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Measurement Uncertainty and Information Enabled Decision Making

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ABSTRACT It is widely accepted that in many industrial and commercial applications, as well as in the areas of health, safety, environmental protection, and forensics, decisions are mostly based on measurement results that are compared with one or more threshold values to state whether the measured quantity conforms to tolerance limits, or maximum admissible limits set by Standards or Laws. It is also known that the risk of making the wrong decision always exists, and is closely related to the credibility of the data entered in the decision-making process. When the input data are measurement results, their credibility is evaluated in terms of measurement uncertainty. Different decision rules have been proposed in literature and in the Standards, the ISO Std. 14253-1 and ISO Guide 98-4 above all, which consider measurement uncertainty, evaluated according to the recommendations of the Guide to the expression of Uncertainty in Measurement (GUM), in conformity assessment. However, given the tolerance limit, they do not fully explore the relationship between the measured value, measurement uncertainty, and the risk of wrong decision, and how any of them can be set, given the two other ones. This article aims to discuss how to establish suitable decision rules, fully explain the underlying assumptions about measurement uncertainty and risk, and show how the proper definition of the decision rule may increase efficiency by reducing costs and may, consequently, represent a step forward in the direction of sustainability.

INDEX TERMS Decision rules, measurement uncertainty, risk evaluation.

I. INTRODUCTION

It IS widely recognized that the outcome of decisions influences almost every aspect of our lives, from industrial strategies to environmental protection, health care, business, legal decisions, and everyday life. Therefore, it is critically important to base decisions on trustworthy evidence, as clearly stated also by the ISO Std. 9000:2015 [1] in its sixth quality management principle. The rationale for this principle states: *decision-making can be a complex process and it always involves some uncertainty*.

This statement recognizes that the data on which decisions are based are affected by uncertainty and, consequently, they may influence the validity of the resulting decision. In other words, there is a risk that the decision is not correct, and this risk is related to uncertainty in the input data [2], [3].

This awareness of the critical role of uncertainty in decision-making processes has led to numerous attempts to identify different contributions to uncertainty, as clearly shown by a vast literature. A good example of how confusing an approach to uncertainty definition can be when it is not driven by strict technical consideration can be found in [4], where 13 dimensions of uncertainty and 8 different approaches to managing it are considered in the business field. It can be readily understood that such an approach can hardly provide a unique decision-making procedure.

In the technical field, fewer dimensions are assigned to uncertainty, and its analysis focuses on the distinction between *epistemic* uncertainty and *variability* or *aleatoric* uncertainty [3].

In this framework, epistemic uncertainty is defined as the contribution to uncertainty that depends on our lack of knowledge and can be reduced by conducting additional research and/or performing empirical experiments. On the other hand, aleatoric uncertainty is defined as the contribution of uncertainty related to the intrinsic variability of the considered data [3], [5]. This same framework has also been considered in the more recent and critical field of decisions based on artificial intelligence (AI), in which AI agents are employed to obtain a model from a great amount of information and use the model to suggest optimal decisions [6], [7].

Although distinguishing between epistemic and aleatoric uncertainty appears attractive, from a philosophical point of view, a few concerns arise about its practical utility, since it is not always possible to unambiguously assign the different contributions to uncertainty to any of the two classes [5], or the way uncertainty is treated in decision analysis is not fully compatible with this distinction [8], or it is not always possible to combine these two contributions in a straightforward way [9].

The above statement can be better perceived by considering that this classification shows strong similarities with the categorization of measurement errors into systematic and random contributions. The criticality of such classifications lies in the fact that assigning a contribution to one of the two classes (epistemic/aleatoric or systematic/random) is not always unique, since it depends on the performed activity. For instance, if a measurement is performed with a single instrument, the prevailing contribution to uncertainty generated by the instrument is systematic, since it does not change when the measurement is repeated. On the other hand, if the same measurement is performed by several, randomly chosen instruments, the same contribution shows random behavior. On the other hand, a random error affecting a measurement result may become systematic when an indirect measurement is obtained through a nonlinear transformation of that result. For instance, if a quadratic transformation is considered, the original random contribution becomes a bias on the transformation output, and hence a systematic contribution.[1]

A further concern is related to the lack of uniformity with which these contributions to uncertainty are considered by different applications, sometimes even in the same field [11], as well as the lack of uniformity with which these contributions are quantified. In complex systems, such as those based on advanced AI agents, where a huge amount of data, often related to phenomena that have origin in different fields, is processed and, consequently, multiple sources of uncertainty may be present, this lack of uniformity may prevent, partially or even totally, to combine all contributions to uncertainty into a unique value assigned to the suggested decision.

