

# UNIVERSITY OF TRENTO 

## DEPARTMENT OF INFORMATION AND COMMUNICATION TECHNOLOGY

38050 Povo - Trento (Italy), Via Sommarive 14
http://www.dit.unitn.it

CDMA RADIO NETWORK PLANNING:
A COMPARISON OF DIFFERENT
INTERFERENCE EVALUATION METHODS

Danilo Severina, Renato Lo Cigno

November 2003

Technical Report \# DIT-03-079

# CDMA Radio Network Planning: A Comparison of Different Interference Evaluation Methods 

Danilo Severina, Renato Lo Cigno

November 2003


#### Abstract

The target of this work is the comparison of different approximation thechniques for the evaluation of the level of interference, and the consequent errors on data frames transmitted on the channel, generated by the sum of interfering signals. The interference, also known as MAI (Multiple Access Interference), is caused by presence of other users in the same cell of the reference user or in neighboring cells.

The system considered is a generic spread-spectrum communication system, whose spreading technique can be described as CDMA (Code Division Multiple Access), with of without rake receivers, with a direcr or indirect spreading technique.

Although the analysis techniques considered are extremely "broad" in their application, mainly as a consequence of their high level of abstraction and approximation, all the examples and numerical evaluation are done in the context of the UTRAN (UMTS-Terrestrial Radio Access NEtwork) interface, the which uses a DS-CDMA (Direct Spreading CDMA) technique with a superposition of spreading and scrambling codes.

The analisy of behaviour of DS-CDMA system is done with different approximation: SGA, IGA, SEIGA and Fourier Series Expansion.

The SGA (Standard Gaussian Approximation) is the most diffused approximation: it considers one useful signal while all the others are considered as white Gaussian noise over the band in exam. This algorithm is based on Central Limit Theorem: a sum of a large number of independent and identically distributed (iid) variables is a Gaussian variable [34]. Is a rough approximation, but it allows easy and fast computation.

The IGA (Improved Gaussian Approximation) is based on Central Limit Theorem too. In this case the MAI is a Gaussian variable conditionally to users states, defined by delay and phase.

The SEIGA (Simplified Expression of Improved Gaussian Approximation) is similar to IGA and introduces simplification in the integral computation using Taylor Series and differences rather than derivatives [15].

The Fourier Series Expansion ([6], [36], [37], [38], [39], [40]) is not based on Central Limit Theorem and does not use Gaussian Approximation for the MAI. In


this case the hypothesis of a big number of users in a cell is not necessary and the approximation allows to study multi-service systems. The algorithms that use Gaussian approximation can not be used for multi-service systems because the hypothesis of iid variable can not be satisfied.

These approximations allow to define parameters as SIR (Signal to Interference Ratio) and BER (Bit Error Rate) to estimate performances of communication on up-link channel. For each of these approximations two different channel models are considered: no-fading (AWGN, Additive White Gaussian Noise) one and fading one.For each of these channel models three different classes of power control are considered: perfect one, imperfect one and absent one.

## 1 Introduction

The introduction of UMTS (Universal Mobile Telecommunication System) as a standard for third generation mobile networks promises circuit and packet switched mobile access with fair data rates. In addition to traditional voice communication this system enables a wide range of new data services including mobile Internet. It is not yet clear how this service mix will look like in real UMTS system.

Everything concerned Third Generation (3G) platforms is described in ITU (International Telecommunication Union) standard IMT-2000 (International Mobile Telecommunication for the year 2000): this standard describes the radio access techniques and defines different platform (Figure 1). Besides it tries to create compatibility among different mobile networks and between mobile and fixed networks.

It is not easy to describe 3 G systems because the aspects defined in IMT-2000 are different for each kind of system. A generic 3G system can be described as follow:

- Do not exist only one 3G platform, technology or application.
- Each system has different technologies, different frequency bands, different channel bandwidths and different modulations.
- 3G describes fixed and wireless applications that can use data transmissions with different bit rates: $144 \mathrm{kbit} / \mathrm{s}$ for vehicular users, $384 \mathrm{kbit} / \mathrm{s}$ for mobile pedestrian users and $2 \mathrm{Mbit} / \mathrm{s}$ for not mobile users.

In systems like UMTS the radio resource is a scarce and precious resource so its use has to be controlled. Systems with distributed multiple access can use different techniques to permit users to access to the medium. The access techniques supported by platforms described in IMT-2000 are:

FDMA (Frequency Division Multiple Access). The spectrum is divided in frequency bands separated by guard bands. Each band can be used by only one user. There is an exclusive allocation of the resource.

TDMA (Time Division Multiple Access). A user occupies the whole frequency band to transmit information, but only for a determinate period of time. The channel is divided in time slots and these are periodically allocated to one user. There is an exclusive allocation of the resource too.

CDMA (Code Division Multiple Access). Every users use the whole band and transmit without temporal limits: their communications are separated by orthogonal codes.

A CDMA system can use an extra frequency or time division for the access of users to the medium. In fact, the UTRAN (UMTS Terrestrial Radio Access Network), the access network for UMTS systems, uses a CDMA technique with FDD (Frequency Division Duplex), [5]. For UMTS the CDMA employed technique is DS-CDMA (Direct Spreading $C D M A$ ): the data bits are multiplied for a high rate sequences to allow separation of users.


Figure 1: Connections among platforms described by IMT-2000 Standard

This operation is called "spreading". At the receiver to recovery the information, the received signal must be multiplied for the same spreading sequences employed in the transmission.

### 1.1 Power Control

An efficient power control algorithm is very important to upgrade performances in CDMA systems, where every signals are transmitted on the same frequency band and in the same time. Power control algorithms are necessary both on the up-link and on the down-link channel, although for different reasons.

On uplink channel every signals should arrive to BS (Base Station) with the same power level and so every UEs (User Equipment) cannot transmit with the same power. If each terminal transmits with tha same power there is the near-far effect: respect to a defined BS, the nearest UE should hide another farer one.

On down - link channel the power control algorithms are used to reduce the interference level among BSs. The signals transmit to one BS are all synchronous and so they do not have problems about cross-interference, but if a $\mathrm{BS}\left(\mathrm{BS}_{0}\right)$ transmits signals with high power level the adjacent cells $\left(\mathrm{BS}_{i}\right)$ hear a non-orthogonal signals that detect as interference. If the level of the detected signal by $\mathrm{BS}_{i}$ is very high, the $\mathrm{BS}_{i}$ increases the level of its transmitted signals and generates a high interference level for other adjacent cells. So, there is a series of events that produces an unstable condition.

In UMTS there are two different power control algorithms: open loop and closed loop [2], [14].

### 1.1.1 Open Loop Power Control

The transmitter detects the interference level on the down-link channel and changes the power transmitted level properly. If the UTRAN uses a DS-CDMA-FDD technique the problem is that up-link and down-link work on different frequencies and are not correlated, so this method provides right power value only in terms of average value on long time interval. Instead, if the UTRAN uses a DS- CDMA-TDD technique the fading process are correlated and this power control has good performances.

### 1.1.2 Closed Loop Power Control

Both the BS and the UE are involved in the measure of the channel parameters. For the up-link, the BS monitors the strength of received signals from every UEs connected with it and notifies to the each UE the power level to use. The BS does this control for each UE separately and notifies to it not the exact power level to use, but sends information for increasing or decreasing of the transmitted power level.

An imperfect power control can generate a variation of the cell area (effect knows as breathing cell): if the interference level grows, there should be a condition where the farthest UE from BS cannot be served. The effective cell radius decreases, the farther user
loses the communication (or well, it should roam to another BS) and so the interference level decreases because the source of interference is reduced. The decrease of cell radius and interference level should permit the BS to serve again the far user, previous eliminated. So the cell has a dimension that changes periodically and the consequences can be controlled difficulty.

### 1.2 Spreading and Scrambling Code

In UMTS systems, the data sequences subject to two operations: spreading and scrambling. For the first one OVSF (Orthogonal Spreading Variable Factor) codes are used and for the second one PN (Pseudo Noise) codes are used [7]. In both cases the codes are binary codes, so at the receiver to recover the sent information the same code employed in transmission must be used.

### 1.2.1 Spreading

In this process the data sequence is multiplied by a sequence OVSF with higher bit rate; the chip rate for UMTS is 3.84 Mega chips per second (Mchip/s). The number of chip elements of OVSF code in a data bit is called Spreading Factor (SF). The spreading causes a expansion (spreading) of the signal band and the distribution of the signal power over a wider band has various advantages: good resistance to interference and noise and good protection versus willing intruders.

The up-link spreading codes are generated by dedicated algorithms that produce code trees ([1], [4], [19]). The spreading factors are included between 4 and 256 for up-link codes and between 4 and 512 for down-link codes. Each level in the code tree defines spreading codes of length SF , corresponding to a particular spreading factor, equals to SF . The number of codes for a given SF is equal to the SF itself. All the codes of the same level constitute a set and they are orthogonal to each other. Two codes of different level (so they have different length) are orthogonal each other only if one of them is not father of the other one.

In up-link OVSF codes cannot be used to separate different users because they have good cross-correlation properties only if they are synchronized. In the down-link they should be used to separate users (the transmission from the BS is synchronous), but the limited number of OVSF codes involves that the same code should be used in adjacent cells, so the interference level grows.

Even if spreading codes are used to spread the spectrum both in up-link and in downlink, in the up-link OVSF codes are assigned for each user, while in down-link the management is more difficult because the same tree code employed in up-link is used: the BS must not stay without usable codes, and one code can be used for only one active user.

### 1.2.2 Scrambling

The spreading procedure causes a spread of the spectrum, but it is not able to separate the users, so a procedure of scrambling is necessary. The scrambling procedure is a XOR
between a spread signal and a pseudo-noise sequence.
The scrambling codes in UMTS are of two types: long scrambling codes and short scrambling codes. In both cases there are standard algorithms for generating them. The short ones are used with advanced receiver employing multi-user detection or interference cancellation. Instead the long ones are used with simple receiver or simple Rake receiver.

In up-link these codes help maintain separation among different UEs, while in the down-link they are used to maintain cell or sector separation. In both case they are constructed using primitive polynomials and the resulting sequences constitute segments of Gold sequences. The primitive polynomial for the employed codes in the two channel are different.

In down-link the possible codes are 512 and are divided into 32 code groups with 16 codes in each group. The grouping is done to make easier and faster cell research by the mobile.

### 1.2.3 Synchronization Codes

Some down-link channels use neither OVSF nor PN codes, but they use synchronization codes. There are two types of these codes: primary codes and secondary codes. The first one are used in primary channels, while the second ones are used in secondary channels.

The primary code is unique in a single cell and the UEs use it in the search of the best BS to connect to. The secondary code can change frequently, but the UEs can recognize the primary code employed in the cell using this secondary one.

### 1.3 Correlation Properties

The autocorrelation measures the correlation between one signal and its temporal shift sequences. The autocorrelation of a sequence with N symbols has a zero shift autocorrelation coefficient equal to N [28]. The values of non-zero shift autocorrelation can be between $-N$ and $+N$. In case where the sequence symbols are complex the absolute value of the autocorrelation is used: in this case the presence of all non-zero shift values lesser than the zero shift peak show good autocorrelation properties.

Sequences with good autocorrelation properties are used to synchronize, to separate users or multi-paths of the same signal.

The cross-correlation measures the correlation between two different signals. Using the definition by Pursley [28], two sequences with N symbols can have absolute crosscorrelation values between a minimum ( 0 ) and a maximum ( $N$ ). Neither for zero shift nor for non-zero shift the cross-correlation values cannot be determined. Low crosscorrelation values show that the codes generate low interference level.

## 2 Network Planning

Network planning is the major task for operator. It is time consuming, labor-intensive and expensive. The quality of network planning process has a direct influence on operator's profits. An inadequate planning can cause call blocking for the not sufficient capacity. Otherwise an under-utilization can cause economical damage for operator that bought unnecessary and expensive equipments.

