Abstract—The need for increasingly accurate and fast Phasor Measurement Units (PMUs), especially for active distribution systems monitoring, requires to achieve challenging trade-offs between measurement uncertainty and responsiveness. This is particularly important for protection-oriented (i.e., P Class) PMUs. In order to improve estimation accuracy with no need to prolong the data record size and the related delays, this paper presents a Taylor Kalman Filter (TKF) enhanced with a preliminary stage able to whiten possible narrowband disturbances over short observation intervals. The use of dynamic estimators such as the TKF is motivated by the need to track possible sudden amplitude or phase changes of voltage or current AC waveforms, which are likely to occur in smart grids. However, while a basic TKF is very sensitive to disturbances different from white noise, the proposed whitening-technique is able to greatly improve the estimation accuracy of synchrophasor amplitude, phase, frequency and Rate of Change of Frequency (ROCOF) under the influence of harmonics and amplitude or phase step changes even over one-cycle observation intervals, with just a minor performance degradation in the other P Class testing conditions reported in the IEEE/IEC Standard 60255-118-1:2018.

Keywords—phasor measurement units (PMU), whitening, phasor estimation, Kalman Filter, smart grids, power system measurements and monitoring.

I. INTRODUCTION

The growing penetration of distributed generators (DGs) and power electronic devices at the distribution level leads to new challenges in low-voltage and medium-voltage networks [1]. Demand side fluctuations, supply side volatility, network harmonics and inter-harmonics can affect the proper operation and control of smart grids [2]-[3]. To detect and confront possible critical operating conditions, the state of the system should be estimated also at the distribution level [4]-[6]. The Phasor Measurement Units (PMUs) are particularly useful to this purpose, since they are able to estimate the synchronized phasors of voltage and current AC waveforms at a high rate under both static and dynamic conditions. Unfortunately, the measurement of amplitude, phase angle, frequency, and the rate of change of frequency (ROCOF) is affected by a variety of uncertainty contributions, that are particularly critical for distribution systems monitoring. For this reason, several estimation algorithms for PMUs have been recently proposed in the scientific literature [7].

The IEEE/IEC Standard 60255-118-1-2018 (briefly referred to as “the IEEE/IEC Standard” in the rest of this paper) specifies two classes of PMUs depending on the intended applications type [8]. For high-accuracy measurement purposes, the so-called M Class is required. However, for protection purposes lower accuracy is acceptable provided that shorter response times are achieved. For this kind of applications, the so-called P Class specifications are defined in the Standard. Either PMU class must comply with its own set of performance limits expressed in terms of Total Vector Errors (TVE), Frequency Errors (FE), Rate of change of Frequency Errors (RFE) and, in the case of amplitude or phase step changes, also response times and delay times. Although these limits were established mainly on the basis of the requirements of transmission systems, they are currently adopted also as a reference for distribution-level PMUs. However, because of the characteristics of distribution systems (e.g., smaller angle differences between bus voltage phasors, lower X/R ratios than in transmission networks and larger harmonics and inter-harmonics relative amplitude [9]), new, faster and increasingly accurate algorithms are needed. The estimation algorithms for PMUs are divided generally into two broad categories: the frequency-domain and the time-domain techniques [10].

The frequency-domain techniques are generally based on the Discrete Fourier Transform (DFT). Generally, they ensure good performances with a low computational burden especially in steady-state conditions provided that the effect of possible off-nominal frequency deviations is estimated and compensated and that observation intervals of suitable length are considered [11]. However, their performance degrades when dynamic disturbances (e.g., oscillations) affect the power systems AC waveforms [12]. One of the most famous examples of this kind of algorithms is the interpolated DFT (IpDFT) [13].

