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Maira Aguiar • Carlos Braumann • Bob W. Kooi • Andrea Pugliese • Nico Stollenwerk • Ezio Venturino Editors

# Current Trends in Dynamical Systems in Biology and Natural Sciences



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## Preface

The Ninth Edition of the International Workshop "Dynamical Systems Applied to Biology and Natural Sciences (DSABNS)" was held at the Department of Mathematics of the University of Torino, Italy, from February 7th to 9th, 2018. The workshop program included both theoretical methods and practical applications, covering research topics in population dynamics, epidemiology of infectious diseases, eco-epidemiology, molecular and antigenic evolution, and methodological topics in the natural sciences and mathematics.

Since 2010, the DSABNS workshop, which was upgraded to a conference in 2019, has been organized by the Mathematical Biology Group of the Center for Mathematics, Fundamental Applications and Operations Research (CMAF-cIO) of Lisbon University, in collaboration with professors and researchers from Portugal, Italy, and the Netherlands. From 2010 to 2015, the event was held at Lisbon University, during which time it acquired a broad organizational structure and attracted an increasing number of participants. From 2016 to 2017, the workshop was held in Évora (Portugal) and it then moved to Italy, being held in Torino in 2018 and Naples in 2019. As a traditional "no registration fee" scientific event, the DSABNS attracts researchers and students from different countries around the world who draw on their own funding to attend and present their recent scientific results. A book of abstracts (with ISBN number) is also published at the end of each event.

The Ninth DSABNS 2018 in Torino attracted the participation of 133 delegates from 30 countries. There were 13 plenary talks, 10 invited talks, 58 contributed talks, and a poster session. In this book, we have collected papers based on the research topics presented during DSABNS 2018, centering mainly on topics involving ecology and epidemiology but even touching on waste recycling and a genetic application. Some contributions also involve the application of numerical techniques to problems of structured populations.

In ecology, the contributions range from a theoretical investigation aimed at reconstructing the interactions of populations from a niche theory to other issues as the study of suitable techniques for the assessment of the patterns generated by invasive species in the spatiotemporal domain.

In the former case, chapter "Modelling Ecological Systems from a Niche Theory to Lotka-Volterra Equations", the concept of fitness landscape allows a stochastic description of species dynamics and the introduction of the notion of fitness potential for the evolution of a mutual ecosystem. Feasibility of its thermodynamic equilibrium, whose distribution is a multinomial negative distribution, is provided by the study of a master equation. In chapter "Accurate Recognition of Spatial Patterns Arising in Spatio-Temporal Dynamics of Invasive Species", it is remarked that being able to distinguish between the patchy spatial density patterns and continuous front spatial density patterns is essential for the implementation of control measures against invasive species. A model consisting of two integrodifference equations is proposed to investigate various spatial density distributions. With it, several topological characteristics are generated, among which it is found that the number of objects in the visual image of a spatial distribution offers the most reliable conclusion for distinguishing between continuous and patchy spatial structures. The two most relevant features of the monitoring protocol are found, namely the threshold density value and the number of sampling locations.

More abstract problems related to population theory are studied in the next two chapters. In chapter "Collocation Techniques for Structured Populations Modeled by Delay Equations", an improved numerical scheme is proposed based on piecewise polynomial collocation to reduce delay systems to systems of ordinary differential equations or to approximate a periodic solution. For realistic models of structured populations, the proposed method substantially improves performances in comparison with the existing ones that rely on an external ordinary differential equations solver. Its adaptability for the computation of periodic solutions is demonstrated.

A view differing from the classical predator-prey models is taken in chapter "Herding Induced by Encounter Rate, with Predator Pressure Influencing Prey Response", where the effects of herding are investigated, observing that populations living together have less than well-mixed interactions. A range of models is thus obtained for a single population, specifically hyperbolic models which exhibit intermediate growths between the exponential and the logistic ones. In the context of Lotka-Volterra intermingling populations, this formulation stabilizes coexistence. For predators, predation pressure is reduced, as well as access to resources. The latter is modeled via a reduction in carrying capacity with increasing predator pressure, while predator escape is formulated in terms of the degree of herding. The latter is the stronger, the larger the predator pressure becomes. Hopf bifurcations are possible, leading to stable limit cycles for specialist predators and unstable ones when generalist predators are considered.

