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## Original Research Paper

# State of the art and computational aspects of time-dependent waiting models for non-signalised intersections

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## HIGHLIGHTS

- In-depth review of the main time-dependent queue models at non-signalised intersections.
- Three equivalent queue models as “basic” cases for time-dependent demand and capacity.
- Three pairs of recursive queue length and delay equations directly usable in practice.
- Greater reliability and adherence to real cases compared to the most common formulas.

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## ABSTRACT

Time-dependent models are of great importance in road engineering as they are appropriate for evaluating waiting times and queue lengths at intersections, which are integral parts of various activities in planning, verification and decision support for infrastructure. After reviewing the literature of the main time-dependent models based on the coordinate transformation method and a discussion about some computational issues in time-evolution profiles for non-signalised intersections, the paper identifies the requirements these models have to satisfy in order to be used as “basic” cases for analysing complex evolutionary situations. Three “basic” cases are presented with their time-dependent equations for vehicle waiting times and vehicle number; they have been completed and dimensionally homogenised in this paper. As they are recursive, these formulas can be applied for sequential intervals in the time domain in both vehicles and passenger car units. The closed-form expressions for state variables show to be mutually equivalent in comparison with discrete event simulation models and imbedded Markov chain results. For all the three models the paper presents a common deterministic simplification for average waiting time, with good approximation results in the tested cases. The proposed time-dependent formulas will contribute to a better adherence to the real phenomena, compared to the extremely simplified and unrealistic methods suggested by the international manuals for level of service assessment. The proposed formulas will be useful for current applications and possible future development in order to meet the emerging needs of road and transport engineering.

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## 1. Introduction

According to the most up-to-date international manuals, American HCM 2016 and German HBS 2015, levels of service (LOS) at non-signalised intersections can be estimated in practical cases by using waiting times and consequent average delays suffered by cars at intersection arms as principal measure of effectiveness (MOE).

As well known in transportation engineering, waiting times and queue lengths can be evaluated through the results from the waiting theory, but the probabilistic queue theory is also known to be inadequate to fully respond to any situation where an intersection may be found, thus generating realistic results only in stationary situations that are fairly distant from high saturation levels (Mauro, 2010).

Time-dependent models in queue analysis have been studied in the past and are still being investigated, especially in the form generated by the coordinate transformation method (Catling, 1977; Kimber and Hollis, 1979), because of great interest in road engineering. Such models are useful for cases which cannot be tackled by simply using the results of the stationary queue theory due to statistical non-equilibrium conditions, in whatever way they may occur.

Considering that also HCM and HBS manuals propose time-dependent formulas for MOEs evaluation (i.e., queue state variables) at non-signalised intersections, the objective of the paper is to direct the reader's attention to some models available in the literature and to overcome some drawbacks of those currently used for describing real-life situations (Brilon, 2015; Wu, 2006, 2009). It should be emphasized that the assessment related to the LOS analysis is of primary importance in many applications regarding the planning and design activities for road and transport systems. Moreover, from this point of view, the formulas for estimating MOEs in LOS analysis require models that, although conventional, result to be well justifiable in their genesis, reliable and quickly computable.

As will be clarified below, in international manuals LOS evaluation at non-signalised intersections concerns a defined time slice, during which demand and capacity are assumed to be constant. The proposed formulations, in fact, are inadequate to bind the situation in the current time frame to that preceding or following it with variations in demand and capacity. The need for a reliable evaluation requires the intersection performance to be continuously measured over periods with variations in traffic and capacity. Since no closed-form expression can be used for continuous MOE and LOS evaluations, the alternatives can be the numerical and simulative approaches. Numerical approaches, for example by means of Markovian chains, are generally characterized by a complex implementation of the basic model which,

however, develops complications that make it unsuitable for current professional practice. Also simulation techniques, such as micro-simulations or discrete event simulations, may lead to inconvenience in use, for instance, i) difficulty in generalizing, and consequently the necessity of proceeding on a case-by-case model; ii) the burden of implementation, verification, calibration and validation of the models, and iii) the need to reiterate the simulation for a sufficiently large number of times in order to produce statistically significant results.

Thus, this work is motivated by the need to identify, among the models developed for being directly implemented in applications, those that i) have a closed-form expression for time-dependent analyses; ii) can replace the currently used formulas in order to achieve results closer to real cases and, at the same time, get over critical points; iii) are not too sophisticated but easy to calculate; iv) can show great versatility when appropriately connected in sequence, in order to study complex cases as sequences of elementary or "basic" cases.

This work will indicate important technical applications to many aspects. Witness, for example, the assessment of waiting times and queue lengths; the quantitative characterization of transport networks for traffic demand assignment in transport planning; reconfiguration and management for intersection design; more generally, as a support for formulating implementation decisions in cost-benefit analyses.

Clearly, these aspects involve both traditional issues and emerging questions in road and transport engineering. For instance, among the latter, the issues concerning the optimal organization of intersections in the presence of a mix of traffic components are certainly interesting in terms of safety and operational functionality. Although the analysis here has taken only the vehicular component into account, the results can be used as a basis to address innovative topics related to modelling the effects on the intersection performance in terms of safety, capacity, queue lengths, waiting times and pedestrian-vehicle interaction (Gorrini et al., 2018; Helbing and Molnar, 1998; Zeng et al., 2014), to introducing intelligent transportation systems (ITS) and, especially, to mixing traditional vehicles and connected vehicles (CVs) (Guler et al., 2014; Xu et al., 2018; Yang et al., 2016).

The paper is structured as follows. Section 2 introduces the time-dependent approach to queue phenomena and incorporates the essential features of the coordinate transformation method. Section 3 reviews and discusses the main time-dependent models in the literature for non-signalised intersections based on the coordinate transformation method, highlighting the hypotheses for their inference, the links with the formulas currently used and related drawbacks. Section 4 clarifies some computational aspects in time-evolution profiles and explores the possibility of their evaluation by recursively

applying closed-form equations to appropriate “basic” cases. Section 5 deals with the modularity of three identified “basic” cases, completing three pairs of equations which can be applied sequentially. Sections 6 and 7 present two numerical approaches, respectively the imbedded Markov chain (IMC) and the stochastic discrete event simulation (SDES), for the time-dependent evaluation of a waiting system. Section 8 achieves and discusses the recursive equation results for the “basic” cases compared with IMC and reiterated SDES, examining four time evolution profiles for the entry demand and capacity. Section 9 contains the concluding remarks.

## 2. The coordinate transformation method in time-dependent queue models

The basic model for a non-signalised intersection is shown in Fig. 1. It refers to a single secondary arm  $i$ , easily generalisable for multiple arms. In this system, the first space occupied by a vehicle immediately near the stop line can be considered as the point of service, or service counter. In the further waiting positions behind the service point, the actual queue development can be observed. The vehicle number in the system  $L_{is}$  is given by the vehicle number in the queue  $L_{ic}$  plus the vehicle in the service counter. In parallel, with a time  $s_i$  waiting by a vehicle in the service counter called service time and with a queue waiting time  $d_i$ , an user queuing at the arm  $i$  accumulates a waiting time in the system that is  $w_i = s_i + d_i$ .

We define traffic intensity, coefficient or degree of saturation  $\rho_i$  for the secondary arm  $i$  as the quotient of the traffic demand  $Q_{ei}$  over the entry capacity of the same arm  $C_i$  ( $\rho_i = Q_{ei}/C_i$ ). For arm  $i$  we can distinguish the following cases: the traffic demand is less than the entry capacity (under-saturation  $\rho_i < 1$ ); the traffic demand is equal to the entry capacity (saturation  $\rho_i = 1$ ); the traffic demand is greater than the entry capacity (oversaturation  $\rho_i > 1$ ).

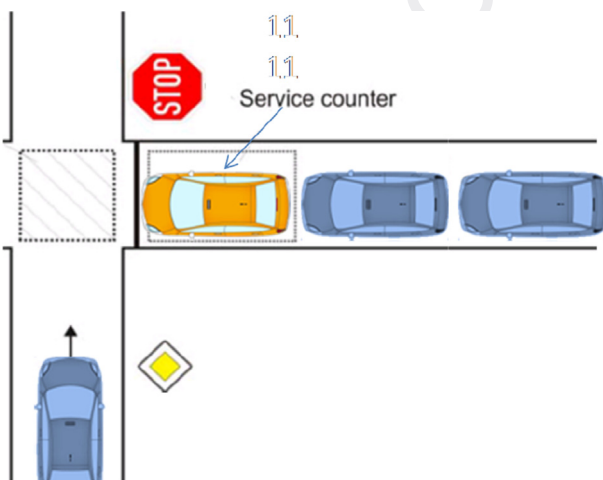


Fig. 1 – Priority system in a simple non-signalised intersection.

Furthermore, we assume the constancy of the average or median values of the critical interval and of the sequence time for the secondary traffic flow. For technical purposes, as it is known, if the time period of analysis  $T$  is sufficiently large and if  $Q_{ei}$  and  $C_i$  are constant during the same period, we can operate as follows.

- If the entrance is under-saturated, for the applications we can use the results of the probabilistic queue theory models for statistical equilibrium. These results are even more realistic the more the entry arm is far from saturation ( $\rho_i \ll 1$ ). Starting from values of  $\rho_i$  generally higher than 0.6–0.8, the probabilistic queue theory provides queue length determinations and waiting times rapidly tending to infinity with  $\rho_i \rightarrow 1$ . Since the interval  $T$  has finite amplitude, however extensive, the aforementioned results for  $\rho_i \rightarrow 1$  appear unrealistic (Webster, 1958).
- If the entrance is saturated ( $\rho_i = 1$ ) or over-saturated ( $\rho_i > 1$ ), it is not possible to use the probabilistic queue theory for statistical equilibrium. Furthermore, using the probabilistic queue theory for conditions other than statistical equilibrium leads to overly complex results for practical applications.
- If the entry is over-saturated, however, we can use the deterministic theory of waiting phenomena, which treats the traffic flow as a continuous fluid (May and Keller, 1967), with more reliable results as the saturation increases ( $\rho_i \gg 1$ ).

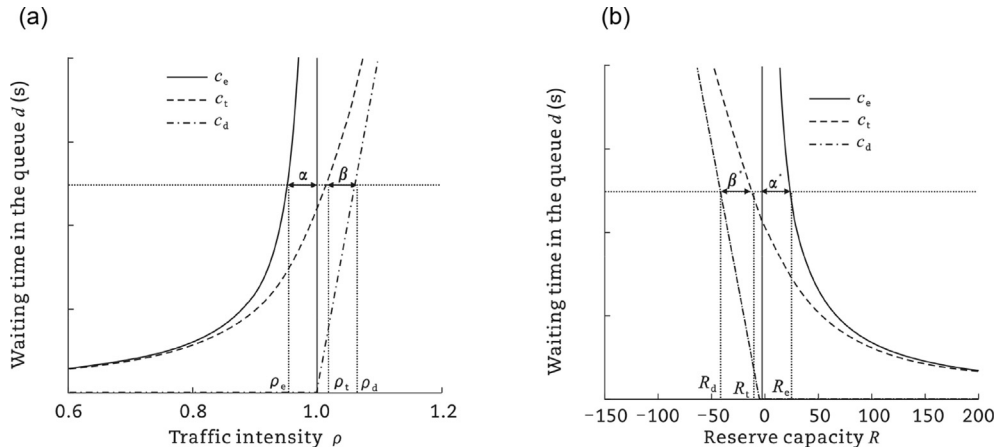
If  $Q_{ei}$  and  $C_i$  vary over time, whatever the status of the entrance (i.e.,  $\rho_i$  values), the results of the queue equilibrium for statistical non-equilibrium are difficult to apply with technical purposes. For over-saturated conditions the deterministic approach can be applied, whatever  $Q_{ei}$  and  $C_i$  vary over  $T$ . The higher the over-saturation levels ( $\rho_i \gg 1$ ) are, the more reliable the results become.

In conclusion, as the entrance can be sub-saturated but close to saturation, saturated or over-saturated with a saturation degree  $\rho_i$  not significantly greater than 1, the probabilistic theory does not apply to the former case, and the deterministic one does not apply to the other two.

In order to treat all the saturation cases at entrances through a unified approach for waiting phenomena at non-signalised intersections, the so-called “time-dependent” models have long been developed. Among the first contributions we can mention Catling (1977) and Kimber and Hollis (1979). As explained further below in the paper, these models make it possible to define some heuristic relationships in order to conveniently combine the solutions deriving from the stationary probabilistic queue theory with those from the deterministic theory of the waiting phenomena. More specifically, with a “time-dependent” model, the probabilistic model is best approximated for  $\rho_i \ll 1$ , modifying the trend in order to tend asymptotically to the deterministic model for  $\rho_i \gg 1$ .

A common approach for time-dependent models is known as “coordinate transformation” or “shared queue” method, developed at the Transportation Research Laboratory initially by Whiting for the TRANSYT program (Robertson, 1979) and later extended by Kimber and Hollis (1979). A similar approach





**Fig. 2 – Transitional curve for waiting time (dash line) inferred with coordinate transformation between deterministic (dash dot line) and probabilistic (solid line) cases. (a)  $\rho$ . (b)  $R$ .**

is due to [Doherty \(1977\)](#), extended and generalized by [Catling \(1977\)](#). In general, the method allows to determine heuristically the transition curve  $c_t$  for the queue length or for the waiting time (dependent variables) by performing a coordinate transformation with respect to traffic intensity  $\rho$  or reserve capacity  $R = C_i - Q_{ei}$  (independent variables) for the curve that represents the relationship under steady-state conditions  $c_e$ , modifying its trend in order to have the deterministic curve  $c_d$  as asymptote. More recently, [Wong et al. \(2003\)](#) propose a review of the coordinate transformation theory, with a general formulation of the transition curves, parameterized with respect to each specific state variable.

In general, with reference to [Fig. 2\(a\)](#), if we use the coordinate transformation method with respect to  $\rho$ , the transition curve  $c_t$  is obtained by imposing the conditions  $\alpha = \beta$  or  $\alpha/1 = \beta/\rho_d$  in relation to  $c_e$  that provides the solutions for the queue system in statistical equilibrium (steady-state queue), and the asymptote  $c_d$  that describes the state of the system in a deterministic way. With a selected value for a generic state parameter (represented in [Fig. 2\(a\)](#) by the waiting time in the queue  $d$ ), we can consider  $\alpha$  as the distance between the fixed value on  $c_e$  and the vertical asymptote for  $\rho = 1$ , and  $\beta$  as the distance between the curve  $c_t$  that is to be determined and the half-line representing the oblique asymptote of the curve  $c_d$ . If we indicate with  $\rho_e$ ,  $\rho_d$  and  $\rho_t$  the values of the independent variable  $\rho$ , uniquely identifiable by the fixed value of the state parameter on  $c_e$ ,  $c_d$  and  $c_t$ , we obtain  $\alpha = 1 - \rho_e$  and  $\beta = \rho_d - \rho_t$ , and the two equations for the coordinate transformation regarding  $\rho$  are

$$\rho_e = \begin{cases} \rho_t - (\rho_d - 1) & \alpha = \beta \\ \rho_t / \rho_d & \alpha/1 = \beta/\rho_d \end{cases} \quad (1)$$

If we consider  $R$  as the independent variable, with reference to [Fig. 2\(b\)](#), the curve  $c_t$  is obtained by imposing the conditions  $\alpha^* = \beta^*$  (a second condition similar to that imposed for  $\rho$ , i.e.,  $\alpha^*/1 = \beta^*/R_d$  would not make sense ([Brilon, 2008](#))) in relation to  $c_e$  and to the asymptote  $c_d$ . Fixing a generic value to the state parameter (represented in [Fig. 2\(b\)](#) by  $d$ ), we can consider  $\alpha^*$  as the distance between the fixed value on  $c_e$  and the vertical asymptote for  $\rho = 1$ ,

and  $\beta^*$  as the distance between  $c_t$  that is to be determined and the half-line representing the oblique asymptote  $c_d$ . If we indicate with  $R_e$ ,  $R_d$  and  $R_t$  the values for  $R$ , uniquely identifiable by the fixed value of the status parameter on  $c_e$ ,  $c_d$  and  $c_t$ , we obtain  $\alpha^* = R_e$  and  $\beta^* = |R_d - R_t|$ , and the equation for the transformation of the coordinates with respect to  $R$  results

$$R_e = |R_d - R_t| \quad \text{if} \quad \alpha^* = \beta^* \quad (2)$$

The coordinate transformation provides the system status in terms of vehicle number and waiting times experienced during a specified observation period, with constant average demand and capacity, thus allowing the management of saturation conditions at the entry. The method can be seen as an approach capable of evaluating time-dependent state variables, whatever their specification, if appropriately applied in succession over several consecutive time slices  $T_k$ , with the constraint that the final queue  $L_{CT,k}$  of each slice  $T_k$  represents the initial queue  $L_{C0,k+1}$  of the following  $T_{k+1}$ , and allows us to approximate and process any entry demand and capacity profile, evaluating the evolving characteristics of queues and waiting times accordingly.

