A novel Nonlinear Model Predictive Controller for Power Maximization on Floating Offshore Wind Turbines.

Luca Pustina

Department of Engineering, Roma Tre University, Rome, Italy

E-mail: luca.pustina@uniroma3.it

Francesco Biral

Department of Mechanical and Structural Engineering, University of Trento, Trento, Italy

Jacopo Serafini

Department of Engineering, Roma Tre University, Rome, Italy

Abstract. Reducing the Levelized Cost of Energy is the main objective of wind turbine industry, in particular for the emerging sector of floating offshore turbines. In this work, a novel Economic Nonlinear Model Predictive Control (ENMPC) strategy is developed to maximize the power production of floating offshore wind turbines. The control problem is solved through an indirect method, which achieves the computational efficiency required to apply it in real world cases. A non-linear Reduced Order Model of the floating turbine predicts aerodynamic power, generator temperature and platform motions inside the controller. A set of constraints, including a bound on the generator temperature, the thrust and platform velocities are imposed. High-fidelity simulations using the open-source tool OpenFAST on the 5MW NREL wind turbine supported by the OC3 spar buoy platform [1] are performed to validate the turbine model and then to assess the controller performances in realistic wind and sea state conditions. With respect to the standard controller, a 4.3% increase of generated power is achieved with a more stable generator temperature.

1. Introduction

An important goal in wind turbines is to increase the annual energy production (AEP) without exceeding safe operation restrictions (e.g. maintaining the correct generator temperature or avoiding excessive rotor speed). The standard control strategy of modern large wind turbines is segmented in regions: region 1, below cut-in speed where the generator is inoperative; region 2, above cut-in speed and below rated wind speed where the control strategy is aimed at maximizing the generated power (lower than generator rated power), increasing generator torque with rotor speed; region 3, over rated wind speed, where the generator torque is set to maintain the rated generator power and the controller acts on collective blade pitch to maintain the rated rotor speed, reducing blade sections incidence as velocity increase. Usually, the controller switches from region 2 to 3 depending on a set of conditions on the low-pass filtered rotor speed and on the collective blade pitch are met [2]. Due to the delay mainly caused by rotor inertia and by the low-pass filter applied to the rotor speed measure, this approach could lead in leakage of power production.

A completely different approach is the Perturbation and Observation ($P\&O$) method [3] where the gradient of produced power with respect to control variables is defined during functioning and used for the optimization of power production. The estimation of the gradient is a complex task, and this approach is bound to fail in wind turbines with high inertia subjected to rapid changes of wind speed [3].

The approach proposed in this work is the Economic Nonlinear Model Predictive Control (ENMPC) that systematically evaluate the blade pitch and generator torque to optimize power production in the near future and can include nonlinear effects, constraints and complex cost function. The drawbacks of the ENMPC are that it requires a prediction of the exogenous inputs (wind and wave-induced forces in the case of a floating turbine) and an accurate Reduced Order Model (ROM) of the turbine. Moreover, the model must be sufficiently simple to be solved faster than real-time in order to continuously update the control law.

In this work a novel and more accurate nonlinear rotor ROM is developed including a widely-used dynamic inflow formulation [4] derived from the Pitt-Peters model [5], in addition to the standard one degree of freedom equation for rotor speed [6, 7]. Moreover, the turbine model is coupled with a platform ROM to include the platform motion contribution to the nacelle velocity. The estimate of the platform motion is not only fundamental to increase power estimation accuracy but also to impose limits on platform velocities, avoiding high loads on mooring lines and electric cable. The proposed model is completed with an equation for the generator thermal dynamics. Indeed, monitoring the temperature allows to override generator rated power without damaging it, obtaining a higher power, especially about rated wind speed.

2. Floating Wind Turbine Reduced Order Model

Usually, a nonlinear, single degree of freedom (DoF) generator model with steady aerodynamics is used to estimate power production for control synthesis [6, 7]). This modelling approach is not accurate in case of wind gusts, rotor speed variations or, more in general, of unsteady aerodynamics loads. This is mainly caused by the neglect of rotor wake memory effect due to the released vortexes dynamics. In this paper, this effect is included through the application of the dynamic inflow formulation [4], derived from the Pitt-Peters model [5].