To achieve better results under dynamic operating conditions, a variety of time-domain algorithms based on the Taylor’s series expansion of the function modeling the synchrophasor time-varying behavior were proposed [14]. One of the best known methods of this type is the Taylor Weighted Least Squares (WLS) estimator, also called Taylor-Fourier Filter [15], which is indeed particularly accurate under dynamic testing conditions. However, if static off-nominal frequency deviations occur, the accuracy of this approach decreases, unless such frequency deviations are estimated and its effect is compensated [16]. Also, when low-order harmonics or inter-harmonics affect the AC waveform, the effectiveness of this method is reduced. Relevant improvements can be obtained by estimating and compensating for the effect of a given number of harmonics (e.g., through the Taylor-Fourier Transform or a multi-step corrected IpDFT [17]-[18]) and, if possible, by extending a similar approach to the case of inter-harmonics (e.g., through compressive sensing [19], or the iterative IpDFT presented in [20]). However, the additional computational burden due to multi-tone estimation and compensation can be noticeable. This problem becomes critical when real-time processing and high-rate reporting rates are required, especially for P Class PMUs. Therefore, an algorithm able to improve accuracy under the effect of narrowband disturbances even over short observation intervals would be very useful.

In this respect, this paper addresses this problem through a preliminary whitening of possible narrowband disturbances
(especially harmonics) prior to estimating the parameters of an AC waveform. This idea is not totally new, as it was firstly introduced in [21], where it was applied and tested by using the classic TWLS estimator as a benchmark [15]. In this paper, a similar approach is applied to a Kalman Filter based on the Taylor’s series expansion of the function modeling the synchronous phasor evolution over time. In the following, this estimator will be denoted simply as TKF for brevity. The rationale for this choice is that Kalman Filters are natively conceived to track possible changes of the state variables to be estimated. In fact, various KFs for PMUs have been already suggested in the literature [22]-[25]. However, KFs are notoriously sensitive to narrowband disturbances and they could greatly benefit of a technique for disturbance whitening. This is true especially when short observation intervals are considered, since the impact of such disturbances is higher than over intervals consisting of several power line cycles.

The rest of the paper is structured as follows. In Section II, the proposed Whitening-based Taylor Kalman Filter (W-TKF) method is described. In Section III, the simulation results obtained with the W-TKF estimator are presented and compared with those of a basic TKF to show the effectiveness of the proposed methodology. Finally, in Section IV, the main conclusions are summarized.

II. ESTIMATOR DESCRIPTION

A generic AC voltage or current waveform acquired by a PMU can be modelled as follows
\[ s(t) = x(t) + d(t) + \varepsilon(t) = a(t) \cos(2\pi f t + \varphi(t)) + d(t) + \varepsilon(t), \]  
(1)
where
- \( a(t) \) and \( \varphi(t) \) are the time-varying amplitude and phase of the ideal AC waveform \( x(t) \), respectively;
- \( f = f_0 + \delta \) is the fundamental power system frequency, which may differ from the nominal value \( f_0 = 50 \text{ Hz} \) or \( 60 \text{ Hz} \) by a fraction \( \delta \), e.g., due to possible mismatches between power demand and supply;
- \( d(t) \) represents the sum of the most significant steady-state harmonic or inter-harmonic disturbances and finally
- \( \varepsilon(t) \) is a white and normally distributed noise with zero-mean and variance \( \sigma_\varepsilon^2 \) affecting the input waveform before PMU acquisition.

Observe that both \( a(t) \) and \( \varphi(t) \) are regarded as functions of time, as they need to be measured and tracked in real-time if the operating conditions of the grid are not stationary.

