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PEAK SIDELOBE LEVEL REDUCTION WITH A HYBRID APPROACH BASED ON A GAS AND DIFFERENCE SETS

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January 2011

Technical Report # DISI-11-009

Peak Sidelobe Level Reduction with a Hybrid Approach based on GAs and Difference Sets

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***Abstract* -- This paper presents an approach for the optimization of the beam pattern produced by massively thinned arrays. The method, which combines the most attractive features of a genetic algorithm and those of a combinatorial technique (namely, the Difference Sets Method), is aimed at synthesizing massively thinned antenna arrays in order to suitably reduce the peak side-lobe level. Selected numerical results are presented in order to assess the effectiveness and reliability of the proposed approach.**

***Index Terms* − Array antennas, massively thinned arrays, difference sets, genetic algorithms, side-lobe control.**

I. Introduction

Filled antenna arrays are composed of radiating elements placed on a uniform lattice being half-wavelength the distance between adjacent points [1]. In order to reduce the element count, cost, weight, power consumption and heat dissipation, a thinning is performed by removing a percentage (called *thinning percentage*) of array elements according to a suitable strategy. Massively thinned arrays have fewer than half the elements of their filled counterparts. For a fixed antenna size, the massive thinning produces antenna arrays much cheaper than completely filled arrays, both in terms of hardware and computational complexity. Moreover, although for a drastic thinning the main lobe width remains approximately unaltered, however, it usually results in a reduced antenna gain and in a loss of side-lobe level control.

In the past, in order to overcome these drawbacks, many approaches have been proposed. The properties of random antenna arrays were studied [1], and thinning algorithms proposed [2][3] with limited effectiveness concerning the side-lobe control.

Recently, stochastic optimizers like simulated annealing (SA) [4] or genetic algorithms (GAs) [5] have been successfully applied by several authors [6][7], showing that an efficient side-lobe control can be attained achieving at the same time a high thinning percentage.

On the other hand, the applicability of cyclic difference sets (CDSs), which are combinatorial mathematics tools [8], to sparse antenna array synthesis has been deeply investigated [9][10] and the method has been applied to the design of massively thinned arrays, attaining array configurations characterized by uniform spatial coverage over the array aperture. Moreover, in correspondence with an infinite array having the displacement of the elements specified by the replication of a CDS, the arising power pattern of the array factor presents leveled side-lobes [10]. However, as suggested by Leeper, further improvements in the framework of real arrays could be achieved combining CDSs with stochastic optimization methods. Following this approach, this paper presents a new hybrid procedure whose effectiveness is assessed by means of a comparative analysis with standard GAs (SGAs) and CDS-based methods.

Moreover, Section II briefly introduces the antenna array notation. In Section III, the most attractive properties and current limitations of CDS and GA-based methods are discussed. In the same section, the application of the integrated approach is shown. This approach overcomes the limitations of each method when applied independently. Finally, in Section IV, a numerical assessment is performed in order to evaluate the effectiveness of the hybrid approach in dealing with the design of linear and planar antenna arrays.

II. Antenna Arrays Notation

The array factor for a linear half-wavelength spaced lattice array is defined as:

$$f(u)=\sum_{m=0}^{V-1} a_m e^{j(\pi m u)} \tag{1}$$

where $u=\sin\theta$, θ being the angular shift with respect to the normal direction, V is the number of $\frac{\lambda}{2}$-spaced lattice locations, $a_m=1$ if an element is present at m-*th* lattice location, otherwise $a_m=0$. Similarly, the array factor for a $V_x \times V_y$ planar half-wavelength spaced array is given by:

$$f(u,v)=\sum_{m=0}^{V_x-1}\sum_{n=0}^{V_y-1} a_{m,n} e^{j[\pi(mu+nv)]} \tag{2}$$

where $u=\sin\theta\cos\phi$ and $v=\sin\theta\sin\phi$ are the direction parameters and $a_{m,n}=1$ or 0 if an element is present or not at the location (m,n).

According to this notation, the peak side-lobe level (PSL) is defined as the maximum value of the array power pattern (the array factor multiplied by the element pattern) in the side-lobe region [1].

