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38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

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ADS-Based Y-Shaped Arrays for Interferometry and Radio Astronomy Applications

Giacomo Oliveri, Federico Caramanica, Paolo Rocca, and Andrea Massa
ELEDIA Group, DISI-University of Trento, Trento, ITALY
andrea.massa@ing.unitn.it

Correlator arrays have been a topic of research since the 1960s due to their applications in the field of radio astronomy [1][2]. Unlike conventional sum arrays, the data gathering efficiency of correlator arrays is related to their spatial filtering behavior [2]. As a consequence, the design of a correlator array is essentially an optimal sampling problem [2] in which the positions of the antennas are chosen in order to ensure optimal performances regarding all possible observation situations [1][2]. In order to obtain such features, correlator arrays have to show either a maximal coverage in the spatial frequency (or u−v) domain, or a minimum peak sidelobe level (PSL) in the angular (or l−m) domain [3]. Towards this end, many different design principles have been proposed, including minimum redundancy, pseudorandomness, power laws, difference set arrangements, minimization of the holes in the sampling, and stochastic optimization methods [1][2]. On the contrary, well-established optimization-based sum-array design techniques cannot be directly applied, since, unlike in traditional sum arrays, the responses in both the u−v and the l−m domains have to be considered in the design procedure [1][2]. Since the evaluation of the array spatial coverage, the Earth rotation effects and the l − m synthesized beam in the synthesis procedure is a time consuming procedure [2], efficient design techniques, requiring a low number of trial solution evaluation, are of interest, especially if large interferometers are considered.

Towards this end, this paper proposes an analytical technique for the design of interferometric arrays which takes advantage of a-priori available information regarding low correlation binary arrangements in order to reduce the search space for an (exhaustive) optimization procedure. The proposed methodology exploits recently introduced binary sequences with almost ideal autocorrelation properties, named Almost Difference Sets (ADSs) [5][6]. This approach is motivated by the fact that (*a*) ADSs exhibit correlation properties which are very close to that of DSs [4] (whose effectiveness for the design of correlator arrays has already been shown [1]); (*b*) ADSs can be defined in a much wider set of cases with respect to DSs [5][6][7]; (*c*) ADSs have already proven effective and computationally efficient as a tool for the design of sparse sum arrays [8].

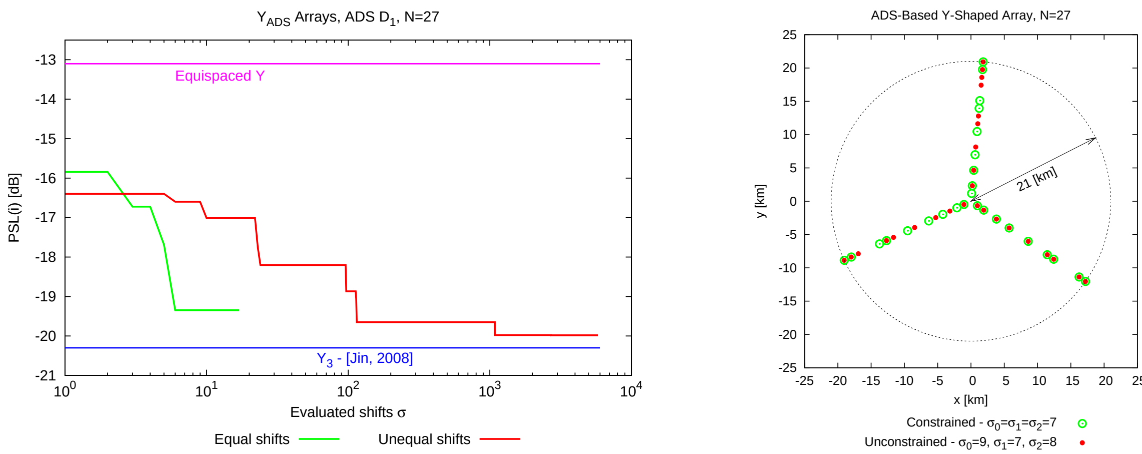
In order to investigate the suitability of ADS sequences in the design of correlator array, their application to a reference problem already considered in the literature [2] is carried out. Towards this end, an Y-shaped geometry is taken into account, and an ADS-based exhaustive search approach (enabled by the cyclical properties of ADSs [5]) is considered. More specifically, assuming that D is a (N, K, Λ, t)-ADS, i.e. a set of K unique integers d_k in the range $0,\ldots,N-1$ whose associated binary sequence exhibits a three-level periodic autocorrelation function [5][6], the following design procedure is introduced

