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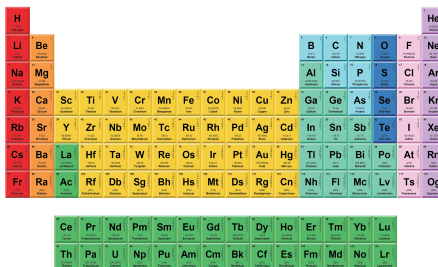
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Quantum states generation and manipulation in a programmable silicon-photonics four-qubit system with high-fidelity and purity

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Jong-Moo Lee,^{1,a)} Jiho Park,¹ Jeongho Bang,¹ Young-Ik Sohn,² Alessio Baldazzi,³ Matteo Sanna,³ Stefano Azzini,³ and Lorenzo Pavesi³

AFFILIATIONS

¹ ETRI, 218 Gajeong-ro, Daejeon 34129, South Korea

² KAIST, 291 Daehak-ro, Daejeon 34141, South Korea

³ Department of Physics, University of Trento, via Sommarive 14, 38123 Trento, Italy

^{a)} Author to whom correspondence should be addressed: jongmool@etri.re.kr

ABSTRACT

We present a programmable silicon photonic four-qubit integrated circuit for the generation and manipulation of diverse quantum states. The silicon photonic chip integrates photon-pair sources, pump-reducing filters, wavelength-division-multiplexing filters, Mach-Zehnder interferometer switches, and single-qubit arbitrary gates, enabling versatile state preparation and tomography. We measure Hong-Ou-Mandel interference with an impressive 98% visibility using four-photon coincidence, laying the foundation for high-purity qubits. Our analysis involves estimating the fidelity and purity of distinct quantum states through maximum-likelihood estimation applied to tomographic measurements. In our experimental results, we showcase the following achievements: a heralded single qubit achieving 98.2% fidelity and 98.3% purity, a Bell state reaching 95.2% fidelity and 94.8% purity, and a four-qubit system with two simultaneous Bell states exhibiting 87.4% fidelity and 84.6% purity. Finally, a four-qubit Greenberger-Horne-Zeilinger (GHZ) state demonstrates 85.4% fidelity and 81.7% purity. In addition, we certify the entanglement of the four-photon GHZ state through Bell's inequality violations and a negative entanglement witness.

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I. INTRODUCTION

Multipartite entanglement of photons on a photonic integrated chip (PIC) is a valuable resource for quantum communication and computing.^{1–7} PICs leverage compact, scalable designs for generating multipartite entangled states crucial in quantum algorithms and gates. Various material platforms have been considered for PICs, such as silica,³ silicon,^{4–9} and silicon nitride.^{10,11} Each one has advantages and disadvantages.^{12,13} Among these, quantum silicon photonics has attracted much interest due to the potential of manufacturing integrated devices within standard microelectronic foundries.¹⁴ Quantum silicon photonics is based on the integration of single photon sources (either deterministic^{11,15} or probabilistic¹⁶),

quantum circuits,⁸ and single photon detectors in a silicon PIC (SiPIC).¹⁷

Here, we showcase a SiPIC capable of preparing and manipulating four-photon path-encoded qubits. The SiPIC incorporates photon-pair sources, pump-reducing filters (PRFs),^{18,19} wavelength-division-multiplexing (WDM) filters, Mach-Zehnder interferometer (MZI) switches, and single-qubit arbitrary gates (Gates) for four-qubit tomography. In our previous work,¹⁸ we compared the visibility of two-photon Hong-Ou-Mandel (HOM) experiments with degenerate photon pairs from a pair of rings against a pair of spirals in the very same SiPIC. While 99% HOM visibility was demonstrated with degenerate photon pairs from the spirals, this visibility does not depend on the purity of the photon states because

the two sources are not independent. On the contrary, HOM visibility with two separable photons individually heralded by a four-photon coincidence also incorporates the purity of the quantum state. The new SiPIC, here discussed, allows measuring quantum state purity.

Integrated into the SiPIC, four waveguide spirals are utilized to generate high-purity four-photon qubits via non-degenerate spontaneous four-wave mixing (SFWM). Four-photon coincidence measurements involving four photons simultaneously generated by an equal number of photon-pair sources were carried out. We experimented on HOM interference,²⁰ Greenberger–Horne–Zeiling (GHZ) states,²¹ and Bell's inequality violations observed in the four-photon qubit states.²² These states were prepared and analyzed by adjusting the phase difference in the MZIs and implementing additional phase control for the Gates within the SiPIC.

With respect to our previous work,¹⁸ the improved design and fabrication enable the proper matching of design parameters, resulting in high-quality entangled photon sources. Integration of PRFs and the use of low-loss band-pass filters (BPFs) in front of the single-photon detectors reduce noise and improve the coincidence-to-accidental ratio (CAR). At the setup level, tailored BPFs (wide/narrow on the pump, high throughput on the idler/signal photons) are employed. Simultaneously, a spectrally

broad laser pump is used to increase the purity of the generated photons and the source brightness and to reduce the CAR. This is achieved by using a wide BPF on the pump laser and a narrow BPF on the idler/signal photons. These adjustments allow for reaching significant four-photon coincidence counts (4CC). Associated with the stability provided by the integrated SiPIC, this allows for performing four-photon tomography measurements, which require 81 sets of Pauli measurements. In addition, the SiPIC allows Pauli measurements to estimate the violation of general Bell's inequality for our four-photon entangled states.

Our work significantly improves the figures of merit for multipartite entangled states reported in previous reports based on SiPIC^{6,7} as well as showing that SiPIC is a valid contender for quantum photonics even when operated with probabilistic single photon sources. The paper is organized as follows: In Sec. II, a detailed description of the experimental setup is provided. In Sec. III, the results of the heralded HOM interference of our silicon-photonics four-qubit four-photon system are reported. Tomographic measurements assessing the fidelity and purity of the various quantum states are found in Sec. III, together with certification measurements of the generated entangled states. While a thorough discussion about the significance of our work is carried out in Sec. V, conclusions are finally drawn in Sec. VI.

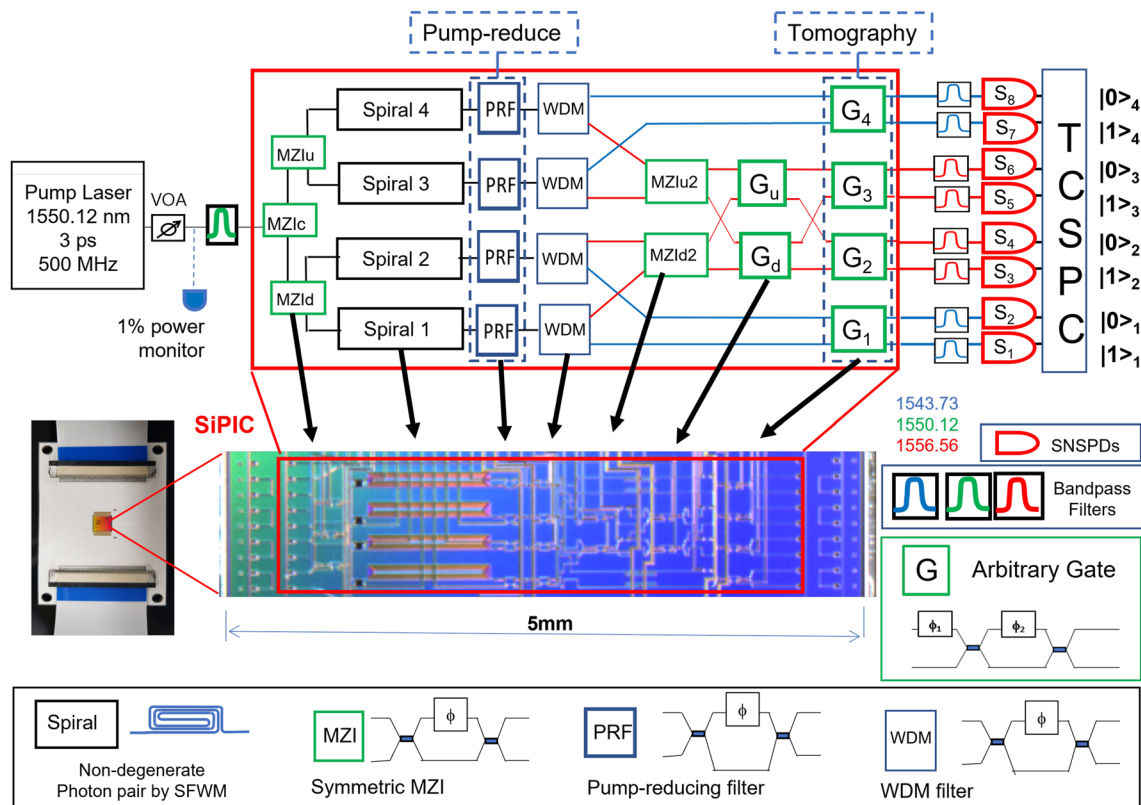


FIG. 1. Experimental setup to measure the quantum states of four-photon path-encoded qubits prepared using the programmable photonic circuits in the SiPIC. TCSPC refers to a time-correlated single photon counter. The last four Gates, labeled $G_1 \dots G_4$, are employed specifically for tomography purposes. Eight SNSPDs (superconducting nanowire single-photon detectors) are assigned for the four path-encoded qubits. For instance, S_1 is associated with $|1\rangle_1$, while S_2 corresponds to $|0\rangle_1$.