III. Hybrid Synthesis Method

In this section, the key-features of CDSs and GA-based methods are analyzed in order to effectively define a way of combining the two approaches.

A. Cyclic Difference Sets

A CDS, $D^{(V,K,\Lambda)}$, (defined by a triple of integer numbers (V,K,Λ)) is a set of integers $D=\{0\le d_k\le(V-1),k=1,...,K-1;\}$, such that $\{(d_h-d_l)_{\mathrm{mod}\,V};h\ne l\}$ appears exactly Λ times. It is well known [10], that the power pattern of the array factor of an infinite CDS-based array (obtained by placing the elements at the locations specified by the difference set) is a two-valued train of impulses presenting perfectly leveled side-lobes. As far as real arrays are concerned, the arising power pattern of the array factor shows undesired ripples. The greatest ripple is generally located in the neighborhood of the main lobe. However, the CDS method guarantees more effective sub-optimal array synthesis in terms of PSL with respect to random elements placement. It results that

$$PSL_{RAND}^{1D}-PSL_{DS}^{1D}\cong 3+10\log(1-K/V)^{-1}\ \mathrm{dB} \tag{3}$$

and

$$PSL_{RAND}^{2D}-PSL_{DS}^{2D}\cong 1.5+10\log(1-K/V)^{-1}\ \mathrm{dB} \tag{4}$$

for linear and planar arrays, respectively [10].

B. Genetic Algorithms

GAs are stochastic optimization methods extensively applied to antenna array optimization problems [6][7][11][12]. The basic working strategy can be summarized as follows. The approach starts by constructing a population N_{pop} trial solutions, $P=\{\Phi_p^{trial},p=1,...,N_{pop}\}$, which are coded into binary strings (called *chromosomes*) indicated by

$$\Phi^{trial}=\{a_m,m=1,...,V\} \tag{5}$$

for the linear case, and by

$$\Phi^{trial}=\{a_{m,n};m=1,...,V_x;n=1,...,V_y\} \tag{6}$$

for the planar displacement. The population undergoes the iterative application of the genetic operators, (namely, the crossover, ξ, the reproduction, ζ, and the mutation, μ) according to a trial solution's validity measure called *fitness*, F, until a stable value for the fitness function has been reached or a maximum number of iterations is achieved, I.

As far as the array synthesis is concerned, the fitness function is defined as follows

$$F(\Phi^{trial})=\max_{u\in S}\left\{\frac{|f(u)|^2}{\max_{\forall u}\left\{|f(u)|^2\right\}}\right\} \tag{7}$$

$$F(\Phi^{trial})=\max_{(u,v)\in S}\left\{\frac{|f(u,v)|^2}{\max_{\forall(u,v)}\left\{|f(u,v)|^2\right\}}\right\} \tag{8}$$

for linear and planar array, respectively; S denotes the side-lobes region.

GAs demonstrated their effectiveness in synthesizing highly optimized solutions. However, an accurate tuning is necessary and generally the convergence rate considerably reduces in the neighborhood of the optimal solution. These drawbacks could be partially avoided if all the available *a-priori* information is efficiently taken into account.

C. Hybrid Optimization Strategy

As suggested by Leeper [10], the combinatorial and stochastic methods could be combined, in order to take advantage from their good characteristics and to compensate for their drawbacks.

Therefore, ripples formation caused by CDS could be corrected in some way by GA search capabilities, while the uniform spatial coverage of CDS-optimized arrays could be helpful to speed up the convergence of the genetic procedure. One possible way of implementing this approach is to consider CDS-based arrays as *a-priori* knowledge to be inserted in the genetic search process in order to improve its efficiency. To this end, let us consider the following steps aimed at transferring good CDS-derived schemata [5] into the GA population. At the initialization step ($i = 0$, i being the iteration index), the GA population is composed as follows

$$\left[\Phi_p^{trial}\right]_{i=0} = \begin{cases} D_p^{(V,K,\Lambda)} & 0 \leq p \leq V-1 \\ \mu\left\{D_p^{(V,K,\Lambda)}\right\} & V \leq p \leq \left(N_{pop}-1\right) \end{cases} \quad (9)$$

