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Advanced Materials for Applications at High Temperature: Fatigue Assessment by Means of Local Strain Energy Density

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The interest on fatigue assessment of steel and other alloys at high temperature has increased continuously in the last years. However, high cycle fatigue of notched components at high temperature has not been deeply investigated experimentally and theoretically.

The present paper investigates the accuracy of Strain Energy Density (SED) averaged over a control volume approach when applied to high temperature fatigue data from notched components. In the present work a large bulk of high temperature fatigue data, taken from literature and regarding notched components made of different advanced materials, is reanalysed by means of the SED approach. In detail: C45 carbon steel at 250°C, Inconel 718 at 500°C and directionally solidified superalloy DZ125 at 850°C are considered. This validation allowed proving that the proposed approach can be a reliable design method in practical applications when dealing with high temperature. The main advantage of SED averaged over a control volume is that different geometries can be summarised in a single narrow scatter band. From an industrial viewpoint, where different components are continuously redesigned, the use of a geometry independent parameter (and fatigue curve) leads to a considerable advantage in terms of time and cost.

1. Introduction

The interest on fatigue assessment of steels and different alloys at high temperature has increased continuously in the last years. The applications in which fatigue phenomenon is affected by high temperature are of considerable interest and involve different industrial sectors such as transportation, energy and metal-manufacturing (e.g. jet engine components, nuclear power plant, pressure vessel, hot rolling of metal). To provide as optimum performance as possible in these high demanding conditions, it is necessary to be aware of the application and of proper tools to perform the fatigue assessment at high temperature.

The state of the art regarding high temperature fatigue of advanced materials employed in such high demanding conditions, suggests that problem has been addressed mainly in terms of strain, focusing the attention on the plastic part of the cyclic deformation. However, it must be pointed out that real components present a complex shape, and in some applications they must be designed for high number of cycles fatigue regime. From these last considerations, it emerges the importance to investigate the high cycles fatigue regime and notch effect at elevated temperatures. As recently underlined by the present authors,[1] the high-temperature notch fatigue problem has been mainly investigated considering only low and medium fatigue regime. On the contrary, little research work has been carried out in order to formalise and validate appropriate design methods suitable for designing against high-cycles fatigue of notched metallic components experiencing high temperature during in-service operations.

Just to name few works, Hamada et al.[2] discussed the effects of non-proportional loading on low cycle fatigue (LCF) lives for Type 304 stainless steel at 923 K. In fact, high temperature applications sometimes involve multiaxial damage rather than uniaxial. Fifteen proportional and non-proportional strain paths at a strain rate of 0.1%/s were employed. They concluded that non-proportional straining drastically decreased low cycle fatigue lives of the considered steel. A parameter α quantifying the amount of additional hardening at high temperature was proposed. However, only low cycle fatigue was considered.

An interesting work was presented by Prasad et al.[3] Low cycle fatigue deformation behaviour of forged turbine disc of IN 718 superalloy of different sections, and thus geometries, was studied under asymmetrical waveforms (slow-fast and fast-slow) at 650°C. The superalloy exhibited dynamic strain aging by showing serration during the lower strain rate plastic region of the hysteresis loop under asymmetrical waveforms. Irrespective of different sections, the superalloy showed a marginally reduction of fatigue life under slow-fast waveform as compared to fast-slow waveform, because of the LCF damage accumulation.

Numerous experimental as well as theoretical investigations have been conducted also to quantify the effect of stress/strain concentration on the fatigue behaviour of different advanced materials experiencing high temperature in service,[4-7] considering also single-crystal components.[8,9]

Because of the lack of literature on high cycle fatigue behaviour of notched components at high temperature, the present authors, in the last years, have devoted some efforts in experimental testing of different materials (of industrial relevance) at high temperature. Moreover, the Strain Energy Density (SED) approach, successfully applied at room temperature in the past, has been extended to high temperature considering the obtained data.

Recently, 40CrMoV13.9 steel has been considered and tested at different temperatures, up to 650°C.[1] Smooth specimens and plates weakened by symmetric V-notches have been considered. A total of 60 new experimental data were provided. At 500°C, the steel showed a drastic decrement of the fatigue strength, while, up to 500°C, no differences were observed with respect to the room temperature data. All new data from room temperature up to 500°C were also summarized by means of SED averaged over a control volume. Regarding instead the temperature of 650°C, a specific energy master curve was proposed. Plates with central hole made of the same steel were also recently considered, investigating the effect of different surface roughness on the fatigue strength.[10]

The present authors have summarized the results form uniaxial tension load-controlled fatigue tests on notched specimens made of Cu-Be alloy.[11] Plates with central hole, as well as smooth specimens have been considered and tested at 650°C. A decrement of 39% in the stress range at 2 million cycles was detected comparing the fatigue strength at high temperature and at room temperature. It was also shown a good microstructural stability of this material when subjected to long time exposure at high temperature. The fatigue curves, expressed in terms of stress range, were presented in terms of SED averaged over a control volume, with very good results. In fact, regardless of the specimen geometries, all the fatigue data have been summarized in a single narrow scatter band.