This scenario changes significantly when uncertainty associated with measurement results is considered. In such cases, a unique framework has been set by the Guide to the expression of Uncertainty in Measurement (GUM) [10], jointly issued in 1995 by BIPM,[2] IEC,[3] IFCC,[4] ILAC,[5] ISO,[6] IUPAC,[7] IUPAP,[8] and OIML.[9]

[1]This effect is considered in the note to art. 3.3.3 of the GUM [10], to which the reader is addressed for further details.

Differently from all other frameworks, the method proposed by the GUM is aimed to be *universal*, that is, as stated by the GUM: *the method should be applicable to all kinds of measurements and to all types of input data used in measurements* [10].

As it will be better discussed in the next section, the GUM approach to uncertainty represents all contributions in the same mathematical way, and contributions are classified according to their origin and not their nature (epistemic or aleatoric).

Consequently, as it will be shown in the following part of this article, under given assumptions often fulfilled in the measurement field, a unique way can be defined to evaluate the risk of taking the wrong decision when the decision is based on experimental data provided by sensor or instrument readings.

To prove the above statements, this article is organized as follows. Section II will briefly recall the GUM approach to uncertainty in measurement. Section III will show how uncertainty can be correctly used to compare a measurement result with a given threshold value. Section IV will discuss a general decision-making procedure recommended by [12], and show some examples developed according to the proposed method. Finally, Section V will draw some conclusions.

The following sections will extensively refer to the basic concepts of metrology, such as *standard uncertainty*, *expanded uncertainty*, *combined standard uncertainty*, and some basic concepts of probability, such as *probability density function*, *cumulative probability distribution*, *coverage interval*, and *coverage probability*. These concepts are well known to readers who are familiar with metrology and measurement terms and are widely covered by textbooks. Summarizing them is out of the scope of a scientific paper, so the readers who are not familiar with them, but are still interested in how they can be applied in decision-making processes based on measurement results, are addressed to the referenced documents [13], [14], [15], [16], [17], [18].

II. GUM APPROACH

The GUM approach is grounded on a few basic assumptions [10], [13].

1) The true value of the measurand, that is the quantity intended to be measured [19], is unknown and unknowable. Therefore, a single measured quantity

[2]Bureau International des Poids et Mesures.
[3]International Electrotechnical Commission.
[4]International Federation of Clinical Chemistry and Laboratory Medicine.
[5]International Laboratory Accreditation Cooperation.
[6]International Organization for Standardization.
[7]International Union of Pure and Applied Chemistry.
[8]International Union of Pure and Applied Physics.
[9]International Organization of Legal Metrology.

value [19] cannot represent the measurand, and additional information is needed.
2) The significant systematic effects are identified and the proper corrections are applied.
3) The dispersion of the values that can be attributed to the measurand is due only to random effects. Therefore, a measurement result can be mathematically expressed by a random variable with zero mean.

The first assumption is probably the strongest one, and finds its root in the finite number of known details about the measurand itself, the mutual interactions between measurand, environment and measuring system, and the measuring system itself [13].

The International Vocabulary of Metrology (VIM) [19], jointly issued, as the GUM, by BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML, whose third edition was published in 2008 and revised in 2012, identifies two major contributions to uncertainty: the *definitional uncertainty* and the *instrumental uncertainty*.

The definitional uncertainty is defined as the *component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand* [19].

By recognizing that full knowledge (an infinite amount of details) about the measurand is not available, the nature of the definitional uncertainty contribution can be considered epistemic, although it may also consider contributions that manifest themselves in an aleatoric way, such as an unknown variability of a measurand that is supposed to be constant, or the effect on the measurand of the variability of some influence quantities.

Since this component of uncertainty reflects our lack of full knowledge about the measurand, it also represents *the practical minimum measurement uncertainty achievable in any measurement of a given measurand*, as stated by the VIM in a note to the definition of definitional uncertainty [19].

This component also quantifies the capability of a measurement method or system to provide, *in principle*, a measurement result suitable for the intended use. Taking into account that epistemic uncertainty is generated by the lack of complete knowledge of the measurand, it can be directly related to the definitional uncertainty. Conversely, aleatoric uncertainty can be related to definitional uncertainty if the lack of knowledge about the measurand manifest itself with data variability.

The instrumental uncertainty is defined as the *component of measurement uncertainty arising from a measuring instrument or measuring system in use* [19].

Therefore, this component of uncertainty quantifies the capability of the whole measuring system, including the operator [13], to provide, *in practice*, under the actual measurement conditions, a measurement result suitable for the intended use.