where $D_0^{(V,K,\Lambda)}$ is the cyclic difference set, $D_{p\in[1,V-1]}^{(V,K,\Lambda)}$ the p-th cyclic shift of $D_0^{(V,K,\Lambda)}$, and $\mu\left\{D_p^{(V,K,\Lambda)}\right\}$ a randomly mutated cyclic shift. Moreover, during the iterative loop of the GA ($i \geq 1$), the mutation occurs in order to introduce new unexplored solutions into the search space. However, in order to keep higher order CDS-derived schemata, trial solutions having binary configurations belonging to higher-order schemata are mutated only in chromosome positions out of the schemata locations. These mechanisms are aimed at constraining the GA to synthesize array configurations similar to CDS-based ones, but with limited ripple amplitudes thanks to evolutionary capabilities.

Hereinafter we will refer to this hybrid GA as difference set genetic algorithm (DSGA).

IV. NUMERICAL RESULTS

In the following, CDSs based numerical results will be compared with those obtained by stochastic optimizers, pointing out the advantages of the combination of the two approaches. The assumed parameters for the GA-based procedures are: $I = 200$, $N_{pop} = 100$, $P_{\xi} = 0.9$ (crossover probability), and $P_{\mu} = 0.05$ (mutation probability).

A. Application to Linear Arrays

As a first test case, the reference CDS defined by $V = 63$, $K = 32$, and $\Lambda = 16$ is considered. This configuration has been investigated in [10] as a representative test case of CDS properties.

As suggested by Leeper, by applying a number of $V-1$ cyclic shifts to a valid CDS, it is possible to generate other valid sets whose properties can potentially be superior to those of the original set.

To this end, a (63,32,16) difference set has been calculated and the mentioned shift operations performed. The best-produced CDS (whose binary sequence generates a PSL of –13.53 dB) has been considered the final solution. This result is representative of the good capabilities of CDSs for array optimization since it is really close to random array average value [1].

Consequently, in order to evaluate the effectiveness of SGA in facing with the same synthesis problem, many algorithm executions have been performed and the results stored in order to allow detailed comparisons. By using the SGA-based method, it has been possible to achieve a PSL equal to –14.60 dB. As far as this test case is concerned, the performance of SGA slightly overcomes that of CDS, although the latter is a good method as well, which is able to provide optimized results as confirmed by the statistics reported in Table I.

However, by combining the two approaches, it has been possible to obtain a lower PSL. Figure 1 shows the arising power patterns in correspondence with the element displacements (Fig. 2) synthesized according to the CDS (Fig. 1(*a*)), the SGA (Fig. 1(*b*)), and the DSGA (Fig. 1(*c*)) method, respectively. The DSGA-optimized array is characterized by a PSL of –15.39 dB.

The advantage of combining the two methods resides in the following consideration. The ideal behavior of an infinite length CDS-based power pattern does not appear in the power pattern of Figure 1(*a*) because of the infinite sequence used to specify array elements locations had to be truncated to a single difference set, thus causing side-lobes to raise. Clearly, GAs can help in keeping under control the unwanted ripple, mixing the good side-lobes properties of CDS arrays with the evolutionary search capabilities, as confirmed by the statistics reported in Table I.

