Uncertainty Measures in Evidence Theory: A Genealogy

Tommaso Pellegrini and Matteo Brunelli

***Abstract*— We consider the problem of quantification of uncertainty in the framework of evidence theory. A large number of uncertainty measures have been proposed in the last 40 years to perform this task, and contributions, albeit numerous, are scattered in the literature, and it is difficult to have an idea of how many measures exist, let alone to know their formal definitions and their evolution in time. In this article, we provide a self-contained exposition of 65 uncertainty measures—out of which 43 were proposed in the last decade—and then we consider their development from the temporal point of view, as well as the lineage of each one from previous ones. These two analyses help us see that 2016 was an *annus mirabilis*, since the three measures proposed in that year would later on become extremely influential. Moreover, our analysis offers at least three insights that may hopefully open the floor for future research. First, the recent proliferation of uncertainty measures has not always been accompanied by sufficiently deep analyses on their formal properties. Second, the widely accepted behavioral requirement that the total uncertainty must be the sum of the contributions of non-specificity and conflict appears questionable in light of its necessary conditions. Third, and last, the choice of the most suitable measure cannot be detached from the interpretation that is given to the belief function.**

***Index Terms*— Belief entropy, Dempster–Shafer theory, evidence theory, uncertainty measures.**

I. Introduction

According to one of the most widely accepted classifications, uncertainty can be either epistemic or aleatoric [1]. Aleatoric uncertainty arises from the random nature of the underlying event, e.g., rolling a die, whereas epistemic uncertainty derives from lack of knowledge of the event, e.g., determining whether a painting is original or counterfeit. Although probability remains the undisputed theory to represent aleatoric uncertainty, a number of other theories were presented to describe epistemic uncertainty [2].

One of the most widely accepted and adopted theories is the Dempster–Shafer theory of evidence, which does not compete against (subjective) probability but instead generalizes it. This theory was first outlined by Dempster [3] and later on revisited by Shafer [4], and it has, since then, become an independent field of research. Its importance is also due to the fact that it offers a common framework to accommodate both probability theory and possibility theory, where the latter one can be seen as an interpretation for fuzzy sets [5].

A basic difference between probability theory and evidence theory is that in probability theory, subjective beliefs are assigned to elementary events, whereas in evidence theory, subjective beliefs can be assigned to any subset of the event space. This greater freedom in the assignment phase yields two measures, belief and plausibility, which coincide (and become a probability measure) only in the special case when the chosen subsets are all singletons.

Similar to what happened in probability theory, where some measures of entropy were introduced to quantify the information contents related to probability distributions, the need to quantify the uncertainty of uncertain events modeled with evidence theory has triggered the proliferation of uncertainty measures. The role of uncertainty measures extends beyond their basic function of quantifying uncertainty. They have been actively employed in practical applications such as information fusion from multiple sources (e.g., sensors) [6], and in group decision making to weigh the preferences of experts [7]. Given their practical importance, it is essential to understand both their definitions and the underlying principles. However, while numerous new uncertainty measures have been proposed in the literature, comparatively less effort has been devoted to their systematic analysis and classification. Recent studies have begun to address this gap by adopting comparative approaches. For instance, Dezert and Tchamova [8] reviewed the definitions of 45 uncertainty measures and highlighted several limitations, while Urbani et al. [9] analyzed 31 measures from a numerical perspective to identify potential similarities. Similarly, Barhoumi et al. [6] focused on the application of uncertainty measures in information fusion and reviewed 25 relevant measures. Jiroušek and Kratochvíl [10] used numerical simulations to study 25 uncertainty measures and identify those that do not satisfy some of their requirements. Mao et al. [11] followed a pragmatic approach and created a neural network classifier based on evidence theory and ranked seven uncertainty measures according to their performance within that system using some real-world datasets.

Although existing comparative studies are valuable, they remain incomplete in terms of coverage and do not explore the temporal evolution of uncertainty measures, which could provide further insights. In this work, we compile and review the definitions of 65 uncertainty measures—43 of which were introduced in the last decade, and 10 within the past three

Received 19 September 2025; accepted 20 December 2025. Date of publication 31 December 2025; date of current version 18 February 2026. This article was recommended by Associate Editor X. Su. *(Corresponding author: Matteo Brunelli.)*

The authors are with the Department of Industrial Engineering, University of Trento, 38123 Trento, Italy (e-mail: matteo.brunelli@unitn.it).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TSMC.2025.3648198.

Digital Object Identifier 10.1109/TSMC.2025.3648198

years—tracing their chronological development. This historical perspective allows us to examine how these measures have influenced one another since the introduction of the earliest ones over 40 years ago. Through this analysis, we aim to take stock of the current state of the field and identify potential directions for future research and development.

The article is organized as follows. Section II presents the necessary notions of evidence theory. Next, Section III presents a self-contained introduction to 65 uncertainty measures. Section IV presents the analysis of these 65 uncertainty measures by means of a timeline and a Hasse diagram. Section V concludes the article with a general discussion, identifies some research gaps, and suggests ideas for future research.

II. Preliminaries

In evidence theory, the *frame of discernment* is a non-empty finite set of n elementary events and is denoted as $X = \{x_1, \ldots, x_n\}$. A *mass assignment*—sometimes called basic belief assignment or basic probability assignment—is a function $m\colon 2^X \to [0, 1]$ such that $m(\emptyset) = 0$ and $\sum_{A\subseteq X} m(A) = 1$. The value $m(A)$ represents an estimate of the degree of belief that the true event is in A. We denote with $\mathcal{F} = \{A|A \subseteq X, m(A) > 0\}$ the *set of focal elements*. Furthermore, $\cup\mathcal{F} = \bigcup_{A\in\mathcal{F}} A$ and $\cap\mathcal{F} = \bigcap_{A\in\mathcal{F}} A$.

Two measures, in the measure theoretic sense, are associated with a mass assignment. The first, *belief* is a superadditive measure defined as

$$\mathrm{Bel}(A) = \sum_{B\subseteq A} m(B) \quad \forall A \subseteq X$$

and represents the sum of the evidence directly supporting the event A. The second, *plausibility*, is subadditive, and is defined as

$$\mathrm{Pl}(A) = \sum_{B|A\cap B\neq\emptyset} m(B) \quad \forall A \subseteq X$$

and is the sum of the evidence that cannot disprove event A. Together, they form a dual pair of measures, i.e., $\mathrm{Pl}(A) = 1 - \mathrm{Bel}(\bar{A})$, where $\bar{A}$ is the complement of A with respect to X. Interestingly, for any pair Bel and Pl, there is always a probability measure Pr such that

$$\mathrm{Bel}(A) \leq \Pr(A) \leq \mathrm{Pl}(A) \quad \forall A \subseteq X \tag{1}$$

We call $\mathcal{P}_m$ the *credal set* of all the probability distributions compatible with m according to (1). Note that, hereafter, we may abuse notation and omit curly brackets for singletons, e.g., $\mathrm{Pl}(x_i) = \mathrm{Pl}(\{x_i\})$. Another significant quantity connected to m is the commonality function Q defined as

$$Q(A) = \sum_{B|A\subseteq B} m(B) \quad \forall A \subseteq X.$$

Indeed, m, Bel, Pl, and Q are all related to each other and knowing one is sufficient to deduce the others.

In some cases, it is necessary to map evidential information into a representative probability distribution. Functions mapping evidential information into probability distributions are commonly called probabilistic transformations. The most prominent one is the *pignistic transformation* [12]

$$\mathrm{Bet}(x_i) = \sum_{\substack{A\subseteq X\\ x_i\in A}} \frac{m(A)}{|A|} \quad \forall x_i \in X. \tag{2}$$

Another approach was pushed forward by Cobb and Shenoy [13] when they proposed the *plausibility transformation*

$$\mathrm{Pl}_T(x_i) = \frac{\mathrm{Pl}(x_i)}{\sum_{j=1}^{n} \mathrm{Pl}(x_j)} \quad \forall x_i \in X \tag{3}$$

to estimate a representative probability distribution.

III. Uncertainty Measures in Evidence Theory

In classical set theory, given a set X, a fundamental measure of its uncertainty is Hartley's measure

$$\mathsf{H}_0 = \log_2 |X| \tag{4}$$

that connects the uncertainty to the cardinality of the reference set. Shannon's entropy can be seen as a generalization of Hartley's measure for the case where elements of the set A have different probabilities. Given a set X and a probability distribution p on the same set, Shannon entropy was defined as

$$\mathsf{H}_s = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i). \tag{5}$$

In evidence theory, an uncertainty measure is an index U whose value is a numerical quantification of the intrinsic uncertainty of the available evidence, regardless of whether it is expressed as a mass function m, a belief measure Bel, a plausibility measure Pl, or a commonality function Q.

The numerical quantification of uncertainty coming from the application of U has been used for many purposes. For example, uncertainty of different mass functions has been used to weigh them to favor more conclusive (less uncertain) ones when they need to be aggregated.

Four key factors have created a favorable environment for the development of uncertainty measures within evidence theory.

1) Evidence theory provides a more general framework than probability theory, allowing many concepts from probability theory to be extended in various ways. As a result, uncertainty measures in evidence theory offer greater flexibility in their composition compared to those in probability theory.
2) In evidence theory, uncertainty arises from two main sources: *non-specificity* and *conflict*. Non-specificity stems from assigning evidence to larger subsets $A \subseteq X$, which reflects a higher degree of uncertainty. The two extremes are: assigning all evidence to a single element, $m(\{x_i\}) = 1$, representing complete certainty, and assigning all evidence to the entire frame of discernment, $m(X) = 1$, which corresponds to total ignorance. Conflict, on the other hand, relates to how belief is distributed across focal elements. For instance, distributing belief uniformly over singletons, such as $m(\{x_i\}) = 1/n$, indicates greater uncertainty than assigning all belief

to a single singleton. It is now widely accepted that any effective measure of uncertainty in evidence theory should account for both non-specificity and conflict.

3) Several plausible properties have been proposed to characterize well-behaved uncertainty measures. However, unlike Shannon entropy in probability theory—which is uniquely defined by a clear set of axioms—these properties in evidence theory tend to be less restrictive and have often been overlooked or challenged by segments of the research community. As a result, uncertainty measures have frequently been accepted even when they fail to satisfy some, or even many, of these proposed properties.
4) In many instances, it is relatively easy to construct a numerical example where a particular uncertainty measure exhibits counterintuitive behavior. Consequently, as we will see, numerous uncertainty measures have been proposed as refinements of earlier ones, aiming to correct their shortcomings in specific cases.

The conjoint effect of these four factors was determinant for the proliferation of new proposals of uncertainty measures. In this section, we shall briefly recall the 65 uncertainty measures that will be considered in our analysis. It is important to reckon that each measure of uncertainty represents a distinct definition of the concept of uncertainty: all measures agree on the definition of certainty, but each provides a different perspective and definition of uncertainty, which is a matter of degrees and is measured by the uncertainty measure.

Höhle's confusion measure: Höhle [14] introduced the first uncertainty measure tailored for belief functions. His measure, often referred to as *confusion*, was defined as

$$\mathsf{U}_H = -\sum_{A\in\mathcal{F}} m(A)\log_2 \mathrm{Bel}(A) \tag{U1}$$

and it is a clear extension of Shannon's entropy.

Smets' commonality measure: Smets [15] introduced an alternative approach to measuring uncertainty in belief functions. His measure, known as the *commonality-based* entropy, modifies *Höhle's confusion measure* (U1) by leveraging the commonality function instead of the belief function. Smets' commonality measure is defined as

$$\mathsf{U}_S(m) = -\sum_{A\in\mathcal{F}} c(A)\log_2 Q(A) \tag{U2}$$

where the function $c(A)$ is a weight assigned to each subset A, which Smets [15] originally set as $c(A) = 1$ as "the most natural choice."