Since the titanium has been recognized for its strategic importance and lightweight, high strength alloy, high temperature fatigue tests were conducted at room temperature and 500°C considering semi-circular notches and plates weakened by symmetric V-notches.[12] A different behaviour has been observed for the two considered geometries: the semi-circular notches presented a reduction of 44% of the stress range at two million cycles with respect to the fatigue strength at room temperature, while the V-notches specimens showed a negligible decrement. Also in this case, a synthesis based on the SED approach was obtained, and all the fatigue data, regardless of the notch geometries, were summarised in a single narrow scatter band. Other attempts to extend such a method to high temperature have been made by the present authors in the last years, considering also other high temperature effect such as creep.[13-17] Some authors have gone further on this topic, investigating the interactive creep-fatigue crack growth, obtaining very good results.[18]

Kawagoishi and co-workers[19,20] investigated the strength of nickel-base superalloy Inconel 718 under rotating banding loading at room temperature and 500°C in air. The applicability of linear notch mechanics to the evaluation of notched fatigue strength at elevated temperature was assessed in terms of the fatigue limit for crack initiation and that for crack growth. The effect of temperature on the fatigue strength and notch sensitivity of Inconel 718 was examined. They showed that linear notch mechanics is applicable not only at room temperature but also at elevated temperature, as long as the small-scale yielding condition is satisfied.

Shi et al.,[21] investigated the effect of notch on fatigue behaviour of a directionally solidified superalloy DZ125 at 850°C. Single-edge notched specimens with V and U type geometries were tested, with a stress ratio R=0.1. It was underlined that for the same applied nominal stress, the fatigue resistance decreased with K_t increasing from 1.76 to 4.35. Moreover, the impact of U-type and V-type notch on the fatigue behaviour was close. It has been proposed that K_t can be regarded as a key parameter to dominate the notch fatigue when absolute dimensions of notched specimens are similar. Different fatigue curves were obtained, as well as a deep analysis of ratcheting effect, crack initiation and failure mechanics at high temperature.

Recently, Louks and Susmel[22] investigated the accuracy of Theory of Critical Distances (TCD) in estimating high-cycle fatigue strength of notched metallic materials experiencing elevated temperatures during in-service operations. They checked the accuracy of TCD against a number of experimental results generated, under uniaxial loading, by testing at 250°C notched specimens made of carbon steel C45, as well as considering many data taken from literature. It has been proved that the TCD can be used as a fatigue assessment parameter at high temperature.

The aim of the present paper is to validate the application of the SED approach as a reliable tool for high-cycle fatigue assessment of notched components, at high temperature. Different data taken from the literature have been considered and re-analysed by means of the strain energy density averaged over a control volume. In detail, C45 carbon steel tested at 250°C,[22] Inconel 718 tested at 500°C[19,20] and directionally solidified superalloy DZ125 at 850°C[21] have been considered.

This validation allowed proving that the proposed approach can be a reliable design parameter in practical applications when dealing with high temperature fatigue. The main advantage of the SED averaged over a control volume is that different geometries can be summarised in a single narrow scatter band, regardless of the notch radius and opening angle. From an industrial viewpoint, where components are usually redesigned, the use of a parameter (and fatigue curve) that is not dependent on the geometrical shape leads to a considerable advantage in terms of time and costs.

2. Fundamentals of the Strain Energy Density Averaged Over a Control Volume

The averaged Strain Energy Density (SED) method, that has been formalized by Lazzarin and co-workers for sharp[23] and blunt notches,[24] derived from Neuber's concept of elementary volume, and local mode I concept proposed by Erdogan and Sih.[25] When dealing with fracture assessment of cracked and notched components, a clear distinction should be done between large and small bodies[26-28]. In fact, the design rules applied to large bodies, assume that local inhomogeneities can be averaged because of the large volume surface ratio involved. In small bodies, instead, the high ratio within surface and volume makes not negligible the local discontinuities and an adoption of multiscaling scheme is needed.[26,27] The averaged SED method, under the hypothesis of large bodies, combines the concept of energy criterion with the advantages tied to the definition of a material-dependent structural volume. The fundamental basis and late developments of the strain energy density approach have been summarized in some recent contributions.[29-31] In those works, the analytical frame and the main applications of the averaged SED approach are revisited in detail. More than eighty papers published over the last fifty years are recalled and reviewed. Such a method has been extensively used in the literature and its predictive capability, especially when dealing with fatigue of notched components, has been largely proofed. In details, it has been applied successfully in static and fatigue assessment of cracks, blunt and V notches geometries and also considering welded joints made of different alloys.[32] This approach is based on the idea that under tensile stresses, failure occurs when the strain energy density averaged over a given control volume, $\overline{W}$, reaches the critical energy value W_c (that depends on the selected material). In plane problems, the control volume becomes a circle or a circular sector with a radius R_c. **Figure 1** shows example of control areas. In the case of blunt notches, the area assumes a crescent shape. Considering the mixed-mode loading, the control area is no longer centred with respect to the notch bisector, but rotated and centred on the point where the maximum principal stress reaches its maximum value.