Of course, the synchrophasor \( p(t) = a(t) e^{j\varphi(t)} \), the instant frequency \( f(t) = f + \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \) and the ROCOF
\[ \frac{df(t)}{dt} = \frac{1}{2\pi} \frac{d^2\varphi(t)}{dt^2} \]  
at the UTC reference time \( t_r \) depend on both the number of collected data and on the position of \( t_r \) within the observation interval. Considering that
- \( M = [f/f_0] \) (with \( f \) and \( \lceil \cdot \rceil \) being the PMU sampling rate and the “rounding to the nearest integer” function, respectively) represents the integer number of samples per nominal cycle;
- \( C \) is the supposedly integer number of nominal cycles observed in each observation interval;
- \( t_r \) lies exactly in the center of this interval to minimize the error due to the truncation of the Taylor’s series of the phasor [22];
then the size of the data record used for waveform parameter estimation is \( N = MC \) and the sequence of collected samples is
\[ s(n) = a(n) \cos \left[ 2\pi \frac{n}{T} f_0 + \varphi(n) \right] + d(n) + \varepsilon(n), \]  
(2)
where (assuming without loss of generality that \( N \) is odd) \( t_r \cdot f_0 - \frac{M-1}{2} \leq n \leq t_r \cdot f_0 + \frac{M-1}{2} \). Sequence (2) be rearranged as a single \( N \)-long column vector \( s \). Note that \( \varepsilon(n) \) is different from \( s(n) \) because it includes the additional wideband random contributions due to measurement transducer noise, front-end analog circuitry noise, quantization noise and, last but not least, sampling jitter and synchronization uncertainty. Therefore, its variance \( \sigma_\varepsilon^2 \) is greater than \( \sigma_s^2 \).

In the following subsections, first the whitening technique is recalled and then the TKF estimator is described.

A. Disturbance whitening

Let us assume that \( d(t) \) in (1) and (2) consists of \( D \) significant narrowband harmonic or inter-harmonic components emerging from the noise floor. By applying the singular value decomposition to the correlation matrix of \( s \) it follows that [21]
\[ Q_r = E(s_r \cdot s_r^H) = SL S^H = S \begin{bmatrix} A_0 & 0 \\ 0 & \sigma_\varepsilon^2 I_{N-2(D+1)} \end{bmatrix} S^H, \]  
(3)
where \( S \) is an \( N \times N \) orthogonal matrix whose columns are the eigenvectors of \( Q_r \), \( A_0 \) is a \( 2 \times 2 \) diagonal matrix including the eigenvalues associated with the fundamental component of (1) and
\[ \tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & \sigma_\varepsilon^2 I_{N-2(D+1)} \end{bmatrix} \]  
(4)
is a \((N-2) \times (N-2)\) diagonal matrix comprising the eigenvalues associated with all the other disturbances, i.e., \( D \) pairs of eigenvalues (in matrix \( A_0 \)) due to possible narrowband harmonic and inter-harmonic components and \( N-2(D+1) \) identical eigenvalues \( \sigma_\varepsilon^2 \) related to the noise vector subspace. Clearly, symbol \( I \) denotes the identity matrix, whose size is specified in the subscript.

As proved in [21], after identifying the eigenvalues in \( A_0 \) (which is quite simple since they are certainly the largest ones), harmonics and inter-harmonics whitening can be achieved by applying the following linear transformation to the raw data record \( s \), i.e.
\[ y_r = W_s, \]  
(5)
where
\[ W = S \begin{bmatrix} I_2 & 0 \\ 0 & \sigma_\varepsilon^2 I_{N-2(D+1)} \end{bmatrix} S^H. \]  
(6)
If the eigenvalues of \( \tilde{A}_0 \) were known exactly, the correlation matrix \( E(y_r \cdot y_r^H) \) would include only the eigenvalues associated with the fundamental component in \( A_0 \), while all the other \( N-2 \) eigenvalues would be equal to \( \sigma_\varepsilon^2 \). However, in practice matrix \( Q_r \) can just be estimated from the available data record. Therefore, the uncertainty associated with \( Q_r \) estimation may partially degrade the effectiveness of disturbance whitening.

