Majority Effect in Cooperative localisation of Mobile Agents using Ranging Measurements

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***Abstract*—Multi agent systems are becoming pervasive in modern industrial and service applications. Either being autonomous robotic agents or human beings performing a collaborative task, relative and/or absolute localisation of the team agents is a mandatory task to be efficiently solved for any meaningful application. In this paper, we will analyse the cooperative localisation problem from the perspective of the number and quality of the available measurements, rooting this analysis to a practical example of ranging measurements for robot localisation. We will make use of a known distributed Kalman filtering technique to highlight a trade off between number and quality of measurements, which is a known effect in estimation theory but never analysed for distributed estimation problems. What is surprising is that the net effect is quite similar to what happens in opinion formation for social groups, which is sometimes referred to as *majority effect*.**

***Index Terms*—Kalman Filter, Cooperative Localisation Algorithm, Majority Effect, Distributed Estimation**

I. Introduction

Precise localisation is one of the main requirements for mobile robot autonomy. Indoors and outdoors, mobile robots need to know their exact position and orientation in order to perform their required tasks. There have been numerous approaches to the localisation problem utilising different types of sensors and a variety of techniques. The key idea behind most of the current localisation schemes is to optimally combine measurements from proprioceptive sensors that monitor the motion of the vehicle with information collected by exteroceptive sensors, that allow to detect and identify other robots moving in the vicinity and measure relative displacements [1].

There area a plethora of different exteroceptive sensing system that can be adopted for localisation of *agents* (i.e. either robots or human beings) even when Global Navigation Satellite Systems (GNSS) is not allowed, as typically happens in indoor environments. To mention a few, there are solutions based on laser beams [2], ultrasonic stimuli [3], cameras [4] or even radars [5]. One of the most effective and low cost solutions relies on ranging sensors, which comes using different approaches, such as Received Signal Strength (RSS) measurements and fingerprinting of standard wireless signals (e.g. WiFi) [6], Radio Frequency Identification Systems (RFID) [7] to time-of-flight (ToF) values of Ultra-Wide Band (UWB) pulses [8], [9]. When applied to wheeled mobile robots, the localisation problem can be solved by fusing the exteroceptive information with proprioceptive data, such as dead reckoning techniques (e.g. based on odometry or inertial measurement units) [10].

When the localisation problem is extended to multiple agents, distributed solutions, such as [11], can be efficiently used to design collaborative or cooperative localisation algorithms that prove to be effective in different contexts [12]. In those cases, the use of ranging sensing systems is heavily adopted, due to their flexibility and efficiency in measuring relative displacements among the agents. For example, in [13] a fully wireless localisation system based on time of arrival/angle of arrival (TOA/AOA) is adopted. In those cases, the Extended Kalman Filter (EKF) remains a valid algorithm [14], either as a distributed sensing fusion solution [11] or as a collaborative localisation approach by means of an Interlaced Extended Kalman Filter (IEKF) [15].

Apart from the several available solutions, there are still some issues that should be addressed for cooperative localisation. In particular, this paper analyses this problem and 1) propose a statistical proof of the relevance of the distributed EKF against individual localisation; 2) present the trade-off between the number of information available and their uncertainties, making an interesting parallel between the problem at hand and social opinion formation, with special reference to the *majority effect* [16].

The paper is organised as follows: Section II describes the distributed localisation problem in a general way, clearly stating the problem at hand and introducing the distributed algorithm adopted, while Section III presents the models adopted for the sensing system and the agents. We then present the statistical analysis of the obtained results and the main message of the paper in Section IV where we also made explicit the connection with social behaviours. Finally, we draw our conclusions and present future research directions in Section V.

II. Background Material and Problem Formulation

To introduce the problem at hand, we will first start with the description of a positioning problem, that is when the agent state $q_k \in \mathbb{R}^n$ at time kT_s (where T_s is the sampling time induced by the sensors and $k \in \mathbb{Z}$) can be entirely estimated using only the measurements $z_k \in \mathbb{R}^m$ at time kT_s, i.e. the state q_k is statically observable from the measurements. To this end, let us define the generic nonlinear measurement function at time kT_s as

