$

$$L_{C,rad} = \sqrt{\frac{2\Delta s}{\left(\frac{1}{R_S}\right) + \left(\frac{1}{R_M}\right) + \left(\frac{1}{R}\right) - \left(\frac{1}{r}\right)}} \quad (20)$$

For the validation of the proposed model, laboratory experiments have been carried out on Pb–Sn soft alloys and the results of radial and axial forming forces have been compared both with the analytical prediction and with numerical simulation implemented in Simufact.Forming 14. Additional simulations study cases have been also carried out in order to prove that the developed algorithm is suitable for the prediction of both radial and axial forming force either for relatively small or large rings, with the outer diameter ranging from 800 mm to 2000 mm.

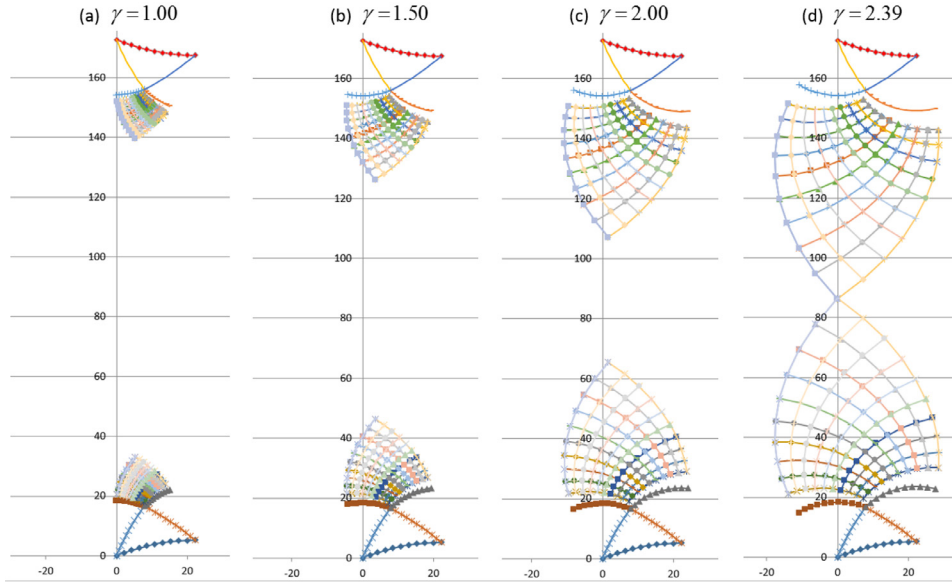


Fig. 11. Iterative calculation of the pressure factor, graphical representation.

6. Numerical model implementation

In the present research work, numerical simulations have been carried out by utilizing the ring rolling module available in the commercial software Simufact.Forming 14. For all the numerical simulations, a full dynamic-explicit thermo-mechanical simulations model has been employed. Dies have been considered as rigid with heat exchange and modeled with the 55NiCrMoV6 tool steel whereas the environment temperature has been set to 50 °C.

In the numerical simulations, as previously done in other authors' works [4,7,18], the shear stress friction model has been utilized, Eq. (21), setting the friction factor to $m = 0.85$ for the contact between the ring and both the mandrel and the main roll and to $m = 0.6$ for the contact between the ring and both axial rolls and centering rolls. The same friction factors have been utilized both in the numerical simulations and for the setting of the analytical model. The definition of these values for the friction between tools and rings during the process has been carried out according to the range made available in the work of Sun et al. [27] and considering that the friction in the mandrel-main roll deformation gap is higher than that between in the axial rolls gap due to the fact that the thickness draft is normally largely bigger than the height draft. For this reason, the heavy contact conditions normally lead to the arising of local descaling on the ring surface those make the friction to have a strong influence on the process. Differently, in the axial rolls gap, due to a lower deformation, the arising of surface descaling on the ring top and bottom surfaces is more unlikely to happen thus a lower friction factor has been set. Concerning the side rolls, their contribution in the process is only to avoid any misalignment between the center of the ring and the tools of the ring rolling mill thus their contact with the ring is normally just instantaneous and they are normally well lubricated.

These fact are accounted in the numerical model by setting a higher friction factor in the contact between ring and mandrel and main roll and a lower one for the contact with axial and sided rolls.

In addition to that, the concept of utilizing a constant friction factor has been already employed in the work of Parvizi et al. [5].

$$m = \frac{\tau_i}{k} \quad (21)$$

The utilized solver is the MUMPS (MULTifrontal Massively Parallel sparse direct Solver) direct solver which performs a Gaussian factorization and is particularly indicated for the case of large dimensions,

square, sparse matrixes. The node position evolution is controlled according to the updated Lagrangian approach, suitable in case of large deformation, as in the ring rolling process. FE models were all meshed with 3D arbitrarily distorted 8-nodes brick hexahedral elements. In the validation study case simulation with the following element size: 4.0 mm (axial), 4.0 mm (radial), 4.0 mm (tangential). Instead, in the study cases presented in the results section, with the following element dimensions: 16.0 mm (axial), 12.0 mm (radial), 16.0 mm (tangential).

The difference in the mesh size is justified by the larger dimensions of the rings of the study cases in comparison to the validation one. In both simulation models, additional numerical simulations have been carried out considering larger and smaller mesh sizes, considering the same ratios between axial, radial and tangential element sizes, in order to avoid any influence of the mesh in the quality of the results. The chosen element sizes are those which assured the best compromise between computational time and accuracy of the results.

Concerning the contact, a node is considered to be in contact with a surface if its position is less than 1/20 the side length of the smallest element in the mesh. In addition, if the relative velocity between two elements is less than 0.001 mm/s, a sticking condition between them is considered. In the numerical models, two different convergence criteria for two consecutive iterations of the calculation are adopted: the first one on the displacement, with a threshold of 0.25 mm and a second one on the nodal force, with a threshold of 0.35 N.

