Preprint UTM 742 - November 2010

State independence for tunneling processes through Black Hole horizons and Hawking radiation

Valter Moretti[1,a] and **Nicola Pinamonti**[2,b]

[1] Dipartimento di Matematica, Università di Trento and Istituto Nazionale di Fisica Nucleare – Gruppo Collegato di Trento, via Sommarive 14 I-38050 Povo (TN), Italy.

[2] Dipartimento di Matematica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma - Italy.

E-mail: [a]moretti@science.unitn.it, [b]pinamont@mat.uniroma2.it

Abstract. Tunneling processes through black hole horizons have recently been investigated in the framework of WKB theory discovering interesting interplay with the Hawking radiation. In this paper we instead adopt the point of view proper of QFT in curved spacetime, namely, we shall use some scaling limit techniques in order to obtain the leading order of the correlation functions related with tunneling processess through a Killing horizon. The computation will be done with respect to a certain large class of reference states for scalar fields. In the limit of sharp localization either on the external side or on opposite sides of the horizon, the quantum correlation functions are of thermal nature, where in both cases the characteristic temperature is the Hawking one. Our approach is valid for every stationary charged rotating non extremal black hole, however, since the computation is completely local, it covers the case of a Killing horizon which just temporarily exist in some finite region too. These results give a strong support to the idea that the Hawking radiation, which is detected at future infinity and needs some global structures to be defined, is actually related to a local phenomenon which holds also for geometric structures (local Killing horizons) existing just for a while.

1 Introduction

As is known, the Hawking radiation [Ha75] is detected at *future null infinity* of a spacetime containing collapsing matter giving rise to a black hole. At least in case of spherical symmetry, the existence and the features of that radiation is quite independent from the details of the collapse, although the type of the short-distance behaviour of the two-point function of the reference state employed to describe the modes of the radiation plays a relevant role [FH90]. In recent years attention has been focused on *local* properties of models where the Hawking radiation is manifest, here *local* means in a neighborhood of a point on the event horizon [PW00, ANVZ05]. In this second approach the radiation is related to some thermal effects shown by some tunneling process through the horizon, namely the tunneling probability, computed in the framework of semiclassical WKB approach, has the characteristic thermal shape e^{-kE/T_H}, where T_H is the Hawking temperature and E the energy of the particle crossing the horizon. More precisely, the exponential thermal factor arises when taking a limit towards the horizon for an endpoint of the path of the classical particle.

This approach is interesting since it deals with local aspects only and, in this sense, is more general than the standard framework. Actually it may be applied to pictures where some geometric structure interpreted as the horizon exists "just for a while", without extending into a true global structure up to the future null infinity (where, traditionally, the Hawking radiation is detected). Actually, in [DNVZZ07, HDVNZ09], even the case of a spherically symmetric black hole *in formation* was analyzed, where no proper horizon structure exists, being replaced by a *dynamical horizon*. Other interesting results, also considering the backreaction, can be found in [KM07, MV05].

Within these new remarkable approaches it is however difficult to understand if the found properties are state independent. This is essentially due to the fact that they are discussed at the *quantum-mechanical* level rather than the *quantum-field-theory* level. Indeed, in the mentioned papers it is assumed that there is some preferred notion of quantum particle whose wavefunction satisfies the Klein-Gordon equation – treated however as a Schrödinger equation when dealing with transition probabilities within the WKB framework. However, in curved spacetime there is no natural definition of particle associated to a quantum field, unless fixing a preferred quasifree reference state and building up the associated Fock-Hilbert space. Furthermore the procedure exploited in [PW00, ANVZ05, DNVZZ07, HDVNZ09] needs a Feynman perscription to make harmless a divergence that pops up when performing the above-mentioned limit toward the horizon. As a matter of fact, that procedure turns the real-axis-computation into a complex-plane-computation and the very imaginary part of the WKB amplitude, arising that way, leads to the wanted factor e^{-kE/T_H}. The choice of that Feynman perscription does not seem to be well motivated at quantum mechanical level, barring the fact that "a posteriori" it produces the wanted remarkable result. It seems palusible that the choice of the Feynman regularization procedure at quantum mechanical level is a remnant of the choice of a preferred reference state, at quantum-field-theory level, whose two-point function has a short-distance divergence close to that of Minkowski vacuum.

While sticking to the local aspects associated with states showing the Hawking radiation, differently from the references quoted above, in this paper we shall deal with a definite framework at the quantum-field-theory level. More precisely, we shall focus on the two-point correlation function $\omega(\Phi(x)\Phi(y))$ of a quantum field Φ settled in a (not necessarily quasifree) state ω whose short-distance divergence is, essentially, of *Hadamard type*. This is one of the hypotheses exploited in [FH90] and actually encompasses a huge class of states, those that are supposed to have a clear physical meaning [Wa94] especially in relation with the problem of the renormalization of the stress-energy tensor and the computation of the quantum backreaction on the metric. In the one-particle space $\omega(\Phi(x)\Phi(y))$ corresponds, up to normalization, to a probability amplitude and, in this sense, it measures the tunneling probability through the horizon when x and y are kept at the opposite sides of the horizon.

From the geometric viewpoint we shall assume to work in a sufficiently small neighborhood $\mathcal{O}$

of a local *Killing horizon* structure $\mathcal{H}$, also assuming that the *surface gravity* κ is *nonvanishing* and *constant* along the horizon. We stress that, indifferently, the structure may be part the future Killing event horizons of a stationary black hole, even charged and rotating in the full Kerr-Newmann family (obtained by the collapse of matter) or it may be completely local and ceasing to exists in the future of $\mathcal{O}$ in view of the general dynamics of the matter and the fields in the considered spacetime. The requirement that the surface gravity is constant on the local horizon means that, at least locally, a thermal equilibrium has been reached, since a constant surface gravity corresponds to the validity of the zero-law of black-hole thermodynamics.

The existence of a timelike Killing vector K defining $\mathcal{H}$ provides the preferred notions of time and energy we intend to consider. (In [DNVZZ07, HDVNZ09], dealing spherically-symmetric black holes in formation, the notion energy was referred to the so called *Kodama-Hayward* vector field that, in those backgrounds, extends the notion of Killing field.)

Exploiting general technical achievements about Killing horizons established in [KW91] and [RW92], we shall prove that, independently from the choice of the quantum state (in above-mentioned class), when the supports of the test functions centered on the two arguments x, y of $\omega(\Phi(x)\Phi(y))$ become closer and closer to the horizon – in a precise mathematical sense we shall specify later – the two-point function acquires a thermal spectrum with respect to the notion of time and energy associated with the Killing field. More precisely, if both arguments stay on the same side of the horizon, the Fourier transform of the two-point function presents the very Bose-Einstein shape driven by the Hawking temperature. Conversely, whenever the two arguments are kept at the opposite sides of the horizon, the resulting spectrum is different, it is however in agreement with Boltzmann's distribution at the Hawking temperature for high energies. In both cases, in order to catch the leading contribution to the two-point function, we shall exploit a suitable scaling limit procedure [Bu96, BV95] towards the horizon. Oparting this way, the local thermal nature of the correlation functions becomes manifest as a state independent feature.

The paper is organized as follows. In the next section we shall present our geometric hypotheses also reminding some technical results established in [KW91] and [RW92]. We assume the reader is familiar with the standard notions of differential geometry of spacetimes [Wa84] (in the remaining part of the work "submanifold" means smooth embedded submanifold). In the subsequent section we shall compute the two-point function $\omega(\Phi(x)\Phi(y))$ and its limit for x, y approaching the horizon. The last section presents a summary and some general remarks.

