

Article

Decision Uncertainty from Strict Preferences in Sequential Search Scenarios with Multiple Criteria

Debora Di Caprio ¹, Yolanda Durán Durán ² and Francisco Javier Santos-Arteaga ^{2,*}¹ Department of Economics and Management, University of Trento, 38122 Trento, Italy; debora.dicaprio@unitn.it² Departamento de Economía Financiera y Actuarial y Estadística, Universidad Complutense de Madrid, 28003 Madrid, Spain; yolandur@ucm.es

* Correspondence: fransant@ucm.es

Abstract: The standard expected utility model applied by economists and decision scientists assumes both that decision makers (DMs) are rational and that their information retrieval behavior and choices are determined by the observed and potential values of the multiple characteristics defining the alternatives. In this regard, if DMs can formalize the information acquisition structures determined by the main postulates of expected utility theory, they should also be able to perform standard operations regarding the potential combinatorial outcomes that may be obtained when evaluating the alternatives. We define an information retrieval scenario where DMs account for the different combinatorial possibilities arising among the realizations of the characteristics defining the alternatives before evaluating them. We demonstrate the indifference that arises among risk-neutral DMs endowed with standard expected utilities within sequential information acquisition environments such as those defined by online search engines. We also illustrate the reticence of DMs to acquire information on new alternatives when increasing their aversion to risk or modifying the relative importance assigned to the different characteristics defining the alternatives. The main strategic consequences that follow from the enhanced information retrieval scenario proposed are also analyzed.

Keywords: multi-attribute sequential search; uncertainty; expected utility; indifference; risk attitude

MSC: 91B06; 91B16; 91B08



Academic Editor: Yu-Wang Chen

Received: 26 March 2025

Revised: 14 April 2025

Accepted: 21 April 2025

Published: 22 April 2025

Citation: Di Caprio, D.; Durán Durán, Y.; Santos-Arteaga, F.J. Decision Uncertainty from Strict Preferences in Sequential Search Scenarios with Multiple Criteria. *Mathematics* **2025**, *13*, 1368. <https://doi.org/10.3390/math13091368>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction and Contribution

The information retrieval behavior of decision makers (DMs) when dealing with the evaluation of alternatives composed of more than one characteristic is generally formalized using the expectation operator as the benchmark determining their choices [1,2]. That is, a DM who considers the effect of future evaluations when defining his current choices does so using the expected value obtained from a set of potential realizations together with their associated probabilities [3,4]. The verification of these predefined expectations conditions the satisfaction derived from the choices made [5]. Similarly, decision trees, defined to summarize and describe the rational and optimal behavior of DMs facing sequential information retrieval and evaluation environments, are solved assuming that DMs consider the expected value arising from any set of potential realizations arising at each node [6,7].

The use of expected utility as a reference benchmark extends into uncertain decision settings formalized across a wide variety of scientific disciplines [8], including the evolution of strategic international relations [9]. Alternatives to expected utility—such as the regret and dual processing theory—have been defined in medical environments where DMs are highly conditioned by subjective judgments and cognitive processes [10]. However, these variants do not consider the combinatorial interactions across characteristics that could be assessed by DMs when dealing with the uncertainty inherent to their evaluation processes.

The sequential search process of DMs endowed with well-defined preferences requires deciding constantly between evaluating an alternative further or discarding it and retrieving information on a new one [11]. DMs tend to seek more variety in their purchases when assisted by Artificial Intelligence agents, particularly within low-involvement environments [12]. In this regard, recommendation agents simplify purchase decisions while triggering higher uncertainty and lower choice satisfaction, with risk aversion limiting the impact of both effects [13]. Trustworthiness mitigates the risk perceived by DMs when using technology to evaluate alternatives [14], while consumer ambivalence increases with the effort and risks perceived when browsing through an online platform [15]. The information retrieval model introduced in the current paper is sufficiently flexible to incorporate these effects while also illustrating how indifference may arise among DMs endowed with well-defined strict preferences.

Consider a DM who retrieves information from a set of alternatives defined by two types of independent characteristics. This scenario is standard within online information retrieval environments and the current Artificial Intelligence-enhanced recommender systems [16,17]. Facing uncertainty, the DM assigns a uniform density to their respective domains and derives an expected value for each characteristic [18,19]. Intuitively, we tend to assume that the DM observes the second set of characteristics from those alternatives whose initial realizations are higher than the expected value of the first characteristic [20]. Given the value of the first observation, the realization of the second characteristic eliminates any uncertainty from the evaluation of the alternative, allowing the DM to make a completely informed decision [21,22]. Thus, DMs should indeed be able to evaluate the whole set of potential combinations that may be defined across characteristics before selecting the next observation to gather.

The current paper addresses a research question generally omitted among scholars studying information retrieval processes and rational choice, namely, if DMs are able to formalize and implement information acquisition and decision frameworks determined by the main postulates of expected utility, they should also be able to perform standard operations regarding the potential combinatorial outcomes that may be obtained when evaluating the alternatives. Enhancing the capacity of DMs to evaluate combinatorial scenarios that go beyond the automatic implementation of the expected value operator should have immediate effects on their information retrieval incentives and consequences when studying sequential and intertemporal decision scenarios.

We analyze the consequences derived from implementing this type of combinatorial retrieval behavior within a sequential information acquisition environment such as the one defined by online search engines [23,24]. We demonstrate that risk-neutral DMs retrieving information from alternatives whose characteristics have identical supports become indifferent between continuing to evaluate a given alternative or discarding it for realizations higher than or equal to the certainty equivalent value. That is, our model illustrates the indifference that arises among risk-neutral DMs endowed with standard expected utilities when considering the main combinatorial possibilities derived from their information retrieval decisions.