The two equation of rotor ROM then reads:

$$\begin{cases} J_{rot}\dot{\Omega} &= \frac{1}{2\Omega}\rho\pi R^2(V_w-\lambda-V_{nac})^3C_P^u-\tau_{gen} \\ m_a\dot{\lambda} &= -2\rho\pi R^2\lambda(V_w-\lambda-V_{nac})+\frac{1}{2}\rho\pi R^2(V_w-\lambda-V_{nac})^2C_T^u \end{cases} \quad (1)$$

where J_{rot} is the rotor inertia, ρ the air density, V_w the wind speed, Ω the rotor speed, τ_{gen} the generator torque, λ the mean induced velocity, V_{nac} the nacelle velocity, $m_a = 0.637\rho(4\pi R^3/3)$ an aerodynamic mass and C_P^u and C_T^u the power and thrust coefficients.

The rotor ROM is coupled to the platform ROM that provides the V_{nac} estimation. Both the aerodynamic thrust and torque are functions of the induced velocity and the nacelle velocity, since the dynamic pressure is evaluated with the relative velocity orthogonal to the disc $(V_w-\lambda-V_{nac})$. The C_P^u and C_T^u coefficients are evaluated in steady conditions, namely

$$\begin{aligned} C_P^u &= \frac{P}{1/2\rho\pi R^2(V_w-\lambda^s)^3} \\ C_T^u &= \frac{T}{1/2\rho\pi R^2(V_w-\lambda^s)^3} \end{aligned} \quad (2)$$

where λ^s is the mean inflow velocity in steady condition that can be obtained by solving the inflow equation imposing $\dot{\lambda}^s = 0 = 2\rho\pi R^2\lambda^s(\lambda^s-V_w)+T$. The maps of P_{aero} and T has been evaluated using the open-source OpenFAST tool [8] varying Ω, β and V_w. In the analyses, an uniform wind profile is considered and a constant rotor speed is maintained. Then, a Radial-Basis-Functions interpolator is used to obtain the analytic representations of the C_P^u and C_T^u functions in terms of Ω, β and V_w. Note that, since the objective of the controller is to maximize the aerodynamic power, it is plausible that only a low-frequency blade pitch and generator torque actuation is required, and then high-frequency effects (such as tower or blades elasticity or tower shadow) are not included in ROM. Instead, the inclusion of the low-frequency wake inflow memory effect, which adds important aerodynamic inertia to the model, improves the accuracy of the model.

2.1. Simplified generator thermal model

Usually, the generator temperature is not considered by the controller. However, the generator can override the rated power for a considerable amount of time and a simple thermal model for the temperature estimation helps to maximize the power production avoiding generator damage. A basic one-ODE model for the generator temperature is proposed:

$$\dot{\theta} = -k(\theta - \theta_0) + cP_{gen} \tag{3}$$

where θ_0 is the external temperature, k is the heat coefficient and c is the inverse of heat capacity. Through simple engineering considerations, the values have been hypothesized as $\theta_0 = 20°C$, $k = \frac{1}{40}s^{-1}$ and $c = 0.0667°C/(sMW)$.

3. Platform Reduced Order Model for FOWTs

Platform motions, in particular surge and pitch motions, modify hub velocity and the apparent wind. Since the aerodynamic power depends on the third power the apparent wind speed, the contribution due to platform motion may be relevant. Moreover, it is desirable to impose constraints on platform motions to avoid high loads on mooring lines and electric cables.

To estimate the nacelle velocity (V_{nac}), two second-order equations for platform surge and pitch dynamics are added to the model. The platform (including the tower) is forced by mooring lines, by hydrodynamic and hydrostatic loads, and by the rotor thrust. Hydrodynamic loads can be separated into diffraction ones (exogenous, depending only to waves) and radiation/viscous loads (depending only on platform state). Then, in the coupled platorm-rotor model, the only external inputs are the wind velocity and the diffraction loads. Note that, in a actual controller application, the FOWT should be be equipped with a sea waves height sensor (for example a wave radar [9]) that would become the input to a diffraction force model predicting diffraction forces. In this work, the platform multiphysics linear model is identified through MATLAB System Indentification Toolbox from high-fidelity simulations performed with OpenFAST.

The same approach could be also followed using experimental scaled or full-scale data.