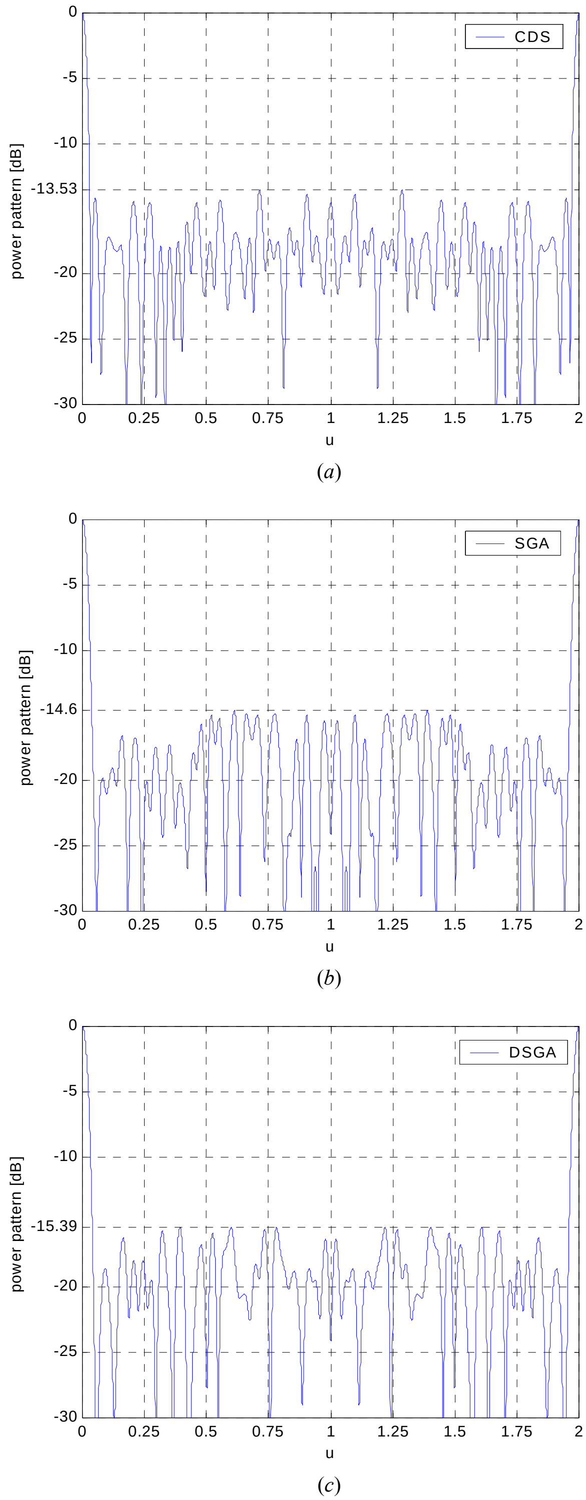


Fig. 1. Linear array V = 63, K = 32, Λ = 16 - Power patterns produced by arrays optimized through: (*a*) CDS, (*b*) SGA, and (*c*) DSGA.

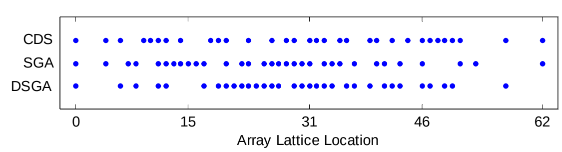
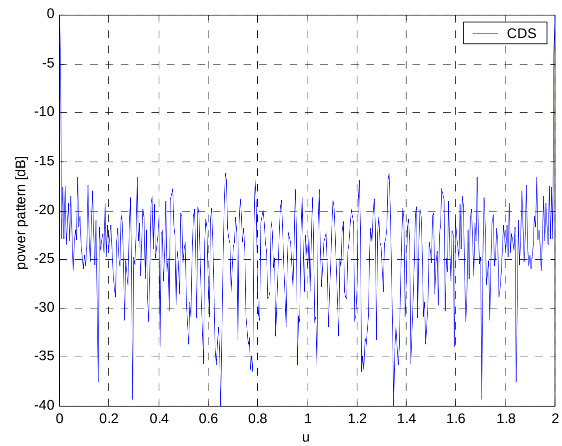


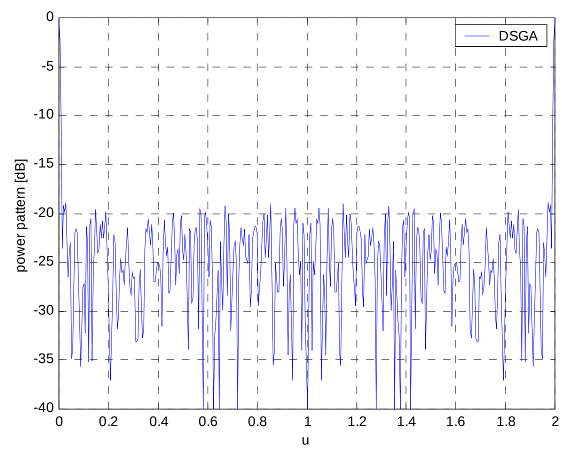
Fig. 2. Linear array V = 63, K = 32, Λ = 16 – Element displacement for generating the power patterns depicted in Fig. 1

In order to further assess the capabilities of the proposed method, a comparison with the CDS-based approach and state-of-the-art GAs [6,11] in thinning a 200 elements array, has been performed.

