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Domestication of the geometrical eye: unpacking geometry with the GGbot drawing robot

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In this paper, using Radford's Theory of Objectification, we analyze 2 students' learning activity that show how the features of the drawing robot (GGbot) affect student's sensuous cognition, forging their theoretical perception for the learning of geometry.

Keywords: Eye as a theoretician, GGbot, geometry, theory of objectification, sensuous cognition.

Introduction

Digital technologies open new possibilities in the teaching and learning of mathematics. In line with the theory of objectification (TO), we conceive learning as the student's sensuous encounter with systems of thinking and action that have been historically and culturally constituted (Radford, 2021). In this paper, we analyze the role of the drawing robot GGBot (abbreviation of GREATGeometryBot) in the learning of geometry in primary school. We report an exploratory study carried out with grade 3 students performing an activity that involved the use of the GGBot in the solution of geometrical tasks. The aim of the contribution is to understand the impact of the GGBot in transforming the individual's perception (which here we intend being visual, tactile, kinesthetic, and imaginative) in the learning of geometry.

Conceptual framework

The TO, embedded in sociocultural perspectives, stems from a profound intertwining between culture and the individuals' activity. Culture is an intrinsic component of mathematical thinking and learning, and activity is the ontological category of the TO as it realizes the consubstantiality between individuals and their culture. In the stance of the TO, mathematical thinking and learning are not processes confined in the mind but they are intertwined with individuals' social activity. Signs and artefacts play an important role, beyond the role of something that stands for something else or as mediators of activity. In the TO, they are considered an integral part of human thinking and human activity (Radford, 2021). The issue of learning is rooted in the dialectics between the individual and their culture. Learning is a movement pushed by the intrinsic differential between the individual and cultural knowledge. In fact, in attending to knowledge the student has to cope with something that in the beginning is different from him, an alterity that challenges, resists and opposes him. Learning is the process that erases such a difference to make sense of cultural knowledge and transform it into something familiar that allows new forms of action, thinking, imagination and feeling. In order to reduce the distance between the individual and cultural knowledge, activity as a specific human endeavor is required on the part of the student. Radford (2021) conceives learning as a social process of becoming aware of cultural-historical systems of thinking and doing, through our bodily, sensory, and artefactual semiotic activity. We remark that, according to the TO, signs and artifacts are constitutive of the activity that leads students to notice mathematical knowledge. They are bearers of an embodied intelligence and culturally endowed with specific patterns of activity that individuals use in their meaning-making processes and to carry out their actions (Radford, 2021). The outcome of the learning process is the encounter with mathematical (cultural) objects and their transformation into objects of consciousness. From this standpoint, learning has a strong phenomenological nature where noticing occurs in an enlarged notion of mind and consciousness, termed by the TO sensuous cognition, that includes not only ideal and mental features but also embodied ones such as perception, feelings, and kinesthetic activity. In light of the dialectic-materialist approach underpinning the TO, the basic tenet behind the notion of sensuous cognition is that the body, the senses, and the objects of sensation are not a priori entities but mutually transformed by cultural-historical activity entangled with the use of signs and artefacts. The relations of mind and body to the world are historical intertwines with material and ideational culture, and our senses change and develop along with the changes of the cultural-historical dimension (Radford, 2021). From the standpoint of sensuous cognition, human perception is, in the words of Wartofsky (1984, p. 865), “a cultural artefact shaped by our own historically changing practices”. In this regard, perception deploys cultural forms of seeing, touching, hearing etc. that characterize our relation with the world. Within sensuous cognition, the issue of learning is identified with the manner in which perception is transformed into a theoretical cultural form of perception, in progressively noticing and endowing with meaning cultural-historical systems of thinking and doing. How do students change their perception from “spontaneous” forms of attending to objects to a mathematical and theoretical one? To answer this question, we must consider learning as a “domestication of the eye” (Radford, 2021), a long process that allows students, in cultural-historical activity intertwined with the use of signs and artifacts, to transform the eye (and other senses) into sophisticated theoretical organs able to notice and make sense of certain things in mathematical manners - for example recognizing numerosity, algebraic structures, geometric invariants etc. We remark the co-variational nature of sensuous cognition in that learning processes entail a transformation of perception along with the transformation of the perceived cultural object into an object of consciousness. We underline the multimodality entailed with the “domestication of the eye”, both in the various sensorial channels and the richness of signs and artefacts interwoven with cultural-historical activity involved in the transformation of perception (Radford, 2021).