The state space form of the identified model is

$$\begin{aligned} \dot{\mathbf{x_p}} &= \mathbf{A}\,\mathbf{x_p} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\,\mathbf{x_p} \end{aligned} \tag{4}$$

where the vector $\mathbf{x_p}$ collects a subset of the platform states, which includes the most relevant for the definition of output $\mathbf{y}$, namely the vector collecting pitch and surge motion. For platforms on which the hydrodynamic radiation memory effect is relevant, also hydrodynamic states may be added. In this work, the states has been considered coincident with outputs.

4. Issues of standard controllers on offshore applications

In a FOWT, platform dynamics makes impossible to use the usual gains of standard controller briefly outlined in section 1. Indeed, if an onshore wind turbine controller is directly used on an offshore wind turbine, a poorly damped or unstable pitch and surge motion comes up. This is caused by the lower frequency of the pitch motion with respect to the first fore-aft tower bending mode. Usually, lower proportional and integral gains are adopted (see [1]) to put the closed-loop rotor frequency below the platform critical frequency (pitch in this case), in order to ensure stability. Moreover, to increase the closed-loop damping, a constant generator torque is adopted in operating region 3. A shortcoming of this approach is that a poorer quality, oscillating generated power output is obtained.

5. Definition of the control optimization problem

In this work, a ENMPC is used on a constrained problem where the objective function is the extracted energy over a moving time horizon. This approach doesn't even requires different control regions as in the standard controller. The same control strategy can be applied also on different objectives like the mitigation of vibratory loads. If the integral of the generated power is directly chosen as the objective function, the optimizer is inclined to increase it by reducing rotor kinetic energy at the end of the optimization window. This leads to optimal power production only in the first time window, but the high reduction of rotor speed at the first window ends causes a sub-optimal overall control action. Then the integral of aerodynamic power is instead chosen as the objective function, namely

$$J = \int_{T_0}^{T_0+T_w} P_{aer} \, \mathrm{d}t \tag{5}$$

where T_w is the length of the optimization window. In such a way, the controller acts to extract maximum energy from air flow, which can be transformed in the electric generator or stored in rotor kinetic energy. A set of constraints to guarantee the feasibility of the optimal control law are imposed and reported in table 1.

Since the generator has a great thermal capacity, the rated generator power can be overridden for a considerable amount of time, whereas electronics small time constant doesn't allow even smaller overriding transient. Hypothesizing that a power electronics with a greater rated power than generator is installed on the turbine, the proposed control strategy allow to override generator temperature for a significant time. However, to avoid unrealistic high power peaks on electronics, a constraint on power (20% higher the rated) is imposed. Thus, the controller override the generator up to when one the two following conditions is met: the generator temperature reaches the upper limit (60 °C) or the power reaches the aforementioned upper limit. In a realistic wind condition (especially about the rated wind speed) this approach can lead to higher power production, safeguarding the generator and the electronics. Since no constraints are imposed on the aerodynamic power, a constraint on rotor thrust is imposed to avoid too high loads on the structure. Moreover, constraints on platform surge and pitch velocities are imposed to avoid large platform motion, since it would lead to high stress on mooring lines and electric cables.

In the application of the controller, after the optimal control law is evaluated on a time window, the evaluation process may be suddenly restarted starting from current system state, improving the performance of the controller. This is related to the fact that the predictions of the ROM model used in the controller evaluation may significantly diverge from the actual system evolution. Note that part of the state (Ω, θ and $\mathbf{x_p}$) is considered measurable, whereas the inflow λ must be observed. Moreover, wind velocity over the optimization time window must be estimated, as well as wave-induced forces.

In the optimization process over a new time horizon, the remaining part of the previous one (i.e. the length of the time horizon minus the update time of the controller) is used as the first guess for the controller, for the last part of the new time horizon, the first guess is kept constant at the last available value of control variables. Using this first guess and thanks to the OCP performance, the optimization procedure on a standard laptop requires usually less than 15 s. In order to maintain the continuity and the derivative continuity (C1) when the control law is updated, the ROM is augmented with four states, namely the first and second derivative of the two control inputs. The initial conditions for these states are chosen according to the past control window to achieve a C1 control law (for the first control window they are chosen null).