2 Spacetime Geometry

2.1. Local geometry. We start our discussion fixing the basic geometric setup that we shall use in the present paper. We henceforth consider a 4-dimensional (smooth) time-oriented spacetime (M, g). Furthermore, we shall require that the following local geometric properties hold. (Notice that these are the same as in [RW92]).

Definition 2.1. *Let $\mathcal{O}$ be an open set contained in M, the* **local general geometric hypotheses** *hold if it exists a smooth vector field K on $\mathcal{O}$ such that:*

(a) *K is a Killing field for g in $\mathcal{O}$.*

(b) *$\mathcal{O}$ contains a connected 3-submanifold $\mathcal{H}$, the* **local Killing horizon**, *that is invariant under the group of local isometries generated by K and $K^a K_a = 0$ on $\mathcal{H}$.*

(c) *The orbits of K in $\mathcal{O}$ are diffeomorphic to some open interval contained in $\mathbb{R}$ and $\mathcal{H}$ admits a smooth 2-dimensional cross section which intersects each orbit of K exactly once.*

(d) *The* **surface gravity** – i.e. *the function $\kappa : \mathcal{H} \to \mathbb{R}$ such that, in view of (a) and (b) $\nabla^a (K_b K^b) = -2\kappa K^a$ – turns out to be strictly positive*[1] *and constant on $\mathcal{H}$.*

As we said above the local Killing horizon $\mathcal{H}$ may denote a horizon which exists "just for a while", without extending into a true global structure reaching the future null infinity. However, our hypotheses are particularly valid [RW92] in a neighborhood of any point on a black hole horizon, once that, after the collapse, the metric has settled down to its stationary (not necessarily static) form of any non-extreme black hole in the charged Kerr-Neumann family. In particular our hypotheses and our results are valid for the Kerr black hole. There K is the Killing vector defining the natural notion of time in the external region of the black hole and $\mathcal{H}$ is (part of) the event horizon.

With the hypotheses (a) and (b), the integral lines of K along $\mathcal{H}$ can be re-parametrized to segments of null geodesics and that $\nabla^a (K_b K^b) = -2\kappa K^a$ on $\mathcal{H}$ where the **surface gravity**, $\kappa : \mathcal{H} \to \mathbb{R}$, is constant along each fixed geodesic [Wa84]. The requirement (d) is not as strong as it may seem at first glance. Indeed, it is possible to prove that whenever a spacetime admitting a Killing horizon satisfies Einstein equations and the dominant energy condition is verified, the surface gravity must be constant on the horizon [Wa84]. However, independently form the dominant energy condition, κ turns out to be constant on the Killing horizon of a stationary black hole [Wa84]. At least in the case $\kappa > 0$ this is the *zero-th law of black hole thermodynamics* [Wa84], taking into account that $\frac{\kappa}{2\pi}$ amounts to the *Hawking temperature* of the black hole.

2.2. Killing and Bifurcate Killing horizons. We shall now discuss the relation of the previously introduced local geometric hypothesis with the more rigid case of bifurcate Killing horizon. A Killing field K determines a **bifurcate Killing horizon** [Bo69] when it vanishes on a connected 2-dimensional acausal space-like submanifold $\mathcal{B} \subset M$, called the **bifurcation surface** and K is light-like on the two K-invariant 3-dimensional null submanifods $\mathcal{H}_+, \mathcal{H}_- \subset M$ generated by the pairs of null geodesic orthogonally emanated by $\mathcal{B}$. In particular $\mathcal{H}_+ \cap \mathcal{H}_- = \mathcal{B}$ and the null geodesics forming $\mathcal{H}_+ \cup \mathcal{H}_-$ are re-parametrised integral lines of K on $(\mathcal{H}_+ \cap \mathcal{H}_-) \setminus \mathcal{B}$. By definition, on $\mathcal{H}_+$ the field K in the future of $\mathcal{B}$ is directed outgoing $\mathcal{B}$. The simplest example of a bifurcate Killing horizon is that realized by the Lorentz boost K in Minkowski spacetime. Other more interesting cases are those shown in maximally extended black hole geometries like the Kruskal extension of the Schwarzschild even including the non-extreme charged rotating case.

For our purposes it is important to notice that, in the case of a bifurcate Killing horizon, any neighborhood $\mathcal{O}$ of a point on $\mathcal{H}_+$ with empty intersection with the bifurcation $\mathcal{B}$, satisfies the local general hypotheses stated above. It is very remarkably for physical applications and for our subsequent discussion in particular, that such a result can be partially reversed as established by

[1] What actually matters is $\kappa \neq 0$, since $\kappa > 0$ can always be obtained in that case by re-defining $K \to -K$.

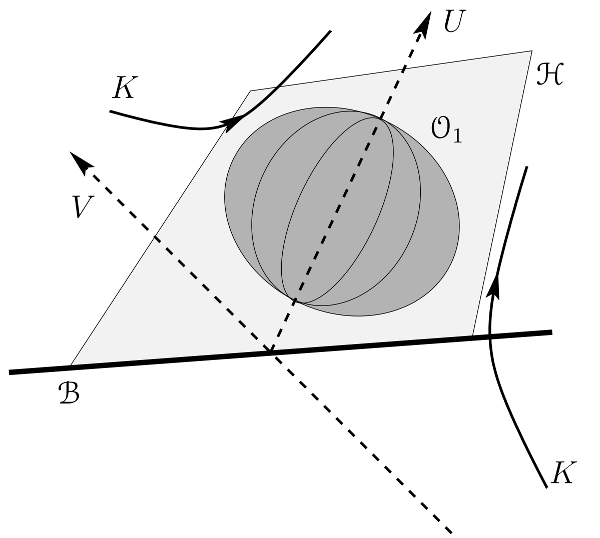

Figure 1: The region painted in light gray correspond to the horizon $\mathcal{H}$ while the region in dark gray is the open set $\mathcal{O}_1$

Racz and Wald [RW92, RW96]. Actually, if the local general geometric hypotheses are fulfilled on $\mathcal{H}$, shrinking $\mathcal{O}$ around $\mathcal{H}$ if necessary, the spacetime outside $\mathcal{O}$ can be smoothly deformed preserving the geometry inside $\mathcal{O}$ and extending K and $\mathcal{H}$ to the whole bifurcate Killing horizon in the deformed spacetime. Thus, when studying local properties, the bifurcation $\mathcal{B}$ can be "added" also to those spacetimes without bifurcation as is the case for black holes formed by stellar collapse. One can than take advantage of the various technical properties of the bifurcate Killing horizon as we shall do in the rest of the paper.

2.3. Killing vector and geodesical distance in $\mathcal{O}$. Let us focus on a relevant coordinate patch [KW91] defined in a neighborhood of $\mathcal{H}_+$ for a bifurcate Killing horizon generated by a Killing vector field K (a similar construction can be made for $\mathcal{H}_-$). Let U denote an affine parameter along the null geodesics forming $\mathcal{H}_+$ fixing the origin at the bifurcation $\mathcal{B}$, thus, each point $p \in \mathcal{H}_+$ is determined by the pair (U, s), where $s \in \mathcal{B}$ denotes the point which is intersected by the null geodesic generator through p.