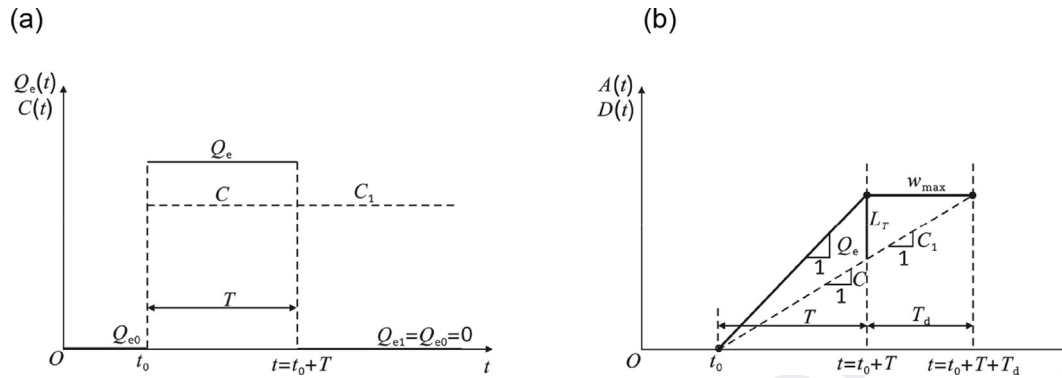
In the following sections we describe some models that make use of the capacity reserve  $R$  or of the saturation coefficient  $\rho$  to compare, among other things, the substantial differences that occur when they are used operationally.

### 3. Models review and discussion

#### 3.1. Akçelik and troutbeck model

The simplest time-dependent model for calculating the waiting time and queue lengths in such a system as in [Fig. 1](#) is given by the equation due to [Akçelik and Troutbeck \(1991\)](#), referring to the situations for entry traffic demand  $Q_{ei}$  and capacity  $C_i$  defined in [Fig. 3\(a\)](#).

According to [Akçelik and Troutbeck \(1991\)](#) the expression for the waiting time in the queue  $d_T$  (s), as the average value during the interval  $T$  (s), is given by the following equation



**Fig. 3 – Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991). (a) Traffic flow  $Q_e(t)$  and capacity  $C(t)$ . (b) Cumulative arrivals  $A(t)$  - departures  $D(t)$  for the deterministic queue.**

$$d_{iT} = \frac{T}{4} \left[ (\rho_i - 1) + \sqrt{(\rho_i - 1)^2 + \frac{8\rho_i}{TC_i}} \right] \quad (3)$$

where the entry traffic flow  $Q_{ei}$  and capacity  $C_i$  are constant during  $T$  and expressed in veh/sec. According to Eq. (3), since service time  $s_i$  is equal to the reciprocal of  $C_i$ , the average waiting time in the system  $w_{iT}$  becomes

$$w_{iT} = \frac{1}{C_i} + \frac{T}{4} \left[ (\rho_i - 1) + \sqrt{(\rho_i - 1)^2 + \frac{8\rho_i}{TC_i}} \right] \quad (4)$$

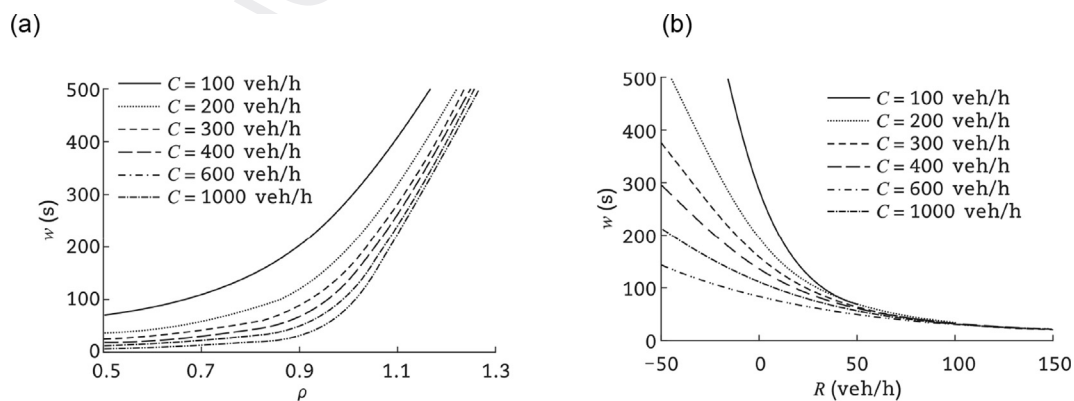
Regarding Eq. (4), Brilon (1995) suggests using  $R$  as the independent variable, highlighting that for a long period  $T$  (equals to 3600 s, for example) the curves expressing  $w_{iT} = f(R_i; C_i)$  appear to be very close to each other, with a trend almost independent of normal delays that can be recorded in practical cases less than 60 s (Fig. 4(b)). From this point of view, using  $R$  makes it easier to express some inter-relationships with other theoretical aspects of non-signalised intersections (Brilon, 1995). The same thing does not happen using  $\rho$ , getting curves  $w_{iT} = f(\rho_i; C_i)$  which appear to be much more dispersed when  $C_i$  varies (Fig. 4(a)).

Brilon (2007, 2008) demonstrates that Eq. (3) can be obtained by applying the coordinate transformation with the condition  $\alpha = \beta$  as in Fig. 2(a),  $Q_{ei}$  and  $C_i$  as in Fig. 3(a), and it

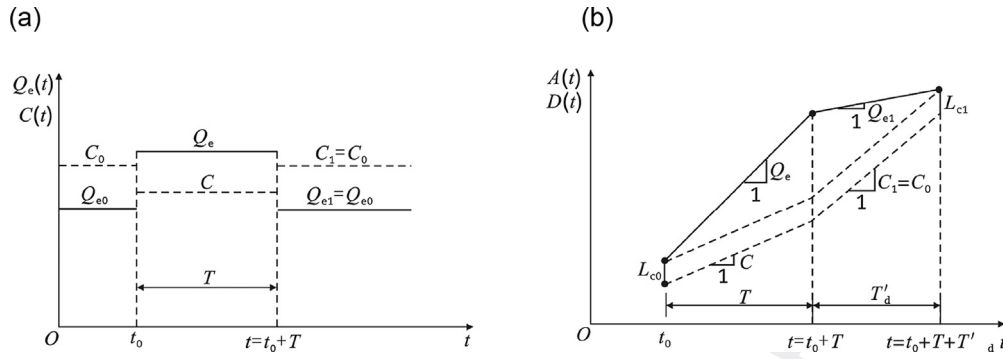
can identify an M/M/1 model for the queuing system in statistical equilibrium and with a deterministic curve, to which the transformation tends, as in Fig. 3(b).

The coordinate transformation that allows obtaining Eq. (3) considers the waiting time in the queue  $d_i$  for the steady-state queue, and the waiting time in the system  $w_i$  for the deterministic queue. However, this shows a certain degree of inconsistency since the approximation converges on terms of a different nature ( $d_i$  and  $w_i$ ) for the two extremes of the independent variable domain ( $\rho_i \rightarrow 0$  and  $\rho_i \rightarrow \infty$ ). Brilon (2007, 2008) demonstrates that Eq. (3) can be obtained by applying the coordinate transformation imposing  $\alpha/1 = \beta/\rho_d$  as in Fig. 2(a) for an M/M/1 queuing system in equilibrium with entry flow/capacity condition and cumulative counts for arrivals and departures, as shown in Fig. 3(a) and (b) respectively. This transformation takes place in homogeneous terms, considering  $d_i$  for both cases  $\rho_i \rightarrow 0$  and  $\rho_i \rightarrow \infty$ .

Taking Eq. (4) again into consideration, it should be noted that this equation is proposed by the American Highway Capacity Manual (HCM) from 1994 until the last edition of 2015. The last two HCM editions (i.e., 2010 and 2015) consider a further term  $d' = 5$  s to consider the time needed to decelerate up to the queued vehicle speed and to accelerate up to the free flow speed, once the stop line has been crossed (Eqs. (19)–(64) in HCM 2010, Eq. (128) in NCHRP



**Fig. 4 – Graphic representations of waiting time in the system with Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991). (a)  $w_{iT} = f(\rho_i; C_i)$ . (b)  $w_i = f(\rho_i; R_i)$ .**



**Fig. 5 – Kimber and Hollis model (Kimber and Hollis, 1979). (a) Traffic flow  $Q_e(t)$  and capacity  $C(t)$ . (b) Cumulative arrivals  $A(t)$  - departures  $D(t)$  for the deterministic queue.**

report (Richard et al., 2016)). The additional term becomes  $d' = 5\min(\rho_i, 1)$  for roundabouts, considering that vehicles need an extra time to stop and start again as  $\rho_i$  increases, while for low saturation levels vehicles can cross the priority line without stopping completely (Eqs. 17–21 in HCM 2010, Eq. (143) in NCHRP report (Richard et al., 2016)). It can also be noted that the German HBS capacity manual (FGSV, 2015) suggests the Akçelik and Troutbeck model in the form of Eq. (4) for non-signalised intersections in its latest edition (FGSV, 2015).

Nevertheless, Eq. (4) shows a serious drawback because it assumes that the traffic before and after the observation period  $T$  is null. This hypothesis is obviously unrealistic. It should be noted, however, that in Troutbeck and Brilon (2000) further sophistications of Eq. (3) and consequently of Eq. (4) are given which allow positive traffic flows to be considered before and after  $T$ . These authors provide an expression of the original Akçelik and Troutbeck time-dependent model (Akçelik and Troutbeck, 1991) which considers the presence of users in the system at the beginning of the observation period ( $L_{is0} \neq 0$ ). The equation is also shown in Cvitanic et al. (2007). A very similar formula proposed by Heidemann (2002) applies the coordinate transformation with the condition  $\alpha/1 = \beta/\rho_d$  as in Fig. 2(a) for an M/G/1 queue system, obtaining the following

$$w_{iT} = \frac{1}{C_i} + \frac{1}{2} \left\{ \frac{L_{is0} + (\rho_i - 1)T}{C_i} + \sqrt{\left[ \frac{L_{is0} + (\rho_i - 1)T}{C_i} \right]^2 + \frac{2\rho_i}{C_i} T\gamma} \right\} \quad (5)$$

where  $\gamma = (1 + \sigma^2 C_i^2)/2$  and  $\sigma^2$  is the service time variance.

Eq. (5) is reported in the form of Heidemann (2002). Troutbeck and Brilon (2000) and Cvitanic et al. (2007) show its variant

$$w_i = \frac{1}{C_i} + \frac{1}{2} \frac{L_{is0} + (\rho_i - 1)T}{C_i} + \sqrt{\left[ \frac{L_{is0} + (\rho_i - 1)T}{2C_i} \right]^2 + \frac{\rho_i}{2C_i} T\epsilon}$$

with  $\epsilon$  constant approximately equals to 1.

If the queue system in statistical equilibrium is of the type M/M/1, it can be assumed  $\gamma = 1$ , being  $\sigma^2 = 1/C_i^2$ .

Considering that, in any case, Eq. (4) is recommended by the most recent international manuals, it is necessary to keep in mind another drawback highlighted by Wu (2009). The author highlights a dimensional inconsistency deriving from the fact that the shared equations have different

dimensions for the stationary/deterministic waiting times (s/veh) for  $\rho_i \rightarrow 0$  (stochastic formulation) and (s) for  $\rho_i \rightarrow \infty$  (deterministic formulation). This incongruity generates further problems due to the homogenization of the vehicle classes into passenger car units (pcu), made by using the equivalence coefficients recommended by the HCM. Wu (2009) solves the dimensional inconsistency by making a distinction between the capacity of the waiting system in statistical equilibrium  $C_{i,st}$ , expressed as reciprocal of the average service time of the stationary queue and therefore in (1/s), and the deterministic one  $C_i$  expressed in (veh/hr). The distinction allows Eq. (4) to be written as follows

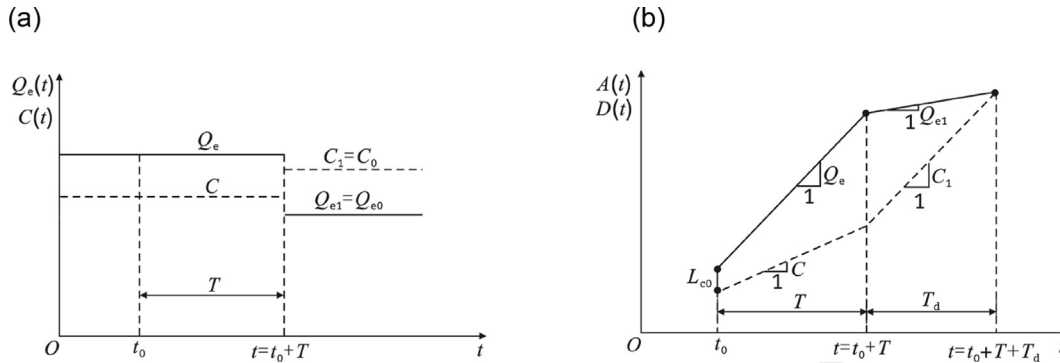
$$w_{iT} = \frac{3600}{C_{i,st}} + 900T \left[ (\rho_i - 1) + \sqrt{(\rho_i - 1)^2 + \frac{3600f_o\rho_i}{450TC_i}} \right] \quad (6)$$

where  $w_{iT}$  is expressed in s,  $T$  in h,  $C_{i,st}$  in 1/s,  $C_i$  in veh/h or pcu/h,  $\rho_i = Q_{ei}/C_i$  and  $f_o$  is a constant that takes into account how  $Q_{ei}$  and  $C_i$  are expressed ( $f_o = 1$  veh, if they are expressed in veh/h,  $f_o = f$  in unit of pcu, if they are expressed in pcu/h with  $f$  corresponding to the global equivalence coefficient used and being equal to the ratio between pcu/h and veh/h).