6. Numerical solution of the optimal control problem

Nowadays, complex practical Optimal Control Problems OCPs, like that presented in section 5, can be solved also for online applications thanks to the recent advances in theory, the availability of more efficient optimization algorithms and the increase of computational power. Solution methods can be divided into dynamic programming, indirect and direct methods Dynamic programming [10] can be easily applied to systems having discrete states and control spaces. The major drawback of this approach is the exponential growth of computational cost with the number of states, that limits the applicability of the method to problems with few states. More of 90% of available software and solution schemes proposed in the literature are based on direct methods. Direct methods transform the optimal control problem into a large constrained optimization problem (so-called Non Linear Programming NLP). A large number of NLP solvers are available and a wide number of NLP solvers comparison review articles [11] are present in literature.

The indirect method relates to Pontryagin Minimum Principle [12] to derive the necessary conditions of optimality [13]. Solution methods span from single shooting to multiple shooting and different types of collocation [14] being the last ones the most robust and fast converging of indirect methods [15].

In this work, PINS [15], a code developed at Trento University and available for free for academic use under request to the authors, has been used. PINS is based on an indirect method where the Two-Points Boundary Value Problem (TPBVP) is solved through the collocation approach. A custom designed, robust and fast non linear solver that exploits the problem structures of the discretized TPBVP is used in PINS.

In [15] PINS has been successfully compared with other direct and indirect solvers.

7. Numerical results

The proposed controller is tested on the NREL-5MW turbine [2] supported by the OC3 spar buoy platform [1].

The high fidelity simulations are performed by the open-source software OpenFAST [8] considering six platform rigid-body motion DoFs, four elastic modes for the description of the tower deformation and three elastic modes for the description of each blade deformation (namely, two flap modes and one lead-lag mode). The wake inflow is estimated by the Generalized Dynamic-Wake (GDW) model [16] that includes tip losses, hub losses, and skewed wakes effects, while the aerodynamic loads are determined by the Beddoes-Leishman dynamic stall formulation [16]. The simple generator thermal model described in section 2.1 is also included in OpenFAST.

First, the coupled ROM has been validated against high fidelity simulation considering a realistic wind and sea condition. The main focus was on the capability of the ROM to estimate low-frequency power production, wind turbine motions and loads.

Then, to estimate the performance of the proposed controller, a set of realistic wind turbine operating conditions are investigated. The developed controller is tested in different operating regions of the wind turbine and it is compared with the reference controller in terms of power production, structural loads and control effort.

7.1. Validation of the coupled ROM.
To assess the accuracy of the coupled ROM it is compared against high fidelity simulations.

An offshore wind condition near the rated wind one is considered, following the IEC 61400-3 standard [17]. Specifically, the mean wind speed is equal to 12 m/s, and an IEC Kaimal spectral model with a 'C' standard IEC category of turbulence and a power-law mean wind profile are applied [18]. A realistic multi-directional short-crested sea state characterized by a JONSWAP [19] spectrum with a significant wave height of 3 m and a peak at 0.1 Hz is chosen to simulate a realistic sea conditions.

The reference controller is applied to both the high fidelity model and the ROM in order to have a stable system. Since the ROM require the knowledge of the effective wind speed, the wind speed field has to be processed to obtain a single effective value for each time-step. Indeed, the power and thrust coefficient maps are obtained with a constant spatial wind condition, and then the mean axial wind speed on the rotor disk (of the considered realistic wind condition) is evaluated for each time-step and used as the input for the ROM. Finally, the same surge and pitch diffraction forces of the high fidelity model are considered for the ROM. The objective is to verify that the ROM to be used for the control law synthesis provides a good approximation of the objective function (the aerodynamic power) and of the constraints (the thrust, rotor speed, generator temperature, generated power and platform motions).

In fig. 1, which depicts the generated and aerodynamic power comparison, the ROM provides a good low-frequency approximation of all the monitored quantities, also in presence of significant (nonlinear) deviation from reference condition due to wind fluctuations and waves.