II. EXPERIMENTAL SETUP

Figure 1 shows the SiPIC and the experimental setup for measuring the four path-encoded qubits. Briefly, the integrated PRFs and WDMs are based on asymmetric MZIs, while the variable switches and arbitrary Gates are based on symmetric MZIs. The control of the MZIs and Gates allows us to manipulate the four-photon qubit states and route them along different optical paths. The waveguide crossings shown in Fig. 1 route the waveguides for a path-encoding signal separately from idler and fusion operations between two qubits. The waveguide crossing uses a two-step-etched taper structure, and the insertion loss of the waveguide crossing is typically less than 0.3 dB/crossing.

Five symmetric MZIs serve as variable switches, controlled by the phase ϕ in each MZI. In addition, four asymmetric MZIs function as PRFs to extract the residual pump photons after the waveguide spirals. The pass-band in each PRF is adjusted by tuning the phase ϕ .¹⁸ Furthermore, four more asymmetric MZIs act as WDM filters to separate the signal and idler photons. Similar to the PRFs, the transmission peak of the WDM filter is adjusted by controlling the phase ϕ . Six symmetric MZIs function as Gates to control the state of qubits. Two phases denoted as ϕ_1 and ϕ_2 enable the rotation of the quantum state on the corresponding Bloch sphere, analogous to applying the Pauli operators. The last four Gates ($G_1 \dots G_4$) are specifically employed for on-chip tomography of the 4 qubit state (see Sec. IV). The detailed characteristics of the waveguide structure, grating coupler, and MZIs in the SiPIC were described in our previous report.¹⁸

A mode-locked laser (UOC supplied by Pritel) operating around 1550.12 nm with a 3-dB bandwidth of 0.88 nm serves as the pump at a repetition rate of 500 MHz. The temporal width of the laser pulse is estimated at about 3 ps, taking into account the time-bandwidth product of 0.315.²³ The spectrum of the pump laser is shown in Fig. 2. A BPF labeled BPF400G_pump with a 3-dB bandwidth of 2.7 nm (shown in Fig. 2) is employed as a pump filter before the SiPIC to eliminate the noise floor of amplified spontaneous emission (ASE) in the laser. In Sec. III, we also utilized another BPF, BPF100G_pump, with a 3-dB bandwidth of 0.48 nm (shown in Fig. 2) instead of the BPF400G_pump. The substitution allowed us to compare the influence of the bandwidth of the pump light on HOM interference.²⁴

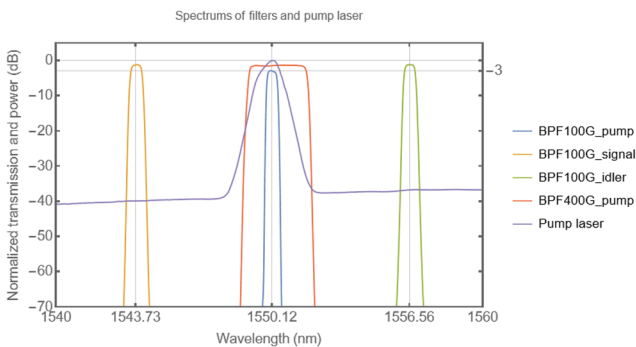


FIG. 2. Normalized transmission of the band-pass filters (BPFs) and the spectrum of the pump laser employed in the experimental setup in Fig. 1.

The output photons from the SiPIC are, first, filtered by BPFs designed for the signal centered at 1543.73 nm and the idler centered at 1556.56 nm, respectively, and, second, transferred through optical fibers to eight-channel superconducting nanowire single-photon detectors (SNSPDs, supplied by Single Quantum). The SNSPD single-photon events are counted by a time-correlated single photon counter (TCSPC, Logic16 from UQdevices) and analyzed by logical post-selection. Note that the 3-dB bandwidths of the BPFs are 0.53 nm (see the transmission spectra of BPF100G_signal and BPF100G_idler in Fig. 2). Consequently, the bandwidth of the pump laser is broader by 165% with respect to the bandwidth of the signal or idler photons. This broad bandwidth of the pump is a crucial factor for enhancing the spectral purity of the heralded single photons, as discussed in Sec. III.

III. FOUR-PHOTON COINCIDENT HOM INTERFERENCE

HOM interference²⁵ is a valuable method for characterizing the identity of two individual photons. In this context, we estimate the visibility of on-chip interference of two separable photons individually heralded by four-photon coincidence. The measured HOM visibility provides insight into the purity level of the quantum state in the SiPIC.¹⁶

A. Heralded single photons

Before assessing HOM interference, we conduct a thorough characterization of the heralded single photons, depending on the pump power. We used the signal photons (toward $|0\rangle_1$ or $|1\rangle_1$ in Fig. 1) as the heralder and the idler photons (toward $|0\rangle_2$ or $|1\rangle_2$ in Fig. 1) as the heralded photons. The results of this characterization are presented in Fig. 3, which includes the measured coincidence-to-accidental ratio (CAR),^{16,19} the two-photon coincidence rate, the $g^{(2)}(0)$ of the heralded photons, or $g_h^{(2)}(0)$,¹⁶ and finally the four-photon coincidence rate. The coincidence time window of the TCSPC is set to 0.4 ns through this experiment. We control the MZIs and Gates in Fig. 1 appropriately for each measurement, as shown in Table I.

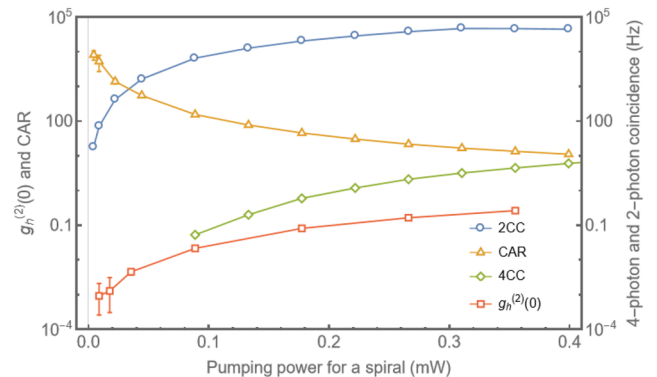


FIG. 3. Four-photon coincidence rate (4CC) and two-photon coincidence properties [heralding rates (2CC), CAR, and $g_h^{(2)}(0)$] measured with the photons from the SiPIC. The phase conditions for the measurements are shown in Table I.

TABLE I. Phase conditions to measure the heralded single photons (2CC: two-photon coincidence count and 4CC: four-photon CC).

	MZ1c	MZ1d	MZ1d2	$G_d(\phi_2)$	$G_1(\phi_1)$	$G_1(\phi_2)$	$G_2(\phi_1)$	$G_2(\phi_2)$
2CC								
and CAR	π	0	π	π	π	π	π	π
$g_h^{(2)}(0)$	π	0	π	π	π	π	π	$\pi/2$
4CC	π	$\pi/2$	π	π	π	π	π	π

The pump power in Fig. 3 is the average power coupled to a spiral, estimated from the 1% power monitor in Fig. 1. The peak power P_{peak} of the pump pulse can be roughly estimated using the relation $P_{peak} \simeq P_{average} / (500 \text{ MHz} \times 3 \text{ ps}) \simeq P_{average} \times 667$. Therefore, $P_{peak} \simeq 67 \text{ mW}$ when $P_{average} = 0.1 \text{ mW}$ in Fig. 3.

In Fig. 3, the CAR exhibits an increase to $\sim 10^4$, while $g_h^{(2)}(0)$ decreases to around 10^{-3} . The measured CAR and $g_h^{(2)}(0)$ closely align with the best-reported values from a SiPIC,²⁶ confirming the high quality of the single photons generated in this integrated circuit.

The two-photon coincident rate rises to 46 kHz with increasing pump power but saturates, as shown in Fig. 3. The main reason for the saturation is regarded as two-photon absorption (TPA), and the saturation starts gradually with low pumping power and becomes severe at 0.2 mW for a spiral. In addition, the four-photon coincident rate is shown as growing to 8 Hz as the average pump power rises to 0.4 mW per spiral. We must strike a balance between quality and coincident rate by optimizing the pump power. Accordingly, we fine-tuned the pump power to $\sim 0.1 \text{ mW}$ in average power per spiral for the following experiments of HOM interference and the tomography of entangled states.

B. Experiment and simulation on HOM interference

Figure 4 shows HOM interference measured by four-photon coincidence for 0.1 and 0.4 mW per spiral average pump power, respectively. The phase conditions for the HOM measurement are akin to the 4CC condition outlined in Table I, with the exception that the phase $G_2(\phi_2)$ is varied from 0 to 2π . The measured HOM visibility is 98% for the 0.1 mW pump power, while it decreases to 80% for the 0.4 mW, mainly due to degradation in the CAR at higher pump power.