We will also use the model to describe numerically the main consequences derived from modifying the relative importance of the characteristics defining the alternatives and the risk attitudes of DMs on their information retrieval incentives. The simulations illustrate the reticence of DMs to acquire information on new alternatives when increasing their aversion to risk or the relative importance assigned to the first characteristics defining the alternatives.

The paper proceeds as follows: The next section describes the basic assumptions required to define the expected search utilities introduced in Section 3. Section 4 analyzes the resulting information retrieval framework, which is illustrated numerically together with a variety of alternative scenarios in Section 5. Section 6 concludes and suggests potential extensions.

2. Basic Notations and Main Assumptions

The evaluation model determining the basic retrieval incentives of DMs is defined by Di Caprio and Santos Arteaga [25]. We summarize the main notations and basic assumptions below.

Let $\succsim$ be a preference relation defined on the nonempty set X . Let X_1 and X_2 represent the sets of all the potential realizations that may be observed when evaluating the first and second characteristics of an alternative. We will assume that for every $k = 1, 2$, there exist $x_k^m, x_k^M > 0$, with $x_k^m \neq x_k^M$, such that $X_k = [x_k^m, x_k^M]$, where x_k^m and x_k^M are the minimum and maximum value of the characteristic X_k . The preferences of DMs are represented using an additive utility function u whose components $u_k : X_k \rightarrow \mathbb{R}$, with $k = 1, 2$, are continuous and strictly increasing utility functions on X_k , such that $\forall \langle x_1, x_2 \rangle \in X_1 \times X_2$, $u(\langle x_1, x_2 \rangle) = u_1(x_1) + u_2(x_2)$.

DMs assign a continuous probability density to each X_k , $\mu_k : X_k \rightarrow [0, 1]$, with $k = 1, 2$, whose support, $S(\eta_k)$, consists of the set $\{x_k \in X_k : \mu_k(x_k) \neq 0\}$. The densities $\mu_k(x_k)$, $k = 1, 2$, describe the subjective beliefs of DMs regarding the probability that a randomly selected alternative displays a value of $x_k \in X_k$ as its k -th characteristic. We will assume that the densities μ_1 and μ_2 are independent, though conditional relations across characteristics could be easily introduced in the model.

DMs consider the k -th certainty equivalent defined through μ_k and u_k as the benchmark determining their information retrieval behavior. In particular, the certainty equivalent induced by μ_k and u_k , denoted by ce_k , $k = 1, 2$, is the value in X_k providing the DM with the same utility as the expected one derived from μ_k and u_k . That is, for each $k = 1, 2$, $ce_k = u_k^{-1}(E_k)$, thus, $u(ce_k) = u(u_k^{-1}(E_k)) = E_k$, where E_k corresponds to the expected value of u_k . The continuity and strict increasingness of u_k guarantee the existence and uniqueness of ce_k , respectively.

We will assume through most of the analysis that DMs are risk-neutral; that is, they evaluate the realizations of the different characteristics using linear utility functions, $u_k(x_k) = x_k$, $k = 1, 2$. This assumption will be relaxed when comparing different evaluation scenarios numerically in Section 5, where the information retrieval incentives of risk-averse DMs will be studied. Following the standard decision-theoretical approach [2], the attitude of DMs towards risk will be defined in terms of the coefficient of relative risk aversion

$$R_k(x_k) = \frac{-x_k u_k''(x_k)}{u_k'(x_k)}, \quad k = 1, 2. \quad (1)$$

3. Expected Search Utilities

DMs evaluate the first characteristic from an alternative and must decide whether to continue retrieving information from the same alternative or to discard it and start acquiring

information on a new alternative. We will initially assume that both characteristics are equally important for the DM. This assumption will be modified to illustrate numerically the case where the first characteristic is more important than the second one, providing the DM with a higher expected utility.

The retrieval incentives of DMs are determined by two real-valued expected utility functions defined on X_1 . The sum of the expected utilities defined by $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$, $E_1 + E_2$, constitutes one of the main benchmark values determining the behavior of these functions.

After evaluating the first characteristic from an alternative, x_1 , the DM may continue evaluating the second characteristic of the alternative, x_2 . In this case, the gain in expected utility over the reference expected value $E_1 + E_2$ is determined by the value of x_1 . That is, for each $x_1 \in X_1$, defined by

$$\Delta^+(x_1) = \{x_2 \in X_2 \cap S(\eta_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \quad (2)$$

and

$$\Delta^-(x_1) = \{x_2 \in X_2 \cap S(\eta_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\} \quad (3)$$

the set of x_2 values whose combination with those of x_1 provide the DM with a utility above or below that of an alternative chosen randomly, respectively.

The resulting continuation function $C : X_1 \rightarrow \mathfrak{R}$ is defined as follows:

$$C(x_1) \stackrel{\text{def}}{=} \int_{\Delta^+(x_1)} \eta_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + \int_{\Delta^-(x_1)} \eta_2(x_2)(E_1 + E_2) dx_2. \quad (4)$$

To provide some intuition, note that the continuation function evaluated at the certainty equivalent value under risk neutrality equals

$$C(ce_1) \stackrel{\text{def}}{=} \int_{E_2}^{x_2^M} \eta_2(x_2)(u_1(ce_1) + u_2(x_2)) dx_2 + \int_{x_2^m}^{E_2} \eta_2(x_2)(E_1 + E_2) dx_2. \quad (5)$$

Given the uniform densities defined on both characteristics and the fact that $u_1(ce_1) = E_1$, the previous function simplifies to

$$C(ce_1) \stackrel{\text{def}}{=} \int_{E_2}^{x_2^M} \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + u_2(x_2)) dx_2 + \int_{x_2^m}^{E_2} \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) dx_2. \quad (6)$$

The continuation function describes the expected utility received when observing x_2 for every value of x_1 . The decision made by the DM is determined by the relative values of $u_1(x_1) + u_2(x_2)$ and $E_1 + E_2$. Clearly, if the DM would not consider the potential interactions between both characteristics when evaluating an alternative, the decision would be based on the values of $u_1(x_1)$ and E_1 .

