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AN INNOVATIVE MICROWAVE IMAGING TECHNIQUE FOR NON  
DESTRUCTIVE EVALUATION: APPLICATIONS TO CIVIL STRUCTURES  
MONITORING AND BIOLOGICAL BODIES INSPECTION.

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# **An Innovative Microwave Imaging Technique for Non Destructive Evaluation: Applications to Civil Structures Monitoring and Biological Bodies Inspection.**

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***Abstract*** – *Industrial and biomedical applications of microwave imaging techniques based on inverse scattering integral relations have become more and more important. In order to reduce the computational costs, allowing a quasi real-time processing, an innovative inversion procedure based on the use of a Genetic Algorithm and on the Sherman-Morrison-Woodbury matrix inversion method is presented. Selected numerical results, concerning various scenarios and scatterer dimensions, are presented in order to give some indications on the effectiveness but also current limitations of the proposed approach.*

***Keywords:*** *NDT/NDE, Microwave Imaging, Inverse Scattering, Genetic Algorithms.*

## I. INTRODUCTION

Non-invasive diagnostics using microwaves are very appealing in several applications [1][2] ranging from civil and industrial engineering [3][4], to geophysical monitoring and biomedical analysis [5][6] (for a general and complete overview, see [7] and the references cited therein).

In this framework, some microwave tomographic methods for nondestructive evaluation (NDE) have been recently proposed [8]-[11]. These approaches are aimed at inspecting dielectric objects by means of interrogating electromagnetic waves. Indeed, according to inverse scattering relations [12], scattered electric fields depend on dielectric properties of the structure under test. Consequently, in order to retrieve geometric and dielectric parameters of the investigation domain starting from the knowledge of the scattered field, a nonlinear inverse scattering problem has to be solved. On the other hand, since in NDE applications a complete image of the scatterer under test is not required, such methods are designed in order to fully exploit all the a-priori knowledge of the scenario under test.

In this paper, an innovative microwave imaging method is proposed. Unlike strategies described in [8] and [10][11], the proposed approach computes the unknown solution taking into account the dependence of the electric field on the scatterer characteristics.

The paper is organized as follows. After a description of the mathematical formulation of the proposed approach (Sect. II), a set of selected numerical results concerning different non-invasive applications is presented in Section III. Finally, some conclusions are drawn.

## II. MATHEMATICAL FORMULATION

Let consider a two dimensional scenario (Fig. 1) where a structure under test  $D_{inv}$  is successively illuminated by a set of  $V$  TM uniform plane waves ( $E_{inc}^v(x_n, y_n)$  being the related electric field). Outside the investigation domain, where the scatterer is located, a circular arrangement of probing antennas is used to collect the scattered electric field  $E_{scatt}^v(x_m, y_m)$  ( $(x_m, y_m)$  being the coordinates of the position of the  $m$ -th receiver).

Starting from the knowledge of the scattered field collected in the external observation domain and of the incident field measured in the investigation domain, the proposed technique is aimed at locating and reconstructing an unknown defect in the structure under test. Towards this end, the unknown defect is parameterized by considering the following ‘‘descriptors’’

$$\bar{g}_{crack} = \{L, W, \alpha, x_0, y_0\} \quad (1)$$

Moreover, the electric field distribution in the structure is to be determined as well

$$\bar{g}_{field} = \{E_{tot}^v(x_n, y_n); v = 1, \dots, V; n = 1, \dots, N\} \quad (2)$$

Then, by considering the inverse-scattering equations that relate the unknown quantities to the measured data [12], the problem solution is recast into the minimization of the cost function  $\Gamma$

$$\Gamma(\bar{g}) = \Gamma_{Data}(\bar{g}) + \Gamma_{State}(\bar{g}) \quad (3)$$

where

$$\Gamma_{Data}(\bar{g}) = \frac{\sum_{v=1}^V \sum_{m=1}^M |E_{scatt}^v(x_m, y_m) - \Omega_{Data}^m\{\bar{g}\}|^2}{\sum_{v=1}^V \sum_{m=1}^M |E_{scatt}^v(x_m, y_m)|^2}$$

and

$$\Gamma_{State}(\bar{g}) = \frac{\sum_{v=1}^V \sum_{n=1}^N |E_{inc}^v(x_n, y_n) - \Omega_{State}^n\{\bar{g}\}|^2}{\sum_{v=1}^V \sum_{n=1}^N |E_{inc}^v(x_n, y_n)|^2}.$$

being  $\bar{g} = \{\bar{g}_{crack}, \bar{g}_{field}\}$ .

Because of the nonlinear nature of (3), a global minimization procedure is needed for the solution of the minimization problem in hand. However, due to the large search space, a real-time processing is generally difficult to be obtained. In order to overcome this drawback, an effective method based on the numerical computation of the Green’s function has been proposed

in [11]. The inhomogeneous Green's function-based approach (IGA) strongly reduces the search space but the estimation of the electric field is obtained indirectly during the genetic evolution of the algorithm.