A HOM visibility of 98% implies that the purity of the photon qubit can reach 98%, as the two heralded photons used in the HOM interference are separable, in contrast to the non-fully separable pair used in our previous report.¹⁸

Figure 4(b) shows the dependence of HOM visibility on CAR. The measured HOM visibility degrades as the CAR decreases with higher pump power, also because of multi-photon effects contributing to the increase in accidental events.

Another factor that influences the HOM visibility, i.e., the spectral purity of the heralded photons, is the spectral width of the pump laser.^{24,27,28} To verify this, we filtered the pump laser with two BPFs: the first labeled BPF400G_pump with a spectral bandwidth of 2.7 nm, and the second labeled BPF, BPF100G_pump, which has a 3-dB bandwidth of 0.48 nm. The first has a bandwidth larger than the laser bandwidth, while the second is narrower than the laser bandwidth.

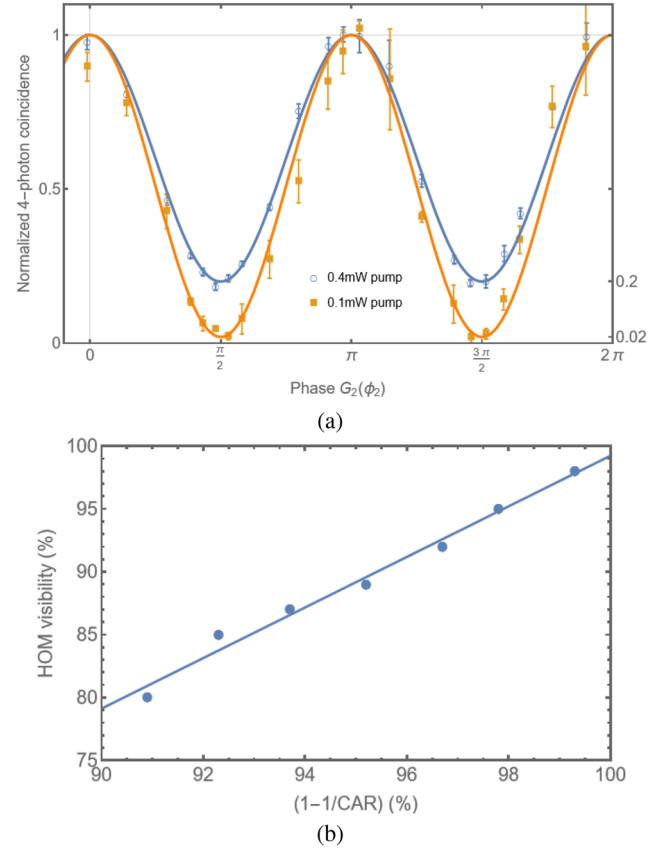


FIG. 4. HOM interference measured by four-photon coincidence for the average pump powers of 0.1 and 0.4 mW per spiral, respectively, in (a). Measured HOM interference visibility depending on CAR in (b). The measured visibility is 98% for the 0.1 mW pump; however, it declines to 80% at the 0.4 mW pump due to degradation in both CAR and $g_h^{(2)}(0)$.

Figure 5 shows the results of HOM interference measurements conducted with the utilization of two different BPFs for comparison. In particular, this pertains to two different ratios of the spectral bandwidth of the pump laser, denoted as Pbw, to the detected photon BPF bandwidth, denoted as Sbw. The HOM visibility decreases to 90% for Pbw/Sbw at 90%, compared to 98% for Pbw/Sbw at 165%. Note that the average pump power used was not identical, but it was optimized for the best HOM visibility. In particular, it was 0.2 mW for the BPF100G_pump case and 0.1 mW for the BPF400G_pump case. Considering the time-bandwidth product,²³ the two peak powers were effectively close.

Given the pump laser bandwidth, the filter response, and the phase matching parameter, we simulate the Joint Spectral Intensity (JSI) of the generated bi-photon state.^{16,18,29}

We point out that the bandwidth of the detection filters centered at 1543.73 and 1556.56 nm coincides with the bandwidth of the generated twin photons at such wavelengths. Indeed, the intramodal SFWM in the spiral generates the pairs in a continuous interval around the pump energy, and the detection filters determine their bandwidth, purity, and generation efficiency.^{24,27}

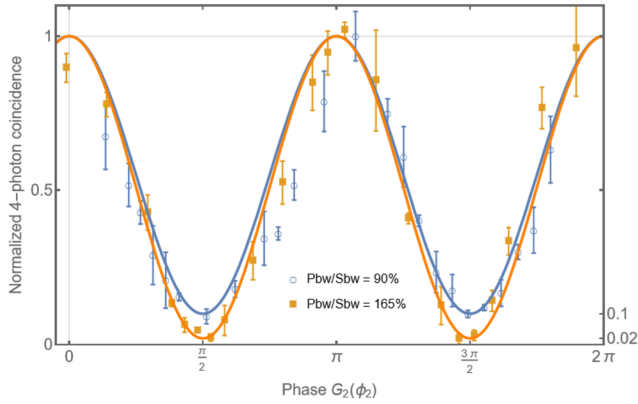


FIG. 5. HOM interference measured by four-photon coincidence depending on the ratio of the bandwidth of the pump laser (Pbw) to the detected signal and idler photon (Sbw). The HOM visibility is enhanced as the bandwidth of the pump laser is wider than the detected photon.

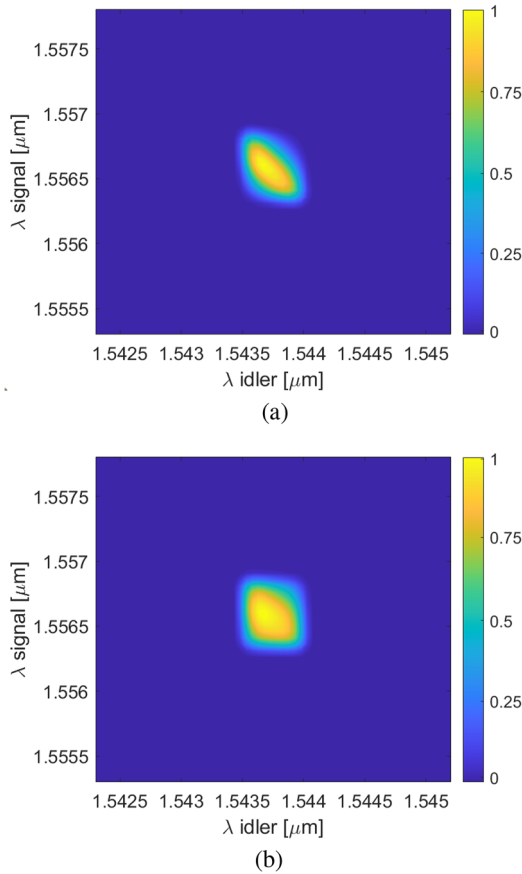


FIG. 6. Simulated JSIs as a function of the signal and idler wavelengths with two different 3-dB bandwidths of the pump's BPF: 0.48 nm (Pbw/Sbw = 90%) (a) and 2.7 nm (Pbw/Sbw = 165%) (b). The corresponding purity parameters are 91.2% (a) and 99.1% (b). The color bars on the right are normalized with respect to the JSI maximum. (a) Pbw/Sbw = 90% and (b) Pbw/Sbw = 165%.

The simulated JSIs are shown in Fig. 6. The simulated purities in the two cases are (labels refer to the figure)

$$P_a = 91.2\%, \quad P_b = 99.1\%.$$

As expected, we found that a broader pump gives a higher value of purity.^{24,27} Such values can be compared to the visibility of the HOM interference reported in Fig. 5. Such a value of purity quantifies the amount of uncertainty in the heralded photon states with respect to single photon states: thus, it is an important intrinsic property of the sources.

IV. QUANTUM-STATE TOMOGRAPHY

Quantum tomography is a technique used to reconstruct the quantum state of a system by performing measurements on it. Quantum tomography is crucial for gaining a comprehensive understanding of quantum states, enabling the validation and optimization of quantum systems, and providing the necessary information for the development of practical quantum technologies.^{30–32}

A density matrix of a quantum state with N_Q qubits, denoted as ρ , is represented by a $2^{N_Q} \times 2^{N_Q}$ matrix. The 4^{N_Q} complex components of the density matrix can be reconstructed from $4^{N_Q} - 1$ sets of real values obtained by 3^{N_Q} sets of Pauli measurements.^{6,21,30–32} Pauli measurements are a way to find a vector's position for a qubit on Bloch's sphere. These measurements are associated with the Pauli matrices, which are a set of three 2×2 matrices: Pauli-X (σ_X), Pauli-Y (σ_Y), and Pauli-Z (σ_Z). Each matrix corresponds to a different basis and represents a measurement along a specific axis on Bloch's sphere. The measurement outcomes correspond to the eigenvalues of these matrices. For example, Pauli-X (σ_X) measures the qubit along the X-axis.³²

The Pauli measurements in quantum state tomography can be affected by non-orthogonal alignments between the measurement axes, unbalanced optical losses to the final detection between channels, statistical errors arising from fluctuations in pump power, phase variations in MZIs, etc.^{21,32} The density matrix reconstructed from Pauli measurements with errors cannot guarantee physically reasonable normalization. To address this issue, maximum-likelihood estimation (MLE)^{31,32} provides a way to obtain a reasonably normalized density matrix.