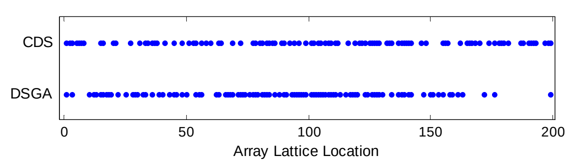
To this end, the (199, 99, 49) difference set has been considered, whose best cyclic shift produces a SLP of –15.99 dB. This difference set represents the a-priori knowledge to be inserted in the genetic loop of DSGA. Figure 3 shows the achieved result when the number of array elements is fixed to $K = 99$. For completeness, the best CDS has been also reported (b) jointly with the array placement (c).



(a)



(b)



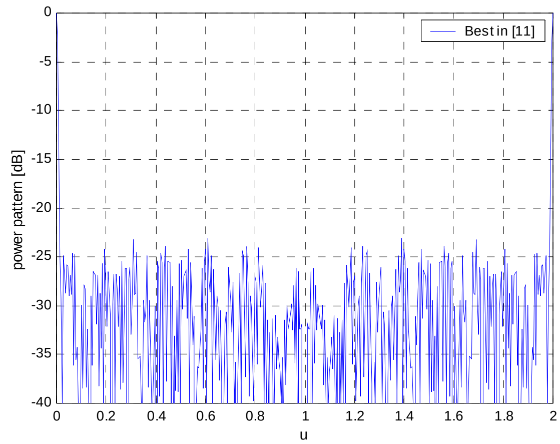
(c)

Fig. 3. Linear array $V = 199$, $K = 99$, $\Lambda = 49$ - Power patterns produced by arrays optimized through: (a) CDS and (b) DSGA. (c) Element displacement.

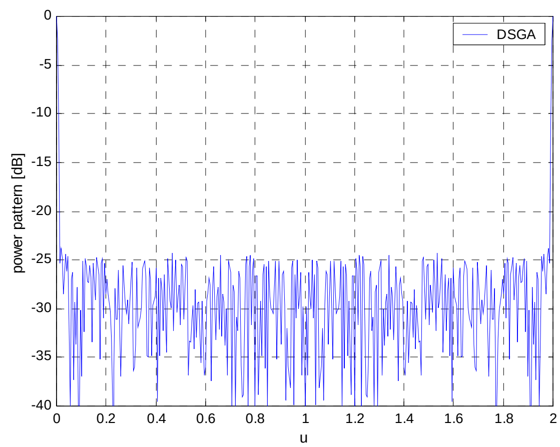
The best DSGA synthesized array yields a SLP of –19.24 dB, showing that the proposed integration has succeeded in lowering the SLP of CDS-based array of more than 3 dB.

Up to now, according to the aim of the proposed method, only massively thinned arrays have been considered. However, in order to allow a comparison with published literature [6][11], the effectiveness of DSGA-based method is also assessed in synthesizing arrays with an inferior thinning percentage.

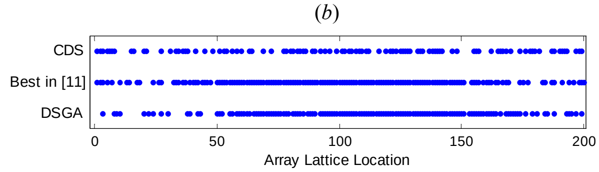
The best result in the literature for the thinning of a 200 elements array with isotropic elements is reported in [11] (SLP = –22.79 dB; $K = 154$). On the other hand, the use of a $K = 150$ directional elements array ($\cos(\theta)$ being the corresponding element pattern) yields a SLP equal to –23.69 dB [6].