The discard function is determined by the potential improvements that may be observed relative to the partial evaluation of a given alternative x_1 ; that is, this function defines the expected utility derived from discarding the alternative observed and retrieving information on a new one while remaining with the uncertainty derived from not observing the initial alternative fully and obtaining only partial information about the new alternative.

If the DM discards an alternative, he must consider evaluating a new one while remaining with unresolved uncertainty regarding its second characteristic. We can denote

by y_1 the first characteristic of a new alternative different from the one being evaluated, and then, for each value $x_1 \in X_1$, we can define

$$\Phi^+(x_1) = \{y_1 \in X_1 \cap S(\eta_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \quad (7)$$

and

$$\Phi^-(x_1) = \{y_1 \in X_1 \cap S(\eta_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\} \quad (8)$$

as the sets of y_1 values improving upon or lacking with respect to the maximum between x_1 and the expected utility derived from a random choice, E_1 , respectively.

The discard function $D : X_1 \rightarrow \Re$ is defined as follows:

$$D(x_1) \stackrel{\text{def}}{=} \int_{\Phi^+(x_1)} \eta_1(y_1)(u_1(y_1) + E_2) dy_1 + \int_{\Phi^-(x_1)} \eta_1(y_1)(\max\{u_1(x_1), E_1\} + E_2) dy_1. \quad (9)$$

The function evaluated at the certainty equivalent value equals

$$D(ce_1) \stackrel{\text{def}}{=} \int_{E_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m} \right) (u_1(y_1) + E_2) dy_1 + \int_{x_1^m}^{E_1} \left(\frac{1}{x_1^M - x_1^m} \right) (E_1 + E_2) dy_1. \quad (10)$$

Whenever both functions cross; that is, if $\exists x_1^* \in X_1$ such that $C(x_1^*) = D(x_1^*)$, we obtain a threshold value that modifies the information acquisition behavior of DMs. That is, the continuation and discard functions formalize and compare the incentives of DMs to either continue acquiring information on a partially observed alternative and complete its evaluation or discard it and partially observe a new alternative. If the functions cross within the domain of the first characteristic, the retrieval incentives of DMs can shift through the evaluation process.

The next section studies the existence of x_1^* values and the subsequent partition of X_1 into intervals determining the retrieval behavior of DMs within a risk-neutral environment. In particular, we will illustrate the indifference arising whenever the intervals defining the potential realizations of the characteristics are identical.

4. Thresholds and Indifference

The existence of threshold values determining the information retrieval behavior of DMs can be guaranteed by imposing standard assumptions [25]. Through this section, we demonstrate that risk-neutral DMs retrieving information from alternatives whose characteristics have identical supports become indifferent between continuing and discarding for realizations higher than or equal to the certainty equivalent value.

Proposition 1. Consider evaluation intervals X_1 and X_2 with identical domains, that is, $[x_1^m, x_1^M]$ and $[x_2^m, x_2^M]$ such that $x_1^m = x_2^m$ and $x_1^M = x_2^M$. Assume complete uncertainty about the realizations of the characteristics, leading DMs to assign a uniform probability function to each evaluation interval. If DMs are risk-neutral, defining identical linear utilities on both characteristic spaces, then $C(x_1) = D(x_1)$, $\forall x_1 \geq ce_1$.

Proof. We consider the value of the continuation, $C(x_1)$, and discard, $D(x_1)$, functions when $x_1^m = x_2^m$ and $x_1^M = x_2^M$. To simplify the presentation, we eliminate the subscripts from the corresponding domain values and refer to x^m and x^M through the proof.

$$C(x_1) = \int_{x_2 > E_1 + E_2 - x_1} \left[\frac{1}{x^M - x^m} \right] (x_1 + x_2) dx_2 + \int_{x_2 \leq E_1 + E_2 - x_1} \left[\frac{1}{x^M - x^m} \right] (E_1 + E_2) dx_2. \quad (11)$$

$$D(x_1) = \int_{y_1 > x_1} \left[\frac{1}{x^M - x^m} \right] (y_1 + E_2) dy_1 + \int_{y_1 \leq x_1} \left[\frac{1}{x^M - x^m} \right] (x_1 + E_2) dy_1. \quad (12)$$

We solve the corresponding integrals and unify the notation of the discard function

$$C(x_1) = \left[\frac{1}{x^M - x^m} \right] \left(x_1 x^M + \frac{(x^M)^2}{2} - x_1(E_1 + E_2 - x_1) - \frac{(E_1 + E_2 - x_1)^2}{2} \right) + \left[\frac{E_1 + E_2 - x_1 - x^m}{x^M - x^m} \right] (E_1 + E_2). \quad (13)$$

$$D(x_1) = \left[\frac{1}{x^M - x^m} \right] \left(\frac{(x^M)^2}{2} + x^M E_2 - \frac{x_1^2}{2} - x_1 E_2 \right) + \left[\frac{x_1 - x^m}{x^M - x^m} \right] (x_1 + E_2). \quad (14)$$

After some basic algebra, we obtain

$$C(x_1) = x_1 x^M + (E_1 + E_2 - x_1) \left[\frac{E_1 + E_2 - x_1 - 2x^m}{2} \right]. \quad (15)$$

$$D(x_1) = (x^M - x^m) E_2 + \frac{x_1^2}{2}. \quad (16)$$

Given the assumptions regarding identical domains, densities, and utilities, it trivially follows that $E_1 = E_2$. Substituting in the above equations and operating, we obtain

$$C(x_1) = 2x_1 x^M + (2E_1 - x_1)[2E_1 - x_1 - 2x^m] \quad (17)$$

$$D(x_1) = 2(x^M - x^m)E_1 + x_1^2 \quad (18)$$

which, after some algebra, simplifies to

$$C(x_1) = 2E_1^2 - E_1 x^m \quad (19)$$

$$D(x_1) = x^M E_1 \quad (20)$$

Clearly, for $C(x_1) = D(x_1)$, we just require that

$$2E_1^2 = x^M E_1 + E_1 x^m \quad (21)$$

Given the fact that $E_1 = \frac{x^M + x^m}{2}$, we derive the final equality between both functions, namely, $E_1(x^M + x^m) = 2E_1^2$, which completes the proof. $\square$

The following corollary follows immediately from Proposition 1.