### 3.2. Kimber and Hollis model

Temporally, the Kimber and Hollis model (Kimber and Hollis, 1979) is the first among the time-dependent models based on the coordinate transformation method, developed by the authors. The model is derived by imposing the condition  $\alpha = \beta$  as in Fig. 2(a) for entry flow and capacity showed in Fig. 5(a), considering an M/G/1 model for the queuing system in equilibrium, and cumulative counts for arrivals and departures as in Fig. 5(b). Now we can point out how the hypothesis for  $Q_{ei}$  and  $C_i$  - which are at the basis of Kimber and Hollis inference (Kimber and Hollis, 1979) - appears more realistic than those underlying Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) expressed in Eq. (4).

As shown in Fig. 5, before and after the peak period  $T$  during which  $Q_{ei} > C_i$ , traffic volumes  $Q_{ei0}$  and  $Q_{ei1}$  are different from zero (with  $Q_{ei0} = Q_{ei1}$ ) and lower than their respective capacities  $C_{i0}$  and  $C_{i1}$  (with  $C_{i0} = C_{i1}$ ). According to Kimber and Hollis (1979), with traffic flow and capacity in



**Fig. 6 – Brilon model (Brilon, 2007, 2008). (a) Traffic flow  $Q_e(t)$ , capacity  $C(t)$ . (b) Cumulative arrivals  $A(t)$  - departures  $D(t)$  for the deterministic queue.**

veh/h and  $T$  in h, the expression for  $d_{iT}$  in s is given by the following set of equations (Brilon et al., 1997; Mauro, 2010).

$$d_{iT} = \left[ \frac{1}{2} \left( \sqrt{F^2 + G} - F \right) + E \right] 3600 \quad (7)$$

$$F = \frac{1}{C_{i0} - Q_{ei0}} \left[ \frac{T}{2} (C_i - Q_{ei}) z + \gamma \left( z - \frac{H}{C_i} \right) \right] + E \quad (8)$$

$$G = \frac{2Tz}{C_{i0} - Q_{ei0}} \left[ \gamma \frac{Q_e}{C_i} - (C_i - Q_{ei}) E \right] \quad (9)$$

$$E = \frac{\gamma Q_{ei0}}{C_{i0} (C_{i0} - Q_{ei0})} \quad (10)$$

$$H = C_i - C_{i0} + Q_{ei0} \quad (11)$$

$$z = 1 - \frac{H}{Q_{ei0}} \quad (12)$$

where  $\gamma = (1 + \sigma^2 C_i^2) / 2$  and  $\sigma^2$  is variance for  $s_i$ . According to the same authors, it can be assumed that  $\gamma = 1$  for non-signalised intersections (for an M/M/1 queue model it holds  $\sigma^2 = 1/C_i^2$ ) or  $\gamma = 0.5$  for signalised intersections (for an M/D/1 queue model it holds  $\sigma^2 = 0$ ). In the latter case, however, the use of empirical values between 0.5 and 0.6, which reflect the non-perfect uniformity of the service, is pointed out by some authors (Branston, 1978; Catling, 1977; Kimber and Hollis, 1979; Wong et al., 2003). It should also be noted that  $\gamma$  takes into account only the service process by means of its variation coefficient  $c_b = \sigma C_i$ .

Kimber et al. (1986) propose an alternative formulation to take into account also the variation coefficient of arrivals  $c_a$ , in such a way as to allow a heuristic approximation for non-Poissonian arrivals. It should be noted that the original formula of the model considers a parameter  $\gamma' = \epsilon \gamma$  (Brilon et al., 1997; Mauro, 2010), with an  $\epsilon$  value introduced as a result of validation and calibration, to take into account peak periods  $T$  of very short duration. For short periods of about 2 or 3 min, a value  $\epsilon = 2$  is suggested, while for longer time periods, e.g., between 10 and 15 min, that are more usual in technical evaluations, it is necessary to consider  $\epsilon = 1$ , thus avoiding unrealistic results.

As noted by Mauro (2010), Eq. (7) represents the average waiting time in the queue which is regarded as

conventional: despite being related to all vehicles affected by the traffic peak (vehicles arrived in the peak period  $T$  plus the following  $T_d$ , necessary to bring back the queue  $L_{ic1}$  to the value  $L_{ic0}$  occurring before the peak), it is distributed by considering only the vehicles arrived during  $T$ . In order to get the average waiting time in the system  $w_i$ , the average service time  $s_i$  must be added to Eq. (7). This value should be calculated as the average of the reciprocals for  $C_{i0}$  and  $C_{i1}$  weighed considering the respective application intervals. In a simplified way, Mauro (2010) proposes to consider only the reciprocal of  $C_i$  during  $T$ , so the waiting time in the system in s becomes

$$w_{iT} = \left[ \frac{1}{2} \left( \sqrt{F^2 + G} - F \right) + E + \frac{1}{C_i} \right] 3600 \quad (13)$$

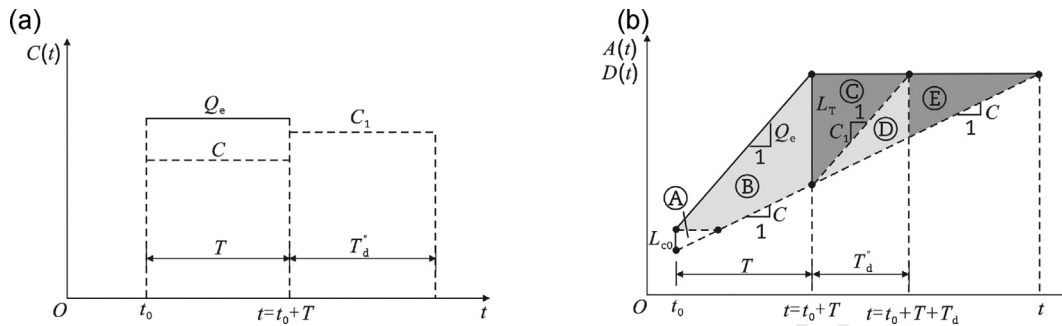
with Eqs. 8–12 still valid. Eq. (13) in the variables  $F$ ,  $G$ , and  $E$  expressed by Eqs. 8–12 is recommended in the 2001 edition of the German HBS capacity manual (FGSV, 2001). Bildgleichungen 7–19). Also regarding this model, Wu (2006) finds a dimensional inconsistency, similar to that indicated in the Akçelik and Troutbeck model, if flows and capacities are expressed in equivalent vehicles considering a global equivalence coefficient  $f$ . The inconsistency is solved by operating with the parameter  $\gamma' = \epsilon \gamma f_0$ , for a non-signalised intersection ( $\gamma = 1$ ) and a peak period of about 10–15 min ( $\epsilon = 1$ ) it results  $\gamma' = f_0$ . With the global equivalence coefficient  $f$ , it arises  $f_0 = f$  in unit of pcu by assigning the correct dimension of equivalent vehicles; if  $f_0 = 1$  veh, then the correct dimension of vehicles can be assigned.

### 3.3. Brilon's models

Brilon (2007, 2008) proposes a time-dependent model based on the coordinate transformation method, inferred for traffic flows and capacities as in Fig. 6(a) for an M/M/1 equilibrium queue model, and with cumulative counts for arrivals and departures for the deterministic queue as in Fig. 6(b).

Also in this case the basis hypotheses for the inference appear more realistic than those in the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) expressed with Eq. (4), with non-zero flows before  $T$ , at the beginning  $t_0$  of which system a queue  $L_{ic0} \neq 0$ . As shown in Fig. 6, before and after  $T$  during which  $Q_{ei} > C_i$ , traffic flows  $Q_{ei0}$  and  $Q_{ei1}$  assume non-





**Fig. 7 – Brilon model (Brilon, 2007, 2008). (a) Traffic flow  $Q_e(t)$  and capacity  $C(t)$ . (b) Cumulative arrivals  $A(t)$  - departures  $D(t)$  for the deterministic queue for the total time interval until the departure of the last vehicle arrived during the peak interval  $T$ .**

zero values that are lower than their relative capacities  $C_{i0}$  and  $C_{i1}$ . Compared to the inference of Kimber and Hollis (1979) in Section 4, furthermore, conditions of flow and capacity equality before and after the peak are removed ( $Q_{ei0} \neq Q_{ei1}$  and  $C_{i0} \neq C_{i1}$ ).

Considering the conventionality in inferring waiting times which, despite being related to all vehicles affected by the traffic peak (vehicles arrived during  $T$  and in the following  $T_d$ ), are only distributed on vehicles arrived at the peak interval  $T$ , in this case the period  $T_d$  does not represent the time necessary to bring back the queue  $L_{ic1}$  to the value  $L_{ic0}$  at the beginning of the peak  $T$  (in  $t_0$ ) like  $T_d'$  in Kimber and Hollis (1979) (Fig. 5), but it represents the time necessary to reset the deterministic queue  $L_{icT}$  from the end of the peak period ( $t_0 + T$ ). According to Brilon (2007, 2008), with flows and capacities in veh/s and time period in s, the expression  $w_i$  in s as a function of  $\rho_i$  or  $R_i$  is

$$w_{iT} = d_{iT} + \frac{T + T_d}{C_i T + C_{i1} T_d} \quad (14)$$

with

$$d_{iT} = \frac{D}{(C_i - R_i)T} = \frac{D}{\rho_i C_i T} \quad (15)$$

$$T_d = \frac{L_{icT}}{C_{i1} - (1 - \rho_{i1})} = \frac{L_{icT}}{R_{i1}} \quad (16)$$

The term  $D$  is determined using the equation set (17)–(21) if  $R$  is considered as an independent variable, or else using the equation set (22)–(26) if the independent variable is  $\rho$ .

$$D = D_c + \frac{L_{icT}^2}{2} \frac{(C_{i1} - R_{i1})}{C_{i1} R_{i1}} \quad (17)$$

$$D_c = D_0 - \frac{(C_i - R_i)T}{4C_i} \left( \Delta R T + 2 - \sqrt{(\Delta R T - 2)^2 + 8C_i T} \right) \quad (18)$$

$$D_0 = \frac{L_{ic0}}{2C_i} \quad (19)$$

$$\Delta R = R_i - \frac{L_{ic0}(2C_i T - L_{ic0})}{T(C_i T - L_{ic0})} \quad (20)$$

$$L_{icT} = \max[L_{ic0} - C_i(1 - \rho_i)T; 0] \quad (21)$$

$$D = D_c + \frac{L_{icT}^2}{2} \frac{\rho_{i1}}{C_{i1} - (1 - \rho_{i1})} \quad (22)$$

$$D_c = D_0 + \frac{T}{4} C_i \rho_i \left( \Delta \rho_i + \sqrt{\Delta \rho_i^2 + \frac{8\rho_i}{C_i T}} \right) \quad (23)$$

$$D_0 = \frac{L_{ic0}}{2C_i} \quad (24)$$

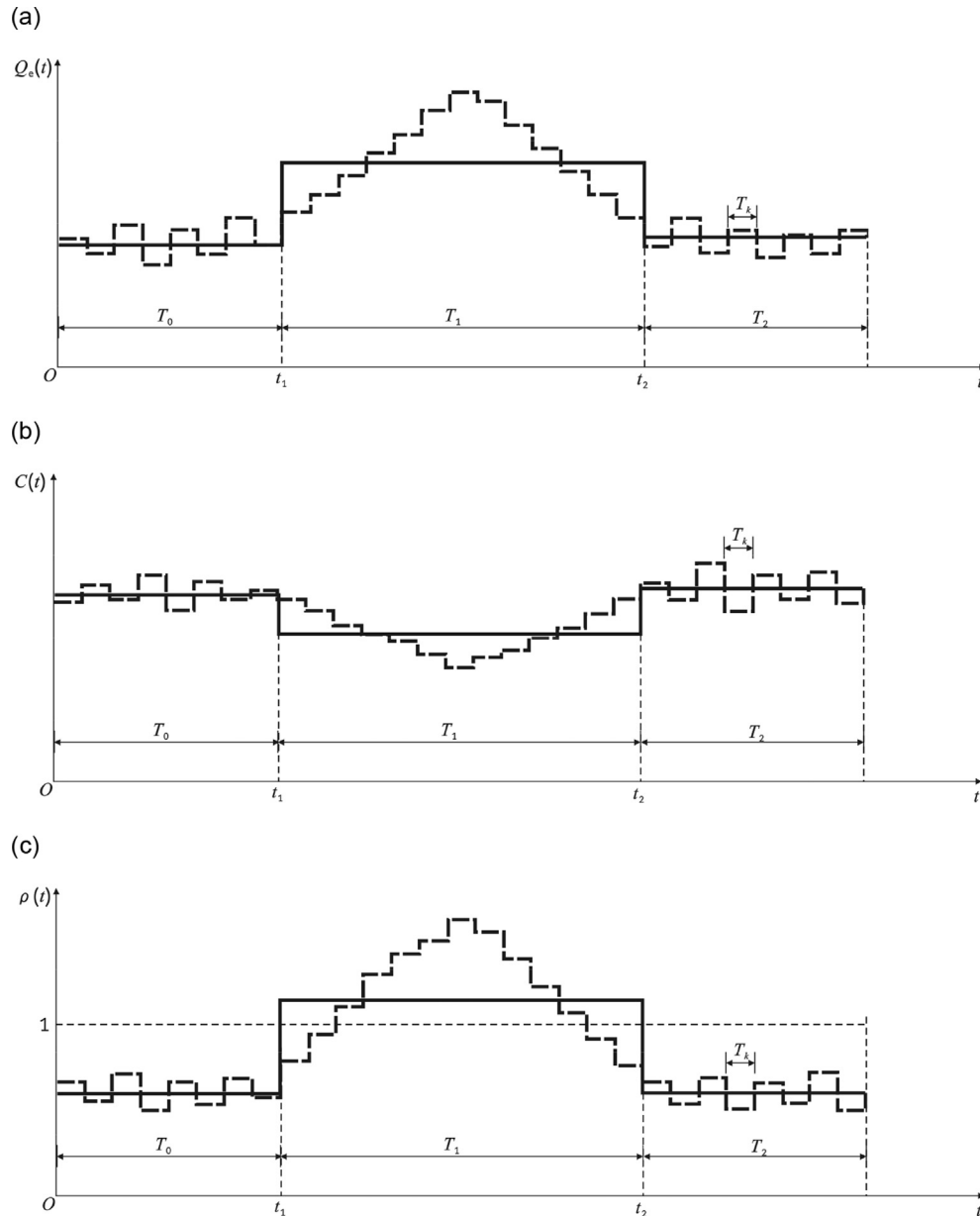
$$\Delta \rho_i = \rho_i - 1 + \left( \frac{1}{C_i T} + \frac{1}{C_i T - L_{ic0}} \right) L_{ic0} \quad (25)$$

$$L_{icT} = \max[L_{ic0} - R_i T; 0] \quad (26)$$

The transition curve for  $d_i$  is obtained by imposing the condition  $\alpha/1 = \beta/\rho_d$  (Fig. 2(a)) with  $\rho$  (Eqs. 17–21), or  $\alpha^* = \beta^*$  (Fig. 2(b)) with  $R$  (Eqs. 22–26) as independent variables. It is also worth underlining that the above model is inferred from Brilon's operating in terms of total queue waiting time  $D$ , and considering in the deterministic case ( $\rho_i \rightarrow \infty$  or  $R_i \rightarrow -\infty$ ) the total waiting time as the area under cumulative arrivals and departures (Fig. 6(b)). The total waiting time in  $T + T_d$  is then conventionally distributed over all the vehicles arrived during the peak period  $T$ , as already highlighted by Mauro model (Mauro, 2010) for Kimber and Hollis model (Kimber and Hollis, 1979) (Section 4.2).