Yager's dissonance measure: Similar to Höhle, Yager [16] introduced an uncertainty measure that extends *Shannon entropy* as

$$\mathsf{U}_{\mathrm{Yd}} = -\sum_{A\in\mathcal{F}} m(A)\log_2 \mathrm{Pl}(A). \tag{U3}$$

Like *Höhle's confusion measure* (U1), it quantifies conflict but relies on plausibility instead of belief.

Yager's non-specificity measure: Alongside his work on *dissonance*, Yager [16] introduced a measure to quantify the *specificity* of belief functions, assessing how mass assignments contribute to precise or imprecise information. Yager's specificity measure is defined as

$$S(m) = \sum_{A\in\mathcal{F}} \frac{m(A)}{|A|}.$$

Later, George and Pal [17] reformulated this concept, defining *non-specificity* as its complement

$$\mathsf{U}_{\mathrm{Yns}} = N(m) = 1 - S(m). \tag{U4}$$

Nguyen's uncertainty measure: Nguyen [18] introduced an alternative approach to quantifying uncertainty and is probably the most straightforward extension of Shannon entropy to belief functions, and was defined as

$$U_N = -\sum_{A\in\mathcal{F}} m(A)\log_2 m(A). \tag{U5}$$

Dubois and Prade's uncertainty measure: Dubois and Prade [19] refined a possibilistic measure introduced by Higashi and Klir [20] and formalized the uncertainty measure

$$\mathsf{U}_{\mathrm{DPu}} = \sum_{A\in\mathcal{F}} m(A)\log_2(|A|). \tag{U6}$$

Dubois and Prade [19] stated that $\mathsf{U}_{\mathrm{DPu}}$ "can be viewed as a Hartley measure of information weighted by the basic probability assignment $m(A)$." Hence, it is closely related to *Hartley entropy* (4).

Dubois and Prade's index of fuzziness: Dubois and Prade [21] introduced the *index of fuzziness*

$$\mathsf{U}_{\mathrm{DPf}} = -\ln\left(\sum_{A\in\mathcal{F}} m(A)\ Q(A)\right) \tag{U7}$$

as an uncertainty measure designed to quantify *conflict*.

Dubois and Prade's entropy-like index: In the same study where Dubois and Prade [21] introduced the *index of fuzziness*,[1] they formalized another measure, the entropy-like index

$$\mathsf{U}_{\mathrm{DPe}} = -\sum_{A\in\mathcal{F}} m(A)\ln Q(A). \tag{U8}$$

This measure is based on *commonality-based entropy measure of Smets* (U2), replacing the function $c(A)$ with the BPA $m(A)$.

Dubois and Prade's imprecision measure: Similar to *Yager's non-specificity measure* (U4), Dubois and Prade [22] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{DPi}} = \sum_{A\in\mathcal{F}} m(A)|A| \tag{U9}$$

to quantify non-specificity.

Lamata and Moral's lower entropy measure: The uncertainty measures presented so far were devised to capture only some aspects of uncertainty: (U1)–(U3), (U5), (U7), and (U8) are measures of conflict and (U4), (U6), and (U9)

[1] As correctly pointed out by Dubois and Prade [21], what are commonly referred to as uncertainty measures in this manuscript and in the literature should, from a formal standpoint, be called uncertainty indices, as they do not satisfy the formal requirements of measure theory.

are measures of non-specificity. A big leap happened when Lamata and Moral [23] introduced the first total uncertainty measure, namely the lower entropy measure, which they defined as a "global measure of uncertainty" in [23]. This measure integrates both conflict and non-specificity, providing a comprehensive quantification of uncertainty. Lamata and Moral [23] defined their lower entropy measure as

$$\mathsf{U}_{\mathrm{LMl}} = \underbrace{-\sum_{A\in\mathcal{F}} m(A)\log_2\left(\frac{\mathrm{Pl}(A)}{|A|}\right)}_{\mathsf{U}_{\mathrm{Yd}}+\mathsf{U}_{\mathrm{DPu}}}. \tag{U10}$$

As can be observed, this measure is the sum of *Yager's dissonance measure* (U3) and *Dubois and Prade's imprecision measure* (U6), and thus derives directly from them.

Lamata and Moral's upper entropy measure: Together with the *lower entropy measure* [23] introduced another "global measure of uncertainty". Therefore, also this measure accounts for both conflict and non-specificity. Lamata and Moral [23] defined their upper entropy measure as

$$\begin{aligned}\mathsf{U}_{\mathrm{LMu}} = &-\sum_{A\in\mathcal{F}} m(A)\sup\left\{\log_2 \mathrm{Pl}(x_i) \mid x_i\in A\right\}\\ &+\log_2\left(\sum_{A\in\mathcal{F}} m(A)|A|\right).\end{aligned} \tag{U11}$$

The influence of three key measures can be observed in this formulation, two of which stem from studies conducted by Dubois and Prade that are: *Dubois and Prade's imprecision measure* (U9) and *Dubois and Prade's index of fuzziness* (U7). In the first part, the influence of *Yager's dissonance measure* (U3) can be seen.

Klir and Ramer's discord measure: Klir and Ramer [24] introduced the discord measure

$$\mathsf{U}_{\mathrm{KR}} = -\sum_{A\in\mathcal{F}} m(A)\log_2\left(\sum_{B\in\mathcal{F}} m(B)\frac{|A\cap B|}{|B|}\right) \tag{U12}$$

an uncertainty measure designed to quantify conflict in evidence theory. This measure builds directly upon *Shannon entropy*.

Klir and Ramer's total uncertainty measure: Alongside their study on the *discord measure*, Klir and Ramer [24] also introduced the total uncertainty measure

$$\begin{aligned}\mathsf{U}_{\mathrm{KRt}} = &\overbrace{\sum_{A\in\mathcal{F}} m(A)\log_2(|A|)}^{\mathsf{U}_{\mathrm{DPu}}}\\ &\underbrace{-\sum_{A\in\mathcal{F}} m(A)\log_2\left(\sum_{B\in\mathcal{F}} m(B)\frac{|A\cap B|}{|B|}\right)}_{\mathsf{U}_{\mathrm{KR}}}\end{aligned} \tag{U13}$$

as a combination of *Dubois and Prade's uncertainty measure* (U6) and *Klir and Ramer's discord measure* (U12), and thus directly derives from them.

Inuiguchi et al.'s uncertainty measure: Inuiguchi et al. [25] proposed the following direct generalization of *Shannon entropy*

$$\mathsf{U}_{\mathrm{HK}} = \max_{p\in\mathcal{P}_m}\left\{-\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)\right\} \tag{U14}$$

where $\mathcal{P}_m$ is the credal set associated with m. This measure was later on popularized by Harmanec and Klir [26].

Klir and Parviz's uncertainty measure: Klir and Parviz [27] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{KP}} = -\sum_{A\in\mathcal{F}} m(A)\log_2\left(\sum_{B\in\mathcal{F}} m(B)\frac{|A\cap B|}{|A|}\right). \tag{U15}$$

This measure was introduced to reconsider *Klir and Ramer's discord measure* (U12), maintaining its entropy-based framework while modifying its normalization to account for the degree of specificity in mass assignments.

Pal et al.'s uncertainty measure: Pal et al. [28] introduced an uncertainty measure that integrates conflict and non-specificity. This measure

$$\mathsf{U}_{\mathrm{PBH}} = \underbrace{-\sum_{A\in\mathcal{F}} m(A)\log_2\left(\frac{m(A)}{|A|}\right)}_{\mathsf{U}_{\mathrm{DPu}}+\mathsf{U}_{N}} \tag{U16}$$

builds upon previous work by explicitly combining *Dubois and Prade's uncertainty measure* (U6), which quantifies non-specificity, and *Nguyen's uncertainty measure* (U5), which accounts for conflict.

Maeda and Hichihashi's uncertainty measure: Maeda and Ichihashi [29] introduced an uncertainty measure formulated as a "global uncertainty measure." In their study, they considered

$$\begin{aligned}\mathsf{U}_{\mathrm{MH}} = &\overbrace{\max_{p\in\mathcal{P}_m}\left\{-\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)\right\}}^{\mathsf{U}_{\mathrm{HK}}}\\ &+\underbrace{\sum_{A\in\mathcal{F}} m(A)\log_2(|A|)}_{\mathsf{U}_{\mathrm{DPu}}}\end{aligned} \tag{U17}$$

that is the sum of two existing measures: Inuiguchi's measure (1991), which, as previously outlined, corresponds to *Inuiguchi et al.'s uncertainty measure* (U14), and *Dubois and Prade's uncertainty measure* (U6).

George and Pal's uncertainty measure: A new conflict measure was introduced by George and Pal [17]. Unlike previous uncertainty measures, which were either derived from *Shannon entropy* or *Hartley entropy*, their measure

$$\mathsf{U}_{\mathrm{GP}} = \sum_{A\in\mathcal{F}}\sum_{B\in\mathcal{F}} m(A)m(B)\left(1-\frac{|A\cap B|}{|A\cup B|}\right) \tag{U18}$$

represents a distinct approach to quantifying conflict.

Maluf's uncertainty measure: Maluf [30] introduced an uncertainty measure based on the concept of *dissonance*. Maluf's uncertainty measure was defined as

$$\mathsf{U}_{M} = -\sum_{A\subseteq X} \mathrm{Pl}(A)\log_2 \mathrm{Bel}(A) \tag{U19}$$

Maluf [30] described his measure as a "symmetrical function to the dissonance measure," specifically referring to *Yager's dissonance measure* (U3).

Klir's Shannon-like measure: Klir [31] proposed another uncertainty measure aimed at extending Shannon entropy to the Dempster–Shafer framework. This measure was defined as

$$\mathsf{U}_{\mathrm{Ksl}} = -\frac{1}{c}\sum_{x_i\in X}\left(\mathrm{Bel}(x_i)\log_2 \mathrm{Bel}(x_i) + \mathrm{Pl}(x_i)\log_2 \mathrm{Pl}(x_i)\right) \tag{U20}$$

where the term c is defined as $c = \sum_{x_i\in X}(\mathrm{Bel}(x_i) + \mathrm{Pl}(x_i))$.

Yager's Shapley entropy measure: Yager [32] introduced the *Shapley entropy measure*, an uncertainty measure that incorporates concepts from cooperative game theory, particularly *Shapley values*, to quantify the contribution of each element in the frame of discernment

$$\mathsf{U}_{\mathrm{Yse}} = -\sum_{x_i\in X}\left[\left(\sum_{\substack{A\in\mathcal{F}\\ x_i\in A}}\frac{m(A)}{|A|}\right)\ln\left(\sum_{\substack{A\in\mathcal{F}\\ x_i\in A}}\frac{m(A)}{|A|}\right)\right]. \tag{U21}$$

The measure derives from *Shannon entropy* and *Yager's non-specificity measure* (U4).

Jousselme et al.'s uncertainty measure: Jousselme et al. [33] presented a study on an uncertainty measure that was originally introduced by Dezert et al. [34]. However, this measure, i.e.,

$$\mathsf{U}_J = -\sum_{x_i\in X}\mathrm{Bet}(x_i)\log_2 \mathrm{Bet}(x_i) \tag{U22}$$

is commonly referred to in the literature as *Jousselme* et al.*'s uncertainty measure*. This measure differs from *Yager's Shapley entropy measure* (U21) only in the choice of the logarithm's base, making it a closely related extension of Yager's approach.