Corollary 1. Consider a risk-neutral DM assigning a uniform density function to the domains defining both characteristics $[x_1^m, x_1^M]$ and $[x_2^m, x_2^M]$. Assume that these domains are identical, that is, $x_1^m = x_2^m$ and $x_1^M = x_2^M$. The $C(x_1)$ and $D(x_1)$ functions cross at the point $x_1 = ce_1$.

Within the current setting, the continuation and discard functions do not cross again. To illustrate this feature, we first prove that $C(x_1^m) < D(x_1^m)$.

Proposition 2. Consider evaluation intervals X_1 and X_2 with identical domains, that is, $[x_1^m, x_1^M]$ and $[x_2^m, x_2^M]$ such that $x_1^m = x_2^m$ and $x_1^M = x_2^M$. Assume complete uncertainty about the realizations of the characteristics, leading DMs to assign a uniform probability function to each evaluation interval. If DMs are risk-neutral, defining identical linear utilities on both characteristic spaces, then $C(x_1^m) < D(x_1^m)$.

Proof. At x_1^m , the continuation, $C(x_1^m)$, and discard, $D(x_1^m)$, functions are equal to

$$C(x_1^m) = \int_{ce_1+ce_2-x_1^m}^{x_2^M} \left(\frac{1}{x_2^M - x_2^m} \right) (u_1(x_1^m) + u_2(x_2)) dx_2 + \int_{x_2^m}^{ce_1+ce_2-x_1^m} \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) dx_2. \quad (22)$$

$$D(x_1^m) = \int_{ce_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m} \right) (u_1(x_1) + E_2) dx_1 + \int_{x_1^m}^{ce_1} \left(\frac{1}{x_1^M - x_1^m} \right) (E_1 + E_2) dx_1. \quad (23)$$

After solving the corresponding integrals, we obtain

$$C(x_1^m) = \left(\frac{1}{x_2^M - x_2^m} \right) \left(x_2^M x_1^m + \frac{(x_2^M)^2}{2} - (ce_1 + ce_2 - x_1^m) x_1^m - \frac{(ce_1 + ce_2 - x_1^m)^2}{2} \right) + \left(\frac{ce_1 + ce_2 - x_1^m - x_2^m}{x_2^M - x_2^m} \right) (E_1 + E_2). \quad (24)$$

$$D(x_1^m) = \left(\frac{1}{x_1^M - x_1^m} \right) \left(\frac{(x_1^M)^2}{2} + x_1^M E_2 - \frac{ce_1^2}{2} - ce_1 E_2 \right) + \left(\frac{ce_1 - x_1^m}{x_1^M - x_1^m} \right) (E_1 + E_2). \quad (25)$$

Operating after some basic algebra, we obtain

$$C(x_1^m) = \left(x_2^M x_1^m + \frac{(x_2^M)^2}{2} - (ce_1 + ce_2 - x_1^m) x_1^m - \frac{(ce_1 + ce_2 - x_1^m)^2}{2} \right) + (ce_2 - x_2^m)(E_1 + E_2). \quad (26)$$

$$D(x_1^m) = \left(\frac{(x_1^M)^2}{2} + x_1^M E_2 - \frac{ce_1^2}{2} - ce_1 E_2 \right). \quad (27)$$

Comparing both functions, the expected utility from discarding is higher than the one derived from continuing if

$$\begin{aligned} D(x_1^m) &= \left(x_1^M E_2 - \frac{ce_1^2}{2} - ce_1 E_2 \right) > \\ C(x_1^m) &= \left(x_2^M x_1^m - (ce_1 + ce_2 - x_1^m) x_1^m - \frac{(ce_1 + ce_2 - x_1^m)^2}{2} \right) + (ce_2 - x_2^m)(E_1 + E_2). \end{aligned} \quad (28)$$

Given the assumptions regarding identical domains, densities, and utilities, it trivially follows that $ce_1 = E_1 = E_2 = ce_2$. Substituting in the above equation and operating, we obtain

$$\begin{aligned} &\left(x_1^M E_2 - \frac{E_1^2}{2} - E_1 E_2 \right) > \\ &\left(x_2^M x_1^m - (E_1 + E_2 - x_1^m) x_1^m - \frac{(E_1 + E_2 - x_1^m)^2}{2} \right) + (E_2 - x_2^m)(E_1 + E_2) \end{aligned} \quad (29)$$

which leads to $2x_1^M E_1 + 4E_1 x_1^m > 2x_1^M x_1^m + 3E_1^2 + (x_1^m)^2$.

Given the fact that $E_1 = \frac{x_1^M + x_1^m}{2}$, we derive the following expression

$$2x_1^M \left(\frac{x_1^M + x_1^m}{2} \right) + 4 \left(\frac{x_1^M + x_1^m}{2} \right) x_1^m > 2x_1^M x_1^m + 3 \left(\frac{x_1^M + x_1^m}{2} \right)^2 + (x_1^m)^2 \quad (30)$$

which simplifies to $(x_1^M)^2 + (x_1^m)^2 - 2x_1^M x_1^m > 0$, trivially implying that $(x_1^M - x_1^m)^2 > 0$. $\square$

We conclude by illustrating that the slope of the continuation function decreases through the $[x_1^m, ce_1]$ interval while that of the discard function remains constant.