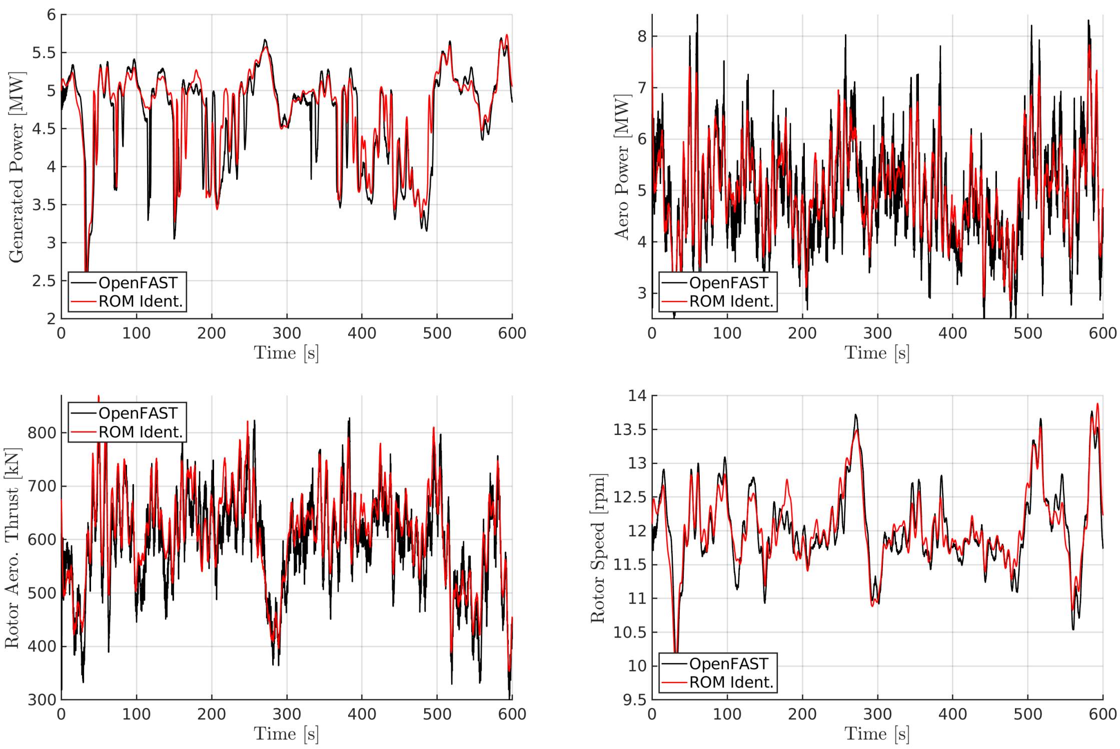

Figure 1. Generated power, aerodynamic power, rotor speed and thrust comparison.

As demonstrated by fig. 2 (which depicts the comparison of the nacelle velocity), also the platform rigid motion is estimated with a high degree of accuracy (for the sake of conciseness, platform surge and pitch motions are not shown separately).

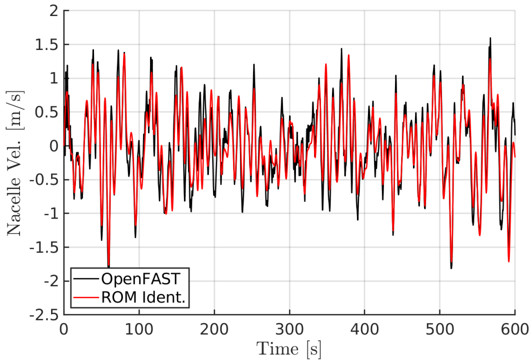

Figure 2. Nacelle velocity comparison.

Concluding, the validation tests on the ROM model for power generation purposes have given fully satisfactory results.

7.2. Numerical Testing of the Economic Non-linear Model Predictive Controller for Power Production Maximization

To assess the controller effectiveness, the developed ENMPC has been coupled with the high-fidelity solver OpenFAST. In particular, the control law is updated every 5 s and the rotor speed, generator temperature platform motions and velocities are corrected with the lecture of high-fidelity values. Since mean disk inflow velocity is a non-measurable quantity, the ENMPC itself acts as a non-linear observer. A time horizon of $120s$, significantly longer than the update time is chosen, to give to the controller the sensibility to long term effects of the control law. Indeed, rotor speed degree of freedom have a long characteristic period due to the large rotor inertia.