Yang et al.'s non-specificity measure: Yang et al. [35] proposed the following non-specificity measure

$$\mathsf{U}_{\mathrm{YHD}} = \sum_{x_i\in X}(\mathrm{Pl}(x_i) - \mathrm{Bel}(x_i)) \tag{U23}$$

which can be seen as the first attempt to use belief intervals to quantify uncertainty. Moreover, they showed that it can be rewritten as

$$\mathsf{U}_{\mathrm{YHD}} = \sum_{\substack{A\subseteq X\\ |A|\geq 2}} m(A)\frac{|A|}{|X|}.$$

Yang and Han's distance-based uncertainty measure: A new concept of uncertainty measure was introduced in 2016 by Yang and Han [36], who proposed the distance-based total uncertainty measure

$$\mathsf{U}_{\mathrm{YH}} = 1 - \frac{1}{|X|}\sqrt{3}\sum_{x_i\in X} d^I\left([\mathrm{Bel}(x_i), \mathrm{Pl}(x_i)], [0,1]\right) \tag{U24}$$

where d^I is defined as a distance between the two intervals.

Deng entropy: Deng [37] introduced a new total uncertainty measure known as *Deng entropy*

$$\mathsf{U}_D = -\sum_{A\in\mathcal{F}} m(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}\right). \tag{U25}$$

This measure builds upon *Pal et al.'s uncertainty measure* (U16).

Wang and Song's uncertainty measure: Wang and Song [38] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{WS}} = \sum_{x_i\in X}\left[-\frac{\mathrm{Bel}(x_i)+\mathrm{Pl}(x_i)}{2}\log_2\left(\frac{\mathrm{Bel}(x_i)+\mathrm{Pl}(x_i)}{2}\right) + \frac{\mathrm{Pl}(x_i)-\mathrm{Bel}(x_i)}{2}\right] \tag{U26}$$

that builds upon *Yang* et al.*'s non-specificity measure* (U23) and *Klir's Shannon-like entropy* (U20).

Zhou et al.'s belief entropy: Zhou et al. [39] introduced the measure

$$\mathsf{U}_{\mathrm{ZTJ}} = -\sum_{A\in\mathcal{F}} m(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right) \tag{U27}$$

by modifying *Deng entropy* (U25) to incorporate an exponential factor.

Tang et al.'s weighted belief entropy: Another total uncertainty measure was introduced by Tang et al. [40]. This measure

$$\mathsf{U}_T = -\sum_{A\in\mathcal{F}}\frac{|A|}{|X|}m(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}\right) \tag{U28}$$

also builds upon *Yang et al.'s non-specificity measure* (U23) and *Deng entropy* (U25).

Deng improved distance-based uncertainty measure: Deng et al. [41] considered (U24) and proposed the belief interval-based measure

$$\mathsf{U}_{\mathrm{DXD}} = \sum_{i=1}^{n}\left(1 - d_E([\mathrm{Bel}(x_i), \mathrm{Pl}(x_i)], [0,1])\right) \tag{U29}$$

where d_E is the Euclidean distance between two intervals.

Pan and Deng's new belief entropy: Pan and Deng [42] introduced the total uncertainty measure

$$\mathsf{U}_{\mathrm{PD}} = -\sum_{A\in\mathcal{F}}\frac{\mathrm{Bel}(A)+\mathrm{Pl}(A)}{2}\log_2\left(\frac{\mathrm{Bel}(A)+\mathrm{Pl}(A)}{2(2^{|A|}-1)}\right). \tag{U30}$$

that builds upon *Deng entropy* (U25) and *Wang and Song's uncertainty measure* (U26).

Jiroušek and Shenoy's uncertainty measure: Jiroušek and Shenoy [43] introduced a new total uncertainty measure as the sum of two distinct terms:

$$\mathsf{U}_{\mathrm{JS}} = \underbrace{\sum_{x_i\in X}\mathrm{Pl}_T(x_i)\log_2\left(\frac{1}{\mathrm{Pl}_T(x_i)}\right)}_{\mathsf{H}_s(\mathrm{Pl}_T)} + \underbrace{\sum_{A\in\mathcal{F}} m(A)\log_2(|A|)}_{\mathsf{U}_{\mathrm{DPu}}}. \tag{U31}$$

The first term takes inspiration from *Jousselme* et al.*'s uncertainty measure* (U22) but, in this case, employs the plausibility transformation to account for conflict. The second term corresponds to *Dubois and Prade's uncertainty measure* (U6), which quantifies non-specificity.

Jiroušek and Shenoy's decomposable entropy measure: Jiroušek and Shenoy [44], [45] introduced another uncertainty measure

$$\mathsf{U}_{\mathrm{JSq}} = \sum_{A\subseteq X} (-1)^{|A|} Q(A) \log_2 Q(A) \tag{U32}$$

based on the commonality function, making it conceptually aligned with *Smets' commonality-based entropy measure* (U2).

Mambé uncertainty measure: Mambé et al. [46] introduced the total uncertainty measure

$$\mathsf{U}_{\mathrm{Mb}} = -\sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|}-1}\, e^{\frac{|A|-1}{2^{|X|}}} \right) \tag{U33}$$

by modifying *Zhou* et al.*'s belief entropy* (U27).

Cui et al.'s uncertainty measure: Cui et al. [47] took inspiration from *Zhou* et al.*'s belief entropy* (U27) and formalized the uncertainty measure

$$\mathsf{U}_C = -\sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|}-1} \exp\left(\sum_{\substack{B\in\mathcal{F}\\ B\neq A}} \frac{|A\cap B|}{2^{|X|}-1} \right) \right). \tag{U34}$$

Li et al.'s uncertainty measure: Li et al. [48] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{LGD}} = \sum_{A\in\mathcal{F}} \left(\frac{m(A)}{2^{|A|}-1} \right)^2 \tag{U35}$$

as a generalized expression for information quality within the Dempster–Shafer framework. Their approach is directly connected to *Deng entropy* (U25).

Pan et al.'s uncertainty measure: Pan et al. [49] introduced a TU measure, conceptually linked to *Jiroušek and Shenoy's uncertainty measure* (U31) and, like the latter, it is composed of two distinct terms:

$$\begin{aligned}\mathsf{U}_P = &\sum_{A\subseteq X} m(A) \log_2 \left(\frac{1}{\sum_{x_i\in A} \mathrm{Pl}_T(x_i)} \right)\\ &+ \underbrace{\sum_{A\in\mathcal{F}} m(A) \log_2(|A|)}_{\mathsf{U}_{\mathrm{DPu}}}. \end{aligned}\tag{U36}$$

Chen et al.'s improved belief entropy: Chen et al. [50] introduced a new measure that builds upon *Deng entropy* (U25). Their measure is defined as

$$\mathsf{U}_{\mathrm{CDS}} = -\sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|}-1} \frac{|A|}{|\cup\mathcal{F}|} \right). \tag{U37}$$

Gao et al.'s Tsallis entropy: Gao et al. [51] proposed the following extension of Tsallis entropy that considers the same denominator of *Deng entropy*

$$\mathsf{U}_G = \sum_{A\subseteq X} \frac{\left(2^{|A|}-1\right) m(A) \left(1 - \left(\frac{m(A)}{2^{|A|}-1}\right)^{|A|-1}\right)}{|A|-1}. \tag{U38}$$

Zhao et al.'s improved belief entropy: Zhao et al. [52] presented the uncertainty measure

$$\begin{aligned}\mathsf{U}_Z = &-\sum_{x_i\in X} \frac{\mathrm{Bel}(x_i)+\mathrm{Pl}(x_i)}{2} \log_2 \Bigg(\frac{\mathrm{Bel}(x_i)+\mathrm{Pl}(x_i)}{2}\\ &\times e^{-(\mathrm{Pl}(x_i)-\mathrm{Bel}(x_i))} \Bigg)\\ &- \sum_{\substack{A\in\mathcal{F}\\ |A|\geq 2}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|}-1} e^{-(\mathrm{Pl}(A)-\mathrm{Bel}(A))} \right)\end{aligned} \tag{U39}$$

refines uncertainty quantification by leveraging *Deng entropy* and incorporating the belief interval. The first component of the measure is similar to *Pan and Deng's new belief entropy* (U30) and the second component to *Deng entropy* (U25). The argument of the exponent is linked to *Yang* et al.*'s non-specificity measure* (U23).

Buono and Longobardi's Deng Extropy: Buono and Longobardi [53] considered the concept of eXtropy and merged it with *Deng eXtropy* obtaining the following measure of uncertainty

$$\mathsf{U}_{\mathrm{BL}} = \sum_{\substack{A\subset X\\ m(A)>0}} (1-m(A)) \log_2 \left(\frac{1-m(A)}{2^{|\bar{A}|}-1} \right). \tag{U40}$$

Yan and Deng's uncertainty measure: Yan and Deng [54] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{YD}} = -\sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{m(A)+\mathrm{Bel}(A)}{2(2^{|A|}-1)} \right) e^{\frac{|A|-1}{|\cup\mathcal{F}|}} \tag{U41}$$

conceptually linked to *Zhou* et al.*'s belief entropy* (U27) and *Pan and Deng's new belief entropy* (U30).

Qin et al.'s uncertainty measure: Qin et al. [55] introduced the uncertainty measure

$$\mathsf{U}_Q = \sum_{A\in\mathcal{F}} \frac{|A|}{|X|} m(A) \log_2(|A|) + \sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{1}{m(A)} \right). \tag{U42}$$

The first term uses *Dubois and Prade's uncertainty measure*, whereas the second term employs *Nguyen's uncertainty measure*. The scaling factor is, instead, borrowed from *Yang* et al.*'s non-specificity measure* (U23).

Li and Pan's new belief entropy: Li and Pan [56] introduced a similar uncertainty measure

$$\mathsf{U}_{\mathrm{LP}} = \sum_{A\in\mathcal{F}} |X| m(A) \log_2(|A|) + \sum_{A\in\mathcal{F}} m(A) \log_2 \left(\frac{1}{m(A)} \right) \tag{U43}$$

that uses *Dubois and Prade's uncertainty measure* and *Nguyen's uncertainty measure* as well as a normalization factor similar to *Tang* et al.*'s weighted belief entropy*.

Wen et al.'s uncertainty measure: Wen et al. [57] introduced the uncertainty measure

$$\mathsf{U}_W = \frac{e - \sum_{x_i \in X} m(x_i) e^{m(x_i)} - \sum_{\substack{A \in \mathcal{F} \\ A \neq \{x_i\}}} \frac{m(A)}{|A| \cdot |\cup \mathcal{F}|} e^{\frac{m(A)}{|A| \cdot |\cup \mathcal{F}|}}}{e - \frac{1}{|X|} e^{\frac{1}{|X|^2}}} \tag{U44}$$

that is, in their words, "completely different from the existing measures".