Proposition 3. Consider evaluation intervals X_1 and X_2 with identical domains, that is, $[x_1^m, x_1^M]$ and $[x_2^m, x_2^M]$ such that $x_1^m = x_2^m$ and $x_1^M = x_2^M$. Assume complete uncertainty about the realizations of the characteristics, leading DMs to assign a uniform probability function to each evaluation interval. If DMs are risk-neutral, defining identical linear utilities on both characteristic spaces, then $\frac{\partial C(x_1)}{\partial x_1} < \frac{\partial D(x_1)}{\partial x_1}$, $\forall x_1 \leq ce_1$.

Proof. Note first that the discard function defines a straight line within the $[x_1^m, ce_1]$ interval

$$D(y_1) = \int_{ce_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m} \right) (u_1(x_1) + E_2) dx_1 + \int_{x_1^m}^{ce_1} \left(\frac{1}{x_1^M - x_1^m} \right) (E_1 + E_2) dx_1, \quad \forall y_1 < ce_1, \quad (31)$$

implying that $\frac{\partial D(y_1)}{\partial y_1} = 0$.

The continuation function decreases whenever x_1 shifts from ce_1 towards x_1^m , where its slope equals zero. Given the definition of the function $\forall x_1 < ce_1$

$$C(x_1) = \int_{ce_1+ce_2-x_1}^{x_2^M} \left(\frac{1}{x_2^M - x_2^m} \right) (u_1(x_1) + u_2(x_2)) dx_2 + \int_{x_2^m}^{ce_1+ce_2-x_1} \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) dx_2, \quad (32)$$

and applying Leibniz's rule, we obtain

$$\begin{aligned} \frac{\partial C(x_1)}{\partial x_1} = & \int_{ce_1+ce_2-x_1}^{x_2^M} \frac{\partial}{\partial x_1} \left(\frac{1}{x_2^M - x_2^m} \right) (u_1(x_1) + u_2(x_2)) dx_2 + \left(\frac{1}{x_2^M - x_2^m} \right) (u_1(x_1) + u_2(x_2^M)) \frac{dx_2^M}{dx_1} - \\ & \left(\frac{1}{x_2^M - x_2^m} \right) (u_1(x_1) + u_2(ce_1 + ce_2 - x_1)) \frac{d(ce_1 + ce_2 - x_1)}{dx_1} + \\ & \int_{x_2^m}^{ce_1+ce_2-x_1} \frac{\partial}{\partial x_1} \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) dx_2 + \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) \frac{d(ce_1 + ce_2 - x_1)}{dx_1} - \\ & \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) \frac{dx_2^m}{dx_1} \end{aligned} \quad (33)$$

which simplifies to

$$\frac{\partial C(x_1)}{\partial x_1} = \int_{ce_1+ce_2-x_1}^{x_2^M} \left(\frac{1}{x_2^M - x_2^m} \right) dx_2 + \left(\frac{1}{x_2^M - x_2^m} \right) (u_2(ce_1 + ce_2)) - \left(\frac{1}{x_2^M - x_2^m} \right) (E_1 + E_2) \quad (34)$$

and

$$\frac{\partial C(x_1)}{\partial x_1} = \left(\frac{x_2^M - (ce_1 + ce_2 - x_1)}{x_2^M - x_2^m} \right) \geq 0 \quad (35)$$

The above equation is trivially satisfied for $x_1 = ce_1$. The $x_1 < ce_1$ case only requires $x_2^M + x_1 - (ce_1 + ce_2) \geq 0$, which follows immediately from $x_2^M + x_1 - \frac{x_1^M + x_1^m + x_2^M + x_2^m}{2} \geq 0$, since this inequality simplifies to $x_1 - x_1^m \geq 0$ given the assumptions of the model. $\square$

Together with Corollary 1, $C(ce_1) = D(ce_1)$, and Proposition 2, $C(x_1^m) < D(x_1^m)$, Proposition 3 implies that $C(x_1)$ and $D(x_1)$ do not cross for values of $x_1 < ce_1$, with the discard function remaining above the continuation one throughout the corresponding interval.

5. Discussion

The formal environment analyzed applies to most online decision settings where the main characteristics defining the alternatives are evaluated within a common structured

numerical framework [26]. Thus, risk-neutral DMs browsing through online evaluation and selection environments may become indifferent between the discard and continue options, implying that DMs may discard alternatives with high values of their first characteristic that should otherwise be accepted. Figure 1 illustrates this feature numerically within the $u(x_k) = x_k$, $S(\eta_k) = [0, 10]$, $k = 1, 2$, retrieval framework. Indifference and a potentially suboptimal behavior can be the results derived from optimizing the information retrieval process of a rational DM whose expected utilities satisfy the standard postulates commonly assumed among decision theorists.

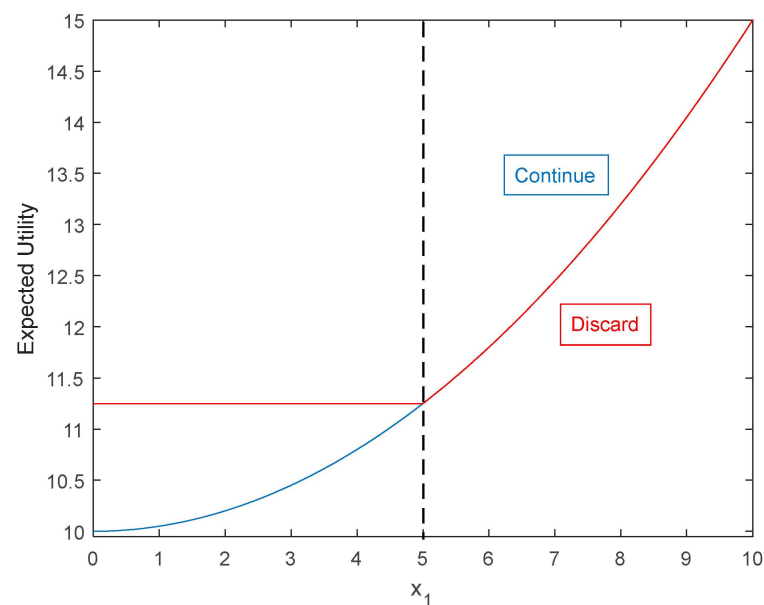


Figure 1. Continuation and discard functions: $u(x_k) = x_k$, $k = 1, 2$, on the domain $[0, 10]$.