We shall now extend those coordinate system on a neighborhood of $\mathcal{H}_+$. To this end, for each point $q \in \mathcal{B}$, let us indicate by n the unique future pointing null vector which is orthogonal to $\mathcal{B}$ and has inner product $-1/2$ with $\frac{\partial}{\partial U}$. We extend n on all of $\mathcal{H}_+$ by parallel transport along the null generators of $\mathcal{H}_+$. Let V denote the affine parameter along the null geodesics determined by n, with V= 0 on $\mathcal{B}$. It is clear that (V, U, s) characterizes points in a sufficiently small neighborhood of $\mathcal{H}_+$. We are thus in place of introducing the sought coordinate patch. If (x^3, x^4) denote coordinates defined on a open neighborhood (in $\mathcal{B}$) of a point in $\mathcal{B}$, a coordinate patch (V, U, x^3, x^4), we call **adapted to** $\mathcal{H}_+$, turns out to be defined in corresponding open neighborhoods (in M) of points on $\mathcal{H}_+$. In these coordinates:

$$g\restriction_{\mathcal{H}_+} = -\frac{1}{2}dU \otimes dV - \frac{1}{2}dV \otimes dU + \sum_{i,j=3}^{4} h_{ij}(x^3, x^4)dx^i \otimes dx^j \,, \qquad (1)$$

where metric h is that induced by g on $\mathcal{B}$, it is thus *positive defined*, and it does not depend on V, U. We stress once more that, in view of Racz-Wald's result, such a coordinate system always exits provided the local geometric hypotheses hold for a sufficiently small set $\mathcal{O}$.

In the rest of the paper, referring to this geometric structure in $\mathcal{O}$, we shall employ the following notation. We shall indicate by $\mathcal{S}_{V,U}$ the cross section of $\mathcal{O}$ at V, U constant, moreover, $s(p) \in \mathcal{B}$ will be the point with coordinates (x^3, x^4) when $p \in \mathcal{O}$ has coordinates (V, U, x^3, x^4). The set $G_\delta(p, V', U') \subset \mathcal{S}_{V',U'}$ is defined as the set whose image $s(G_\delta(p, V', U')) \subset \mathcal{B}$ under s coincides with the open h-geodesical ball centered on $s(p)$ with radius δ. Finally we shall denote by $\sigma(p, p')$ the squared g-geodesic distance between points $p, p' \in \mathcal{O}$ taken with sign between any couple of points p, p' contained some g-geodesically convex neighborhood and by $\ell(s, s')$ the squared (signed) h-geodesic distance between points s, s' in some h-geodesically convex neighborhood contained in $\mathcal{B}$. Making use of the preceding definitions and the introduced notations, we are in place of presenting the following useful proposition, which is also based on achievements in [KW91].

Proposition 2.1. *Let $\mathcal{O}$ be a set on which the local general geometric hypotheses hold, and, let $\mathcal{O}_1$ be another open set such that $\overline{\mathcal{O}}_1 \subset \mathcal{O}$ is compact and it has vanishing intersection with $\mathcal{H}$. Suppose that $\mathcal{O}$ is covered by coordinates adapted to $\mathcal{H}_+$, so that $p \in \mathcal{O}$ has coordinates (V, U, x^3, x^4) and $p \in \mathcal{H}$ iff $V = 0$. The following holds.*

- **(a)** *In $\mathcal{O}$, the decomposition $K = K^1 \frac{\partial}{\partial V} + K^2 \frac{\partial}{\partial U} + K^3 \frac{\partial}{\partial x^3} + K^4 \frac{\partial}{\partial x^4}$ is valid and, if $p \in \mathcal{O}_1$:*

$$K^1(p) = -\kappa V + V^2 R_1(p) , \quad K^2(p) = \kappa U + V^2 R_2(p) , \quad K^i(p) = V R_i(p) , \ i = 3, 4 , \quad (2)$$

  where R_1, R_2, R_i are bounded smooth functions defined on $\mathcal{O}_1$.
- **(b)** *If $\mathcal{O}_1$ is sufficiently small and included in a g-geodesically convex and if $p, p' \in \mathcal{H} \cap \mathcal{O}_1$, then $\sigma(p, p') = \ell(s(p), s(p'))$.*
- **(c)** *If $\mathcal{O}_1$ is as in (b), there exist $\delta > 0$ such that for every fixed $p \in \mathcal{O}_1$, the smooth map $G_\delta(p, V', U') \ni p' \mapsto \sigma(p, p')$ has vanishing s-gradient in a unique point $q(p, V', U')$ attaining its minimum there. In particular $s(q(p, V', U')) = s(p)$ if $p \in \mathcal{H}$.*
- **(d)** *For p and $q = q(p, V', U')$ as in (c):*

$$\sigma(p, q) = \ell(s(p), s(q)) - (U - U')(V - V') + R(p, V', U') , \quad (3)$$

  where $R(p, V', U') = AV^2 + BV'^2 + CVV'$, for some bounded smooth functions A, B, C of p, V', U'.

Proof. **(a)** $\nabla_a K_b + \nabla_b K_a = 0$ and $\nabla^a(K_b K^b) = -2\kappa K^a$ on $\mathcal{H}$ imply that $\nabla_K K = \kappa K$ on $\mathcal{H}$, so that $K\restriction_{\mathcal{H}} = \kappa U \frac{\partial}{\partial U}$ because U is an affine parameter and K vanishes on $\mathcal{B}$ where $U = 0$. If x^a is any of $x^1 = V, x^2 = U, x^3, x^4$, exploiting the parallel transport used to define the coordinates:

$$\Gamma^a_{21}\restriction_{\mathcal{H}} = \Gamma^a_{12}\restriction_{\mathcal{H}} = \Gamma^a_{22}\restriction_{\mathcal{H}} = \Gamma^a_{11}\restriction_{\mathcal{H}} = \Gamma^1_{2a}\restriction_{\mathcal{H}} = \Gamma^1_{a2}\restriction_{\mathcal{H}} = \Gamma^2_{a1}\restriction_{\mathcal{H}} = \Gamma^2_{1a}\restriction_{\mathcal{H}} = 0 . \quad (4)$$

(The fifth one is equivalent to $g(\frac{\partial}{\partial U}, \nabla_{\frac{\partial}{\partial U}} \frac{\partial}{\partial x^a}) \restriction_{\mathcal{H}} = 0$ and it arises from $g(\frac{\partial}{\partial U}, \frac{\partial}{\partial x^a}) = -n_a$ and $\nabla_{\frac{\partial}{\partial U}} \frac{\partial}{\partial U} = 0$ on $\mathcal{H}$, the seventh one arises similarly.). Next, taking the first-order Taylor expansion in V of both K^1 and g^{ab} about $V=0$, we have, for some smooth function $V^2 R_1(p)$ bounded in view of the compacteness of $\overline{\mathcal{O}_1}$,

$$K^1 = K^1\restriction_{\mathcal{H}} + V\left(g^{1b}\restriction_{\mathcal{H}} \frac{\partial K_b}{\partial V}\restriction_{\mathcal{H}} + K_b\restriction_{\mathcal{H}} \frac{\partial g^{1b}}{\partial V}\restriction_{\mathcal{H}}\right) + V^2 R_1(p)\,. \quad (5)$$