$$z_k = h(q_k, \overline{\eta}_k),$$

where $\overline{\eta}_k$ is the sequence of random effects acting on the measurement process and generating uncertainties, which is customarily assumed as a zero mean white sequence with covariance matrix $\overline{R}_k$. In the general case, a solution to this problem can be given by the nonlinear Gauss solution to the Weighted Least Squares (WLS). Intuitively, the more measurements are available, the less would be the uncertainty on the estimates $\hat{q}_k$. The proof comes from the linear description of the measurement process, that is when the measurement forward map is given by

$$z_k = H_k q_k + G_k \overline{\eta}_k,$$

the solution is given by the WLS solution $\hat{q}_k = P_k H_k^T R_k^{-1} z_k$, where $P_k = (H_k R_k^{-1} H_k^T)^{-1}$ is the covariance matrix of the estimation error $\tilde{q}_k = q_k - \hat{q}_k$ and $R_k = G\overline{R}_k G^T$ is the covariance matrix of the noises $\eta_k = G\overline{\eta}_k$ affecting the measurements. Notice that the WLS is the Best Linear Unbiased Estimator (BLUE), which turns to a Minimum Variance Unbiased Estimator (MVUE) when the noises η_k are Gaussian. Furthermore, when the measurements z_k are given by m independent sensors, the noise sequences η_k can be considered as independent and, further assuming all the sensors of the same type, we have $R_k = \sigma_{\eta_k}^2 I_m$, where I_m is the identity matrix of order m. In such a case, the WLS solution $\hat{q}_k$ turns out to be proportional to the arithmetic mean that implies an inverse proportionality between m and P_k.

Similar arguments still hold for a Bayesian filter, such as the Kalman Filter (KF) or its nonlinear extension Extended Kalman Filter (EKF), where the model now comprises the system state dynamic as well, i.e.

$$\begin{aligned} q_{k+1} &= f(q_k, u_k, \varepsilon_k), \\ z_k &= h(q_k, \overline{\eta}_k), \end{aligned} \tag{1}$$

where $u_k \in \mathbb{R}^p$ are the model inputs and ε_k are the model and/or actuation uncertainties, usually generated by a zero mean white stochastic process. For the localisation problem, the filter acts as a sensor fusion algorithm fusing (in general) *proprioceptive* sensor readings (usually adopted in the model of the system and, hence, affected by ε_k) during the *prediction step* and then using *exteroceptive* measurements in the *update step* involving the measurements z_k and its uncertainties $\overline{\eta}_k$. In the case of linear dynamics and linear measurements, the KF turns to be BLUE, while if the noises are Gaussian, it is also the MVUE. Notice that for a correct solution of the localisation problem, the exteroceptive measurements usually involve *absolute* measures, that is measurement results expressed with respect to a known fixed reference frame (e.g., GPS readings). Nevertheless, when a team of agents is considered, *relative* measurements, i.e. measurements involving relative values and expressed in relative coordinates (e.g. the distance between two cars taken from a radar), can be adopted to reduce the estimation uncertainty.

As shown in the next sections, relative measurements highly correlates the agents state estimates, which may be detrimental for the estimation process and may generate a trade-off. Indeed, either for the static or dynamic filters, "the more measurements z_k are available, the less is the uncertainty affecting the estimates $\hat{q}_k$" is a rule of thumb that not always hold. For example, if $m = 3$ measurements are collected, but one is linearly dependent from the other two (i.e. one has correlation coefficient equals to one with the other two), the estimation uncertainty remains the same for $m = 2$ or $m = 3$. In other words, the higher is the correlation coefficient, the less is the information that the measurement carries to the estimation process.

A. Distributed EKF

The EKF implemented in this paper is conceived for being distributed among the team of N agents, but it is presented with a centralised manner for ease of analysis. Once the centralised version is available, the distributed version can be immediately obtained following [11]. In order to identify the collaborative nature of the solution, please refer to [1].

a) Propagation step: For the model equations of the EKF we use those reported in (1), one for each agent, i.e.