(*a*)



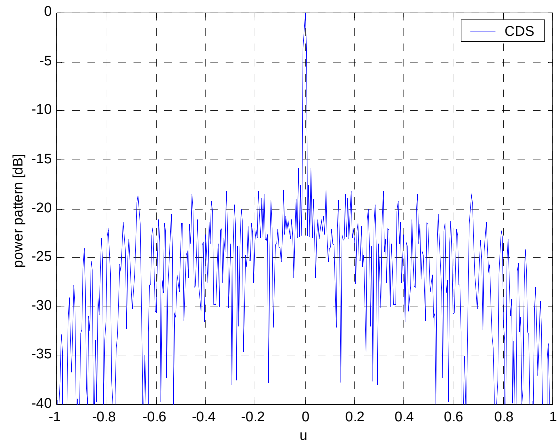
(*b*)



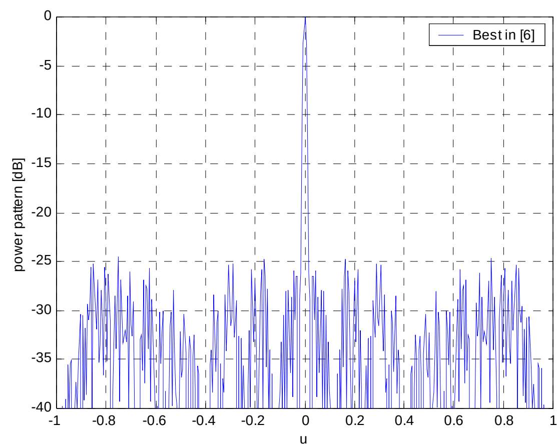
(*c*)

Fig. 4. 200 elements array - Isotropic elements
Power pattern of the array factor: (*a*) [11] and (*b*) DSGA array.
(*c*) Elements displacement.

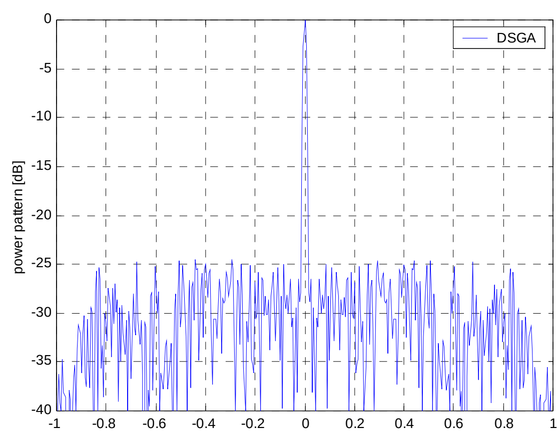
Thanks to the integration between GAs and CDS, better performance has been obtained in both situations. Figure 4 shows patterns and element locations of the synthesized arrays when isotropic elements are used. In more detail, CDS array presents a SLP equal to –15.99 dB, while the SLP of the DSGA synthesized array is equal to –23.70 dB by using 139 elements.



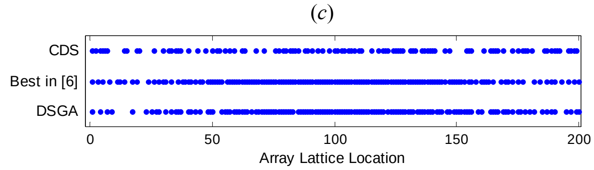
(*a*)



(*b*)



(*c*)



(*d*)

Fig. 5. 200 elements array - $\cos(\theta)$ element pattern

Power pattern of the array factor: (*a*) CDS array, (*b*) [6] and (*c*) DSGA array. (*d*) Elements displacement.

For completeness, Figure 5 shows the power patterns and elements displacements when an $\cos(\theta)$ element pattern is taken into account. CDS-based array yields a SLP of –17.10 dB, while DSGA produced a 132-element configuration with a SLP of –24.77 dB, that favorably compares with literature results as shown in Table 2.

