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Iterative Multi Scaling-Enhanced Inexact Newton-Method for Microwave Imaging

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Introduction

In the last years, several approaches have been proposed for solving inverse problems arising in microwave imaging [1] and related applications including non-invasive diagnostics, biomedical imaging, remote sensing, and subsurface prospecting [1]-[4]. In a microwave imaging problem, the targets are illuminated by incident waves and scattered field samples are measured outside the investigation area [1]-[4]. In order to retrieve the unknwon objects from the measurements, different stochastic [1][3] and deterministic [2][4] approaches have been proposed. As for these latter, they are usually based on iterative procedures such as gradient or Newton-type methods. In this framework, an approach based on an Inexact-Newton method (IN) has been recently proposed for solving inverse scattering problems formulated through electric field integral equations (EFIEs) [4]. Such a method has been validated on synthetic and experimental results as well as extended to contrast source formulations [5] showing several advantages in terms of stability, accuracy, and convergence rate with respect to state-of the art techniques [4]. However, it can suffer from local minima because of its deterministic nature. In order to overcome/mitigate such a drawback, the iterative multiscaling approach (IMSA) introduced in [2] for conjugate-gradient methods is considered in this paper. The IMSA is a synthetic zoom procedure that, thanks to an efficient exploitation of the available information from scattering data, guarantees higher resolution and enhanced reconstruction with respect to the corresponding "bare" approaches whatever the inversion technique [2][3]. Thanks to these features, it represents a candidate solution for improving the performances of IN and avoiding some intrinsic drawbacks caused by the limited amount of indipendent data and the deterministic nature of the same approach. In the following, the integration of the IMSA with the IN method (*IMSA-IN* technique) will be described and its performances will be compared to those of the standard IN implementation (*Bare-IN*).

Mathematical Formulation

The Inexact Newton method (IN) [4] is an iterative regularization technique aimed at solving nonlinear and ill-posed problems. Under the assumption of cylindrical scatterers and Transverse-Magnetic (TM) polarization of the incident fields with respect to the axes of the scatterers, the retrieval of the dielectric properties, $\varepsilon_r(\mathbf{r})$ and $\sigma(\mathbf{r})$, of an investigation region D_{inv} can be recast as the solution of the following integral equations

$$E_s^v(\mathbf{r}) = k_0^2 \int_{D_{inv}} \tau(\mathbf{r}')E^v(\mathbf{r}')G(\mathbf{r};\mathbf{r}')d\mathbf{r}', \qquad \mathbf{r} \in D_{meas}^v \tag{1}$$

$$E^v(\mathbf{r}) = E_i^v(\mathbf{r}) - k_0^2 \int_{D_{inv}} \tau(\mathbf{r}')E^v(\mathbf{r}')G(\mathbf{r};\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in D_{inv} \tag{2}$$

where $\tau(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - 1 - j\dfrac{\sigma(\mathbf{r})}{\omega\varepsilon_0}$ is the contrast function, *v* is the *v*-th illumination, *G* denotes the free-space Green function. Moreover, E^v, E_s^v, and E_i^v are the total electric field in D_{inv}, the scattered electric field in the observation domain D_{meas}^i, and the incident field, respectively.

By introducing the unknown array, $\mathbf{x} = [\tau, E^1, \ldots, E^V]$, and the known array, $\mathbf{y} = [E_s^1, \ldots, E_s^V, E_i^1, \ldots, E_i^V]$, the inverse problem can be written as

$$\mathbf{T}(\mathbf{x}) = \mathbf{y} \tag{3}$$

where $\mathbf{T}$ is the nonlinear operator defined by (1) and (2).

The Bare-IN method discretizes D_{inv} in N_{BARE} subdomains, and iteratively linearizes the nonlinear problem (3) around the current solution $\mathbf{x}_j$ by means of the Fréchet derivative $\mathbf{T}|_{\mathbf{x}_j}$ of $\mathbf{T}$ and updates $\mathbf{x}_j$ as follows

$$\mathbf{x}_{j+1} = \mathbf{x}_j + \mathbf{h}_j \tag{4}$$

("outer" IN loop [4]) where $\mathbf{h}_j$ is found by using the truncated Landweber method [6] as a regularized solution of the linear problem

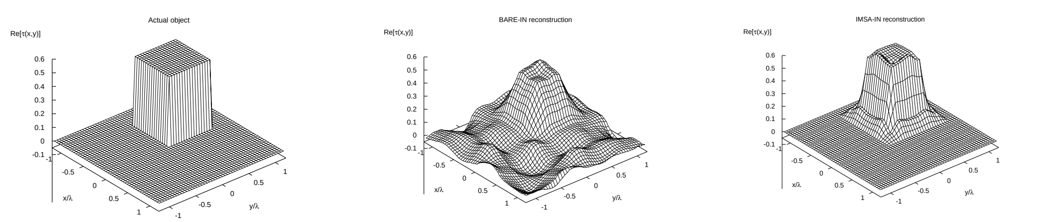
$$\mathbf{T}|_{\mathbf{x}_j}\mathbf{h}_j = \mathbf{y} - \mathbf{T}(\mathbf{x}_j) \tag{5}$$

("inner" IN loop [4]). The Bare-IN "outer" and "inner" loops stop when $\mathbf{x}_j$ is a satisfactory solution according to the user-defined convergence criterion or when a maximum number of iterations (I_{out} and I_{in}, respectively) is reached.