Figure 2 illustrates how the introduction of risk aversion modifies the shape of the continuation and discard functions, leading to a retrieval behavior similar to the one that should be observed in a standard expected utility setting where the certainty equivalent defines the main threshold value [27]. That is, higher values of $R_k(x_k)$, $k = 1, 2$, increase the incentives of DMs to continue acquiring information on the alternative being evaluated.

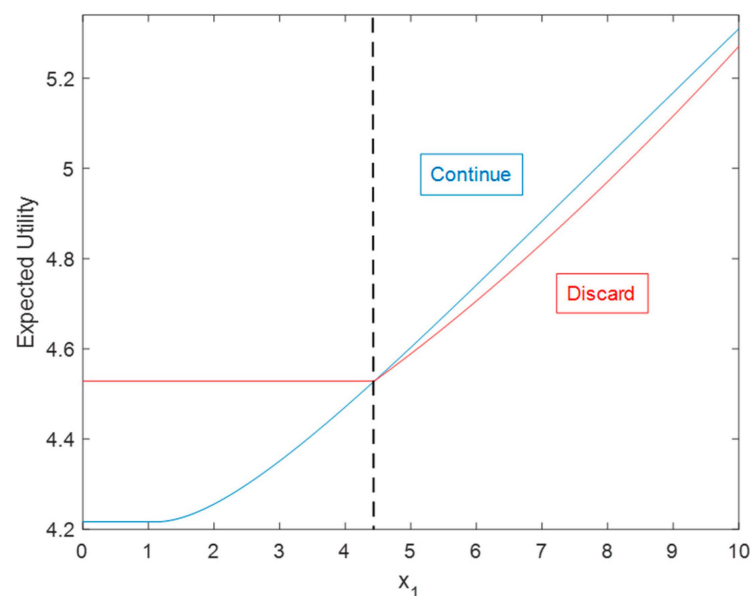


Figure 2. Continuation and discard functions: $u(x_k) = \sqrt{x_k}$, $k = 1, 2$, on the domain $[0, 10]$.

At the same time, the combination of risk aversion with the stochastic dominance of the first alternative—relative to the second—shifts the threshold below the certainty equivalent value. As illustrated in Figures 3 and 4, these features would lead DMs to continue acquiring information on the initial product for increasingly lower values of the first characteristic. In this case, eliminating indifference from the retrieval behavior of DMs may lead to regret if they must purchase the product to observe the set of secondary experience characteristics [28,29].

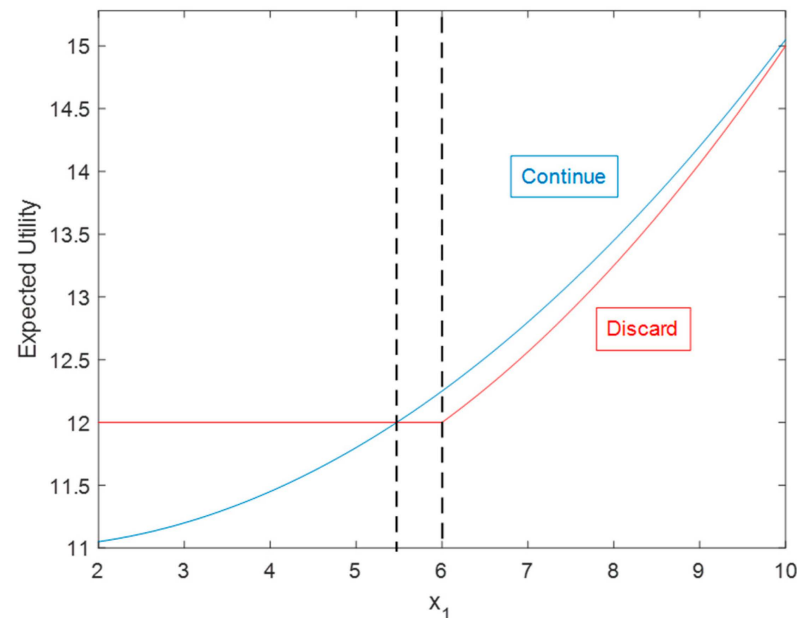


Figure 3. Continuation and discard functions: $u(x_k) = x_k$, $k = 1, 2$, with $S(\eta_1) = [2, 10]$ and $S(\eta_2) = [0, 10]$.

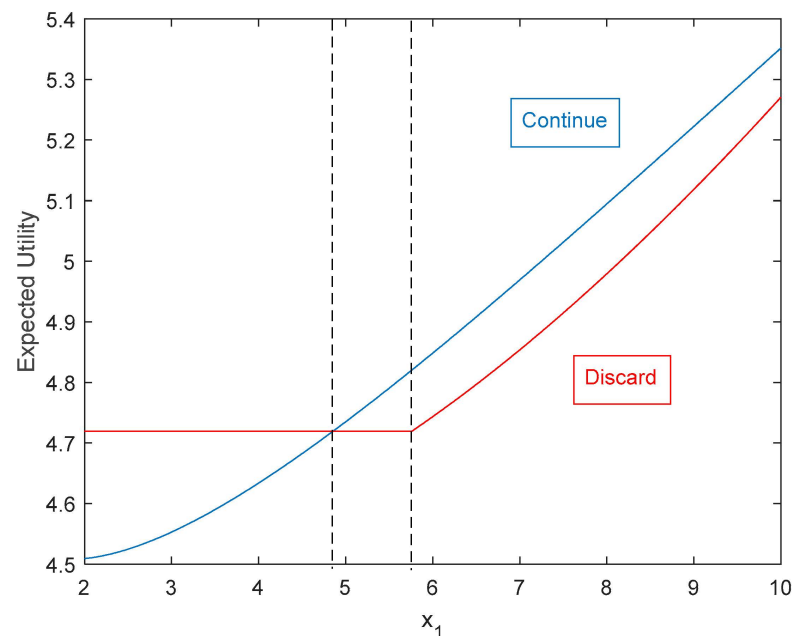


Figure 4. Continuation and discard functions: $u(x_k) = \sqrt{x_k}$, $k = 1, 2$, with $S(\eta_1) = [2, 10]$ and $S(\eta_2) = [0, 10]$.