$$\hat{\mathbf{q}}_{k+1}^{-} = \begin{bmatrix} \hat{q}_{k+1}^{(1)-} \\ \hat{q}_{k+1}^{(2)-} \\ \vdots \\ \hat{q}_{k+1}^{(N)-} \end{bmatrix} = \begin{bmatrix} f^{(1)}(\hat{q}_k^{(1)}, u_k^{(1)}, \varepsilon_k^{(1)}) \\ f^{(2)}(\hat{q}_k^{(2)}, u_k^{(2)}, \varepsilon_k^{(2)}) \\ \vdots \\ f^{(N)}(\hat{q}_k^{(N)}, u_k^{(N)}, \varepsilon_k^{(N)}) \end{bmatrix} = \mathbf{f}(\hat{\mathbf{q}}_k, \mathbf{u}_k, \boldsymbol{\varepsilon}_k) \tag{2}$$

where the superscript $\cdot^-$ stands for "the predicted value" (i.e. using the model information only), the bolded symbols represents the collection of the quantities for all the N agents and where we use the superscript $\cdot^{(i)}$ to identify the quantities of the i–th agent. It is worthwhile to note that each agent dynamic is independent from the other, hence the Jacobian matrices $A_k = \frac{\partial \mathbf{f}(\mathbf{q}_k, \mathbf{u}_k, \boldsymbol{\varepsilon}_k)}{\partial \mathbf{q}_k}$, $B_k = \frac{\partial \mathbf{f}(\mathbf{q}_k, \mathbf{u}_k, \boldsymbol{\varepsilon}_k)}{\partial \mathbf{u}_k}$, and $C_k = \frac{\partial \mathbf{f}(\mathbf{q}_k, \mathbf{u}_k, \boldsymbol{\varepsilon}_k)}{\partial \boldsymbol{\varepsilon}_k}$ will be all diagonal. As a consequence, the model noise covariance will be a block diagonal matrix $\mathbf{Q}_k = C_k \overline{\mathbf{Q}}_k C_k^T$, where $\overline{\mathbf{Q}}_k$ is the covariance matrix of $\boldsymbol{\varepsilon}_k$.

The estimation error $\tilde{\mathbf{q}}_{k+1} = \mathbf{q}_{k+1} - \hat{\mathbf{q}}_{k+1}$ covariance propagation equation then becomes

$$\mathbf{P}_{k+1}^{-} = A_k \mathbf{P}_k A_k^T + \mathbf{Q}_k,$$

with initial values

$$\mathbf{P}_0 = \text{diag}(P_0^{(11)}, P_0^{(22)}, \ldots, P_0^{(NN)}), \tag{3}$$

since each agent may only know its own position and covariance in the reference coordinates frame. Of course, the block diagonal entry $P_k^{(ii)}$ refers to the error covariance matrix of the i-th agent.

b) Update step with absolute measurements: Using again (1) with additive noises (a customary assumption) and when only absolute measurements are considered, the vector of observations taken by the different agents can be written as

$$\mathbf{z}_k = \begin{bmatrix} z_k^{(1)} \\ \vdots \\ z_k^{(N)} \end{bmatrix} = \begin{bmatrix} h^{(1)}(q_k^{(1)}) \\ \vdots \\ h^{(N)}(q_k^{(N)}) \end{bmatrix} + \begin{bmatrix} \eta_k^{(1)} \\ \vdots \\ \eta_k^{(N)} \end{bmatrix} = \mathbf{h}(\mathbf{q}_k) + \boldsymbol{\eta}_k, \quad (4)$$

whose Jacobian is given by $\mathbf{H}_k = \frac{\partial \mathbf{h}(\mathbf{q}_k)}{\partial \mathbf{q}_k}$, which is a block diagonal matrix as the covariance in (3) and with diagonal block $H_k^{(ii)}$ following the same naming convention as in $\mathbf{P}_0$. Similarly, the covariance matrix $\mathbf{R}_k$ of the noises $\boldsymbol{\eta}_k$ will be block diagonal with entries $R_k^{(ii)}$. It then follows that by applying the standard equations of the EKF, i.e.

$$\begin{aligned} \mathbf{S}_{k+1} &= \mathbf{H}_{k+1}\mathbf{P}_{k+1}^{-}\mathbf{H}_{k+1}^{T} + \mathbf{R}_{k+1}, \\ \mathbf{K}_{k+1} &= \mathbf{P}_{k+1}^{-}\mathbf{H}_{k+1}^{T}\mathbf{S}_{k+1}^{-1}, \\ \hat{\mathbf{q}}_{k+1} &= \hat{\mathbf{q}}_{k+1}^{-} + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - h(\hat{\mathbf{q}}_{k+1}^{-})), \\ \mathbf{P}_{k+1} &= (I - \mathbf{K}_{k+1}\mathbf{H}_{k+1})\mathbf{P}_{k+1}^{-}, \end{aligned} \quad (5)$$

an updated of the estimates can be given. Straightforwardly, the covariance matrix of the estimation error $\mathbf{P}_k$ remains block diagonal $\forall k$.