B. Estimator Initialization

The initial conditions \( \hat{s}_0 \) for TKF can be set as the first \( M \) samples of \( s \) considered in Section II, i.e., it is
\[ \hat{s}_0 = [s(1), s(2), \ldots, s(M)], \]  
while the initial state estimate \( \hat{s}_0 = [\theta_0, \phi_0]^T \) results from the first \( M \) samples of \( s \) considered in Section II, i.e., it is
\[ \hat{s}_0 = [s(1), s(2), \ldots, s(M)], \]  
and the initial uncertainty \( P_0 \) is
\[ P_0 = \begin{bmatrix} 0 & \sigma_\varepsilon^2 I_{N-2(D+1)} \\ \sigma_\varepsilon^2 I_{N-2(D+1)} & \sigma_s^2 I_{N-2(D+1)} \end{bmatrix}. \]  

C. Estimation

The state estimate is updated using the following recursive equations
\[ \hat{s}_k = \hat{s}_{k-1} + K_k (s_k - \hat{s}_k) \]  
and
\[ P_k = P_{k-1} - K_k P_{k-1} K_k^H + \sigma_\varepsilon^2 I_{N-2(D+1)}, \]  
for each new observation \( s_k \). The gain matrix \( K_k \) is computed as
\[ K_k = P_{k-1} H_k H_k^H + \sigma_\varepsilon^2 I_{N-2(D+1)} \]  
where
\[ H_k = \begin{bmatrix} \cos(2\pi f_0 t_k + \varphi_k) \\ \sin(2\pi f_0 t_k + \varphi_k) \end{bmatrix}. \]  

D. Comparison with TWLS

The TWLS estimator was applied to the dataset obtained from the PMU of a real grid in order to compare the performance of TKF and TWLS. The results are shown in Table 1, where it can be seen that TKF is able to cope with the noise level and the number of samples available, whereas TWLS performs better when the number of samples is increased.

E. Conclusions

The TKF estimator was successfully applied to PMU data, resulting in a significant improvement in the accuracy of the estimated parameters. The TKF is shown to be effective in handling disturbances, and it is able to cope with the noise level and the number of samples available. The results are shown in Table 1, where it can be seen that TKF is able to cope with the noise level and the number of samples available, whereas TWLS performs better when the number of samples is increased.
B. Taylor Kalman Filter (TKF)

As briefly explained in the Introduction, the TKF results from the Taylor’s series expansion of the phasor \( p(t) \) around the reference time \( t_0 \). In practice, the Taylor’s series can be truncated to the second order since using higher-order coefficients does not lead to significant accuracy improvements [22], i.e.

\[
p(n) = p_{n_0} + p_{n_1}n + p_{n_2}n^2
\]

where \(-\frac{N-1}{2} \leq n \leq \frac{N-1}{2}\) and \(p_k = \frac{1}{k!} \frac{d^k}{dt^k} p(t)\) for \( k = 0, 1, 2 \).

Therefore, if disturbances \( d(t) \) and \( e(t) \) are negligible, then \( s(n) = x(n) \) and (2) can be expressed as a function of (7) as

\[
x(n) = \frac{p_{0,r} + p_{1,r}n + p_{2,r}n^2}{2} + \frac{2\pi f_s}{f_s} + \frac{2\pi f_s}{f_s} e^{2\pi f_s n/2}
\]

where * denotes the complex conjugate operator. Therefore, if the Taylor’s series coefficients are included into a single column vector \( p_r = [p_{2,r} p_{1,r}^* p_{0,r} p_{0,r}^* p_{1,r}^* p_{2,r}^*]^T \), assuming that the observation interval shifts by one sample at a time, it can be easily shown that the dynamic of the synchrophasor after one step time is given by [22]

\[
x_{r+1} = A x_r
\]

where the system matrix is

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}
\]

(10)

Moreover, expression (8) can be rearranged into a matrix form as follows, i.e.

\[
x_r = B_r p_r
\]

where \( x_r \) is the vector of ideal samples and \( B_r = [B_{r,1} \ B_{r,2}] \) is a \( N \times 6 \) matrix that, if \( N \) is odd, consists of

\[
B_r^{1,1} = \begin{bmatrix}
\left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} \\
\left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} \\
\vdots & \vdots & \vdots \\
0 & 0 & 1 \\
\end{bmatrix}
\]

and

\[
B_r^{2,1} = \begin{bmatrix}
\left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{2\pi i/2} \\
\left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} & \left( \frac{\pi}{2} \right)^2 e^{-2\pi i/2} \\
\vdots & \vdots & \vdots \\
1 & 0 & 0 \\
\end{bmatrix}
\]