It is also noted that setting  $C_i = C_{i1}$  and  $Q_{ic0} = Q_{ic1} = 0$  (and therefore  $L_{ic0} = L_{ic1} = 0$ ) from Eqs. (13), (15), (16) and (22) – (26) we obtain the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) in Eq. (4). With the same constraints for capacity and flow in the initial and final situations, from Eqs. (13), (15), (16) and (22) – (26), a different solution is obtained for the analogous case in Brilon (1995), since in the latter the solution is obtained by non-consistently operating a transition from the waiting time in the system for the case of M/M/1 stationary queue versus the queue waiting time for the deterministic case.





**Fig. 8 – Time evolution for entry traffic demand  $Q_e(t)$ , capacity  $C(t)$  and intensity  $\rho(t)$  for sequential time slices  $T_k$ .**

Brilon (2007, 2008) proposes another model (Fig. 7) that he defines as more suitable for its use in technical applications (“which traffic engineers usually define, when they estimate delays by empirical methods” (Brilon, 2008)). This model, on the one hand, considers the waiting time experienced beyond the peak period  $T$  only by the vehicles reaching the intersection at the peak interval, and on the other, it excludes the waiting time experienced by the vehicles arrived when the peak interval has elapsed (A + B + C area in Fig. 7(b)). In such terms, therefore,  $T_d''$  is the interval from the instant  $t_0 + T$  at the end of the peak up to  $T_d''$ , the

instant when the last vehicle arrived during the peak  $T$ , is discharged from the queue.

By operating in terms of total waiting time in queue  $D$  conventionally distributed on vehicles arriving during  $T$ , with flows and capacities in veh/s and time in s, the average waiting time in the system in s as a function of  $\rho$  or  $R$  is

$$w_{iT} = d_{iT} + \frac{T + T_d''}{C_i T + C_{i1} T_d''} \quad (27)$$

with

$$d_{iT} = \frac{D}{(C_i - R_i)T} = \frac{D}{\rho_i C_i T} \quad (28)$$

$$T_d' = \frac{L_{ic0} - C_i(1 - \rho_i)T}{C_{i1}} \quad (29)$$

In this case  $D$  is calculated with the set of Eqs. 17–26 if we consider  $R$  as the independent variable or, alternatively, with the set of Eqs. 22–26 if the independent variable is  $\rho$ , with  $D = D_c$  being  $R_{i1} = C_{i1}$  by effect of  $Q_{i1} = 0$ . Also for the present case the transition curve for  $d_i$  is obtained by imposing the condition  $\alpha/1 = \beta/\rho_d$  (Fig. 2(a)) with  $\rho$  as independent variable or, alternatively, with  $\alpha^* = \beta^*$  (Fig. 2(b)) as a function of  $R$ .

The latter case, again with reference to the situation in Fig. 7, is also included in (Brilon, 2015). The model is inferred by operating in terms of average waiting time in the system, instead of the total queue waiting time in (Brilon, 2007, 2008), and by excluding from the calculation the time spent in the system by vehicles already queued at the beginning of  $T$  ( $L_{ic0}$ ) ( $B + C$  areas in Fig. 7(b)). The solution for  $w_i$  is obtained by considering the simplification hypothesis  $C_{i1} = C_i$  ( $B + C + D + E$  areas in Fig. 7(b)) and operating the coordinate transformation with the condition  $\alpha^* = \beta^*$  (Fig. 2(b)) for  $R$  independent variable. In order to take into account that it can result  $C_{i1} \neq C_i$  and to avoid computational complications that the coordinate transformation would generate, the correction on the average waiting time in the system, due to the change in service capacity at the end of the observation period  $T$  ( $D + E$  areas in Fig. 7(b)), is approximately approached by calculating a correction term  $\Delta w$ . The term  $\Delta w$ , as compensation for the bias due to the approximation, is linked to a multiplicative calibration coefficient  $\varphi$ . For the general case with  $C_{i1} \neq C_i$  we get

$$w_{iT} = \frac{1}{4C_i} \left[ -A + \sqrt{A^2 + 8MC_i T} \right] - \varphi \Delta w \quad (30)$$

where

$$\Delta w = \frac{(L_{ic0} - R_i T)^2}{2T(C_i - R_i)} \left( \frac{1}{C_i} - \frac{1}{C_{i1}} \right) \quad (31)$$

$$A = R_i T - 2L_{ic0} - 2 = C_i T(1 - \rho_i) - 2L_{ic0} - 2 \quad (32)$$

$$M = \begin{cases} 1 + \frac{L_{ic0}^2 R_i}{2C_i T C_i} & R_i > 0 \\ 1 & \text{otherwise} \end{cases} \quad (33)$$

From calibrations made on real data, Brilon (2015) suggests a value  $\varphi = 1.1$  to be applied if  $C_{i1} \neq C_i$ . In general, being possible to result  $C_i > Q_i$ , with  $T$  as under-saturation period, the queue  $L_{icT} = L_{ic0} - R_i T$  could be null. If  $L_{ic0} > R_i T$ , therefore,  $w_{iT}$  is calculated with Eqs. 30–33 with  $\varphi = 1$ ; if on the contrary,  $L_{ic0} = R_i T$ ,  $w_i$  is calculated with Eqs. (30), (32) and (33) for  $\Delta w = 0$ .

Moreover, from the above equations it is clear that the different models proposed by Brilon (2007, 2008, 2015) depend on  $L_{ic0}$ , i.e., the queue length at the beginning of  $T$ , and on  $L_{icT}$ , i.e., its value at the end of the same period, assessed deterministically with Eq. (21) or Eq. (26).

The analysis of the equations shows, as in the previous two models, a dimensional inconsistency if flows and capacities are expressed in equivalent vehicles considering a global equivalence coefficient  $f$ .

#### 4. Computational aspects in time evolution profiles

Time-dependent models are essential tools for analysing waiting phenomena in case of near-saturation, over-saturation and/or when, however, demand and capacity are not constant. In practical cases, transiency characterizes these situations, with an increase in queues and waiting times that are nevertheless contained within finite values. The use of time-dependent functions  $\vartheta(t)$  for the instantaneous description of the waiting phenomena during an observation period  $T$ , although more accurate in testing state-variable variations, is scarcely feasible in that the same functions can hardly be integrated even by numerical means. For this reason, it is preferable to operate by reformulating the instantaneous approach in discrete terms.

As already noted, if properly inferred, the coordinate transformation models can manage all the saturation conditions at the entrance, providing the system status in terms of vehicle number and waiting times during an observation period  $T$ , i.e., specified with constant average entry flows and capacities. If flow and capacity cannot be considered constant during  $T$ , their evolutionary profile can be approximated in terms of gradual variations over a certain number of time slices  $T_k$  (with  $k = 1, 2, \dots, n$ ), thus dividing  $T$ . Thus, changes in flow and capacity (and consequently their quotient  $\rho$  or their difference  $R$ ) can gradually be approximated over  $T$ , assuming that they are constant for each time slice  $T_k$  (Fig. 8).

Heydecker and Verlander (1998) focus on some theoretical issues related to this approach, highlighting how the coordinate transformation results depend on the extension of the observation period with respect to which the initial queue  $L_{ic0}$  is identified. If we operate by considering a succession of time slices  $T_k$ , at the beginning of each slice the queuing process is re-initialized to the initial value for the current interval while taking into account the final status of the previous one. As shown by Heydecker and Verlander (1998) in this situation the rate at which the queue evolves depends on the duration of  $T_k$ , highlighting the non-transitivity of the problem, with a queue that grows less rapidly as the duration of  $T_k$  increases. The authors highlight, therefore, the substantial arbitrariness in the choice of the duration of the re-initialization time slices, which in turn affects the evolutionary profile of the queue by placing itself between two borderline cases: a single sub-interval (without re-initialization) throughout the observation period; a number of sub-intervals that tends to infinity and of length tending to zero (instant re-initialization).

As suggested by Heydecker and Verlander (1998), the two extreme possibilities are respectively not satisfactory in that they remove the possibility of actually taking into account

evolutionary profiles of demand and capacity, and involve the resolution of differential equations. To the complex search for the differential solution with instant re-initialization, which should produce more accurate estimates, Taylor (2014) adds that the same data are often available as average values over periods of a given duration. Together with the unclear entity of the effects of this non-transitivity on state-variable estimates (Taylor, 2014), this makes the time slice approach to be still widely used.

In the terms described above, by the successive application of “basic” time-dependent models over several consecutive time slices  $T_k$  to follow traffic demand and capacity evolution at the generic intersection arm  $i$ , and with the constraint that the final state of a given slice  $T_k$  represents the initial state of the next one  $T_{k+1}$ , any saturation time profile can be approximated and treated, thus evaluating the evolutionary characteristics of the queue length and waiting times.

As noted by Taylor (2014), when we estimate the evolution of the waiting phenomena from a sub-interval to the next one, the average value of the queue length is usually considered as the transfer variable between the two intervals. In this way, for a time slice  $T_k$  we consider an initial queue represented by its average value  $L_{ic0,k}$  coinciding with the average value  $L_{icT,k-1}$  of the final queue at the end of the immediately preceding slice  $T_{k-1}$ . Taylor himself in several works (2007a, b, 2014, 2018, 2015) highlights how the existence of an initial queue represents a problematic element for the coordinate transformation method, already identified by Kimber and Hollis (1979). With an initial queue significantly different from zero, these last authors propose two variants (i.e., “divided model”) depending on whether the queue is increasing (with a realignment of the temporal origin from  $t_{k0}$ , i.e., the initial instant of the interval  $T_k$ , with  $L_{ic0,k}$  to  $-t_{k0}$  in which the queue is null), or decreasing (with a simplified model of linear decay). Taylor (2007a, b) specifies that it should be more appropriate to transfer the average value of the queue as well as its entire distribution of probabilities from one sub-interval to the next.

If the probability distribution of the queue length can be sufficiently simple in steady-state condition, the same thing does not happen in the more general case. Taylor (2007a, b, 2014, 2018, 2015) proposes a method for correcting the coordinate transformation so as to consider the queue variance and not just its average value, based on the inference of a general variance formula for the time-dependent queue. Since the shared transformation for the variance as obtained for the average value is not possible, the author works by identifying corrections in the model that can satisfy the asymptotic behaviour. The solutions proposed by Taylor – whose original publications we refer to Taylor (2007a, b, 2014, 2018, 2015) – are characterized by a significant complexity especially in the perspective of technical application cases, requiring a rapid computability. Under these conditions, we maintain here the approach that only considers the propagation of the mean value between sequential time slices, neglecting any consideration about the probability distribution and higher order moments of propagation.

## 5. “Basic” cases modularity in time evolution profiles

In order to operate computationally, the “basic” case from which the time-dependent model is inferred needs adequate versatility to be applied in sequence for analysing “complex” time-varying profiles. Compared to the basic models examined in Section 3, compatibility features for the treatment of time evolution profiles for the entry demand and capacity can be found in the expression of the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) (Section 3.1), referred to by Troutbeck and Brilon (2000), Heidemann (2002) and Cvitanic et al. (2007), according to Eq. (5). We also find the same characteristics in the model suggested by Brilon (2015) discussed in Section 3.3 with reference to the situation in Fig. 7 according to Eqs. 30–33. On the other hand, the Kimber and Hollis model (Kimber and Hollis, 1979) for the situation in Fig. 5 (Section 3.2) and the Brilon models (Brilon, 2007, 2008) for the situation in Fig. 6 (Section 3.3), cannot be directly used as “basic” cases for calculating consecutive intervals.

It should be noted that in the two cases compatible as “basic” situations, however, the explicit time-dependent model only regards the average waiting times in the queue or in the system without explaining a similar model for the number of users, either in the queue or in the system. Despite the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) in the formulation proposed by Troutbeck and Brilon (2000) and Heidemann (2002) or the Brilon (2015) model is compatible with a sequential application, the time-dependent model cannot explain the number of queued users and thus it cannot be used directly and express concretely the final queue  $L_{isT,k}$ , depending on the initial queue  $L_{is0,k}$  in  $T_k$ , which moves as the initial queue for the next interval  $T_{k+1}$ . It is worth underlining how the coordinate transformation method can be also used for the queue length. When composing several consecutive time intervals, the availability of time-dependent models for both status variables allows a joint evaluation of the evolutionary characteristics of both waiting times and queues, and makes available the average queue value from one time slice to the next.

This aspect is dealt with by Mauro (2010) who revises the model by Kimber and Hollis (1979) in Section 4.2 and presents a time-dependent model suitable for representing a “basic” case in the meaning specified above. The author makes explicit both equations for calculating the average waiting time in the system and users' number in the queue and in the system at the end of the observation period. The model applies to entry flow and capacity showed in Fig. 7(a), considering an M/M/1 model for the queuing system in equilibrium and with cumulative counts for arrivals and departures for the deterministic queue as in Fig. 7(b), with the condition  $C_{i1} = C_i$ .

The transition curves for  $L_{icT}$  and  $L_{isT}$ , respectively users in the queue and in the system at the end of the observation period  $T$ , and  $w_i$ , average time in the system for vehicles

**Table 1 – Complete and dimensionally homogeneous time-dependent equations for “basic” cases.**

Akçelik and Troutbeck model with initial queue (ATIQ) (Akçelik and Troutbeck, 1991)

$$w_{iT,k} = \frac{f_o}{C_{i,k}} + \frac{1}{2} \left\{ \frac{L_{isT,k-1}}{C_{i,k}} + \frac{(\rho_{i,k} - 1)T_k}{2} + \sqrt{\left[ \frac{L_{isT,k-1}}{C_{i,k}} + \frac{(\rho_{i,k} - 1)T_k}{2} \right]^2 + \frac{2f_o\rho_{i,k}T_k}{C_{i,k}}} \right\}$$

$$L_{isT,k} = \frac{L_{isT,k-1}}{2} + \frac{(\rho_{i,k} - 1)C_{i,k}T_k}{2} + C_{i,k} \sqrt{\left[ \frac{L_{isT,k-1}}{2C_{i,k}} + \frac{(\rho_{i,k} - 1)T_k}{2} \right]^2 + \frac{f_o\rho_{i,k}T_k}{C_{i,k}}}$$

Brilon model (BRILON) (Brilon, 2015)

$$w_{iT,k} = \frac{1}{4C_{i,k}} [-A + \sqrt{A^2 + 8MC_{i,k}T_k}] - 1.1\Delta w_{i,k}$$

$$\Delta w_{i,k} = \frac{(L_{icT,k-1} - R_{i,k}T_k)^2}{2T_k(C_{i,k} - R_{i,k})} \left( \frac{1}{C_{i,k}} - \frac{1}{C_{i,k+1}} \right)$$

$$A = R_{i,k}T_k - 2L_{icT,k-1} - 2f_o = C_{i,k}T_k(1 - \rho_{i,k}) - 2L_{icT,k-1} - 2f_o$$

$$M = \begin{cases} f_o + \frac{L_{icT,k-1}^2}{2C_{i,k}T_k} \frac{R_{i,k}}{C_i} & R_{i,k} > 0 \\ f_o & \text{otherwise} \end{cases}$$

$$C_{i,k+1} = C_{i,k} \text{ if } k = n$$

$$L_{icT,k} = \frac{1}{2} [\sqrt{D^2 + E} - D]$$

$$D = \frac{R_{i,k}C_{i,k}T_k^2 - C_{i,k}T_kL_{icT,k-1} + 2(L_{icT,k-1} + C_{i,k}T_k - R_{i,k}T_k)}{C_{i,k}T_k - f_o}$$

$$E = \frac{4f_o(L_{icT,k-1} + C_{i,k}T_k - R_{i,k}T_k)^2}{C_{i,k}T_k - f_o}$$

Kimber and Hollis (1979) recalculated by Mauro (2010) (KH-M)

$$w_{iT,k} = \frac{1}{2} [\sqrt{J^2 + M} - J]$$

$$J = \frac{T_k}{2} (1 - \rho_{i,k}) - \frac{1}{C_{i,k}} (L_{isT,k-1} + f_o)$$

$$M = \frac{4}{C_{i,k}} f_o \left[ \frac{T_k}{2} (1 - \rho_{i,k}) + \frac{1}{2} \rho_{i,k} T_k \right]$$

$$L_{isT,k} = \frac{1}{2} [\sqrt{A^2 + B} - A]$$

$$A = (1 - \rho_{i,k})C_{i,k}T_k + f_o - L_{isT,k-1}$$

$$B = 4f_o(L_{isT,k-1} - \rho_{i,k}C_{i,k}T_k)$$

arrived during  $T$  (considering the total time in the queue until the departure from the system of the last vehicle arrived in  $T$ , given by the areas  $B + C + D + E$  in Fig. 7(b) and broken down on all vehicles arrived during  $T$ ) can be obtained by imposing the condition  $\alpha = \beta$  with  $\rho$  as independent variable (Fig. 2(a)) as in Kimber and Hollis (1979).