The network-planning processes in 3G WCDMA networks and in 2G GSM networks have fundamental differences: in GSM a lot of work is done with frequency planning, while in UMTS there is no frequency planning because all base stations work on the same frequencies. In WCDMA systems different frequencies are used, but typically for different network hierarchies. Each operator has two to five $5-\mathrm{MHz}$ frequency channels. For example, an operator can use one frequency for macrocells, one for microcells and another one for picocells, so he uses a hierarchical structure to coverage a region. Also in this case a frequency planning is neither required nor possible. The areas where is need a hierarchical structure are areas where there are different kinds of users that have different behaviors. Macrocells handle fast- moving UE to reduce the number of handovers. Its radius size is typically calculated in kilometers. Microcells are used in cities to increase network capacity and its typical users are pedestrian and not mobile users like office workers. Its diameter size is a few hundred meters. Picocells are used in hot spot, like offices or shops, where there are not fast-moving users, otherwise the amount of handovers should be excessive and should be exhaust the network's signal-processing capacity. This kind of cell is often used in indoor locations. The radius size of these cells is only tens of meters.

The IMT-2000 standard foresee also the presence of higher hierarchical cells: satellite ones. The radio access technology for satellite cells will not be the same for terrestrial ones and the infrastructure of the two systems must be planned to support inter-system handovers.

In WCDMA systems, the cell sizes are not fixed, but depend on the required capacity: breathing cells. The coverage and the capacity parameters are dependent each other, so both of them have to be planned together. If more capacity is needed in an existing WCDMA network, new base station must be added to ease the capacity storage. When a new base station is added to the network, its influence will reach surrounding base stations and their parameters have to be changed. These changes influence the neighboring cells so the adding of a base station will reach even distant cells.

### 2.1 Performance Parameters and Network Planning Terminology

To determinate the performances of a CDMA system, you can use different parameters:
SNR (Signal to Noise Ratio) defines the ratio between the signal level and the noise level.
SIR (Signal to Interference Ratio) defines the ratio between the signal level and the interference level.
$\mathbf{P}_{e}$ the error probability, also known as BER (Bit Error Rate), define the probability to receive a mistaken bit. Similar is FER (Frame Error Rate).
$\mathbf{P}_{\text {out }}$ the outage probability is the likelihood of a radio network not fulfilling a specific QoS (Quality of Service) target. The outage condition is reached when the interference level gets a prefixed threshold upon noise level. Usually this condition is closely correlated to system capacity.
$\mathbf{P}_{\text {block }}$ the block probability is the likelihood that a new incoming call is denied. This can happen because the whole resource is busy or because there is not sufficient resource to support the requested service.

Capacity defines the number of voice calls that the system is able to hold. This concept can be extended to multi-service system with different kinds of services.
The following terms explain some fundamental concept of network planning.
Traffic intensity is measured in Erlang. One Erlang is equivalent to one call lasting one hour. Thus the traffic intensity can be calculated as
$\frac{\text { Number of call in one hour } \cdot \text { average call duration }}{3600}$
Traffic density measures the number of calls per square kilometer [Erlang/ $\mathrm{Km}^{2}$ ].Usually this is used for circuit- switched voice calls, while for data services, the traffic density is measured using [Mbps/Km²].

Spectral efficiency is defined as the amount of traffic that can be handle within a certain bandwidth and area.

$$
\frac{\text { Traffic intensity [Erlang] }}{\text { Bandwidth } \cdot \text { Area }}=\frac{\mathrm{bps}}{\mathrm{MHz} \cdot \mathrm{~km}^{2}}
$$

Cell loading defines the relative occupancy of the cell. This is given as a percentage of maximum capacity.

Loading factor defines the amount of interference loaded into the cell by surrounding cells. It is given by ratio of power received by a base station from other cells (external interference) to power that it receives from UEs in its own cell (internal interference and useful signal).

### 2.2 Network Planning Process

The network planning is not a frequency planning, but it is a much wider process. At the highest level it can be divided into two major areas: radio-network planning and network dimensioning.

Network-planning process includes traffic estimation, figuring the proper number of cell, placement of base stations and frequency planning. Before to perform a detailed network planning phase a high- level network planning phase (network dimensioning) is needed.

### 2.2.1 Network Dimensioning

Network dimensioning is a process aims to estimate the amount of equipment that the telecommunication network needs: for WCDMA networks this includes infrastructures both for the radio access and for the core. This process includes calculating radio link budgets, capacity and coverage. The output of this process should be an estimation of required equipment and a rough placement of base station. The estimating network needs to satisfy the above requirements.

The estimation procedures start with the calculation of radio link budget. The radio link budget is a crucial point for planning process: it is derived assuming a particular noise rise and it is combined with a particular path loss model to determinate cell range. To determinate the maximum path loss in a particular environment different parameters must be considered: frequency, bearer rate, coverage probability, cell loading, transmitted power, transmitted and received antenna gains, average noise, average $\mathrm{E}_{b} / \mathrm{N}_{0}$ (bit energy to noise ratio) and fading parameters ([44],[44]). In this estimation task the maximum allowable system loading must be set. It is important to remember that in a WCDMA system the capacity is not limited by the number of channel elements, but by the amount of noise and interference in air interface (UTRAN). In this phase the human network planner must decide the maximum allowable loading of the system: this load factor is a fraction of pole capacity (the maximum theoretical system capacity). Usually values between 0.4 and 0.6 are used [17]. These values are used to include interference margin and power control margin.

The interference margin is required to prevent breathing cells. In WCDMA systems the cell loading influence its dimension: higher the load is, smaller the cell size is. If the interference level is high the cell size decreases and this means that some users will lose their connection. This causes a lowering of interference and the cell size should increase: hence a pulsating cell. Interference margin lesser than one prevent this effect, because the margin can keep cell sizes unchanged while interference level changes.

The power control margin is required to give the UE the possibility to perform closedloop power control (fast power control). The maximum cell size is achieved if a mobile transmit with full power, but so fast power control could not be used for this UE.

### 2.2.2 Radio-network Planning

The detailed radio-network planning includes the exact design of radio network. This task is not done by human planner, but by dedicated computer tool: the placement of base station is not a simple procedure because the found spots for BS can be located in restricted area or in unusable area, so quite often a optimum cell positioning is impossible. To find possible site for base stations the dedicated computer tools need digital map of area (usually 3D map to include presence of obstacle like hills, buildings, woods... to have a detailed characterization of radio environment), population and traffic informations.

To obtain satisfying results, gains and losses by soft handovers, hard handover, presence of hierarchical cells, features of antennas (sectorization, adaptative and/or smart antennas) and features of receivers (classic or rake) should be considered .

## 3 Techniques of Analysis

To determinate error probability, blocking probability, signal to noise ratio or whatever performance parameter an approximation must be used for the MAI (Multiple Access Interference). We supposed that in a cell managed by base station $0\left(\mathrm{BS}_{0}\right)$ there are more than one user: the $\mathrm{UE}_{0}$ is the useful user, while the other users in the same cell generate interference (internal interference). The MAI includes the grants of both active user equipments above described and the active UEs in the adjacent cells (external interference).

### 3.1 Approximation for MAI

In literature there are some possible approximation: Gaussian, Log-normal and ChiSquare.

### 3.1.1 Gaussian Approximation

This is the most common approximation for MAI. The signal of each user that arrives at the receiver is modeled as a random Gaussian process with average power equals to the sum of each user's power. To warrant this kind of approximation the Central Limit Theorem is used. To apply this theorem is necessary that variables are identically distributed and the number of this process is large. In a UMTS cell can be only two or three active users so this approximation could seem rough. Furthermore it can be used to study single service systems, because in case of multi-service systems the process could not be identically distributed.

This approximation is rough, but it is the most used, because allows mathematical simplifications: sum of Gaussian variables is still a Gaussian variable with mean equal sum of mean of single processes and variance equal sum of varince of single processes. To find the BER, the Marcum function $Q(\cdot)$ can be used.

$$
Q(x)=\frac{1}{\sqrt{2 \pi}} \int_{x}^{\infty} \exp \left(-\frac{1}{2} u^{2}\right) d u
$$

### 3.1.2 Log-normal Approximation

In this case the received signal (sum of signal transmitted by active users in the cell) is assumed log-normally distributed conditioned on the number of active users. This approximation follows from the fact that a finite sum of log-normal random variables is approximately log-normal. This approximation assumes that if the received signal $Z$ is log-normally distributes conditioned on the number of active voice users and active data users, the process $\tilde{Z}=\ln (Z)$ is conditionally Gaussian. The log-normal approximation is more accurate than the Gaussian one when the number of users in the cell is small and also it can be used for multi-service systems.

### 3.1.3 Chi-Square Approximation

This approximation does not use Central Limit Theorem so it can be used also with a small number of active users in the cell. The expression of $\chi^{2}$ (Chi-Square) characteristic function is

$$
T(\omega)=\left(1-j \frac{\sigma^{2}}{\mu} \omega\right)^{\left[n\left(\mu^{2} / \sigma^{2}\right)\right]}
$$

For a large number of users this characteristic function is the Gaussian one, so the two approximations are equal.

### 3.2 Channel Model

The wireless channel is characterized by three different models:
AWGN: the channel is modeled with an additive random process $n(t)$

$$
\begin{equation*}
h_{k}(t)=\mathrm{e}^{\jmath \phi_{k}} \delta\left(t-\tau_{k}^{\prime \prime}\right) \tag{1}
\end{equation*}
$$

Flat Fading: the model includes the path gains and these coefficients are Rayleigh variable

$$
\begin{equation*}
h_{k}(t)=\alpha_{k} \mathrm{e}^{\jmath \phi_{k}} \delta\left(t-\tau_{k}^{\prime \prime}\right) \tag{2}
\end{equation*}
$$

Selective Fading: the model includes path gains and multi-paths

$$
\begin{equation*}
h_{k}(t)=\sum_{m_{k}=0}^{M_{k}} \alpha_{k, m_{k}} \mathrm{e}^{\jmath \phi_{k, m_{k}}} \delta\left(t-\tau_{k, m_{k}}^{\prime \prime}\right) \tag{3}
\end{equation*}
$$

where $\phi_{k}, \phi_{k, m_{k}}$ and $\tau_{k}^{\prime \prime}, \tau_{k, m_{k}}^{\prime \prime}$ are the phase delays and time delays introduced by the channel and are uniform variable over $(2, \pi]$ and $\left[o, T_{\text {max }}\right.$ ], respectively, with $T_{\text {max }}$ the maximum allowed time delay. The coefficients $\alpha_{k}$ and $\alpha_{k, m_{k}}$ are path gain components with Rayleigh distribution:

$$
\begin{equation*}
f_{\alpha}(\alpha)=\frac{\alpha}{\sigma_{\alpha}^{2}} \exp \left\{-\frac{\alpha^{2}}{\sigma_{\alpha}^{2}}\right\} . \tag{4}
\end{equation*}
$$

### 3.3 Signal Model

Using the notations as in [11], [28] e [29], the expression of transmitted signal is

$$
\begin{equation*}
s_{k}(t)=\sqrt{P_{k} / 2} b_{k}\left(t-\tau_{k}^{\prime}\right) a_{k}\left(t-\tau_{k}^{\prime}\right) \cos \left(\omega_{c} t+\theta_{k}\right) \tag{5}
\end{equation*}
$$

where $P_{k}$ is the power of transmitted signal, $\omega$ is the frequency carrier, $\theta_{k}$ is the phase offset, $b_{k}(t)$ e $a_{k}(t)$ are the data bit and the spreading sequence respectively and $\tau_{k}^{\prime}$ is a random up-link channel delay caused by the asynchronism of transmitted users (it is measured to a reference signal).