In this framework, we aim to analyze the impact of the GGBot, with its artifactual and semiotic features, which combine the well-known strengths and opportunities offered by the modern visual programming language with those of Papert’s original robotic drawing-turtle (Papert, 1980). The GGBot is composed of two wheels, a marker-holder at each end, where one can insert markers to let GGBot draw (Figure 1a), and SNAP!, the visual programming language used to provide commands to the GGBot. We show some of the available commands in Figure 1b, and we refer to Baccaglini-Frank and colleagues (2020) for more details on how the GGBot works.

a) b)

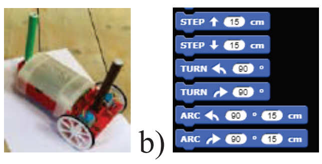


Figure 1: a) view of the GGbot; b) SNAP! Commands list

This contribution focuses on the dialectical movement between primary school students, geometrical knowledge and cultural-historical activity that pivots around the use of GGBot as they learn geometry. More precisely, the present study aims at answering the following research question: how does the GGBot with its related multimodal artifactual and semiotic features (the physical robot, the SNAP! commands, movement, drawing, gesturing, natural language) affect students' sensuous cognition? That is, how does the GGBot "domesticate the eye" (seeing, touching, kinesthetic activity, imagination) for the learning of geometrical figures in primary school?

Methodology, data and analysis

We consider data collected during one session of a sequence of three that was conducted by Anna Baccaglini-Frank (Baccaglini-Frank & Mariotti, in press). Each session lasted a fixed time-frame and involved one class of grade 3 students, the classroom teacher, and the researcher. The students were asked to carry out various types of tasks. First, a technical exploration of the GGBot guided by questions is conducted. Then, students continued working in pairs and they were asked to use SNAP! to code the movement of the GGBot that would lead it to draw with the marker a certain given figure (figure-to-code tasks). The students were involved in another type of task as well, a collective task where they could answer in turn. Starting with a given SNAP! code, they were asked to predict the GGBot's movement and, consequently, foresee the trace that the marker would have left on the paper (code-to-figure tasks). Given the potentialities of predictive tasks in providing insight into the learning process in geometry (Miragliotta, 2020), in this paper, we focus on the code-to-figure tasks. We consider some video recorded sequences where a group of students are predicting the GGBot's drawing outcome of a given code displayed on the projector (Figure 2a). For the reader's convenience, we show in Figure 2b the commands explanation and the expected figure drawn by the GGBot according to the code alongside (what is shown in Figure 2b was not projected nor shared with students during the experiment).

a) b)

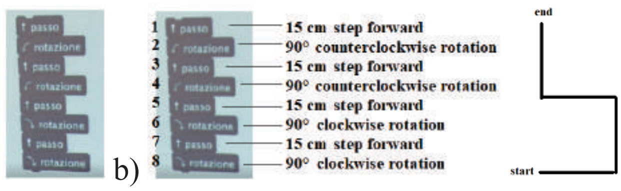
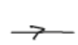
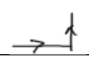
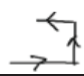
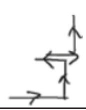


Figure 2: a) SNAP! Commands list (projected); b) Commands explanations and the expected figure