¿From $\frac{\partial g^{ab}}{\partial x^c} = -g^{bd}\Gamma^a_{cd} - g^{ad}\Gamma^b_{cd}$ and and (4), exploiting (1) and $K^1 = 0$ on $\mathcal{H}$, the identity (5) semplifies: The last derivative vanishes and $K^1 = -V\frac{\partial K_2}{\partial V}\restriction_{\mathcal{H}} + V^2 R^1(p)$. On the other hand, $\nabla_2 K_1 + \nabla_1 K_2 = 0$ evaluated on $\mathcal{H}$ and using (4) leads to $\frac{\partial K_2}{\partial V}\restriction_{\mathcal{H}} = -\frac{\partial K_1}{\partial U}\restriction_{\mathcal{H}}$, so that:

$$K^1 = -V\frac{\partial K_1}{\partial U}\restriction_{\mathcal{H}} + V^2 R_1(p) = V\left(g_{1a}\restriction_{\mathcal{H}} \frac{\partial K^a}{\partial U}\restriction_{\mathcal{H}} + K^a\restriction_{\mathcal{H}} \frac{\partial g_{1a}}{\partial U}\restriction_{\mathcal{H}}\right) + V^2 R_1(p)\,.$$

The last derivative vanishes in view of (4), (1) and the know identity $\frac{\partial g_{ab}}{\partial x^c} = g_{bd}\Gamma^d_{ca} + g_{ad}\Gamma^d_{cb}$. Therefore the first identity in (2) holds in view of (1) and $K\restriction_{\mathcal{H}_R} = \kappa U\frac{\partial}{\partial U}$. The second one can be proved with the same procedure noticing that the Killing identity $\nabla_V K_1 = 0$, on $\mathcal{H}$, becomes $\frac{\partial K_1}{\partial V}\restriction_{\mathcal{H}} = 0$ in view of (4). The last identity in (2) is obvious.

(b) Since the geodesically convex neighborhoods form a base of the topology and the projection $\pi : (V, U, s) \mapsto (0, 0, s)$ is continuous, if $\mathcal{O}_1$ is chosen to be sufficiently small, we have that $\mathcal{O}_1$ is contained in a geodesically convex neighborhood while, at the same time, $\pi(\mathcal{O}_1)$ is contained in a h-geodesically convex neighborhood in $\mathcal{B}$. Without loosing generality, we can further assume that the latter is included in a g-geodesically convex neighborhood of M. Thus $\sigma(p, p')$, $\sigma(s, s')$ and $\ell(s, s')$ are simultaneously defined for $p, p' \in \mathcal{O}_1 \cap \mathcal{H}$ for a sufficiently small $\mathcal{O}_1$. We notice that, $\sigma(p, p')$ is invariant under the action of the Killing isometry. Hence, for any $p, p' \in \mathcal{H} \cap \mathcal{O}_1$ we get the identity $\sigma(p, p') = \sigma(s, s')$ taking the limit towards $\mathcal{B}$ of the flow generated by the Killing field K applied to p, p'. Finally $\sigma(s, s') = \ell(s, s')$ because $\mathcal{B}$ is totally geodesic as it can be proved by direct inspection.

(c) Let $(V, U, s) \equiv p$ and $(V', U', s') \equiv p'$. Whenever both points p and p' are contained on the horizon, namely $V = V' = 0$, the thesis holds in view of (b) and the fact that $\ell(s, s')$ is positive defined, with positively-defined Hessian matrix in the coordinates x'^3, x'^4 of s'. Furthermore, in this case $s(q(p, 0, U')) = s(p)$. By continuity, that Hessian matrix remains positively defined if p, p' stay close to $\mathcal{H}$, so that, any zero $q(p, V', U')$ of the x'^3, x'^4-gradient of $\mathcal{S}_{V',U'} \ni p' \mapsto \sigma(p, p')$ determines a minimum of $\sigma(p, p')$. Taking the Taylor expansion of $\nabla_{x'^i}\sigma(p, p')$ $(i = 3, 4)$ centered on a point in $\mathcal{H} \times \mathcal{H}$ with respect to all the coordinates of p and p', the equation for $q(p, V', U')$ can easily be handled by exploiting Banach's fix point theorem, proving the existence and the uniqueness of $q(p, V', U')$ for $p \in \mathcal{O}_1$ sufficiently shrinked around $\mathcal{H}$, and p' varying in a neighborhood $G_\delta(p, V', U')$ of $(0, U', s)$ in $\mathcal{S}_{V',U'}$. We recall that $G_\delta(p, V', U')$ is the preimage through $\mathcal{S}_{V',U'} \ni p' \mapsto s(p')$ of a geodesic ball on $\mathcal{B}$ centered on $s(p)$. The compactness of $\overline{\mathcal{O}_1}$ and a continuity argument assures that $\delta > 0$ can be chosen uniformly in p.

(d) Keeping U, U', x^3, x^4 fixed, the expansion (3) is nothing but the first-order (V, V')-Taylor expansion of $\sigma(p, q(p, V', U'))$ at $V = V' = 0$, paying attention to the fact that the coordinates x'^1, x'^2 of $q(p, V', U')$ depend on V through the dependence of $q(p, V', U')$ from p. $\square$

3 Correlations accross the Killing horizon

3.1. General outset. We wish to compute the correlation functions of a real scalar quantum field, Φ, for field observables localized in a region $\mathcal{O}$ containing a Killing field K and which satisfies the local general geometric hypotheses. We suppose in particular that $\mathcal{O}$ can be covered by *coordinates adapted to* $\mathcal{H}_+$ and that that U and V increase toward the future. All the remaining cases can be treated similarly. Finally we restrict $\mathcal{O}$ to a subregion $\mathcal{O}_1$ as in (b) of proposition 2.1 because we want to use the expression (3) for the geodesic distance. The region $\mathcal{O}_1$ considered above – in coordinates (V, U, s) – can always be taken of the form $(-a, a)\times(b, c)\times S$, where $S \subset \mathcal{B}$ is an open relatively compact subset. Shrinking $\mathcal{O}_1$ around the region of $\mathcal{H} \cap \mathcal{O}_1$ determined by $(b, c) \times S$ means taking $a > 0$ smaller and smaller.

In view of (a) in proposition 2.1, for a sufficiently small $\mathcal{O}_1$, the Killing vector K can be assumed to be spacelike in $\mathcal{O}_s \equiv \{p \in \mathcal{O}_1 \mid V(p) > 0\}$, whereas is taken to be timelike in $\mathcal{O}_t \equiv \{p \in \mathcal{O}_1 \mid V(p) < 0\}$. Referring to stationary black holes, $\mathcal{O}_1$ can be interpreted as a sufficiently small region around a point on the future horizon, the only horizon existing when the black hole is produced by collapsing matter. There $\mathcal{O}_s$ is part of the *internal* region, containing the singularity, while $\mathcal{O}_t$ stays in the *external* region, stationary with respect to the Killing time associated to K. In this way a notion of *energy* related to K can be defined in $\mathcal{O}_t$ at least and we will take advantage of it shortly. Inspired by the ideas proper of the *scaling-limit procedure* [BV95, Bu96], we are going to compute the limit:

$$\lim_{\lambda\to 0^+} \omega\left(\Phi(f_\lambda)\Phi(f'_\lambda)\right)$$

where ω denotes the reference state and f_λ and f'_λ are smooth functions supported in $\mathcal{O}_1$ whose supports become closer and closer to the horizon as long as $\lambda \to 0^+$. Since only the short distance behavior of the two-point function of the reference state is relevant for our computation, it is not necessary to specify the equation of motion satisfied by the quantum field Φ. Conversely, we assume that the two-point function of ω is a distribution of $\mathcal{D}'(M \times M)$ defined as

$$\omega\left(\Phi(f)\Phi(f')\right) = \lim_{\epsilon\to 0^+} \int_{M\times M} \omega_\epsilon(x, x') f(x) f'(x') dx dx'$$