Chen et al.'s weighted belief entropy: Chen et al. [58] introduced a modification of *Tang* et al.*'s weighted belief entropy* (U42) in the form

$$\mathsf{U}_{\mathrm{CTL}} = -\sum_{A \subseteq X} \frac{|A|}{|X|} \overline{m}(A) \log_2 \frac{\overline{m}(A)}{2^{|A|} - 1} \tag{U45}$$

where $\overline{m}$ is the negation of m and it is defined for all $A \in \mathcal{F}$ as

$$\overline{m} = \frac{1 - m(A)}{|\mathcal{F}| - 1}.$$

Li and Cai's uncertainty measure: Li and Cai [59] introduced an uncertainty measure that builds upon *Li* et al.*'s uncertainty measure* (U35)

$$\mathsf{U}_{\mathrm{LC}} = \sum_{A \in \mathcal{F}} \left(\frac{m(A)}{2^{|A|} - 1} \right)^2 e^{\left(\sum_{\substack{B \in \mathcal{F} \\ B \neq A}} \frac{|A \cap B|}{|X|} \right)}. \tag{U46}$$

Chen and Luo's uncertainty measures: Chen and Luo [60] proposed the following extension of Renyi entropy that considers the same denominator of *Deng entropy*

$$\mathsf{U}_{\mathrm{CL}} = \frac{n \log_2 \sum_{A \subseteq X} \left(\frac{m(A)}{2^n - 1} \right) \left(1 + \left(\frac{m(A)}{2^n - 1} \right)^{n-1} - \frac{1}{2^n - 1} \right)}{1 - n}. \tag{U47}$$

Deng and Wang's uncertainty measure: Deng and Wang [61] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{DW}} = \sum_{x_i \in X} \left(1 - \sqrt{\left(\sqrt{\mathrm{Bel}(x_i)} \right)^2 + \left(1 - \sqrt{\mathrm{Pl}(x_i)} \right)^2} \right) \tag{U48}$$

that derives from *Yang and Han's distance-based uncertainty measure* (U24).

Zhang et al.'s uncertainty measure: Zhang et al. [62] proposed a measure designed to overcome the limitations of *Yang and Han's uncertainty measure* (U24)

$$\mathsf{U}_{\mathrm{ZLT}} = \sum_{x_i \in X} \frac{4}{3} \left(\frac{1}{(1 + d(x_i))^2} - \frac{1}{4} \right) \tag{U49}$$

where the distance d is calculated as follows:

$$d(x_i) = \sqrt{[\mathrm{Bel}(x_i) - 0]^2 + [\mathrm{Pl}(x_i) - 1]^2}.$$

Zhang et al.'s entropy: Zhang et al. [63] proposed the entropy measure

$$\mathsf{U}_{\mathrm{Ze}} = \sum_{\substack{A \subseteq X \\ \mathrm{Pl}(X) > 0}} m(A) \log_2 \frac{2^{f(|A|)}}{\mathrm{Pl}(A)} \tag{U50}$$

where f is an increasing function with $f(1) = 0$. This can be seen as a modification of (U10).

Li et al.'s uncertainty measure: Li et al. [64] introduced their measure

$$\mathsf{U}_{\mathrm{LCL}} = \sum_{x_i \in X} \left(\frac{2}{1 + d(x_i)} - 1 \right) \tag{U51}$$

to extend *Zhang* et al.*'s uncertainty measure* (U49).

Zhou and Deng's fractal-based entropy: Zhou and Deng [65] introduced a measure based on a probability transformation inspired by fractal theory and by *Deng entropy* (U25). Their fractal-based entropy was defined as

$$\mathsf{U}_{\mathrm{FB}} = \sum_{\substack{A \subseteq X \\ m_F(A) > 0}} m_F(A) \log_2 m_F(A) \tag{U52}$$

where $m_F(A)$ is calculated as follows:

$$m_F(A) = -\sum_{B | A \subseteq B} \frac{m(B)}{2^{|B|} - 1}.$$

Zhou et al.'s generalized belief entropy: Zhou et al. [66] introduced the uncertainty measure that extends the foundational ideas present in *Deng entropy* (U25) and *George and Pal's conflict measure* (U18), (U53), as shown at the bottom of the next page.

Xue and Deng's decomposable Deng entropy: Xue and Deng [67] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{XD}} = \sum_{\substack{A \subseteq X \\ Q(A) > 0}} (-1)^{|A|} Q(A) \log_2 \left(\frac{Q(A)}{2^{|A|} - 1} \right) \tag{U54}$$

that builds upon both *Deng entropy* (U25) and *Jiroušek and Shenoy's decomposable entropy measure* (U32).

Dezert Tchamova's extended bet entropy: Dezert and Tchamova [8] introduced the following uncertainty measure by adding *Jousselme* et al.*'s uncertainty measure* (U22) to *Dubois and Prade's uncertainty measure* (U6)

$$\mathsf{U}_{\mathrm{DT}} = \underbrace{-\sum_{x_i \in X} \mathrm{Bet}(x_i) \log_2 \mathrm{Bet}(x_i)}_{\mathsf{U}_J} + \underbrace{\sum_{A \in \mathcal{F}} m(A) \log_2(|A|)}_{\mathsf{U}_{\mathrm{DPu}}}. \tag{U55}$$

Dutta and Shome's uncertainty measure: Dutta and Shome [68] proposed

$$\mathsf{U}_{\mathrm{DS}} = -\sum_{A \in \mathcal{F}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|} - 1} \cdot \frac{|A|}{|\cup \mathcal{F}|} \cdot \frac{\exp\left(1 + \frac{|\cap \mathcal{F}|}{|\cup \mathcal{F}|} \right)}{|\cup \mathcal{F}|} \right) \tag{U56}$$

taking inspiration from *Chen* et al.*'s improved belief entropy* (U37).

Chen and Deng's new belief entropy: Chen and Deng [69] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{CD}} = -\sum_{A\in\mathcal{F}} m(A)\log_2\left(\frac{m(A)}{\sum_{k=1}^{|A|}\binom{|A|}{k}(2^k-1)}\right) \tag{U57}$$

by changing the denominator of the argument of $\log_2$ in *Deng entropy* (U25).

Cui and Deng's plausibility entropy: Cui and Deng [70] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{CDn}} = -\sum_{x_i\in X}\mathrm{Pl}(x_i)\log_2\mathrm{Pl}_T(x_i). \tag{U58}$$

Such a measure shares similarities with the measures introduced after *Höhle's confusion measure* (U1).

Kavya et al.'s uncertainty measure: Kavya et al. [71] introduced the uncertainty measure

$$\mathsf{U}_{\mathrm{KCP}} = \sum_{x_i\in X}\frac{-\mathrm{Pl}(x_i)\log_2\mathrm{Pl}(x_i)}{e^{\mathrm{Pl}(x_i)-\mathrm{Bel}(x_i)}} + (\mathrm{Pl}(x_i)-\mathrm{Bel}(x_i)) \tag{U59}$$

that is linked to *Zhao* et al.*'s uncertainty measure* (U39) through the use of a similar exponential factor. The second term employs *Yang* et al.*'s non-specificity measure* (U23).

Zhou and Deng's belief entropy: Zhou and Deng [72] introduced the uncertainty measure

$$\begin{aligned}\mathsf{U}_{\mathrm{ZD}} = -&\sum_{\substack{x_i\in X\\ \mathrm{Pl}(x_i)>0}}\left(\frac{\mathrm{Pl}(x_i)+\mathrm{Bel}(x_i)}{\sum_{x_j\subseteq X}\mathrm{Pl}(x_j)+\mathrm{Bel}(x_j)}\log_2\right.\\ &\left.\times\frac{\mathrm{Pl}(x_i)+\mathrm{Bel}(x_i)}{\sum_{x_j\subseteq X}\mathrm{Pl}(x_j)+\mathrm{Bel}(x_j)}\right)\\ &+\underbrace{\sum_{\substack{x_i\in X\\ \mathrm{Pl}(x_i)>0}}(\mathrm{Pl}(x_i)-\mathrm{Bel}(x_i))}_{\mathsf{U}_{\mathrm{YHD}}}\end{aligned} \tag{U60}$$

the first term builds upon *Wang and Song's uncertainty measure* (U26), whereas the second term employs *Yang* et al.*'s non-specificity measure* (U23).

Deng et al.'s plausibility extropy: Deng et al. [73] introduced a new total uncertainty measure that is conceptually linked to *Cui and Deng's plausibility entropy* (U58)

$$\begin{aligned}\mathsf{U}_{\mathrm{Dpe}} = -\sum_{x_i\in X}&\left[\left(\sum_{x_j\in X}\mathrm{Pl}(x_j)-\mathrm{Pl}(x_i)\right)\log_2\right.\\ &\left.\times\frac{\sum_{x_j\in X}\mathrm{Pl}(x_j)-\mathrm{Pl}(x_i)}{\sum_{x_j\in X}\mathrm{Pl}(x_j)}\right].\end{aligned} \tag{U61}$$

Zhang et al.'s belief interval Euclidean distance entropy: Zhang et al. [74] introduced a measure that builds upon *Deng entropy* (U25) and integrates a Euclidean distance-based correction term, inspired by *Deng improved distance-based uncertainty measure* (U29). Their uncertainty measure was defined as

$$\mathsf{U}_{\mathrm{ZCC}} = -\sum_{A\in\mathcal{F}} m(A)\log_2\frac{d_M(A)}{2^{|A|}-1} \tag{U62}$$

where

$$d_M(A) = \begin{cases}\displaystyle\sum_{\substack{B\subseteq A\\ B\in\mathcal{F}}}\frac{\left[(\mathrm{Bel}(B)-0)^2+(\mathrm{Pl}(B)-1)^2\right]-1}{2(m(B)-1)}, & m(A)<1\\ 1, & m(A)=1.\end{cases}$$

Zhou et al.'s information volume-based entropy: Zhou et al. [75] considered the novel concept of information volume of evidential information and proposed the uncertainty measure

$$\mathsf{U}_{\mathrm{ZLD}} = \sqrt{\mathrm{IV}(m)\cdot\left(-\sum_{i=1}^{n}\mathrm{Pl}_T(x_i)\log_2\mathrm{Pl}_T(x_i)\right)} \tag{U63}$$

where

$$\mathrm{IV}(m) = \lim_{p\to\infty}\left(\sum_{t=0}^{p}\sum_{A\in\mathcal{F}} m^{(t)}(A)\log_2\frac{m(A)}{2^{|A|}-1} + \sum_{\substack{A\in\mathcal{F}\\ |A|>1}} m^{(n)}(A)\right).$$

and $m^{(n)}$ is defined as suggested by Kang and Deng [76]. This measure contains the normalization factor of *Deng entropy* (U25) and the same treatment of the plausibility transformation proposed by Jiroušek and Shenoy (U31).

Barhoumi et al.'s Entropy Measure: Barhoumi et al. [6] defined their uncertainty measure as

$$\mathsf{U}_B = -\sum_{x_i\in X}\mathrm{RBetP}_m(x_i)\log_2\mathrm{RBetP}_m(x_i) \tag{U64}$$

where the exponentially weighted pignistic transformation $\mathrm{RBetP}_m(x_i)$ is given by

$$\mathrm{RBetP}_m(x_i) = \sum_{\substack{A\subseteq X\\ x_i\in A}}\frac{m(A)}{|A|}\cdot e^{\left(1-2^{\frac{|X|-|A|}{|A|}}\right)}.$$

$$\mathsf{U}_{\mathrm{Zbe}} = \begin{cases}\displaystyle\frac{1}{|\mathcal{F}|-1}\sum_{\substack{A_i,A_j\in\mathcal{F}\\ i<j}}\left(1-\frac{|A_i\cap A_j|}{|A_i\cup A_j|}\right)\left[-m(A_i)\log_2\frac{m(A_i)}{2^{|A_i|}-1}-m(A_j)\log_2\frac{m(A_j)}{2^{|A_j|}-1}\right], & |\mathcal{F}|\geq 2\\ \displaystyle m(A_j)\log_2\frac{m(A_j)}{2^{|A_j|}-1} & |\mathcal{F}|=1\end{cases} \tag{U53}$$

This measure was introduced to enhance the uncertainty measure (U22) proposed by Jousselme et al. [33].