Still in the context of ecology, in chapter "Harvesting Policies with Stepwise Effort and Logistic Growth in a Random Environment" constant and variable effort harvesting policies to maximize the expected total discounted profit are investigated over a finite horizon in the presence of stochastic fluctuations naturally occurring in real-life ecological situations. Due to the inapplicability and other shortcomings of the optimal variable effort policy, constant effort policies were considered. They are easy to implement, have no such shortcomings, and surprisingly provide a profit that is only slightly lower. The paper then studies variable effort stepwise strategies, where the effort is kept constant over one or two years and then updated. These stepwise policies are easy to implement at the cost of reducing the already low profit advantage of the optimal variable effort strategy.

In chapter "Mathematical Modeling of the Population Dynamics of Long-Lived Raptor Species: Application to Eurasian Black Vulture Colonies", a stochastic approach is also employed for the investigation of the population dynamics of raptor species. The long-lived Eurasian black vulture colonies are examined via discrete-time branching models, identified by time rather than by generation. A distinguishing feature in the population is the consideration of the coexistence of individuals from different generations. The most informative reproductive parameters are estimated in a non-parametric statistical setting using a Bayesian estimation procedure. Real data coming from the region of Extremadura (Spain) are used in the simulations. Specifically, the colonies used for the sampling represent two of the largest breeding colonies worldwide. They are located in the National Park of Monfragüe and in the Sierra San Pedro.

Control theory is also employed for waste recycling in chapter "On the Role of Inhibition Processes in Modeling Control Strategies for Composting Plants", in particular for the composting process of biocells. It allows optimization of the ways to provide air when inhibition due to a high concentration of oxygen occurs, thereby guaranteeing that the aerobic biodegradation process proceeds smoothly. Special attention is devoted to the assessment of the minimal cost of the control policy thus devised.

A further application of control is presented in chapter "Optimal Control of Invasive Species with Budget Constraint: Qualitative Analysis and Numerical Approximation". It concerns the optimal removal of invasive species, addressing the best temporal resource allocation strategy to achieve it. The optimality system in the state and control variables is derived, and phase-space analysis is used to provide qualitative insights about the behavior of the optimal solution. In particular, a practical situation involving plants is considered. The problem is reduced to a boundary-valued nearly-Hamiltonian system which is solved by suitable exponential Lawson symplectic approximations. An application to a real plant ground-reclaiming case is finally provided.

Control theory also represents the link with the second part of the contributions, describing investigations performed in the domain of epidemiology. A stabilization problem for an epidemic model, described by a reaction-diffusion system with feedback, is considered in chapter "A Shape Optimization Problem Concerning the Regional Control of a Class of Spatially Structured Epidemics: Sufficiency Conditions", where sanitation measures are envisaged. The main aim is the assessment of control programs administered only in a given subdomain of the region of interest that induce an effective disease eradication in the whole habitat. The sufficient optimality conditions are obtained and an approximate conceptual algorithm is discussed.

Vaccination, as an explicit disease control measure accounting for people's behavior, is considered in chapter "The Interplay Between Voluntary Vaccination

and Reduction of Risky Behavior: A General Behavior-Implicit SIR Model for Vaccine Preventable Infections". Two broad classes of behavior-implicit SIR models are reviewed: prevalence-dependent vaccination and prevalence-dependent contact rate. Then behavior-dependent and nonlinear and linear forces of infection are set in a general framework that also encompasses epidemic memory. These two different issues are here combined in a single unified approach that allows an assessment of the complicated interplay between the different behavioral responses due to various epidemiological parameters. As a result, sustained oscillations of vaccine coverage, risky behavior, and infection prevalence are obtained.

In epidemiology, a fundamental concept is the disease basic reproduction number  $R_0$ . In the presence of parameter uncertainties, the sensitivity estimation of the stochastic model is allowed by suitable numerical methods using polynomial chaos expansions. Evaluation of Sobol indices by polynomial chaos-based methods are presented in chapter "PC-Based Sensitivity Analysis of the Basic Reproduction Number of Population and Epidemic Models", showing how  $R_0$  is affected by varying the input parameters. The newly developed computational model for  $R_0$  introduced here allows for the efficient and versatile treatment of rather complex epidemic models.