The signals produced by every UEs are sent on a wireless channel and the received signal can be expressed as

$$
\begin{equation*}
r(t)=\sum_{s=0}^{S-1} \sum_{k=0}^{K_{s}-1} \sum_{m_{k}=0}^{M_{k}-1} \sqrt{P_{k} / 2} \alpha_{k, m_{k}} b_{k}\left(t-\tau_{k, m_{k}}\right) a_{k}\left(t-\tau_{k, m_{k}}\right) \cos \left(\omega_{c} t+\phi_{k, m_{k}}\right)+n(t) \tag{6}
\end{equation*}
$$

where $n(t)$ is Additive White Gaussian Noise (AWGN) with power density equal to $N_{0} / 2$; the variable $\phi_{k, m_{k}}$ includes the value of offset transmitted carrier $\theta_{k}$ and the variable $\tau_{k, m_{k}}$ includes the channel delays $\tau_{k}^{\prime}$ and $\tau_{k, m_{k}}^{\prime \prime}$. This is the most general expression for a received signal because it considers both multi-path and multi-service system.

The receiver tries to decode the received signal and tries to recognize a desider user with correlators and filters. Without loss of generality the user zero $\left(K_{0}\right)$ is the desired user while the other $K_{u}-1$ users are interference. The receiver takes back the signal to baseband and then multiplies it by the spreading signal of desired user $\left(a_{0}(t)\right)$ and integrates on the bit period. The receiver is supposed to be synchronized (in phase and in time) with the multi-path zero. The decision maker for $m$-th bit can be described as in [35] and [11]

$$
\begin{align*}
Z_{0}(m) & =\int_{m T_{b}}^{(m+1) T_{b}} r(t) a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t \\
& =b_{0}(m) \alpha_{0,0} \sqrt{\frac{P_{0}}{2}} T_{b}+\sum_{k=0}^{K_{u}-1} \sum_{\left[m_{k}=0, m_{0} \neq 0\right]}^{M_{k}-1} I_{k, m_{k}}+\nu \tag{7}
\end{align*}
$$

where $b_{0}(m)$ is the m -th bit of the user 0 .
The noise and interference terms can be expressed as

$$
\begin{align*}
\nu & =\int_{m T_{b}}^{(m+1) T_{b}} n(t) a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t  \tag{8}\\
I & \triangleq \sum_{k=0}^{K_{u}-1} \sum_{\left[m_{k}=0, m_{0} \neq 0\right]}^{M_{k}-1} I_{k, m_{k}}  \tag{9}\\
& =\sum_{k=0}^{K_{u}-1} \sum_{\left[m_{k}=0, m_{0} \neq 0\right]}^{M_{k}-1} \int_{m T_{b}}^{(m+1) T_{b}} \alpha_{k, m_{k}} s_{k}\left(t-\tau_{k, m_{k}}\right) \mathrm{e}^{\jmath \phi_{k, m_{k}}} a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t
\end{align*}
$$

The noise term is a Gaussian random variable with zero mean and variance equal to $\sigma_{\nu}^{2}=N_{0} T_{b} / 4$, while the MAI includes both signals from user $k_{i}$ with $i=1, \ldots K_{u}-1$ (with their multi-paths) and the multi-paths of the signal of the user $k_{0}$ except the path number zero (the useful signal).

The function that describes the statistic decision of the receiver (7) should be expressed in short way as

$$
\begin{align*}
Z_{0}(m) & =D_{0}(m)+I+\nu  \tag{10}\\
& =D_{0}(m)+I_{O}+I_{I}+\nu
\end{align*}
$$

where $D_{0}(m)$ is the desired user and $I$ is the MAI term that can be divided into two independent terms, the internal interference $I_{I}$ and the external one $I_{O}$.

The performances of a CDMA system can be estimated using the SIR parameter. Using the expression (10) the SIR can be write as

$$
\begin{equation*}
S I R=\frac{E\left\{D_{0}(m)^{2}\right\}}{E\left\{(I+\nu)^{2}\right\}}=\frac{E\left\{D_{0}(m)^{2}\right\}}{E\left\{I_{0}^{2}\right\}+E\left\{I_{I}^{2}\right\}+E\left\{\nu^{2}\right\}} \tag{11}
\end{equation*}
$$

The useful term is given by [30]

$$
\begin{equation*}
E\left\{D_{0}(m)^{2}\right\}=E\left\{\left(b_{0}(m) \sqrt{\frac{P_{0}}{2}} \alpha_{0,0} T_{b}\right)^{2}\right\}=\frac{P_{0} T_{b}^{2}}{2} E\left\{\alpha_{0,0}^{2}\right\} \tag{12}
\end{equation*}
$$

or by

$$
\begin{equation*}
E\left\{D_{0}(m)^{2} \mid \alpha_{0,0}\right\}=\frac{P_{0} T_{b}^{2}}{2} \alpha_{0,0}^{2} \tag{13}
\end{equation*}
$$

The AWGN term is given by [30]

$$
\begin{equation*}
E\left\{\nu^{2}\right\}=\sigma_{\nu}^{2}=\frac{N_{0} T_{b}}{4} \tag{14}
\end{equation*}
$$

The interference term depends on channel model used, so there are three expressions [35], [30] , [23], [41].

AWGN Channel

$$
\begin{equation*}
E\left\{I^{2} \mid P_{k}\right\}=\left.\sigma_{I}^{2}\right|_{\left\{P_{k}\right\}}=\frac{G_{p} T_{c}^{2}}{6} \sum_{k=1}^{K_{u}-1} P_{k} \tag{15}
\end{equation*}
$$

Flat Fading Channel

$$
\begin{equation*}
E\left\{I^{2} \mid P_{k}, \alpha_{k}\right\}=\left.\sigma_{I}^{2}\right|_{\left\{P_{k}, \alpha_{k}\right\}}=\frac{G_{p} T_{c}^{2}}{6} \sum_{k=1}^{K_{u}-1} \alpha_{k}^{2} P_{k} \tag{16}
\end{equation*}
$$

## Selective Fading Channel

$$
\begin{align*}
E\left\{I_{O}^{2}\right\} & =E\left\{\alpha_{k_{0}, 0}^{2}\right\}\left(K_{u}-1\right) \frac{A^{2} T_{b}^{2}}{3 G_{p}}+E\left\{\alpha_{k_{0}, m_{k 0}}^{2}\right\}(M-1) K_{u} \frac{A^{2} T_{b}^{2}}{3 G_{p}} \\
& =\frac{A^{2} T_{b}^{2} 2 \sigma^{2}(M K-1)}{3 G_{p}}  \tag{17}\\
E\left\{I_{I}^{2}\right\} & =E\left\{\alpha_{k_{1}, m_{k 1}}^{2}\right\} E\left\{\beta_{k_{1}}^{4}\right\} M K \frac{A^{2} T_{b}^{2}}{3 G_{p}} \\
& =2 \sigma^{2} \frac{M K}{5} \frac{A^{2} T_{b}^{2}}{3 G_{p}} \tag{18}
\end{align*}
$$

where $\alpha_{k_{0}, m_{k 0}}$ is the path gain for user connected to $\mathrm{BS}_{0}$ (under exam), $\alpha_{k_{1}, m_{k 1}}$ is the path gain for user not connected to $\mathrm{BS}_{0}, \beta_{k_{1}}=\frac{r_{1, k 1}}{r_{0, k 0}}$ is the ratio between the distance from $k-1$-th user to its $\mathrm{BS}\left(\operatorname{not} \mathrm{BS}_{0}\right)\left(r_{1, k 1}\right)$ and the distance from the same user to $\mathrm{BS}_{0}\left(r_{0, k 1}\right)$.

If path gains are identically distributed Rayleigh variable and $\beta_{k_{1}}$ is equally distributed variable over $(0,1]$, the expression of $E\left\{\alpha_{k_{x}}^{2}\right\}$ and $E\left\{\beta_{k_{1}}^{4}\right\}$ are

$$
\begin{equation*}
E\left\{\alpha_{k_{0}, 0}^{2}\right\}=E\left\{\alpha_{k_{0}, m_{k 0}}^{2}\right\}=E\left\{\alpha_{k_{1}, m_{k 1}}^{2}\right\}=2 \sigma^{2} \tag{19}
\end{equation*}
$$

and

$$
\begin{equation*}
E\left\{\beta_{k_{1}}^{4}\right\}=\frac{1}{5} \tag{20}
\end{equation*}
$$

### 3.4 Standard Gaussian Approximation (SGA)

To establish SIR and BER for CDMA systems this approximation esteems the received signal as a Gaussian process. Using the equation (10) and rating $i$ and $\nu$ as two independent variables, a new variable can be defined as $\xi=I+\nu$. So $Z_{0}(m)$ is a Gaussian variable with $D_{0}$ mean and $\sigma_{\xi}^{2}$ variance (variance of $\xi$ ). The evaluation of SIR and BER is computed using the three different channel models and three models for power control.

### 3.4.1 Channel without Fading

Using the property of independence between noise and MAI the quantity $\sigma_{\xi}^{2}$ is given by

$$
\begin{align*}
\sigma_{\xi}^{2} & =\sigma_{I}^{2}+\sigma_{\nu}^{2} \\
& =\frac{G_{p} T_{c}^{2}}{6} \sum_{k=1}^{K_{u}-1} P_{k}+\frac{N_{0} T_{b}}{4} \tag{21}
\end{align*}
$$

The parameters BER and SIR are computed as

$$
\begin{align*}
B E R & =Q\left(\frac{\left|D_{0}\right|}{\sigma_{\xi}^{2}}\right)=Q\left(\sqrt{\frac{P_{0} T_{b}^{2}}{2 \sigma_{\xi}^{2}}}\right)=Q(\sqrt{2 S I R})  \tag{22}\\
S I R & =\frac{P_{0} T_{b}^{2}}{2 \sigma_{\xi}^{2}} \\
& =\frac{0.5}{\frac{1}{3 G_{p}} \sum_{k=1}^{K_{u}-1} \frac{P_{k}}{P_{0}}+\frac{N_{0}}{2 T_{b} P_{0}}} \tag{23}
\end{align*}
$$

### 3.4.2 Flat Fading Channel

When the channel has loss, the Rayleigh path gains $\alpha_{k}$ must be considered. in this scenario SIR and BER are conditioned on path gains and received power and are given by [35]

$$
\begin{equation*}
\left.S I R\right|_{\left\{\alpha_{k}, P_{k}\right\}}=\frac{1}{\frac{1}{3 G_{p}} \sum_{k=1}^{K_{u}-1} \frac{P_{k} \alpha_{k}^{2}}{P_{0}} \frac{N_{0}}{\alpha_{0}^{2}}+\frac{N_{0}}{2 T_{b} P_{0} \alpha_{0}^{2}}} \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
\left.B E R\right|_{\left\{\alpha_{k}, P_{k}\right\}}=Q\left(\sqrt{\frac{1}{\frac{1}{3 G_{p}} \sum_{k=1}^{K_{u}-1} \frac{P_{k}}{P_{0}} \frac{\alpha_{k}^{2}}{\alpha_{0}^{2}}+\frac{N_{0}}{2 T_{b} P_{0} \alpha_{0}^{2}}}}\right) \tag{25}
\end{equation*}
$$

### 3.4.3 Selective Fading Channel

The SIR for a system in a selective fading environment is given by [23]

$$
\begin{equation*}
\left.S I R\right|_{\alpha_{0,0}}=\frac{\left(A \alpha_{0,0} T_{b}\right)^{2}}{E\left\{I_{O}^{2}\right\}+E\left\{I_{I}^{2}\right\}+E\left\{\nu^{2}\right\}} \tag{26}
\end{equation*}
$$

where $A=\sqrt{2 P_{0}}$ is the amplitude of useful signal.
With perfect power control, a conventional receiver and $N_{c}$ interference cells, using (14), (17), (18) the SIR and BER are given by

$$
\begin{align*}
\left.S I R_{M}\right|_{\alpha_{0,0}} & =\frac{\alpha_{0,0}^{2}}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{27}\\
\left.B E R_{M}\right|_{\alpha_{0,0}} & =Q\left(\sqrt{S I R_{M}}\right) \\
& =Q\left(\sqrt{\frac{\alpha_{0,0}^{2}}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}}\right) \tag{28}
\end{align*}
$$

Then after a media over Rayleigh variables $\alpha_{0,0}$, SIR and BER is given by

$$
\begin{align*}
S I R_{M} & =\frac{2 \sigma^{2}}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{29}\\
B E R_{M} & =\frac{1}{2}\left[1-\frac{1}{\sqrt{1+\frac{N_{0}}{2 E_{b} \sigma^{2}}+\frac{2}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}}\right] \tag{30}
\end{align*}
$$

If a Rake receiver is used the above expressions change as [23]

$$
\begin{align*}
\left.S I R_{R M}\right|_{x} & =\frac{x}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{31}\\
\left.B E R_{R M}\right|_{x} & =Q\left(\sqrt{\frac{x}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}}\right) \tag{32}
\end{align*}
$$

where

$$
\begin{equation*}
x=\sum_{m_{1}=1}^{M} \alpha_{m_{1} i}^{2} \tag{33}
\end{equation*}
$$

is a $\chi^{2}$ process with pdf:

$$
\begin{equation*}
f(x)=\frac{x^{M-1} e^{-x / 2 \sigma^{2}}}{\left(2 \sigma^{2}\right)^{M}(M-1)!} . \tag{34}
\end{equation*}
$$

After the media over distribution of $x$, the SIR and BER are given by

$$
\begin{align*}
S I R_{R M} & =\frac{2 M \sigma^{2}}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{35}\\
B E R_{R M} & =\int_{0}^{+\infty} Q\left(\sqrt{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}\right) f(x) d x \tag{36}
\end{align*}
$$

### 3.4.4 Imperfect Power Control

In the previous analysis a perfect power control is supposed. If the power control is not perfect, the signal of $k$-th user could be a random variable with mean equal to its nominal value $A_{k}$.