An immediate strategic quality can be derived from the simulations presented in these figures. An increment in the aversion to risk of DMs when evaluating alternatives

whose first characteristics are relatively more important than the second ones increases their incentives to focus on the initial alternatives being evaluated. Thus, fostering any of these qualities may induce DMs to limit their information acquisition incentives to a subset of alternatives strategically displayed in the initial positions of their evaluation processes.

6. Conclusions

The standard expected utility approach, commonly used in economics and decision sciences, assumes both that DMs are rational and that their decisions are determined by current and future events, the latter being assimilated through the expected values of the different potential realizations. Our model has demonstrated the indifference that may arise among risk-neutral DMs endowed with standard expected utilities when considering the main combinatorial possibilities derived from their information retrieval decisions. We have also illustrated numerically the aversion to search that may be induced when increasing the aversion to risk of DMs and the relative importance assigned to the first characteristics defining the alternatives. The malleability of the retrieval framework presented allows us to consider a variety of potential extensions. For instance, we have ignored both cognitive biases [30,31] and the memory capacity of DMs and their ability to modify their reference benchmark whenever the characteristics observed are located above the corresponding certainty equivalent values [32,33].

Further extensions could aim at increasing the complexity of the sequential information retrieval structure by introducing a third set of characteristics and operating within the subsequent three-dimensional environment [34]. While plausible from a theoretical viewpoint, the complexity of the resulting information retrieval and evaluation processes would require DMs to be endowed with a substantial computational capacity [35]. In this regard, Cheek and Goebel [36] illustrated that DMs who aim at maximizing their utilities tend to experience decision difficulty, displaying a negative relation between their search for alternatives and decisiveness. At the same time, signals and learning processes could be introduced to formalize strategic environments that account for a variety of interactions taking place among multiple DMs and information senders [37].

Author Contributions: Conceptualization, D.D.C. and F.J.S.-A.; methodology, D.D.C. and F.J.S.-A.; software, F.J.S.-A.; validation D.D.C., Y.D.D. and F.J.S.-A.; formal analysis, D.D.C. and F.J.S.-A.; investigation, D.D.C., Y.D.D. and F.J.S.-A.; resources, D.D.C., Y.D.D. and F.J.S.-A.; writing—original draft preparation, D.D.C., Y.D.D. and F.J.S.-A.; writing—review and editing, D.D.C., Y.D.D. and F.J.S.-A.; funding acquisition, F.J.S.-A. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: No new data were created or analyzed in this study.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Qi, J.; Han, Z.; Zhong, H. Behavioural economics theories in information-seeking behaviour research: A systematic review. *J. Librariansh. Inf. Sci.* **2024**, 09610006231219246. [[CrossRef](#)]
2. Kahneman, D.; Tversky, A. *Choices, Values, and Frames*; Cambridge University Press: Cambridge, UK, 2000.
3. Zhang, C.X.; Zhao, M.; Cai, M.Y.; Xiao, Q.R. Multi-stage multi-attribute decision making method based on online reviews for hotel selection considering the aspirations with different development speeds. *Comput. Ind. Eng.* **2020**, *143*, 106421. [[CrossRef](#)]
4. Bearden, J.N.; Connolly, T. Multi-attribute sequential search. *Organ. Behav. Hum. Decis. Process.* **2007**, *103*, 147–158. [[CrossRef](#)]
5. Khalek, S.A.; Dey, D.; Chakraborty, A.; Samanta, T. “From expectations to frustrations”: Dissecting negative experiences to understand negative word-of-mouth in online grocery services. *J. Retail. Consum. Serv.* **2025**, *84*, 104221. [[CrossRef](#)]