However, if the perturbed object differs only in a limited region characterized by  $P \ll N$  cells, an alternative solution is available. This solution consists in relating the field unknowns to the defect descriptors taking into account an efficient and computationally effective "direct" procedure (i.e., a procedure aimed at computing the electric field in the investigation domain starting from the knowledge of dielectric profile of the scenario under test). The "direct" problem requires the solution of the following matrix equation (which represents the numerical counterpart of the inverse scattering "state equation" [12])

$$[E_{tot}^v] = [\Theta]^{-1} [E_{inc}^v] \quad (4)$$

where  $[\Theta]^{-1} = \{[I] - [G][\tau]\}$  and the diagonal matrix  $[\tau]$  describes the dielectric distribution in the investigation domain. In order to solve Eq. (4), a computationally-effective procedure based on the Sherman-Morrison-Woodbury formula (SMWA) [13] is adopted. More in detail, firstly the "state" operator  $[\Theta_{un}]^{-1}$  related to the unperturbed geometry (i.e., without the defect) is computed

$$[E_{tot(un)}^v] = [\Theta_{un}]^{-1} [E_{inc}^v] \quad (5)$$

where  $[\Theta_{un}]^{-1} = \{[I] - [G][\tau_d]\}$ ,  $[\tau_d]$  is the matrix related to the unperturbed geometry and  $E_{tot(un)}^v$  the corresponding electric field.

Then, the "perturbed" operator is determined according to the following relationship

$$[\Theta]^{-1} = [\Theta_{un}]^{-1} + [\Theta_{un}]^{-1} [U] \{ [I] - [V]^T [\Theta_{un}]^{-1} [U] \}^{-1} [V]^T [\Theta_{un}]^{-1} \quad (6)$$

where

- $[\Theta] - [\Theta_{un}] = [U][V]^T$  ;
- $[U]$  is an  $N \times P$  matrix whose columns are the  $P$  non-zero columns of  $[\Theta] - [\Theta_{un}]$ ;
- $[V]$  is an  $N \times P$  matrix whose elements are defined as follows

$$v_{np} = \begin{cases} 1 & \text{if } (p = q) \text{ and } (n = j_q) \\ 0 & \text{otherwise} \end{cases}$$

where  $j_q$  is the  $q$ -th element of a vector of  $P$  elements, which indicates the number of the  $q$ -th perturbed cell in the investigation domain of  $N$  cells.

Consequently, the reconstruction process operates according to a genetic strategy [14] as described in [11], but at each step  $k$  ( $k = 1, \dots, K$ ), the total electric field  $\bar{g}^{k,l}_{field} = \{E_{tot,k,l}^v(x_n, y_n); v = 1, \dots, V; n = 1, \dots, N\}$  related to the  $l$ -th trial solution or defect configuration  $\bar{g}^{k,l}_{crack}$ , is computed according to the SMW-based procedure. More in detail, it turns out that

$$\bar{g}^{k,l}_{field} = [\Theta_{k,l}]^{-1} [E_{inc}^v] \quad (7)$$

where  $[\Theta_{k,l}]^{-1}$  is obtained according to (6) rewritten as follows

$$[\Theta_{k,l}]^{-1} = [\Theta_{un}]^{-1} - [\Theta_{un}]^{-1} [U_{k,l}] \{ [I] - [V_{k,l}]^T [\Theta_{un}]^{-1} [U_{k,l}] \}^{-1} [V_{k,l}] [\Theta_{un}]^{-1} \quad (8)$$

As can be noticed, while the matrices  $[U_{k,l}]$  and  $[V_{k,l}]$  are computed at each iteration for every  $l$ -th trial solution,  $[\Theta_{un}]^{-1}$  is computed once and off-line during the initialization of the iterative process. This allows a non-negligible computational saving with respect to a standard direct algorithm and an improvement of the accuracy in the electric field prediction.

### III. NUMERICAL VALIDATION

To assess the effectiveness and current limitations of the proposed approach, several test cases concerned with synthetic data have been analyzed and selected representative results will be presented in the following. In particular, different applications will be shown in order to point out the versatility of the proposed technique. The one is related to the monitoring of civil structures, the other is concerned with a biomedical tomographic application.

As a first representative example and for a comparative study, the performances of the proposed approach (indicated as SMWA in the following) have been compared with those of the IGA method [11] by considering a reference test case where a void square crack ( $\varepsilon_R^{crack} = 1$ ,  $\sigma^{crack} = 0 \text{ S/m}$ ,  $A_c = 0.04 \lambda^2$ ) is located at  $\{x_c = 0.15\lambda, y_c = 0.10\lambda\}$  inside an cylindrical host medium of square cross-section ( $\varepsilon_R^{host} = 2$ ,  $\sigma^{host} = 0 \text{ S/m}$ ). Towards this purpose, various dimensions of the host dimensions and different levels of noise have been taken into account. In particular the area of the host medium  $A_D$  has been varied in the range  $\{0.25; 2.25\} \lambda^2$  and the Signal to Noise Ratio (SNR) from 2.5 up to 30.0 dB, being

$$SNR = 10 \log_{10} \frac{\sum_{v=1}^V \sum_{m=1}^M |E_{scatt}^v(x_m, y_m)|^2}{2MV\sigma_{noise}^2} \quad (9)$$

As far as the probing electromagnetic source is concerned, a plane wave illuminated from  $V = 4$  orthogonal directions the investigation domain and the scattered field has been collected at  $M = 50$  measurement points located in the observation domain as shown in Fig. 1.