According to Mauro's (2010) recalculations for Kimber and Hollis model (Kimber and Hollis, 1979), with entry flows and capacities (veh/s) and time (s), the expression for  $w_i$  (s) as a function of  $\rho$  is

$$w_{iT} = \frac{1}{2} \left( \sqrt{J^2 + M} - J \right) \quad (34)$$

with the following auxiliary variables

$$J = \frac{T}{2} (1 - \rho_i) - \frac{1}{C_i} (L_{is0} + 1) \quad (35)$$

$$M = \frac{4}{C_i} \left[ \frac{T}{2} (1 - \rho_i) + \frac{1}{2} \rho_i T \right] \quad (36)$$

Again according to Mauro (2010), Eqs. (37) and (40) give respectively  $L_{isT}$  and  $L_{icT}$  (veh) as  $\rho$  functions, with auxiliary variables provided by Eqs. (38) and (39), and by Eqs. (41) and (42)

$$L_{isT} = \frac{1}{2} (\sqrt{A^2 + B} - A) \quad (37)$$

where

$$A = (1 - \rho_i)C_iT + 1 - L_{is0} \quad (38)$$

$$B = 4(L_{is0} - \rho_iC_iT) \quad (39)$$

and

$$L_{icT} = \frac{1}{2} [\sqrt{D^2 + E} - D] \quad (40)$$

where

$$D = \frac{(1 - \rho_i)(C_iT)^2 - C_iTL_{ic0} + 2(L_{ic0} + \rho_iC_iT)}{C_iT - 1} \quad (41)$$

$$E = \frac{4(L_{ic0} + \rho_iC_iT)^2}{C_iT - 1} \quad (42)$$

It is worth observing that Eqs. 34–36, as well as Eqs. 37–39, appear also in Heidemann (2002) for a more general case with an M/G/1 model for the queuing system in equilibrium.

The above equations contain all the elements to evaluate the evolutionary characteristics over several consecutive time



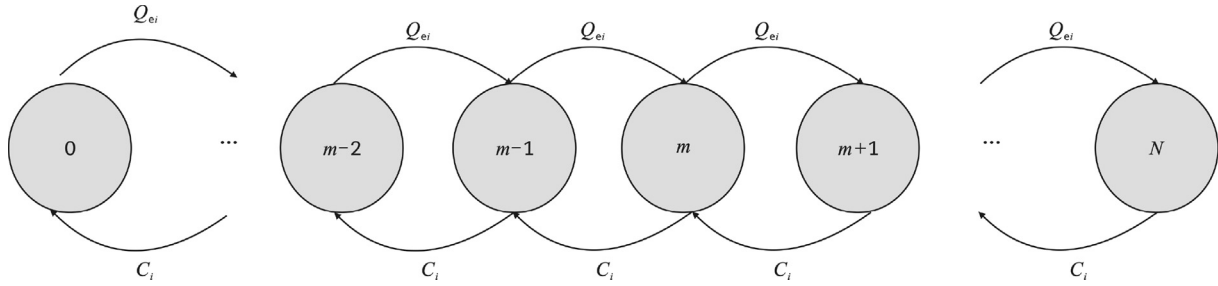


Fig. 9 – IMC state transition diagram for an M/M/1/N queue system.

intervals. Remembering that the equations reported in Mauro (2010) contain the constraint  $C_{i1} = C_i$  (for  $T_k$  it holds  $C_{i,k} = C_{i,k+1}$ ) we can observe how the same constraint could be removed by operating as suggested by Brilon (2015), i.e., subtracting from Eq. (34) the approximate term  $\Delta w$  due to the change in service capacity at the end of the observation period ( $D + E$  areas in Fig. 7(b)) according to Eq. (31).

Following the previous discussion, in this paper we propose the time-dependent equations also in the “basic” cases represented by the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) with the initial queue, following the formulation proposed by Heidemann's model (Heidemann, 2002) and Brilon's models (Brilon, 2015). Regarding the Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) with the initial queue, whose equation of average waiting time in the system depending on the initial vehicle number in the system  $L_{is0}$  is given by Eq. (5), we can operate with coordinate transformation considering the condition  $\alpha/1 = \beta/\rho_d$  as in Fig. 2(a) with an M/M/1 model for the queuing system in equilibrium, and get the following relationship for the vehicle number in the system  $L_{isT}$  at the end of a time interval  $T$

$$L_{isT} = \frac{L_{is0}}{2} + \frac{(\rho_i - 1)C_i T}{2} + C_i \sqrt{\left[ \frac{L_{is0}}{2C_i} + \frac{(\rho_i - 1)T}{2} \right]^2 + \frac{\rho_i}{C_i} T} \quad (43)$$

Regarding the Brilon (2015) model, whose equation of average waiting time in the system depending on the initial vehicle number in the queue  $L_{ic0}$  is given by Eq. (30) through Eqs. 31–33, we can operate considering Eqs. 40–42 reported by Mauro (2010) expressed with  $R$  as independent variable, and get the following relationship for the queue length  $L_{icT}$  at the end of  $T$

$$L_{icT} = \frac{1}{2} \left[ \sqrt{D^2 + E'} - D' \right] \quad (44)$$

where

$$D' = \frac{R_i C_i T^2 - C_i T L_{ic0} + 2(L_{ic0} + C_i T - R_i T)}{C_i T - 1} \quad (45)$$

$$E' = \frac{4(L_{ic0} + C_i T - R_i T)^2}{C_i T - 1} \quad (46)$$

In so doing, the inference of the two “basic” cases is completed, making it possible to apply them sequentially to study the temporal evolution profiles of demand and capacity.

In conclusion, if we can divide the observation period  $T$  into  $n$  sub-intervals or time slices  $T_k$  (with  $k = 1, 2, \dots, n$ ) in which

$Q_{ei,k}$  and  $C_{i,k}$  are constant, we can analyze for each  $k$  the evolutions over  $T$  of the vehicles waiting at the end of each  $T_k$  ( $L_{is,k}$  or  $L_{ic,k}$  as appropriate) and its average time  $w_{i,k}$  applying the equations shown in Table 1.

In particular, the equations reported in Table 1 for the three “basic” models have all been revisited here to take into account the necessary dimensional homogeneity of the terms already discussed in Wu (2006, 2009). Introducing a constant  $f_0$  so that it results  $f_0 = 1$  veh if  $Q_{ei}$  and  $C_i$  are expressed in veh/h, and  $f_0 = f$  in unit of pcu if they are expressed in pcu/h where  $f$  corresponds to the global equivalence coefficient used (i.e., the ratio between pcu/h and veh/h), dimensional homogeneity is guaranteed both for  $L_{isT,k}$  (in pcu or veh) and  $w_{iT,k}$  (in h).

By applying the equations for queue lengths and delays, reported in Table, with traffic units for traffic demand and capacity that can be expressed in vehicular units (veh) or passenger car units (pcu), the congruence relationships  $L_{iT,k}(\text{pcu}) = L_{iT,k}(\text{veh})f$  and  $w_{iT,k}$  for passenger car units =  $w_{Ti,k}$  for vehicles in Wu (2009) are satisfied.

## 6. Markov chains analysis

While it is difficult to produce exact closed-form solutions for the state-variable evolution in a waiting system and the coordinate transformation method provides only approximate solutions, numerical solutions based on the properties of Markov systems can be a useful tool for testing the obtained results. Imbedded Markov chain (IMC) approach represents a recognized and efficient tool for analysing waiting systems. An IMC is a discrete Markov chain over time for which there is a transition between two successive states  $X_n = m$  and  $X_{n+1} = l$  based on the time  $\tau_k$  spent in the state  $X_n = m$ . The discrete sequence  $\{X_n\}$  represents, therefore, all the consecutive states visited after each transition leading to the state change, according to the transition probabilities expressed by

$$p_{ml} = P(X_{n+1} = m | X_n = l) \quad (47)$$

With an M/M/1 waiting system, the arrivals on the intersection arm are Poissonian with rate  $Q_{ei}$  and are served according to the first-in-queue-first-out-of-queue (FIFO) discipline with service times distributed exponentially with average  $1/C_i$ , where  $Q_{ei}$  and  $C_i$  are expressed in (veh/s). In the logic of IMC, the transition from a state  $X_n = m$  to the next  $X_{n+1} = l$  can be identified in the variation of user number in the system. This variation may consist alternatively in around

one unit, depending on whether an arrival or departure occurs. This situation represents a process of birth and death in which the only possible transitions that the process can perform are exclusively between neighbouring states, as shown in the relative state transition diagram in Fig. 9. Thus, each of the two independent events “completion of a service” or “arrival of a new vehicle” contributes to identify the time  $\tau_k$  spent in the state  $X_n = m$ .

If we indicate with  $t = 1, 2, \dots, t_{\max}$  the discrete instants in the observation period  $T$  in which a status change occurs, the probability that the vehicle number in the queue changes from  $m$  to  $l$  is  $p_{ml}(t)$ . The state probability  $p_m(t+1)$  to find  $m$  vehicles in the system for any  $t+1$  is calculated on the basis of the analogous  $p_m(t)$  for  $t$  by the following expression

$$p_m(t+1) = \sum_{j=0}^N p_j(t) p_{jm}(t) \quad (48)$$

With  $N$  significantly larger than the maximum possible queue for the system. In  $t$ , therefore, the expected value of the user number in the system  $L_{is}(t)$  results to be

$$L_{is}(t) = \sum_{m=0}^N m p_m(t) \quad (49)$$

Whereas the expected value of the time  $\tau_k$  spent in the state  $X_n = m$  is equal to the expected value of the minimum between the arrival time and the service time, exponentially distributed with media  $1/Q_{ei}$  and  $1/C_i$  respectively, the temporal step  $\Delta t$  (s) with which the IMC is defined is given by

$$\Delta t = E(\tau_m) = \int_0^{\infty} e^{-(Q_{ei}+C_i)t} dt = \frac{1}{Q_{ei} + C_i} \quad (50)$$

with  $Q_{ei}$  and  $C_i$  supposed to be constant during the interval  $T$ . Eq. (50) is valid for  $k \geq 1$ . If  $m = 0$ , then  $\Delta t = \frac{1}{Q_{ei}}$ , which considers only an arrival.

The conventional average waiting time in the system  $w_{iT}'$  over  $T$  is calculated as the ratio between the sum of the waiting times accumulated by the vehicles in the system in  $T$  and the total of the same arrivals in  $T$ . By defining  $t_{\max}$  as the number of status changes observed during  $T$ , with  $t_{\max} = \text{int}[T(Q_{ei} + C_i)]$ , we have

$$w_{iT}' \approx \frac{\sum_{t=0}^{t_{\max}} L_{is}(t) \Delta t}{\sum_{t=0}^{t_{\max}} Q_{ei}(t) \Delta t} \quad (51)$$

where the approximation is due to the non-integer part of  $T(Q_{ei} + C_i)$ . Being  $Q_{ei}$  and  $\Delta t$  constant over  $T$ , we have

$$w_{iT}' \approx \frac{\Delta t}{Q_{ei} T} \sum_{t=0}^{t_{\max}} L_{is}(t) \quad (52)$$

For a similar reason, we get an approximate value also for  $L_{isT}$ .

It remains to express the general form of the matrix  $P$  of the transition probabilities  $p_{ml}$ , considering that a transition takes place depending on a new arrival or a new service. If we consider the state transition diagram of birth and death process in Fig. 9, the expressions in Table 2 result for  $p_{ml}$ , i.e., the probability that the vehicle number in the system will change from  $m$  to  $l = m+1$  or  $l = m-1$  during the time step  $\Delta t$  of the chain. If  $m = 0$ , i.e., in the initial state with an empty system, there is the probability  $Q_{ei}/(Q_{ei} + C_i)$  that the system makes a transition to have one user ( $l = 1$ , i.e., an arrival occurs) and therefore there is a probability  $1 - Q_{ei}/(Q_{ei} + C_i)$  that the system continues to be empty ( $l = 0$ , i.e., no arrival occurs, since no departure is feasible). Based on Eq. (48), using the expressions in Table 2 and applying Eq. (49) we obtain the evolution of the user number in the system during  $T$  and, therefore, on the basis of Eq. (51) the value of the average waiting time in the system for the M/M/1 queue is calculated.

About  $N$ , we said that this value must be chosen so as to be suitably greater than the maximum possible value of the waiting users. In this regard, Taylor (2014) identifies a minimum threshold value for  $\rho < 1$  such that  $N+1 \geq L_{ic0} + 7/\sqrt{\rho}$ , while for  $\rho > 1$  the author recommends to consider the maximum possible value for  $N$ , compatibly with the constraints of the calculation.

It should be noted that the waiting time calculated with Eq. (51), and therefore Eq. (52), considers the total time in the system for vehicles already present at the beginning of the observation period  $T$  ( $L_{is0}$ ), and excludes the time spent by vehicles that are still in the system at the end of  $T$  ( $L_{isT}$ ), normalized on the total vehicles arrived during  $T$ .

With reference to the areas in Fig. 7(b), the average waiting time in the system  $w_i'$  takes into account the total time spent represented by the sum of the areas A and B and a homogeneous realignment is required in order to compare the time-dependent models summarized in section 5. With the realignment an approximate value for  $w_i$  can be obtained by considering a deterministic approach which aims, with reference to the areas highlighted in Fig. 7(b), to reduce the waiting quota for area A (vehicles already present at the beginning of the observation period  $T$ ) and to include area C (vehicles that are still in the system at the end of  $T$ ). By so doing, starting from  $w_{iT}'$  for the considered IMC, we can approximate  $w_{iT}$  (which we indicate as a value for realignment IMC\*) with

$$w_{iT} \approx w_{iT}' + \frac{1}{2Q_{ei}T} \left[ \frac{L_{isT}^2}{C_{i1}} - \frac{L_{is0}^2}{C_i} \right] \quad (53)$$

**Table 2 – IMC state transition probabilities for an M/M/1 queue system.**

Values for $m$ and $l$	Transition probability $p_{kl}$
$m \geq 1$	$p_{mm} = 0$ $p_{mm+1} = \frac{Q_{ei}}{Q_{ei} + C_i}$ $p_{mm-1} = \frac{C_i}{Q_{ei} + C_i}$

## 7. Stochastic discrete event simulation

Another useful tool to test the results obtained through the coordinate transformation is the SDES. This type of approach simulates the evolution of the state variables in a waiting system by randomly generating the arrivals at the intersection arm and the departures from the service counter by means of appropriate probability distribution functions.