This component of uncertainty can be evaluated through proper calibration of the measuring system and, therefore, uncertainty associated with the references used in the calibration operation represents a lower bound of instrumental uncertainty [19].

Instrumental uncertainty can be either of epistemic or aleatoric nature. It is epistemic when the measuring system is operating under stable conditions, though different from the calibration ones. On the other hand, it is aleatoric when the measuring system is used under significant random variations of the quantities that affect the instrument behavior, the so-called influence quantities.

The ambiguity in classifying the different uncertainty contributions as epistemic or aleatoric gives further evidence that a classification based on their origin, and not their characteristics, is more effective and unambiguous.

Having identified the origin of the different uncertainty components, a suitable mathematical model is needed to express and evaluate uncertainty. The second and third assumptions provide the framework for defining this model.

The second assumption states that all significant systematic effects must be identified and corrected for. According to the GUM [10], a systematic effect generates a systematic measurement error, that is defined by the VIM as *component of measurement error that in replicate measurements remains constant or varies in a predictable manner*.

Typical systematic effects include, for instance, instrument drifts from calibration conditions, variations in environmental conditions from the reference ones, and approximations in the employed measurement model. If not identified and compensated for, they may generate large measurement errors and, for this reason, the best measurement practice, to which the GUM refers [10], requires making *every effort* to identify and compensate them.

The GUM also states that *the result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects* [10].

According to the GUM, a random effect generates a random measurement error, that is defined by the VIM as *component of measurement error that in replicate measurements varies in an unpredictable manner* [19].

This consideration justifies the third assumption listed at the beginning of this Section, so that the dispersion of the values that could reasonably be attributed to a measurand as a measurement result can be mathematically expressed as a *random variable*, characterized by a suitable *probability density function*. It is also well known that such a dispersion can be quantified by the *standard deviation* of the probability density function, so that the GUM defines the *standard uncertainty* as *uncertainty of the result of a measurement expressed as a standard deviation* [10].

As an example, Fig. 1 shows the measurement result of the resistance of a resistor, when the measured value is $x_m = 100\ \Omega$ and the dispersion of values that could reasonably be attributed to the measurand is supposed to follow a normal probability distribution with a standard

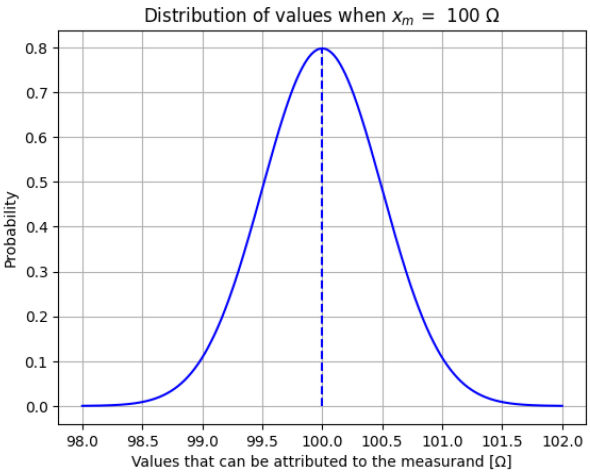

FIGURE 1. **Example of measurement result expressed by a normal probability density function. A measured value $x_m = 100\ \Omega$ is considered, with a standard uncertainty $u(x_m) = 0.5\ \Omega$.**

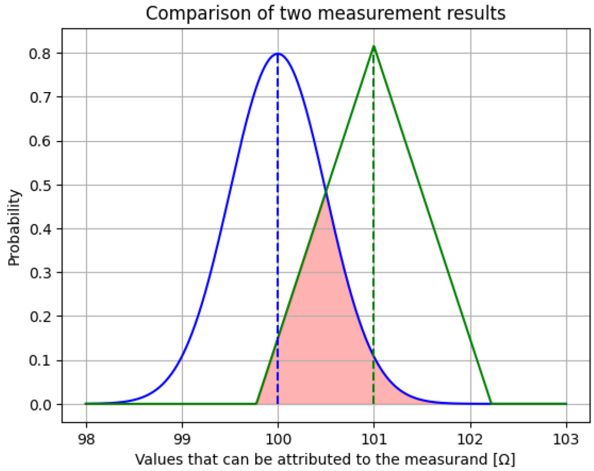

FIGURE 2. **Comparison of two measurement results. The red area shows the probability that values that can be attributed to the measurand showing a higher measured value (101 Ω (green line)) are lower than those that can be attributed to the measurand showing a lower measured value (100 Ω (red line)).**

deviation $\sigma = 0.5\ \Omega$. According to the GUM, a standard uncertainty $u(x_m) = \sigma = 0.5\ \Omega$ is assigned to the measurand.