Under plane strain conditions, R_c for static loading can be easily evaluated by using the following equation[33] as a function of the fracture toughness and the ultimate tensile stress:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{1}$$

In case of fatigue loading, R_c depends on smooth specimen fatigue limit $\Delta\sigma_0$ and on the threshold behaviour ΔK_{th} for metallic materials:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{2}$$

When $\nu=0.3$, Equation (2) gives $R_c=0.845a_0$, where $a_0 = (1/\pi)(\Delta K_{th}/\Delta\sigma_0)^2$ is the El Haddad-Smith-Topper parameter.[34]

With reference to plane stress conditions, simple algebraic considerations give for static and fatigue loading, respectively:

$$R_c = \frac{(5-3\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{3}$$

$$R_c = \frac{(5-3\nu)}{4\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{4}$$

Resulting in $R_c=1.025a_0$ for $\nu=0.3$.

If the material behaviour is ideally brittle, then the critical energy value W_c can be evaluated by using the conventional ultimate tensile strength σ_t for static cases and the fatigue strength $\Delta\sigma_0$ for fatigue loading:

$$W_c = c_w \frac{\sigma_t^2}{2E} \tag{5}$$

$$\Delta W_c = c_w \frac{\Delta\sigma_0^2}{2E} \tag{6}$$

The parameter c_w is introduced to take into account the possible influence of the nominal load ratio on the variation of the deviatoric energy given to the material in one cycle, according to the simple rule reported in literature.[35] It is derived analytically, by simple considerations on the evaluation of the strain energy density range that is nothing but the area under the σ-ε curve. In other words, the parameter c_w represents the ratio within ΔW in case of generic R and ΔW(R=0), which is assumed as a reference value. It was derived in particular c_w=1.0 for R=0 and c_w=0.5 for R=-1. In fact, as a result of the reduction of the effective SED, the fatigue strength range for R=-1 should theoretically increase according to a factor $\sqrt{2}$ with respect to the case R=0. More details regarding this parameter can be found in the given references.

Dealing with blunt notches under fatigue loading, the following expression can be used to evaluate the strain energy density range:[24]

$$\Delta\overline{W} = c_w F(2\alpha) \times H(2\alpha, \frac{R_c}{\rho}) \times \frac{K_{t,n}^2 \Delta\sigma_n^2}{E} \tag{7}$$

Here $\Delta\sigma_n$ is the stress range, $K_{t,n}$ is the theoretical stress concentration factor (both referred to the net sectional area), E is the Young's modulus. F is a function that depends on the notch opening angle. Finally H depends on the notch angle and on the critical radius-notch tip radius ratio. Values of the function H and F for numerous Poisson's ratios and opening angles are given in literature.[24] The coefficient c_w takes into account the different loading ratio, as already explained before.

Once defined the control volume, the SED can be easily evaluated through a finite element analysis. As opposed to the evaluation of the NSIFs that needs very refined meshes and high computational effort, the mean value of the elastic SED on the control volume can be determined with high accuracy by using very coarse meshes.[36] The SED, in fact, can be derived directly from the nodal displacements, that are accurately evaluated also when coarse mesh is employed. Other important advantages can be achieved by using the SED approach. The most relevant are:

- It permits consideration of the scale effect which is fully included in the NSIF approach;
- It permits consideration on the contribution of different modes;[37,38]
- It permits consideration of the cycle nominal load ratio;

- It overcomes the problem tied to the different NSIF units of measure in the case of different notch opening angles;
- It directly takes into account the T-stress for thin structures;
- It directly includes the loading effects thanks to the analytical parameter c_w cited previously;
- It directly includes three-dimensional effect and out-of-plane singularities.[39,40]

Under the hypothesis that all the material inhomogeneities can be averaged,[26,27] the SED approach showed to be a powerful tool both for static and fatigue strength assessment of notched and welded structures.