Su al.'s Entropy Measure: Su et al. [77] introduced the total uncertainty measure

$$\mathsf{U}_{\mathrm{SZ}} = -\sum_{A\in\mathcal{F}} m(A)\log_2 \frac{\sum_{B\in\mathcal{F}} m(B)\cdot D(A,B)}{2^{|A|}-1} \tag{U65}$$

where D is a similarity measure between the two sets, e.g., the Jaccard index. It resembles Deng's entropy (U25), but the use of a similarity between two sets was already used in (U18).

A. On Their Applications

It is hardly possible to provide an exhaustive presentation of the applications of uncertainty measures. Deng [78] analyzed his entropy measure (U25)—by far the most used in the literature—and discussed a number of fields of application. The four most relevant ones, at least in terms of publications, were multisensor information fusion, fault diagnosis, failure mode and effects analysis, and decision making. It may be worth discussing some of their applications that go beyond the mere quantification of information content and may show how uncertainty measures can be operationalized.

A number of aggregation rules have been proposed in the literature as alternatives to the Dempster rule of combination [79], and some of them allow for the use of weights to account for the relative importance of the sources of information. Many uncertainty measures, e.g., (U25), (U27), (U28), (U41), (U42), (U48), (U51), (U56), have been used for this purpose.

Uncertainty measures have also been used to quantify the informativeness of preferential information provided by experts in decision-making processes. For example, Liang et al. [7] considered an extension of the Best-Worst method, where uncertainty preferences were expressed by means of mass assignments. In this context, in this context, an uncertainty measure—in this case (U14)—was paired with a measure of internal consistency of preferences to obtain an aggregate measure of global reliability of preferences. Revision of preferences was suggested when such a global measure exceeded a given threshold.

As discussed by Klir [80], depending on the definition of the problem, it may be the case that uncertainty shall be minimized or maximized, according to the appropriate "principle of uncertainty." An example is the sequential acquisition of evidence, which may be relevant in the medical field where alternative tests may be available, and for which uncertainty maximization and uncertainty minimization strategies were proposed [81] as leading principles for the selection of new focal elements.

B. Formal Properties

The quest for meaningful and mathematically sound uncertainty measures has generated a literature on the analysis of their formal properties. There is no consensus on which properties should be adopted, but it is safe to say that the following five are among the most widely accepted and studied ones [82].[2]

Property 1 (Probabilistic Consistency): If all focal elements are singletons, then an uncertainty measure should collapse into Shannon entropy, $\mathsf{U}(m) = \sum_{x\in X} m(x)\log_2 m(x)$;

Property 2 (Set Consistency): When a set $A \subseteq X$ exists such that $m(A) = 1$, then an uncertainty measure should collapse into the Hartley measure $\mathsf{U}(m) = \log_2 |A|$;

Property 3 (Subadditivity): Let m be a BPA on $X \times Y$ and m_X and m_Y its marginal BPAs on X and Y, then $\mathsf{U}(m) \leq \mathsf{U}(m_X) + \mathsf{U}(m_Y)$;

Property 4 (Additivity): Let m be a BPA on $X \times Y$, and m_X and m_Y its marginal BPAs on X and Y. If the marginals are not interacting ($m(A \times B) = m_X(A)m_Y(B)$ with $A \subseteq X$, $B \subseteq Y$ and $m(C) = 0$ if $C \neq A \times B$), then $\mathsf{U}(m) = \mathsf{U}(m_X) + \mathsf{U}(m_Y)$.

Property 5 (Monotonicity): An uncertainty measure U is *monotone* if, for $n \geq 2$

$$\mathrm{Bel}_1 \leq \mathrm{Bel}_2 \Rightarrow \mathsf{U}(m_1) \geq \mathsf{U}(m_2) \quad \forall \mathrm{Bel}_1, \mathrm{Bel}_2 \in \mathcal{B}_n \tag{6}$$

where $\mathcal{B}_n$ is the set of all belief measures on a set X with $|X| = n$.

Other properties are compliance with the Dempster–Shafer theory, range, monotonicity with respect to the cardinality of the frame of discernment, and strict additivity [43]. In particular, compliance with Dempster–Shafer theory refers to the compatibility of the measure with the Dempster rule of combination, and range refers to the fact that the measure should have the same range as Shannon entropy in probability theory. Some other qualitative characteristics have been discussed in the literature [83]: *complexity* as the existence of efficient algorithms to compute the measure; *separability* as the possibility to split the measure into the contributions of conflict and non-specificity; *sensitivity* as the capacity of the measure to be sensitive to changes in the mass assignment; *extensibility* as the capacity of the measure to be extended to generalizations of evidence theory. On the subject of computational complexity, let us add that, if we exclude truly extreme cases, today even the most complex uncertainty measure, (U14), can be efficiently computed [84].

Table I reports an analysis of Properties 1–5 for some uncertainty measures.

For a large number of uncertainty measures—e.g., (U34), (U35), and (U44)—the original contributions did not discuss or check their formal properties, and currently it is hard to know anything about them other than what transpires from their formulation. For some other measures, properties were allegedly validated using some examples, but this does not constitute proof, and it is still unsure whether they satisfy those properties. At present, the analysis of the formal properties of uncertainty measures, especially with respect to the more recent ones, appears largely incomplete.

[2]Consider that, in spite of their importance, the desirability of even these basic properties have been questioned. For example, Deng and Wang [61] argued that "it makes no sense to discuss the probability consistency and the set consistency" and that an uncertainty measures "does not require to satisfy the additivity and subadditivity."

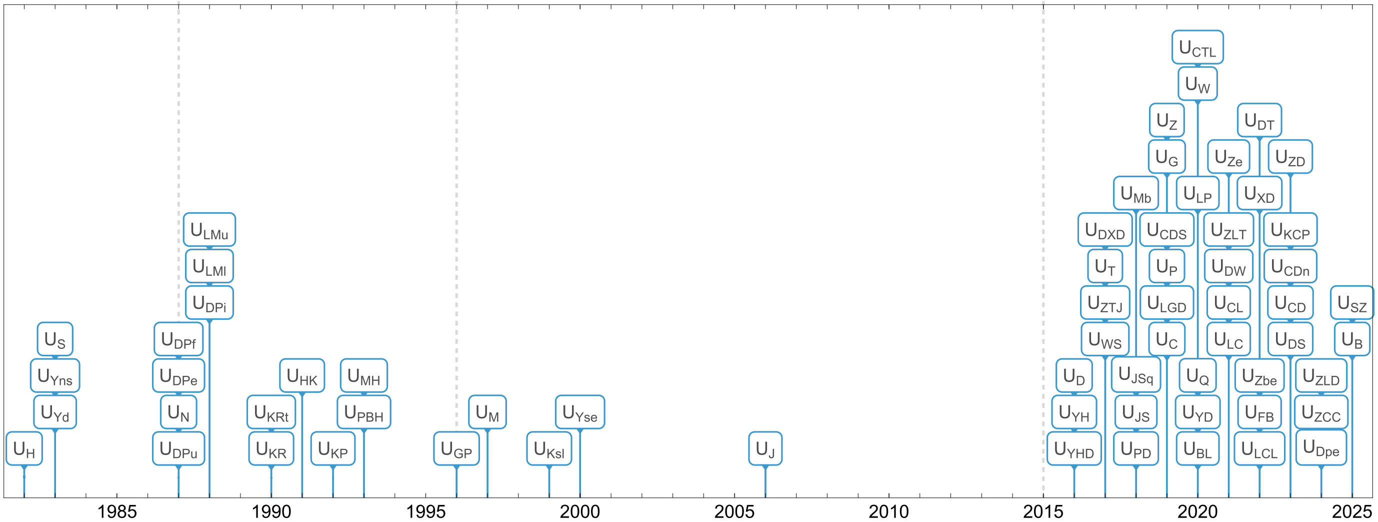


Fig. 1. Timeline of uncertainty measures in evidence theory. Extension of the plot by Urbani et al. [9].

TABLE I

ANALYSIS OF FORMAL PROPERTIES OF SOME UNCERTAINTY MEASURES. ✓: SATISFIED; ✗: NOT SATISFIED; —: UNKNOWN OR INSUFFICIENT EVIDENCE; [REF.]: REFERENCE(S) TO THE EVIDENCE SUPPORTING THE CLAIM OR TO A MANUSCRIPT REPORTING THIS RESULT

Measure	(eq.)	P1	P2	P3	P4	P5	[Ref.]
U_{H}	(U1)	✓	✗	✓	✗	✗	[49, 10]
U_{Yd}	(U2)	✓	✗	✓	✗	✗	[49, 10]
U_{S}	(U3)	✗	✗	✗	✗	✗	[49, 10]
U_{N}	(U5)	✓	✗	✓	✗	✗	[49, 10]
U_{DPu}	(U6)	✗	✓	✓	✓	✓	[49, 10]
U_{LMl}	(U10)	✓	✓	✓	✗	✗	[49, 10]
U_{KRt}	(U13)	✓	✓	✓	✗	✗	[10]
U_{HK}	(U14)	✓	✓	✓	✓	✓	[26, 10]
U_{KP}	(U15)	✓	✓	✓	✗	✗	[49, 10]
U_{PBH}	(U16)	✓	✓	✓	✗	✗	[49, 10]
U_{MH}	(U17)	✓	✗	✓	✓	✓	[82]
U_{GP}	(U18)	✗	✗	✗	✗	✗	[10]
U_{M}	(U19)	✗	—	✗	✗	✗	[10]
U_{J}	(U22)	✓	✓	✓	✗	✗	[10]
U_{YHD}	(U23)	✗	✗	✗	✗	✓	[35, 85]
U_{YH}	(U24)	✗	✓	✗	✗	✗	[83]
U_{D}	(U25)	✓	✗	✗	✗	✗	[86]
U_{WS}	(U26)	✓	✓	✗	✗	✗	[83]
U_{ZTJ}	(U27)	✓	✗	✗	✗	✗	[85]
U_{DXD}	(U29)	✗	✓	✗	✗	✓	[83]
U_{PD}	(U30)	✓	✗	✗	✗	—	[6]
U_{JS}	(U31)	✓	✗	✓	✗	✗	[43, 10, 87]
U_{C}	(U34)	✓	✗	✗	✗	✗	[85]
U_{P}	(U36)	✓	✓	✓	✗	—	[49]
U_{CDS}	(U37)	✗	✗	✗	✗	✗	[6]
U_{G}	(U38)	✓	✗	✓	✗	—	[6]
U_{Z}	(U39)	✓	✗	✗	✗	✓	[87]
U_{YD}	(U41)	✗	—	✗	✗	✗	[10]
U_{Q}	(U42)	✓	✓	✗	✗	✗	[55, 10]
U_{LP}	(U43)	✓	—	✗	✗	—	[56]
U_{W}	(U44)	✗	✗	✗	✗	—	[6]
U_{DW}	(U48)	—	—	—	—	✓	[61]
U_{LCL}	(U51)	✗	—	✗	✗	—	[64]
U_{KCP}	(U59)	✓	✗	✗	✗	—	[71]

IV. GENEALOGY

We are ready to present two key visual representations in the form of a timeline and a Hasse diagram to provide a comprehensive overview of the evolution of, and the interconnections between, different uncertainty measures.

A. Updated Timeline

First of all, we analyze the temporal evolution of uncertainty measures by considering the timeline proposed by Urbani et al. [9], where the year of publication of 29 uncertainty measures was shown on a timeline. Fig. 1 presents an extension of that timeline to the case of the 65 measures presented in Section III.