In the numerical model, for each tool of the ring rolling mill (see Fig. 5a for the reference names) different degrees of freedom have been set, as schematically summarized in Table 1 where "X" means fixed, or no allowed movements, "O" means free to move, or neither fixed nor controlled by any press (i.e. tabular motion) whereas "T" means controlled by a press system (i.e. tabular motion). The two centering rolls are set in order to stay in contact with the ring during its expansion and to release contact in case the contact pressure exceeds 50 MPa.

The backward movement of the axial rolls along the y-direction is controlled by the expansion of the ring and it is set in order to always keep the contact point between inner surface of the ring and axial rolls always in the position they have at the beginning of the process. The feeding laws for mandrel and upper axial rolls for the 9 study cases are reported in Section 8 of the paper.

In relation to the material utilized for the nine study cases, with the exception of the validation one, the 42CrMo4 steel alloy properties have been utilized. The material model, according to GMT-Matilda®, de-

Table 1
DoF of the implemented numerical model.

Tool	X-transl.	Y-transl.	Z-transl.	X-rot.	Y-rot.	Z-rot.
Main roll	X	X	X	X	X	T
Mandrel	X	T	X	X	X	T
Upper axial roll	X	T	T	X	T	X
Lower axial roll	X	T	X	X	T	X
Centering roll 1 (right)	T	T	X	X	X	O
Centering roll 2 (left)	T	T	X	X	X	O

Table 2
42CrMo4 alloy material model constants and validity ranges.

Parameters	Value
Temperature range for the model (°C)	800–1250
Strain range for the model (–)	0.05–2
Strain rate range for the model (1/s)	0.01–150
C_1	5290.47
C_2	–0.0036967
n_1	–0.000334025
n_2	0.20612
L_1	–8.26584e–5
L_2	0.0289085
m_1	0.000300752
m_2	–0.156181

Table 3
Ring rolling mill characteristics, process settings, and 42CrMo4 material information.

Parameters	Value
Radius of the main roll (mm)	325
Radius of the mandrel (mm)	125
Length of the axial rolls (mm)	595.9
Half of cone vertex angle (°)	17.5
Temperature of the environment (°C)	50
Tool temperature (°C)	150
Density (kg/m ³)	7847
Specific heat capacity (J/kg K)	According to Fig. 12c
Emissivity (–)	0.25
Heat transfer coefficient (conduction) (W/m ² K)	20,000
Heat transfer coefficient (convection) (W/m ² K)	50

rived from the Hansel–Spittel formulation, is shown in Eq. (22) whereas the relevant model constants are reported in Table 2. In the analytical model, the value of the flow stress calculated according to Eq. (22) has divided by $\sqrt{3}$, according to the von Mises yield criterion, for the calculation of the yield shear stress.

$$\sigma = C_1 e^{(C_2 \cdot T)} \epsilon^{(n_1 \cdot T + n_2)} e^{\left(\frac{L_1 \cdot T + L_2}{\epsilon}\right)} \dot{\epsilon}^{(m_1 \cdot T + m_2)} \quad (22)$$

The dimensions of the ring rolling mill considered for the nine study cases, those results are presented in Section 8 of the paper, are summarized, together with additional material properties and process conditions, in following Table 3 and Fig. 12. The material properties of the 42CrMo4 steel alloy have been acquired from the MATILDA® (Material Information Link and Database Service) database available in Simufact.Forming 14.

The same material model equation, material model constants, ring rolling mill geometry and process conditions have been inputted both in analytical and numerical models, in order to have a fair comparison between the results. In addition, in all the simulations, the ring has been initially heated up to 1200 °C and no intermediate re-heating operations have been considered.

Table 4
Material model constants for the Pb75–Sn25 material.

Parameters	Value	Parameters	Value
K_0	92	b_0	0.015
a_0	0.1	b_1	0.17
a_1	0.03		

Table 5
Ring rolling test machine data, ring dimensions, and process conditions.

Parameters	Value
Mandrel diameter (mm)	50
Main roll diameter (mm)	200
Axial rolls working plane length (mm)	190
Axial rolls vertex angle (°)	15.166
Guide rolls diameter (mm)	60
Main roll rolling speed (rpm)	60
Mandrel maximal pushing force (kN)	55
Upper axial roll maximal pushing force (kN)	25
Ring initial outer diameter (mm)	155
Ring initial inner diameter (mm)	105
Ring initial height (mm)	42
Ring final outer diameter (mm)	195
Ring final inner diameter (mm)	153
Ring final height (mm)	37
Ring initial temperature (°C)	25
Mandrel initial feeding speed (mm/s)	0.05
Mandrel final feeding speed (mm/s)	0.11
Process time (s)	80

7. Validation experiments

For the validation of both the developed analytical model and the numerical simulation implemented in Simufact.Forming 14, study cases have been carried out on rings made of the soft alloy Pb75–Sn25 utilizing the laboratory size ring rolling machine shown in Fig. 13.

The material properties of the Pb75–Sn25 alloy have been determined, at room temperature process conditions and for different strain rates, by means of laboratory compression test experiments and the relevant flow curves are shown in Fig. 14. The plastic behavior of the material has been described utilizing the formulation defined by Kim et al. [14], reported in (23) and the relevant material model constants, derived from the inverse calibration of the experimental results, are listed in Table 4. The dimensions of the laboratory ring rolling machine and the relevant process settings are listed in Table 5.

$$\sigma = K_0 (a_0 + \epsilon)^{a_1} (b_0 + \epsilon)^{b_1} \quad (23)$$

For a fair comparison among the results, the process settings utilized during the laboratory experiments have been utilized for the setting of both the analytical model and numerical simulation. In the analytical model, the flow stress is analytically calculated according to the estima-