3. Validation of Strain Energy Approach Applied to High Temperature Fatigue

Recently, Louks and Susmel[22] presented some results from high temperature fatigue tests conducted on C45 notched specimens. The aim was to check the accuracy of linear-elastic TCD in estimating high-cycle fatigue strength of notches metals, when experiencing in service high temperature. Plain and notched samples of C45 were considered and tested under uniaxial loading at 250°C. The load ratio, R, was set equal to 0.1 and the load frequency to 15 Hz. C45 is a very common and conventional structural steel that presents lower mechanical properties with respect to high-performance alloys employed in high temperature applications. However, there are situations of practical interest (e.g. vehicle engines, engine beds) where conventional structural steels experience medium/high temperatures. For C45 steel, the fatigue damage reaches its maximum value at a temperature in the range of 200-250 °C,[41] which is the motivation to choose 250°C as the testing temperature. The geometries are shown in detail in **Figure 2**. Plain, Blunt U-Notches, Sharp U-Notches and Sharp V-Notches were considered. **Table 1** reports all the values of the tested geometries.

The strain energy density of the plain specimens has been evaluated through Equation (6). At 250°C, C45 steel presents a Young's modulus E=190 GPa. The evaluation of the control radius for the notched geometries, instead, has been evaluated analytically following the existing link within the theory of the critical distance and the SED approach. Lazzarin and Berto[42] showed that the critical radius can be also expressed as function of $a_0=(1/\pi)(\Delta K_{th}/\Delta\sigma_0)^2$ that is the El Haddad-Smith-Topper parameter.[34] Considering a value of the Poisson's ratio $\nu=0.3$ and following Equation (4), one obtains the final relationship:

$$R_c = \frac{0.845}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 = 0.845 a_0 \tag{8}$$

To perform the high-cycle fatigue assessment of notched components, TCD makes use of the range of an effective stress which is calculated by taking into account a suitable material length scale parameter. This material characteristic length, called L, is a material property and defined as follows:[43-45]

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{9}$$

From Equations (8) and (9), the critical radius can be easily expressed as a function of L:

$$R_c = 0.845 a_0 = 0.845 L \tag{10}$$

Louks and Susmel[22] evaluated the value of the material characteristic length of the considered C45 steel, that has been found to be 0.252 mm. This value, according to Equation (10), returns a critical radius R_c=0.21 mm. This critical radius has been adopted here to reanalyse the same data at high temperature in terms of strain energy density averaged over a control volume. Briefly recalling the considered geometry (see Table 1 and Figure 2): plain specimens, blunt U-notches, sharp U-notches and sharp V-notches. The obtained curve in terms of SED averaged over a control volume is given in **Figure 3**. The fatigue data were statistically re-analysed by using a lognormal distribution and are plotted in terms of SED range referred to the net area. The run-out specimens (marked by tilted arrows) were excluded from the statistical analysis. A vertical line is drawn in correspondence of five hundred thousand cycles where the mean value of the SED range is given to make the comparison easier. Details about inverse slope k and the scatter index T_w are provided in the figure. It is evident that, thanks to SED approach, all the fatigue data, regardless of the specimen geometries, can be summarized in the same narrow scatter band. The value of the scatter index T_w is equal to 3.00 in terms of energy, and becomes T_σ=1.73 if reconverted in terms of local stress range.

Shi et al.[21] provided fatigue test results conducted at 850°C, considering flat specimens with a single lateral notch made of Directionally Solidified (DS) superalloy DZ125. It is a DS Ni-base superalloy used for turbine blade and vanes. It was characterized by the following elastic constants at 850°C:[21] E_1(L)=91 GPa, E_2(T)=117 GPa. The loading direction was parallel to the direction of solidification and the load ratio R was kept constant and equal to 0.1. The specimens were tested in air. U-notches and V-notches were considered, with different notch tip radii and opening angles:

- U-notched geometry with root radius of 0.4 mm and 0.2 mm, called U3 and U4 respectively;
- V-notched geometry with opening angle equal to 120° and tip radius of 0.3 mm, called V1;
- V-notched geometry with opening angle equal to 60° and tip radius of 0.2 mm called V2.

The detailed geometrical specifications are reported in **Table 2**. Refer to Figure 2 for the identification of the parameters. The control radius has been determined through Equation (10). Considering a value of the material characteristic length L equal to 0.452 mm,[22] a value of R_c=0.38 mm was obtained. The fatigue data reanalysed in terms of SED averaged over a control volume are given in **Figure 4**. The same procedure presented before has been adopted for the statistical analysis: data were statistically re-analysed by using a lognormal distribution; the run-out specimens (marked by tilted arrows) were not considered and a vertical line is drawn in correspondence of one million cycles. Figure 4 shows that all the fatigue data, regardless of the specimen geometries, can be summarized in the same narrow scatter band. The value of the scatter index T_w is equal to 3.18 in terms of energy, and becomes T_σ=1.78 if reconverted in terms of the local stress range.