In order to quantitatively estimate the reconstruction accuracy, the following error figures are defined. The localization error  $\delta_c$

$$\delta_c = \frac{\sqrt{(x_c - \hat{x}_c)^2 + (y_c - \hat{y}_c)^2}}{d_{max}} \times 100 \quad (10)$$

where  $d_{max}$  is the maximum error equal to the diagonal of the investigation domain. The occupation error  $\delta_a$

$$\delta_a = \frac{A_c - \hat{A}_c}{A_c} \times 100 \quad (11)$$

and the error in the estimating the electric field distribution

$$\Delta E_{tot} = \frac{1}{NV} \left( \sum_{v=1}^V \sum_{n=1}^N \left\{ \frac{\left\| \hat{E}_{tot}^v(x_n, y_n) - E_{tot}^v(x_n, y_n) \right\|}{\left\| E_{tot}^v(x_n, y_n) \right\|} \right\} \right) \times 100 \quad (12)$$

where the superscript  $\hat{\phantom{x}}$  differentiates estimated from actual quantities.

As regards to the localization error (Fig. 2), both the methods achieve good results with a mean error  $\langle \delta_c \rangle$  of 5.04 % for the SMWA and 5.93 % for IGA. Concerning the occupation error, Fig. 3 shows a dependence of the error on the SNR value. Moreover, it is pointed out that the SMWA overcomes the IGA also in correspondence with a larger investigation domain. On average, it turns out that  $\langle \delta_a \rangle_{SMWA} = 12.45\%$  versus  $\langle \delta_a \rangle_{IGA} = 14.56\%$ . Such a behavior is further confirmed when the field error is taken into account. As shown in Fig. 4, the SMWA significantly outperforms the IGA ( $\langle \Delta E_{tot} \rangle_{SMWA} = 3.78\%$  vs.  $\langle \Delta E_{tot} \rangle_{IGA} = 6.64\%$ ).

According to the indications drawn from the previous analysis, which seem to confirm the effectiveness of the SMWA in dealing with NDE/NDT problems, two different and possible applications concerned with more realistic scenarios have been addressed. The first application refers to the civil engineering and it deals with the monitoring of the scenario sketched in Fig. 5. The investigation domain is a squared cylinder  $L_d = 4.167\lambda$  in side where the inhomogeneous host medium is characterized by a homogeneous cement paste ( $\varepsilon_R^{host} = 2.37$ ,  $\sigma^{host} = 5.7 \times 10^{-4} S/m$ ) [15] with four steel bars ( $\varepsilon_R^{steel} = 1.0$ ,  $\sigma^{steel} = 1.1 \times 10^6 S/m$ ) inserted.

A preliminary assessment has been carried out by considering the dependence of the reconstruction accuracy of the SMW-based approach on the dimensions of the defect (i.e., the ratio  $[L_c/\lambda]$  has been varied between 0.1 and 1.0,  $L_c$  being the side dimension of the square defect). As can be observed (Fig. 6), the SMW-based technique overcomes the IGA method both in terms of localization and area estimation as confirmed by the error values:  $\delta_c \leq 20\%$  and  $\delta_a \leq 40\%$ .

The second test case (Fig. 7) deals with a biological structure. In this case, the SMWA has been used to localize a malignant tissue inside a tomographic cross section of a human thorax. In particular, the “defect” (with respect to the safe configuration) has been supposed to belong to the “kidney” tissue [Fig. 7(a)] or to the “muscle” tissue [Fig. 7(b)] and the size of the square “defect” has been varied between  $7 \times 10^{-3}$  to  $37 \times 10^{-3} \lambda$ . For comparison purposes, the localization error has been computed for the SMWA as well as for the IGA (Fig. 8). The achieved results still show the effectiveness of the proposed approach in locating the defect and the error values turn out to be lower than 25 % whatever the configuration under test.

#### IV CONCLUSIONS AND FUTURE DEVELOPMENTS

In this paper, an innovative optimization procedure for microwave non-destructive applications has been proposed. The approach is based on an inverse scattering formulation developed in the spatial domain. The approximate retrieval of an unknown defect in a known host medium has been obtained by minimizing a suitable cost function through a suitable genetic-based iterative procedure. Since the dimension of the defect is generally small when compared to the area of the host medium, an effective procedure for the estimation of the induced electric field has been considered, thus further reducing the number of unknowns of the problem in hand and allowing a more accurate reconstruction of the field distribution. Selected numerical results, concerned with biomedical diagnostics as well as monitoring of civil structures, have been reported. In particular, the effects of the dimensions of the defect on the reconstruction accuracy have been considered.

Certainly, further analysis should be performed for a complete assessment of the proposed technique. Nevertheless, some potential features of the approach seem to be very attractive for an industrial experimentation.