c) Update step with relative measurements: When two agents fall within the relative sensing range, a relative measurements can be collected. Let us assume that, at time kT_s, the i-th agent measure the j-th agent, in such a case (4) turns to

$$z_k^{(ij)} = h^{(ij)}(q_k^{(i)}, q_k^{(j)}) + \eta_k^{(ij)}, \quad (6)$$

that yields a linearised relative measurements matrix as $H_k^{(ij)} = \frac{\partial h^{(ij)}(q_k^{(i)}, q_k^{(j)})}{\partial q_k^{(i)}} + \frac{\partial h^{(ij)}(q_k^{(i)}, q_k^{(j)})}{\partial q_k^{(j)}}$. Similarly, the covariance matrix of the noise $\eta_k^{(ij)}$ is generically represented as $R_k^{(ij)}$.

The net effect of the use of the matrix $H_k^{(ij)}$ in the innovation covariance $\mathbf{S}_{k+1}$ computation, i.e. the covariance of the uncertainties related to the innovation brought by the relative measurements, is the generation of correlations in the covariance matrix of the estimates, i.e. $\mathbf{P}_k$ is no more block diagonal as in (3) but becomes

$$\mathbf{P}_k = \begin{bmatrix} P_k^{(11)} & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots & & \vdots & \\ 0 & \dots & P_k^{(ii)} & \dots & P_k^{(ij)} & \dots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \dots & P_k^{(ji)} & \dots & P_k^{(jj)} & \dots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 & \dots & P_k^{(NN)} \end{bmatrix},$$

where correlations among the agent estimates is generated. These new mixing elements represent the shared knowledge in the agents team that should be included in the computations of the next propagation and update steps. Although evident from this analysis, these terms are sometimes neglected in the literature, as for example in the interlaced filter defined in [15] and analysed in [17], and may lead to filter inconsistency depending on the ratio of absolute measurements in comparison with relative ones.

Remark 1: With only relative measurements, the system is not observable, that is it cannot be localised in the selected reference frame. Indeed, only the relative positions between agents can be recovered, while their absolute positions remains unknown. As a consequence, there is at least one eigenvalue of $\mathbf{P}_k$ that grows unbounded. This fact makes even more grounded the proposed analysis, as expressed in the next section.

d) General updates with mixed measurements: In general, the overall set of measurements is given by

$$\mathbf{z}_k = \begin{bmatrix} z_k^{(1)} \\ \vdots \\ z_k^{(N)} \\ z_k^{(ij)} \\ \vdots z_k^{(pl)} \end{bmatrix} = \begin{bmatrix} h^{(1)}(q_k^{(1)}) \\ \vdots \\ h^{(N)}(q_k^{(N)}) \\ h^{(ij)}(q_k^{(i)}, q_k^{(j)}) \\ \vdots \\ h^{(pl)}(q_k^{(p)}, q_k^{(l)}) \end{bmatrix} + \begin{bmatrix} \eta_k^{(1)} \\ \vdots \\ \eta_k^{(N)} \\ \eta_k^{(ij)} \\ \vdots \\ \eta_k^{(pl)} \end{bmatrix} = \begin{bmatrix} \mathbf{h}^a(\mathbf{q}_k) \\ \mathbf{h}^r(\mathbf{q}_k) \end{bmatrix} + \boldsymbol{\eta}_k, \quad (7)$$

whose Jacobians are given by

$$\mathbf{H}_k^a = \frac{\partial \mathbf{h}^a(\mathbf{q}_k)}{\partial \mathbf{q}_k} \text{ and } \mathbf{H}_k^r = \frac{\partial \mathbf{h}^r(\mathbf{q}_k)}{\partial \mathbf{q}_k}, \quad (8)$$

and where we used the superscript a or r to explicitly distinguish between absolute and relative measurements. As a consequence, the output matrix to be used in (5) at time kT_s will be given by $\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_k^a \\ \mathbf{H}_k^r \end{bmatrix}$.