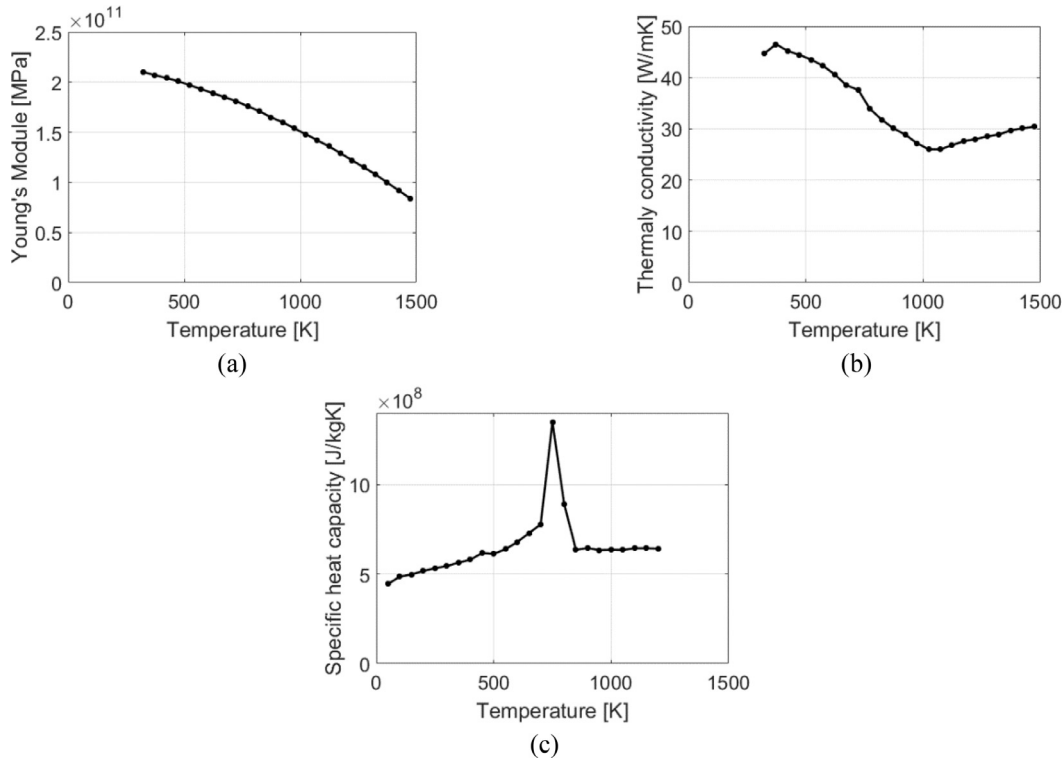


Fig. 12. Thermal properties of the 42CrMo4 steel alloy.



Fig. 13. Laboratory-size ring rolling machine utilized for the experiments.

tion of strain, strain rate and temperature carried out by utilizing authors' previous works [18,4]. In the numerical model, the flow curves have been directly inputted into the material property section.

In order to enhance the strength of the validation of the developed numerical model, by utilizing the previous authors' analytical model for the estimation of the ring geometry expansion during the process [7], the outer diameter of the ring has been estimated throughout the process and the results have been compared with those of the relevant experiment and numerical simulation, for the validation study case. The cross-comparison of the results is shown in Fig. 15.

In the beginning of the process, the numerical simulation and the experiment data matches well whereas the analytical model has a steepest slope. This fact is attributed to the slippage which normally occurs in the first stages of the ring rolling process, those cannot be taken into account by the analytical model but is well simulated by the numerical one. Concerning the estimation of the radial and axial forming force for the validation case, the prediction obtained with the analytical model

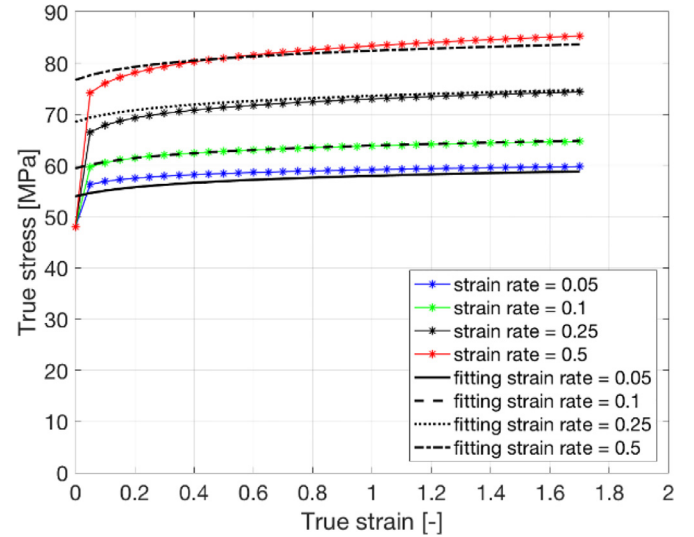


Fig. 14. Pb75-Sn25 alloy flow curves.

presented in this paper have been compared with the results of both numerical simulation and laboratory experiments and are presented in Figs. 16 and 17.

In the estimation of both radial and axial forming force, although the shape of the curve resulting from the predictions are different in the beginning and in the end of the process time, mainly due to slippage and dynamic effect which cannot be taken into account by the analytical formulation, the developed algorithm is able to catch both the trend of the forming force curves throughout the rolling time as well as predict, in a precise way, the maximum value of the relevant forces. The cross-comparison of the results for the validation case are presented in following Table 6, where the experimental value has been taken as refer-

Table 6
Ring rolling test machine data, ring dimensions, and process conditions.