**Table 3 – Test profiles parameters ( $T_k$  in sec;  $Q_{ei,k}$  and  $C_{i,k}$  in veh/h).**

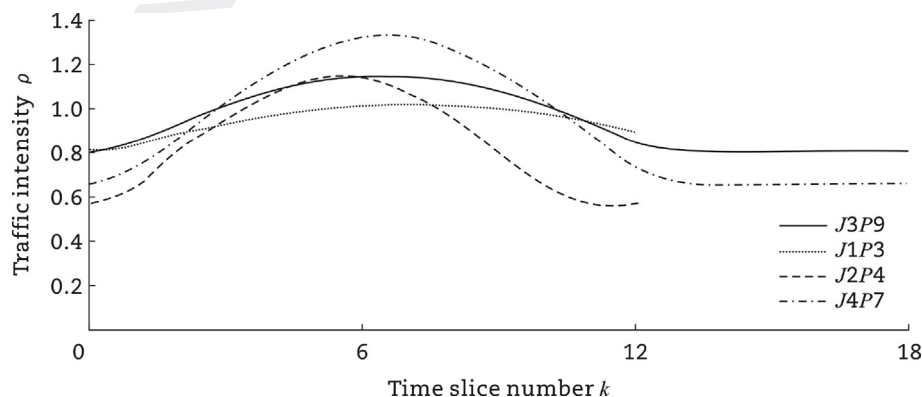
k	JxPy profiles															
	J1P3				J2P4				J3P9				J4P7			
	$T_k$	$Q_{ei,k}$	$C_{i,k}$	$\rho_{i,k}$	$T_k$	$Q_{ei,k}$	$C_{i,k}$	$\rho_{i,k}$	$T_k$	$Q_{ei,k}$	$C_{i,k}$	$\rho_{i,k}$	$T_k$	$Q_{ei,k}$	$C_{i,k}$	$\rho_{i,k}$
EQUI	$\infty$	461	572	0.806	$\infty$	546	954	0.572	$\infty$	1143	1414	0.808	$\infty$	1200	1800	0.667
1	540	477	571	0.835	540	602	930	0.647	600	1185	1390	0.853	600	1270	1730	0.734
2	540	505	570	0.887	540	726	882	0.823	600	1260	1344	0.938	600	1403	1597	0.878
3	540	529	568	0.931	540	800	840	0.952	600	1324	1306	1.014	600	1518	1482	1.025
4	540	549	567	0.968	540	867	810	1.071	600	1374	1275	1.077	600	1611	1389	1.160
5	540	563	566	0.995	540	903	793	1.138	600	1409	1255	1.123	600	1676	1324	1.267
6	540	573	566	1.012	540	903	793	1.138	600	1426	1245	1.146	600	1710	1290	1.326
7	540	576	565	1.019	540	867	810	1.071	600	1426	1245	1.146	600	1710	1290	1.326
8	540	574	566	1.015	540	800	840	0.952	600	1409	1255	1.123	600	1676	1324	1.267
9	540	566	566	1.001	540	708	882	0.803	600	1374	1275	1.077	600	1611	1389	1.160
10	540	553	567	0.976	540	602	930	0.647	600	1324	1306	1.014	600	1518	1482	1.025
11	540	535	568	0.941	540	546	954	0.572	600	1260	1344	0.938	600	1403	1597	0.878
12	540	512	569	0.899	540	536	936	0.572	600	1185	1390	0.853	600	1270	1730	0.734
13	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667
14	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667
15	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667
16	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667
17	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667
18	—	—	—	—	—	—	—	—	600	1143	1414	0.809	600	1200	1800	0.667

In the simulation model that has been set up for the purposes of this work, the discrete states in which the system is evaluated coincide with the arrivals at the arm. At any arrival instant the instant departure of the arriving vehicle will be evaluated, taking into account the presence of queued vehicles and the service time distribution for all vehicles in the system. In this case, a simulation model has been prepared for an M/M/1 system, with an exponential distribution for inter-arrival intervals and times at the service counter.

For each new vehicle  $v$  on the arm, which represents an evolution of the system state, within a given period of observation  $T$  the inter-arrival interval is generated randomly with respect to the previously arrived vehicle, simulating the random extraction of an exponentially distributed variable with a constant parameter  $Q_{ei}$  for the whole period. For the same vehicle  $v$ , the service time is generated by simulating the random extraction of an exponentially distributed variable with parameter  $C_i$ , which is also constant throughout the entire period.

Evaluating the end of the service in the previous state of the system (i.e., the instant when the previous vehicle  $v - 1$  disengages the service counter) and taking into account the arrival time and service time for the newly arrived  $v$ , the end-of-service time is obtained for this latter vehicle. By so doing, the arrival and departure times are simulated for each vehicle, thus obtaining the evolution of the average vehicle number in the queue and in the system, and of the average waiting time in the system. If we consider any evolutionary profile of supply and demand and extend the analysis to multiple intervals, the evolution of the waiting phenomena can be simulated in terms of vehicles and times.

It should be emphasized that in order to get statistically significant results, the simulation process must be iterated a high number of times, so that in every system state (and therefore for each vehicle arrived) the probability distribution of arrival headways and service times is respected. Each alternative evolution of the waiting system, represented by a different set of arrival and departure times sampled by the relative distributions, constitutes a single iteration (i.e., a

**Fig. 10 – Time evolution of  $\rho_{i,k}$  for the four JxPy profiles.**

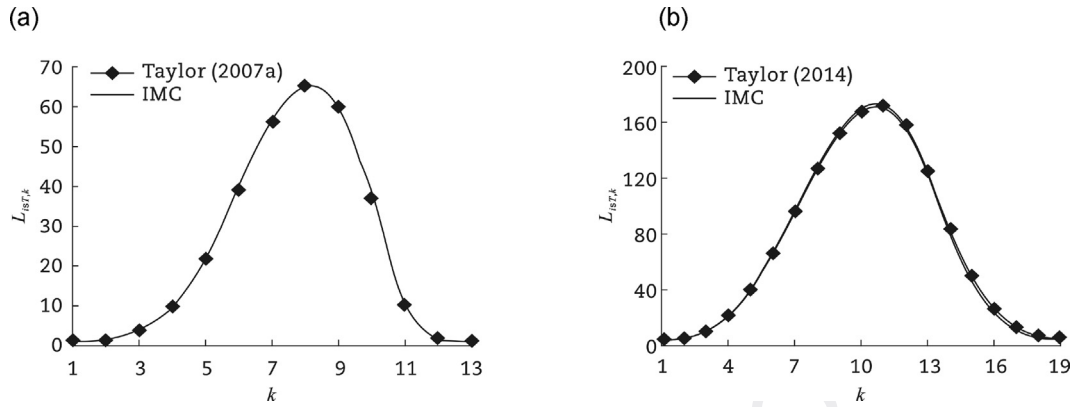


Fig. 11 –  $L_{isT,k}$  comparisons. (a) IMC versus Taylor and Heydecker (2015) for J2P4. (b) IMC versus Taylor (2014) for J3P9.

trial) of the simulation  $Q_{ei}$  and  $C_i$  values, which are constant on a single interval but may be variable on sequences of successive intervals, and are common to all simulation trials.

In the case study, with a waiting system M/M/1, the inter-arrival interval and service time are simulated by considering the inverse function of the exponential distribution, with parameter  $Q_{ei}$  and  $C_i$  respectively. So for the generic vehicle  $v$ , the time headway regarding the previous vehicle is given by Eq. (54) and the service time by Eq. (55), both expressed in (s)

$$x_v = -1 / Q_{ei} \ln(1 - u') \quad (54)$$

$$s_v = -1 / C_i \ln(1 - u'') \quad (55)$$

with  $u', u'' \sim U(0,1)$ , i.e., distributed according to an uniform variable with values between 0 and 1, where  $Q_{ei}$  and  $C_i$  are both expressed in veh/sec.

It is worth noting that, contrary to the Markov chain analysis that calculates the time in the system of vehicles already present at the beginning of the observation period  $T$  ( $L_{is0}$ ) and excludes that of vehicles remaining in the system at the end of  $T$  ( $L_{isT}$ ), the stochastic simulation of arrivals and departures makes it possible to evaluate significantly different situations.

Compared to the greater computational burden for evaluating a single vehicle in terms of arrival time, time spent in the

system and moment of disengagement from the service counter, the stochastic simulation allows, for instance, the average waiting time to be evaluated also in the case for time-dependent models as summarized in Section 6, with no need for the IMC to be realigned. The availability of arrival/departure times for each simulated vehicle, in fact, allows the direct calculation of the conventional average waiting time in the system which is represented by the sum of areas B and C in Fig. 7(b).

## 8. Results from applicative comparisons

In order to draw a numerical comparison between the results from the “basic” models, four temporal evolution profiles of demand and capacity have been built up. The tests refer to the cases formalized by Kimber et al. (1986) and reconstruct the trend of 34 types of symmetrical Gaussian profiles analytically. These types represent the combination of four hypothetical priority intersections,  $J = 1$ : major/minor junction;  $J = 2$ : small roundabout entry;  $J = 3$ : average roundabout entry;  $J = 4$ : very large roundabout entry) and twelve demand profiles ( $P = 1, \dots, 12$ ), and they also define some equations linking entry capacity with traffic demand. These last two quantities are considered to be constant in equal-duration intervals  $T_k$ , with  $k = 1, \dots, n$ , thus

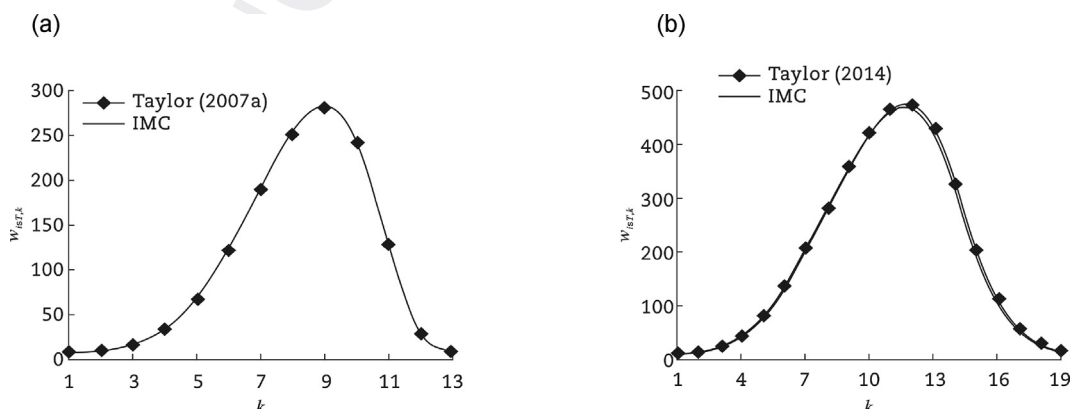
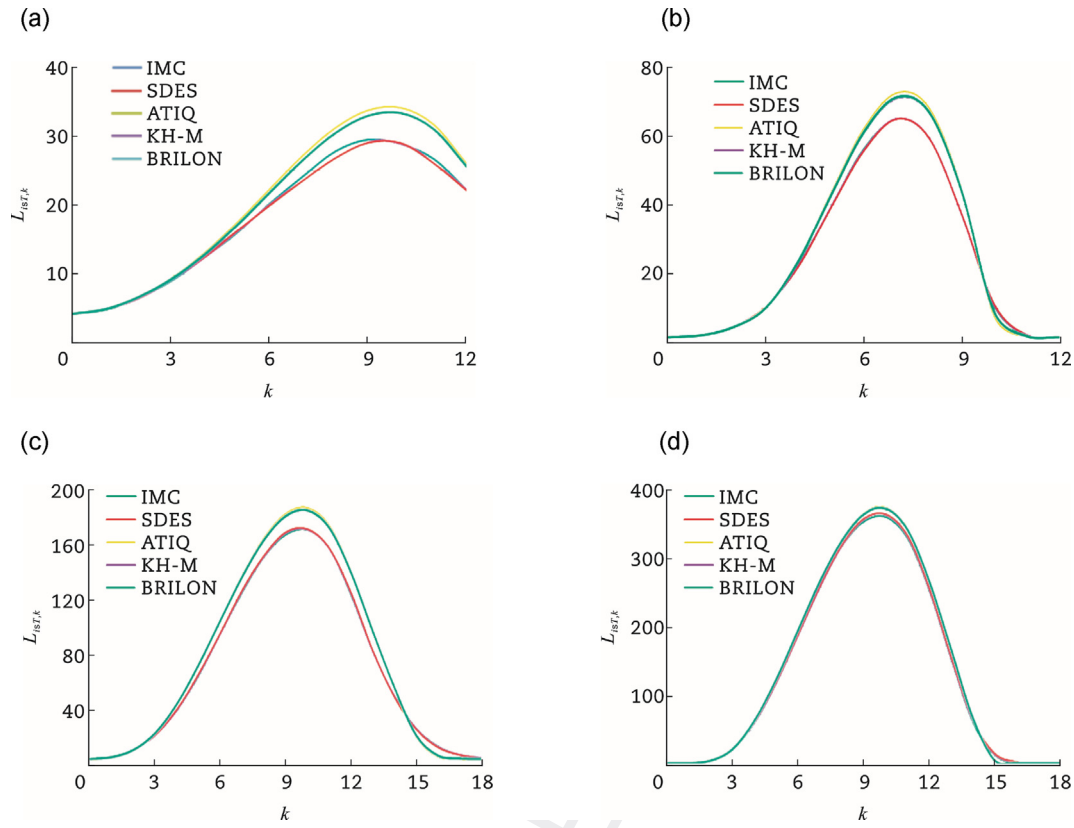


Fig. 12 –  $w_{isT,k}$  comparisons. (a) IMC versus Taylor and Heydecker (2015) for J2P4. (b) IMC versus Taylor (2014) for J3P9.





**Fig. 13** –  $L_{isT,k}$  for each time slice  $T_k$  according to: IMC; SDES; ATIQ; KH-M; BRILON. (a) J1P3. (b) J2P4. (c) J3P9. (d) J4P7.

subdividing the observation period  $T$  ranging from 45 to 120 min, and they also produce transient oversaturation states of varying duration. Regarding the nomenclature in Kimber et al. (1986), J1P3 and J4P7 are generated for this study, resuming J2P4 and J3P9 from Taylor (2007a, 2014), as references for validation.

Table 3 shows the main features for each profile related to each time slice  $k$  in which the observation period  $T$  splits ( $k = 1, \dots, 12$  lasting 540 s for J1P3 and J2P4, and  $k = 1, \dots, 18$  lasting 600 s for J3P9 and J4P7). Fig. 10 shows the evolution of the saturation degree  $\rho_{i,k}$ . For each profile, the system is assumed to reach the first time slice  $T_1$  with a queue in statistical equilibrium.

The analysis shows that, with respect to IMC and SSDE,  $L_{isT,k}$  and  $w_{iT,k}$  results for J2P4 and J3P9 perfectly overlap Taylor and Heydecker (2015) and Taylor (2014) values obtained by means of a Markov simulation, thus highlighting a useful reference for validation (Figs. 11 and 12).

Fig. 13 shows  $L_{isT,k}$  evolution at the end of each time slice  $T_k$ , according to the equations proposed for the three “basic” models and summarised in Table 1, assuming  $f_o = 1$ . For each of the test profiles, the results from the “basic” models are compared with the values obtained by IMC and SDES average with 500 trials. Table 4 shows the corresponding values of the percentage error, with respect to the corresponding IMC estimations. It shows a good overlap between  $L_{isT,k}$  values by IMC calculation and SDES values, thus testifying to the adequacy of the 500-trail target for stochastic simulations.