We remark that with the code-to-figure predictive task, students are asked to manage three separate, but connected, passages that entail theoretical forms of seeing, touching, movement and imagination. The first goes from the given code to the prediction of the GGBot movement in their imaginary dimension; the second, from the imagined movement to the prediction of the figure that such movement would trace on the paper; the third, from the imagined trace to its external outcome shared with the use of words, gestures, and drawings. The analysis of the nodes of such a chain of passages allow us to understand how the students' sensuous cognition evolves in the interplays with GGBot features. Our analysis focus is on the mutual transformation of perception and mathematical objects in activities whose outcome is the "domestication of the eye". We consider variables, specific to the task at stake, concerning perception, signs and artifacts, and geometric knowledge: in regard to perception, seeing, touching, movement and imagination; with respect to signs and artifacts, gestures, drawings, natural language and SNAP! commands; in regards to knowledge we focus on the egocentric and allocentric system of references (SoR) – whose involvement in similar activities is well documented in the literature (e.g. Baccaglini-Frank et el., 2014) - the notion of angle and the (mis)matching between the drawing of the figure resulting from the GGBot movement and the one primary school students would have performed using paper, pencil and possibly a ruler. We remark that in the code-to-figure task the GGBot is not physically present as in the figure-to-code ones. Nevertheless, the GGBot, in relation to the previous activities, is present in the imaginative perception as students coordinate gestures, natural language, drawings, and SNAP! commands.

Data and their analysis. The students, the researcher and the teacher are arranged in a circle around a big piece of paper on the floor where there are some drawings of previous activities (familiarization with the robot's functionality and figure-to-code task to draw a square). We focus on 2 students (Angela and Vanessa) and the following tables contain excerpts of the transcription of the video recording significant of the three passages described above (code to imagined movement, imagined movement to imagined figure and imagined figure to external shared the figure). In order to be faithful to the synchronicity in the use of signs and artifacts, we present the data in three columns: one for the gestures and non-verbal signs, the second for the utterances, and the third for the drawings (in Table 1, the drawings are just for the reader's convenience since Angela traced the figure with a finger on the paper). In the transcription: R stands for the researcher, and we numbered the lines using the same number to indicate simultaneity. In the analysis we enumerate both the transcript line (TL) and the SNAP! command line (SL, see Figure 2b).

Angela, who traces the figure with her finger on the paper		
Gestures and non-verbal signs	**Utterances**	**Drawings**
[1] Angela points her index finger on a white area of the paper and then she traces a first segment in the horizontal direction (according to her egocentric SoR) towards her right 	[1] R: like that	[1] 
[2] Angela goes on with a second segment in the vertical direction (according to her egocentric SoR) moving away from herself and articulating the movement in two steps		[2] 
	[3] Angela: ehm	
[4] Angela traces a third segment in the horizontal direction (according to her egocentric SoR) towards her left. This segment is shorter than the first two.		[4] 
[5] Angela traces a fourth segment in the horizontal direction (according to her egocentric SoR) towards her right. This segment is shorter than the first two as well.		[5] 
[6] Angela traces a fifth segment in the vertical direction (according to her egocentric SoR) moving away from herself. Then, she stops moving and gazes at R.		[6] 
[7] Angela gazes at her finger on the paper		

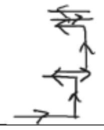
[8] Angela moves horizontally her finger right and left various times	[8] Angela: and like…	[8-9] 
	[9] Angela: and like so and so…	
[10] R points her finger towards Angela	[10] R: So it is an excellent idea. It is a sort of stair, did you see?	

Table 1: Angela

Analysis. Angela traces correctly with her finger the first three segments of the figure (TL 1-4 that correspond to her interpretation of the SL from 1 to 5). When reaching the 6 SL, she is not able to correctly handle the change of direction in the rotation. Therefore, she interprets the change in the direction of the rotation as a reverse direction along the same segment (TL 5). At the 7SL, Angela traces with her finger the vertical segment (TL 6). Then, the last command puzzles her even more and she goes back and forth with her finger (TL 8-9). In her activity, Angela resorts only to gestures and the SNAP! commands. She is able to notice in her imaginative perception the corresponding movement of the GGBot related to the first 3 segments of the figure. Nevertheless, when it comes to the third rotation, her perception is not able to grasp the angle of the figure as it is conveyed by the movement of the GGBot. The coordination of gestures and the SNAP! icons requires a transformation of imaginative perception to notice the angle of the figure in terms of step and rotation of the GGBot and not as the portion of the plane delimited by the two half-lines (i.e. the two sides of the figure). Furthermore, it requires a transformation of imaginative perception able to consider also the connection between egocentric (Angela's) and allocentric (GGBot's) SoR that does not emerge in the use of pencil and paper. Her perception does not encompass the change of direction in the rotation due to both the new way of encountering the angle of the figure as a rotation of the GGBot and the conflict between the egocentric and allocentric SoR. The back-and-forth gesturing along the side of the figure (TL 8-9) and the global absence of structured natural language are tokens of Angela's blurred perception and her struggle in "domesticating the eye" to transform her perception of the angle with respect to the one she learnt before. The process of "domestication of the eye" does not make the necessary leap to handle both the angle of the figure and the SoR, thus missing in the imaginative perception the expected figure drawn by the GGBot corresponding to the SNAP! commands.