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## FIGURE CAPTIONS

- Figure 1.** Problem Geometry.
- Figure 2.** Test case 1 – Localization Error (*a*) SMWA, (*b*) IGA
- Figure 3.** Test case 1 – Occupation Error (*a*) SMWA, (*b*) IGA
- Figure 4.** Test case 1 – Field Estimation Error (*a*) SMWA, (*b*) IGA
- Figure 5.** Scenario “Civil Structure Model”
- Figure 6.** Scenario “Civil Structure Model” – Error figures. (*a*) Localization Error, (*b*) Occupation Error
- Figure 7.** Scenario “Biological-Structure Model” – (*a*) Test case “*Kidney*”, (*b*) Test case “*Muscle*”
- Figure 8.** Scenario “Biological-Structure Model” – Localization Error. (*a*) Test case “*Kidney*”, (*b*) Test case “*Muscle*”

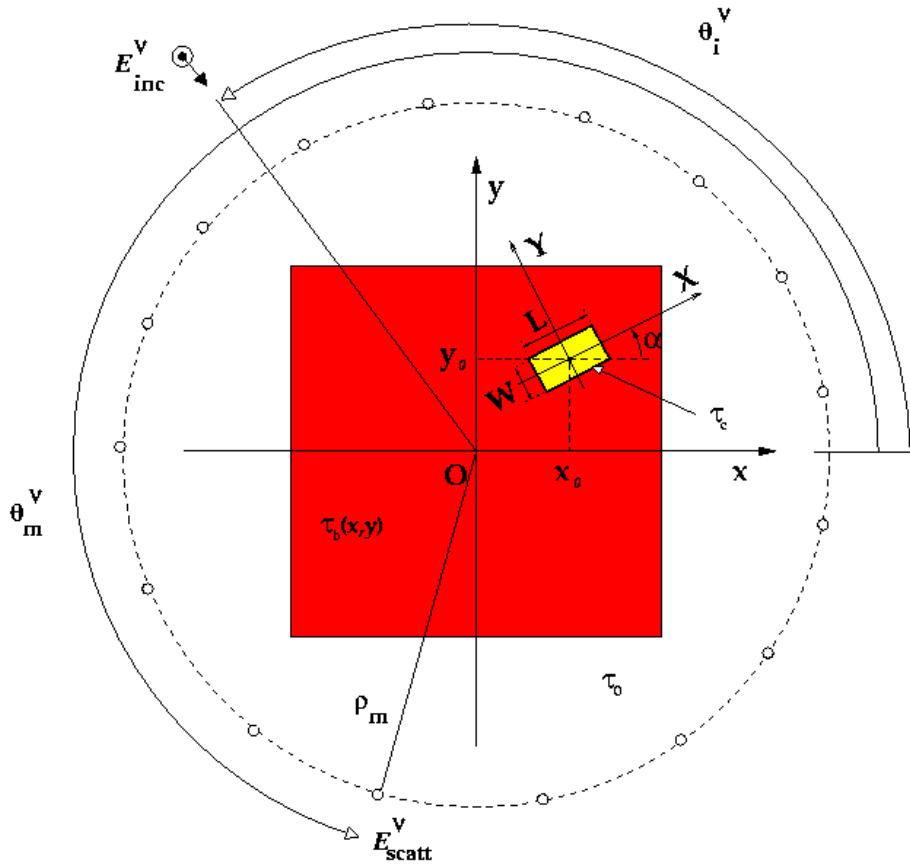
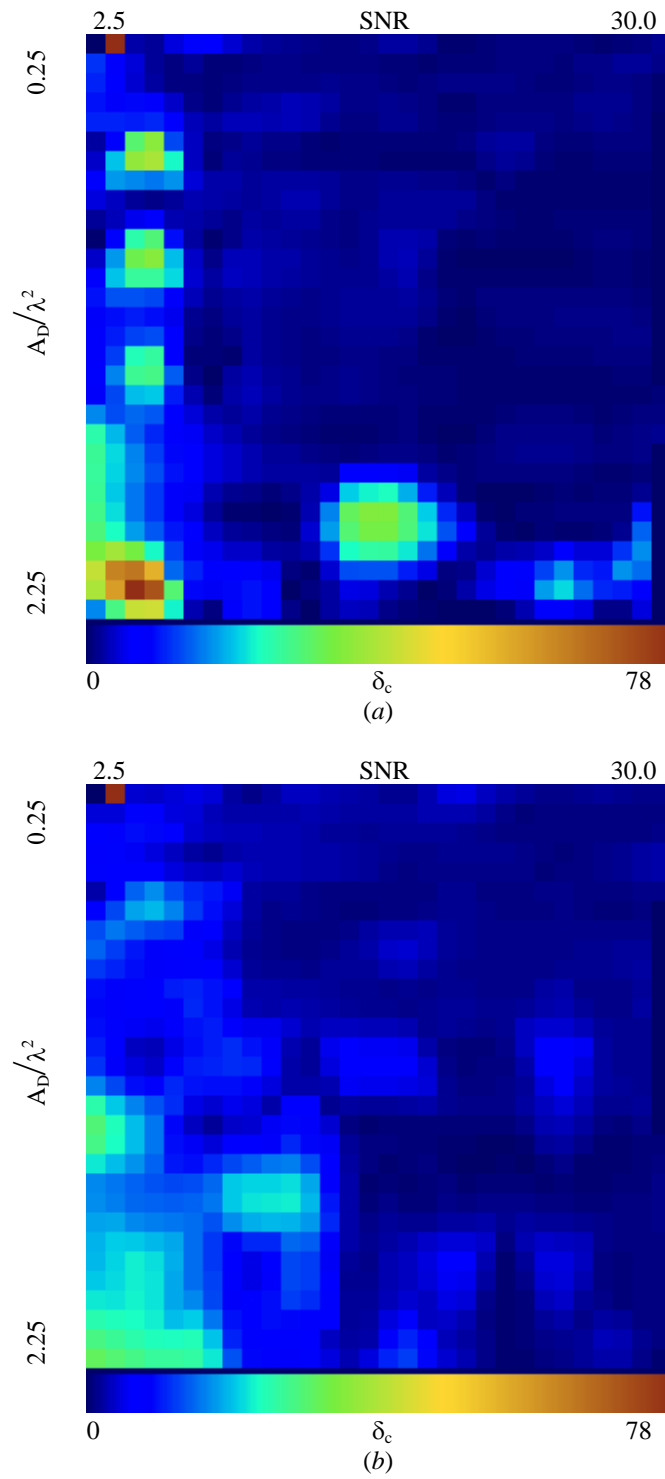


