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A convex optimization-based inversion method for the synthesis of monopulse linear arrays

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Abstract. In this work, the design of monopulse linear arrays generating sum and difference patterns with a partially shared beam forming network is addressed as an inverse problem and solved through a suitable inversion strategy. The array tails are organized into sub-arrays controlled by amplitude weights, while the remaining array elements have two independent amplitude coefficients, one for each beam. The optimal values of both common and independent excitations are determined by means of a convex optimization technique able to retrieve the globally optimal solution whether existing. Such a synthesis method allows one to include user-defined upper power bounds on the secondary lobes as well as to arbitrarily decide the layout (i.e., the number and the sizes of the sub-arrays) of the common part. Representative results are reported and discussed to assess the effectiveness and the efficiency of the proposed technique.

1. Introduction

Monopulse radars require antenna systems able to generate two kinds of beam patterns, sum and difference, which are jointly used to track the position of moving targets [1]. The excitations of the array elements that provide good monopulse performance are different for the sum (e.g., Taylor) and difference (e.g., Bayliss) mode [2]. Accordingly, two sets of independent beam-forming weights would be necessary, unavoidably increasing the antenna complexity and costs [3].

To overcome such a problem, compromise architectures have been proposed to yield optimal trade-off between the complexity of the array structure and the performances of the radiated sum and difference patterns. In such a framework, it is worth mentioning the solution with independent excitations for the sum pattern, while the difference one is generated by aggregating the elements into sub-arrays and using amplifiers at the sub-array level to reduce the overall number of control points [4][5].

In [6], a method for synthesizing sum and difference patterns in continuous linear apertures having a common aperture zone has been proposed. The approach was inspired by the observation that in some cases the current distributions generating the sum and difference patterns are similar in the outer part of the aperture and it is based on suitable perturbations of the Bayliss roots [2] defined by means of a Simulated Annealing technique. Such a method has been successively used for discrete arrays by clustering the elements not in common into sub-arrays properly weighted to generate the difference pattern [7].



The case of linear arrays has been solved in [8] with a Convex Programming (*CP*) procedure whose main advantages are: (*i*) the formulation of the problem as the minimization of a linear function over a convex sets which guarantees the retrieval of the globally optimal solution if existing; (*ii*) an easy inclusion of arbitrary upper power bounds on the secondary lobes of the two antenna modes; (*iii*) the deterministic behaviour (*CP*-based [9][10]) of the optimization technique.

In this work, the method in [10] is customized to deal with simplified linear array architectures where, unlike [7], the sub-arrays are in the array tails. Because of the linearity of all the synthesis constraints [11], a Linear Programming (*LP*) procedure is exploited since more computationally efficient than the *CP*.

2. Mathematical Formulation

Let us consider a N -element linear array arrangement with electromagnetic radiators uniformly spaced by d along the x -axis. The array factor of the sum mode is given by

$$AF_{\Sigma}(\vartheta) = \sum_{n=-M; n \neq 0}^M \alpha_n \exp \left\{ j \left[k \left(n - \frac{\text{sgn}(n)}{2} \right) d \sin(\vartheta) \right] + \varphi_n \right\} \quad (1)$$

while that of the difference mode turns out to be

$$AF_{\Delta}(\vartheta) = \sum_{n=-M; n \neq 0}^M \beta_n \exp \left\{ j \left[k \left(n - \frac{\text{sgn}(n)}{2} \right) d \sin(\vartheta) \right] + \varphi_n \right\} \quad (2)$$

where α_n and β_n , $n = \pm 1, \dots, \pm M$ are the amplitude coefficients generating the sum and difference patterns, respectively, characterized by a symmetric (i.e., $\alpha_n = \alpha_{-n}$) and an anti-symmetric (i.e., $\beta_n = -\beta_{-n} = \beta_{-n} \exp \{j\pi\}$) distribution. In (1) and (2), φ_n , $n = \pm 1, \dots, \pm M$ are the phase weights enabling the beam steering for the target tracking, $k = \frac{2\pi}{\lambda}$, λ being the free-space wavelength, and ϑ the angular direction measured from broadside. Let the sub-set of elements in the array tails having $|n| \geq \eta$, $\eta \in [1, M-1)$ being an integer threshold, be clustered into Q sub-arrays ($Q/2$ for each half of the array) made of C ($C \geq 2$) elements

$$\alpha_{n+(q+1)C} = \alpha_{n+(q+1)C+c}; \quad q = 1, \dots, Q; \quad c = 1, \dots, C-1 \quad (3)$$

$$\beta_{n+(q+1)C} = \beta_{n+(q+1)C+c}; \quad q = 1, \dots, Q; \quad c = 1, \dots, C-1. \quad (4)$$