B. Application to Planar Arrays

CDSs can be used to produce low sidelobes planar arrays too, provided that binary sequences are arranged on a two-dimensional lattice such that good autocorrelation properties are preserved. A way to do this was proposed in [10], and it has been used for evaluating the (63,32,16) CDS used as the reference solution for the first case of linear array optimization. Accordingly, an array lattice characterized by $V_x \times V_y = 9 \times 7$ positions has been considered.

Again, the evaluation of all the CDS and their V-1 cyclic shifts identifies the best array element configuration. Consequently, the numerical analysis indicated a minimum PSL equal to –12.47 dB, that is, a much better result than those obtained by random array placements.

On the other hand, SGA has been executed without any use of *a-priori* knowledge, in order to assess the effectiveness of a bare genetic algorithm procedure. SGA synthesized an optimal array configuration yielding a PSL equal to –13.69 dB. Moreover, the worst result achieved during the multiple running of the SGA is very close to the best obtained with CDS-based array (see Table I).

Finally, DSGA has been used in order to point out the capabilities of the combined approach. The final synthesis presents a PSL equal to -14.26 dB.

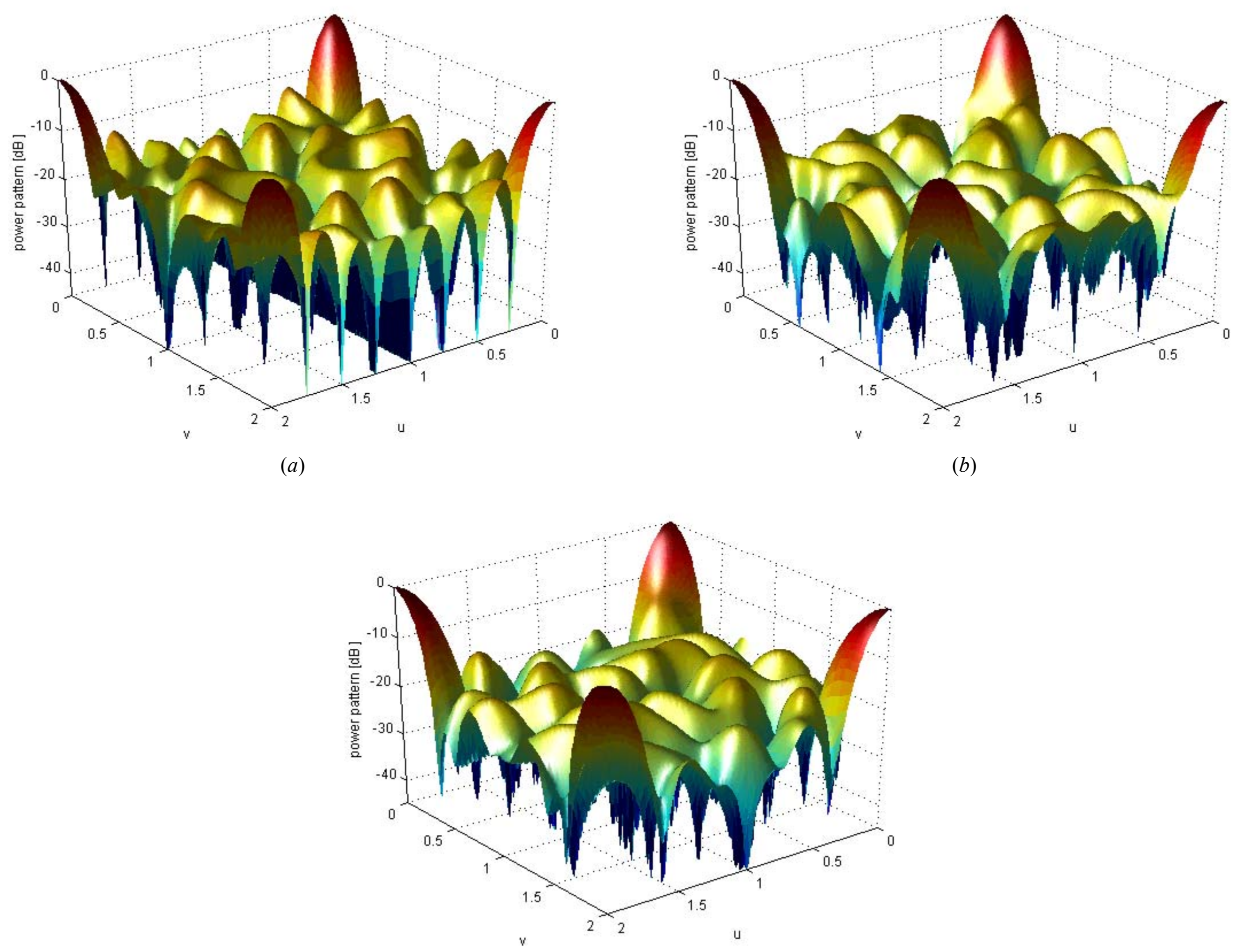


Fig. 6. Planar array power patterns produced by arrays optimized through: (*a*) CDS, (*b*) SGA, and (*c*) DSGA.

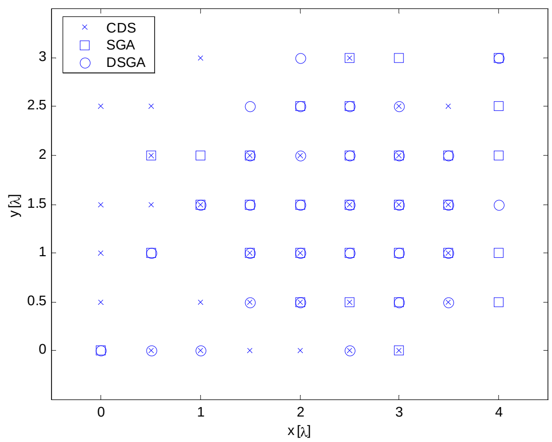


Fig. 7. Planar array element displacements
(× CDS method, □ SGA method, ○ DSGA method).

The resulting array power patterns are shown in Figure 6 and the corresponding element displacement patterns are reported in Figure 7.