Fig. 1 – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”



**Fig. 2** – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”

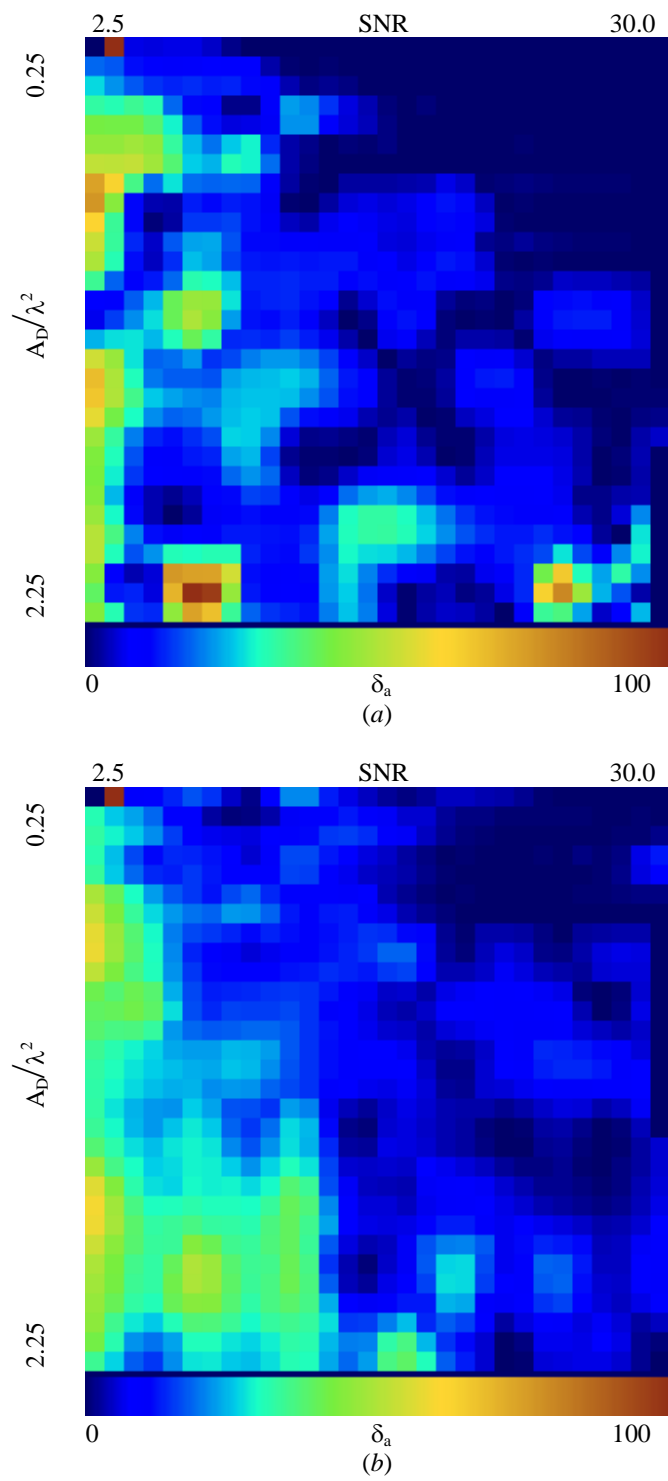
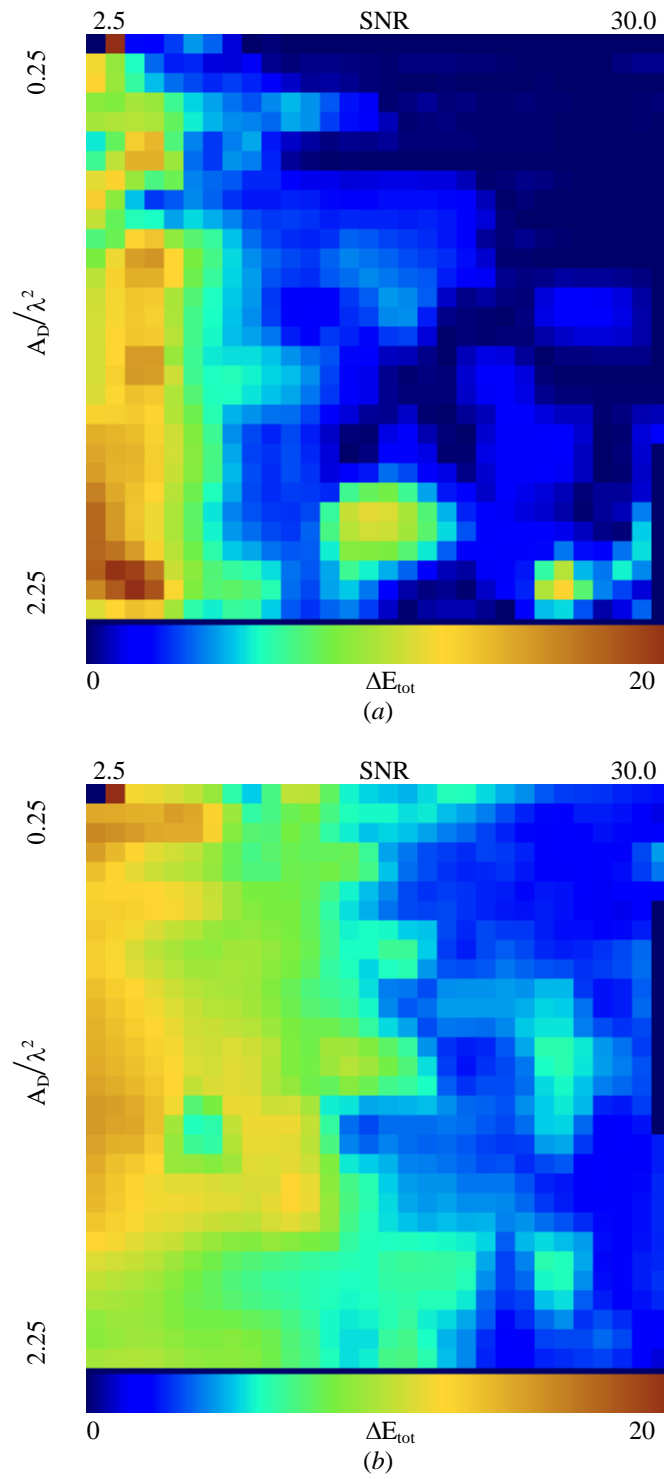