$$\begin{cases} x_i = \sin\left(\frac{\pi a}{3}+\varphi\right)\frac{L\left[1+\left(d_k+\sigma_a\right)_{\mathrm{mod}\,N}\right]}{N} \\ y_i = \cos\left(\frac{\pi a}{3}+\varphi\right)\frac{L\left[1+\left(d_k+\sigma_a\right)_{\mathrm{mod}\,N}\right]}{N} \end{cases} \quad i = Ka+k, k=0,\ldots,K-1, a=0,1,2 \tag{1}$$

where σ_a is the cyclic shift applied to the a-th arm, L is the arm length, and φ is the geometry inclination with respect to the north-south direction. In order to evaluate the ADS-based correlator arrays, a software analyzer was developed in order to analyze the snapshot u-v coverage, the tracking u-v coverage and the synthesized beam of the resulting arrangements. As a numerical experiment, the PSL optimization of an ADS-based correlator array is carried out considering the same reference geometry and geographical settings of [2], and exploiting for the ADS-based synthesis the (18,9,4,13)-ADS [7]. Two different target geometries are analyzed:

- *Unconstrained ADS-Based Geometry*: in this case, each arm can show a different σ_a; as a consequence, up to N^3 different arrangements have to be analyzed;
- *Constrained ADS-based Geometry*: in this case, the same σ_a is considered in all arms; this choice results in up to N different arrangements from a single ADS.

The results of the extensive analysis regarding the two ADS-based design approaches are shown in Figure 1 and 2. As it can be noticed, the exploitation of ADS-based approaches for the Constrained PSL minimization allows one to obtain a sub-optimal arrangement in a very low number of iterations [only 7 different arrangements are analyzed to reach a PSL value of -19.58 dB – Fig. 1(*a*)].



(*a*) (*b*)

Figure 1. ADS-based correlator array design. (*a*) Behaviour of the PSL as a function of σ_a for an exhaustive ADS-based PSL optimization procedure. (*b*) Geometry of the resulting Constrained and Unconstrained ADS-based arrays.

Such results confirm that ADSs can be applied to the synthesis of sub-optimal correlator arrays in a very computationally efficient way (similar conclusions would hold also for larger arrangements). On the other side, higher performance configurations (*PSL*=-19.9 dB) can be obtained by means of the Unconstrained ADS-based approach, but with increased computational costs, due to the larger number of configurations to be analyzed [Fig. 1(*a*)]. However, also in this case the required number of analysis is very low (around 2.1×10^3 different configurations are evaluated to reach convergence) if compared to state-of-the-art optimization techniques [2]. Moreover, thanks to Eq. (1), the resulting arrangements show favorable geometrical characteristics in terms of regularity and construction simplicity, since the elements are displaced on each arm according to a regular lattice of *N* positions [Fig. 1(*b*)]. As a final observation, it is worth noticing that ADS-based correlator arrays exhibit a very regular sidelobe pattern (as it is shown in Fig. 2 for the Constrained geometry): as a consequence, such arrangement are suitable to obtain low-noise observations of distant radio sources.

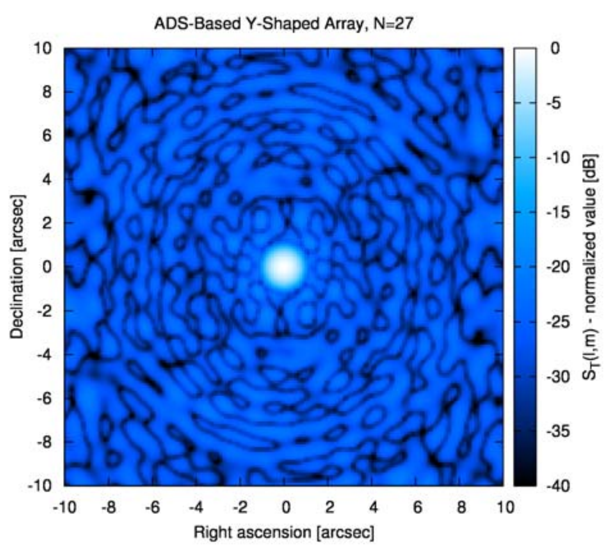


Figure 2. Syntesized beam for the Constrained Y-shaped ADS-Based array in Fig. 1(*b*).

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