Moreover, the same sub-array amplitude weighting is used for both the sum and difference patterns such that

$$\alpha_n = \beta_n \quad \text{for} \quad |n| \geq \eta. \quad (5)$$

It is worth noticing that both constraints (3)-(5) are linear with respect to the problem unknowns. The synthesis of the excitations generating the sum and difference patterns is then addressed as an inverse problem, by minimizing the cost function

$$\Psi(\alpha_n; n = \pm 1, \dots, \pm M) = -\text{Re} \{ AF_{\Sigma}(\vartheta_0) \} \quad (6)$$

subject to the additional constraints for the difference pattern

$$\text{Im} \left(\frac{\partial AF_{\Delta}}{\partial \vartheta} \right) \Big|_{\vartheta=\vartheta_0} > A \quad (7)$$

$$AF_{\Delta}(\vartheta_0) = 0 \quad (8)$$

and the upper power bounds along a set of S angular directions in the sidelobe region for both beams

$$-\sqrt{SLM_{\Sigma}(\vartheta_s)} \leq AF_{\Sigma}(\vartheta_s) \leq \sqrt{SLM_{\Sigma}(\vartheta_s)}; \quad s = 1, \dots, S \quad (9)$$

$$-\sqrt{SLM_{\Delta}(\vartheta_s)} \leq AF_{\Delta}(\vartheta_s) \leq \sqrt{SLM_{\Delta}(\vartheta_s)}; \quad s = 1, \dots, S. \quad (10)$$

where SLM_{Σ} and SLM_{Δ} denote the sidelobe masks for the sum and difference patterns, respectively (i.e., the maximum power admitted in the sidelobes directions). More in detail, Equation (6) is devoted to enforce the maximization of the amplitude and, as a consequence, the peak power of the sum beam along the target direction ϑ_0 , while (7) and (8) impose that the slope of the difference pattern is higher than a user-defined threshold, A , fixed on the basis of the desired angular resolution, and the beam has a null along ϑ_0 , respectively.

Since (6) is linear with respect to the excitation amplitudes of the sum mode and all other constraints are linear either with respect to α_n or β_n , $n = \pm 1, \dots, \pm M$, the synthesis problem turns out being a *LP* one and the *linprog* MATLAB routine is used for its solution.

3. Numerical Results

Two representative results are reported to validate the proposed *LP* synthesis method. In the first example, an array of $N = 16$ elements and $d = \lambda/2$ has been taken into account with 50 % (i.e., $\eta = 5$) of the array aperture shared between the two modes and $A = 10$. In the common part, the array elements are clustered into $Q = 4$ sub-arrays of $C = 2$ elements each.

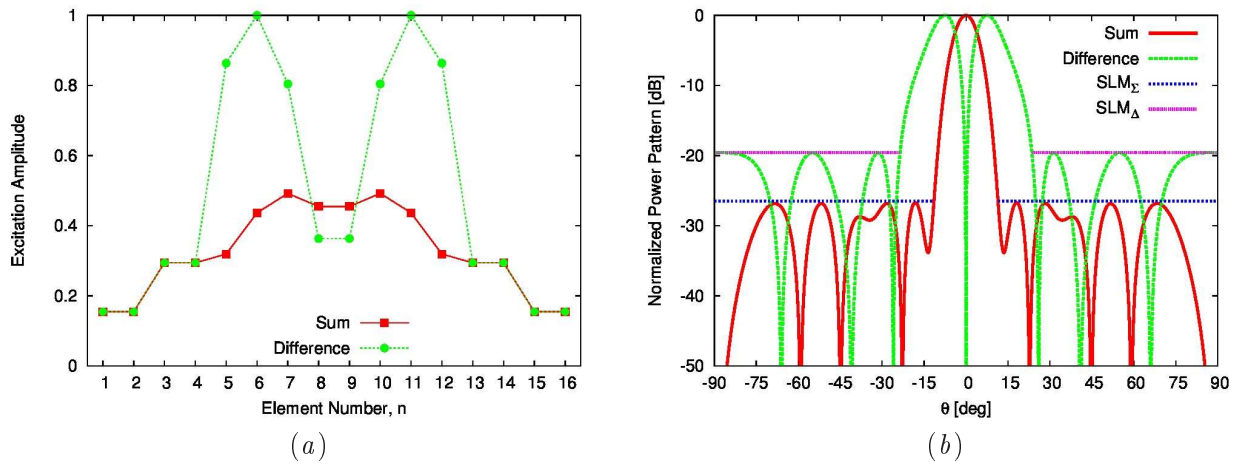


Figure 1. *Numerical Assessment* ($N = 16$, $Q = 4$, $C = 2$) - Synthesized sum and difference modes with 50% of common amplitude coefficients; (a) Amplitude coefficients; (b) Power patterns.