In table 1 the constraints for the optimal control problem are summarized. No constraint is imposed to the aerodynamic power, and instead an upper limit is set on the rotor thrust to limit the aerodynamic loads. Although an upper limit on the generator temperature is imposed, to avoid peaks in generator power that can cause electronics damages, also a limit on generated power is considered. Blade pitch and rotor speed are constrained to remain inside power and thrust coefficient maps boundaries avoiding unexpected wind turbine outputs. Finally, the control inputs derivative and second derivative are constrained to help the optimization algorithm convergence and to obtain a smooth law (the second derivative limits are box constraints). Note that the blade pitch velocity limit chosen (1.5 deg/s) is much lower than the maximum allowed for this turbine (8deg/s), resulting in a reduced control effort. Finally, to avoid platform instability and high loads on mooring lines and electric cables constraints on platform maximum pitch and surge velocities are imposed. Note that the constraints proposed values are first guess and may be fine-tuned to improve controller performance.

Constraint	min	max	Constraint	min	max
Rot. Thrust. [kN]	//	900	Blade pitch acc. [deg/s^2]	-1.5	1.5
Gen. Temp. [$C°$]	//	60	Gen. Tq. [kNm]	0	43
Gen. Pow. [MW]	//	6	Gen. Tq. 1st der. [kNm/s]	-10	10
Rot. Speed [rpm]	2	15	Gen. Tq. 2nd der. [kNm/s^2]	-3	3
Blase pitch [deg]	-5	22	Platf. Surge Vel. [m/s]	-0.8	0.8
Blase pitch vel. [deg/s]	-1.5	1.5	Platf. Pitch Vel. [deg/s]	-0.5	0.5

Table 1. Constraints of the optimal control problem

Concerning operating conditions, the same realistic wind and sea state conditions used for ROM validation is considered also in this case. A measurement of the mean wind speed and of sea-waves diffraction forces on all the 120-seconds time window has been hypothesized. In real-world applications, there are several possibilities for measuring incoming wind, for example using LIDAR sensor [20], or a predictor based on previous wind measurements [21]. It is also possible to measure the incoming waves with a wave radar [9] and easily obtain an estimation of the future diffraction forces.

Figure 3 depicts the generated and aerodynamic power time-series for the reference and the proposed controller. In the figure, it is also reported the ROM estimation (updated for every control law update). As discussed in section 7.1, the ROM is fully capable to reconstruct the low-frequency behaviour of the generated and aerodynamic power. The high fidelity simulation predicts a 4.31% increase of power production using the ENMPC instead of the reference controller. The imposed constraint for the generated power of $6MW$ is only slightly violated, probably owing to the the estimation error provided by the ROM. This problem may be addressed adding a safety factor to the constraint value to take into account ROM uncertainties.

Note that the selected wind scenario at about rated wind speed is very common during wind turbine life, since this is the reference design point. At lower or higher wind speed (in full region 2 or 3) the ENMPC controller has demonstrated to have the same performance of the standard controller in terms of power production. This is due to the fact that the thermal management of the generator does not give particular advantages where the power production is constantly below or above rated one. It should be taken into account, however, that in the ENMPC a unique control strategy is adopted for operating regions 2, $2\frac{1}{2}$ and 3 avoiding the necessity to switch between different control strategies.

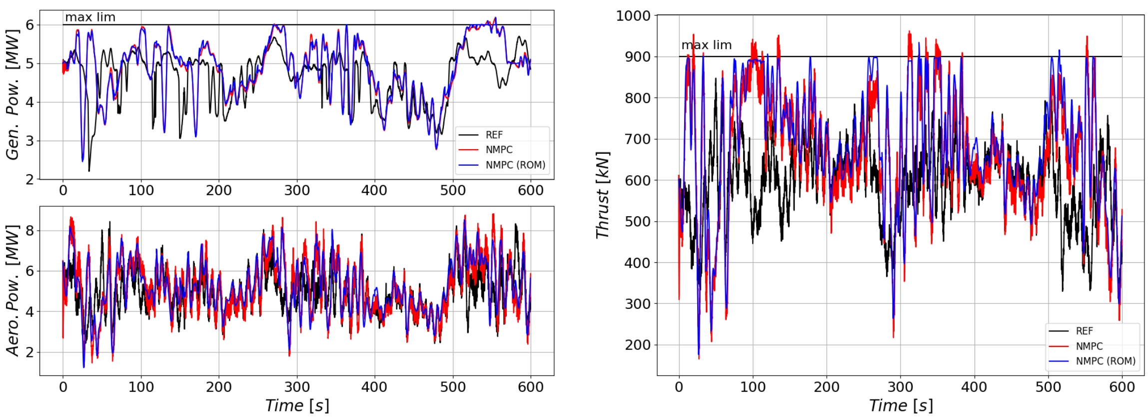

Figure 3. Generated and aerodynamic power, aerodynamic rotor thrust for the reference and the ENMPC (in red the high-fidelity and in blue the ROM).