Fig. 3 – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”



**Fig. 4** – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”

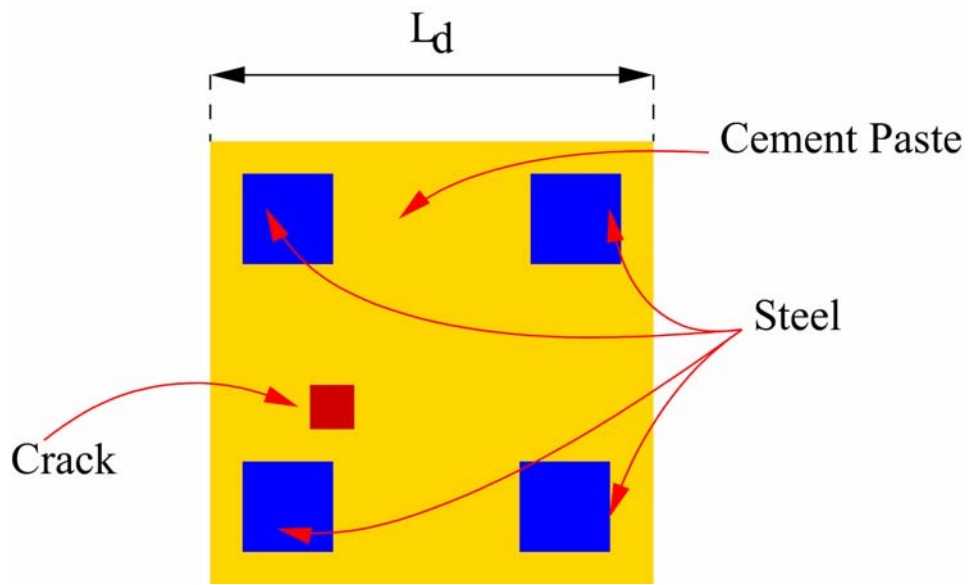