The fidelity of the measured density matrix, denoted as ρ_{exp} , can be calculated by the trace of the matrix product of ρ_{exp} and the density matrix of a pure state, denoted as ρ_{th} , that is, Fidelity = $\text{Tr}(\rho_{\text{th}}\rho_{\text{exp}})$.³³ In addition, the purity of the measured density matrix can be calculated by tracing the square of the measured density matrix, that is, Purity = $\text{Tr}(\rho_{\text{exp}}\rho_{\text{exp}})$.

Phase conditions used in the following experiments are listed in Tables II and III.

TABLE II. Phase conditions used to set quantum states.

State\Phase	MZIC	MZIU	MZID	MZIU2	MZID2	$G_u(\phi_2)$	$G_d(\phi_2)$
$(00\rangle + 11\rangle)^{\otimes 2}$	$\pi/2$	$\pi/2$	$\pi/2$	π	π	π	π
$ 0000\rangle + 1111\rangle$	$\pi/2$	$\pi/2$	$\pi/2$	π	π	π	0

TABLE III. Phase conditions for Pauli measurements.

Axis\Phase	$G_1(\phi_1)$	$G_1(\phi_2)$	$G_2(\phi_1)$	$G_2(\phi_2)$	$G_3(\phi_1)$	$G_3(\phi_2)$	$G_4(\phi_1)$	$G_4(\phi_2)$
ZZZZ	π	π	π	π	π	π	π	π
XXXX	π	$\pi/2$	π	$\pi/2$	π	$\pi/2$	π	$\pi/2$
YYYY	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$

A. Tomography of a heralded single qubit

The density matrix of a single qubit state is represented by a 2×2 matrix, and the four complex components of the matrix can be reconstructed from three sets of real values obtained by three sets of Pauli measurements. We measured a heralded single qubit $|1\rangle$ state heralded by $|11\rangle$ pair for the three Pauli-measurement sets.

Figure 7 shows the measured density matrix before and after MLE for the real and imaginary parts, respectively. The fidelity of the measured density matrix is 99% before MLE and 98.2% after MLE, compared to the $|1\rangle$ state heralded by the $|11\rangle$ pair after the maximum likelihood estimation (MLE). The purity of the measured state is 98.3%.

B. Tomography of a Bell state

The density matrix of a two-qubit state is represented by a 4×4 matrix, and the 16 complex components of the matrix can be reconstructed from 15 ($= 4^2 - 1$) sets of real values obtained by nine ($= 3^2$) sets of Pauli measurements. We measured a two-qubit $(|00\rangle + |11\rangle)$ Bell state for the nine Pauli-measurement sets.

Figure 8 shows the measured real and imaginary parts of the density matrix before and after MLE, respectively. The fidelity of the

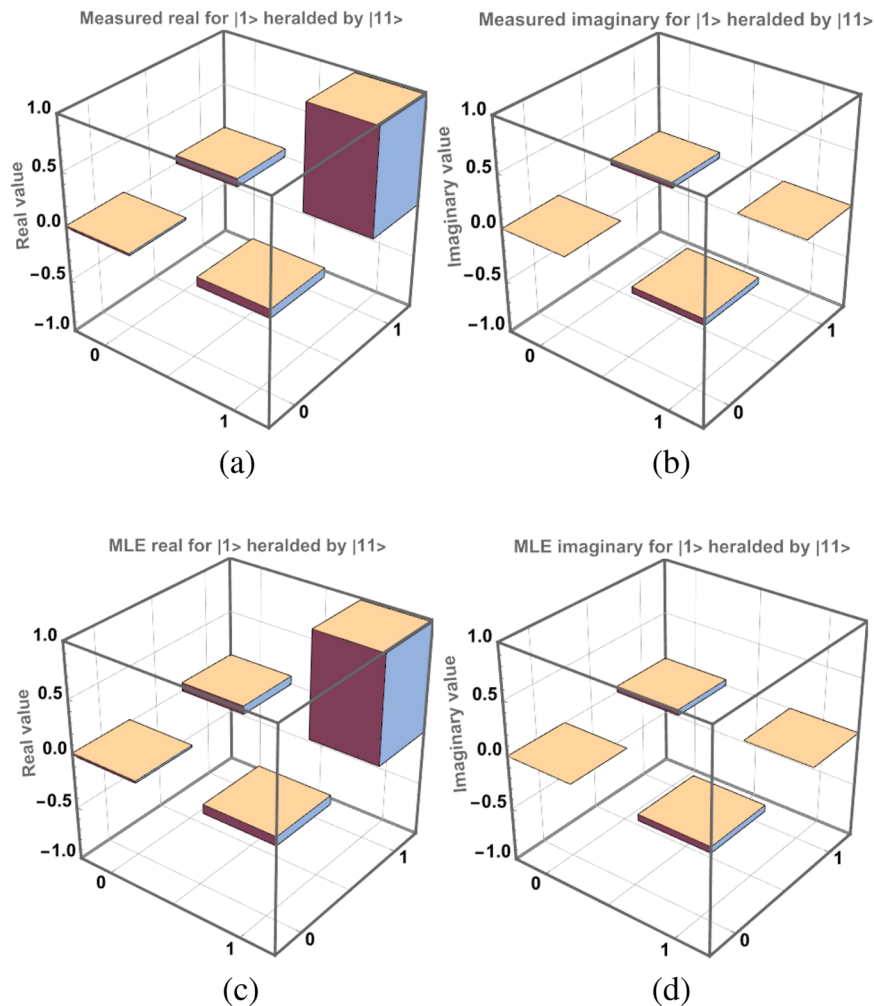


FIG. 7. Single-photon state: Real and imaginary components of the density matrix reconstructed from the on-chip tomography results of the SiPIC. The fidelity of the measured density matrix is 99% before MLE and 98.2% after MLE compared to the $|1\rangle$ state. The purity of the measured state is 98.3%. (a) Real raw data, (b) imaginary raw data, (c) real after MLE, and (d) imaginary after MLE.

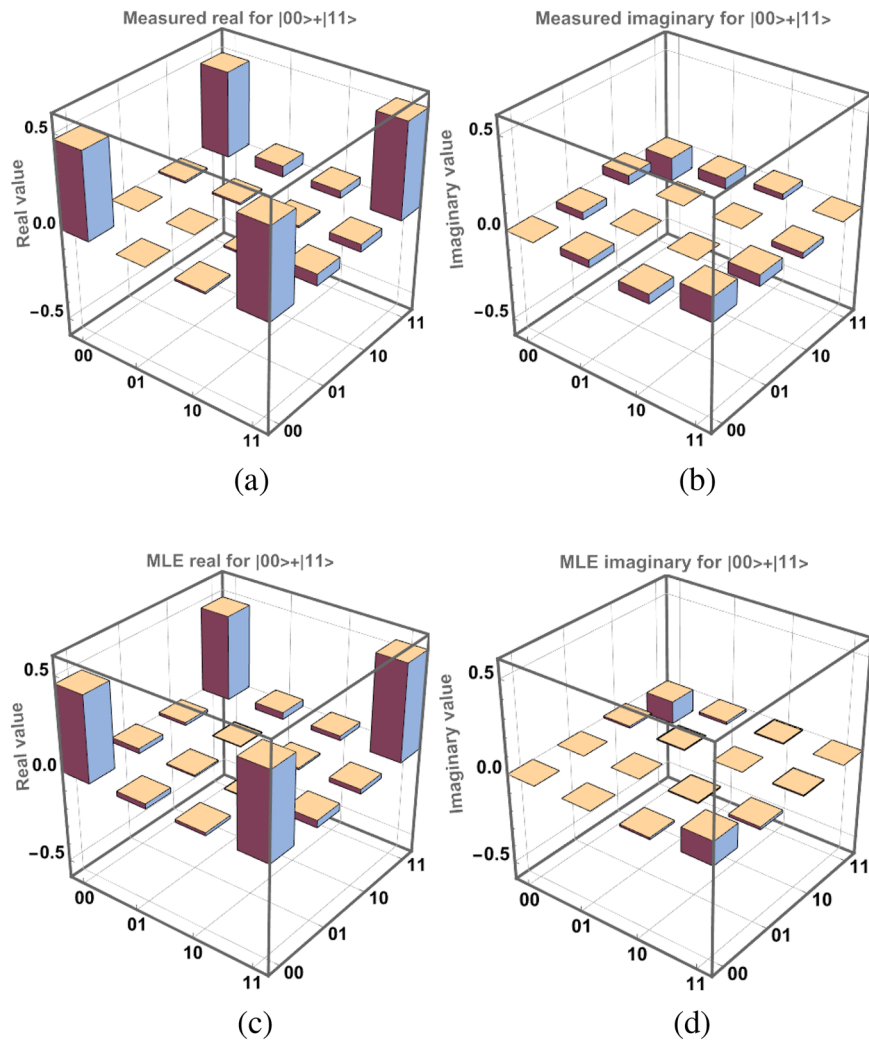


FIG. 8. Two-photon Bell state: Real and imaginary components of the density matrix reconstructed from the on-chip tomography results of the SiPIC. The fidelity of the measured density matrix is 96% before MLE and 95.2% after MLE compared to the $(|00\rangle + |11\rangle)$ state. The purity of the measured state is 94.8%. (a) Real raw data, (b) imaginary raw data, (c) real after MLE, and (d) imaginary after MLE.

measured density matrix is 96% before MLE and 95.2% after MLE compared to the $(|00\rangle + |11\rangle)$ Bell state after MLE. The purity of the measured state is 94.8%. The measured purity of the Bell state is quite high but lower than that of a single qubit. This decrease in purity can be due to the accumulation of noise for each single qubit, unbalanced detection between qubits, and statistical errors in the Pauli measurements.