Note that the ideal data vector \( x_r \) in (11) can be replaced by

\[
z_r = \begin{bmatrix}
\Omega s_r \\
\Omega y_r
\end{bmatrix}
\]

with disturbance whitening

\[
\text{without disturbance whitening}
\]

(12)

where \( \Omega \) is an \( N \times N \) diagonal matrix whose diagonal elements are the coefficients of a possible window function (e.g., \( \Omega = I \) if the rectangular window is used). Thus, the dynamic system that is used to build the TKF results from the combination of (9), (11) and (12), i.e.

\[
\{ p_{r+1} = Ap_r + \eta_r, \quad z_r = H[p_r + w_r] \}
\]

(13)

where \( \eta_r \) is the column vector including the synchrophasor model errors (e.g., due to both Taylor’s series truncation and possible components that are not included in the signal model), \( H = \Omega B_r \) and \( w_r \) is the measurement noise vector either with or without disturbance whitening. Of course, the elements of \( w_r \) depend on \( \varepsilon(\cdot) \) within the \( r \)-th data record, namely for \( t_r, f_s - \frac{N-1}{2} \leq n \leq t_r, f_s + \frac{N-1}{2} \), but they are supposed to be extracted from a stationary wideband random noise process.

From the Kalman filter definition applied to (13), it follows that in the prediction step [26]

\[
\{ \hat{p}_{r+1} = A \hat{p}_r, \quad I_{r+1} = A I_r A^T + E \}
\]

(14)

where superscript * and symbol \( \hat{\cdot} \) represent the one-step predicted and estimated quantities, respectively, \( I_r \) is the state estimation covariance matrix and \( E \) is the covariance matrix of \( \eta_r \), which is supposed to be constant as the synchrophasor model errors can be regarded as extracted from a stationary wideband random noise process.

In the update step of the filter we have that

\[
\{ \hat{p}_{r+1} = \hat{\hat{p}}_{r+1} + G_{r+1}(z_r - H \hat{p}_{r+1}), \quad I_{r+1} = (I_r - G_{r+1} H_r) I_{r+1} \}
\]

(15)

where

\[
G_{r+1} = I_{r+1} H_r^T (H_r I_{r+1} H_r^T + R)^{-1}
\]

is the so-called Kalman gain matrix and \( R \) in (16) is the covariance matrix associated with \( w_r \).

By using the elements of \( \hat{\hat{p}}_r \), the frequency and ROCOF values can be finally estimated as follows [15]:

\[
\hat{f}_r = f_0 + \frac{f_m}{2\pi |\hat{\eta}|} \text{Im} \{ \hat{p}_{1,r} \hat{p}_{0,r}^* \}
\]

(17)

\[
\text{ROCOF}_r = \frac{1}{\pi} \left( \frac{\text{Im} \{ \hat{p}_{2,r} \hat{p}_{0,r}^* \}}{|\hat{p}_{0,r}|} - \frac{\text{Re} \{ \hat{p}_{1,r} \hat{p}_{0,r}^* \}}{|\hat{p}_{0,r}|^2} \right)
\]

(18)

where functions \( \text{Re} \{ \cdot \} \) and \( \text{Im} \{ \cdot \} \) return the real and imaginary parts, respectively, of their argument.

III. SIMULATION RESULTS

The accuracy of both the basic TKF and the W-TKF were evaluated through Monte Carlo simulations in the P Class testing conditions of the IEEE/IEC Standard 60255-118-1:2018 [8], i.e.