Chen at al.[19] investigated the high-cycle fatigue behaviour of V-notched and smooth cylindrical specimens of Inconel 718, which is a precipitation hardened nickel-base superalloy widely used in the critical components of jet and gas turbine engines. The samples were tested under rotating bending (R=-1) at 500°C. The net diameter w_{net} of the samples was kept constant and equal to 8 mm, three different values of the notch root radius ρ were considered: 1 mm, 0.1 mm and 0.05 mm. The opening angle was kept constant and equal to 60°. By the way, only the fatigue curve of plain and V-notched geometry with a tip radius ρ of 0.05 mm was reported in detail. For this reason, only that geometry and the plain specimens were considered herein for the final synthesis based on SED. The details of the V-notched specimens considered in the present investigation are reported in **Figure 5** and listed as follows: tip radius $\rho=0.05$, opening angle $2\alpha=60°$, $w_{net}=8$ mm, gross diameter $w=9$ mm and total length $H=80$ mm. The plain cylindrical specimens presented a net section of 8 mm.

In order to reanalyse the data by means of the strain energy, it is necessary to evaluate the critical radius as already done for the previous considered advanced materials. Also in this case, as already explained before, the critical radius can be expressed as a function of the material characteristic length $L=0.154$ mm determined by Louks and Susmel.[22] R_c results to be equal to 0.13 mm. Considering the equations given above for the SED evaluation, it must be pointed out that the weighting parameter c_w[35] must be equal to 0.5 since the fatigue tests were carried out under rotating bending condition and then the load ratio R was equal to -1. The fatigue data reanalysed in terms of the SED averaged over a control volume are presented in **Figure 6**. The same procedure presented before was employed for the statistical analysis. All the details about the scatter index, the inverse slope k and other information related to the re-analysis are provided in the figure. It appears evident that both plain and notched specimens have been summarized in the same narrow scatter band characterized by a scatter index equal to 2.83 in terms of local energy and 1.68 if reconverted in terms of the stress range. The obtained scatter is not so far from the value obtained for the other series considered here and those previously analysed in literature.[32,46]

4. Discussion

High performance applications mostly required advanced and innovative materials, because of the high strength at high temperature and corrosive resistance possessed by them. Most researchers focused on creep and low cycle fatigue at high temperature but neglecting high cycle fatigue that is very important in many applications. Meanwhile, the design criteria and fatigue assessment methods for this kind of application have become more and more complicated, especially when dealing with low-cycle fatigue. Because of the lack of literature on high cycle fatigue of advanced materials at elevated temperature and the lack of simple reliable methods, some authors suggested to extend, if possible, linear notch mechanics also at high temperature, as asserted in the references given in the introduction. Following this idea, the present paper presented a synthesis by means of the Strain Energy Density averaged over a control volume of high temperature fatigue data taken from the literature, regarding advanced materials (DZ125, Inconel 718) and C45 steel.

The SED method has been successfully applied in the past and recent literature to room temperature fatigue data of notched components and welded joints,[24,30] and some attempts to extend the method also to high temperature has been done by the present authors considering different advanced materials such as Cu-Be alloy, Titanium[11,12] as well as other structural materials. The main idea is that the linear elastic approach can be applied not only at room temperature but also at elevated temperature, as long as the small-scale yielding condition is satisfied, as suggested by Nisitani and collaborators.[19]

In the results presented here, all the fatigue data taken from literature have been successfully summarised in terms of the SED, and this validation exercise strongly support the idea that, also at high temperature, some linear approaches can be applied neglecting non-linearities. In fact, as suggested by Lazzarin and Zambardi,[47] the linear-elastic energy equals the elasto-plastic one when they are averaged over the entire fatigue process zone. The advantage of the energy approach, moreover, becomes evident if comparing the SED data with the original high temperature fatigue data expressed in terms of stress range: regardless of the specimen geometry, all the results collapse in a single curve, maintaining a narrow scatter-band. The obtained curves presented here can be used as master curves to design against fatigue for the considered materials at elevated temperature. From an industrial view point where different components are redesigned several times, the SED approach can lead to an easy and fast way to design against fatigue. Recently, other similar results have been obtained considering thermo-mechanical tests by other authors, employing a nonlocal invariant area-averaging method to collapse all experimental data in a single trend.[48]

Another interesting aspect is that the SED can be easily evaluated through the given formulas or through a linear elastic finite element analysis with very coarse mesh.[30] Moreover, it can be linked to different well established method such as Theory of Critical Distances (as shown in the present paper) but also to J-Integral.[49-52]

The results are very promising and the extension of the method also to multiaxial fatigue at high temperature or under thermomechanical fatigue has been seriously considered. In addition, with the aim to better support the reliability of the method proposed here, more experimental tests and theoretical developments are on-going. More work need to be done in the field of high temperature fatigue at high number of cycles, in order to give an easy and fast tool to design against fatigue and to better theoretically support the extension of SED approach at elevated temperature.