where the integral kernels ω_ϵ have the following from

$$\omega_\epsilon(x, x') = \frac{\Delta(x, x')^{1/2}}{4\pi^2\sigma_\epsilon(x, x')} + w_\epsilon(x, x')\,, \tag{6}$$

whenever the test functions are supported in a fixed, relatively compact, geodesically convex neighborhood. Furthermore, in the previous expression, $\sigma_\epsilon(x, x') = \sigma(x, x') + 2i\epsilon(T(x) - T(x')) + \epsilon^2$ and T is any fixed time function, the smooth strictly-positive function Δ is the so-called Van Vleck-Morette determinant [Wa94, KW91]. We shall also assume that w_ϵ leads to a less singular distribution in the $\epsilon \to 0$ limit, namely w_ϵ are required to be bounded uniformly in ϵ by an M^2-integrable function whose limit toward $\mathcal{H}$ exists and gives an $\mathcal{H}^2$-integrable function there. For example (however there are further different possibilities):

$$w_\epsilon(x, x') = v(x, x') \ln \sigma_\epsilon(x, x') + w(x, x') \tag{7}$$

with v and w any fixed pair of smooth functions. That ultraviolet behaviour is a straightforward generalization of the short distance structure of the two-point function of Minkowski vacuum and it is fulfilled by all the (quasifree) states of *Hadamard type* [Wa94] defined by the requirements (6)-(7) together with further requirements on v. Those states are supposed to be the most significant states in QFT in curved spacetime [Wa94] and are very often employed in the rigorous description of thermal properties of quantum fields in the presence of black holes [KW91, FH90, DMP09].

We recall that, in the GNS representation of a state ω, the expectation value of the product of two fields $\omega\left(\Phi(f)\Phi(f')\right)$ is equal to $\langle\widehat{\Phi}(f)\Psi_\omega|\widehat{\Phi}(f')\Psi_\omega\rangle$, where Ψ_ω is the cyclic vector of the GNS triple and where $\widehat{\Phi}(f)$ is the Hermitean field operator. Hence, up to normalization, $|\omega\left(\Phi(f)\Phi(f')\right)|^2$ can be interpreted as a transition probability between the states $\widehat{\Phi}(f)\Psi_\omega$ and $\widehat{\Phi}(f')\Psi_\omega$ for a particle associated to the field. Whenever f and f' are localized on the opposite sides of the horizon – the regions $\mathcal{O}_s$ and $\mathcal{O}_t$ – the correlation function $\omega\left(\Phi(f)\Phi(f')\right)$ provides a measures of the *transition probability* through the horizon.

In order to obtain the leading order to that probability we shall consider some sequences of smearing functions f_λ and f'_λ whose support become closer and closer to the horizon $\mathcal{H}$ in the limit $\lambda \to 0^+$. We build this sequences as follows. Let f and f' be some smooth functions with compact support contained respectively in the regions $\mathcal{O}_s$ and $\mathcal{O}_t$, then the functions f_λ, f'_λ are defined as follows,

$$f_\lambda(V,U,x^3,x^4) := \frac{1}{\lambda} f\left(\frac{V}{\lambda},U,x^3,x^4\right)\ , \quad f'_\lambda(V,U,x^3,x^4) := \frac{1}{\lambda} f'\left(\frac{V}{\lambda},U,x^3,x^4\right)\ , \quad \lambda > 0\ , \tag{8}$$

where the pre factor λ^{-1} is introduced in order to keep the result finite. In order to avoid divergences due to zero-modes[2] in the limit $\lambda \to 0^+$, as those modes are invariant under rescaling of the coordinate V, we assume that f, f' are of the form:

$$f = \frac{\partial F}{\partial V}\ , \quad f' = \frac{\partial F'}{\partial V}\ , \quad \text{for fixed } F, F' \in C_0^\infty(\mathcal{O}_1). \tag{9}$$

Alternatively, sticking to general smooth compactly supported f, f', the divergent contribution of zero-modes has to be subtracted at the end of the computations. The $\lambda \to 0^+$ limit of $\omega\left(\Phi(f_\lambda)\Phi(f'_\lambda)\right)$ is precisely our notion of *scaling limit of* $\omega\left(\partial_V\Phi(x)\partial_V\Phi(y)\right)$ *towards the horizon.* It computes the first contribution to the sought *transition probability* in an ideal asymptotic expansion for small λ.

3.2. Computation. We shall now present the most important result of this paper. Notice that in the proof of the following theorem we shall make use of techniques similar to those employed in the Appendix B of [KW91], however, our result differs from those presented there because we are interested in computing the scaling limit toward the whole horizon and not just one of its spatial sections. Furthermore, we are not interested in obtaining the restriction of the

[2]Indeed, given $f \in C_0^\infty(\mathcal{O}_1)$, an $F \in C_0^\infty(\mathcal{O}_1)$ with $f = \frac{\partial F}{\partial V}$ exists if and only if $\int_{\mathbb{R}} f(V,U,x^3,x^4)dV = 0$ on $\mathcal{O}_1$, namely, if and only if $f(\cdot,U,x^3,x^4)$ has no zero modes referring to the V-Fourier transform.

states on the horizon but we would like to compute the scaling limit of the whole state. Actually the relevant part for our computation is somehow the contribution of the state orthogonal to those that survive after projecting the wavefunctions on the horizon. In this respect there are some partial similarities also to the analysis performed in [FH90] although here we do not restrict ourself to the spherically symmetric case.

Theorem 3.1. *Assuming the general local geometric hypotheses on $\mathcal{O}$ (covered by coordinates adapted to $\mathcal{H}_+$), suppose that $\mathcal{O}_1 \subset \mathcal{O}$ is a sufficiently small open neighborhood of a point on $\mathcal{H}$ with $\overline{\mathcal{O}_1} \subset \mathcal{O}$ compact. If the state ω has two-point function given by a distribution satisfying (6)-(7), f, f' are taken as in (8)-(9) and μ is the measure associated to the 2-metric on the bifurcation $\mathcal{B}$:*

$$\lim_{\lambda\to 0^+} \omega\left(\Phi(f_\lambda)\Phi(f'_\lambda)\right) = \lim_{\epsilon\to 0^+} -\frac{1}{16\pi}\int_{\mathbb{R}^4\times\mathcal{B}} \frac{F(V,U,s)F'(V',U',s)}{(V-V'-i\epsilon)^2} dUdVdU'dV'd\mu(s)\,. \tag{10}$$

Proof. Since ω_ϵ is of the form (6)-(7), it is sufficient to consider the contribution (where dp is a shortcut for the measure induced by the metric):

$$\lim_{\lambda\to 0^+}\lim_{\epsilon\to 0^+}\int_{\overline{\mathcal{O}_1}\times\overline{\mathcal{O}_1}} \frac{\Delta^{1/2}(p,p')f_\lambda(p)f'_\lambda(p')}{4\pi^2\sigma_\epsilon(p,p')}\, dpdp' \tag{11}$$

because the contribution due to w_ϵ vanishes for small λ as it can easily be veryfied following the same procedure we are empoying in the rest of the proof. Fixing $\delta > 0$ and p, V', U', consider the neighborhood $G_\delta(p,V',U') \subset \mathcal{S}_{V',U'}$ as in (c) of proposition 2.1, and define a smooth map $\mathcal{B} \ni s \mapsto \chi_\delta(s,V',U',p) \geq 0$ with support completely included in $G_\delta(p,V',U')$ and $\chi_\delta(s',V',U',p) = 1$ for $0 \leq \sqrt{\lambda(s(p),s')} \leq \frac{\delta}{2} + \frac{1}{2}\sqrt{\lambda(s(p),s(q(p,V',U')))}$. In view of the smoothness of all considered functions it is possible to arrange these functions in order that $(s',V',U',p) \to \chi_\delta(s',V',U',p)$ is jointly smooth. Finally decompose the integral in (11) as:

$$\begin{aligned}&\int_{\overline{\mathcal{O}_1}} dp\left(\int_{\overline{\mathcal{O}_1}} \frac{\Delta^{1/2}(p,p')f_\lambda(p)f'_\lambda(p')}{4\pi^2\sigma_\epsilon(p,p')}\chi_\delta(s',V',U',p)\sqrt{|\det g(p')|}dV'dU'ds'\right)\\ &+\int_{\overline{\mathcal{O}_1}} dp\left(\int_{\overline{\mathcal{O}_1}} \frac{\Delta^{1/2}(p,p')f_\lambda(p)f'_\lambda(p')}{4\pi^2\sigma_\epsilon(p,p')}(1-\chi_\delta(s',V',U',p))\sqrt{|\det g(p')|}dV'dU'ds'\right)\,,\end{aligned} \tag{12}$$

where $ds' = dx'^3dx'^4$. Let us focus on the second integral. It is simply proven that, for a fixed $\eta > 0$, it is possible to shrink $\mathcal{O}_1$ in order that $\sqrt{\sigma(p,p')} \geq \eta/2$ if $\sqrt{\ell(s(p),s(p'))} > \eta$ for $p,p' \in \mathcal{O}_1$, as consequence of the compactness of $\overline{\mathcal{O}_1}$ the continuity of σ and (b) in proposition 2.1 (notice that the limit in $\lambda \to 0^+$ in (11) allows us to take $\mathcal{O}_1$ as small as we need). By definition of $G_\delta(p,V',U')$, the integrand second integral in (12) is jointly smooth in all variables including ϵ, even for $\epsilon = 0$, since $\sigma_{\epsilon=0}(p,p') \geq \delta^2/16$ when $1-\chi_\delta \neq 0$. Then, in view of Lebesgue's dominated convergence theorem, the limit in ϵ can be computed simply taking $\epsilon = 0$ in the integrand of that integral and the subsequent limit as $\lambda \to 0^+$ converges to 0 because, at fixed U,s, the function $V \mapsto f_\lambda(V,U,s)$ weakly tends to $\delta(V)\int dx\partial_x F(x,U,s) = 0$ in the limit as

$\lambda \to 0^+$ and the same happens for f'_λ. We conclude that only the former integral in (12) may survive the limits in (11). Let us focus on that integral. Making use of (c) in proposition 2.1, in each set $G_\delta(p, V', U') \subset \mathcal{S}_{V',U'}$ we define the fuction $\rho(p') \geq 0$ such that:

$$\sigma(p, p') = \rho(p')^2 + \sigma(p, q(p, V', U')) .$$

In view of (c) in proposition 2.1, the pair ρ, θ, where $\theta \in (-\pi, \pi)$ is the standard polar angle in geodesic polar coordinates centered on $q(p, V', U')$, determines an allowable local chart to determine $p' \in G_\delta(p, V', U')$ (see also the appendix B of [KW91]), that is smooth barring the usual conical singularity for $\rho = 0$. Notice that, due to the last statement in (c) of proposition 2.1, when $p \in \mathcal{H}$ and $V' = 0$, ρ coincides to the standard geodesic radial coordinate centered on $s(p) \in \mathcal{B}$. In the following we shall employ that coordinate system in each $G_\delta(p, V', U')$. Making finally use of (d) in proposition 2.1, choosing $T = (U + V)/2$ we can re-arrange the former integral in (12) so that:

$$\lim_{\lambda \to 0^+} \omega\left(\Phi(f_\lambda)\Phi(f'_\lambda)\right) = \lim_{\lambda \to 0} \lim_{\epsilon \to 0^+} \int \frac{\Delta'^{1/2}(p, p') f_\lambda(p) f'_\lambda(p')}{\rho^2 - (V - V' - i\epsilon)(U - U' - i\epsilon) + R(p, V', U')} \frac{dp' dp}{4\pi^2} , \quad (13)$$

where, $\Delta'^{1/2}(p, p') := \Delta^{1/2}(p, p')\chi_\delta(p, p')$ and *it is from now on understood that the integral in p' is performed before that in p.* Using this coordinate system the integral in the right-hand side of (13) can be rewritten[3] as

$$\int \Delta'^{1/2}(p, p') f_\lambda(p) f'_\lambda(p') \frac{\partial}{\partial \rho} \ln\left(\rho^2 + \sigma(p, q(p, V', U'))\right) \frac{\sqrt{|\det g|}}{8\pi^2 \rho} d\rho d\theta dU' dV' dp$$

where $\det g$ is the determinant of the metric of the coordinates ρ, θ, V', U', parametrically depending on p. Notice that, the domain of integration in ρ is bounded by the support of the function $\chi_\delta(p, p')$ embodied in Δ'. For $V = V' = 0$, the metric takes the form (1) on $\mathcal{B}$ which does not depend on U, U', V, V' anymore while R vanishes. By direct inspection one sees that $\frac{\sqrt{|\det g|}}{\rho}$ is continuous (tends to $1/2$ when $p, p' \in \mathcal{H}$ and $p' \to p$) and its ρ-derivative is continuous for $\rho \neq 0$ it being however bounded there. If Δ'_λ, R_λ, $\det g_\lambda$, dp_λ are respectively defined as Δ', R, $\det g$ and dp with V and V' rescaled by λ, changing coordinates $(V, V') \to (\lambda V, \lambda V')$ the integral in the right-hand side of (13) can be rearranged as

$$\int \partial_V F(p) \partial_{V'} F'(p') \Delta'^{1/2}_\lambda(p, p') \frac{\partial}{\partial \rho} \ln\left[\rho^2 - (\lambda V - \lambda V' - i\epsilon)(U - U' - i\epsilon) + R_\lambda(p, V', U')\right]$$
$$\frac{\sqrt{|\det g_\lambda|}}{8\pi^2 \rho} d\rho d\theta dU' dV' dp_\lambda .$$

Leaving unchanged the remaining integrations, we can first integrate by parts in the polar coordinate ρ. We obtain two boundary terms (integrals in the remaining variables) evaluated at $\rho = 0$ and $\rho = \rho_0 > 0$ sufficiently large respectively, and an integral in all the variable including

[3] We henceforth assume that the cut $Re z < 0$ in the complex plane to define the function $\ln z$.