Concerning the other constrained state and control quantities (angular velocity and temperature, generator torque and pitch) the high fidelity simulations highlight a marginal thrust limit exceeding, as shown in fig. 4. Also in this case, a safety factor may to be used. Concerning the control law, a smooth C1 function is obtained. The control power associated to pitch actuation is similar to that of the reference controller.

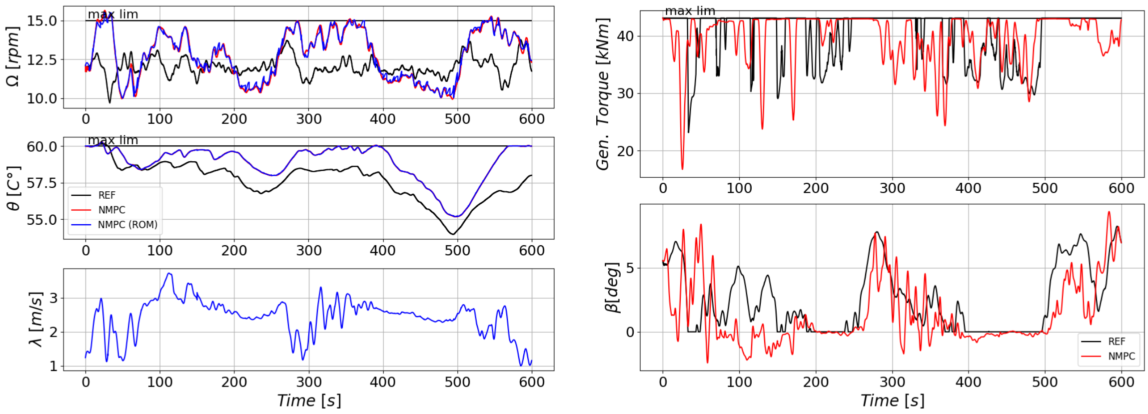

Figure 4. On the right ROM states time-series for the reference and the ENMPC (in red the high-fidelity and in blue the ROM). On the left the control law.

Figure 5 depicts the surge and pitch motion for the reference controller and the ENMPC and in both case a stable platform dynamics is obtained. A small violation of the constraints is observed as in the other cases.

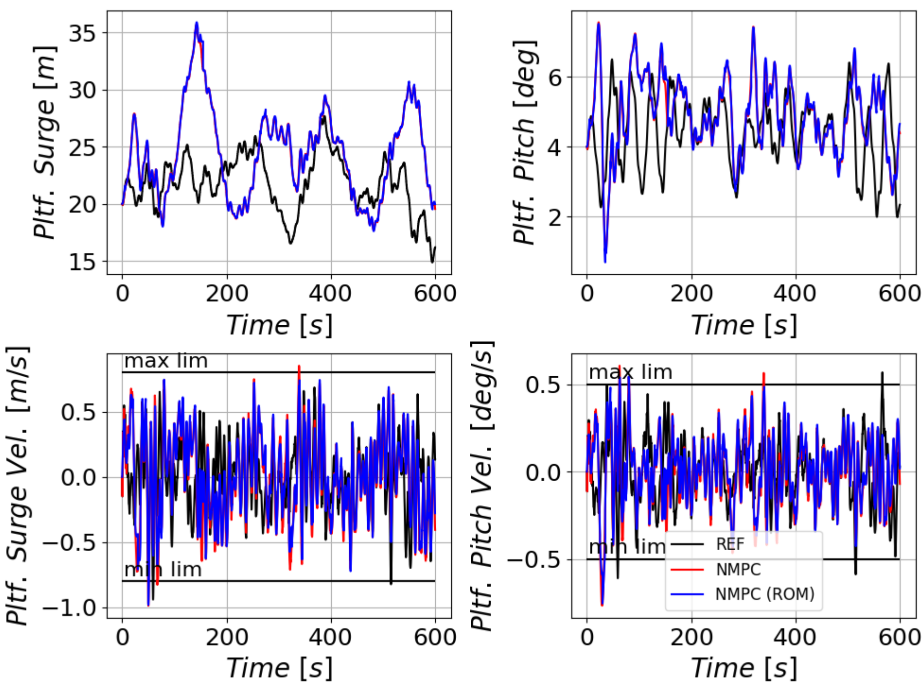

Figure 5. Platform surge and pitch motions time-series for the reference and the ENMPC (in red the high-fidelity and in blue the ROM).