The distribution of $A_{k}$ can be assumed as uniform process

$$
\begin{equation*}
f\left(A_{k}\right)=\frac{1}{2 V} \quad A_{0}-V \leq A_{k} \leq A_{0}+V \tag{37}
\end{equation*}
$$

where $V$ is the maximum semi-range of signal variance.
For a traditional receiver, with $N_{c}$ interference cell, M multi-path for each signal and fading channel, the SIR and BER are given by [23]

$$
\begin{align*}
S I R_{I M} & =\frac{2 \sigma^{2}}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left(1+\frac{V^{2}}{3 A_{0}^{2}}\right)\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{38}\\
B E R_{I M} & =\frac{1}{2}\left[1-\frac{1}{\sqrt{1+\frac{N_{0}}{2 E_{b} \sigma^{2}}+\frac{2}{3 G_{p}}\left(1+\frac{V^{2}}{3 A_{0}^{2}}\right)\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}}\right] \tag{39}
\end{align*}
$$

In a configuration like the previous one, the SIR and BER are given by [23]

$$
\begin{align*}
\left.S I R_{I R M}\right|_{x} & =\frac{x}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left(1+\frac{V^{2}}{3 A_{0}^{2}}\right)\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}  \tag{40}\\
\left.B E R_{I R M}\right|_{x} & =Q\left(\sqrt{\frac{x}{\frac{N_{0}}{2 E_{b}}+\frac{2 \sigma^{2}}{3 G_{p}}\left(1+\frac{V^{2}}{3 A_{0}^{2}}\right)\left[\left(1+\frac{N_{c}}{5}\right) M K_{u}-1\right]}}\right) \tag{41}
\end{align*}
$$

The SIR and BER are found using an uniform process for strength signal: this is a rough approximation that allows simple computation. More accurate model can be used to include imperfections of power control algorithms.

### 3.4.5 Absence of Power Control

If there is no power control in the BS, the received power depends by the distance between the UE and the BS. The received signal is a function of $r^{n}$, where $r$ is the distance between BS and UE and $n$ is the path-loss exponent, which can assume value between $n=2$ (for free space) and $n=4$ (for urban environment). In case with absence of power control the effect near-far is much important.

If the user are supposed uniformly distributed over the cell area, the distribution of variable $r$ is

$$
\begin{equation*}
f_{r}(r)=\frac{2 r}{R_{c}^{2}-r_{o}^{2}}, \quad r \in\left[r_{o}, R_{c}\right] \tag{42}
\end{equation*}
$$

where $R_{c}$ is the cell radius (the cell shape is assumed circular) and $r_{0}$ is the minimum distance between UE and BS.

The amplitude of the received signal is given by

$$
\begin{equation*}
A=\frac{2 A_{0}}{(\sqrt{r})^{n}} \tag{43}
\end{equation*}
$$

and the probability density function $f_{A}(a)$ can be derived by cumulative distribution $F_{A}(a)$.

$$
\begin{align*}
F_{A}(a) & =\operatorname{Pr}\{A<a\}=\operatorname{Pr}\left\{\frac{A_{0}}{r^{n / 2}}<a\right\}= \\
= & \operatorname{Pr}\left\{r>\left(\frac{A_{0}}{a}\right)^{2 / n}\right\}=1-\operatorname{Pr}\left\{r \leq\left(\frac{A_{0}}{a}\right)^{2 / n}\right\}  \tag{44}\\
F_{A}(a) & =\left\{\begin{array}{lll}
0 & \tilde{r}>R_{c}, & a<\frac{A_{0}}{R_{c}^{n / 2}} \\
1-\int_{r_{o}}^{\tilde{r}} f_{r}(r) d r & \tilde{r} \in\left(0, R_{c}\right], & \frac{A_{0}}{R_{c}^{n / 2}}<a<\frac{A_{0}}{r_{o}^{n / 2}} \\
1 & r<r_{o}, & a>\frac{A_{0}}{r_{o}^{n / 2}}
\end{array}\right. \tag{45}
\end{align*}
$$

where $\tilde{r}=\left(\frac{A_{0}}{a}\right)^{2 / n}$.
The probability density function (pdf) $f_{A}(a)$ si

$$
f_{A}(a)=\frac{d F_{A}(a)}{d a}= \begin{cases}\frac{4 A_{0}^{4 / n}}{n\left(R_{c}^{2}-r_{o}^{2}\right)} a^{-\frac{4+n}{n}} & \frac{A_{0}}{R_{c}^{n / 2}<a<\frac{A_{0}}{r_{o}^{n / 2}}}  \tag{46}\\ 0 & \text { otherwise }\end{cases}
$$

The values of mean and variance of found pdf are given by

$$
\begin{align*}
& \mu_{A}=E\{A\}= \begin{cases}\frac{4 A_{0}}{(n-4)\left(R_{c}^{2}-r_{o}^{2}\right)}\left[r_{o}^{\frac{4-n}{2}}-R_{c}^{\frac{4-n}{2}}\right] & n \neq 4 \\
\frac{A_{0}}{\left(R_{c}^{2}-r_{o}^{2}\right)}\left[\ln \left(\frac{A_{0}}{r_{o}^{n 2}}\right)-\ln \left(\frac{A_{0}}{R_{c}^{n+2}}\right)\right] & n=4 .\end{cases}  \tag{47}\\
& \sigma_{A}^{2}=E\left\{A^{2}\right\}-\mu_{A}^{2}=\frac{2 A_{0}^{2}}{(n-2)\left(R_{c}^{2}-r_{o}^{2}\right)}\left[r_{o}^{2-n}-R_{c}^{2-n}\right]-\mu_{A}^{2} \tag{48}
\end{align*}
$$

These expressions can be used also for adjacent cells: the area where users are distributed uniformly is an area delimited by two concentric circles of radius $R_{c}$ and $r_{0}$, where $R_{c}$ is the maximum distance between UE and BS of reference cell and $r_{0}$ is the radius of the cell served by reference BS .

Similarly in equation (38) and (39) [23], the SIR and BER with no power control can be expressed as

$$
\begin{align*}
S I R_{A M} & =\frac{2 \sigma^{2}}{\frac{2 \sigma^{2}}{3 G_{p}} \frac{1}{\mu_{A}^{2}}\left[E\left\{A_{O}^{2}\right\}\left(M K_{u}-1\right)+\frac{N_{c} M K_{u}}{5} E\left\{A_{I}^{2}\right\}\right]+\frac{N_{0}}{2 E_{b}}}  \tag{49}\\
B E R_{A M} & =\frac{1}{2}\left[1-\frac{1}{\sqrt{1+\frac{N_{0}}{2 E_{b} \sigma^{2}}+\frac{2}{3 G_{p}} \frac{1}{\mu_{A}^{2}}\left[E\left\{A_{O}^{2}\right\}\left(M K_{u}-1\right)+\frac{N_{c} M K_{u}}{5} E\left\{A_{I}^{2}\right\}\right]}}\right] \tag{50}
\end{align*}
$$

where $A_{O}$ is a random variable that defines the strength of received signals from users in the cell served by examined BS and $A_{I}$ is a random variable that defines the strength of received signals from users in neighbor cells.

### 3.5 Improved Gaussian Approximation (IGA)

The expression in 3.4 are valid only if the number of users is large and the power control is perfect. When the power control is imperfect the Gaussian Approximation is not appropriate. In the previous approximation the SIR and BER for a single user channel are evaluated as a deterministic measure of averaged environmental conditions.

A possible different approach is calculating the averaged SIR and BER over environmental statistic. Simulations by [35], [20] show that this approach yields to an improvement of the Gaussian approximation method. The IGA analysis defines the MAI conditioned on particular operating conditions for each user. When this is done, the variable $\psi$ is defined as the conditioned variance of MAI for a specific operating condition [30]

$$
\begin{equation*}
\psi=\operatorname{var}\left(I \mid\left\{\varphi_{k}\right\},\left\{\tau_{k}\right\},\left\{P_{k}\right\}, B\right) \tag{51}
\end{equation*}
$$

In this case the conditional variance of the MAI is a random variable and the SIR is given by

$$
\begin{equation*}
\left.S I R\right|_{\psi}=\frac{P_{0} T_{b}^{2}}{2 \psi} \tag{52}
\end{equation*}
$$

If the distribution $f(\psi)$ of $\psi$ is known, the SIR and the BER can be found by averaging overall possible values of $\psi$ and their expressions can be given by

$$
\begin{align*}
S I R_{I G A} & =\int_{0}^{\infty} d \psi \frac{P_{0} T_{b}^{2}}{2 \psi} f(\psi)  \tag{53}\\
B E R_{I G A} & =\int_{0}^{\infty} d \psi Q\left(\sqrt{\frac{P_{0} T_{b}^{2}}{2 \psi}}\right) f(\psi) \tag{54}
\end{align*}
$$

In case of perfect power control the expressions 53 and 54 yields to accurate results for a very small number of interfering users. This approach could be extended to each case showed for SGA approximation (flat fading channel, selective fading channel, imperfect power control and absence of it), but an explicit expression for $f(\psi)$ must be available.

### 3.6 Simplified Expression of Improved Gaussian Approximation (SEIGA)

The expression (53) and (54) are complicated and required significant computational time to evaluate. Holtzman [15] presents a simplified technique for evaluating these. Liberti [21] extends these ones in the case of imperfect power control, and Sunay and McLane [35] in the case of frequency non-selective fading channel.