Fig. 1 confirms the overview proposed by Urbani et al. [9], who distinguished four phases. In the first phase (1982–1987), measures were introduced heuristically without much consideration for their formal properties. Later on (1988–1995), more comprehensive formulations were suggested to capture all facets of uncertainty. These measures usually went under the name of *total uncertainty* measures. In this phase, uncertainty measures became themselves objects of study, together with their formal properties. In the next 20 years (1996–2015), studies on formal properties were still presented, but, as shown in Fig. 1, there was a practical lack of new proposals. Since 2016, we have been witnessing a deluge of uncertainty measures in the literature. More precisely, due to this "big boom," 43 measures out of 65 (the 66% of them) were proposed in the last 10 years. Dezert and Tchamova [8] discussed various uncertainty measures and recalled that "many of them have bloomed like mushrooms since 2016."

This, possibly still ongoing, phase is characterized by a relatively low interest in formal properties of uncertainty measures, and the measures themselves are not even introduced heuristically anymore, but often they are small variations of existing measures, and whose alleged superiority was shown on some ad hoc generated numerical examples. Last, from the qualitative point of view, it is safe to say that, in most of the cases, new proposals are much more complex than old ones, and sometimes this greater complexity comes at the cost of a reduced interpretability of the measure.

B. Hasse Diagram

From the presentation of the uncertainty measures, it is clear that some of them had an influence on some others, and in some cases, some of them are declared extensions of existing ones. We consider the dependencies highlighted in Section IV-A, and we use a Hasse diagram to visualize them. This diagram represents a finite partially ordered set, where there is an arc $\mapsto$ from measure U_x to measure U_y if and only if U_y is conceptually derived from, or possibly influenced by U_x. To enhance readability, the diagram is presented in a reduced form, which means that only the most immediate dependencies are explicitly shown. Hence, if a measure is linked to multiple predecessors that share a common connection, only the direct link to the most recent predecessor is displayed. This simplification preserves the hierarchical structure while avoiding redundant edges, retaining the ease of interpretation of the diagram. In the big picture, we also consider Hartley and Shannon entropies: their influence on uncertainty measures is undeniable, even if they cannot be used in evidence theory. Additionally, Shannon entropy can be seen as a generalization of Hartley entropy, but we represent them as independent since Shannon entropy was introduced independently and by means of an axiomatic characterization. The Hasse diagram is presented in Fig. 2.

At the root of the diagram, *Shannon entropy* (5) and *Hartley entropy* (4) represent the core concepts of *conflict* and *non-specificity* and emerge as the two fundamental measures from which most of the subsequent developments originate. Other uncertainty measures for the probabilistic framework, like Tsallis and Renyi's entropy, have seen some generalizations, but their relevance cannot be compared to the one gained by Shannon's entropy. As the diagram progresses, branches unfold that show how uncertainty measures have evolved by combining, extending, or modifying these primary principles.

Notably, four measures stand out for their lack of predecessors. *George and Pal's measure* (U18) introduced a novel approach to quantifying *total conflict*, while *Yang* et al.*'s non-specificity measure* (U23) and *Yang and Han's distance-based uncertainty measure* (U24) represented the first attempts to incorporate a distance-based framework into uncertainty quantification. The only measure that remains structurally isolated is *Wen* et al.*'s measure* (U44), which, due to its unique mathematical formulation, does not share conceptual connections with other measures.

Deng's entropy has often been considered the most influential uncertainty measure. The Hasse diagram supports this statement. In fact, out of the 82 arcs in the graph, 15 depart from Deng's entropy U_D, to corroborate its centrality in the literature. The uncertainty measure with the second largest number of outgoing arcs, in this case 7, is *Dubois and Prade's uncertainty measure* $\mathsf{U}_{\mathrm{DPu}}$, thus indirectly remarking, by comparison, the visibility gained by *Deng entropy*. Interestingly, the uncertainty measures $\mathsf{U}_{\mathrm{YHD}}$ and U_{YH}, possibly thanks to their originality and ease of interpretation due to the concept of belief intervals, have also been influential in the literature. U_D, $\mathsf{U}_{\mathrm{YHD}}$, and U_{YH} were the only three new measures introduced in 2016, the year before the 'big boom' of uncertainty measures. In retrospect, we can see their influence, and it is safe to say that they have been determinant for the rapid increase in the formulation of new proposals. Since the introduction of $\mathsf{U}_{\mathrm{YHD}}$ and U_{YH}, the formulations of 11 uncertainty measures have considered the length of the interval $[\mathrm{Bel}(x_i), \mathrm{Pl}(x_i)]$ or its distance from the unit interval. On top of this, we already discussed the influence of Deng entropy, and we conclude that it is hardly possible to find uncertainty measures introduced after 2016 that cannot be seen as descendants of the above-mentioned three measures. One side effect is that, aside from a few exceptions and attempts to unify the two approaches, e.g., (U62), a dichotomy has emerged between uncertainty measures based on belief intervals and those following an entropy-like formulation (commonly referred to as belief entropies).

The extreme influence of some measures may also have some drawbacks. It is well-known, for example, that Deng entropy fails to satisfy some reasonable properties [86], and the same problem has been propagated to a number of its extensions [85]. Such critiques undermined the reliability of U_D so much that Dezert and Tchamova [8] explicitly questioned its effectiveness by stating: "We really wonder about such strong interest in this measure because Deng entropy is obviously not effective." The persistent prominence of criticized, yet popular, uncertainty measures may be an example of status quo bias [88].

V. Discussion and Conclusion

Let us begin with an apology for two potential limitations of our analysis. First, in spite of our goal of presenting a complete genealogy, it may be that some existing measures of uncertainty were not considered in our study. This is not caused by any will to exclude some of them, but more genuinely, it may be caused by the extremely large number of uncertainty measures and the fact that no comprehensive overview of them has ever been attempted so far. Dezert and Tchamova [8] used the term "jungle" to describe the set of all ineffective uncertainty measures, so let alone be their totality. To the best of our knowledge this represents the first attempt to provide an exhaustive and self-contained summary of all the uncertainty measures with a commentary on their evolution. Second, some connections between measures, as shown in Fig. 2, are necessarily a by-product of our understanding of the uncertainty measures: some relations between uncertainty measures are certain, in the sense that the authors of some measures explicitly declared what the inspiring measures were, whereas some connections are more faded and less explicit. In contrast to the above-mentioned potential limitations, we believe that the exclusion of few uncertainty measures and small variations in the Hasse diagram should not undermine the validity of the comments formulated in Section IV.

The following are some comments arising from the genealogical study.

1) On a more general level, we believe that Figs. 1 and 2 show an overabundance of uncertainty measures, which may be reasonably interpreted as the symptom of an overexploited field of research. Nevertheless, the large number of proposals for uncertainty measures has not gone hand-in-hand with the study of their formal properties, and only few uncertainty measures were presented together with a fully-fledged and rigorous study on their formal properties.

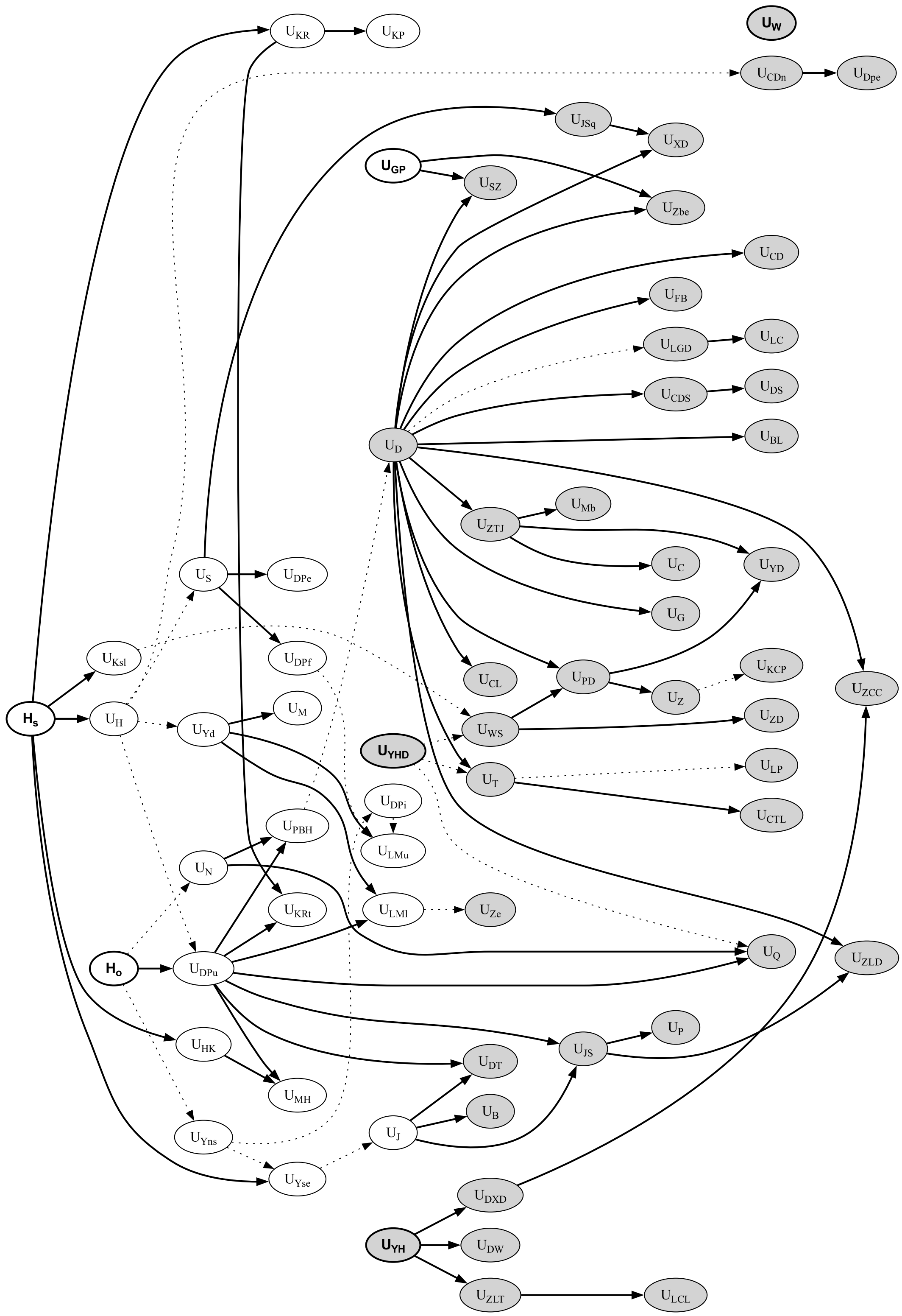


Fig. 2. Transitive reduction of Hasse diagram of the conceptual relations between uncertainty measures. An arrow from one measure to another denotes a direct conceptual dependence of the latter on the former: solid arrows denote the existence of a direct bibliographical reference to the predecessor and dashed lines denote conceptual dependencies without a specific bibliographical reference. Boldfaced nodes have no predecessors and nodes in gray indicate uncertainty measures introduced after 2016.

Formal studies [85], [86] showed that Deng entropy, as well as other measures, fail to satisfy some minimal requirements, and yet this did not seem to raise much awareness. This could be taken as an indicator that the critical inspection of formal properties has not been able to prune branches of the Hasse diagram, which, instead, have proliferated. This can be seen as a signal that the usual survival of the fittest process of the evolution of mathematical models, as well as of scientific theories at large, may not have been working properly. More research should be carried out to verify the formal properties of the most recently introduced uncertainty measures.

We believe and hope that this study brings more evidence to reinforce the point of view already expressed, among others, by Dezert and Tchamova [8], according to which it is desirable that serious scrutiny be given to the formal properties of uncertainty measures. Hopefully, the self-contained exposition of this article could serve as a starting point for an in-depth analysis of uncertainty analysis. A natural consequence is that the inception of new measures should not be done uncritically and should not be motivated only by examples showing that, in simple instances, a new measure outperforms an existing one.