C. Tomography of two simultaneous Bell states

The density matrix of a four-qubit state is represented by a 16×16 matrix, and the 256 complex components of the matrix can be reconstructed from 255 $(= 4^4 - 1)$ sets of real values obtained by 81 $(= 3^4)$ sets of Pauli measurements. We measured the four-

qubit $(|00\rangle + |11\rangle)^{\otimes 2}$ state for the 81 Pauli-measurement sets. In the experiment, we set the pump power at a level where the four-photon coincident rate was about 0.2 Hz (Fig. 3). This was done to reduce the noise from accidental coincident counts.

Figure 9 shows the measured real and imaginary parts of the density matrix before and after MLE, respectively. The fidelity of the measured density matrix is 91% before MLE and 87.4% after MLE compared to the $(|00\rangle + |11\rangle)^{\otimes 2}$ state. The purity of the measured state is 84.6%. Once again, there is a degradation in purity from the two-qubit state discussed in Sec. IV B. This decline can be attributed to the accumulation of noise in every single qubit, unbalanced detection between qubits, and statistical errors in the 81 sets of Pauli measurements. Nonetheless, the achieved 87.4% fidelity and 84.6% purity in the four-qubit tomography is a notable demonstration in a SiPIC with integrated photon sources and quantum gates.

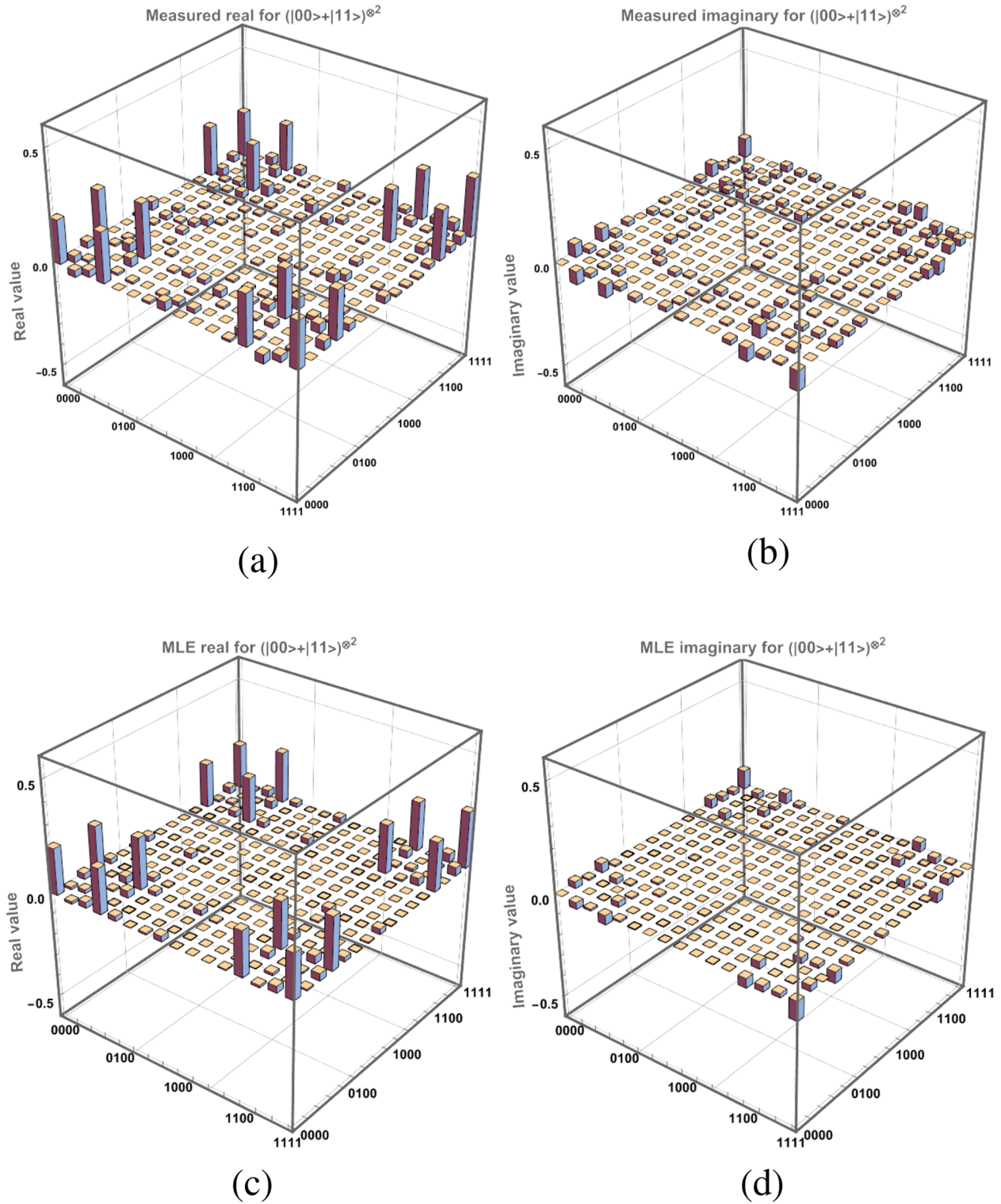


FIG. 9. Two separable two-photon Bell states: Real and imaginary components of the density matrix reconstructed from the on-chip tomography results of the SiPIC. The fidelity of the measured density matrix is 91% before MLE and 87.4% after MLE compared to the $(|00\rangle + |11\rangle)^{\otimes 2}$ state. The purity of the measured state is 84.6%. (a) Real raw data, (b) imaginary raw data, (c) real after MLE, and (d) imaginary after MLE.

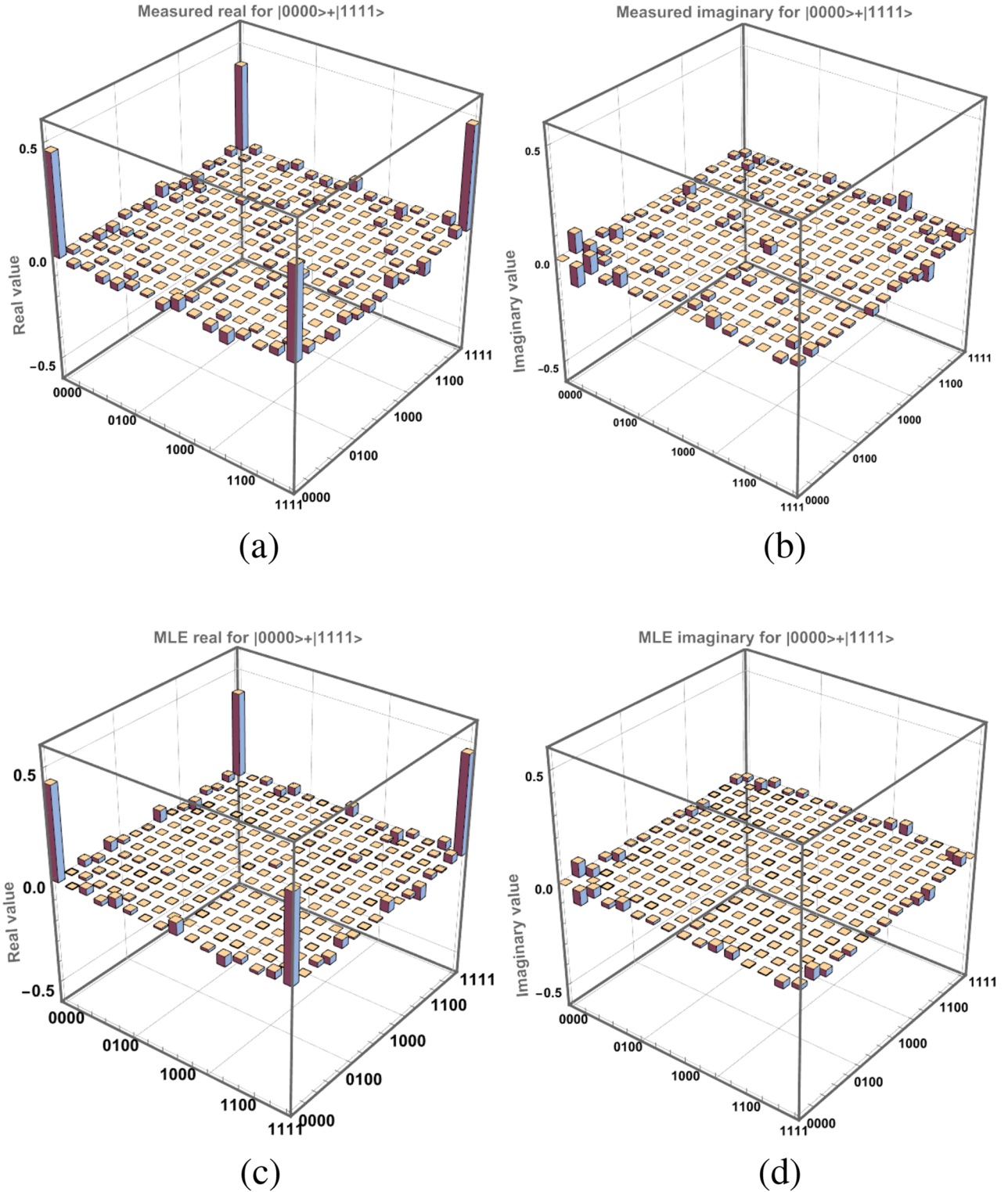


FIG. 10. Four-photon GHZ state: Real and imaginary components of the density matrix reconstructed from the on-chip tomography results of the SiPIC. The fidelity of the measured density matrix is 88.5% before MLE and 85.4% after MLE compared to the $(|0000\rangle + |1111\rangle)$ GHZ state. The purity of the measured state is 81.7%. (a) Real raw data, (b) imaginary raw data, (c) real after MLE, and (d) imaginary after MLE.