6. Di Caprio, D.; Santos Arteaga, F.J.; Tavana, M. An information retrieval benchmarking model of satisficing and impatient users' behavior in online search environments. *Expert Syst. Appl.* **2022**, *191*, 116352. [\[CrossRef\]](#)
7. Hambarde, K.A.; Proença, H. Information retrieval: Recent advances and beyond. *IEEE Access* **2023**, *11*, 76581–76604. [\[CrossRef\]](#)
8. Kydd, A.H. Realism and rational choice. *Crit. Rev.* **2024**, *36*, 560–574. [\[CrossRef\]](#)
9. Lammers, W.; Ferrari, S.; Wenmackers, S.; Pattyn, V.; Van de Walle, S. Theories of uncertainty communication: An interdisciplinary literature review. *Sci. Commun.* **2024**, *46*, 332–365. [\[CrossRef\]](#)
10. Scarffe, A.; Coates, A.; Brand, K.; Michalowski, W. Decision threshold models in medical decision making: A scoping literature review. *BMC Med. Inform. Decis. Mak.* **2024**, *24*, 273. [\[CrossRef\]](#)
11. Zhou, F.; Jiang, Y.; Qian, Y.; Liu, Y.; Chai, Y. Product consumptions meet reviews: Inferring consumer preferences by an explainable machine learning approach. *Decis. Support Syst.* **2024**, *177*, 114088. [\[CrossRef\]](#)
12. Zhang, Y.; Yang, M.; Zhang, Z. Uncertainty of artificial intelligence assistant: The effect of assistant type on variety seeking. *Front. Psychol.* **2022**, *13*, 904302. [\[CrossRef\]](#) [\[PubMed\]](#)
13. Rohden, S.F.; Espartel, L.B. Consumer reactions to technology in retail: Choice uncertainty and reduced perceived control in decisions assisted by recommendation agents. *Electron. Commer. Res.* **2024**, *24*, 901–923. [\[CrossRef\]](#)
14. Sohn, S. Consumer perceived risk of using autonomous retail technology. *J. Bus. Res.* **2024**, *171*, 114389. [\[CrossRef\]](#)
15. Yu, Y.; Davis, F.; Walden, E.; Turel, O. Unpacking conflicting evaluations and ambivalence in online hotel booking: The moderating role of perceived enjoyment in user retention. *J. Retail. Consum. Serv.* **2025**, *85*, 104266. [\[CrossRef\]](#)
16. Rohden, S.F.; Zeferino, D.G. Recommendation agents: An analysis of consumers' risk perceptions toward artificial intelligence. *Electron. Commer. Res.* **2023**, *23*, 2035–2050. [\[CrossRef\]](#)
17. Chang, W.; Park, J. A comparative study on the effect of ChatGPT recommendation and AI recommender systems on the formation of a consideration set. *J. Retail. Consum. Serv.* **2024**, *78*, 103743. [\[CrossRef\]](#)
18. Gu, C.; Wang, Y. Consumer online search with partially revealed information. *Manag. Sci.* **2022**, *68*, 4215–4235. [\[CrossRef\]](#)
19. Zhang, Z.; Nan, G.; Li, M.; Tan, Y. Competitive entry of information goods under quality uncertainty. *Manag. Sci.* **2022**, *68*, 2869–2888. [\[CrossRef\]](#)
20. Qi, J.; Han, Z.; Hansen, P. Information search process model based on costs and benefits: A behavioural economics perspective. *J. Doc.* **2024**, *80*, 1494–1507. [\[CrossRef\]](#)
21. Gönsch, J. How much to tell your customer?—A survey of three perspectives on selling strategies with incompletely specified products. *Eur. J. Oper. Res.* **2020**, *280*, 793–817. [\[CrossRef\]](#)
22. Lin, Y.; Pazgal, A. Effects of information quantity and diversity on consumers under complex uncertainty. *J. Retail. Consum. Serv.* **2024**, *77*, 103652. [\[CrossRef\]](#)
23. Di Caprio, D.; Santos-Arteaga, F.J. Rational satisficing heuristics as determinants of online search behavior. *Int. J. Inf. Manag. Data Insights* **2024**, *4*, 100229. [\[CrossRef\]](#)
24. Di Caprio, D.; Santos-Arteaga, F.J.; Tavana, M. A new algorithm for modeling online search behavior and studying ranking reliability variations. *Appl. Intell.* **2022**, *52*, 7529–7549. [\[CrossRef\]](#)
25. Di Caprio, D.; Santos-Arteaga, F.J. An optimal information gathering algorithm. *Int. J. Appl. Decis. Sci.* **2009**, *2*, 105–150. [\[CrossRef\]](#)
26. Harris, P.; Dall'Olmo Riley, F.; Hand, C. Multichannel shopping: The effect of decision making style on shopper journey configuration and satisfaction. *J. Retail. Consum. Serv.* **2021**, *58*, 102286. [\[CrossRef\]](#)
27. Li, M.; Choudhury, A.H. Using website information to reduce postpurchase dissonance: A mediated moderating role of perceived risk. *Psychol. Mark.* **2021**, *38*, 56–69. [\[CrossRef\]](#)
28. Kim, K. Who reads customer reviews after product choice? Maximizing, satisficing, and post-choice information search. *Pers. Individ. Dif.* **2022**, *184*, 111192. [\[CrossRef\]](#)
29. Nelson, P. Information and consumer behaviour. *J. Polit. Econ.* **1970**, *78*, 311–329. [\[CrossRef\]](#)
30. Gomroki, G.; Behzadi, H.; Fattahi, R. Identifying effective cognitive biases in information retrieval. *J. Inf. Sci.* **2023**, *49*, 348–358. [\[CrossRef\]](#)
31. Chen, T. A systematic integrative review of cognitive biases in consumer health information seeking: Emerging perspective of behavioral information research. *J. Doc.* **2021**, *77*, 798–823. [\[CrossRef\]](#)
32. Gardete, P.M.; Hunter, M. Multiattribute search: Empirical evidence and information design. *Mark. Sci.* **2024**, *43*, 1052–1080. [\[CrossRef\]](#)
33. Yuan, R.; Xi, X.; Liu, Z. Random “decision and experienced utility”, adaptive “consumer memory and choice”: The impact of mind fluctuations and cognitive biases on consumption and classification. *Mathematics* **2023**, *11*, 1470. [\[CrossRef\]](#)
34. Di Caprio, D.; Santos-Arteaga, F.J.; Tavana, M. An optimal sequential information acquisition model subject to a heuristic assimilation constraint. *Benchmarking Int. J.* **2016**, *23*, 937–982. [\[CrossRef\]](#)
35. Chanda, S.S.; Ray, S. Is comprehensiveness in making strategic decisions always helpful? *Aust. J. Manag.* **2023**, 03128962231201497. [\[CrossRef\]](#)

36. Cheek, N.; Goebel, J. What does it mean to maximize? “Decision difficulty,” indecisiveness, and the jingle-jangle fallacies in the measurement of maximizing. *Judgm. Decis. Mak.* **2020**, *15*, 7–24. [[CrossRef](#)]
37. Shi, J.; Lu, H.; Liu, W. Tripartite evolutionary game analysis for manufacturing firms, service providers and consumers in collaborative innovation systems. *Mathematics* **2025**, *13*, 428. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.