This is not the appropriate venue to delve into the details of assigning a probability density function to a measurement result. Specific recommendations are provided by the GUM and its supplements [10], [20], and a substantial body of literature exists to reiterate and explain them [13], [14], [15], [16], [17].

Here, it is worth noting that, having properly defined the probability density function, it is also possible to define an interval, about the measured value, inside which the unknown value of the measurand is supposed to lie with a given probability. Indeed, given a probability density function $p(x)$ and an interval $[x_1;\ x_2]$ with $x_1 \le x_m \le x_2$, the probability that the value of the measurand lies in that interval is given by

$$P_{x_1 \le x < x_2} = \int_{x_1}^{x_2} p(x)\, dx. \tag{1}$$

Taking into account this property, the GUM defines the *expanded uncertainty* as a *quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand* [10]. In a note to this definition, the GUM also states that *the fraction may be viewed as the coverage probability or level of confidence of the interval*, and, in another note, that *to associate a specific level of confidence with the interval defined by the expanded uncertainty requires explicit or implicit assumptions regarding the probability distribution characterized by the measurement result and its combined standard uncertainty.*[10] Moreover, *the level of confidence that may be attributed to this interval can be known only to the extent to which such assumptions may be justified*. Both statements are mathematically expressed by (1).

[10]According to the GUM, the *combined standard uncertainty* is the standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities [10].

Reconsidering the example of Fig. 1, it is possible to define an expanded uncertainty $U(x_m) = 2 \cdot u(x_m) = 1\ \Omega$, so that interval $[x_m - U(x_m);\ x_m + U(x_m)]$ can be defined about the measured value x_m. Being $p(x)$ a normal distribution, (1) returns a probability of 95.45 % that the unknown measurand value lies inside the defined interval.

This brief overview has confirmed the universality of the GUM approach to uncertainty, since, under the given assumptions that reflect the best practice in measurement, all components of uncertainty are handled under the same probabilistic framework and can be combined in a strict mathematical way [10], [20].

III. COMPARISON

Comparing a measurement result with another measurement result or with a numerical value, such as a threshold or a tolerance limit, is a quite complex operation, due to the presence of measurement uncertainty.

According to the consideration reported in the above Section II, comparing two measurement results implies the comparison of two probability density functions.

As an example, let us consider again the resistance measurement shown in Fig. 1, let us suppose that another resistance is measured, and let the measurement result be represented by the green probability density function shown in Fig. 2. For the sake of generality, a triangular probability density function has been considered in this example with the same standard uncertainty as the one considered in the case shown in Fig. 1.

Let us also suppose that the measured value, for this resistance, is 101 Ω. Therefore, if only the measured values are considered, the resistance of this second resistor is greater than that (100 Ω) of the first one.

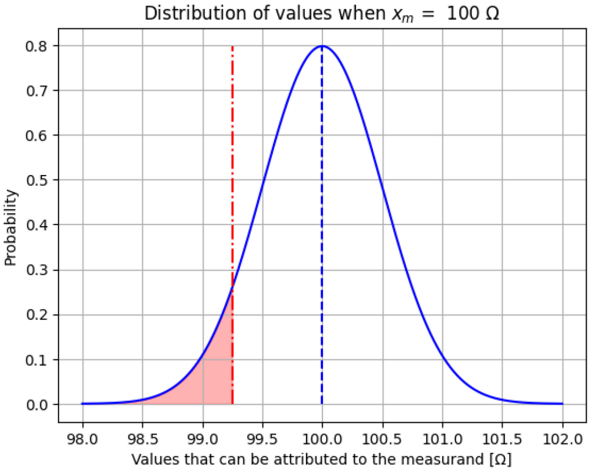

FIGURE 3. Comparison of a measurement result with a threshold (red dashed line). The red area represents the probability of the measurand value being lower than the threshold, while the measured value (blue dashed line) is greater.

On the other hand, if the whole probability density functions shown in Fig. 2 are considered, they are partially overlapping, so that values that can be attributed to the second resistance (green line) are lower than values that can be attributed to the first one (blue line). The red area shown in Fig. 2 quantifies the probability that the second resistance is lower than the first one. In the considered example, this probability is equal to 33%.

If these two measurement results are employed to assess whether one resistor has a greater resistance value than the other, this probability represents the probability, that is the risk, of making the wrong decision.