ρ. When taking the limits as $\epsilon \to 0^+$ and $\lambda \to 0^+$, concerning the boundary term at $\rho = \rho_0 > 0$ and the integral, we can pass both limits under the sign of the integration by straightforward application of Lebesgue's dominated convergence theorem. The result is that, in the integrand, only $\partial_V F$ and $\partial_{V'} F'$ depend on V, V' coordinates, hence, performing the integrations in V and V' both integrals vanish because F and F' are compactly supported. The remaining boundary term leads to the limit (the factor 2π arises by the integration in θ at $\rho = 0$ and we have safely replaced ϵ with $\lambda\epsilon$ in the integral in view of the order of the limits):

$$\lim_{\lambda\to 0^+}\lim_{\epsilon\to 0^+} -2\pi \int \Delta'^{1/2}(\lambda V, U, s, \lambda V', U', s)\partial_V F(V,U,s)\partial_{V'} F'(V',U',s)$$
$$\left(\ln\left(-(V - V' - i\epsilon)(U - U' - i\lambda\epsilon) + \frac{R_\lambda(p, V', U')}{\lambda}\right) + \ln\lambda\right) \frac{\sqrt{|\det g_\lambda|}}{8\pi^2\rho}\bigg|_{\rho=0} dU'dV'dp_\lambda \ .$$

Notice that $|\lambda^{-1}R_\lambda(p, V', U')| < C\lambda$ for some constant C, when $p, p' \in \mathcal{O}_1$ and $\lambda \in [0, \lambda_0)$ in view of (d) of proposition 2.1. The term $\log(\lambda)$ can be dropped as it gives no contribution to the final result since g_0, Δ_0 do not depend on V and $dp_\lambda = \frac{1}{2}(1 + \lambda V z)dUdVd\mu(s)$ for some smooth function $z = z(V, U, s)$ in view of the form (1) of the metric on $\mathcal{H}$, where $d\mu$ is the measure associated to the 2-metric h on $\mathcal{B}$. The limits can be computed, in the given order, exploiting Lebesgue's theorem and eventually obtaining:

$$-\frac{1}{16\pi}\int \partial_V F \partial_{V'} F' \left(i\pi\chi_{E_+}\chi_{A_+} - i\pi\chi_{E_+}\chi_{A_-} + \ln|V - V'||U - U'|\right) dUdVdU'dV'd\mu(s) \ , \quad (14)$$

where $E_\pm$ is the subset of $\mathcal{O}_1$ with, respectively, $(V - V')(U - U') \gtrless 0$ and $A_\pm$ is the analog with, respectively, $U - U' \gtrless 0$, and χ_S is the characteristic function of the set S. We have also used the fact that dp_λ becomes $\frac{1}{2}dUdVd\mu(s)$ for $\lambda = 0$ in view of the form (1) of the metric on $\mathcal{H}$ and that $\Delta' = \Delta = 1$ when $s = s'$ and $V = V' = 0$ (it follows from $\Delta(p, p) = 1$ and, since Δ is invariant under isometries, using an argument similar to that employed to prove (b) of proposition 2.1). For the same reason, in view of the meaning ρ, $\frac{\sqrt{\det|g_\lambda|}}{\rho} \to 1/2$ for $\rho \to 0$ when $\lambda = 0$, when working in coordinates ρ, θ, V, U. The integral in (14) can equivalently be re-written introducing another ϵ-perscription as:

$$\lim_{\epsilon\to 0^+} -\frac{1}{16\pi}\int \partial_V F \partial_{V'} F' \ln(-(V - V' - i\epsilon)(U - U'))dUdVdU'dV'd\mu(s) \ .$$

Summing up and integrating by parts, we have found that:

$$\lim_{\lambda\to 0}\lim_{\lambda\to 0}\omega\left(\Phi(f_\lambda)\Phi(f'_\lambda)\right) = \lim_{\epsilon\to 0^+} -\frac{1}{16\pi}\int_{\mathbb{R}^4\times\mathcal{B}} \frac{F(V,U,s)F'(V',U',s)}{(V - V' - i\epsilon)^2} dUdVdU'dV'd\mu(s) \ . \quad (15)$$

□

3.3. The correlation functions and their thermal spectrum. As is known (e.g., see [Wa84]) a timelike Killing vector field on a hand provides a natural notion of time, which is nothing but the parameter of the integral lines of the field, moreover it gives a natural notion of conserved energy for fields and matter propagating in the region where the Killing vector is present. We are interested in computing the energy spectrum of the correlation functions $\omega(\Phi(f_\lambda)\Phi(f'_\lambda))$ (in the limit $\lambda \to 0^+$) seen by an observer that moves along the curves generated by the Killing field K and computed with respect to the associated Killing time. More precisely, explointing Theorem 3.1 we intend to compute that energy spectrum in the limit of test functions squizeed on the local Killing horizon. As the supports of the test functions are infinitesimally close to the horizon, we have to focus on what happens for $V \sim 0$. Therefore we truncate every component of the formula (2) for the Killing vector at the dominant order in powers of V and we make use of the right-hand side of (10) as definition of correlation two-point function. If τ is the Killing time, namely the integral parameter of the curves tangent to K, in the said approximation, in the first identity in the right-hand side of (2) implies:

$$V(\tau) = -e^{-\kappa\tau} \quad \text{for } V < 0 \text{ (that is in } \mathcal{O}_t) \text{ and } \quad V(\tau) = e^{-\kappa\tau} \quad \text{for } V > 0 \text{ (that is in } \mathcal{O}_s), \quad (16)$$

up to an additive constant in the definition of τ which in principle could depending on the integral curve. Our choice is coherent with the standard definitions of τ in Minkowski or Schwarzschild spacetime where τ is the Killing time in the external region. Notice that we reduce to that case in the limit where the τ-constant 2-surfaces are close to the Killing horizon. We now examine two cases.

(a) *Both the supports of* f_λ *and* f'_λ *stay in* $\mathcal{O}_t$.
In that case, thinking of the functions F, F' as functions of τ, τ' instead of V, V' in view of (16), we can re-arrange the found expression for the correlation function as

$$\lim_{\lambda\to 0} \omega(\Phi(f_\lambda)\Phi(f'_\lambda)) = \lim_{\epsilon\to 0^+} -\frac{\kappa^2}{64\pi} \int \frac{F(\tau, U, x)F'(\tau', U', x)}{(\sinh(\frac{\kappa}{2}(\tau - \tau')) + i\epsilon)^2} d\tau dU d\tau' dU' d\mu(x) \; . \quad (17)$$

where we have used the fact that the functions F and F' are compactly supported by construction even adopting the new coordinate frame. It is known that in the sense of the Fourier transform of the distribution (e.g., see the appendix of [DMP09]) $\int_{\mathbb{R}} \frac{d\tau}{\sqrt{2\pi}} \frac{e^{-iE\tau}}{(\sinh(\frac{\tau}{2})+i0^+)^2} = -\sqrt{2\pi}\frac{Ee^{\pi E}}{e^{\pi E}-e^{-\pi E}}$.
That identity and the convolution theorem lead to

$$\lim_{\lambda\to 0} \omega(\Phi(f_\lambda)\Phi(f'_\lambda)) = \frac{1}{32} \int_{\mathbb{R}^2\times\mathcal{B}} \left(\int_{-\infty}^{\infty} \frac{\overline{\hat{F}(E, U, x)}\hat{F}'(E, U', x)}{1 - e^{-\beta_H E}} E dE \right) dU dU' d\mu(x) \; ,$$

where $\beta_H = 2\pi/\kappa$ is the *inverse Hawking temperature* and we have defined:

$$\overline{\hat{F}(E, U, x)} := \int_{\mathbb{R}} \frac{d\tau}{\sqrt{2\pi}} e^{-iE\tau} F(\tau, U, x) \; . \quad (18)$$

The thermal content of the found correlation function is manifest in view of the Bose factor $(1 - e^{-\beta_H E})^{-1}$ where the Hawking temperature takes place.