5. Conclusion

The present paper investigated the accuracy of the Strain Energy Density (SED) averaged over a control volume approach when applied to high temperature fatigue data of notched components. In the present work a large bulk of high temperature fatigue data taken from literature, regarding notched components of C45 carbon steel at 250°C, Inconel 718 at 500°C and directionally solidified superalloy DZ125 at 850°C have been considered. This validation test allowed proving that the proposed approach can be a reliable design parameter in some practical applications when dealing with high temperature of notched components. The main conclusions can be summarized as follows:

- The Strain Energy Density averaged over a control volume has been extended successfully to high temperature applications;
- The method permitted to summarise in a single narrow scatter band all the fatigue data, regardless of the specimen geometry;
- All the considered fatigue data, taken from the literature, have been re-analysed successfully by means of SED approach, giving master curves for the considered advanced materials that can be used as an easy and fast tool to design against fatigue;
- At high temperature, until small scale yielding condition is satisfied, the linear notch mechanics and linear elastic approaches can be extended to high temperature high cycles fatigue;
- More work needs to be done in the field of high temperature fatigue in the high cycle regime and on the extension of linear elastic approaches (such as SED) also at high temperature.

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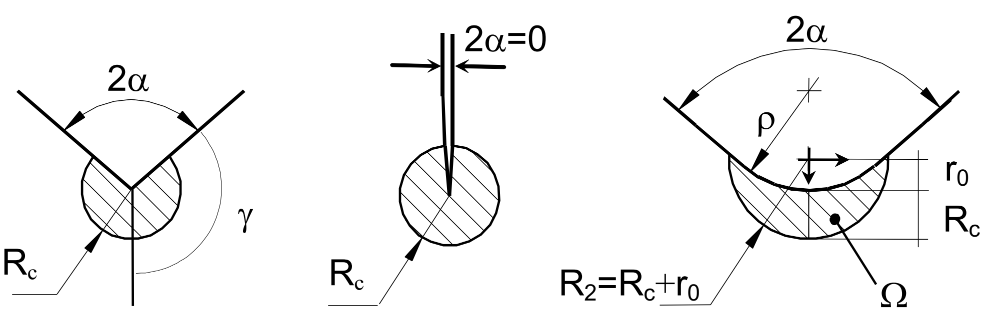

(a) (b) (c)

Fig. 1. Critical volume (area) for sharp V-notch (a), crack (b) and blunt V-notch (c) under mode I loading. Distance $r_0 = \rho \times (\pi - 2\alpha) / (2\pi - 2\alpha)$.

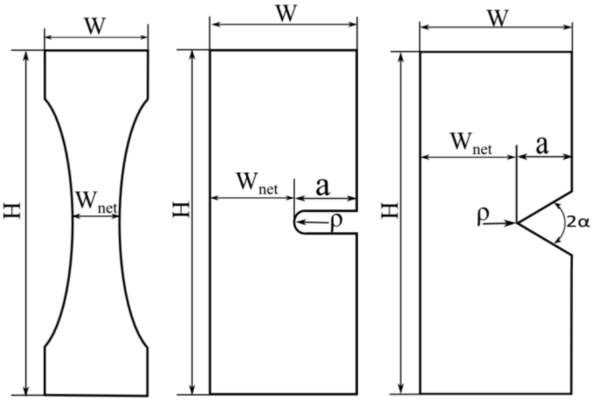

Fig. 2. Specimens geometry details.