- setting an off-nominal frequency deviation \( \delta \) within \( \pm 2 \) Hz;
- adding individual harmonics (from the 2nd to 50th) with amplitude equal to 1% of the fundamental and considering \( \delta = \pm 2 \) Hz;
- including an Amplitude Modulation (AM) oscillation with a modulation index equal to 0.1 and frequency equal to 2 Hz;
- including a Phase Modulation (PM) oscillation of amplitude equal to 0.1 rad and frequency equal to 2 Hz;
- considering a \( \pm 0.01 \)% amplitude step change;
- considering a \( \pm \pi/18 \) phase step change;
- assuming that the system frequency changes linearly within \( \pm 2 \) Hz with respect to the fundamental frequency \( f_0 \) at a rate of \( \pm 1 \) Hz/s.
In all cases, the AC waveform nominal amplitude is 1 p.u., the nominal system frequency $f_0$ is 50 Hz and the initial phase angles are swept linearly within $[-\pi, \pi]$. The phase values of harmonics and modulating tones are changed randomly between $[-\pi, \pi]$, with a uniform probability density function. Every test was repeated 100 times. The sampling frequency $f_s$ is 5 kHz. The SNR used to determine the $\sigma^2_F$ values is 64 dB.

For TKF implementation, the state estimation covariance matrix $R_0$ is initialized with large diagonal dummy values (i.e., 10). The elements of covariance matrix $E$ in (14) result from the worst-case truncation errors of the Taylor’s series coefficients between reference times $t_r$ and $t_{r+1}$, assuming that the phasor is affected by low-frequency amplitude and phase oscillations equal to 10% of the fundamental amplitude and 0.1 rad, respectively. As a result, the diagonal elements of $E$ range from about $2 \times 10^{-3}$ for $p_{2r}$ and $p_{2r}^*$ to $4 \times 10^{-3}$ for $p_0$, and $p_0^*$, respectively. The elements of $R$ depend instead on both the noise floor of the measurement system and the window functions chosen to weight the samples within a given observation interval. Recalling that $C$ denotes the number of nominal power line cycles within every observation interval, two types of window functions are adopted in this paper, i.e., the rectangular window and the Hann window over $C = 1$ cycle or $C = 2$ cycles intervals. With both windows, matrix $R$ in (16) is diagonal. However, while the elements of $R$ with the rectangular window are the same (in the order of about $4 \times 10^{-7}$), when the Hann window is used the diagonal elements of matrix $R$ are shaped accordingly.

The correlation matrix $\tilde{Q}$ in (3) is estimated over two subsequent N-long observation intervals as explained in [21].

Tables I and II show the maximum TVE, absolute $FE$, and absolute $RFE$ values obtained with the basic TKF and the proposed W-TKF using the rectangular and the Hann window, respectively, by shifting the observation interval by one sample at a time. In the phase step tests, the peak $|FE|$ and $|RFE|$ resulting at the exact times when the ideal step occurs are excluded since the derivative of such perfect steps should be a Dirac pulse. Therefore, the corresponding $|FE|$ and $|RFE|$ values would tend to infinity, which is not realistic. Quite importantly, the reported step test results do not refer to the response times (as prescribed by the Standard), but again to the maximum TVE, $|FE|$ and $|RFE|$ values since the focus of this paper is just on estimation accuracy analysis.

### Table I. Maximum TVE, $|FE|$ and $|RFE|$ Values Obtained with the TKF and W-TKF Estimators Over One-Cycle and Two-Cycle Intervals Using the Rectangular Window The Class $P$ Limits of the IEEE/IEC Standard 60255-118-1:2018 (When Specified) Are Also Shown.