	Value at 20s	Value at 40s	Value at 60s	Value at 80s
Ring outer diameter				
Experiment (mm)	161.37	175.99	188.94	194.72
Analytical prediction (mm)	164.81	177.11	190.64	195.00
% Error	-2.13	-0.64	-0.90	-0.14
Numerical simulation (mm)	162.17	175.18	189.99	194.52
% Error	-0.50	0.46	-0.56	0.10
Radial forming force				
Experiment (kN)	28.99	37.62	37.77	33.68
Analytical prediction (kN)	30.51	38.20	36.12	35.37
% Error	-5.24	-1.54	4.37	-5.02
Numerical simulation (kN)	33.09	37.81	39.61	36.35
% Error	-14.14	-0.51	-4.87	-7.93
Axial forming force				
Experiment (kN)	27.09	33.95	35.21	31.55
Analytical prediction (kN)	28.75	35.15	35.39	34.08
% Error	-6.13	-3.53	-0.51	-8.02
Numerical simulation (kN)	27.69	31.73	33.95	35.15
% Error	-2.21	6.54	3.58	-11.41

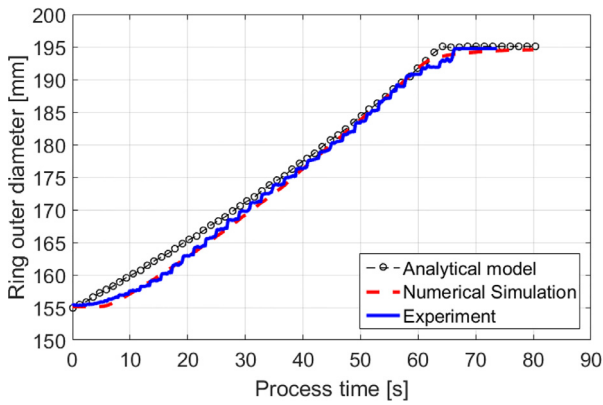


Fig. 15. Cross-comparison between analytical, numerical and experimental data for the outer diameter expansion for the validation study case.

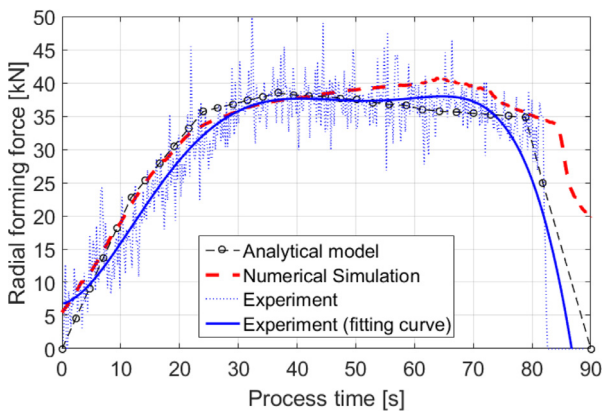


Fig. 16. Cross-comparison between analytical, numerical and experimental data for the radial forming force for the validation study case.

ence one. The errors are calculated by comparing analytical, numerical and experimental results at 20 s, 40 s, 60 s and 80 s of the process time.

Both analytical and experimental model replicate well the trend of the experimental curves and are able to catch the maximum value, located in the center of the curve for the radial and axial force, and in the end of the process time for the outer diameter evolution. At the be-

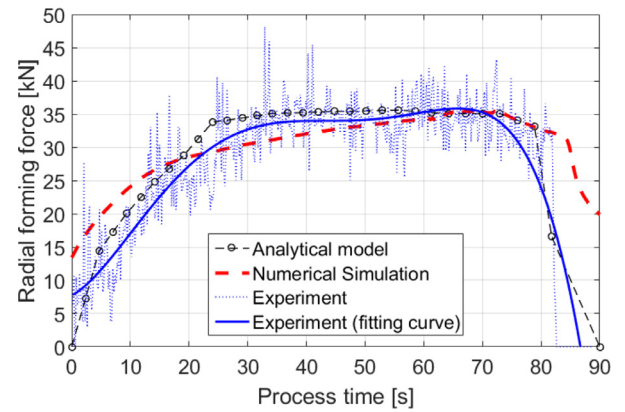


Fig. 17. Cross-comparison between analytical, numerical and experimental data for the axial forming force for the validation study case.

ginning and in the end of the process, mainly due to sliding effects and irregularities which are not replicable neither by the numerical nor by the analytical models, the errors is higher than in the center of the force curves.

8. Simulation study cases settings and results

The developed analytical model has been applied to nine different flat rings study cases, where the outer diameter ranges between 800 mm and 2000 mm. In addition to that, in order to prove that the developed analytical model is able to calculate precise results also for different shapes of the ring cross section, for each considered diameter, three different study cases, with the ratio between ring final height and thickness equal to 1, 2 and 3, have been considered. The summary of the ring pre-form dimensions, calculated according to the algorithm detailed in the authors' previous work [7], and ring final dimensions, are summarized in Table 7.

For the nine study cases, mandrel and upper axial roll feeding speed have been chosen in the feasible process ranges defined according to authors' previous work [7] and are summarized in following Table 8 along with the process times. The initial and final values of the mandrel feeding speed have been utilized as initial and final values of a linear decreasing mandrel feeding speed curve. In Table 8, the process time

Table 7
Ring study cases initial and final geometry.

Case	Ring initial geometry (mm)			Ring final geometry (mm)		
	Outer diam.	Inner diam.	Height	Outer diam.	Inner diam.	Height
1	518	325	195	800	680	180
2	551	325	175	800	645	145
3	575	325	125	800	600	100
4	822	600	215	1400	1270	195
5	876	600	208	1400	1220	180
6	918	600	159	1400	1155	123
7	876	600	226	2000	1875	190
8	945	600	232	2000	1820	180
9	1049	600	225	2000	1700	150

Table 8
Ring study cases initial and final geometry.

Case	Mandrel feeding speed (mm/s)		Upper axial roll feeding speed (mm/s)		Process time (s)
	Initial	Final	Initial	Final	
1	7.70	6.00	3.82	2.00	14.49
2	7.50	5.00	4.83	2.50	14.82
3	6.00	4.50	6.00	4.50	13.90
4	6.00	3.90	3.10	1.30	18.79
5	5.50	3.50	3.65	1.75	20.28
6	4.50	3.00	4.50	3.00	19.37
7	5.50	2.70	3.35	0.89	27.97
8	5.00	2.70	3.71	1.35	31.09
9	4.70	2.70	4.70	2.70	30.02

Table 9
Ring study cases maximum radial and axial forming forces numerical results.