Remark 2: The filter thus discussed does not assume that all the absolute and relative measurements are available at all time kT_s. Instead, the number of measurements can change in time due to the agent trajectory, as also assumed in [17].

B. Problem statement

The problem we are facing in this paper is to statistically evaluate the estimation error $\tilde{\mathbf{q}}_{k+1} = \mathbf{q}_{k+1} - \hat{\mathbf{q}}_{k+1}$ of the distributed EKF (recall that the presented EKF can be distributed among agents using [11] and assuming communication holds among the team) as a function of the number of agents involved in the team and with the presence of relative measurements. In particular, we want to empirically understand if: a) the relative measurements actually plays a role in mitigating the random effects in both the model and the measurements, b) the rule of thumb "the more measurements z_k are available, the less is the uncertainty affecting the estimates $\hat{q}_k$" is preserved at the team level, i.e. replacing z_k with $\mathbf{z}_k$ and $\hat{q}_k$ with $\hat{\mathbf{q}}_k$, and irrespective of the presence of correlations among the agents, c) if this effect is bounded after a certain value.

III. Models and Hypotheses

In order to increase the generality of the approach and clearly state if the relative measurements plays an actual role and to what extent, we select two different motion models for the mobile agents, which are inspired by the human motion models as presented in [18]. Moreover, to identify the trade-off between the number of relative measurements and the performances, we select two different ranging measurement techniques: an UWB approach, which allows lower uncertainties but a reduced sensing range [8], [9], and an RSSI approach [6], which instead ensures a longer sensing range with a reduced uncertainty. For the absolute reference, we again use a ranging system to simplify the analysis, but any possible solution actually applies.

A. Robot models

Using the motion model idea that underlies human dynamics [5], [18], we adopt two different models for the motion of the autonomous agents, which will be referred henceforth as robots. In the first model, the robots are modelled as points moving on a plane with respect to a fixed reference frame $\langle W \rangle$. Therefore, for the i-th robot we have $q_k^{(i)} = [x_k^{(i)}, y_k^{(i)}]^T$ with constant velocity dynamic

$$q_{k+1}^{(i)} = q_k^{(i)} + B \begin{bmatrix} v_{x_k}^{(i)} \\ v_{y_k}^{(i)} \end{bmatrix} + \boldsymbol{\varepsilon}_k, \tag{9}$$

where $v_{x_k}^{(i)}$ and $v_{y_k}^{(i)}$ are the velocities along X_w and Y_w global axes, respectively, and $B = T_s I_2$ describes the input matrix of the system.

The second model, instead, accounts for a smooth trajectory and follows a unicycle like nonlinear dynamic, i.e. $q_k^{(i)} = [x_k^{(i)}, y_k^{(i)}, \theta_k^{(i)}]^T$, which can be expressed for the i-th robot as

$$q_{k+1}^{(i)} = q_k^{(i)} + T_s \begin{bmatrix} \cos(\theta_k^{(i)})(v_k^{(i)} + \varepsilon_{v_k}^{(i)}) \\ \sin(\theta_k^{(i)})(v_k^{(i)} + \varepsilon_{v_k}^{(i)}) \\ (\omega_k^{(i)} + \varepsilon_{\omega_k}^{(i)}) \end{bmatrix} \tag{10}$$

where $v_k^{(i)}$ and $\omega_k^{(i)}$ are the forward and angular velocities, respectively.

For the model (9), three different absolute ranging readings are needed to compute trilateration and, hence, ensure the positioning of the system, while for the modelin (10) two consecutive readings are need to solve the localisation problem [7].

B. Sensors model

a) Absolute measurement: The absolute reference system considered in this paper is composed of m fixed UWB nodes of known position (x_{s_j}, y_{s_j}) used to compute the ranging. The model for the i-th robots with respect to the j-th sensor is then given by

$$z_{k,j}^{(i)} = h_j^{(i)}(q_k^{(i)}) + \eta_{k,j}^{(i)} = \sqrt{(x_k^{(i)} - x_{s_j})^2 + (y_k^{(i)} - y_{s_j})^2} + \eta_{k,j}^{(i)}. \tag{11}$$