A much more frequent situation, in industrial applications as well as in legal and forensic metrology [21], and healthcare, is the comparison of a measurement result with a fixed-value threshold, be it a tolerance limit, a limit set by laws or regulations, or safety margins.

In such situations, the most critical occurrence is when the threshold falls within the values that can be reasonably attributed to the measurand, as shown in Fig. 3. In such a case, some of the values fall below the threshold, while some others fall above the threshold. Therefore, an informed decision can be made if and only if it is possible to evaluate the probability that the measurand value falls below or above the threshold.

Having considered a threshold t and a probability density function $p(x)$, the probability that the measurand value falls below t can be obtained from (1) as

$$P_{x \leq t} = \int_{-\inf}^{t} p(x)\,dx \tag{2}$$

when $p(x)$ as infinite support, and as

$$P_{x \leq t} = \int_{x_1}^{t} p(x)\,dx \tag{3}$$

when $p(x)$ has finite support and x_1 is the lower bound of the support itself.

If the cumulative probability distribution is considered

$$F_X(x) = \int_{-\inf}^{x} p(\tau)\,d\tau. \tag{4}$$

Equations (2) and (3) can be rewritten as

$$P_{x \leq t} = F_X(t). \tag{5}$$

Of course, it is also $P_{x>t} = 1 - P_{x \leq t}$. The computed probabilities quantify the risk of making an incorrect decision. For instance, in the case of Fig. 3, where the measured value is 100 Ω and the threshold is set to 99.25 Ω, the risk of making the incorrect decision by stating that the measured resistance value is greater than the threshold is 6.8%.

It can be readily checked that the closer the measured value is to the threshold, the higher the risk of making an incorrect decision. In the case of symmetric probability density functions, as in most practical situations, if the measured value is equal to the threshold, the risk of making an incorrect decision climbs up to 50%.

According to the above considerations and equations, it is also immediately evident that the risk of making an incorrect decision depends on the measured value, the threshold, the standard uncertainty, and the shape of the probability density function. Equations (2)–(4) allow one to evaluate the risk of an incorrect decision once the relative position of the measured value and the threshold is known, together with the probability density function associated with the measurement result.

However, such an approach might not be the most convenient, especially when the evaluated risk turns out to be excessive for the desired application. The most convenient, more general approach, in such a case, is that of setting the maximum admissible risk and, given the shape of the probability density function, deriving either the threshold position or the standard uncertainty required not to exceed that risk. This requires a more general decision-making procedure, as shown in the next section.

IV. DECISION-MAKING PROCEDURES

Document JCGM106 [12], published by the BIPM in 2012, and encompassed in the ISO Guide 98-4 [22], provides a general procedure, based on *guard bands* to consider measurement uncertainty in conformity assessment.

In principle, this procedure is quite simple. Let us suppose that the value assigned to a given quantity shall remain inside a given tolerance interval whose lower and upper limits are T_L and T_U, respectively, as shown in Fig. 4.

According to the considerations reported in the previous Section III, the closer the measured value is to the tolerance limits, the higher the risk of exceeding those limits even if the measured value is still within the limits.

Such a risk can be reduced if the measured value remains within another interval, narrower than the tolerance interval, called *acceptance interval*, whose lower and upper limits are A_L and A_U, respectively, and are obtained by subtracting

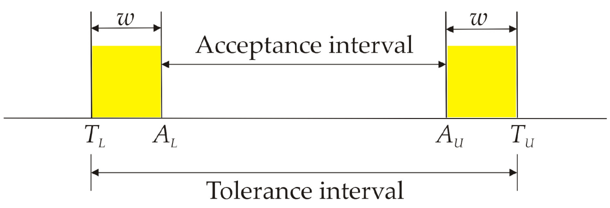

FIGURE 4. Guarded acceptance. Given a tolerance interval, an acceptance interval is defined by inserting guard bands of width *w* (the yellow intervals) inside the tolerance interval.

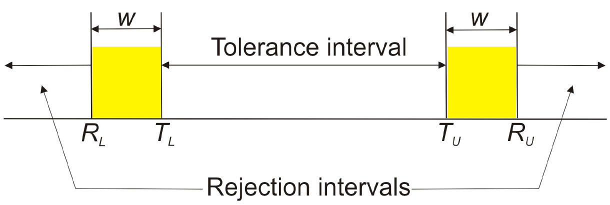

FIGURE 5. Guarded rejection. Given a tolerance interval, a rejection interval is defined by inserting guard bands of width *w* (the yellow intervals) outside the tolerance interval.

a suitable quantity w (the guard band) from the tolerance limits.