The stability of the numerical results (see the statistics reported in Table I), clearly confirms that it is profitable to combine the combinatorial and the evolutionary approaches for the planar array synthesis problem, too.

VI. CONCLUSIONS

Difference sets and genetic algorithms' performances have been evaluated by considering the peak side-lobe level reduction in the power pattern of the array factor generated by massively thinned arrays. It has been shown that the combined use of the two methods allows gaining a good side-lobe control. Moreover, it has been possible to obtain satisfactory results even if the thinning percentage of the original cyclic difference set has been altered. This seems to indicate that arbitrarily long and thinned arrays can be suitably synthesized, provided that a suitable CDS (that is, with similar parameters) be used for boosting the GA performance. Future works, currently under development, will be aimed at further integrating cyclic difference sets with stochastic optimizers in order to speed up the convergence rate of the optimization procedure.

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TABLE I
(63, 32, 16) CDS BASED PEAK SIDELOBE LEVEL OPTIMIZATION – NUMERICAL RESULTS

Method	**PSL Linear**				**PSL Planar**			
	Best	**Worst**	**Avg**	**Var**	**Best**	**Worst**	**Avg**	**Var**
CDS	-13.53	-9.53	-11.44	1.21	-12.47	-7.87	-9.44	1.35
SGA	-14.60	-13.68	-14.10	0.23	-13.69	-12.44	-13.00	0.23
DSGA	-15.39	-14.07	-14.49	0.17	-14.26	-12.55	-13.23	0.39

TABLE II
(199, 99, 49) CDS BASED PEAK SIDELOBE LEVEL OPTIMIZATION – NUMERICAL RESULTS

Method	**Isotropic Elements**		$\cos(\theta)$ **Element pattern**	
	SLP [dB]	**% Thinning (N. el)**	**SLP [dB]**	**%Thinning (N. el)**
CDS	-15.99	>50% (99)	-17.10	>50% (99)
Best in Lit.	-22.79 [11]	23% (154) [11]	-23.69 [6]	25% (150) [6]
DSGA 99 el.	-19.24	>50% (99)	-21.59	>50% (99)
DSGA	-23.70	30.5% (139)	-24.77	34% (132)