Fig. 15 shows  $w_{iT,k}$  evolution in each time slice  $T_k$  with respect to the vehicles arrived during the same period, according to the equations summarised in Table 1 and assuming  $f_o = 1$ . Since in this case the IMC value cannot represent a direct reference but it becomes so only after a deterministic realignment, for the comparison shown in Table 5 we consider the average between the approximate IMC\* value and the estimated SDSE value. Because of the already mentioned misalignment of  $w_i$  values for IMC with respect to  $w_i$  values obtained from applying SDSE homogeneously to the time-dependent “basic” models, by resorting to the IMC\* realignment with Eq. (53), the final values reciprocally overlap as shown in Fig. 14.

The results show a substantial coincidence between the three “basic” models in estimating  $L_{isT,k}$  evolution for all the profiles (Fig. 13 and Table 4). Therefore, we observe a uniformity of behaviour towards the values calculated with IMC and SDES: the “basic” models applied in sequence overestimate the vehicle number in the system during the peak period with over-saturation at the entrance, thus highlighting an underestimation in the final phases with a decreasing queue, leading to stabilisation of  $\rho_{i,k}$  towards under-saturated levels.

Regarding  $w_{iT,k}$  evolution (Fig. 15 and Table 5), we observe an overestimation during the over-saturation phases. The Brilon model (2015) shows values closer to IMC\* and SDES estimations, with differences in the increasing queue phases which appear to be reduced if compared to Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991) with initial

Table 4 – Difference in  $L_{isT,k}$  for each time slice  $T_k$  with respect to IMC (%).

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
J1P3	IMC	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	0.0	2.9	–1.2	2.3	0.6	–1.3	–2.4	–3.4	–1.5	0.6	–2.3	–0.3	–	–	–	–	–	–
	ATIQ	0.0	3.1	3.4	5.0	5.7	10.5	12.7	12.4	14.5	18.4	18.8	17.2	–	–	–	–	–	–
	KH-M	0.0	2.4	1.7	2.8	3.1	7.7	9.9	9.8	12.0	15.9	16.6	15.6	–	–	–	–	–	–
J2P4	BRILON	0.0	2.7	2.3	3.4	3.7	8.2	10.3	10.1	12.2	16.1	16.6	15.5	–	–	–	–	–	–
	IMC	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	–0.7	2.0	–1.6	2.9	–1.5	–1.1	–0.1	–0.1	0.4	6.4	8.0	15.2	–	–	–	–	–	–
	ATIQ	0.0	–1.0	–5.3	–0.2	7.7	9.9	11.3	14.4	19.7	–27.6	–22.8	–0.9	–	–	–	–	–	–
J3P9	KH-M	0.0	–1.5	–7.1	–3.1	4.5	7.1	9.1	12.2	17.6	–14.6	–14.1	–0.3	–	–	–	–	–	–
	BRILON	0.0	–1.1	–6.1	–2.1	5.4	7.8	9.4	12.3	17.4	–17.0	–17.4	–0.6	–	–	–	–	–	–
	IMC	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	0.0	10.3	2.3	1.3	2.4	2.6	1.4	1.2	1.6	1.0	0.8	2.1	1.5	–1.4	–2.2	–6.1	–1.2	–5.7
J4P7	ATIQ	0.0	3.7	1.3	8.8	13.7	13.9	11.8	10.4	9.5	9.8	11.1	14.7	19.4	12.8	–22.2	–54.4	–41.1	–21.5
	KH-M	0.0	3.1	–0.4	6.4	11.5	12.0	10.3	9.1	8.4	8.7	9.9	13.4	18.0	12.0	–19.0	–50.4	–39.8	–21.3
	BRILON	0.0	3.3	0.1	6.9	11.9	12.3	10.5	9.2	8.5	8.7	9.9	13.4	17.8	11.8	–19.5	–50.9	–40.0	–21.3
	IMC	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
J4P7	SDES	0.0	–11.5	–6.5	–1.7	3.1	1.7	1.8	1.2	1.0	0.9	1.4	1.5	2.4	4.0	8.1	6.4	–5.1	–6.9
	ATIQ	0.0	–8.6	–10.0	3.3	8.8	6.6	5.4	4.3	3.6	3.7	4.4	5.9	10.4	10.2	–62.8	–34.0	–2.0	–2.3
	KH-M	0.0	–8.9	–11.3	0.9	6.9	5.3	4.3	3.5	2.9	3.0	3.7	5.1	9.4	10.1	–53.2	–31.5	–1.9	–2.3
	BRILON	0.0	–8.7	–10.8	1.5	7.4	5.6	4.6	3.7	3.0	3.0	3.7	5.0	9.3	9.7	–54.6	–32.2	–1.9	–2.3

queue, and Kimber and Hollis (1979) model as revised by Mauro (2010) with the introduction of the corrective term  $\Delta w_{i,k}$ . The reduced gap between the Brilon model (2015) and IMC\* or SDES results for the intervals with increasing queues is extremely evident for J4P7 profile, as shown in Fig. 12(d). The three models show very similar trends in the decreasing queue phases leading to stabilization of  $\rho_{i,k}$  towards under-saturated levels, thus highlighting an underestimation.

For the three “basic” models, we can observe how  $w_{iT,k}$  at each time slice  $k$  can be estimated deterministically by only using  $L_{isT,k-1}$  and  $L_{isT,k}$  values. This approximation makes it possible to express, in a simplified form and independently from the coordinate transformation model of the “basic” case used for queue length estimation (i.e., for  $L_{isT,k-1}$  and  $L_{isT,k}$ ), the average waiting time in the system at the end of the time slice  $k$  by the expression

$$w_{iT,k} \approx \frac{1}{2Q_{ei,k}T_k} \left[ \frac{L_{isT,k}^2}{C_{i,k+1}} + (L_{isT,k} + L_{isT,k-1})T_k - \frac{L_{icT,k-1}^2}{C_{i,k}} \right] \quad (56)$$

and bearing in mind that for the Brilon model application, it results  $L_{isT,k} = L_{icT,k} + \rho_{i,k}f_0$ .

The evolution of  $w_{iT,k}$  obtained with Eq. (56) based on the progression of  $L_{isT,k}$ , in turn following each of the three “basic” models (Figs. 16 and 17), replicates what is obtained with the relative expressions of the coordinate transformation during the peak period with input over-saturation; for J4P7 the result of Eq. (56) best approximates that one given by IMC\* and SDES during over-saturation. Regarding the phases with a decreasing queue, the comparison among the results obtained with each of the three “basic” models, the deterministic estimate with Eq. (56) and the average value between IMC\* and SDES presents greater variability, showing however a sufficient degree of global approximation for Eq. (56).

## 9. Conclusion

After briefly discussing a few general aspects of non-stationary queue analysis and coordinate transformation method, the paper directs reader's attention to some time-dependent models for non-signalised intersections in the literature:

- Akçelik and Troutbeck model (Akçelik and Troutbeck, 1991), the simplest, proposed by the American HCM from 1994 until the last edition of 2015 and from the German HBS capacity manual in its latest edition of 2015.
- Kimber and Hollis model (Kimber and Hollis, 1979), among the oldest, proposed in the 2001 edition of the German HBS capacity manual.
- Brilon models (Brilon, 2007, 2008, 2015), among the most up-to-date models.

The discussion about Akçelik and Troutbeck model (1991) highlights some drawbacks related to its un-realistic hypotheses (zero traffic demand at the beginning/end and zero queue at the beginning of the observation period). In order to overcome its unrealistic statements, the paper re-proposes a

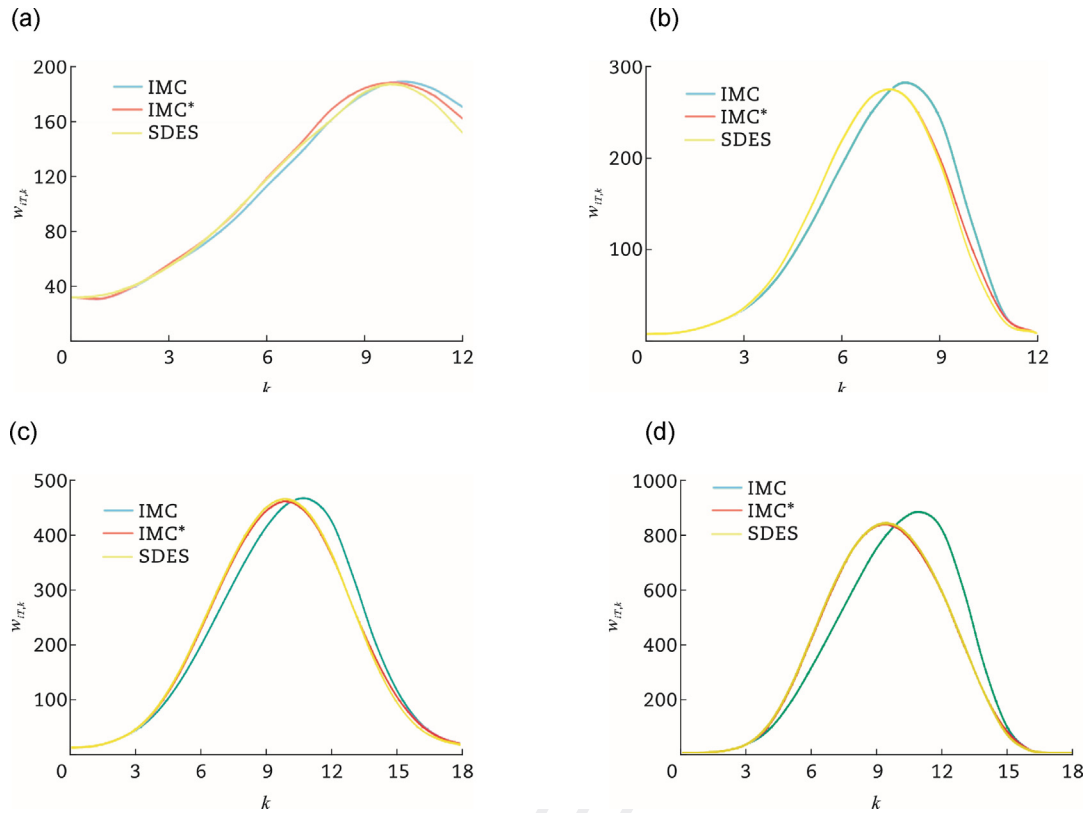


Fig. 14 –  $w_{T,k}$  for each time slice  $T_k$  according to: IMC; IMC\*; SDES. (a) J1P3. (b) J2P4. (c) J3P9. (d) J4P7.

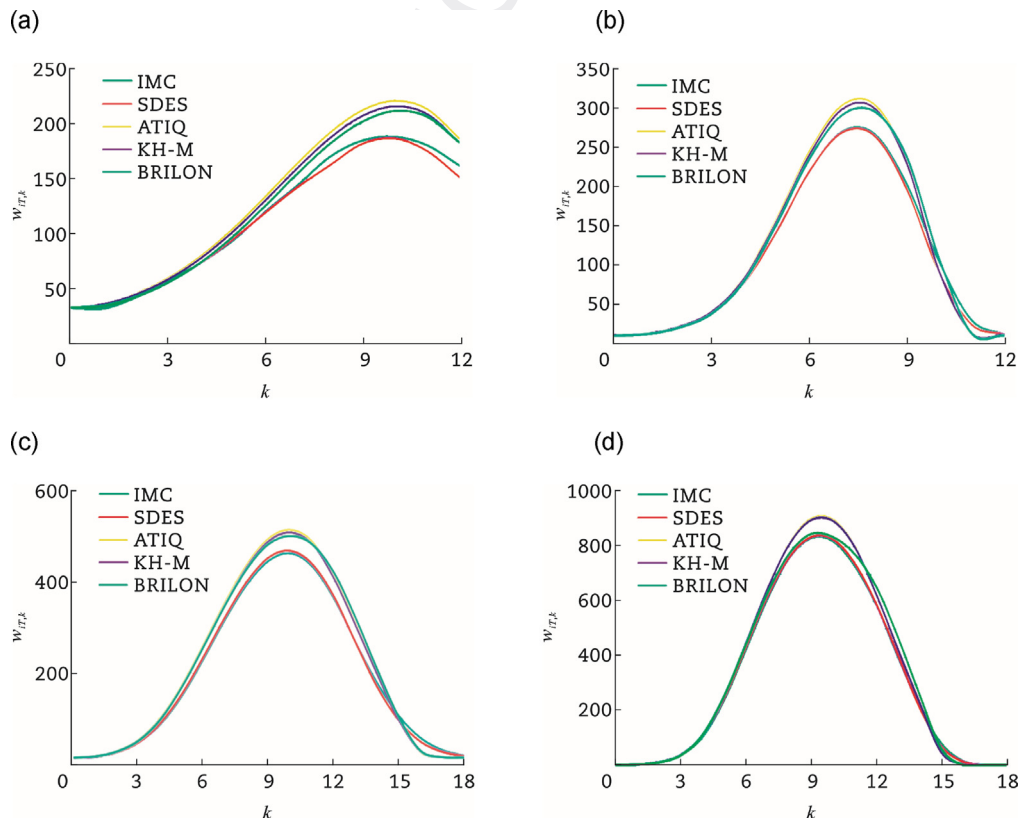


Fig. 15 –  $w_{T,k}$  for each time slice  $T_k$  according to: IMC\*; SDES; ATIQ; KH-M; BRILON. (a) J1P3. (b) J2P4. (c) J3P9. (d) J4P7.

Table 5 – Difference in  $w_{T,k}$  for each time slice  $T_k$  with respect to the average value for IMC\* and SDES.

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
J1P3	IMC*	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	ATIQ	0.0	8.4	7.1	6.2	8.7	12.1	14.9	15.8	16.1	18.2	20.3	19.4	–	–	–	–	–	–
	KH-M	0.0	7.7	5.4	3.9	6.2	9.3	12.1	13.1	13.6	15.7	18.0	17.5	–	–	–	–	–	–
J2P4	BRILON	0.0	2.0	–0.5	–1.9	0.7	3.1	5.0	8.3	10.5	13.3	16.4	17.0	–	–	–	–	–	–
	IMC*	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	ATIQ	0.0	1.6	3.1	6.9	10.6	12.3	13.1	15.0	17.2	–6.6	–60.7	–9.8	–	–	–	–	–	–
J3P9	KH-M	0.0	1.0	0.6	3.5	7.1	9.2	10.8	12.7	14.9	–6.0	–55.4	–9.2	–	–	–	–	–	–
	BRILON	0.0	–1.0	–2.7	–1.3	3.1	6.6	8.2	11.3	19.6	12.2	–53.2	–10.8	–	–	–	–	–	–
	IMC*	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SDES	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
J4P7	ATIQ	0.0	4.0	6.8	11.3	13.3	13.9	12.5	10.4	10.2	10.7	11.9	12.9	15.4	14.8	–4.9	–53.1	–51.9	–27.6
	KH-M	0.0	3.3	4.7	8.8	10.9	11.8	10.8	9.7	9.0	9.5	10.8	11.6	14.1	13.5	–4.6	–48.5	–50.3	–27.3
	BRILON	0.0	0.2	0.8	4.9	8.4	10.4	9.7	8.5	7.3	7.6	10.4	15.1	20.2	20.0	1.8	–46.4	–51.4	–29.1
	IMC*	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
J4P7	SDES	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	ATIQ	0.0	–2.0	1.3	8.0	8.4	5.5	5.9	6.9	8.2	9.0	9.2	6.6	7.1	8.4	–39.1	–66.3	–15.1	–4.9
	KH-M	0.0	–2.4	–0.5	5.1	6.1	4.3	5.0	6.1	7.4	8.3	8.4	5.8	6.2	7.5	–35.5	–64.5	–15.0	–4.9
	BRILON	0.0	–3.8	–3.1	1.6	5.0	3.7	3.0	2.3	1.9	1.7	5.4	11.3	17.6	22.0	–13.6	–64.5	–16.0	–6.0

less widespread sophistication of the model with an initial queue (Akçelik and Troutbeck, 1991; Cvitanic et al., 2007; Heidemann, 2002; Troutbeck and Brilon, 2000). Kimber and Hollis model (Kimber and Hollis, 1979) appears more realistic than Akçelik and Troutbeck's (1991), but it also has a drawback with the hypothesis of final traffic, capacity and queue length values coinciding with the initial ones. The models by Brilon (2007, 2008, 2015) consider more realistic formulations for immediate practical use, with some variations in the observation period and initial/final entry demand and capacity.