The overall set of absolute measurements for the i-th robot is then given by $z_k^{(i)} = [z_{k,j_1}^{(i)}, \dots, z_{k,j_p}^{(i)}]^T$, where $j_1, \dots, j_p \in \{1, \dots, m\}$ and $p \le m$. It then follows that the i-th robot component of $\mathbf{H}_k^a$ in (8) will be given by

$$H_k^{(ii)} = \begin{bmatrix} 0 & \dots & 0 & \frac{\partial z_{k,j_1}^{(i)}}{\partial x_k^{(i)}} & \frac{\partial z_{k,j_1}^{(i)}}{\partial y_k^{(i)}} & 0 & \dots & 0 \\ 0 & \dots & 0 & \frac{\partial z_{k,j_2}^{(i)}}{\partial x_k^{(i)}} & \frac{\partial z_{k,j_2}^{(i)}}{\partial y_k^{(i)}} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & \frac{\partial z_{k,j_p}^{(i)}}{\partial x_k^{(i)}} & \frac{\partial z_{k,j_p}^{(i)}}{\partial y_k^{(i)}} & 0 & \dots & 0 \end{bmatrix},$$

where

$$\frac{\partial z_{k,j}^{(i)}}{\partial x_k^{(i)}} = \frac{x_k^{(i)} - x_{s_j}}{\sqrt{((x_k^{(i)} - x_{s_j})^2 + ((y_k^{(i)} - y_{s_j})^2}},$$

$$\frac{\partial z_{k,j}^{(i)}}{\partial y_k^{(i)}} = \frac{y_k^{(i)} - y_{s_j}}{\sqrt{((x_k^{(i)} - x_{s_j})^2 + ((y_k^{(i)} - y_{s_j})^2}}.$$

Since the different anchors can be seen as independent, we can safely assume that $R^{(ii)} = \sigma_a^2 I_p$, where σ_a^2 is the uncertainty affecting the absolute ranging measurement.

b) Relative measurements: As regards the relative measurements, i.e when the j-th robot is in the ranging area of robot i, we assume that the robots are also able to communicate and thus to exchange data. This way, robot i measure the relative position with respect to robot j and exchange this information with it. The nonlinear sensing function $h(q_k^{(i)}, q_k^{(j)})$ generically defined in (6) can then be specified as

$$h(q_k^{(i)}, q_k^{(j)}) = \sqrt{(x_k^{(i)} - x_k^{(j)})^2 + (y_k^{(i)} - y_k^{(j)})^2}$$

from which we can derive the linearised measurement matrix of $\mathbf{H}_k^r$ in (8) for the i-th robot as

$$H_k^{(ij)} = \begin{bmatrix} \dots & \frac{\partial z_k^{(ij)}}{\partial x_k^{(i)}} & \frac{\partial z_k^{(ij)}}{\partial y_k^{(i)}} & \dots & \frac{\partial z_k^{(ij)}}{\partial x_k^{(j)}} & \frac{\partial z_k^{(ij)}}{\partial y_k^{(j)}} & \dots \end{bmatrix}, \tag{12}$$

where

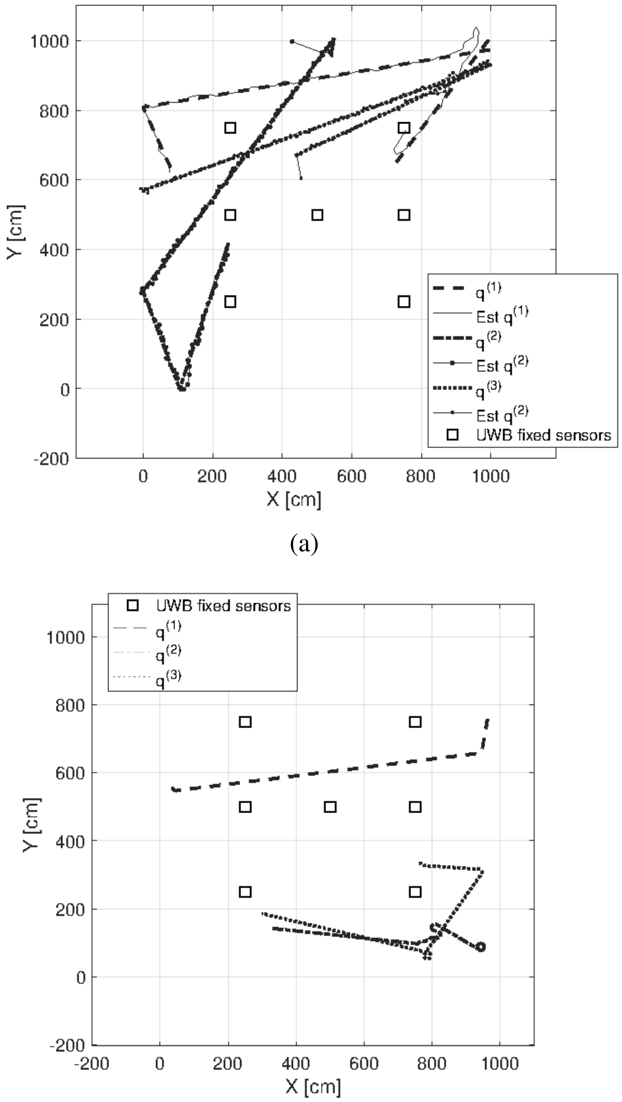
$$\frac{\partial z_k^{(ij)}}{\partial x_k^{(i)}} = \frac{x_k^{(i)} - x_k^{(j)}}{\sqrt{(x_k^{(i)} - x_k^{(j)})^2 + (y_k^{(i)} - y_k^{(i)})^2}} = -\frac{\partial z_k^{(ij)}}{\partial x_k^{(j)}},$$