D. Tomography of a four-qubit GHZ state

A multipartite entanglement GHZ state can serve as a photonic qubit for a photonic quantum computer,^{1,2} and efforts to realize integrated circuits able to engineer four-photon GHZ states have been reported both on silicon as well as glass-based photonic platforms.^{6,21} The four-photon GHZ state can be prepared through a fusion operation involving two Bell states.⁶ In our SiPIC, the fusion operation can be performed by simply controlling the phase ϕ_2 of the gate G_d , from zero to π , to interchange the path-encoded input $|1\rangle$ (the lower input) to G_2 and G_3 in Fig. 9, together with post-selection. Therefore, we programmed our SiPIC for the fusion operation between the two Bell states discussed in Sec. IV C and conducted tomography on the resulting four-qubit state.

Figure 10 shows the real and imaginary components of the density matrix reconstructed from the Pauli measurements of a $(|0000\rangle + |1111\rangle)$ GHZ state.

The fidelity of the density matrix is estimated at 88.5% for the raw data and 85.4% after MLE compared to the $(|0000\rangle + |1111\rangle)$ GHZ state. The purity of the state is estimated at 81.7%. The full tomography result of the four-photon GHZ state with a fidelity of 85.4% and a purity of 81.7% is a valuable demonstration achieved in a SiPIC.

There is a degradation in purity from the four-qubit dual Bell state discussed in Sec. IV C. This decline can be attributed to the accumulation of errors in the fusion operation because of the non-ideal behavior of the integrated components. Throughout the experiment, the optical loss from photon-pair generation to detection was ~ 6 dB, and the detector efficiency was around 75%. We anticipate an enhancement in both fidelity and purity by reducing optical loss in future experiments.

E. Entanglement certificates

Violation of Bell's inequality serves as an intriguing test distinguishing quantum physics from classical physics.^{22,34} Bell's inequality is widely recognized for its application in entanglement certification, and various reports have also explored Bell's inequality test with multipartite systems, such as a four-qubit GHZ state.^{22,35–37} In this context, we apply Zukowski–Brukner (ZB) inequalities^{22,37} and Mermin–Ardehali–Belinskii–Klyshko (MABK)^{35,36} inequalities to our Pauli-measurement data for the four-qubit GHZ state tomography.

The S-parameter of ZB inequality in the GBI form of the four-partite GHZ state can be described as follows:³⁷

$$S = \frac{1}{16} \sum_{s_1, \dots, s_4} \left| \sum_{k_1, \dots, k_4} f(s_1, \dots, s_4, k_1, \dots, k_4) E(k_1, \dots, k_4) \right|, \quad (1)$$

where $s_j = -1, 1, k_j = 1, 2$, $f(s_1, \dots, s_4, k_1, \dots, k_4) = s_1^{k_1-1} s_2^{k_2-1} s_3^{k_3-1} s_4^{k_4-1}$, and $E(k_1, \dots, k_4)$ denotes the correlation function. Parameter k_j denotes the Pauli measurement settings, such that $k_j = 1$ corresponds to Pauli-X and $k_j = 2$ corresponds to Pauli-Y. The classical prediction of the S-parameter for local realism (LR) in Eq. (1) is $S_{LR} \leq 1$. In contrast, according to quantum physics, the maximum value of the S-parameter is predicted to be between 2 and $2\sqrt{2}$, depending on the phase of the multipartite state, with 2 for phase 0 and $2\sqrt{2}$ for phase $\pi/4$.³⁷

TABLE IV. Entanglement certificates.

	Value	Condition
ZB	1.717 ± 0.067 (10.7 σ)	>1
MABK	4.677 ± 0.076 (35.0 σ)	>2
EW	-0.367 ± 0.012 (15.3 σ)	<0

The Bell's inequality test by ZB inequality in this experiment shows a clear violation with the S-parameter of 1.77, which exceeds the classically predicted maximum value of 1, as shown in Table IV.

In the MABK inequality, the S-parameter is defined in a relatively simple form compared to that of the ZB inequality by focusing on the target state. The S-parameter of the MABK inequality for the four-qubit GHZ state can be described as follows:³⁶

$$S = \frac{1}{2} \left| \langle (\sigma_x \sigma_x \sigma_x - \sigma_x \sigma_y \sigma_y + \sigma_y \sigma_y \sigma_x) (\sigma_a + \sigma_b) \rangle \right| + \frac{1}{2} \left| \langle (\sigma_y \sigma_y \sigma_y + \sigma_x \sigma_y \sigma_x - \sigma_x \sigma_x \sigma_y) (\sigma_a - \sigma_b) \rangle \right|, \quad (2)$$

where $\sigma_a = (\sigma_x + \sigma_y)/\sqrt{2}$ and $\sigma_b = (\sigma_x - \sigma_y)/\sqrt{2}$. The classical prediction of the S-parameter for MABK is $S_{LR} \leq 2$. In contrast, according to quantum physics, the maximum value of the S-parameter in Eq. (2) is predicted to be $4\sqrt{2}$.³⁶

The Bell's inequality test by MABK inequality in this experiment also shows a clear violation with the S-parameter of 4.721, which exceeds the classically predicted maximum value of 2, as shown in Table IV. Both results of Bell's inequality exceed the classically predicted values.

In addition, we calculated an entanglement witness to verify the genuine entanglement of the GHZ state. An observable W is considered an entanglement witness (EW) if $\text{Tr}[WQ_E] < 0$ for an entangled state Q_E .^{37–39} We use the following entanglement as a witness for the GHZ state:³⁷

$$W_{GHZ} = \frac{I}{2} - |GHZ^{(4)}\rangle\langle GHZ^{(4)}| = \frac{I}{2} - \frac{1}{2} (|0\rangle\langle 0|^{\otimes 4} + |1\rangle\langle 1|^{\otimes 4} + |0\rangle\langle 1|^{\otimes 4} + |1\rangle\langle 0|^{\otimes 4}), \quad (3)$$

where I denotes the identity operator. Experimental data obtained based on Eq. (3) is estimated to have a negative value of -0.367 , as shown in Table IV. This result implies that the generated state is a genuine four-partite entanglement state.

V. DISCUSSION

High-fidelity and high-purity quantum states are essential for quantum information processing tasks. In a simple model of a quantum circuit where each gate introduces some error, the purity and fidelity of a state decay with the number of operations, i.e., the depth of a quantum circuit depends on the purity and fidelity. Therefore, to increase the circuit depth, error correction codes have been introduced. Most of them are based on the use of many physical qubits to protect a single logical qubit. For example, in integrated photonics, it has been proposed to use multipartite entangled states.¹ Our results, summarized in Table V, show that a purity greater than 80% can be

TABLE V. Fidelity and purity parameters for four-qubit entangled states in SiPICs.

References	State	Fidelity (%)	Purity	Entanglement certificates
4	$ S_4\rangle$	78	...	...
6	GHZ ⁽⁴⁾	73.5	...	EW − 0.183 < 0
This work	GHZ ⁽⁴⁾	85.4	81.7%	EW − 0.367 < 0, ZB 1.717 > 1
This work	Bell ^{⊗2}	87.4	84.6%	...

achieved for two- and four-dimensional bipartite entangled states in a SiPIC. This yields a potential circuit depth larger than, e.g., 10 with a gate error of 0.01, assuming a simple depolarizing noise model.⁴⁰ We also compare our results to other reports on four-qubit SiPIC.

To get high fidelity and purity, we concentrated on optimizing the set-up heralding efficiency (HE) and the source CAR. In fact, the single photon states are obtained by the heralding method, which is characterized by a heralding efficiency (HE), i.e., the probability that the measurement of the herald photon is followed by the measurement of the heralded photon.¹⁶ We can estimate the HE of the setup in Fig. 1 by considering the total optical loss from the spiral waveguide sources to the SNSPD detectors. In our case, we estimate a HE of 12% by considering a SiPIC device loss of 2 dB, a coupling loss of 3.5 dB, a BPF filter loss of 2 dB, and a SNSPD detector efficiency of 70%, which varies depending on the detection rate. The efficiency degrades at a high detection rate, and the maximum detection rate (MDR) of the SNSPDs is limited to 50 MHz in this experiment.