A similar problem occurs when it is necessary to assess whether the value assigned to a quantity falls outside the given tolerance interval. In such a case, the risk of rejecting a value that is indeed inside the tolerance interval can be reduced by defining a *rejection interval*, whose limits R_L and R_U are defined as shown in Fig. 5.[11] These limits are obtained by subtracting a suitable quantity w (the guard band) from the tolerance limits.

Summarizing, when the measured value falls within the acceptance or rejection intervals a decision can be made with a known probability of incorrect decision. Conversely, when the measured value falls inside the guard band, the risk of incorrect decision is higher than the desired one and, hence, a more accurate measuring procedure and/or the measuring system needs to be employed.

More generally, the problem to be solved is, hence, that of defining a suitable guard band w to obtain an acceptance (or rejection) interval such that the risk of an incorrect decision is the desired one.

While the JCGM106 document [12] offers the theoretical considerations to solve the problem, it provides an explicit solution only in the very particular case when the measurement result is represented by a normal probability density function. In such a case, it is suggested to set $w = U$, where $U = 2u$ is the expanded uncertainty that defines an interval about the measured value with a coverage probability of 95.45%. It can be readily checked that, in this case, the probability of exceeding the tolerance limits is 2.3% when the measured value is the same as the acceptance (or rejection) limits. This is also the particular case considered by the ISO 14253-1 Standard [23], of which the JCGM106 document is a generalization.

Practical solutions to set the acceptance (or rejection) limits, having defined a maximum allowable risk, are given in [24] for some common probability density functions and in [25] for the more general case when the probability density function is numerically estimated by a MonteCarlo simulation [20]. In particular, Ferrero et al. [24] derived useful closed-form formulas that can also be used to provide the standard uncertainty value required not to exceed a given risk to exceed the tolerance limits when the acceptance (or rejection) limits are set.

A. EXAMPLES

To fully understand how the recommendations provided by [12] can be extremely useful in designing a reliable decision-making procedure, let us consider the simple example of a resistor, with rated resistance value $R_n = 100\ \Omega$ and a tolerance interval of $\pm 2\%$ about the rated value. This means, using the same notation as the one recommended by [12] and above recalled, that $T_L = 98\ \Omega$ and $T_U = 102\ \Omega$.

Let us now suppose that the available measuring system provides measurement results with a standard uncertainty $u(R_n) = 0.50\ \Omega$, and that it is desired not to exceed a Maximum Admissible Risk $MAR = 5\%$ of accepting an out-of-tolerance resistor. Therefore, in this example, it is required to evaluate the pertaining guard band w that leads to define the A_L and A_U limits of the acceptance interval.

If the probability density function associated with the measurement result is supposed to be normal, it can be proved [24] that the desired guard band w is given by

$$w = \sqrt{2} \cdot u(R_n) \cdot \operatorname{erfinv}(1 - 2 \cdot \mathrm{MAR}) \tag{6}$$

where erfinv(•) is the inverse error function [24].

It is then possible to evaluate $A_L = T_L + w$ and $A_U = T_U - w$ so that the following values are obtained: $A_L = 98.82\ \Omega$ and $A_U = 101.18\ \Omega$. Fig. 6 shows the obtained acceptance interval and also shows that in the worst case, when the measured value coincides with the acceptance limits, the risk of exceeding the tolerance limits, represented by the colored areas, is lower or equal to the desired 5%.

As a further example, let us now suppose that the probability function associated with the measurement result is trapezoidal, symmetrical with respect to the measured value, with a standard deviation $\sigma = u(R_n) = 0.5\ \Omega$ and a ratio between the two bases $\beta = 0.75$. Under such conditions, the standard deviation is related to the major base a by

$$\sigma = \frac{a}{\sqrt{\frac{1+\beta}{24}}}. \tag{7}$$

It can also be proved [24] that the desired guard band is still directly related to standard uncertainty and is given by

$$w = u(R_n) \cdot \sqrt{\frac{6}{1+\beta}} \cdot \left(1 - \sqrt{2 \cdot MAR \cdot \left(1 - \beta^2\right)}\right). \tag{8}$$

[11] Although in most practical cases the same guard band is considered to define the acceptance and rejection intervals, for the sake of generality and to follow the JCGM106 document approach [12], two different values and two separate figures are used here.