$$\frac{\partial z_k^{(ij)}}{\partial y_k^{(i)}} = \frac{y_k^{(i)} - y_k^{(j)}}{\sqrt{(x_k^{(i)} - x_k^{(j)})^2 + (y_k^{(i)} - y_k^{(i)})^2}} = -\frac{\partial z_k^{(ij)}}{\partial y_k^{(j)}},$$

and all the other entries are zero.

IV. Simulation results

In this section, we will present the simulation results of the distributed EKF using Monte Carlo simulations over 1000 trials, each lasting 100 s. A variable number of agents, ranging from $N = 2$ to $N = 10$, is considered. The agents are supposed to move inside an empty 10×10 m^2 room. Each agent is supposed to be endowed with a system able to detect the room borders and, in such a case, the vehicle is steered away in border: this behaviour is inspired by off-the-shelf



(a)

(b)

Fig. 1. Example of one trial of the Monte Carlo simulation after 10 s of simulation for $N = 3$ robots using (a) the constant velocity model (9) or (b) the unicycle like vehicle model (10). Thick lines represents the actual trajectories $q_k^{(i)}$, while thin lines in (a) represents the estimated trajectory $\hat{q}_k^{(i)}$ by the distributed EKF.

consumer home cleaners. For the model (9), the velocity components $v_{x_k}^{(i)}$ and $v_{y_k}^{(i)}$ are randomly generated using a Brownian motion and assuming that the forward velocity has always a magnitude equals to $v_d = 1.7$ m/s, i.e.

$$v_d = \sqrt{{v_{x_k}^{(i)}}^2 + {v_{y_k}^{(i)}}^2}.$$

The chosen value of the velocity is the typical pace of normally abled human beings. For the unicycle model (10), the idea is basically the same, with the same v_d, but the generated trajectory are smoother. An example of the generated trajectories can be found in Figure 1. Notice that

x_{s_j} [m]	2.5	7.5	2.5	7.5	5 2.5	7.5
x_{s_j} [m]	2.5	2.5	7.5	7.5	5 5	5

TABLE I
DEPLOYMENT OF THE ABSOLUTE UWB SENSORS.

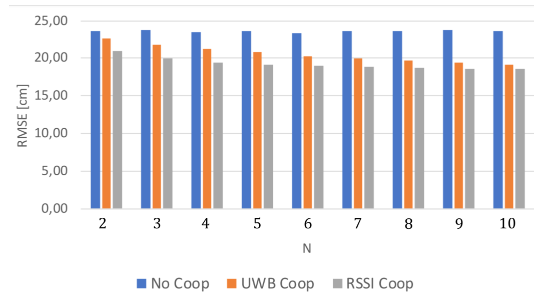


Fig. 2. RMSE of the distance as function of the number N of agents and for the different sensing configurations.

the same algorithm has been applied to avoid collisions among the agents. The sampling time for the simulations has been set to 100 ms, which is realistic for many commercial platforms.