The simplified SIR and BER expressions are based on the fact that if $g(x)$ is a continuous function and $x$ is a random variable with mean value $\mu_{x}$ and variance $\sigma_{x}^{2}$, then the average value $E\{g(x)\}$ can be expressed by making use of the Taylor's expansion as [9]:

$$
\begin{equation*}
E\{g(x)\}=g\left(\mu_{x}+x\right)+\frac{1}{2} \sigma_{x}^{2} g "\left(\mu_{x}\right)+\ldots \tag{55}
\end{equation*}
$$

Computational reduction can be obtained by expanding in differences rather than derivatives

$$
\begin{equation*}
E\{g(x)\} \cong g\left(\mu_{x}\right)+\frac{\sigma_{x}^{2}}{2} \frac{g\left(\mu_{x}+h\right)-2 g\left(\mu_{x}\right)+g\left(\mu_{x}-h\right)}{h^{2}} \tag{56}
\end{equation*}
$$

Choosing $h=\sqrt{3} \sigma_{x}$, (56) becomes [15]

$$
\begin{equation*}
E\{g(x)\} \cong \frac{2}{3} g\left(\mu_{x}\right)+\frac{1}{6} g\left(\mu_{x}+\sqrt{3} \sigma_{x}\right)+\frac{1}{6} g\left(\mu_{x}-\sqrt{3} \sigma_{x}\right) \tag{57}
\end{equation*}
$$

Using this approximation, the expression of SIR and BER when the noise term is significant are given by

$$
\begin{align*}
S I R_{S E I G A} \cong & \frac{2}{3} \frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}+\frac{N_{0} T_{b}}{4}\right)}+\frac{1}{6} \frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}+\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)} \\
& +\frac{1}{6} \frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}-\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}  \tag{58}\\
B E R_{S E I G A} \cong & \frac{2}{3} Q\left(\sqrt{\frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right)+\frac{1}{6} Q\left(\sqrt{\frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}+\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right)+
\end{align*}
$$

$$
\begin{equation*}
\frac{1}{6} Q\left(\sqrt{\frac{P_{0} T_{b}^{2}}{2\left(\mu_{\psi}-\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right) \tag{59}
\end{equation*}
$$

where $\mu_{\psi}$ is the mean value of the variance of the MAI $\psi$, conditioned on a specific set of operating conditions, and $\sigma_{\psi}^{2}$ is the variance of $\psi$.

In absence of fading and with perfect power control, if the received power levels from the $K_{u}-1$ interfering users are identically distributed with mean $\mu_{p}$ and variance $\sigma_{p}^{2}$, the mean and variance of $\psi$ are given by [30]:

$$
\begin{align*}
\mu_{\psi}= & \frac{T_{c}^{2} G_{p}}{6}\left(K_{u}-1\right) \mu_{p}  \tag{60}\\
\sigma_{\psi}^{2}= & \left(K_{u}-1\right) \frac{T_{c}^{4}}{4} \cdot\left[\frac{7 G_{p}^{2}+2 G_{p}-2}{40} \sigma_{p}^{2}+\right. \\
& \left.\left(\frac{23 G_{p}^{2}}{360}+G_{p}\left(\frac{1}{20}+\frac{K_{u}-2}{36}\right)-\frac{1}{20}-\frac{K_{u}-2}{36}\right) \mu_{p}^{2}\right] \tag{61}
\end{align*}
$$

Note that equation (59) is valid only for

$$
\begin{equation*}
\mu_{\psi}+\frac{N_{0} T_{b}}{4}>\sqrt{3} \sigma_{\psi} \tag{62}
\end{equation*}
$$

to ensure that the denominator of the third term is positive. The same condition can be expressed in function of $\sigma_{p}^{2}$ and $\mu_{p}$

For a flat fading channel and perfect power control, the expression of SIR and BER are given by [35]

$$
\begin{align*}
\left.\operatorname{SIR}_{S E I G A}\right|_{\left\{\alpha_{k}\right\}} \cong & \frac{2}{3} \frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}+\frac{N_{0} T_{b}}{4}\right)}+\frac{1}{6} \frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}+\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)} \\
& +\frac{1}{6} \frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}-\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}  \tag{63}\\
\left.B E R_{S E I G A}\right|_{\left\{\alpha_{k}\right\}} \cong & \frac{2}{3} Q\left(\sqrt{\frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right)+\frac{1}{6} Q\left(\sqrt{\frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}+\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right)+ \\
& \frac{1}{6} Q\left(\sqrt{\frac{P_{0} T_{b}^{2} \alpha_{0}^{2}}{2\left(\mu_{\psi}-\sqrt{3} \sigma_{\psi}+\frac{N_{0} T_{b}}{4}\right)}}\right) \tag{64}
\end{align*}
$$

where, in case of perfect power control, the mean and the variance of $\psi$ are

$$
\begin{align*}
\left.\mu_{\psi}\right|_{\left\{\alpha_{k}\right\}}= & \frac{G_{p} T_{c}^{2} P_{0}}{6} \sum_{k=1}^{K_{u}-1} \alpha_{k}^{2}  \tag{65}\\
\left.\sigma_{\psi}^{2}\right|_{\left\{\alpha_{k}\right\}}= & \frac{T_{c}^{4} P_{0}^{2}}{4}\left[\frac{23 G_{p}^{2}+18 G_{p}-18}{360} \sum_{k=1}^{K_{u}-1} \alpha_{k}^{4}\right. \\
& \left.+\frac{G_{p}-1}{36} \sum_{k=1}^{K_{u}-1} \sum_{l=1}^{K u-1} \alpha_{k}^{2} \alpha_{l}^{2}\right] \tag{66}
\end{align*}
$$

In order to find the average SIR and BER for this case, expressions (63) and (64) have to be further averaged over the distribution of the path gains $\left\{\alpha_{k}\right\}$.

Note that equation (64) is only valid for

$$
\begin{equation*}
\mu_{\psi}+\frac{N_{0} T_{b}}{4}>\sqrt{3} \sigma_{\psi} \tag{67}
\end{equation*}
$$

For a no-fading channel and imperfect power control the mean and the variance of $\psi$ are

$$
\begin{align*}
\mu_{\psi}= & \frac{G_{p} T_{c}^{2} P_{0}}{6} \sum_{k=1}^{K_{u}-1} P_{k}  \tag{68}\\
\sigma_{\psi}^{2}= & \frac{T_{c}^{4}}{4}\left[\frac{23 G_{p}^{2}+18 G_{p}-18}{360} \sum_{k=1}^{K_{u}-1} P_{k}^{2}\right. \\
& \left.+\frac{G_{p}-1}{36} \sum_{k=1}^{K_{u}-1} \sum_{l=1}^{K u-1} P_{k}^{2} P_{l}^{2}\right] \tag{69}
\end{align*}
$$

For fading channel and imperfect power control, in 68 and 69 the term $P_{k}$ should be substituted with $P_{k} \cdot \alpha_{k}^{2}$.

In the most general case, the mean and variance of MAI are given by [30]

$$
\begin{align*}
\mu_{\psi}= & \frac{G_{p} T_{c}^{2} P_{0}}{6} \sum_{k=1}^{K_{u}-1} \mu_{p, k}  \tag{70}\\
\sigma_{\psi}^{2}= & \frac{T_{c}^{4}}{4}\left[\frac{23 G_{p}^{2}+18 G_{p}-18}{360} \sum_{k=1}^{K_{u}-1} \mu_{p, k}^{2}\right. \\
& +\frac{G_{p}-1}{36} \sum_{k=1}^{K_{u}-1} \sum_{l=1}^{K u-1} \mu_{p, k}^{2} \mu_{p, l}^{2} \\
& \left.+\frac{7 G_{p}^{2}+2 G_{p}-2}{40} \sum_{k=1}^{K_{u}-1} \sigma_{p, k}^{2}\right] \tag{71}
\end{align*}
$$

To estimate the mean and variance of MAI is necessary to know the mean and variance of power $P$ of received signals. In previous section the distribution of signal amplitude $A$ are shown, so assuming that $P=2 A^{2}$ the mean and variance of power are given by Imperfect Power Control

$$
\begin{align*}
\mu_{p} & =\frac{\left(A_{0}+V\right)^{3}-\left(A_{0}-V\right)^{3}}{3 V}  \tag{72}\\
\sigma_{p}^{2} & =2 \frac{\left(A_{0}+V\right)^{5}-\left(A_{0}-V\right)^{5}}{5 V}-\mu_{p}^{2} \tag{73}
\end{align*}
$$

## Absence of Power Control

$$
\begin{align*}
\mu_{p} & =\frac{4 A_{0}^{2}}{(n-2)\left(R_{c}^{2}-r_{o}^{2}\right)}\left(r_{o}^{(2-n)}-R_{c}^{(2-n)}\right)  \tag{74}\\
\sigma_{p}^{2} & =\frac{4 A_{0}^{4}}{(n-1)\left(R_{c}^{2}-r_{o}^{2}\right)}\left(r_{o}^{2(1-n)}-R_{c}^{2(1-n)}\right)-\mu_{p}^{2} \tag{75}
\end{align*}
$$

### 3.7 Fourier Series Expansion

The approximations as SGA and SEIGA cannot be extended to analyze system with different services: neither of them is able to guarantee that MAI can be approximated with a Gaussian variable. A multi-service system as UMTS can be analyzed using a new approximation based on truncated Fourier series expansion for the error function $Q(x)$.

A model with AWGN channel, perfect power control and a single service is first analyzed. Then with the same approach, the analysis can be extended for multi-service, fading channel and not-perfect power control. In all cases the multi-path is not considered.

### 3.7.1 Single Service and AWGN Channel

Using notation similar to [11], [28] and [29] the received signal can be expressed as in equation (6) (without multi-path and obviously without summation over different services)

$$
\begin{equation*}
r(t)=\sum_{k=0}^{K_{u}-1} \sqrt{2 P_{k}} \alpha_{k, m_{k}} b_{k}\left(t-\tau_{k, m_{k}}\right) a_{k}\left(t-\tau_{k, m_{k}}\right) \cos \left(\omega_{c} t+\phi_{k, m_{k}}\right)+n(t) \tag{76}
\end{equation*}
$$

The decision on m -th bit is given by

$$
\begin{aligned}
Z_{0}(m) & =\int_{m T_{b}}^{(m+1) T_{b}} r(t) a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t \\
& =b_{0}(m) \alpha_{0,0} \sqrt{\frac{P_{0}}{2}} T_{b}+\sum_{k=0}^{K_{u}-1} I_{k}+\nu \\
& =D_{0}+\psi+\nu
\end{aligned}
$$

where $D_{0}$ is the useful term, $\psi$ is the MAI and $\nu$ is the noise term. (It's the equation (7).)
It starts by finding the error probability conditioned on the MAI term, $\psi$. Once conditioned on a particular value of M , the decision statistic $Z_{0}$ becomes exactly Gaussian since the desired term D is deterministic and the AWGN term $\nu$ is a Gaussian random variable. In this case the conditioned error probability can be expressed as

$$
\begin{equation*}
P(E \mid M)=Q\left(\frac{D_{0}-\psi}{\nu}\right)=Q\left(\frac{\sqrt{\frac{P_{0}}{2}} \alpha_{0} b_{0} T_{b}-\psi}{\sqrt{\frac{N_{0} T_{b}}{4}}}\right) \tag{77}
\end{equation*}
$$

where $\alpha_{0}$ is the possible path gain, $b_{0}$ is the transmitted bit $(+1$ or -1$)$ and $T_{b}$ is the time duration of data bit.

The unconditioned error probability is given by

$$
\begin{align*}
P(E) & =B E R=\int_{-\infty}^{+\infty} f_{\psi}(\psi) P(E \mid \psi) d \psi \\
& =\int_{-\infty}^{+\infty} f_{\psi}(\psi) Q\left(\frac{D_{0}-\psi}{\nu}\right) d \psi \tag{78}
\end{align*}
$$

where $f_{\psi}(\psi)$ is the probability function of MAI. The estimation of exact error probability is difficult ([13] and [22]) because the distribution function $f_{\psi}(\psi)$ is not always easy to define.