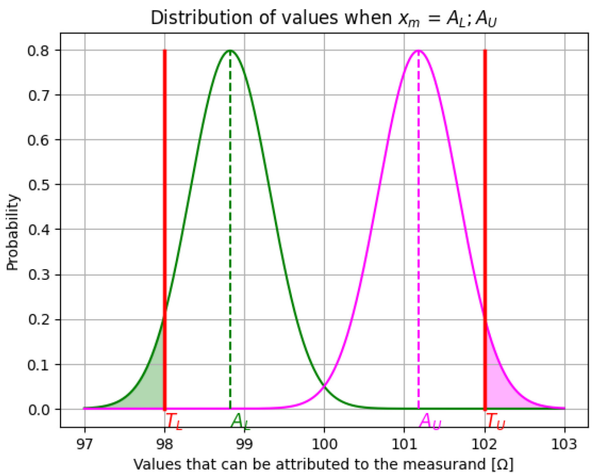

FIGURE 6. Acceptance interval (between vertical dotted lines), given a tolerance interval (between vertical solid lines), when the resistance values are measured with a 0.5 Ω standard uncertainty and the probability density function associated with the measurement result is supposed to be normal. Such distributions are shown when the measured values are A_L (green line) and A_U (pink line). The colored areas represent the residual risk of 5%.

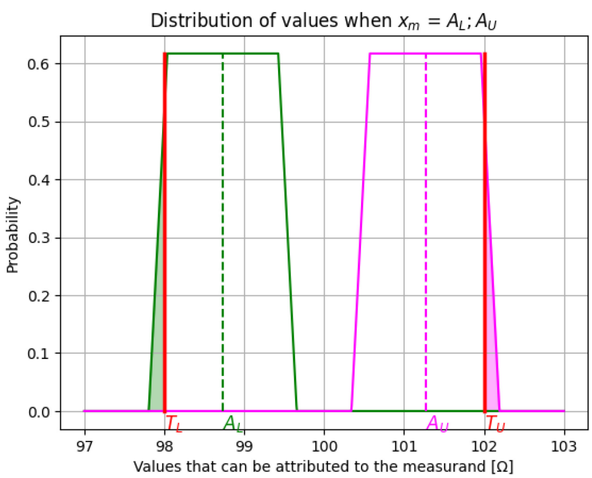

FIGURE 7. Acceptance interval (between vertical dotted lines), given a tolerance interval (between vertical solid lines), when the resistance values are measured with a 0.5 Ω standard uncertainty and the probability density function associated with the measurement result is supposed to be trapezoidal. Such distributions are shown when the measured values are A_L (green line) and A_U (pink line). The colored areas represent the residual risk of 5%.

Similarly to the case of the normal distribution, it is now possible to obtain the limits of the acceptance interval as: $A_L = T_L + w = 98.73\ \Omega$ and $A_U = T_U - w = 101.27\ \Omega$. Fig. 7 shows the obtained acceptance interval and also shows that the risk of exceeding the tolerance limits, represented by the colored areas, is lower or equal to the desired 5%.

Let us now consider a different example, always referring to the same resistor, the same tolerance interval, and the same maximum allowable risk *MAR* of exceeding the tolerance limits. In this example, unlike the previous one in which the measurement uncertainty was given and the acceptance limits had to be found, the acceptance limits are given and set to $A_L = 98.5\ \Omega$ and $A_U = 101.5\ \Omega$, and the standard uncertainty $u(R_n)$ must be evaluated.

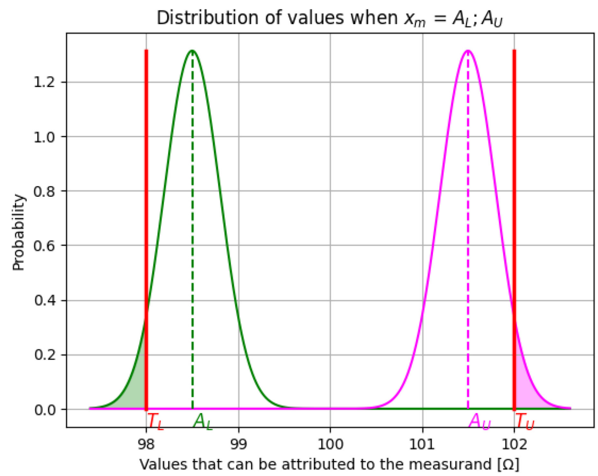

FIGURE 8. Normal probability distributions associate with the resistance measurement results when the measured values are A_L (green line) and A_U (pink line) and the standard uncertainty is provided by (9). The colored areas represent the residual risk of 5%.

This is the typical case when it is requested to identify the metrological characteristics of the measuring system (in this case the standard uncertainty) that satisfy given conditions.