Finally, an application to genetics is presented in chapter "Linear Dynamics of mRNA Expression and Hormone Concentration Levels in Primary Cultures of Bovine Granulosa Cells". The Gene Regulatory Matrices technique is here generalized to encompass also hormones, specifically estradiol (E2) and progesterone (P4), by constructing a directed weighted graph to model the interactions of several mRNA encoding enzymes. This allows the calculation of hormone concentration from the concentration of mRNA. This approach had previously been attempted only via differential equations, which are, however, limited by the need for accurate knowledge of the decay rates of hormones and mRNA. The novel technique with Gene and Hormone Regulatory Matrices allows estimation of the concentration on the whole network by using only a subset of its nodes. The models are constructed from data obtained in experiments providing gene expression and hormone concentration levels for primary bovine granulosa cells.

The collection of selected papers presented in this SEMA SIMAI Springer Series followed the traditionally rigorous reviewing standards of journals that are traditional to this series. The authors are indebted and express their thanks to Luca Formaggia and SIMAI for the kind invitation to contribute to this series.

Trento, Italy Évora, Portugal Amsterdam, The Netherlands Trento, Italy Lisbon, Portugal Torino, Italy May 2019 Maira Aguiar Carlos Braumann Bob W. Kooi Andrea Pugliese Nico Stollenwerk Ezio Venturino

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## **About the Editors**

**Maira Aguiar** is a biologist who has also trained in the mathematical modeling of biological systems, with emphasis on nonlinear dynamics, bifurcation analysis, and biostatistics. Her research investigates problems in public health epidemiology, focusing on the dynamics of vector-borne diseases. She has authored more than 35 papers and is frequently invited as a plenary speaker at international scientific meetings. Since 2018, she has been Vice President of the European Society for Mathematical and Theoretical Biology.

**Carlos A. Braumann** is Emeritus Professor in the Department of Mathematics and a member of the Research Centre in Mathematics and Applications, Universidade de Évora, Portugal, working on stochastic differential equations and biological applications. He has been an elected member of the International Statistical Institute since 1992 and has been President of both the European Society for Mathematical and Theoretical Biology (2009–2012) and the Portuguese Statistical Society (2006–2012).

**Bob W. Kooi**'s main research interests concern interacting populations in Life Sciences—Ecology, Evolution, Epidemiology, and Biochemistry—using mathematical models based on physical/chemical processes at different organizational levels: at the individual level, the Dynamic Energy Budget model, and at higher levels, unstructured/physiologically structured populations and community and ecosystem models. The emphasis is on sensitivity, perturbation, bifurcation, and nonlinear dynamics analysis techniques.

Andrea Pugliese is Professor of Mathematical Analysis and Mathematical Biology at the University of Trento. He obtained a Master's in Mathematics at the University La Sapienza of Rome and a PhD in Ecology and Evolution at the State University of New York at Stony Brook. He is the author of more than 90 scientific publications, mainly in the areas of mathematical epidemiology and ecology. He is an editorial board member for the Journal of Mathematical Biology and the Journal of Biological Dynamics. **Nico Stollenwerk** received his PhD in theoretical physics from the University of Clausthal. He previously worked at the Research Center Jülich, Germany and is currently the Principal Investigator of the Mathematical Biology group at CMAF, Lisbon University, where he has designed a Biomathematics PhD course. He is a coauthor of the book "Population Biology and Criticality" and has also coauthored many articles in international journals and more than 40 book chapters, as well as refereeing contributions in international congresses.

**Ezio Venturino** received his PhD in Applied Mathematics from SUNY at Stony Brook in 1984. He is currently Professor of Mathematics in the Department of Mathematics, University of Turin, Italy. He has visited a number of international institutions worldwide and has a wide scientific collaboration network. His earlier research focused on numerical analysis, mainly methods for integral equations, and he is currently engaged in research on nonlinear models for biological and ecological applications.