At the same time, the CAR in a two-photon coincidence measurement is related to the probability of a coincident detection of both the signal and idler photons, P_{si} . By denoting the probability of measuring the signal photon, P_s , and the probability of measuring the idler photon, P_i , which is approximately equal to P_s , we have $P_{si} \approx HE \times P_s$. Then, considering Refs. 41 and 42, CAR can be related to HE as the following equation:

$$\begin{aligned}
 CAR &\approx \frac{P_{si}}{P_s P_i + C_0 + C_1 \sqrt{P_s P_i} + C_2 P_s P_i} \\
 &\approx \frac{HE \cdot P_s}{P_s^2 + C_0 + C_1 P_s + C_2 P_s^2} \\
 &\approx \frac{HE \cdot P_s}{P_s^2 + C_0},
 \end{aligned} \quad (4)$$

where C_0 is a constant noise offset caused by the ambient light and the dark counts of the SNSPDs. $C_1 \sqrt{P_s P_i}$ is a linear noise caused by the pumping light, such as Raman scattering,¹⁹ and $C_2 P_s P_i$ is a noise caused by photon-pair generation after the spiral. Equation (4) is a simple approximation that neglects multiple pair generation and TPA. We used this approximation since relatively low pump power was used in our experiment. In particular, TPA can be neglected in spiral waveguide sources at low power since they do not exhibit the typical field enhancement effects observed for ring resonators. In our setup, we minimized the noise offset C_0 by blocking ambient light and the noises $C_1 \sqrt{P_s P_i}$ and $C_2 P_s P_i$ by reducing the pumping light with the integrated PRFs. In addition, we used custom-made BPFs to enhance the HE.

Table VI compares the actual measured data with estimates obtained by using Eq. (4). We set the pump noise $C_1 = C_2 = 0$ in

TABLE VI. Comparison of measured coincidence rates with estimated coincidence rates calculated with a CAR obtained via Eq. (4). For experimental values, two rows are reported, which refer to two different pump powers (0.1 and 0.4 mW). Calculation refers to the use of Eq. (4) to simulate our measurement results. Future outlook refers to an estimate of the possibility of up-scaling the SiPICs.

	C_0	P_s	HE (%)	CAR	MDR (MHz)	2cc (kHz)	4cc (Hz)	6cc (Hz)
This work	...	...	12	80	50	12	0.2	...
	...	...	12	11	50	44	8	...
Calculation	10^{-11}	0.001	12	120	50	6	0.7	...
	10^{-11}	0.01	12	11	50	60	72	...
Future outlook	10^{-11}	0.01	95	95	1 GHz	9 MHz	90 kHz	800

Eq. (4), assuming that the PRFs reduce the pump noise enough to neglect such a contribution. The two-folded coincidence count (2cc) can be calculated by $(HE \cdot P_s) \cdot MDR$, and the four-folded coincidence count (4cc) by $(HE \cdot P_s)^2 \cdot MDR$. The six-folded coincidence count (6cc) can be calculated by $(HE \cdot P_s)^3 \cdot MDR$.

In Table VI, order of magnitude agreements are observed by calculation compared with the experiment. This stimulates us to use this equation to evaluate the scaling potential of fully integrated SiPICs. For this, we used MDR enhanced to 1 GHz, a HE of $\approx 95\%$, and considered a full integration of SNSPDs as reported recently,⁴³ in addition to overall improvements in performance to enhance the HE. We expect a six-folded coincidence count (6cc) of 800 Hz in the future from the full integration of photon sources into SNSPDs in a SiPIC.

Table VI shows the limit of state-of-the-art SiPICs in increasing the number of qubits. Moreover, the probabilistic nature of fusion operations used to create GHZ states and, more in general, cluster states appears as another potential issue. However, this can be solved by using redundantly encoded n -photon logical qubits,⁴⁴ e.g., by means of type-II fusion gates that can be attempted more times to increase their success probability. Of course, this translates into a requirement in terms of higher generation rates. Another possibility is to entangle photons in time in addition to space.^{1,45}

VI. CONCLUSION

In summary, we demonstrated high-purity qubits in a SiPIC by four-photon coincidence measurements. We measured 98.3% purity for a path-entangled single qubit ($|1\rangle$) and 94.8% purity for a Bell state ($|00\rangle + |11\rangle$). We demonstrated an impressive 98% HOM visibility with two separable photons heralded by two pairs of photons. Through four-photon tomography, we measured the 87.4% fidelity of a four-qubit dual Bell state with 87.4% purity. By fusion operation to a four-qubit dual Bell state, we measured 85.4% fidelity of the four-qubit GHZ state ($|0000\rangle + |1111\rangle$) with 81.7% purity. Our estimation for the violation of Bell's inequality and negative witness certified the genuine entanglement of the four-photon GHZ state. These results show a promising future for silicon-photonics qubits for various quantum technology applications, including quantum computation and networks.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Jong-Moo Lee: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Jiho Park:** Formal analysis (equal); Investigation (equal); Methodology (equal); Validation (equal); Visualization (equal). **Jeongho Bang:** Funding acquisition (equal); Investigation (equal); Methodology (equal); Validation (equal). **Young-Ik Sohn:** Funding acquisition (equal); Project administration (equal); Validation (equal). **Alessio Baldazzi:** Formal analysis (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Matteo Sanna:** Formal analysis (equal); Validation (equal); Writing – original draft (equal). **Stefano Azzini:** Formal analysis (equal); Funding acquisition (equal); Validation (equal); Writing – original draft (equal); Writing – review & editing (equal). **Lorenzo Pavesi:** Formal analysis (equal); Funding acquisition (equal); Project administration (equal); Validation (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- ¹S. Bartolucci, P. Birchall, H. Bombin, H. Cable, C. Dawson, M. Gimeno-Segovia, E. Johnston, K. Kieling, N. Nickerson, M. Pant, F. Pastawski, T. Rudolph, and C. Sparrow, "Fusion-based quantum computation," *Nat. Commun.* **14**, 912 (2023).
- ²B. Pankovich, A. Kan, K. H. Wan, M. Ostmann, A. Neville, S. Omkar, A. Sohbi, and K. Brádler, "High photon-loss threshold quantum computing using GHZ-state measurements," *arXiv:2308.04192* (2023).