To better address the drawbacks ineherent with the deterministic nature of IN when dealing with nonlinear problems, the IMSA strategy is profitably exploited and integrated with the Bare-IN. Towards this end, the Bare-IN is iteratively applied to reconstruct the dielectric distribution of the region-of-interest (RoI) belonging to the investigation domani (equal to the investigation domani at the first step of the process). At each step, a fixed discretization of the RoI is used by considering N_{IMSA} subdomains ($N_{IMSA} << N_{BARE}$, N_{IMSA} being the number of degrees-of-freedom of the inverse problem and the geometry at hand) and the IN reconstruction is performed. From two successive steps, the RoI is updated exploiting the information on the location and extension of the scatterers acquired by processing the reconstructed profile [3]. The synthetic zooming process is iterated until the stationarines of the RoI is reached [2][3]. The result is that a high resolution IN reconstruction problem (as required to achieve a suitable image of the investigation domain) is recast as a set of low resolution ones [2][3] allowing improved convergence speed and accuracy of the overall inversion as well as an enhanced robustness to local minima problem.

Numerical Results

In the first numerical example, a homogeneous lossless square cylinder of 0.8λ side is considered [2]. The object is located in an investigation domain of $L = 2.4\lambda$ side (free space background) and it is characterized by $\varepsilon_r = 1.5$. A set of *V*=8 line sources equally-spaced on a circle of $\rho_S = 2.4\lambda$ radius is employed. For each source, the total field is measured at $M = 21$ equally spaced detectors located over a circle of $\rho_M = 1.8\lambda$ radius (the noiseless case is considered). The inversion data have been synthetically computed by means of the MoM method and different discretization grids have been adopted for the direct and inverse procedure in order to avoid the "inverse crime problem" [2].



(*a*) (*b*) (*c*)

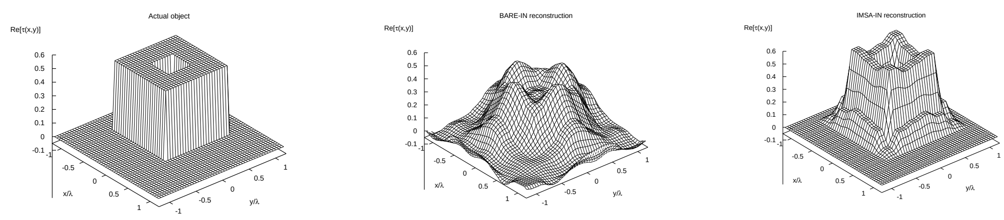
Figure 1 – Square cylinder: (*a*) actual object, (*b*) Bare-IN reconstruction, and (*c*) IMSA-IN reconstruction.

The plots in Fig. 1 show the effectiveness of the IN method ($N_{BARE} = 400$, $I_{out} = 50$, and $I_{in} = 30$) in localizing the object and providing a good approximation of the actual distribution. However, the shape of the object is distorted and some artifacts appear [Fig. 1(*b*)]. Otherwise, the reconstruction obtained by the IMSA-IN approach after $S = 4$ steps ($N_{IMSA} = 36$, $I_{out} = 30$, and $I_{in} = 30$ at each step) confirms the effectiveness of the technique in reducing the reconstruction error [Fig. 1(*c*)]. Such an observation is confirmed by the values of the error indexes (total - ξ_{tot}, internal - ξ_{int}, and external - ξ_{ext} [2]) in Tab. I.

Table I – Reconstruction indexes.

	Bare-IN			*IMSA-IN*		
Object	ξ_{int}	ξ_{ext}	ξ_{tot}	ξ_{int}	ξ_{ext}	ξ_{tot}
Square	1.36×10^{-1}	3.93×10^{-2}	6.37×10^{-2}	7.12×10^{-2}	8.25×10^{-3}	1.69×10^{-2}
Hollow	1.39×10^{-1}	6.86×10^{-2}	9.85×10^{-2}	9.01×10^{-2}	3.37×10^{-2}	4.77×10^{-3}

The second example deals with the reconstruction of a hollow square cylinder with $L_{out} = 1.2\lambda$ and $L_{in} = 0.4\lambda$. The same parameters of the previous example have been employed and the IMSA has been stopped after $S = 3$ steps. As it can be observed (Fig. 2), although the Bare-IN approach provides quite good performances, the shape of the scatterer does not exactly match with the actual one. On the contrary, the IMSA integration allows significant improvements in terms of accuracy of the retrieved profile [Fig. 2(*c*) – Tab. 1] (e.g., $\xi_{tot}^{BARE} = 9.85 \times 10^{-2}$ vs. $\xi_{tot}^{IMSA} = 4.77 \times 10^{-3}$).



(*a*) (*b*) (*c*)

Figure 2 – Hollow square cylinder: (*a*) actual object, (*b*) Bare-IN reconstruction, and (*c*) IMSA-IN reconstruction.

Also from the computational point of view, the IMSA scheme enables a non-negligible reduction of the computational burden. As a matter of fact, the Bare-IN inversion required about 10 minutes on an Intel Core Duo PC, while less than 7.5 seconds were required by the IMSA-IN inversion.

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