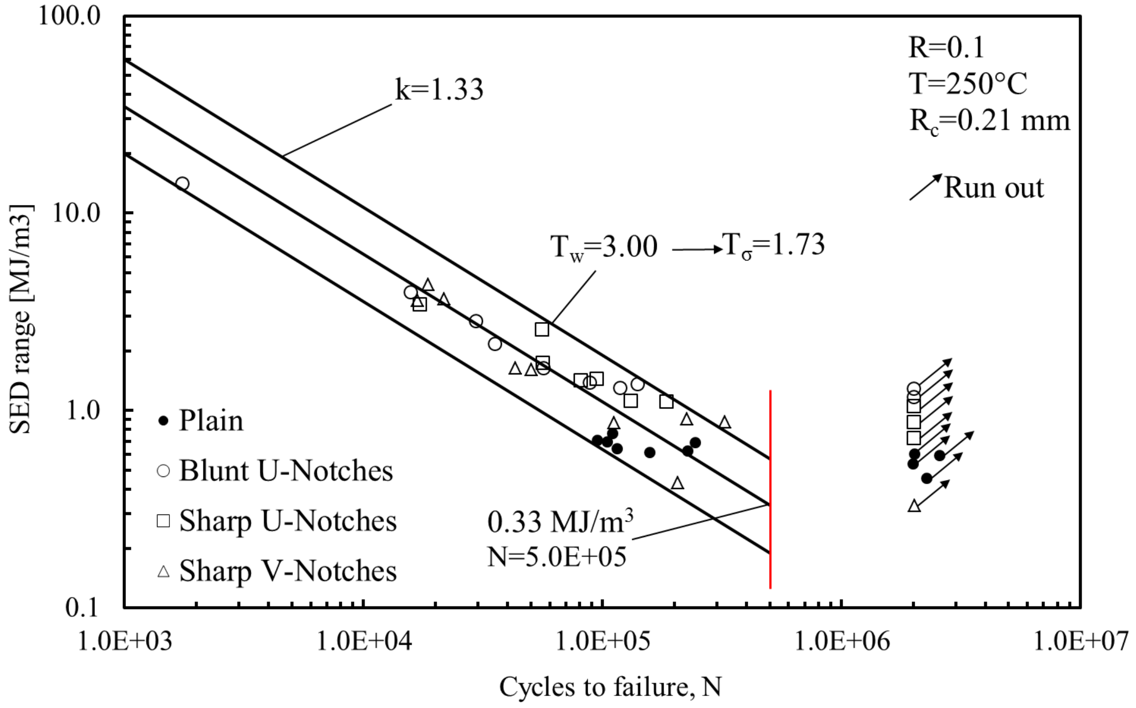

Fig. 3. Synthesis by means of local SED of the C45 steel fatigue data.

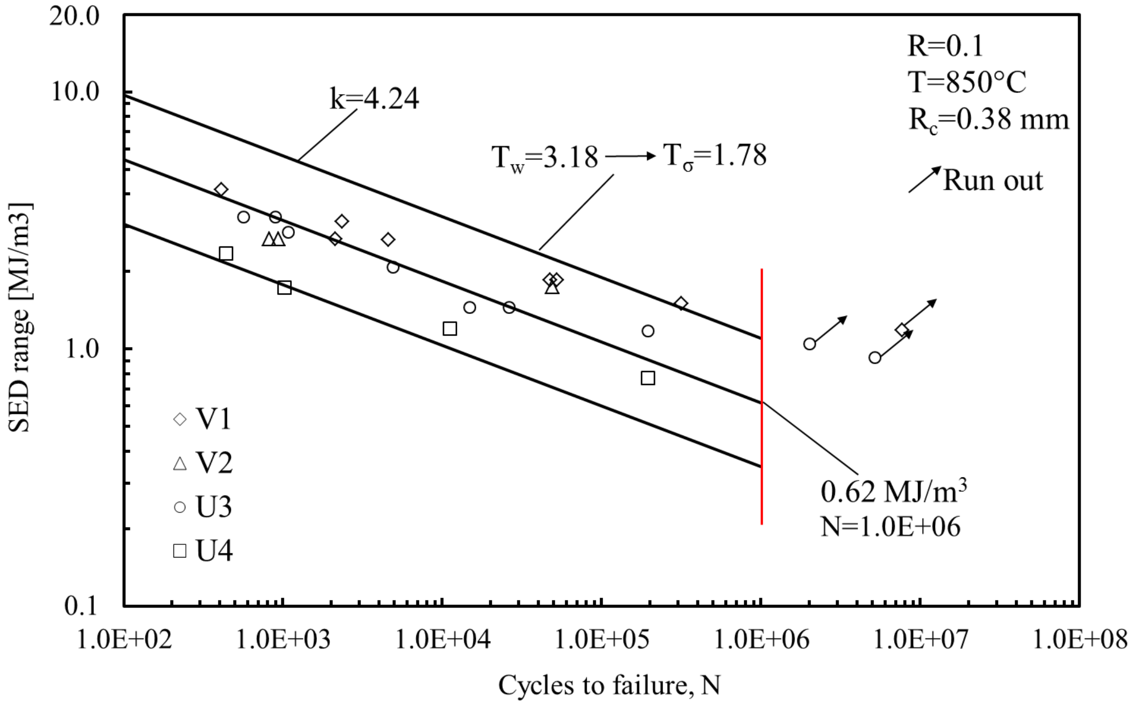

Fig. 4. Synthesis by means of local SED of the DS DZ125 superalloy fatigue data.