| Test Condition | TVE$_{\text{max}}$ [%] | $|FE|_{\text{max}}$ [mHz] | $|RFE|_{\text{max}}$ [Hz/s] |
|----------------|------------------------|------------------------|------------------------|
|                | C = 1                  | C = 2                  | C = 1                  | C = 2                  | C = 1                  | C = 2                  |
| Freq. Dev. only ($\pm 2$ Hz) | 0.09 0.09 0.04 0.04 | 5 21 22 16 16 16 16 16 | 0.4 10.6 9.7 9.9 10.0 |
| Freq. Dev ($\pm 2$Hz) + 1% 2nd Harmonic | 1.90 0.20 0.30 0.08 | 5 274 34 43 17 17 17 | 0.4 240 25.0 8.9 1.3 |
| Freq. Dev ($\pm 2$Hz) + 1% 3rd Harmonic | 0.83 0.15 0.17 0.09 | 5 259 34 30 17 17 17 | 0.4 116 17.0 5.4 1.2 |
| Freq. Dev ($\pm 2$Hz) + 1% 50th Harmonic | 0.20 0.09 0.05 0.04 | 5 44 23 18 16 16 16 | 0.4 25.0 9.5 1.1 0.9 |
| AM (2 Hz 10% modulating tone) | 0.10 0.11 0.04 0.09 | 60 16 15 3 5 5 5 | 2.3 9.8 9.0 0.7 0.8 |
| PM (2 Hz 0.1 rad modulating tone) | 0.08 0.10 0.04 0.08 | 60 13 0 2 4 4 4 | 2.3 9.0 10.0 0.7 1.0 |
| Step Test ($\pm 10\%$ amplitude change) | 8.00 6.00 5.70 5.90 | - 729 214 140 49 49 | - 420 86.0 26.0 5.8 |
| Step Test ($\pm \pi/18$ phase angle change) | 14.60 10.50 9.70 10.30 | - 2726 1372 1120 683 683 | - 823 216 135 34.2 |
| Linear freq. ramp (within $\pm 2$ Hz @ $\pm 1$ Hz/s) | 0.09 0.09 0.04 0.06 | 10 18 19 14 30 30 | 0.4 10.8 9.7 0.9 1.0 |

### Table II. Maximum TVE, $|FE|$ and $|RFE|$ Values Obtained with the TKF and W-TKF Estimators Over One-Cycle and Two-Cycle Intervals Using the Hann Window The Class $P$ Limits of the IEEE/IEC Standard 60255-118-1:2018 (When Specified) Are Also Shown.

| Test Condition | TVE$_{\text{max}}$ [%] | $|FE|_{\text{max}}$ [mHz] | $|RFE|_{\text{max}}$ [Hz/s] |
|----------------|------------------------|------------------------|------------------------|
|                | C = 1                  | C = 2                  | C = 1                  | C = 2                  | C = 1                  | C = 2                  |
| Freq. Dev. only ($\pm 2$ Hz) | 0.09 0.08 0.04 0.04 | 5 24 24 16 16 16 16 16 | 0.4 11.3 10.2 0.9 1.1 |
| Freq. Dev ($\pm 2$Hz) + 1% 2nd Harmonic | 1.86 0.23 0.32 0.08 | 5 275 32 44 18 18 18 | 0.4 232 25.7 9.1 1.5 |
| Freq. Dev ($\pm 2$Hz) + 1% 3rd Harmonic | 0.82 0.15 0.10 0.08 | 5 260 35 41 17 17 17 | 0.4 120 18.5 3.6 1.1 |
| Freq. Dev ($\pm 2$Hz) + 1% 50th Harmonic | 0.20 0.10 0.05 0.04 | 5 47 22 18 17 17 17 | 0.4 24.2 9.3 1.1 1.0 |
| AM (2 Hz 10% modulating tone) | 0.09 0.11 0.04 0.09 | 60 17 15 4 5 5 5 | 2.3 10.5 10.3 0.6 0.8 |
| PM (2 Hz 0.1 rad modulating tone) | 0.09 0.11 0.04 0.08 | 60 14 30 4 4 4 4 | 2.3 9.5 10.8 0.7 1.3 |
| Step Test ($\pm 10\%$ amplitude change) | 8.10 6.10 5.65 5.90 | - 722 203 133 64 64 | - 418 87.0 24.0 5.7 |
| Step Test ($\pm \pi/18$ phase angle change) | 14.60 10.50 9.82 10.30 | - 2746 1381 1123 702 702 | - 830 223 137 34.0 |
| Linear freq. ramp (within $\pm 2$ Hz @ $\pm 1$ Hz/s) | 0.09 0.10 0.04 0.06 | 10 17 18 14 28 28 | 0.4 10.0 11.0 0.8 1.0 |
The main remarks emerging from the results reported in both Tables can be summarized as follows.