To have a simplified expression computationally a truncated approximation of error function $Q(x)$ using Fourier series [6]

$$
\begin{align*}
Q(x) & =\sum_{m=-\infty}^{+\infty} c_{m} \mathrm{e}^{j m \omega x} \\
& =\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega x}+\varepsilon(x) \\
& \simeq \sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega x} \tag{79}
\end{align*}
$$

where $\omega$ is the Fourier series fundamental frequency and $c_{m}$ are the Fourier series coefficients given by [40] and [38]

$$
c_{m}=\left\{\begin{array}{lll}
\frac{1}{j 2 m} \mathrm{e}^{-m^{2} \omega^{2} / 2} & m>0 & m \text { odd }  \tag{80}\\
0 & m>0 & m \text { even } \\
1 / 2 & m=0 &
\end{array}\right.
$$

Using this approximation the expression (79) can be

$$
\begin{equation*}
Q(x) \simeq \frac{1}{2}-\frac{2}{\pi} \sum_{m=1, m O D D}^{+A} \frac{\exp \left(-m^{2} \omega^{2} / 2\right) \sin (x)}{m} \tag{81}
\end{equation*}
$$

The accuracy of this technique is clearly bounded by the truncation of the infinite series. Larger is the number of considered terms lesser is the error of the approximation [6]. To have negligible error with error probability greater than or equal to $10^{-9}$ the first 21 terms of Fourier expansion of $Q(x)$ must be considered and the fundamental frequency must be $\omega=\pi / 25$ [35].

Using the approximation of $Q(x)$ (79) the equation for error probability (78) is given by

$$
B E R=\int_{-\infty}^{+\infty} f_{M}(M)\left\{\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \frac{D_{0}-M}{\nu}}+\varepsilon\left(\frac{D_{0}-M}{\nu}\right)\right\} d M
$$

$$
\begin{align*}
& =\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \frac{D_{0}}{\nu}} \int_{-\infty}^{+\infty} f_{M}(M) \mathrm{e}^{-j m \omega \frac{M}{\nu}} d M \\
& +\int_{-\infty}^{+\infty} f_{M}(M) \varepsilon\left(\frac{D_{0}-M}{\nu}\right) d M \\
& =\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \frac{D_{0}}{\nu}} \int_{-\infty}^{+\infty} f_{M}(M) \mathrm{e}^{-j m \omega \frac{M}{\nu}} d M+\operatorname{Err}(M) \tag{82}
\end{align*}
$$

The swap between the integral and the sum can be done because the series that approximates $Q(x)$ converges uniformly to a finite limit [31].

Using the definition of characteristic function

$$
\begin{equation*}
\Phi_{X}(\omega)=E\left[\mathrm{e}^{j \omega X}\right]=\int_{-\infty}^{+\infty} f_{X}(X) \mathrm{e}^{j \omega X} d X \tag{83}
\end{equation*}
$$

the error probability is given by

$$
\begin{align*}
B E R & =\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \frac{D_{0}}{\nu}} \Phi_{M}\left(-\frac{m \omega}{\nu}\right)+\operatorname{Err}(M) \\
& =\sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \frac{\sqrt{\frac{P_{0}}{2}} b_{0} T_{b}}{\sqrt{\frac{N_{0} T_{b}}{4}}} \Phi_{M}\left(-\frac{m \omega}{\sqrt{\frac{N_{0} T_{b}}{4}}}\right)+\operatorname{Err}(M)} \\
& \simeq \sum_{m=-A}^{+A} c_{m} \mathrm{e}^{j m \omega \sqrt{\frac{2 P_{0} b_{0}^{2} T_{b}}{N_{0}}}} \Phi_{M}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) \tag{84}
\end{align*}
$$

For AWGN channel and single service system, the interference on m-th bit of user 0 is

$$
\begin{equation*}
M(m)=\sum_{k=1}^{K_{u}-1} I_{k}(m) \tag{85}
\end{equation*}
$$

with

$$
\begin{align*}
I_{k}(m) & =\int_{m T_{b}}^{(m+1) T_{b}} s_{k}\left(t-\tau_{k}\right) \mathrm{e}^{j \phi_{k}} a_{0}\left(t-\tau_{0}\right) \cos \left(\omega_{c} t\right) d t \\
& =\sqrt{\frac{P_{k}}{2}} T_{b} a_{0}(m) \omega_{k} \cos \phi_{k}(m)=A_{k}(m) T_{b} \cos \phi_{k}(m) \tag{86}
\end{align*}
$$

where $A_{k}$ is a constant and $\phi_{k}$ is a random variable with distribution uniformly over $[0,2 \pi)$ that include the channel delay.

If the single interference terms are statistically independent, the characteristic function of $M$ is given by
$\Phi_{M}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right)=E\left\{\mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} \sum_{k=1}^{K_{u}-1} A_{k} T_{b} \cos \phi_{k}}\right\}$

$$
\begin{align*}
& =E\left\{\prod_{k=1}^{K_{u}-1} \mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \cos \phi_{k}}\right\}=\prod_{k=1}^{K_{u}-1} E\left\{\mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \cos \phi_{k}}\right\} \\
& =\prod_{k=1}^{K_{u}-1} \Phi_{I_{k}}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) \tag{87}
\end{align*}
$$

If the random variable $I_{k}$ is considered as a product of a constant $A_{k}$ and a random variable $\phi_{k}$, the function $\Phi_{I_{k}}$ is given by [6]

$$
\begin{align*}
\Phi_{I_{k}}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) & =\int_{-\infty}^{+\infty} f_{\phi_{k}}\left(\phi_{k}\right) \mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \cos \phi_{k}} d \phi_{k} \\
& =\frac{1}{2 \pi} \int_{0}^{2 \pi} \mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \cos \phi_{k}} d \phi_{k} \\
& =\frac{1}{\pi} \int_{0}^{\pi} \cos \left(\frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \cos \phi_{k}\right) d \phi_{k} \\
& =J_{0}\left(\frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b}\right) \tag{88}
\end{align*}
$$

where $J_{0}$ is Bessel function of the First Kind.
So the error probability for single service system with AWGN channel can be expressed as

$$
\begin{align*}
B E R \simeq & \frac{1}{2}-\frac{2}{\pi} \sum_{m=1, m O D D}^{+A}\left[\frac{\exp \left(-m^{2} \omega^{2} / 2\right) \sin \left(m \omega \sqrt{\frac{2 P_{0} T_{b} b_{0}^{2}}{N_{0}}}\right)}{m}\right. \\
& \left.\cdot \prod_{k=1}^{K_{u}-1} J_{0}\left(\frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b}\right)\right] \tag{89}
\end{align*}
$$

### 3.7.2 Single Service and Flat Fading Channel

Analyzing still a single service system, if the environment is characterized by a flat fading channel, the received signal are not equal for all the users, but they are a Rayleigh process. The received signal can be expressed as product of a constant $A_{k}$ (previous section) and a Rayleigh variable $\alpha_{k}$.

In this case the expression for error probability is the same, but the expression of characteristic function of MAI changes. Considering the signal of k-th user as

$$
I_{k}=A_{k} T_{b} \alpha_{k} \cos \phi_{k}
$$

and that random variables $\phi_{k}$ and $\alpha_{k}$ are statistically independent and using the series expansion of Bessel function

$$
\begin{equation*}
J_{0}=\sum_{i=0}^{\infty} \frac{(-1)^{i}(x / 2)^{2 i}}{(i!)^{2}} \tag{90}
\end{equation*}
$$

the characteristic function of $I_{k}$ is given by

$$
\begin{align*}
\Phi_{I_{k}}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) & =\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{\phi_{k}}\left(\phi_{k}\right) f_{\alpha_{k}}\left(\alpha_{k}\right) \mathrm{e}^{-j \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} A_{k} T_{b} \alpha_{k} \cos \phi_{k}} d \phi_{k} d \alpha_{k}= \\
& =\sum_{i=0}^{\infty} \frac{(-1)^{i}}{i!} \frac{1}{2^{2 i+1}}\left(\frac{2 m \omega A_{k} T_{b} \sigma_{\alpha}}{\sqrt{N_{0} T_{b}}}\right)^{i} \tag{91}
\end{align*}
$$

For single service system and flat fading channel the complete expression for BER is

$$
\begin{align*}
B E R & \simeq \frac{1}{2}-\frac{2}{\pi} \sum_{m=1, m O D D}^{+A}\left\{\frac{\exp \left(-m^{2} \omega^{2} / 2\right) \sin \left(m \omega \sqrt{\frac{2 P_{0} T_{b} b_{0}^{2}}{N_{0}}}\right)}{m}\right. \\
& \left.\cdot \prod_{k=1}^{K_{u}-1}\left[\frac{1}{2} \sum_{i=0}^{\infty} \frac{(-1)^{i}}{i!2^{2 i+1}}\left(\frac{2 m \omega A_{k} T_{b} \sigma_{\alpha}}{\sqrt{N_{0} T_{b}}}\right)^{i}\right]\right\} \tag{92}
\end{align*}
$$

Also in this case, this expression is an approximation of exact value. The accuracy is bounded by the truncation of infinite series.

### 3.7.3 Multi-service System

If the system is able to support $N_{S}$ different class of service $\left\{s 1, s 2, \ldots, s_{N_{S}}\right\}$ with different bit rates, the interference contribution can be given by

$$
\begin{aligned}
M A I & =I=\sum_{j=1}^{N_{S}} \sum_{k \epsilon s_{j}} I_{k, j} \\
& =\sum_{j=1}^{N_{S}} \sum_{k \epsilon s_{j}} \int_{m T_{b}}^{(m+1) T_{b}} \alpha_{k} s_{k}\left(t-\tau_{k}\right) \mathrm{e}^{\jmath \phi_{k}} a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t
\end{aligned}
$$

With this note, the MAI include all the received signal from BS excluding the useful contribution (user 0).

Without loss of generality, only two users ( 0 and $k$-th) are considered and each of them uses a different service. In this scenario , two possible cases can be thought.
I) $T_{b}>T_{b I}$

The period of data bit of user 0 is greater than the one of $k$-th user (interference user) that uses the j -th service. The interference expression for the term of this user can be expressed as

$$
\begin{aligned}
I_{k, j} & =\int_{m T_{b}}^{(m+1) T_{b}} \alpha_{k} s_{k}\left(t-\tau_{k}\right) \mathrm{e}^{\jmath \phi_{k}} a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t \\
& =\sum_{n=0}^{L S-1} \int_{m T_{b}+n T_{b I}}^{m T_{b}+(n+1) T_{b I}} \alpha_{k} s_{k}\left(t-\tau_{k}\right) \mathrm{e}^{\jmath \phi_{k}} a_{0}\left(t-\tau_{0,0}\right) \cos \left(\omega_{c} t\right) d t
\end{aligned}
$$

$$
\begin{equation*}
=\sum_{n=0}^{L S-1} I_{k, j, n} \tag{93}
\end{equation*}
$$

with

$$
L S= \begin{cases}\left\lceil T_{b} / T_{b I}\right\rceil & \text { underrate of } I_{k, j} \\ \left\lfloor T_{b} / T_{b I}\right\rfloor & \text { overrate of } I_{k, j}\end{cases}
$$

II) $T_{b}<T_{b I}$

The period of data bit of user 0 is lesser than the one of $k$-th user (interference user) that uses the $j$-th service. The interference expression for the term of this user is equal to the expression for a single service system.

For $I$ ), when $T_{b}>T_{b I}$, the characteristic function of MAI can be given by

$$
\begin{align*}
\Phi_{I}(x) & =E\left\{\mathrm{e}^{-j x I}\right\}=E\left\{\mathrm{e}^{-j x \sum_{j=1}^{N_{S}} \sum_{k \epsilon s_{j}} I_{k, j}}\right\} \\
& =\prod_{j=1}^{N_{S}} \prod_{k \in s_{j}} \prod_{n=0}^{L S-1} \Phi_{I_{k, j, n}}(x) \tag{94}
\end{align*}
$$

Over the interval of integration $T_{b}$ the useful signal has a phase delay and an attenuation, $\phi_{0}$ and $\alpha_{0}$ respectively. Over the same interval integration there are different bits for the k -th user and the phase delays $\phi_{k}(m)$ and attenuation coefficients $\alpha_{k}(m)$ can be considered the same for each bit in the interval of integration. So the signal of k-th user over the interval of integration is characterized by two value $\phi_{k}$ and $\alpha_{k}$. With this hypothesis the term $\Phi_{I_{k, j, n}}$ is independent from $n$, so the characteristic function of MAI is given by

$$
\begin{equation*}
\Phi_{I}(x)=\prod_{j=1}^{N_{S}} \prod_{k \in s_{j}}\left[\Phi_{I_{k, j}}(x)\right]^{L S} \tag{95}
\end{equation*}
$$

This can be calculated as showed before, using the proper channel profile.