Vanessa, who draws the figure on the paper with the marker		
Gestures and non-verbal signs	Utterances	Drawings
	[61] Vanessa: So before she (Vanessa is referring to a classmate's answer in a previous figure-to-code task) did a square, ok? [62] R: ok	
[63] Vanessa draws on the paper a line in the vertical direction (according to her egocentric SoR) moving away from herself.	[63] Vanessa: So, he took a step forward, no?	[63] 

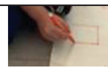
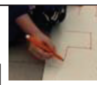
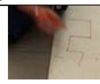
[64] Vanessa draws on the paper a line in the horizontal direction (according to her egocentric SoR) towards her right	[64] Vanessa: a rotation	[64] 
[65] Vanessa separates the marker from the paper and starts to oscillate over the second segment	[65] Vanessa: then another, a ...another step forward, so the rotation…	
[66] Vanessa continues to oscillate repeatedly over the second segment	[66] Vanessa: yes well, the, the step forward	
[67] Vanessa draws a third short segment in the vertical direction (according to her egocentric SoR) towards herself	[67] Vanessa: a rotation	[67] 
[68] Vanessa draws out the line towards herself	[68] Vanessa: and then after the rotation again a step forward	[68] 
[69] Vanessa draws on the paper a line in the horizontal direction (according to her egocentric SoR) towards her right	[69] Vanessa: then he put the rotation the opposite way, and so like this	[69] 
[70] Vanessa draws quickly another two lines	[70] Vanessa: and like this	[70] 

Table 2: Vanessa

Analysis. For Vanessa the conflict starts since the beginning, when she is managing the second SL (TL 64). After she has drawn the first segment, she explicitly links the word rotation with the drawing of the second perpendicular segment. After that, in TL 65, Vanessa should go a step forward with the marker but she is puzzled about where to go due to the previous interpretation of the rotation. Indeed, drawing another step forward in her situation would have resulted in a drawing with a "side doubled" ,i.e., two strokes of the marker (instead of a step, a rotation and then another step). Vanessa's confusion is highlighted by her oscillating the marker over the second segment and uttering "then another, a ...another step forward, so the rotation…" and "yes well, the, the step forward" (TL 65-66). Vanessa is confused because she lives in a conflict between the two ways in which the notion of angle co-emerges with her sensuous act: the angle as the part of the plane between two half-lines and the angle as the rotation of the GGBot. The gesturing with the marker and the utterances described above, testify such a conflict; she is able to draw the consecutive segments, but, when trying to relate them with the SNAP! commands, she is at odds with what she is doing. In TL 67, Vanessa is managing the second rotation of the 4th SL: she avoids the conflict linking the word rotation with a little portion of a perpendicular segment that she extends synchronously with the words "then after the rotation again a step forward" (TL 68). Vanessa's use of natural language is always assertive and explicit: she uses words to scan the imagined movement and the resulting trace. Her coordinated use of natural language and the drawing with the marker (TL 67-68) shows in an evident and interesting way the "domestication of the eye" related to the angle and her struggle to erase the differential between her previous form of noticing and the new (GGBot's) one that is challenging, resisting, and opposing her. We observe Vanessa's difficulty in fully accomplishing the domestication of her sensuous cognition to "see" angles with the "eyes" of the GGBot. Vanessa handles the coordination of the egocentric and allocentric SoR in the direction of the steps, in fact she always says "forward" as if she were in the SoR of the GGBot. However, she is not able to coordinate the two SoR when it comes to the rotations. She systematically shifts the left with the right and vice versa. Notwithstanding the difficulties in coping with the angle and the two SoR, Vanessa arrives at a drawing consistent, apart from the inversions of left and right rotations, with the SNAP! commands and the ensuing movement of the GGBot. This testifies a first transformation of her geometrical perception to conceive of figures both as theoretically perceived in drawings with pencil and paper and in the entanglement between the SNAP! commands and the ensuing movement of the GGBot in terms of steps and rotations. The coordinated use of the marker to draw and as a pointer, natural language and the SNAP! commands allows a transformation of Vanessa's multimodal perception made of seeing, touching, movement and imagination to encompass new ways of noticing the SoR (egocentric and allocentric), the angle of the figure (resulting from steps and rotations), and the geometric figure (in the interplay between SNAP! commands and the imagined movement of the GGBot). Despite her struggle in coping with new ways of attending to the angle and the SoR, Vanessa's "domestication of the eye" allowed her to connect the two meanings of the figure, the one conveyed via the GGbot and SNAP! and the previous one conveyed via drawings and paper and pencil. Thus, Vanessa testifies in her learning process the mutual transformation of perception and the mathematical object.