As for the first two models the paper focuses on the problem of dimensional inconsistency of the equations when traffic and capacity are alternately expressed in vehicles or pcu per hour (Wu, 2006, 2009). The same problem emerges in the Brilon models, thus highlighting how inconsistent the results are in terms of vehicular equivalence coefficients (vehicles to pcu, as usually reported in international manuals).

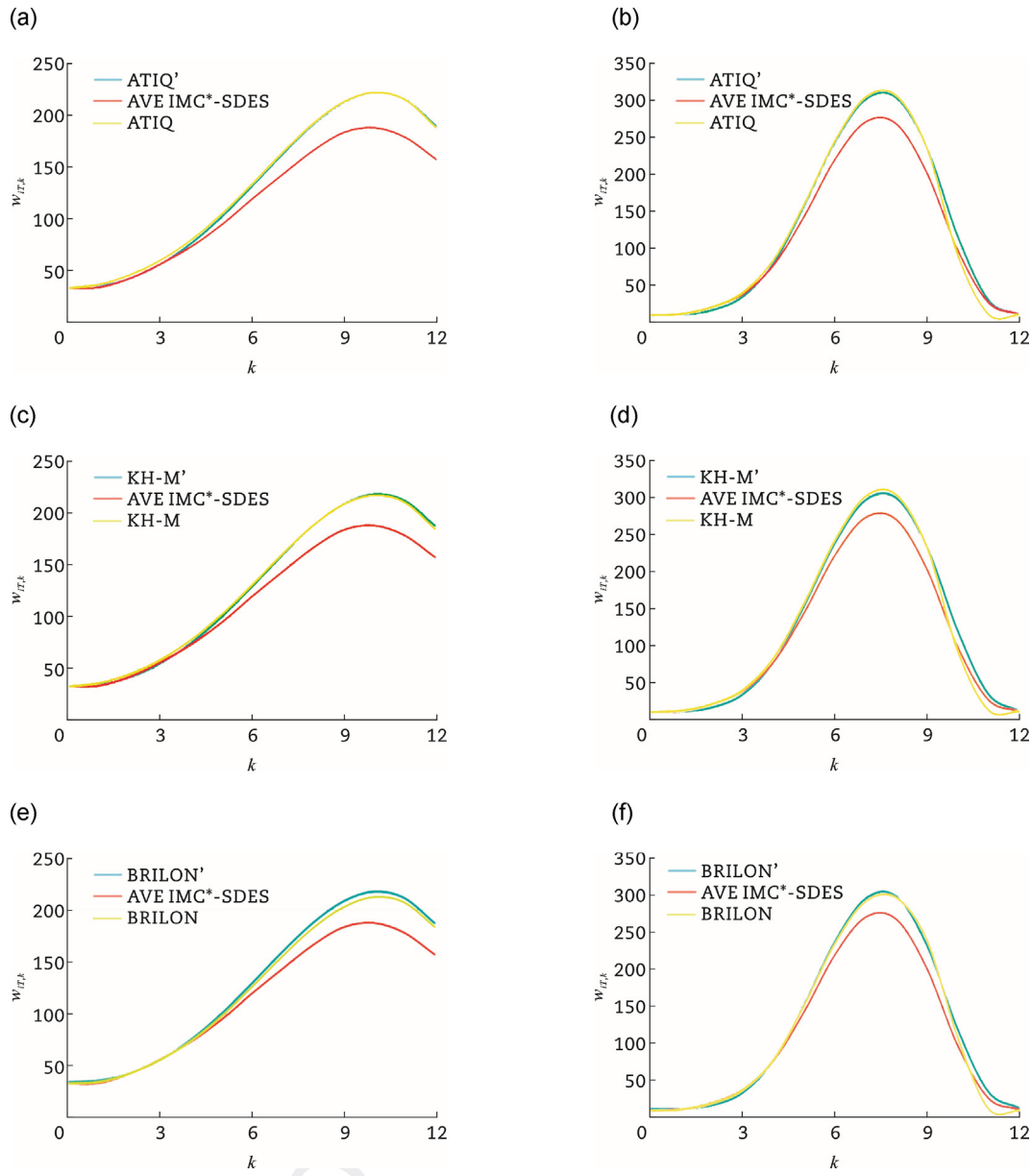
Some computational aspects in temporal evolution profiles for the coordinate transformation method are therefore addressed in order to show that any entry demand and capacity profile can be approximated through the use of “basic” models adequately inferred and applied in succession over several consecutive time slices.

Compatibility features for treating time evolution profiles on the entry demand and capacity appear for the Akçelik and Troutbeck model with the initial queue (Akçelik and Troutbeck, 1991; Cvitanic et al., 2007; Heidemann, 2002; Troutbeck and Brilon, 2000) and for the most recent among Brilon's models (Brilon, 2015). On the contrary, the original Kimber and Hollis model (Kimber and Hollis, 1979) cannot be directly used as a “basic” case for the calculations on consecutive intervals. A reformulation of the Kimber and Hollis model by Mauro (2010) is then adopted, in that it appears suitable as a “basic” case. The latter model, moreover, clarifies the equations for both the average waiting time in the system and the user number in the queue and in the system at the end of the observation period, necessary for recursive applications. Since this does not occur in the other two models, for the Akçelik and Troutbeck model with the initial queue the time-dependent number of vehicles in the system is inferred by applying the coordinate transformation method, while for the Brilon model the equations by Mauro (2010) are re-proposed and readjusted.

The three pairs of equations of the “basic” models presented in this paper are then completed to be directly used for a sequential application at multiple time intervals. Furthermore, in all formulas the recurring dimensional problems in vehicle-pcu transformation are removed to develop an appropriate formulation of queue lengths and delays in both vehicles (veh/h) and passenger car units (pcu/h).

Two numerical approaches are then presented and developed for the time-dependent evaluation of the waiting system: the imbedded Markov chain (IMC) and the stochastic discrete event simulation (SDES). The results obtained by recursively applying the “basic” cases are compared with those got with IMC (namely, its deterministic realignment IMC\*) and SDES, considering four time-evolution profiles for the entry demand and capacity. The results show a substantial coincidence between the three “basic” models in





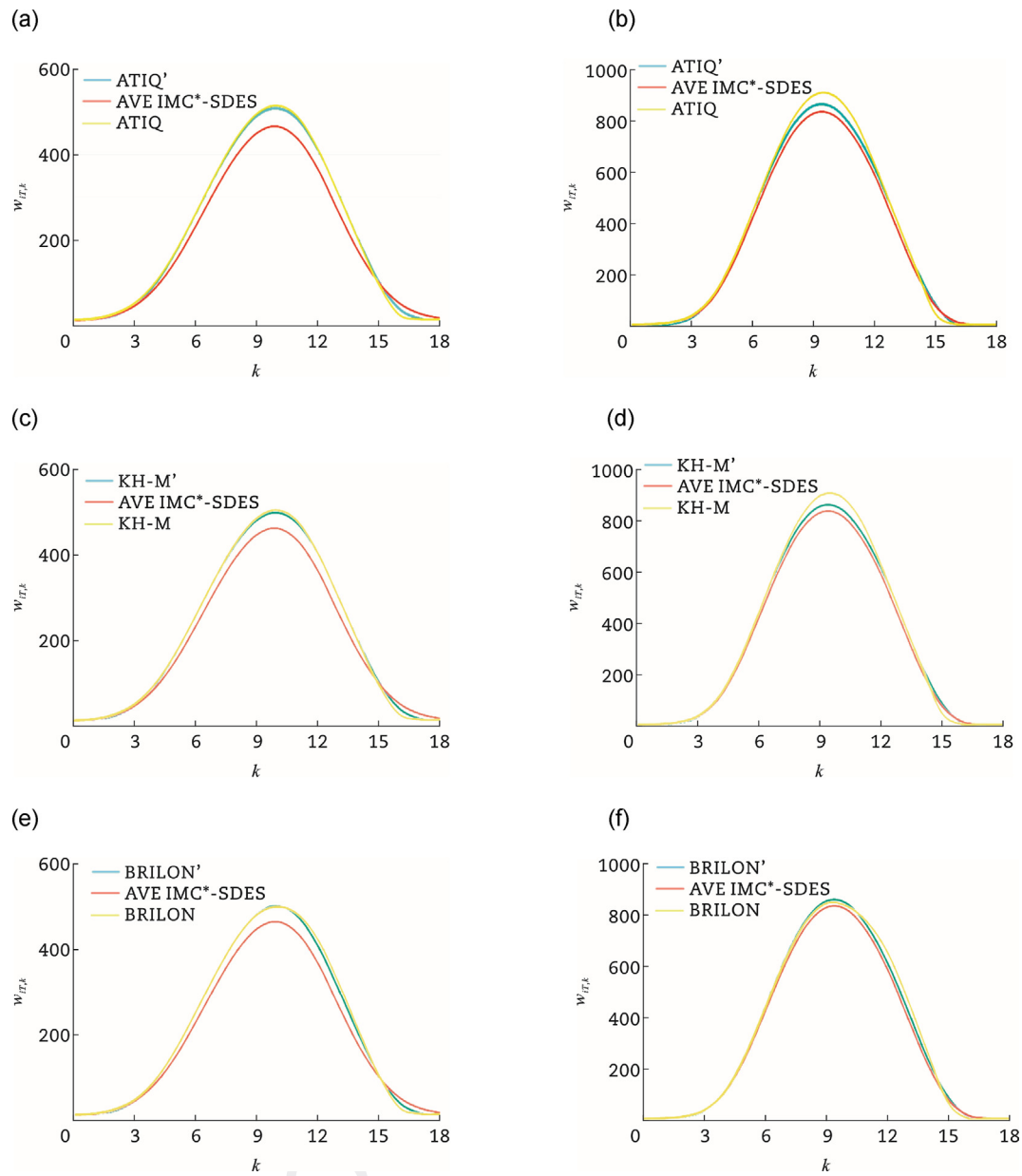
**Fig. 16 – Comparison of the estimated of  $w_{IT,k}$  for each time slice  $T_k$  with “basic” models (ATIQ, KH-M, BRILON) the average value for IMC\* and SDES (AVE IMC\*-SDES), and the deterministic approximation according to Eq. (56) (ATIQ', KH-M', BRILON'). (a) ATIQ for J1P3. (b) KH-M for J1P3. (c) BRILON for J1P3. (d) ATIQ for J2P4. (e) KH-M for J2P4. (f) BRILON for J2P4.**

estimating the evolution of vehicle number in the system for each profile, with an overestimation during over-saturation intervals and, conversely, with an underestimation in the final phase with a decreasing queue. The Brilon model (2015), however, proves to be closer to IMC\* and SDES results, with differences in the increasing queue phases in that they are smaller than the other two “basic” models. This is observed after the introduction of the correction term which evaluates capacity changes from the current to the next time slice. In decreasing queue situations, leading to the stabilisation of the arm on under-saturated levels, the time-dependent models show very similar trends, thus highlighting some underestimation with respect to reference values.

Finally, in terms of average waiting time in the system, the comparison between the average value for IMC\* and SDES and

those obtained with each “basic” model has been proposed, directly by using the relative equations as well as indirectly by proposing a common approximation based only on the initial and final values of queued users for each time slice. This comparison shows how in the test cases the deterministic estimate of the average waiting time can represent a good approximation of the relative value calculated with the coordinate transformation method.

As a conclusion to the general discussion, the research highlights how the simplified equations used for delay calculations at non-signalised intersections - proposed by international manuals (i.e., HCM and HBS in different editions), inferred under definitely unrealistic assumptions and showing some problems of appropriateness and dimensional homogeneity - can be replaced by closed



**Fig. 17 – Comparison of the estimated trend of  $w_{IT,k}$  for each time slice  $T_k$  with “basic” models (ATIQ, KH-M, BRILON) the average value for IMC\* and SDES (AVE IMC\*-SDES), and the deterministic approximation according to Eq. (56) (ATIQ', KH-M', BRILON'). (a) ATIQ for J3P9. (b) KH-M for J3P9. (c) BRILON for J3P9. (d) ATIQ for J4P7. (e) KH-M for J4P7. (e) BRILON for J4P7.**

formulas for queue sizes and waiting times that are more suitable to capture typical aspects of real-world traffic conditions.

The “basic” models discussed in the paper and based on the widespread coordinate transformation method take an initial queue under time-dependent flows and capacities into account, and also consider any transient over-saturation; but especially they confirm the requirements of easy computability and significant versatility for the treatment of any temporal profile of demand and capacity at non-signalised intersections.

The closed-form equations for transient queue assessment are clarified in the paper by taking models from the literature that have been completed and perfected for the purposes of

the research; they show a certain sophistication that, however, does not hinder their applicability in technical cases. These formulas and the common deterministic approximation for delays shown in this paper give acceptable predictions under time-variant demand and capacity, also including conditions of transient over-saturation during traffic peak periods. A congruent formulation of queue lengths and delays using alternatively vehicular units or passenger car units for traffic volumes is also guaranteed.

Together with the aforementioned use for the quantification of MOEs in determining the LOS, if we replace the formulas currently used for getting results closer to real cases, the three pairs of recursive equations can be used in many other traffic engineering applications for the design

and management of non-signalised intersections. These applications range from the quantitative characterization of the network for traffic demand assignment to the design choices for their geometric specification and cost-benefit analyses.

It is worth pointing out that in the current formulation the three pairs of recursive equations allow us to treat only the vehicular component of the traffic, but they represent a useful starting point for investigating situations of coexistence and interaction between different traffic components. From this point of view, in fact, in-depth studies and interesting research analysis may cover some emerging topics on intersection design and management in transport engineering.

For instance, special consideration deserves the analysis of the interactions in terms of both vehicular and pedestrian traffic (Helbing and Molnar, 1998; Gorrini et al., 2018; Zeng et al., 2014) with the aim of optimizing the organization of intersections with users' safety and performance objectives.

Another research area to develop in transport engineering may regard the connected vehicles (CVs) in intelligent transportation system (ITS) environments. The new technologies for information exchange between the various elements of the transport system - generically identified with vehicle to everything (V2X) - will awaken a growing interest for efficient and safe intersection management systems (Guler et al., 2014; Xu et al., 2018; Yang et al., 2016). From this point of view, an investigation field with expected applications could concern the evaluation of the effects determined by CVs platoons in a situation of mixed traffic with traditional vehicles, as well as the intersection performance in terms of both safety and level of service (i.e., capacity, queue lengths and waiting times).

### Conflict of interest

The authors do not have any conflict of interest with other entities or researchers.

### Uncited references

Kouvatsos and Tantos (2006); TRB, 1994; TRB, 1997; TRB, 2000; TRB, 2010; TRB, 2015.

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