Under these conditions, having set the acceptance limits, the width of the guard band is also set to $w = 0.5\ \Omega$ that is smaller than in the previous example. If a normal probability density function is associated with the measurement result, (6) can be solved on $u(R_n)$ so that

$$u(R_n) = \frac{w}{\sqrt{2} \cdot \mathrm{erfinv}(1 - 2 \cdot \mathrm{MAR})} \tag{9}$$

and $u(R_n) = 0.30\ \Omega$ is obtained. Fig. 8 shows the normal probability density functions associated with the measurement results when the measured values are A_L and A_U. The reduction in the required standard uncertainty with respect to the case in Fig. 6 to achieve the same risk of false acceptance is evident.

It is also possible to consider different probability density functions associated with the measurement result. Let us consider a trapezoidal function with the same characteristics as the one considered in the previous example. It is now possible to solve (8) on $u(R_n)$ so that

$$u(R_n) = \frac{w}{\sqrt{\frac{6}{1+\beta}} \cdot \left(1 - \sqrt{2 \cdot MAR \cdot \left(1 - \beta^2\right)}\right)} \tag{10}$$

and $u(R_n) = 0.34\ \Omega$ is obtained. Fig. 9 shows the trapezoidal probability density functions associated with the measurement results when the measured values are A_L and A_U. Since the risk of false acceptance is the same as the one in the case in Fig. 7, the narrower the width of the guard band, the lower the measurement standard uncertainty.

As a final remark, the above examples considered only guarded acceptance. However, similar considerations also hold for guarded rejection.

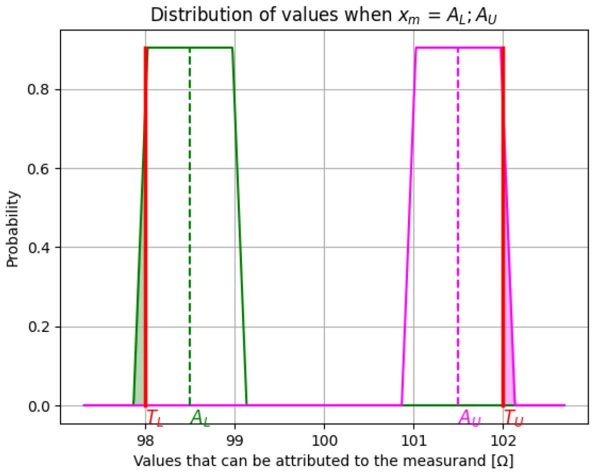

FIGURE 9. **Trapezoidal probability distributions associated with the resistance measurement results when the measured values are A_L (green line) and A_U (pink line) and the standard uncertainty is provided by (10). The colored areas represent the residual risk of 5%.**

V. CONCLUSION

This article has briefly discussed the approach to uncertainty definition proposed by the GUM [10], issued by the Joint Committee on Guides in Metrology (JCGM) and widely accepted in metrology as a best practice. It is well known that any measurement result can only represent an approximation of the unknown true value of the measurand, so that a doubt always remains about how well a measurement result may represent the measurand. Such a doubt may become quite critical whenever the measurement result must be compared with another measurement result or a threshold.

This article has shown how the GUM approach, strictly based on probability, allows one to define suitable decision rules that take into account the risk of making incorrect decisions due to measurement uncertainty. In particular, the decision rule based on guard bands, proposed by document JCGM 106 [12] has been discussed in the previous Section IV.

This discussion and the given examples show how to master the risk of making incorrect decisions either by setting suitable acceptance (or rejection) limits, or by designing the employed measuring system in such a way that the related measurement uncertainty allows one to keep the risk of exceeding the given limits at the desired level.

The procedure proposed in this article has general validity whenever a decision is based on experimental data resulting from sensor or instrument output values and, if consistently adopted, may lead to a reduction of the overall cost due to the cost of measurement and the detrimental effects of incorrect decisions.

Furthermore, the validity of the GUM approach to uncertainty and, more in general, the modern approach to assessing data validity, can be also extended to the evaluation of nonphysical properties, such as, for instance, quality, comfort, and other weakly defined quantities [26] as long as also the related uncertainty is evaluated. Since more and more decisions performed in any human endeavor can leverage on trustworthy data, the approach reported in this article might represent a useful tool toward information-enabled decision-making procedures, with general validity across different fields.

In a future perspective, when decisions might be taken by AI classifiers, a thorough revision of the method presently recommended by the Standard Organizations would be needed, since nowadays only a few classifiers provide an estimate of the probability of right decision, though not obtained by processing measurement uncertainty as shown in this article. Conversely, when systems based on generative AI are concerned, the problem of assessing uncertainty is still open and quite difficult to face.

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