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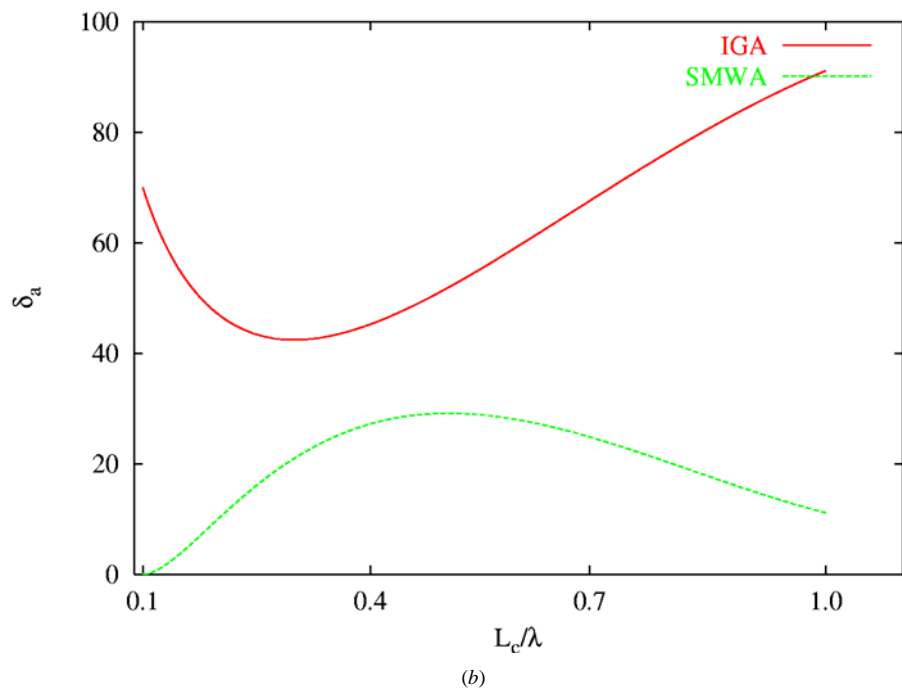
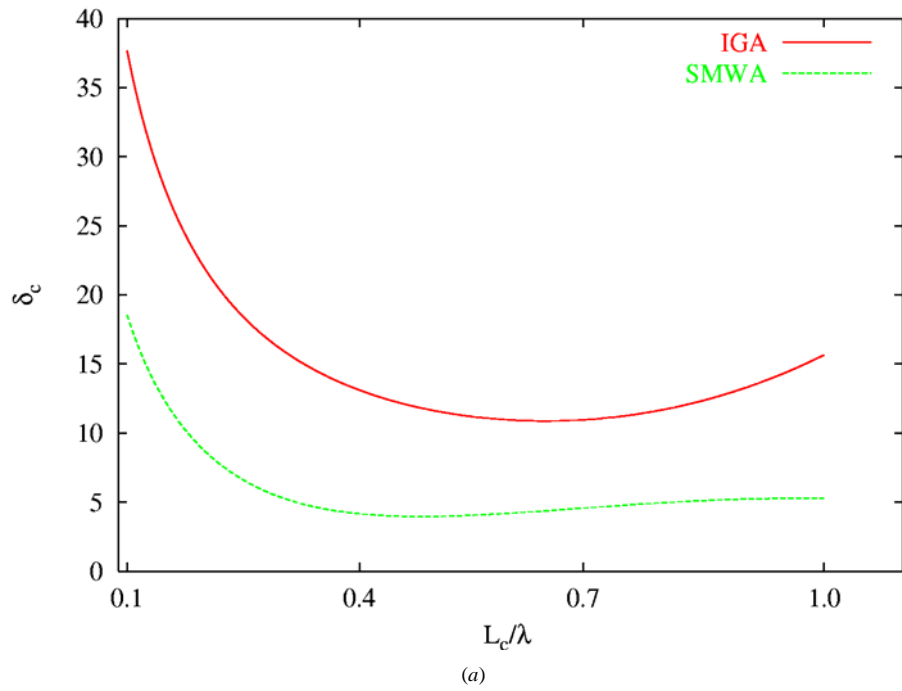
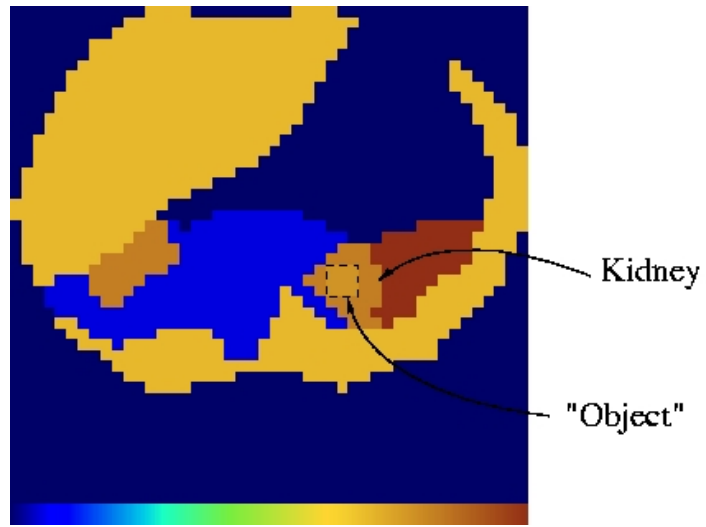
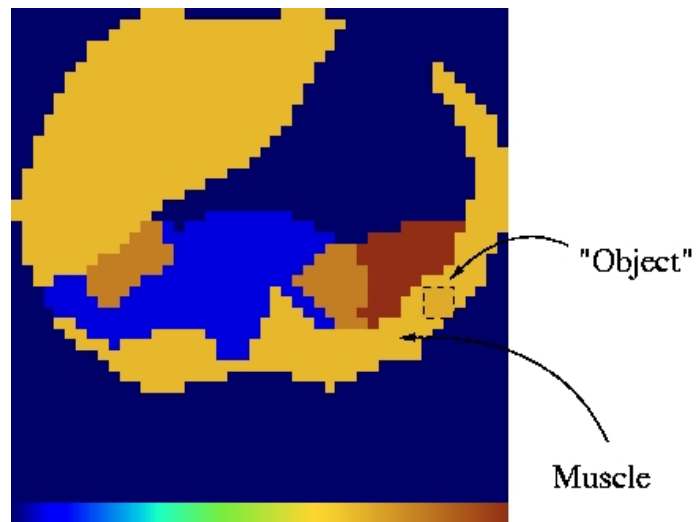