Case	FEM (radial) (kN)	FEM (axial) (kN)
1	1370	364
2	1152	503
3	717	459
4	1241	472
5	1201	637
6	852	754
7	1300	643
8	1417	851
9	1476	127

also included the calibration time, relevant for 2 calibration rounds. In all the cases, the rotational speed of the main roll has been set to 3 rad/s.

All the numerical simulations have been run on an i7-6700 3.4 GHz CPU with 16 GB of RAM computer, utilizing a 4 domain and 2 cores for domain model decomposition (parallelized calculation). For the above-mentioned nine study cases, the computational time ranges between 5 h and 14 h.

The results, in terms of maximum radial and axial forming forces, for the nine study cases, are summarized in Table 9. In addition to that, in order to prove the stability of the developed numerical model, the final rings at the end of the process, showing the contour plot of the equivalent plastic strain, are shown in Fig. 18a–c.

9. Slip line model and numerical simulation results comparison

The developed slip line field model has been applied to the nine study cases summarized in the previous section of the paper and the results, in terms of estimation of the maximum values for the radial and the

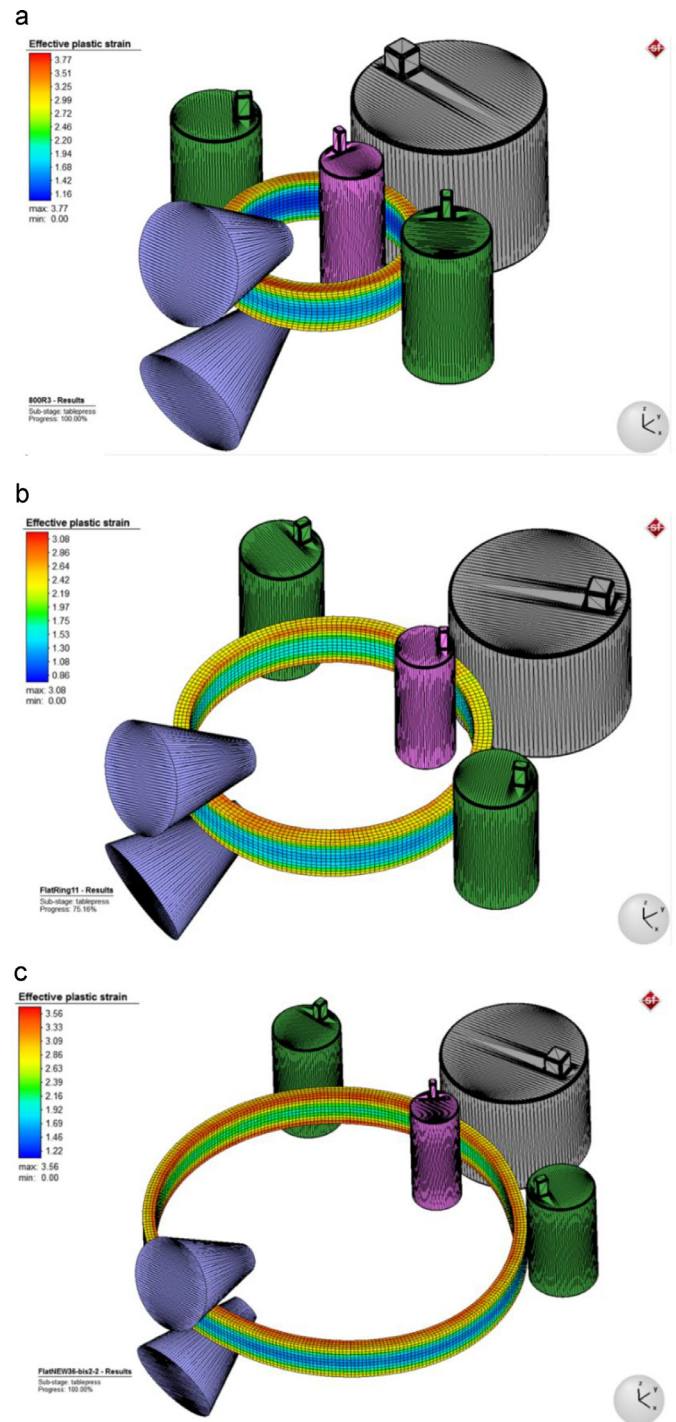


Fig. 18. (a) Equivalent plastic strain at the end of the rolling time for study case 1. (b) Equivalent plastic strain at the end of the rolling time for study case 4. (c) Equivalent plastic strain at the end of the rolling time for study case 7.

axial forming forces, have been compared with those of the numerical simulation, reported in Table 8.

For all the study cases the process forces have been estimated with an interval between 1 s and 2 s, for the whole process time, allowing to identify the maximum values for both forces. The average time for the input into to MS Excel spreadsheet of the required parameters for the calculation of the slip line solution, for each of the selected time steps, has been calculated between 30 s and 40 s. For the calculation of the whole set of data (results of the process forces) for a considered study

Table 10
Ring study cases radial and axial forming force slip line model and errors in comparison to the numerical simulation results.

Case	Slip line model (radial force) (kN)	% Error (radial force)	Slip line model (axial force) (kN)	% Error (axial force)
1	1378	0.61	376	3.19
2	1120	2.78	486	3.22
3	705	1.62	461	0.30
4	1180	4.92	480	1.57
5	1183	1.50	658	3.35
6	881	3.40	738	2.08
7	1350	3.85	605	5.88
8	1472	3.88	863	1.52
9	1541	4.40	1202	5.36

case, the total time varies between 7 min and 9 min, according to the overall length of the ring rolling process.

The summary of the results concerning the maximum radial and axial forming force for the nine analyzed study cases is reported in Table 10, along with the errors in comparison to the results of the numerical simulation, summarized in Table 9.

10. Discussion

As initially stated in the introduction of the paper, the main aim of the research proposed in this paper is the development of a reliable and generalized force model to be utilized for the estimation of radial and axial forming force in the radial-axial ring rolling process of flat rings.