2) Some uncertainty measures—e.g., (U31) and (U42)—were explicitly introduced as the sum of two distinct parts: conflict and non-specificity. Many others are not apparently the sum of the two terms, but they can be reported in such a form. For illustrative purposes, let us consider (U16), which can be rewritten as

$$\mathsf{U}_{\mathrm{PBH}} = \underbrace{\sum_{A \in \mathcal{F}} m(A) \log_2 \frac{1}{m(A)}}_{\text{conflict}} + \underbrace{\sum_{A \in \mathcal{F}} m(A) \log_2 |A|}_{\text{non-specificity}}.$$

Although the possibility of splitting uncertainty measures into two parts complies with the behavioral requirement of separability, there may be some negative implications. Such formulations correspond to an additive aggregation where two terms are considered within a fully compensatory model, which, in turn, is equivalent to a weighted average of the two contributions where the two weights are equal. Some considerations are in order. First, the ranges of the two terms are different, and the implicit choice of equal weights appears arbitrary and questionable. Second, the very same choice of an additive value model could be questioned in light of the conditions that are necessary to apply it [89, Section 5.4], [90, Section 3.6.2] and which have never been discussed in the literature of uncertainty measures. Questioning the applicability of the assumptions leading to the additive value model would imply questioning a large number of uncertainty measures, and may lead to further improvements based on non-additive aggregation functions [91]. This could be seen as a solution to the problem faced by Dezert and Tchamova [8] when they discussed the possibility to develop uncertainty measures "not based on the additive decomposition" and declared that they hoped that "it will appear in a close future."

3) A precise analysis of formal properties remains essential to understand the behavior of uncertainty measures, especially for the most recent measures, which could outperform the old ones, but whose formulations may be hard to interpret. It should also be highlighted, however, that evidence theory can have multiple interpretations. For example, as discussed by Halpern and Fagin [92], beliefs can be interpreted as either generalized probabilities or evidence, and the two interpretations lead to different operations. For example, only the latter interpretation leads to the Dempster rule of combination, whereas the former does not. In this context, properties like monotonicity have been questioned due to their lack of compatibility with the Dempster rule of combination [43, p. 56]. Thus, we may consider that the desirable properties of uncertainty measures are dependent on the interpretation of the belief measure. Hence, the suitability of uncertainty measures could be interpretation-dependent. Nevertheless, it is hardly ever the case that uncertainty measures are introduced, discussing the interpretation to which they refer. If we examine the alignment between uncertainty measures and their interpretations, we can infer that all the uncertainty measures proposed for use in alternative information fusion methods—those differing from Dempster's rule—were, in the minds of their proponents, not originally designed for the Dempster–Shafer interpretation of evidence theory. Future research could explore the idea that the suitability of uncertainty measures is not an inherent property, but rather depends on the interpretation of the belief measure.

REFERENCES

[1] B. M. Ayyub and G. J. Klir, *Ertainty Modeling and Analysis in Engineering and the Sciences*. London, U.K.: Chapman & Hall, 2006.

[2] F. Cuzzolin, "Uncertainty measures: A critical survey," *Inf. Fusion*, vol. 114, Feb. 2025, Art. no. 102609.

[3] A. P. Dempster, "Upper and lower probabilities induced by a multivalued mapping," *Ann. Math. Statist.*, vol. 38, no. 2, pp. 325–339, Apr. 1967.

[4] G. Shafer, *A Mathematical Theory of Evidence*. Princeton, NJ, USA: Princeton Univ. Press, 1976.

[5] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets Syst.*, vol. 100, pp. 3–28, Jan. 1978.

[6] S. Barhoumi, I. K. Kallel, É. Bossé, and B. Solaiman, "An empirical survey-type analysis of uncertainty measures for the fusion of crisp and fuzzy bodies of evidence," *Inf. Fusion*, vol. 121, Sep. 2025, Art. no. 103106.

[7] F. Liang, M. Brunelli, K. Septian, and J. Rezaei, "Belief-based best worst method," *Int. J. Inf. Technol. Decis. Making*, vol. 20, no. 1, pp. 287–320, Jan. 2021.

[8] J. Dezert and A. Tchamova, "On the effectiveness of measures of uncertainty of basic belief assignments," *Inf. Security: Int. J.*, vol. 52, pp. 9–36, 2022.

[9] M. Urbani, G. Gasparini, and M. Brunelli, "A numerical comparative study of uncertainty measures in the Dempster–Shafer evidence theory," *Inf. Sci.*, vol. 639, Aug. 2023, Art. no. 119027.

[10] R. Jiroušek and V. Kratochvíl, "About twenty-five naughty entropies in belief function theory: Do they measure informativeness?" *Int. J. Approx. Reasoning*, vol. 184, Sep. 2025, Art. no. 109454.

[11] K. Mao, Y. Wang, W. Zhou, J. Ye, and B. Fang, "Evaluation of belief entropies: From the perspective of evidential neural network," *Artif. Intell. Rev.*, vol. 58, no. 5, p. 133, Feb. 2025.

[12] P. Smets, "Decision making in the TBM: The necessity of the pignistic transformation," *Int. J. Approx. Reasoning*, vol. 38, no. 2, pp. 133–147, Feb. 2005.

[13] B. R. Cobb and P. P. Shenoy, "On the plausibility transformation method for translating belief function models to probability models," *Int. J. Approx. Reasoning*, vol. 41, no. 3, pp. 314–330, Apr. 2006.

[14] U. Höhle, "Entropy with respect to plausibility measures," in *Proc. 12th IEEE Int. Symp. Multiple Valued Log.*, Paris, Nov. 1982, pp. 167–169.

[15] P. Smets, "Information content of an evidence," *Int. J. Man-Machine Stud.*, vol. 19, no. 1, pp. 33–43, 1983.

[16] R. R. Yager, "Entropy and specificity in a mathematical theory of evidence," *Int. J. Gen. Syst.*, vol. 9, no. 4, pp. 249–260, Jan. 1983.

[17] T. George and N. R. Pal, "Quantification of conflict in Dempster-Shafer framework: A new approach," *Int. J. Gen. Syst.*, vol. 24, no. 4, pp. 407–423, Mar. 1996.

[18] H. T. Nguyen, "On entropy of random sets and possibility distributions," *Anal. Fuzzy Inf.*, vol. 1, pp. 145–156, Jan. 1987.

[19] D. Dubois and H. Prade, "A note on measures of specificity for fuzzy sets," *Int. J. Gen. Syst.*, vol. 10, no. 4, pp. 279–283, Feb. 1985.

[20] M. Higashi and G. J. Klir, "Measures of uncertainty and information based on possibility distributions," *Int. J. Gen. Syst.*, vol. 9, no. 1, pp. 43–58, Dec. 1982.

[21] D. Dubois and H. Prade, "Properties of measures of information in evidence and possibility theories," *Fuzzy Sets Syst.*, vol. 100, pp. 35–49, Jan. 1999.

[22] D. Dubois and H. Prade, "Representation and combination of uncertainty with belief functions and possibility measures," *Comput. Intell.*, vol. 4, no. 3, pp. 244–264, Sep. 1988.

[23] M. T. Lamata and S. Moral, "Measures of entropy in theory of evidence," *Int. J. Gen. Syst.*, vol. 14, no. 4, pp. 297–305, 1988.

[24] G. J. Klir and A. Ramer, "Uncertainty in the Dempster-Shafer theory: A critical re-examination," *Int. J. Gen. Syst.*, vol. 18, no. 2, pp. 155–166, Dec. 1990.

[25] M. Inuiguchi, M. Maekawa, and Y. Kume, "On a global measure of information in Dempster and Shafer's theory," *Trans. Inst. Syst., Control Inf. Engineers*, vol. 4, no. 11, pp. 435–444, 1991.

[26] D. Harmanec and G. J. Klir, "Measuring total uncertainty in Dempster-Shafer theory: A novel approach," *Int. J. Gen. Syst.*, vol. 22, no. 4, pp. 405–419, Sep. 1994.

[27] G. J. Klir and B. Parviz, "A note on the measure of discord," in *Uncertainty in Artificial Intelligence*, D. Dubois, M. P. Wellman, B. D'Ambrosio, and P. Smets, Eds., San Mateo, CA, USA: Morgan Kaufmann, 1992, pp. 138–141.

[28] N. R. Pal, J. C. Bezdek, and R. Hemasinha, "Uncertainty measures for evidential reasoning II: A new measure of total uncertainty," *Int. J. Approx. Reasoning*, vol. 8, no. 1, pp. 1–16, Jan. 1993.

[29] Y. Maeda and H. Ichihashi, "An uncertainty measure with monotonicity under the random set inclusion," *Int. J. Gen. Syst.*, vol. 21, no. 4, pp. 379–392, Jan. 1993.

[30] D. A. Maluf, "Monotonicity of entropy computations in belief functions," *Intell. Data Anal.*, vol. 1, no. 3, pp. 207–213, Jul. 1997.

[31] G. J. Klir, "Uncertainty and information measures for imprecise probabilities: An overview," in *Proc. 1st Int. Symp. Imprecise Probabilities Appl. (ISIPTA)*, Ghent, Belgium, 1999, pp. 234–240.

[32] R. R. Yager, "On the entropy of fuzzy measures," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 4, pp. 453–461, Aug. 2000.

[33] A.-L. Jousselme, C. Liu, D. Grenier, and E. Bosse, "Measuring ambiguity in the evidence theory," *IEEE Trans. Syst., Man, Cybern. - Part A: Syst. Humans*, vol. 36, no. 5, pp. 890–903, Sep. 2006.

[34] J. Dezert, F. Smarandache, and A. Tchamova, "On the blackman's association problem," in *Proc. 6th Annu. Conf. Inf. Fusion*, Cairns, NQ, Australia, Jul. 2003, pp. 1371–1378.

[35] Y. Yang, D. Han, and J. Dezert, "A new non-specificity measure in evidence theory based on belief intervals," *Chin. J. Aeronaut.*, vol. 29, no. 3, pp. 704–713, Jun. 2016.

[36] Y. Yang and D. Han, "A new distance-based total uncertainty measure in the theory of belief functions," *Knowledge-Based Syst.*, vol. 94, pp. 114–123, Feb. 2016.

[37] Y. Deng, "Deng entropy," *Chaos, Solitons Fractals*, vol. 91, pp. 549–553, Oct. 2016.

[38] X. Wang and Y. Song, "Uncertainty measure in evidence theory with its applications," *Appl. Intell.*, vol. 48, no. 7, pp. 1672–1688, Jul. 2018.

[39] D. Zhou, Y. Tang, and W. Jiang, "A modified belief entropy in Dempster-Shafer framework," *PLoS ONE*, vol. 12, no. 5, May 2017, Art. no. e0176832.

[40] Y. Tang, D. Zhou, S. Xu, and Z. He, "A weighted belief entropy-based uncertainty measure for multi-sensor data fusion," *Sensors*, vol. 17, no. 4, p. 928, Apr. 2017.

[41] X. Deng, F. Xiao, and Y. Deng, "An improved distance-based total uncertainty measure in belief function theory," *Appl. Intell.*, vol. 46, no. 4, pp. 898–915, Jun. 2017.

[42] L. Pan and Y. Deng, "A new belief entropy to measure uncertainty of basic probability assignments based on belief function and plausibility function," *Entropy*, vol. 20, no. 11, p. 842, Nov. 2018.