Discussion and conclusion

Data show how the encounter of geometrical objects using the GGBot involves a complex intertwining of signs and artifacts (icons, gesturing, natural language, material objects), perception, and geometric knowledge. The analysis of Angela and Vanessa, exposed to code-to-figure tasks, allows us to delineate how the use of GGBot resists and opposes our two students in their process of "domestication of the eye". In previous activities without the GGBot, students' sensuous cognition had been carried out with material objects, rulers, gestures, natural language etc., on which they had direct perceptual and sensorimotor control. Furthermore, perception took place in their egocentric SoR. The introduction of the GGBot strongly transforms students' perception in new cultural and theoretical modes of attending to geometrical objects. Metaphorically speaking, students have to think, perceive, move and "feel" as if they were the GGBot. In the code-to-figure task, pupils do not have direct control on the robot, and they have to establish, in sensuous cognition, a relation between the SNAP! code, the resulting movement of the robot and the geometrical figure that it would have traced. Since the code-to-figure is a predictive task, this happens in their imaginative perception without the physical presence of the robot but forged by the use of gestures, natural language and the sensorimotor activity, inherited by the previous tasks with it. The code-to-figure task suggests to what extent the students' perception has been theoretically domesticated according to the ideal and material characteristics pivoting around the use of the GGBot, to embrace new and richer encounters with geometric knowledge.

Answer to the research question. From a geometrical point of view, above all, students have to handle different SoR (egocentric and allocentric) and angles conceived as a rotation. The predictive code-to-figure task shows that the introduction of the GGBot, with its correlates of signs and artifacts, requires a "domestication of the eye". The transformation of perception is hindered by the conflict between an already theoretically domesticated eye - which encounters a geometric figure in a single SoR, in terms of segments and angles perceived as portions of a plane between two half-lines - and the new GGBot's theoretical eye - which encounters a geometric figure as something constructed in terms of steps and rotations, and the intertwining of the egocentric (student) and allocentric (GGBot)

SoR. In regard to the features of sensuous cognition, on the one hand the introduction of GGBot requires the students to “see” theoretically the figure as a recomposition of the SNAP! commands and the corresponding steps and rotations of the robot perceived visually and kinesthetically. On the other hand, in previous activities with paper and pencil, the students theoretically “see” the geometric figure as successive segments with different orientations perceived visually and kinesthetically as they trace on the paper. Concerning the task under scrutiny, we highlighted, in connection to the deepest conflicts lived by the students, that their “domestication of the eye” does not flow smoothly. The mutual transformation of perception and the geometric objects emerging from the activity with the GGBot establishes a distance between the individual and knowledge, which is perceived as an alterity. The “domestication of the eye” is the long process of learning that allows the student to erase such a distance, and further studies are needed to deepen the process of domestication of the eye in activity involving the GGBot. We hope that our work can suggest possible future research directions.

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