### 3.7.4 Imperfect Power Control with AWGN Channel

The previous analysis is made using a perfect power control. If this hypothesis is not ideal the received power is not equal for each user: it can be considered as a random variable with a probability function. All this distribution have a media equal to the received power with perfect power control.

## Uniform Distribution

With a distribution as

$$
f\left(A_{k}\right)=\frac{1}{2 V} \quad A_{0}-V \leq A_{k} \leq A_{0}+V
$$

the characteristic function is given by

$$
\begin{align*}
\Phi_{I_{k}}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) & =\int_{-\infty}^{+\infty} f_{A_{k}}\left(A_{k}\right) J_{0}\left(\frac{2 m \omega A_{k} T_{b}}{\sqrt{N_{0} T_{b}}}\right) d A_{k} \\
& =\frac{1}{2 V} \frac{\sqrt{N_{0} T_{b}}}{2 m \omega} \sum_{i=0}^{\infty} \frac{(-1)^{i}}{(i!)^{2} 2^{2 i}(2 i+1)}\left(x_{s}^{2 i+1}-x_{i}^{2 i+1}\right) \tag{96}
\end{align*}
$$

where

$$
\begin{aligned}
x_{s} & =\left(A_{0}+V\right) T_{b} \frac{2 m \omega}{\sqrt{N_{0} T_{b}}} \\
x_{i} & =\left(A_{0}-V\right) T_{b} \frac{2 m \omega}{\sqrt{N_{0} T_{b}}}
\end{aligned}
$$

As in the previous analysis, to solve the integral a truncated series expansion of Bessel function of first kind is used.

## Gaussian Distribution

With a distribution as

$$
f\left(A_{k}\right)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{\left(A_{k}-\mu\right)^{2}}{2 \sigma^{2}}\right) \quad A_{k} \epsilon(-\infty,+\infty)
$$

the characteristic function is given by

$$
\begin{align*}
\Phi_{I_{k}}\left(-\frac{2 m \omega}{\sqrt{N_{0} T_{b}}}\right) & =\int_{-\infty}^{+\infty} f_{A_{k}}\left(A_{k}\right) J_{0}\left(\frac{2 m \omega A_{k} T_{b}}{\sqrt{N_{0} T_{b}}}\right) d A_{k} \\
= & \frac{1}{\sqrt{2 \pi}} \sum_{i=0}^{\infty} \frac{(-1)^{i}}{(i!)^{2}}\left(\frac{2 m \omega}{2 \sqrt{N_{0} T_{b}}}\right)^{2 i} \\
& \cdot \sum_{l=0, \text { leven }}^{2 i}\binom{2 i}{l} \mu^{2 i-l}\left(2 \sigma^{2}\right)^{(l+1) / 2} \Gamma\left(\frac{l+1}{2}\right) \tag{97}
\end{align*}
$$

The function is calculated using a truncated series expansion of Bessel function of first kind $J_{0}(90)$ (sum on index $i$ ), the generic expression of n-power of binomial (sum on index $l$ ) and

$$
\int_{-\infty}^{+\infty} \mathrm{e}^{y^{2}} y^{a} d y= \begin{cases}0 & \text { lodd } \\ 2 \int_{0}^{+\infty} \mathrm{e}^{y^{2}} y^{a} d y & \text { l even }\end{cases}
$$

(sum on even index $l$ )
This method is mathematical hopeful, but unfortunately it has problems about numerical instability. Maybe this is caused by truncation of Fourier and Bessel series expansions.

## 4 Numerical Results

The expressions for SIR and BER found in previous sections are analyzed with MATLAB: here the graph will be shown. Different environment with or without fading are considered and for each of them perfect, imperfect and absent power control are account.

All the simulations are made using the following values:

- Chip rate $R_{c}=3.84 \mathrm{Mchip} / \mathrm{s}$
- Nominal received power $P_{x}=1$
- Signal to noise ratio

$$
E_{b} / N_{0}= \begin{cases}10 & \mathrm{~dB} \\ 20 & \mathrm{~dB} \\ 30 & \mathrm{~dB}\end{cases}
$$

- Process gain

$$
S F=\left\{\begin{array}{l}
10 \\
64 \\
128 \\
256
\end{array}\right.
$$

### 4.1 Channel without Fading

## Perfect Power Control

For SGA algorithm the equations (23) and (22) with $N_{0}=0$ are used for SIR and BER respectively for interference-limited systems. While the same equations are used with $N_{0} \neq 0$ for SIR and BER respectively, if the noise is also considered.

For SEIGA algorithm the equations (58) and (59) are used with $N_{0}=0$ for SIR and BER respectively for interference-limited systems. While same the equations are used with $N_{0} \neq 0$ for SIR and BER respectively, if the noise is also considered. In this last case, to simulate a perfect power control the media (60) and variance (61) of MAI $\psi$ are computed with $\mu_{p}=1$ and $\sigma_{p}^{2}=0$.

## Imperfect Power Control

For SGA algorithm the equations (38)and (39) are used for SIR and BER respectively considering both interference and noise; if interference cells and multi-path are not considered $N_{c}=0$ and $M=0$. The amplitude of the received signals is a random variable uniformly distributed (37). In graphics the value $k=V / A_{0}$ will be indicated, where $A_{0}$ is the mean amplitude of received signals (and the mean of distribution) and $V$ is the semi-amplitude of uniform distribution.

For SEIGA algorithm the equations (58) and (59) are used for SIR and BER respectively, considering both interference and noise. In this last case, to simulate an imperfect
power control the media (60) and variance (61) of MAI $\psi$ are computed using the values of $\mu_{p}$ and $\sigma_{p}^{2}$ obtained by (72) and (73).

For each model the parameter $k$ takes three different value: $0.4,0.7$, and 1 .
In each graphic there are plotted traces provided by equations (23) and (22) for SGA with perfect power control $\left(P_{k}=P_{0}\right)$ and (58) and (59) for SEIGA considering perfect power control and noise. These traces are labeled as "sga-ppc-n" and "seiga-ppc-n".

## Absent Power Control

For SGA algorithm the equations (49)and (50) are used for SIR and BER respectively considering both interference and noise.

For SEIGA algorithm the equations (58) and (59) are used for SIR and BER respectively considering both interference and noise. To simulate an absent power control the media (60) and variance (61) of MAI $\psi$ are computed using the values of $\mu_{p}$ and $\sigma_{p}^{2}$ obtained by (74) and (75).

In each graphic there are plotted traces provided by equations (23) and (22) for SGA with perfect power control $\left(P_{k}=P_{0}\right)$ and (58) and (59) for SEIGA considering perfect power control and noise. These traces are labeled as "sga-ppc-n" and "seiga-ppc-n".

To estimate mean and variance of amplitude and power of received signals, the pathloss exponent $n=4$, cell radius $R_{c}=3000 \mathrm{~m}$ and minimum distance between transmitting and receiving antennas $r_{0}=20 \mathrm{~m}$.

The SEIGA approximation for BER cannot be calculated, because one of the terms under the square root results always negative.

However, one of the fundamental hypotheses under which the SEIGA approximation is developed, is that the system does perfectly implement the power control. When this hypotheses is not satisfied, as in absent power control case, not only the SEIGA approximation would lack in precision, but even its computation results impossible.

### 4.1.1 Perfect Power Control



Figure 2: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 3: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 4: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 5: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 6: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 7: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 8: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 9: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 10: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 11: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 12: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 13: SIR (a) e BER (b) over a non-fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

### 4.1.2 Imperfect Power Control



Figure 14: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 15: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 16: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 17: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 18: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 19: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 20: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 21: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 22: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 23: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 24: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 25: SIR (a) e BER (b) over a non-fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

### 4.1.3 Absent Power Control



Figure 26: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 27: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 28: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 29: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 30: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 31: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 32: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 33: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 34: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 35: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 36: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

### 4.2 Channel with Fading

## Perfect Power Control

For SGA algorithm the equations (24) and (25) are used for SIR and BER respectively, considering both interference and noise for a system in a fading environment but without multi-path ( $\mathrm{M}=1$ ) ("sga-pp- con fading"). The equations (29) and (30) for SIR and BER respectively, are used to trace graphics about performances of a system in a fading environment with multi-path ( $\mathrm{M}=4$ ) ("sga-ppc con fading multi-path"). In both of these case a classic receiver is used. The equations (31) and (32), for SIR and BER respectively, are used to trace graphics about performances of a system in a fading environment with multi-path $(M=4)$ and with a Rake receiver.

For SEIGA algorithm the equations (63) and (64) are used, for SIR and BER respectively, considering both interference and noise. To simulate a perfect power control the mean and variance of MAI $\psi$ are computed using (65) and (65). This approximation does not permit to study the performance of a system in a multi-path environment and/or a Rake receiver.

In this analysis the fading variance is $\sigma^{2}=1$ and the cell is isolated $N_{c}=0$. In both approximations the SIR and BER are computed using a Montecarlo method: many realization are repeated using values of $\alpha_{k}$ generated by a Rayleigh process and at last an average is done on the found results.

In all graphics (both for perfect power control and for imperfect and absent power control) there are plotted traces provided by equations (23) and (22) for SGA SIR and BER with perfect power control ( $P_{k}=P_{0}$ ) and (58) and (59) for SEIGA SIR and BER considering perfect power control and noise. These traces are labeled as "sga-ppc-no fading" and "seiga-ppc-no fading".

## Imperfect Power Control

For SGA algorithm the equations (38) and (39) are used for SIR and BER respectively, considering both interference and noise for a system in a fading environment with multipath $(M=4)$ ("sga-ppc multi-path"). In both of these case a classic receiver is used. The equations (40) and (41), for SIR and BER respectively, are used to trace graphics about performances of a system in a fading environment with multi-path $(M=4)$ and with a Rake receiver ("sga-ppc multi-path rake").

For SEIGA algorithm the equations (63) and (64) are used, for SIR and BER respectively, considering both interference and noise. To simulate an imperfect power control the media (60) and variance (61) of MAI $\psi$ are computed using the values of $\mu_{p}$ and $\sigma_{p}^{2}$ obtained by (70) and (71). As in perfect power control case, this approximation does not permit to study the performance of a systems in a multi-path environment and/or a Rake receiver.

With these two approximations for imperfect power control, the employed semiamplitude of interval of uniform distribution is $k=0.7$.

## Absent Power Control



Figure 37: SIR (a) e BER (b) over a non-fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

For SGA algorithm the equations (49)and (50) are used for SIR and BER respectively considering both interference and noise.

For SEIGA algorithm the equations (63) and (64) are used for SIR and BER respectively considering both interference and noise. To simulate an absent power control the media (70) and variance (71) of MAI $\psi$ are computed using the values of $\mu_{p}$ and $\sigma_{p}^{2}$ obtained by (74) and (75).

To estimate mean and variance of amplitude and power of received signals, the pathloss exponent $n=4$, cell radius $R_{c}=3000 \mathrm{~m}$ and minimum distance between transmitting and receiving antennas $r_{0}=20 \mathrm{~m}$.

As in case of no-fading channel, with absence of power control the SEIGA approximation for BER cannot be calculated, because one of the terms under the square root results always negative.

### 4.2.1 Perfect Power Control



Figure 38: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 39: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 40: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 41: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 42: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 43: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 44: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 45: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 46: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 47: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 48: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 49: SIR (a) e BER (b) over a fading channel with perfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

### 4.2.2 Imperfect Power Control



Figure 50: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 51: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 52: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 53: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 54: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 55: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 56: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 57: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 58: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 59: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 60: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 61: SIR (a) e BER (b) over a fading channel with imperfect power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

### 4.2.3 Absent Power Control



Figure 62: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 63: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 64: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 65: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=10 \mathrm{~dB}$.