[43] R. Jiroušek and P. P. Shenoy, "A new definition of entropy of belief functions in the Dempster–Shafer theory," *Int. J. Approx. Reasoning*, vol. 92, pp. 49–65, Jan. 2018.

[44] R. Jiroušek and P. P. Shenoy, "A decomposable entropy of belief functions in the Dempster-Shafer theory," in *Belief Functions: Theory and Applications* (Lecture Notes in Artificial Intelligence), vol. 11069, S. Destercke, T. Denoeux, F. Cuzzolin, and A. Martin, Eds., Cham, Switzerland: Springer Nature, 2018, pp. 146–154.

[45] R. Jiroušek and P. P. Shenoy, "On properties of a new decomposable entropy of Dempster-Shafer belief functions," *Int. J. Approx. Reasoning*, vol. 119, pp. 260–279, Apr. 2020.

[46] M. D. Mambé, T. N'Takp, N. Georges, and S. Oumtanaga, "A new uncertainty measure in belief entropy framework," *Int. J. Adv. Comput. Sci. Appl.*, vol. 9, no. 11, pp. 600–606, 2018.

[47] H. Cui, Q. Liu, J. Zhang, and B. Kang, "An improved Deng entropy and its application in pattern recognition," *IEEE Access*, vol. 7, pp. 18284–18292, 2019.

[48] D. Li, X. Gao, and Y. Deng, "A generalized expression for information quality of basic probability assignment," *IEEE Access*, vol. 7, pp. 174734–174739, 2019.

[49] Q. Pan, D. Zhou, Y. Tang, X. Li, and J. Huang, "A novel belief entropy for measuring uncertainty in Dempster-Shafer evidence theory framework based on plausibility transformation and weighted Hartley entropy," *Entropy*, vol. 21, no. 2, p. 163, Feb. 2019.

[50] L. Chen, L. Diao, and J. Sang, "A novel weighted evidence combination rule based on improved entropy function with a diagnosis application," *Int. J. Distrib. Sensor Netw.*, vol. 15, no. 1, Jan. 2019, Art. no. 155014771882399.

[51] X. Gao, F. Liu, L. Pan, Y. Deng, and S.-B. Tsai, "Uncertainty measure based on Tsallis entropy in evidence theory," *Int. J. Intell. Syst.*, vol. 34, no. 11, pp. 3105–3120, Nov. 2019.

[52] Y. Zhao, D. Ji, X. Yang, L. Fei, and C. Zhai, "An improved belief entropy to measure uncertainty of basic probability assignments based on Deng entropy and belief interval," *Entropy*, vol. 21, no. 11, p. 1122, Nov. 2019.

[53] F. Buono and M. Longobardi, "A dual measure of uncertainty: The Deng extropy," *Entropy*, vol. 22, no. 5, p. 582, May 2020.

[54] H. Yan and Y. Deng, "An improved belief entropy in evidence theory," *IEEE Access*, vol. 8, pp. 57505–57516, 2020.

[55] M. Qin, Y. Tang, and J. Wen, "An improved total uncertainty measure in the evidence theory and its application in decision making," *Entropy*, vol. 22, no. 4, p. 487, Apr. 2020.

[56] J. Li and Q. Pan, "A new belief entropy in Dempster–Shafer theory based on basic probability assignment and the frame of discernment," *Entropy*, vol. 22, no. 6, p. 691, Jun. 2020.

[57] K. Wen, Y. Song, C. Wu, and T. Li, "A novel measure of uncertainty in the Dempster-Shafer theory," *IEEE Access*, vol. 8, pp. 51550–51559, 2020.

[58] Y. Chen, Y. Tang, and Y. Lei, "An improved data fusion method based on weighted belief entropy considering the negation of basic probability assignment," *J. Math.*, vol. 2020, pp. 1–11, Nov. 2020.

[59] H. Li and R. Cai, "An improved expression for information quality of basic probability assignment and its application in fault diagnosis," *Soft Comput.*, pp. 6681–6690, Feb. 2020.

[60] Z. Chen and X. Luo, "Uncertainty measure of basic probability assignment based on Renyi entropy and its application in decision-making," *IEEE Access*, vol. 9, pp. 130032–130041, 2021.

[61] Z. Deng and J. Wang, "Measuring total uncertainty in evidence theory," *Int. J. Intell. Syst.*, vol. 36, no. 4, pp. 1721–1745, Apr. 2021.

[62] Q. Zhang, H. Li, R. Li, and Y. Tang, "An improved measure for belief structure in the evidence theory," *PeerJ Comput. Sci.*, vol. 7, p. e710, Sep. 2021.

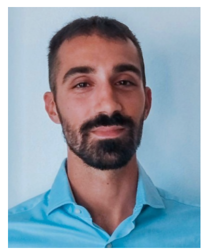
[63] Y. Zhang, F. Huang, X. Deng, and W. Jiang, "A new total uncertainty measure from a perspective of maximum entropy requirement," *Entropy*, vol. 23, no. 8, p. 1061, Aug. 2021.

[64] R. Li, Z. Chen, H. Li, and Y. Tang, "A new distance-based total uncertainty measure in Dempster-Shafer evidence theory," *Appl. Intell.*, vol. 52, no. 2, pp. 1209–1237, Jan. 2022.

[65] Q. Zhou and Y. Deng, "Fractal-based belief entropy," *Inf. Sci.*, vol. 587, pp. 265–282, Mar. 2022.

[66] M. Zhou, S.-S. Zhu, Y.-W. Chen, J. Wu, and E. Herrera-Viedma, "A generalized belief entropy with nonspecificity and structural conflict," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 9, pp. 5532–5545, Sep. 2022.

[67] Y. Xue and Y. Deng, "A decomposable Deng entropy," *Chaos, Solitons & Fractals*, vol. 156, Mar. 2022, Art. no. 111835.
[68] P. Dutta and S. Shome, "A new belief entropy measure in the weighted combination rule under DST with faulty diagnosis and real-life medical application," *Int. J. Mach. Learn. Cybern.*, vol. 14, no. 4, pp. 1179–1203, Apr. 2023.
[69] X. Chen and Y. Deng, "A new belief entropy and its application in software risk analysis," *Int. J. Comput. Commun. Control*, vol. 18, no. 2, Apr. 2023, Art. no. 5299.
[70] Y. Cui and X. Deng, "Plausibility entropy: A new total uncertainty measure in evidence theory based on plausibility function," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 53, no. 6, pp. 3833–3844, Jun. 2023.
[71] R. Kavya, C. Jabez, and P. Subhrakanta, "A new belief interval-based total uncertainty measure for Dempster-Shafer theory," *Inf. Sci.*, vol. 642, Sep. 2023, Art. no. 119150.
[72] Q. Zhou and Y. Deng, "Belief eXtropy: Measure uncertainty from negation," *Commun. Statist. Theory Methods*, vol. 52, no. 11, pp. 3825–3847, Jun. 2023.
[73] X. Deng, S. Xue, W. Jiang, and X. Zhang, "Plausibility extropy: The complementary dual of plausibility entropy," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 54, no. 11, pp. 6936–6947, Nov. 2024.
[74] F. Zhang, Z. Chen, and R. Cai, "A belief interval Euclidean distance entropy of the mass function and its application in multi-sensor data fusion," *Appl. Intell.*, vol. 54, nos. 17–18, pp. 7545–7569, Sep. 2025.
[75] J. Zhou, Z. Li, and Y. Deng, "An improved information volume of mass function based on plausibility transformation method," *Expert Syst. Appl.*, vol. 237, Mar. 2024, Art. no. 121663.
[76] B. Kang and Y. Deng, "The maximum Deng entropy," *IEEE Access*, vol. 7, pp. 120758–120765, 2019.
[77] X. Su, J. Zhong, Z. Hong, H. Qian, and D. Pelusi, "A novel belief entropy and its application in cooperative situational awareness," *Expert Syst. Appl.*, vol. 286, Aug. 2025, Art. no. 128027.
[78] Y. Deng, "Uncertainty measure in evidence theory," *Sci. China Inf. Sci.*, vol. 63, no. 11, Nov. 2020, Art. no. 210201.
[79] K. Sentz and S. Ferson, "Combination of evidence in Dempster-Shafer theory," Sandia National Lab., Albuquerque, NM, USA, Tech. Rep. 2002-0835, 2002.
[80] G. J. Klir, "Principles of uncertainty: What are they? Why do we need them?" *Fuzzy Sets Syst.*, vol. 74, no. 1, pp. 15–31, Aug. 1995.
[81] M. Brunelli, R. P. Jayasuriya Kuranage, and V.-N. Huynh, "Selection rules for new focal elements in the Dempster-Shafer evidence theory," *Inf. Sci.*, vol. 712, Sep. 2025, Art. no. 122160.
[82] J. Abellán and A. Masegosa, "Requirements for total uncertainty measures in Dempster–Shafer theory of evidence," *Int. J. Gen. Syst.*, vol. 37, no. 6, pp. 733–747, Dec. 2008.
[83] S. Moral-García and J. Abellán, "Required mathematical properties and behaviors of uncertainty measures on belief intervals," *Int. J. Intell. Syst.*, vol. 36, no. 8, pp. 1–24, Aug. 2021.
[84] V.-N. Huynh and Y. Nakamori, "Notes on 'reducing algorithm complexity for computing an aggregate uncertainty measure,'" *IEEE Trans. Syst., Man, Cybern. A, Syst. Humans*, vol. 40, no. 1, pp. 205–209, Jan. 2009.
[85] S. Moral-García and J. Abellán, "Critique of modified Deng entropies under the evidence theory," *Chaos, Solitons Fractals*, vol. 140, Nov. 2020, Art. no. 110112.
[86] J. Abellán, "Analyzing properties of Deng entropy in the theory of evidence," *Chaos, Solitons Fractals*, vol. 95, pp. 195–199, Feb. 2017.
[87] S. Moral-García and J. Abellán, "Maximum of entropy for belief intervals under evidence theory," *IEEE Access*, vol. 8, pp. 118017–118029, 2020.
[88] W. Samuelson and R. Zeckhauser, "Status quo bias in decision making," *J. Risk Uncertainty*, vol. 1, no. 1, pp. 7–59, Mar. 1988.
[89] F. S. Roberts, *Measurement Theory* (Encylcopedia of Mathematics and its Applications), vol. 7. Cambridge, U.K.: Cambridge Univ. Press, 1985.
[90] R. L. Keeney and H. Raiffa, *Decisions With Multiple Objectives: Preferences Value Trade-offs*. Cambridge, U.K.: Cambridge Univ. Press, 1993.
[91] M. Grabisch, J.-L. Marichal, R. Mesiar, and E. Pap, *Aggregation Functions* (Encylcopedia of Mathematics and its Applications), vol. 127. Cambridge, U.K.: Cambridge Univ. Press, 2009.
[92] J. Y. Halpern and R. Fagin, "Two views of belief: Belief as generalized probability and belief as evidence," *Artif. Intell.*, vol. 54, no. 3, pp. 275–317, Apr. 1992.



Tommaso Pellegrini received the master's degree from the Department of Industrial Engineering, University of Trento, Italy. His current research interests are in project management, mechanical engineering and uncertainty management.

Matteo Brunelli received the bachelor's and master's degrees from the University of Trento, Italy, and the Ph.D. degree from Åbo Akademi University, Finland. He is an Associate Professor of Mathematical Methods with the Department of Industrial Engineering, University of Trento, Italy. He spent five years as Postdoctoral Researcher at Aalto University, Finland. His research interests include decision analysis, preference modelling, mathematical representations of uncertainty, and fuzzy sets.

Open Access funding provided by 'Università degli Studi di Trento' within the CRUI CARE Agreement