Considering the results presented in Section 7, relevant for the validation experiment carried out on the Pb75–Sn25 soft alloy, and in the previous section, the developed model has shown to be able to catch both the trend as well as the maximum peak of both radial and axial forming force, with a reasonable deviation in comparison to both numerical and experimental results. In addition to that, as proven from the results of the nine study cases, the developed model is able to predict, with a limited error in comparison to the relevant numerical simulation, both the radial and the axial forming force. Especially concerning the three study cases with a final outer diameter equal to 800 mm, the developed model has shown to be an improvement of the previous slip line solution available in the literature. In fact, as demonstrated in the previous authors' work [4], the solution implemented by Hawkyard et al. [2], underestimates the radial forming force for the outer diameter of the ring lower than approximately 1000 mm. Instead, the developed slip line solution is able to predict both the radial and the axial forming force with small variations of the accuracy, as confirmed by the results comparison shown in Table 9.

The comparison carried out on the nine analyzed study cases identifies a maximum error of 4.92% in the estimation of the radial forming force whereas one of 5.88% for the axial force, proving that the developed algorithm is able to effectively predict the maximum forming force for a large variety of ring dimensions. In addition to that, as proven for the validation study case presented in previous section 7, it also allows foreseeing the trend of the forming force along the process. Considering the good agreement between experimental, numerical and slip line model results, as shown for the validation case, likewise the good accordance between slip line model and numerical results shown for the nine study cases, the proposed model is considered to be reliable for the estimation of both radial and axial forming force in the radial-axial ring rolling process of flat rings. In addition to that, as shown for the study cases 1, 2 and 3, it is able to correctly predict the radial forming force also in the case where the previous slip line-based model [2] has shown to underestimate it.

Finally, considering also the computational time, if numerous combinations of process parameters are aimed to be tested, the utilization of the proposed methodology allows sparing a huge amount of computational time, which can be utilized for a more accurate analysis on the

small set of process parameters those have shown to fulfill the force limit requirements, normally related to the available ring rolling mills.

11. Conclusions

In the research work presented in this paper, a slip line based algorithm for the construction of slip line fields in the plastically deformed regions of the material in the ring rolling process is presented in detail and implemented in an MS Excel spreadsheet. Moreover, based on the step-by-step implementation procedure detailed in the paper, its utilization can be extended to every high- to a medium-level programming language.

Taking as solution the value of the stress boundary conditions on the onset and exit contact points between ring and tools, the ratio between this stress-solution value and two times the yield shear stress of the material under plane strain assumptions, is calculated and inputted in the force model equation, for the estimation of the relevant force.

Considering separately the geometry of the two deformation gaps, the mandrel-main roll one and the axial rolls one, two different sets of solutions are calculated throughout the process, allowing to estimate both the trend as well as the maximum values of the radial and axial forming forces. The model has been validated by comparing the results with that of a laboratory experiment carried out by utilizing a Pb–Sn soft alloy. The experimental results also allowed to validate the developed numerical model, implemented in the commercial software Simufact.Forming 14 and utilized for the nine study cases presented in the paper. The comparison between analytical and numerical calculations, in terms of maximum radial and axial forming forces, allowed to conclude that the developed algorithm is able to successfully estimate the radial forming force with an error below 5% and the axial forming force with an error below 6%. Since no specific assumptions have been made during the derivation of the algorithm, the developed slip line procedure can be utilized for the estimation of the forming force in other forging processes where the contact geometry between tools and workpiece can be described as a curved line.

Based on the positive comparison between experimental, numerical and slip line model results, the developed algorithm may be of interest of both academy researcher and industrial process engineers, especially if a large number of the study cases are aimed to be analyzed.

Appendix A. Previous authors' models summary

The algorithm presented in this paper is utilized along with previous authors' algorithms [4,18,7] where models for the estimation of the geometry of the ring, of the strain field of the ring and of the temperature and strain rate during the process have been presented. On the basis of the geometry estimation algorithm [7], the ring is initially subdivided into a certain amount of slices, Fig. A.1, in order to enhance the accuracy of the prediction of its incremental geometry. Since the radial-axial ring rolling pertains to incremental forging processes, the continuous movement of the mandrel towards to the main roll, as well as that of the upper

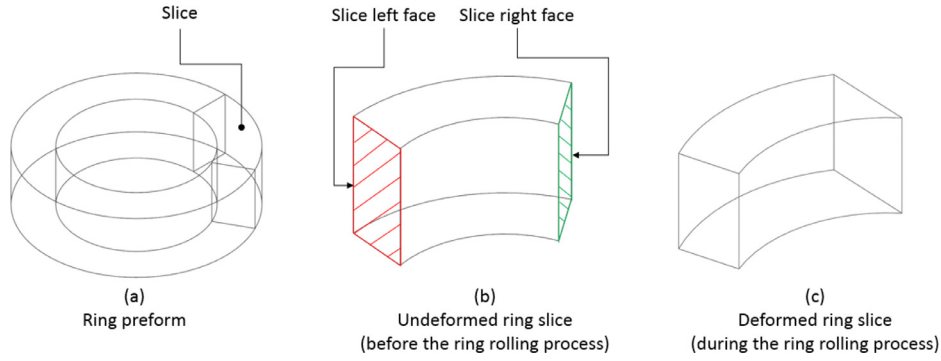


Fig. A.1. Schematic representation (a) of the ring and the slice subdivision, (b) the left and right face of a slice and (c) the deformed slice (with different geometries on the left and right faces).