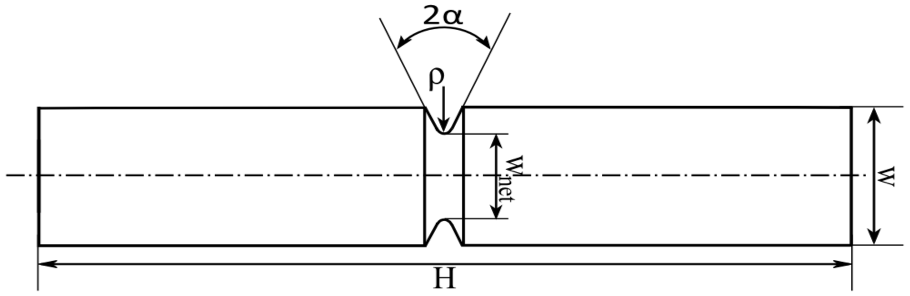

Fig. 5. Cylindrical specimen considered by Chen et al.[19]

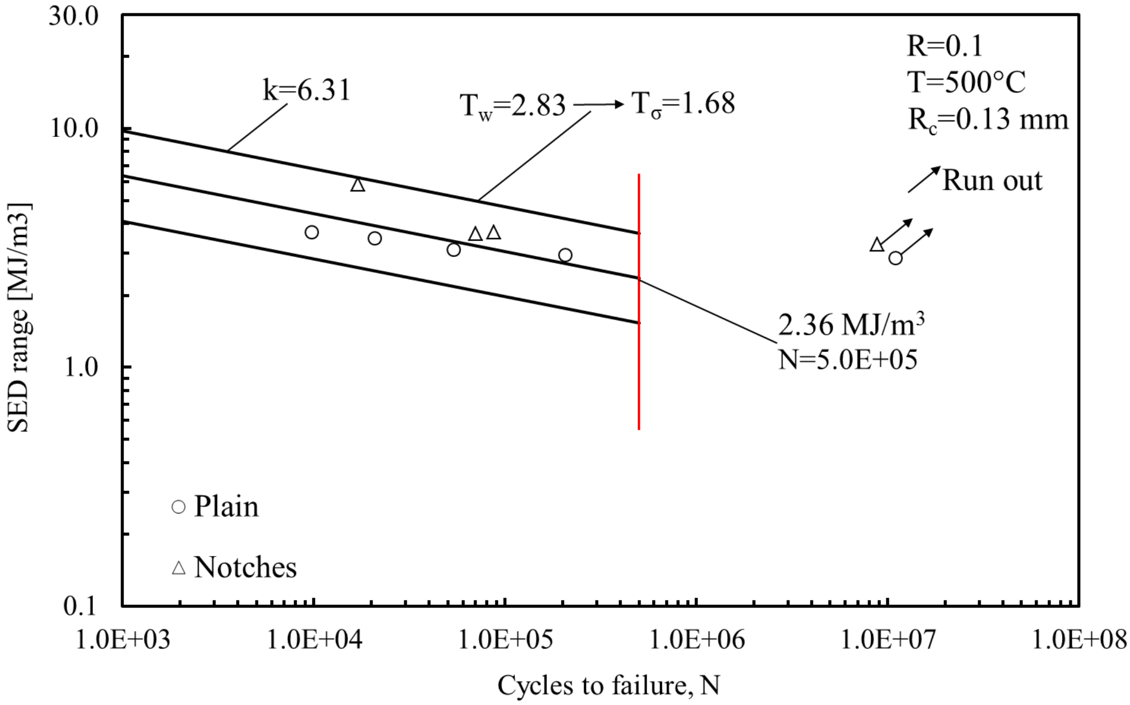

Fig. 6. Synthesis by means of local SED of the Inconel 718 fatigue data.

Table 1. Details of the geometry considered by Louks and Susmel.[22]

Specimens	$K_{t\ net}$	ρ [mm]	a [mm]	2α [°]	W [mm]	W_{net} [mm]	H [mm]
Plain	1.0	-	-	-	25	5	160
Blunt U-notch	6.9	3.00	12	0	25	13	100
Sharp U-notch	10.0	1.00	12	0	25	13	100
Sharp V-notch	26.5	0.15	12	45	25	13	100

Table 2. Details of the geometry considered by Shi.[21]

Notch type	$K_{t\ net}$	ρ [mm]	a [mm]	2α [°]	W [mm]	W_{net} [mm]
U-notch U3	3.0	0.4	0.6	0	6	5.4
U-notch U4	4.4	0.2	0.5	0	6	5.5
V-notch V1	3.0	0.3	0.6	120	6	5.4
V-notch V2	4.3	0.2	0.5	60	6	5.5