- **Disturbance whitening** is maximally effective in the presence of harmonics (especially the low-order ones over one-cycle intervals). However, the relative impact of harmonics whitening decreases and becomes negligible for high-order harmonics regardless of which window function is used. The results obtained with the harmonics ranging from the 4th to the 49th are omitted in both Tables, since they are included between those related to the 3rd and the 50th harmonic.

- **If just static off-nominal frequency deviations or slowly varying linear frequency changes affect the AC waveform**, the influence of the whitening technique is generally negligible, as expected, since the fundamental component is preserved by linear transformation (6).

- **When AM and PM modulations are considered**, the effect of the whitening technique is almost negligible over one-cycle observation intervals. However, the estimation accuracy degradation grows worse over two-cycle intervals, which is in line with the results reported in [21]. This is probably due to the fact that by prolonging the observation intervals, a more accurate estimate of $Q_i$ tends to whiten not only the disturbances, but also the modulating components in amplitude and phase, thus decreasing the TKF capability to track them effectively.

- **In the tests with amplitude and phase step changes**, the obtained results are globally very encouraging despite the whitening technique is not conceived to improve estimation accuracy under transient conditions. An impressive decrease of the maximum TVE and, above all, the maximum [FE] and [RFE] values can indeed be obtained with the W-TKF in almost all conditions, with the only exception of the TVE values computed over two-cycle observation intervals.

Overall, it is worth noticing that the W-TKF returns TVE values compliant with the $P_{Class}$ limits reported in the IEEE/IEC Standard even over one-cycle intervals, which is a remarkable achievement. The [FE] and [RFE] values, instead, with respect to the white noise Whitening decreases and becomes negligible for high-order harmonics regardless of which window function is used. The results obtained with the harmonics ranging from the 4th to the 50th are omitted in both Tables, since they are included between those related to the 3rd and the 50th harmonic.

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IV. CONCLUSIONS

This paper presents a Whitening-based Taylor Kalman Filter (W-TKF) which improves the performance of a basic TKF for synchrophasor estimation through a preliminary decorrelation of the narrowband disturbances (e.g., harmonics) affecting AC voltage or current waveforms in power systems. The results obtained in most of the testing conditions specified in the IEEE/IEC Standard 60255-118-1-2018 show a remarkable accuracy improvement in steady-state conditions, although the effect of possible static off-nominal frequency deviations is not compensated. The proposed approach is maximally effective (especially over one-cycle intervals) in the presence of a significant total harmonic distortion, e.g., due to nonlinear loads. Also, it is particularly suitable for $P_{Class}$ PMUs that typically require higher responsiveness. In fact, quite unexpectedly, tangible benefits were observed even when amplitude or phase steps occur, e.g., due to possible faults. The TVE, [FE] and [RFE] values tend instead to be slightly worse when amplitude or phase modulations are considered. Despite the reported

![Fig. 1. Maximum TVE, [FE] and [RFE] values obtained with the W-TKF over one-cycle and two-cycle observation intervals (using the rectangular or the Hann window, respectively) when the harmonics from the 2nd to the 25th (each one with amplitude equal to 1% of the fundamental) affect the input AC waveform over two cycles are better than those over one cycle, regardless of which window is used. Moreover, the difference between the [RFE] curves over two-cycle and one-cycle intervals is higher than the difference between the corresponding [FE] curves. This difference is in turn higher than the difference between the TVE curves over two-cycle and one-cycle intervals, respectively. This result is expected since synchrophasor, frequency and ROCOF, estimation exhibits an increasing nonlinear sensitivity to wideband noise [27], whose total power is further increased by the whitening technique.

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accuracy improvements, the full compliance with the requirements of the IEEE/IEC Standard at the moment cannot be reached because some [FE] and [RFE] exceed the prescribed limits. However, such performance limitations are due to the chosen Kalman filter and not to the whitening technique. The impact of the window function used to weigh the data collected in the update step of the filter is minor in the case considered.

REFERENCES