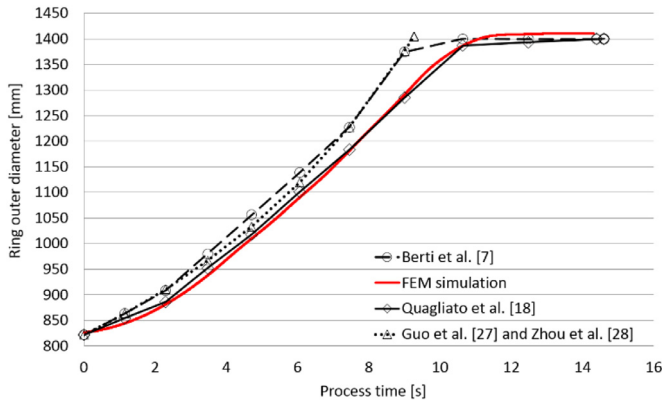


Fig. A.2. Prediction of the outer diameter of the ring according to authors' models [7,18] and previous literature models [28] and [29] (same model).

axial roll towards to lower axial roll, make two consecutive slices of the ring to have different geometrical values. Hence, in order to promote a more efficient and accurate calculation of the geometrical parameters of the ring, namely radii, thickness and height, the incremental geometry is calculated for each slice and for each round of the process. The higher accuracy of the proposed model, in comparison to previous ones, has been shown in [7].

In addition to that, in a more recent work of the authors [18], an improvement to the model proposed in [7] has been proposed, allowing to estimate not only the average geometry of each slice throughout the process but also the different geometries, in terms of radii, thickness and height, for the left and right face of each slice, as shown in Fig. A.1c. An example of the prediction of the outer diameter of the ring throughout

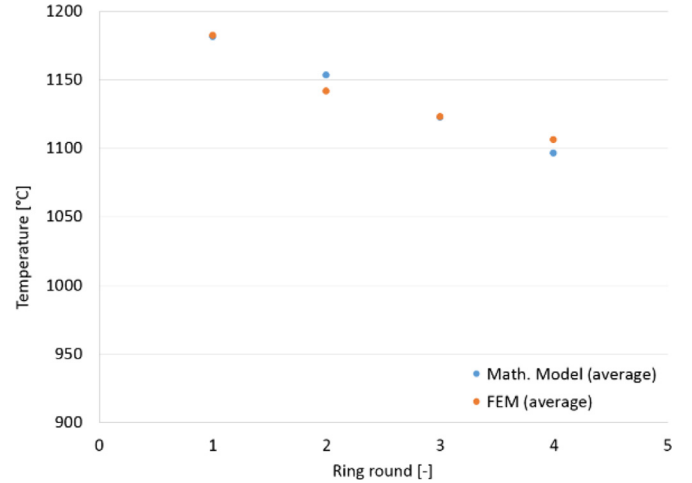


Fig. A.4. Average ring temperature for increasing number of rounds, in comparison to the FEM simulation results.

the process, made by utilizing the authors' initial model [7], authors' improved model [18] and previous literature model [28], in comparison to the relevant numerical simulation, is shown in Fig. A.2.

By utilizing the developed algorithms [7,18], the geometry of the ring throughout the process can be estimated almost in the real time with a deviation lower than 3%, in comparison to the relevant numerical simulation. According to the geometrical estimation of the slice dimensions for each slice of the ring and for each round of the process, the evolution of the strain field in the ring, in terms of strain components along the three principle direction, can be estimated as well, as

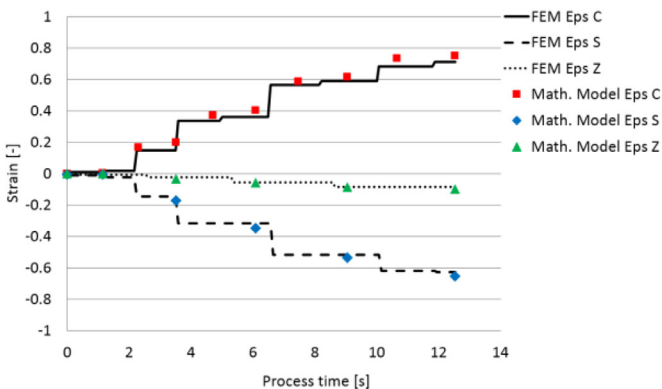


Fig. A.3. Prediction of the 3D strain field of the ring at the (a) inner and (b) outer surfaces of the ring.

shown in Fig. A.3a and b, for the inner and outer surfaces of the ring, respectively.

Finally, in the most recent authors' work concerning the development of analytical and numerical models for the radial-axial ring rolling process [4], an algorithm for the half-round-based estimation of the average temperature of the ring, as a consequence of conductive, convective and heat exchanges, has been proposed and validated. The developed model estimates an average contact temperature for the ring throughout the process and the results have been validated by comparison with those of numerical simulations. In following Fig. A.4 the average temperature of the ring during the expansion phase is estimated and compared with numerical results.

The results shown in Fig. A.2, A.3, and A.4 are related to a ring with initial dimensions equal to 600 mm, 821 mm and 214.7 mm and final dimensions equal to 1270 mm, 1400 mm and 195 mm, for the inner radius, outer radius, and height respectively. Based on the estimation of the three principle strain components, the equivalent plastic strain is directly calculated according to the von Mises criterion. Accordingly, the strain rate is estimated on the basis of the contact times between the tools and each slice of the ring, as defined in [4] and shown in Eq. (A.1).

$$\dot{\epsilon} = \frac{\epsilon_{equiv}}{t_c} \quad (A.1)$$

In Eq. (A.1), the equivalent plastic strain ϵ_{equiv} and the contact time t_c are those relevant for a specific slice passing through one of the two deformation gaps in a specific instant of the process. The full derivations for the estimation of the strain components, as well as of the equivalent plastic strain is detailed in [18] whereas those for the estimation of the contact times, in the two deformation gaps, are reported in [4].

By considering together the analytical estimations for strain, strain rate and temperature, the flow stress of the material flow stress can be analytically estimated for each half-round of the process, following a step-by-step update, as shown in Fig. A.5. For the calculation of the flow stress, the Hansel–Spittel formulation, as shown in previous Eq. (30), has been utilized. By following the proposed approach, the material flow stress is updated at each round of the process, allowing to take into account its variation, especially due to the temperature drop on the ring surfaces.