Figure 66: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 67: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 68: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 69: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=20 \mathrm{~dB}$.


Figure 70: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=10$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 71: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=64$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 72: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=128$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.


Figure 73: SIR (a) e BER (b) over a fading channel with no power control, processing gain $G_{p}=256$ and signal to noise ratio $E_{b} / N_{0}=30 \mathrm{~dB}$.

## 5 Conclusions

In this work the problem of evaluation of interference in systems which use a spread spectrum technique to access the medium. In detail systems as UMTS, which uses a DS-CDMA technique in the UTRAN, are analyzed. The main factors that limit capacity and performances of systems are the available bandwidth and the interference. The target of this work is not the description of a detailed and always valid approximation for the interference, but it is a more organic vision of different possible approximation for the MAI (Multiple Access Interference) caused by presence of other users in the same cell of useful user or in neighbor cell.

An estimation of different techniques proposed in literature is much complex because simulative studies and accurate measurement are difficult to do. The results get from the different approaches do not allow to estimate which of them is better, because the performance of a real system can not be estimated.

For SGA approximation the SIR (Signal to Interference Ratio) and BER (Bit Error Rate) are valid only if the number of active user $K_{u}$ in a cell is high, because the Central Limit Theorem is used. Furthermore this theorem requires the variables are identically distributed and this requirement is satisfied only if every user use the same service and if the power control is perfect. If one of these two is not satisfied the approximation is inappropriate and the provided results can be wrong because the MAI (Multiple Access Interference) is not so accurately modeled as a Gaussian random variable.

For SGA approximation the SIR and BER for a single user are computed as deterministic measure of averaged environmental conditions. A different approach can be followed by calculating the averaged SIR and BER over environmental statistics. This approach yields to an IGA (Improved Gaussian Approximation) method and maybe it produces an better performances than SGA. The IGA approximation can be used only if the characteristic function of MAI is known and has a heavy limit: a big computational time to estimate SIR and BER. To obtain better computational performance the SEIGA (Simplified Expression of IGA) can be used: it is based on same concepts of IGA, introduces an approximation using Taylor's series expansion and uses differences rather than derivatives. SEIGA approximation has the same limits of SGA approximation because it is based on the same hypothesis of SGA approximation.

As far as the BER is concerned, the SGA and SEIGA approximation have similar behavior for large number of user. In other case the error probability differ for some orders. In none of these case is possible to declare which is the best approximation. In the first case, when the two methods provide the same performance, the results can be not reliable, because both of the methods suffering from the introduced approximations. In the second case, when the two methods provide different performances, it's not possible to set the best one because a show of real system is not available. The two found traces can be upper bounds (one better than the other), lower bounds (also in this case, one better than the other) or can be an upper bound and a lower bound. In all these cases, we do not know where the real trace can be plotted.

In literature, there are no comparison among different approximations in terms of SIR.

Analyzing graphics, it's easy to note that SGA provides always higher SIR than SEIGA.
It's important to note that SIR and BER calculated using an imperfect or absent power control (both SGA and SEIGA) are not completely right. These approximation are based on Gaussian hypothesis and Central Limit Theorem (a large number of identically distributed random variables can be approximated by a Gaussian random variable with zero mean and variance which is the sum of variance of all the variables) and since, in case with imperfect or absent power control, the amplitudes of received signals are not perfect equalized the Gaussian supposition for MAI is not completely right.

Although the Gaussian hypothesis could appear a strained operation for some situations (for real situations), it seems however the unique practical, very flexible and very computationally efficient method that has been provided in order to give an estimate of the interference effects in DS-CDMA systems. This approximation is largely used because it leads fundamental simplification of problem formulation, avoiding tedious and cost-inefficient simulations.

One of the biggest limits of Gaussian approximation is the impossibility to manage multi-service systems: as in case of non perfect power control, when users employs different services the Gaussian assumption of MAI is not completely right. For multi-service systems the approximation based on Fourier series expansion can be used, because it allows to split the MAI into different blocks to analyze separately. Unfortunately, with this algorithms meaningful results are not found: maybe the cause can be the approximations introduced with the truncation of Bessel and Fourier series.

## References

[1] 3GPP TS 25.213, v.3.3.0, Spreading and Modulation (FDD), 2000.
[2] 3GPP TS 25.214, v.3.4.0, Physicla Layer Procedures (FDD), 2000.
[3] 3GPP TS 25.215, v.3.4.0, Physicla Layer Measurement (FDD), 2000.
[4] 3GPP TS 25.223, v.3.4.0, Spreading and Modulation (TDD), 2000.
[5] 3GPP TS 25.401, v.3.4.0, UTRAN Overall Description, 2000.
[6] N.C. Beaulieu, The Evaluation of Error Probabilities for Intersymbol and Cochannel Interference, IEEE Trans. on Commmunication, Vol. 39, No. 12, pp. 1740-1849, December 1991.
[7] D.Collins, C.Smith 3G Wireless Networks, New York, Mc Graw Hill, 2002.
[8] G.M. Comparetto, W.A. Foose, An Evaluation of the Gaussian Approximation Technique Applied to the Multiple Input Signal Case of an Ideal Hard Limiter, Military Communication Conference - MILCOM IEEE, October 1990.
[9] G.R. Cooper, C.D. McGillem, Probabilistic Methods of Signal and System Analysis, Holt, Rinehart and Windston, New York, 1986.
[10] G.E. Corazza, G. De Maio, F. Vatalano, CDMA Cellular Systems Performance with Fading, Shadowing, and Imperfeect Power Control, IEEE Trans. on Vehicular Technology, Vol. 47, No. 2, May 1998.
[11] E. Falletti, F. Vipiana, R. Lo Cigno, On SIR \& BER Approximation in DS-CDMA System, Politecnico di Torino, June 2001.
[12] S.V. Hanly, Information Capacity of Radio Networks, PhD Thesis, University of Cambridge, 1993.
[13] C.W. Helstrom, Calculating Error Probabilities for Intersymbol and Cochannel Interference, IEEE Trans. on Commmunication, Vol. COM-24, pp. 430-435, May 1986.
[14] H. Holma, A. Toskala, WCDMA for UMTS: Radio Access for Third Generation Mobile Communication, New York: Wiley, 2000.
[15] J.M. Holtzman, A Simple, Accurated Method to Calculate Spread-Spectrum Multiple-Access Error Probabilities, IEEE Trans. on Commmunication, Vol. 40, No. 3, March 1992.
[16] R. Hoppe, P. Wert, G. Wolfle, F.M. Landstorfer, Consideration of Topography for Wave Propagaztion Modeling in Urban Scenarios, IEEE Trans. on Commmunication, Vol. 49, 2001.
[17] S. Irons, C. Johnson, A. King, D. McFarlane, Supporting the Successful Deployment of Third Generatio Public Cellular, 3G Mobile Communication Technologies Conference, no. 471, London, March 2002.
[18] J.S. Kaufman, Blocking in a Shared Resource Environment, IEEE Trans. on Commmunication, Vol. COM-29, No. 10, October 1981.
[19] J. Korhonen Introduction to $3 G$ Mobile Communications, Norwood, MA: Artech Housel, 2001, pp 507-514.
[20] J. C. Liberti and T. S. Rappaport, Accurate Techniques to evaluate CDMA Bit Error Rates in Multipath Channels with Imperfect Power Control, IEEE Global Telecommunications Conference, 1995, p. 33-37.
[21] J. C. Liberti Jr., CDMA Cellular Communication Systems Employing Adaptative Antenas, Preliminary Draft of Research Including Literature Review and Summary of Work-in-Progress, Virginia Tech, March 1994.
[22] R.W. Lucky, J. Salz, E.J. Weldon Jr., Principles of Data Communication, New York, McGraw-Hill, pp 63-65, 1968.
[23] J. Mar, H. Chen, Performance Analysis of Cellular CDMA Networks Over Frequency-Selective Fading Channel, IEEE Trans. on Vehicular Technology, Vol. 47, No. 4, November 1998.
[24] W. Matragi, S. Nanda, Capacity Analysis of an Integrated Voice adn Data CDMA System, IEEE Trans. on Vehicular Technology, Vol. 47, No. 2, May 1998.
[25] A. Mehrotra GSM Sysyem Engineering, Norwood, MA: Artech Housel, 1997.
[26] T. Ojanpera, R. Prasad, Wideband CDMA for Third Generation Mobile Communication, Norwood, MA: Artech Housel, pp 34-36, 1998.
[27] R. Prasad, W. Mohr, W. Konhauser Third Generation Mobile Communication Systems, Norwood, MA: Artech Housel, 2000.
[28] M.B. Pursley, Performance Evaluation for Phase-Coded Spread-Spectrum MultipleAccess Communication - Part I: System Analysis, IEEE Trans. on Commmunication, Vol. COM-25, No. 8, August 1977.
[29] M.B. Pursley, D.V. Sarwate, Performance Evaluation for Phase-Coded SpreadSpectrum Multiple-Access Communication - Part II: Code Sequence Analysis, IEEE Trans. on Commmunication, Vol. COM-25, No. 28, August 1977.
[30] T. S. Rappaport, Wireless Communication: Principles \& Practice, Prentice Hall PTR, New Jersey, 1996.
[31] W. Rudin, Principles of Mathematical Analysis, New york, McGraw-Hill, pp 184185, 1964.
[32] A. Sampath, N.B. Mandayam, J.M. Holtzman, Erlang Capacity of a Powered Controlled Integrated Voice and Data CDMA System, IEEE Trans. on Vehicular Technology, 1997.
[33] A. Sampath, P. Sarath Kumar, J.M. Holtzman, Power Control and Resource Management for a Multimedia Wireless CDMA System, Proc. PIMRC'95, Canada, September 1995.
[34] H. Stark, J.W. Woods, Probability, Random Variables and Estimation Theory for Engineers, Prentice Hall, Englewood Cliffs, N.J., 1986.
[35] M.O. Sunay, P.J. McLane, Calculating Error Probabilities for DS CDMA Systems: When Not to Use the Gaussian Approximation, IEEE Global Telecommmunications Conference, 1996.
[36] M.O. Sunay, P.J. McLane, Comparison of Biphase Spreading to Quadriphase Spreading in DS CDMA Systems that Employ lLong PN Sequences, Proc. IEEE PIMRC'95, Toronto, Canada, September 1995.
[37] M.O. Sunay, P.J. McLane, Diversity Combining for DS CDMA System with Synchronization Errors, Proc. IEEE ICC'96, Dallas, June 1996.
[38] M.O. Sunay, P.J. McLane, Effects of Carrier Phase and Chip Timing Errors on the Capacity of a Quadriphase Spread BPSK Modulated DS CDMA System, Proc. IEEE GLOBECOM'95, Singapore, November 1995.
[39] M.O. Sunay, P.J. McLane, Performance of Selection Diversity for DS CDMA System with Synchronization Errors, Proc. IEEE ICUPC'95, Tokyo, Japan, November 1995.
[40] M.O. Sunay, P.J. McLane, Sensitivity of a DS CDMA System with Long PN Sequences to Synchronization Errors, Proc. IEEE ICC'95, Seattle, vol. 2, pp. 10291035, June 1995.
[41] B. R. Vojcic, R. L. Pickholtz, and L. B, Milstein, Performance of DS-CDMA with imperfect power control operating over a low earth orbiting satellite link, IEEE J. Selected Areas Communication, vol 12, pp. 560-567, May 1994.
[42] G. Wolfle, R. Hoppe, T. Binzer, F.M. Landstorfer, Radio Network Planning and Propagation Models for Urban andIndoor Wireless Communication Networks, IEEE Trans. on Commmunication, Vol. 49, 2000.
[43] J. Zhang, Link Budget Report for Wideband Testbed, IEEE Trans. on Vehicular Technology, March 2000.
[44] ftp.eurescom.de/~ public-web-deliverables/P900-series/ P921/D2/UMTS\%20Link\%20Budget.xls
[45] www.umtsworld.com/documents/WCDMA」link_budget.xls