- ³T. Meany, M. Gräfe, R. Heilmann, A. Perez-Leija, S. Gross, M. J. Steel, M. J. Withford, and A. Szameit, "Laser written circuits for quantum photonics," *Laser Photonics Rev.* **9**, 363–384 (2015).
- ⁴J. C. Adcock, C. Vigliar, R. Santagati, J. W. Silverstone, and M. G. Thompson, "Programmable four-photon graph states on a silicon chip," *Nat. Commun.* **10**, 3528 (2019).
- ⁵J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, "Integrated photonic quantum technologies," *Nat. Photonics* **14**, 273–284 (2020).
- ⁶D. Llewellyn, Y. Ding, I. I. Faruque, S. Paesani, D. Bacco, R. Santagati, Y.-J. Qian, Y. Li, Y.-F. Xiao, M. Huber, M. Malik, G. F. Sinclair, X. Zhou, K. Rottwitz, J. L. O'Brien, J. G. Rarity, Q. Gong, L. K. Oxenlowe, J. Wang, and M. G. Thompson, "Chip-to-chip quantum teleportation and multi-photon entanglement in silicon," *Nat. Phys.* **16**, 148–153 (2020).
- ⁷C. Vigliar, S. Paesani, Y. Ding, J. C. Adcock, J. Wang, S. Morley-Short, D. Bacco, L. K. Oxenlowe, M. G. Thompson, J. G. Rarity, and A. Laing, "Error-protected qubits in a silicon photonic chip," *Nat. Phys.* **17**, 1137–1143 (2021).
- ⁸J. Bao, Z. Fu, T. Pramanik, J. Mao, Y. Chi, Y. Cao, C. Zhai, Y. Mao, T. Dai, X. Chen, X. Jia, L. Zhao, Y. Zheng, B. Tang, Z. Li, J. Luo, W. Wang, Y. Yang, Y. Peng, D. Liu, D. Dai, Q. He, A. L. Muthali, L. K. Oxenlowe, C. Vigliar, S. Paesani, H. Hou, R. Santagati, J. W. Silverstone, A. Laing, M. G. Thompson, J. L. O'Brien, Y. Ding, Q. Gong, and J. Wang, "Very-large-scale integrated quantum graph photonics," *Nat. Photonics* **17**, 573–581 (2023).
- ⁹J. Huang, X. Li, X. Chen, C. Zhai, Y. Zheng, Y. Chi, Y. Li, Q. He, Q. Gong, and J. Wang, "Demonstration of hypergraph-state quantum information processing," *Nat. Commun.* **15**, 2601 (2024).
- ¹⁰J.-M. Lee, W.-J. Lee, M.-S. Kim, S. Cho, J. J. Ju, G. Navickaite, and J. Fernandez, "Controlled-not operation of sin-photon circuit using photon pairs from silicon-photon circuit," *Opt. Commun.* **509**, 127863 (2022).
- ¹¹Y. Wang, C. F. D. Faurby, F. Ruf, P. I. Sund, K. Nielsen, N. Volet, M. J. R. Heck, N. Bart, A. D. Wieck, A. Ludwig, L. Midolo, S. Paesani, and P. Lodahl, "Deterministic photon source interfaced with a programmable silicon-nitride integrated circuit," *npj Quantum Inf.* **9**, 94 (2023).
- ¹²G. Moody, V. J. Sorger, D. J. Blumenthal, P. W. Juodawlkis, W. Loh, C. Sorace-Agaskar, A. E. Jones, K. C. Balram, J. C. Matthews, A. Laing *et al.*, "2022 Roadmap on integrated quantum photonics," *J. Phys.: Photonics* **4**, 012501 (2022).
- ¹³W. Luo, L. Cao, Y. Shi, L. Wan, H. Zhang, S. Li, G. Chen, Y. Li, S. Li, Y. Wang *et al.*, "Recent progress in quantum photonic chips for quantum communication and internet," *Light: Sci. Appl.* **12**, 175 (2023).
- ¹⁴X. Qiang, X. Zhou, J. Wang, C. M. Wilkes, T. Loke, S. O'Gara, L. Kling, G. D. Marshall, R. Santagati, T. C. Ralph *et al.*, "Large-scale silicon quantum photonics implementing arbitrary two-qubit processing," *Nat. Photonics* **12**, 534–539 (2018).
- ¹⁵A. Senichev, Z. O. Martin, S. Peana, D. Sychev, X. Xu, A. S. Lagutchev, A. Boltasova, and V. M. Shalaev, "Room-temperature single-photon emitters in silicon nitride," *Sci. Adv.* **7**, eabj0627 (2021).
- ¹⁶S. Signorini and L. Pavesi, "On-chip heralded single photon sources," *AVS Quantum Sci.* **2**, 041701 (2020).
- ¹⁷J. Zhang, H. Xu, G. Zhang, Y. Chen, H. Wang, K. H. Tan, S. Wicaksono, C. Sun, Q. Kong, C. Wang *et al.*, "Hybrid and heterogeneous photonic integrated near-infrared InGaAs/InAlAs single-photon avalanche diode," *Quantum Sci. Technol.* **8**, 025009 (2023).
- ¹⁸J.-M. Lee, A. Baldazzi, M. Sanna, S. Azzini, J. T. Ahn, M. L. Lee, Y.-I. Sohn, and L. Pavesi, "Do different kinds of photon-pair sources have the same indistinguishability in quantum silicon photonics?," *Photonics Res.* **11**, 1820–1837 (2023).
- ¹⁹J.-M. Lee, W.-J. Lee, M.-S. Kim, and J. J. Ju, "Noise filtering for highly correlated photon pairs from silicon waveguides," *J. Lightwave Technol.* **37**, 5428–5434 (2019).
- ²⁰A. Ferreri, V. Ansari, C. Silberhorn, and P. Sharapova, "Temporally multimode four-photon Hong-Ou-Mandel interference," *Phys. Rev. A* **100**, 053829 (2019).
- ²¹M. Pont, G. Corrielli, A. Fyrrillas, I. Agresti, G. Carvacho, N. Maring, P.-E. Emeriau, F. Ceccarelli, R. Albiero, P. H. D. Ferreira, N. Somaschi, J. Senellart, I. Sagnes, M. Morassi, A. Lemaître, P. Senellart, F. Sciarrino, M. Liscidini, N.

- Belabas, and R. Osellame, “High-fidelity generation of four-photon GHZ states on-chip,” [arXiv:2211.15626](#) (2022).
- ²²M. Żukowski and Č. Brukner, “Bell’s theorem for general N -qubit states,” *Phys. Rev. Lett.* **88**, 210401 (2002).
- ²³P. Lazaridis, G. Debarge, and P. Gallion, “Time–bandwidth product of chirped sech^2 pulses: Application to phase–amplitude-coupling factor measurement,” *Opt. Lett.* **20**, 1160–1162 (1995).
- ²⁴Z. Vernon, M. Menotti, C. C. Tison, J. A. Steidle, M. L. Fanto, P. M. Thomas, S. F. Preble, A. M. Smith, P. M. Alsing, M. Liscidini, and J. E. Sipe, “Truly unentangled photon pairs without spectral filtering,” *Opt. Lett.* **42**, 3638–3641 (2017).
- ²⁵C.-K. Hong, Z.-Y. Ou, and L. Mandel, “Measurement of subpicosecond time intervals between two photons by interference,” *Phys. Rev. Lett.* **59**, 2044 (1987).
- ²⁶C. Ma, X. Wang, V. Anant, A. D. Beyer, M. D. Shaw, and S. Mookherjee, “Silicon photonic entangled photon-pair and heralded single photon generation with $\text{CAR} > 12,000$ and $g^{(2)}(0) < 0.006$,” *Opt. Express* **25**, 32995–33006 (2017).
- ²⁷C. K. Law and J. H. Eberly, “Analysis and interpretation of high transverse entanglement in optical parametric down conversion,” *Phys. Rev. Lett.* **92**, 127903 (2004).
- ²⁸I. I. Faruque, G. F. Sinclair, D. Bonneau, J. G. Rarity, and M. G. Thompson, “On-chip quantum interference with heralded photons from two independent micro-ring resonator sources in silicon photonics,” *Opt. Express* **26**, 20379–20395 (2018).
- ²⁹K. Zielnicki, K. Garay-Palmett, D. Cruz-Delgado, H. Cruz-Ramirez, M. F. O’Boyle, B. Fang, V. O. Lorenz, A. B. U’Ren, and P. G. Kwiat, “Joint spectral characterization of photon-pair sources,” *J. Mod. Opt.* **65**, 1141–1160 (2018).
- ³⁰D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, “Measurement of qubits,” *Phys. Rev. A* **64**, 052312 (2001).
- ³¹G. M. D’Ariano, M. G. Paris, and M. F. Sacchi, “Quantum tomography,” *Adv. Imaging Electron Phys.* **128**, 205–308 (2003).
- ³²J. B. Altepeter, D. F. James, and P. G. Kwiat, “4 qubit quantum state tomography,” in *Quantum State Estimation*, edited by M. Paris and J. Řeháček (Springer, Berlin, Heidelberg, 2004), pp. 113–145.
- ³³R. Jozsa, “Fidelity for mixed quantum states,” *J. Mod. Opt.* **41**, 2315–2323 (1994).
- ³⁴J. S. Bell, “On the Einstein podolsky Rosen paradox,” *Phys. Phys. Fiz.* **1**, 195–200 (1964).
- ³⁵N. D. Mermin, “Extreme quantum entanglement in a superposition of macroscopically distinct states,” *Phys. Rev. Lett.* **65**, 1838–1840 (1990).
- ³⁶Z. Zhao, T. Yang, Y.-A. Chen, A.-N. Zhang, M. Żukowski, and J.-W. Pan, “Experimental violation of local realism by four-photon Greenberger-Horne-Zeilinger entanglement,” *Phys. Rev. Lett.* **91**, 180401 (2003).
- ³⁷J. Park, J. Ryu, H. Kim, and H. S. Moon, “Violation of bell inequality by four-photon Greenberger-Horne-Zeilinger state with a phase from a warm atomic ensemble,” *Adv. Quantum Technol.* **6**, 2300190 (2023).
- ³⁸D. Chruściński and G. Sarbicki, “Entanglement witnesses: Construction, analysis and classification,” *J. Phys. A: Math. Theor.* **47**, 483001 (2014).
- ³⁹O. Gühne, C.-Y. Lu, W.-B. Gao, and J.-W. Pan, “Toolbox for entanglement detection and fidelity estimation,” *Phys. Rev. A* **76**, 030305 (2007).
- ⁴⁰M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).
- ⁴¹W. Shin, K. Park, H. Kim, D. Lee, K. Kwon, and H. Shin, “Photon-pair generation in a lossy waveguide,” *Nanophotonics* **12**, 531–538 (2023).
- ⁴²S. Kim, Y. Kim, Y.-D. Yoon, S. Jeon, W.-J. Kim, and Y.-I. Sohn, “Simulation framework for integrated nonlinear quantum photonics,” [arXiv:2402.19317](#) (2024).
- ⁴³K. Alexander, A. Bahgat, A. Benyamini, D. Black *et al.*, “A manufacturable platform for photonic quantum computing,” [arXiv:2404.17570](#) (2024).
- ⁴⁴D. E. Browne and T. Rudolph, “Resource-efficient linear optical quantum computation,” *Phys. Rev. Lett.* **95**, 010501 (2005).
- ⁴⁵H. Bombín, I. H. Kim, D. Litinski, N. Nickerson, M. Pant, F. Pastawski, S. Roberts, and T. Rudolph, “Interleaving: Modular architectures for fault-tolerant photonic quantum computing,” [arXiv:2103.08612](#) (2021).