Appendix B. Slip line field governing equations

One of the first and most complete reviews about the theoretical background concerning the slip line theory is probably due to Hill [3], where also several different applications of the developed slip line algorithm are detailed. In the plastically deformed region of the material, the slip lines represent the directions along which the material flows, those also coincide with those of maximum shear strain rate, and always cross themselves with a $\pi/2$ angle. Considering a network of slip lines, a generic portion of material will be enclosed by two-pairs of slip lines, as shown in Fig. B.1. The counterclockwise angle between the tangent to the α -line, in a considered point of the network, and the x-axis is defined as ϑ .

For a generic portion of material enclosed by two-pairs of slip lines, Fig. B.1, the relevant Mohr circle, under plane strain conditions, is defined as in Fig. B.2 and, accordingly, the two stress and the shear stress components can be written as in Eq. (B.1)–(B.3), where $\bar{\sigma}$, k and 2ϑ represent the hydrostatic pressure, the yield shear strength of the material and the angle between the stress vector and the pure shear direction in the Mohr plane, respectively.

$$\sigma_x = -\bar{\sigma} - k \sin 2\vartheta \quad (B.1)$$

$$\sigma_y = -\bar{\sigma} + k \sin 2\vartheta \quad (B.2)$$

$$\tau_{xy} = k \cos 2\vartheta \quad (B.3)$$

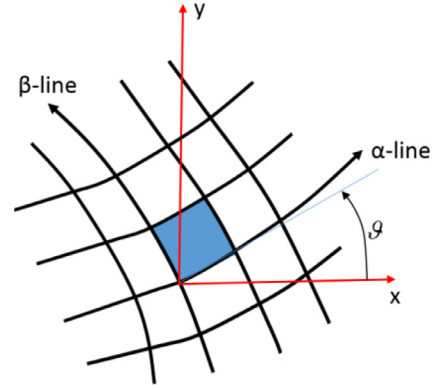


Fig. B.1. Slip line enclosing a generic portion of the material in the plastically deformed region.

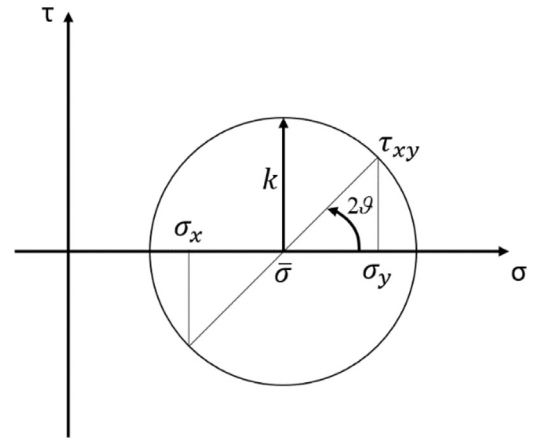


Fig. B.2. Generic Mohr circle for a portion of material enclosed by two-pairs of slip lines.

Under plane strain conditions, the von Mises yield criterion can be written as in (B.4), where Y represents the yield stress of the material under uniaxial tension. In addition to that, neglecting the inertia force, the equilibrium equations, derived from the differential form $\nabla \sigma_{ij} = 0$, are defined as (B.5).

$$\frac{Y}{\sqrt{3}} = \frac{1}{4} (\sigma_x - \sigma_y)^2 + \tau_{xy}^2 = k^2 \quad (B.4)$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad ; \quad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \quad (B.5)$$

Finally, neglecting the elastic contribution and considering only the plastic deformation, the volume conservation principle can be written as in Eq. (B.6), where u_x and u_y define the displacements along the x-direction and the y-direction, respectively.

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \quad (B.6)$$

From the equilibrium Eq. (B.5), it can be immediately verified that in case the stress components along the x-direction and the y-direction are equal, their first-order partial derivative, along their relevant directions, x-direction for σ_x and y-direction for σ_y , are both equal to zero. Substituting the stress state Eqs. (B.1)–(B.3) into $\partial \sigma_x / \partial x = \partial \sigma_y / \partial y = 0$ and considering the direction where the tangent of the α -line and the β -line coincide with x- and y-axis, namely $\vartheta = 0$, it is possible to define a relationship between the variation of the hydrostatic pressure and the angle of the stress vector in the Mohr plane, as defined in (B.7), for the α -line,

and in (B.8), for the β -line.

$$\frac{\partial \sigma_x}{\partial x} = 0 \Rightarrow \frac{\partial \bar{\sigma}}{\partial x} + 2k \frac{\partial \theta}{\partial x} = 0 \quad (\text{B.7})$$

$$\frac{\partial \sigma_y}{\partial y} = 0 \Rightarrow \frac{\partial \bar{\sigma}}{\partial y} - 2k \frac{\partial \theta}{\partial y} = 0 \quad (\text{B.8})$$

The finite forms of differential Eq. (B.7) and (B.8), shown in Eqs. (B.9) and (B.10) respectively, are due to Hencky's and define the equilibrium on the slip line network. Hencky's Eq. (B.9) and (B.10) and equilibrium Eq. (B.5) are, in principle, equal each other but the first ones are defined on the slip line network whereas the second one on the whole body.

$$\bar{\sigma} - 2k\theta = \text{constant on an } \alpha\text{-line} \quad (\text{B.9})$$

$$\bar{\sigma} + 2k\theta = \text{constant on an } \beta\text{-line} \quad (\text{B.10})$$

Once the stress state in one point of the slip line network is known, normally based on the boundary conditions, utilizing (B.9) and (B.10), the stress field, in terms of hydrostatic pressure $\bar{\sigma}$ and angle θ , can be calculated in all the remaining points. One point of the slip line network is defined as the intersection of two slip lines, namely a α -line and a β -line.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at [doi:10.1016/j.ijmecsci.2018.01.025](https://doi.org/10.1016/j.ijmecsci.2018.01.025).

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