Fig. 6 – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”





(a)



(b)

Fig. 7 – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”

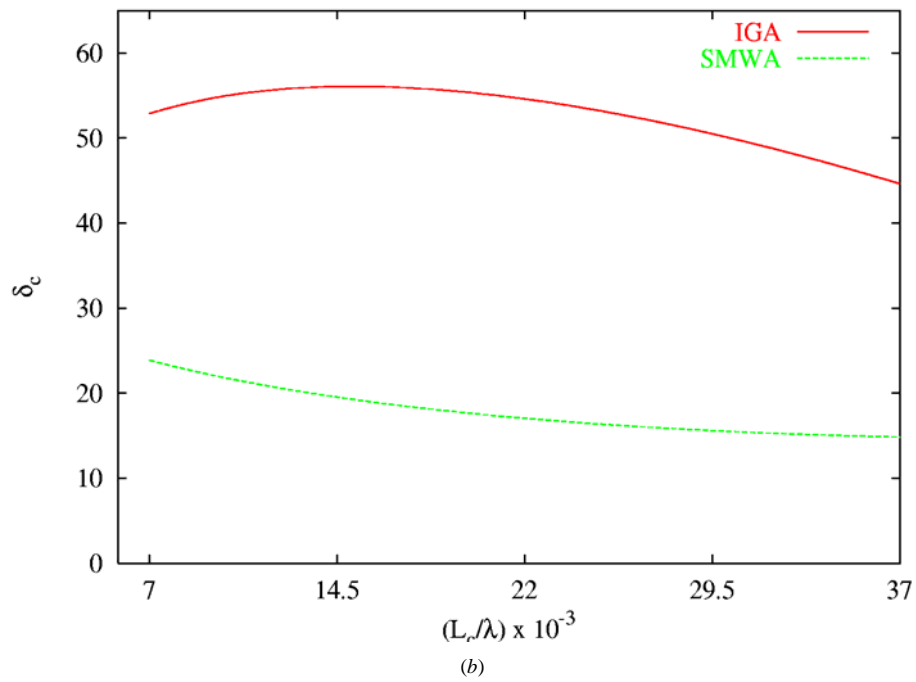
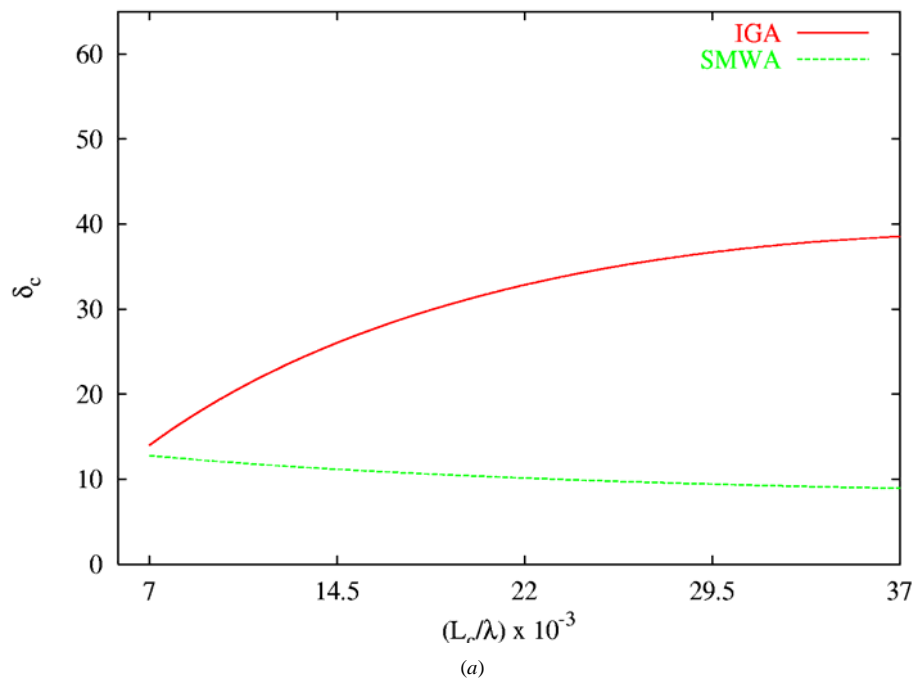


Fig. 8 – M. Benedetti *et al.*, “An Innovative Microwave Imaging Technique...”