(b) *The support of f_λ stays in $\mathcal{O}_s$ while that of f'_λ stays in $\mathcal{O}_t$: Tunneling processes.*
As previously remarked, up to normalization, $|\omega(\Phi(f_\lambda)\Phi(f'_\lambda))|^2$, can now be intepreted as a tunneling probability trough the horizon. Employing (16) once more, we end up with:

$$\lim_{\lambda\to 0}\omega(\Phi(f_\lambda)\Phi(f'_\lambda)) = \lim_{\epsilon\to 0^+}\frac{\kappa^2}{64\pi}\int\frac{F(\tau,U,x)F'(\tau',U',x')}{\cosh(\frac{\kappa}{2}(\tau-\tau')+i\epsilon)^2}d\tau dU d\tau' dU' d\mu(x)\ . \qquad (19)$$

As expected from the fact that, in this case, the support of f_λ is always disjoint from the support of f'_λ, we can directly pass the limit $\epsilon\to 0^+$ under the sign of integration simply dropping $i\epsilon$ in the denominator. Taking advantage of the convolution theorem, the final result reads:

$$\lim_{\lambda\to 0}\omega(\Phi(f_\lambda)\Phi(f'_\lambda)) = \frac{1}{16}\int_{\mathbb{R}^2\times\mathcal{B}}\left(\int_{-\infty}^{\infty}\frac{\overline{\hat{F}(E,U,x)}\hat{F}'(E,U',x)}{\sinh(\beta_H E/2)}EdE\right)dU dU' d\mu(x)\ . \qquad (20)$$

(If the arbitrary additive constant defining τ in $\mathcal{O}_t$ were different from that in $\mathcal{O}_s$ a further exponential $\exp(icE)$ would take place in the numerator for some real constant c.) In this case the energy spectrum does not agree with the Bose law, however considering packets concentrated around to a high value of the energy E_0, (20) leads to the estimate for the tunneling probability:

$$\lim_{\lambda\to 0}|\omega(\Phi(f_\lambda)\Phi(f'_\lambda))|^2 \sim \text{const.}\ E_0^2\, e^{-\beta_H E_0}\ ,$$

in agreement with the ideas in [PW00, ANVZ05]. It is nevertheless worth remarking that the interpretation of E as an energy is questionable for the packet in the internal region $\mathcal{O}_t$ since the Killing vector K is spacelike therein.

4 Conclusions

In this paper we have computed the correlation functions – and the tunneling amplitude trough a Killing horizon in particular – for scalar Klein-Gordon particle states defined with respect to a certain, physically relevant, class of reference states that includes those of Hadamard type, and in the limit of test functions squeezed on the Killing horizon. The considered local Killing horizon with positive constant surface gravity may be a part of the complete horizon of a black hole – including non static black holes as the non extremal charged rotating one – or may just temporarily exist in a finite region and all computations have a completely local nature since they are performed in a sufficiently small neighborhood of the horizon. Moreover the considered states are generally not required to be invariant with respect to the isometry group generated by the Killing field. We have established that in the limit of wavefunctions sharply localized on the opposite sides of the horizon the correlation functions have a thermal nature, namely they have a spectrum which decays exponentially as $\exp\{-\beta_{\text{Hawking}}E\}$ for high energies. The energy E is defined with respect to the Killing field generating the horizon. This achievement is in agreement with the result obtained in other recent papers, although here it is obtained in the framework of the rigorous formulation of quantum field theory on curved spacetime. Furthermore, we have also established that, when both wavefunctions are localized in the external side of the horizon, a full Bose spectrum at the Hawking temperature arises in the expression of the correlation functions, when taking the considered limit. In both cases the computation is completely local, i.e. the nature of the geometry at infinity does not matter and the results do not depend on the employed states provided they belong to the mentioned class. These results give a strong support to the idea that the Hawking radiation, that it is usually presented as a radiation detected at future (lightlike) infinity and needs the global struture of a black hole Killing horizon, can also be described as a local phenomenon for geometric structures (local Killing horizons) existing just "for a while". A fundamental ingredient in our computation was the constance of the nonvanishing surface gravity on the Killing horizon that enabled us to exploit the result in [RW92] and, in turn, some technical constructions of [KW91]. Even if this requirement can easily be physically intepreted as the geometrical description of the thermodynamical equilibrium, it would be interesting to consider from our viewpoint the case of a black hole in formation, where there are no Killing horizons at all. The latter situation has already been investigated, at least in the presence of spherical symmetry, as in [DNVZZ07, HDVNZ09]. In those paper it is used the WKB approach as well as the theory of Kodama-Hayward and the associated notion of dynamical horizon. As a preliminary rigorous result, we notice that the computation of the scaling limit towards the Horizon does not strongly requires the presence of some Killing fields, which could be substituted by some more generic null hypersurface.

Acknowledgments.

We would like to thank Luciano Vanzo and Sergio Zerbini for useful discussions, suggestions and comments on the subject of this paper. The work of N.P. is supported in part by the ERC Advanced Grant 227458 OACFT "Operator Algebras and Conformal Field Theory".

References

[ANVZ05] M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, *"Hawking radiation as tunneling for extremal and rotating black holes,"* JHEP**0505** 014 (2005).

[Bo69] R.H. Boyer, *"Geodesic Killing orbits and bifurcate Killing horizons"*. Proc. Soc. London **A 311**, 245-252 (1969)

[BV95] D. Buchholz, R. Verch, *"Scaling algebras and renormalization group in algebraic quantum field theory,"* Rev. Math. Phys. **7** 1195 (1995).

[Bu96] D. Buchholz, *"Quarks, gluons, color: Facts or fiction?"* Nucl. Phys. **B469** 333 (1996).

[DMP09] C. Dappiaggi, V. Moretti and N. Pinamonti: *"Rigorous construction and Hadamard property of the Unruh state in Schwarzschild spacetime"*. submitted, arXiv:0907.1034 [gr-qc]

[DNVZZ07] R. Di Criscienzo, M. Nadalini, L. Vanzo, S. Zerbini, G. Zoccatelli, *"On the Hawking radiation as tunneling for a class of dynamical black holes".* Phys. Lett. **B657**, 107-111, (2007). arXiv:0707.4425 [hep-th]

[FH90] K. Fredenhagen and R. Haag, *"On the derivation of Hawking radiation associated with the formation of a black hole,"* Commun. Math. Phys. **127** 273 (1990).

[HDVNZ09] S.A. Hayward, R. Di Criscienzo, L. Vanzo, M. Nadalini, S. Zerbini, *"Local Hawking temperature for dynamical black holes".* Class. Quant. Grav. **26** 062001 (2009). arXiv:0806.0014 [gr-qc]

[Ha75] S. W. Hawking, *"Particle Creation by Black Holes,"* Commun. Math. Phys. **43** 199 (1975).

[KW91] B. S. Kay, R. M. Wald, *"Theorems On The Uniqueness And Thermal Properties Of Stationary, Nonsingular, Quasifree States On Space-Times With A Bifurcate Killing Horizon"*, Phys. Rept. **207** 49 (1991).

[KM07] R. Kerner, R. B. Mann, *"Fermions tunnelling from black holes."* Class. Quant. Grav. **25** 095014 (2008).

[MV05] A. J. M. Medved, E. C. Vagenas *"On Hawking Radiation as Tunneling with Back-Reaction"* Mod. Phys. Lett. **A20** 2449-2454 (2005).

[PW00] M. K. Parikh, F. Wilczek, *"Hawking Radiation as Tunneling",* Phys. Rev. Lett. **85** 5042 (2000).

[RW96] I. Racz, R. M. Wald, *"Global extensions of space-times describing asymptotic final states of black holes,"* Class. Quant. Grav. **13** 539 (1996).

[RW92] I. Racz, R. M. Wald, *"Extension of space-times with Killing horizon,"* Class. Quant. Grav. **9** 2643 (1992).

[Wa84] R. M. Wald: *"General Relativity"*, Chicago University Press, (1984).

[Wa94] R. M. Wald: *"Quantum field theory in curved space-time and black hole thermodynamics"*, The University of Chicago Press (1994).