8. Conclusions

The main outcomes of the ENMPC controller development and application to the reference turbine may be summarized as follows:

- The modelling of a wind turbine for power production estimation is significantly improved by the use of a 3-variables power coefficient map instead of the standard 2-variables one.
- At the same time, the inclusion of a first order inflow model significantly improve the model accuracy in unsteady cases.
- although floating offshore wind turbine rigid degrees of freedom are coupled, for power production purposes, it is sufficient to use a 2-DOF (pitch-surge) model of the platform.
- the proposed controller allows a appreciable increase of produced power (4.3%) when the wind speed is about the rated value and it doesn't require the definition of multiple control regions, thanks to the introduction of constraints in the controller definition problem
- the introduction of constraints also allows to obtain a well damped platform motion and a low control effort
- Thanks to the numerical solution strategy of the Optimal Control Problem and to the low computationally effort of the developed ROM, it is possible to use the controller in a real-world applications.

References

[1] Jonkman J 2010 Definition of the floating system for phase iv of oc3 Tech. rep. National Renewable Energy Lab.(NREL), Golden, CO (United States)

[2] Jonkman J, Butterfield S, Musial W and Scott G 2009 Definition of a 5-mw reference wind turbine for offshore system development Tech. rep. National Renewable Energy Lab.(NREL), Golden, CO (United States)

[3] Jeong H G, Seung R H and Lee K B 2012 *Energies* **5** 1339–1354

[4] Leishman G J 2006 *Principles of helicopter aerodynamics with CD extra* (Cambridge university press)

[5] Pitt D M and Peters D A 1981 *Vertica* **5**

[6] Henriksen L C, Poulsen N K and Hansen M H 2011 *IFAC Proceedings Volumes* **44** 551–556

[7] Yan Z, Hall J and Chen D 2013 Mimo control of wind turbine using direct shooting method *2013 American Control Conference* (IEEE) pp 3655–3660

[8] Jonkman J 2013 The new modularization framework for the fast wind turbine cae tool *51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition* p 202

[9] Fucile F, Ludeno G, Serafino F, Bulian G, Soldovieri F and Lugni C 2016 Some challenges in recovering wave features from a wave radar system *The 26th International Ocean and Polar Engineering Conference* (OnePetro)

[10] Bellman R 1954 *Bulletin of the American Mathematical Society* **60** 503–515

[11] Rao A V 2009 *Advances in the Astronautical Sciences* **135** 497–528

[12] Pontryagin L S 1987 *Mathematical theory of optimal processes* (CRC press)

[13] Liberzon D 2011 *Calculus of variations and optimal control theory* (Princeton university press)

[14] Diehl M and Gros S 2011 *Optimization in Engineering Center (OPTEC)*

[15] Biral F, Bertolazzi E and Bosetti P 2016 *IEEJ Journal of Industry Applications* **5** 154–166

[16] Moriarty P J and Hansen A C 2005 Aerodyn theory manual Tech. rep. National Renewable Energy Lab., Golden, CO (US)

[17] Iec 61400-3 (draft january 2006), wind turbines-part 3: Design requirements for offshore wind turbines

[18] Jonkman B J and Buhl Jr M L 2006 Turbsim user's guide Tech. rep. National Renewable Energy Lab.(NREL), Golden, CO (United States)

[19] Hasselmann K, Barnett T P, Bouws E, Carlson H, Cartwright D E, Enke K, Ewing J A, Gienapp H, Hasselmann D E, Kruseman P, Meerburg A, Mller P, Olbers D J, Richter K, Sell W and Walden H 1973 *Ergnzungsheft zur Deutschen Hydrographischen Zeitschrift Reihe* **A8**

[20] Simley E, Pao L, Kelley N, Jonkman B and Frehlich R 2012 Lidar wind speed measurements of evolving wind fields *50th AIAA aerospace sciences meeting including the new horizons forum and aerospace exposition* p 656

[21] Riahy G and Abedi M 2008 *Renewable energy* **33** 35–41