For the relative sensors, we have considered UWB and RSSI based ranging. The UWB noise η_k is supposed to be generated by a stationary and normally distributed white stochastic process, with zero mean and variance $\sigma_{\text{UWB}}^2 = 25$ cm^2. The sensing range has been set to 4 m. On the other hand, the RSSI ranging has a sensing range of 15 m, while its noise η_k is again supposed to be generated by a stationary and normally distributed white stochastic process, with zero mean and variance $\sigma_{\text{RSSI}}^2 = 400$ cm^2. The absolute sensors, which are not under investigation in this paper but are considered to ensure localisation in the $\langle W \rangle$ reference frame, are seven and deployed as reported in Table I and Figure 1.

In all the simulations, the initial covariance matrices for the distributed EKF in (3) are diagonal, i.e. $P_0^{ii} = 10^3 I_n$ where n is equal to 2 for the model (9) and to 3 for the model (10). The covariance matrix of the model noise ε_k is again diagonal and equals to $Q = 36 I_2$ cm^2 (a realistic assumption given the models and the sampling time T_s). An example of the result of the estimated positions using only absolute measurements can be found in Figure 1-a reported with thin lines. Notice that despite the not modelled abrupt change of directions induced by the presence of the environment borders (or of the other agents), the tracking performance are still acceptable.

To clearly show the effect of the relative measurements in the estimation, we report in Figure 2 the average Root Mean Square Errors (RMSE) taken along the trajectories, across the team and across the different 1000 trials. We report the RMSE of the distance error between the estimated position and the actual one considering a variable number of agents ranging form $N = 2$ to $N = 10$. From this summarising results, reported only for the model (9) (since similar results are obtained for the model (10)), we first notice that using only absolute measurements (i.e. no cooperative solution, *No Coop* in the figure) the localisation results are obviously

Case	P-value	Null Hypothesis
No Coop	0.2574	Confirmed
UWB Coop	$2e-3$	Rejected
RSSI Coop	$1.97e-5$	Rejected

TABLE II
RESULT OF THE P-VALUE TEST SHOWING THE STATISTICAL EVIDENCE OF THE RELEVANCE OF THE RELATIVE MEASUREMENTS IN THE LOCALISATION PROBLEM.

independent from the number of robots. On the other hand, when a cooperative localisation strategy is adopted by means of relative measurements (*UWB Coop* and *RSSI Coop* in the figure), the performance are clearly increased and directly proportional with the number of agents N. To enforce this fact, a statistical analysis has been carried out. Specifically, a P-value test has been executed with the following assumptions:

- Null-Hypothesis H_0 : there is no statistical evidence on number of robots and pose estimation performance improvements;
- significance level: $\alpha < 0.05$.

The result of this test are reported in Table II, in which it is evident how the null hypothesis H_0 have been rejected for both the cooperative approaches.

What is of major relevance for this paper is the comparison between the two cooperative approaches: when the number of robots is small, the *RSSI Coop* shares more information among the team (there are on average 3 times more measurements w.r.t. *UWB Coop*) and, even if quite less accurate ($\sigma^2_{RSSI} = 16\sigma^2_{UWB}$), the performance radically increases. When the number of robots increases, the shared information become sufficiently large for the *UWB Coop* as well, hence the performance tend to be the same. In other words, the distributed estimator performance based on relative information appear to be dominated by the number of information that are exchanged rather than by their uncertainties, irrespective of the correlation between the agents. This fact may be surprising at a first look, but this is what actually happens in information sharing among humans in social groups: it is highly recognised that hearing the same opinion from n different persons has a major impact than listen to the same person, even with an undisputed reputation, saying the same argument n times [16], which is what we have called the *majority effect* in this paper.

V. CONCLUSIONS

Collaborative localisation refers to the ability of a group of robots to improve their own estimated positions by using absolute and relative measurements with respect to neighbouring agents. Using ranging measurements, we have shown that relative distance measurements play a fundamental role in the localisation performance. This fact becomes more evident when the probability of exchanging relative information between agents grows. The main result of the paper is on the trade-off between uncertainties and number of measurements: we have shown that having less precise sensors but with a higher ranging distance improves a lot the distributed EKF performance, which resembles the opinion formations in social groups. Further investigation are needed to find an analytic relation among those quantities and to implement the algorithm on an actual testbed.

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