The excitations of the sum and difference modes along with the corresponding normalized power patterns are shown in Fig. 1(a) and Fig. 1(b), respectively, while the key features of the two beams are given in Tab. 1. As it can be observed [Fig. 1(b)], even though half aperture has common control points, a sum and a difference pattern with peak sidelobe level equal to $SLL_{\Sigma} = -26.71$ [dB] and $SLL_{\Delta} = -19.60$ [dB], respectively, have been synthesized¹.

¹ SLL_{Σ} and SLL_{Δ} denote the maximum power in the sidelobes directions for the sum and difference patterns, respectively.

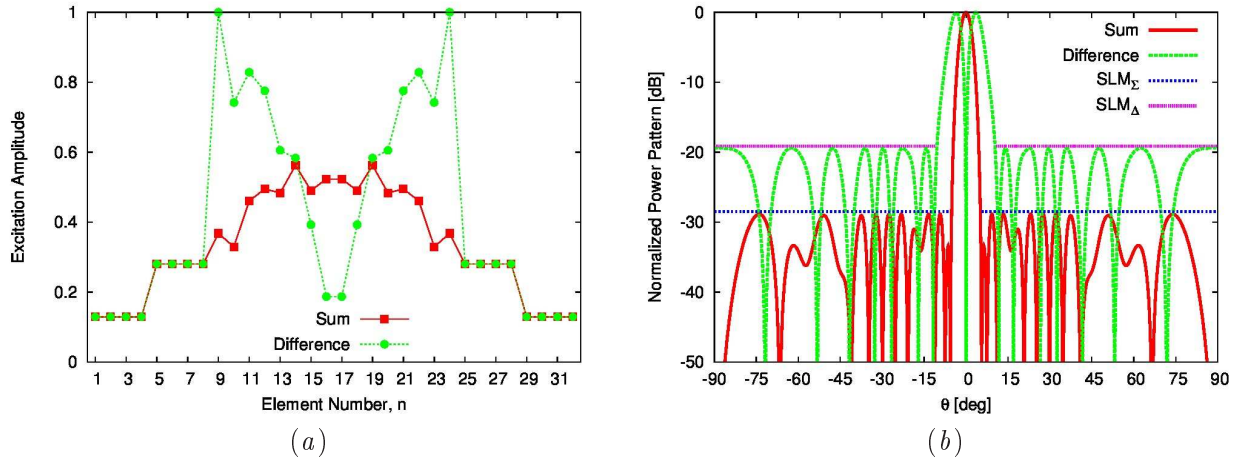


Figure 2. *Numerical Assessment* ($N = 32$, $Q = 4$, $C = 4$) - Synthesized sum and difference modes with 50% of common amplitude coefficients; (a) Amplitude coefficients; (b) Power patterns.

In the second example, a larger array with twice elements ($N = 32$) and same inter-element distance d has been considered. Also in this case, half of the array aperture has been shared, but here the following setup has been chosen: $\eta = 9$, $Q = 4$, and $C = 4$. The solution of the LP -based synthesis ($A = 20$), given in Fig. 2 and Tab. 1, still assesses the reliability and effectiveness of the proposed method.

Table 1. Pattern features.

	Sum Pattern	Difference Pattern
$N = 16$, $Q = 4$, $C = 2$		
D_{\max} [dB]	11.01	7.57
SLL [dB]	-26.71	-19.60
θ_{3dB} [dB]	8.56	9.27
K [rad ⁻¹]	-	0.68
$N = 32$, $Q = 4$, $C = 4$		
D_{\max} [dB]	11.25	10.44
SLL [dB]	-28.85	-19.40
θ_{3dB} [dB]	4.07	4.03
K [rad ⁻¹]	-	0.97

From a computational point of view, it is worth pointing out that both syntheses have been carried out in less than 1 [sec] on a standard laptop (3 GHz PC with 2 GB of RAM memory).

4. Conclusions

The synthesis of sum and difference patterns in linear arrays with common excitation weights in the array tails has been addressed by means of a LP -based inversion computational method. To simplify the complexity of the feeding network, sub-arrays are used in the common part of the array aperture. Besides its effectiveness and the computational efficiency, the main advantage of the proposed approach is the possibility to efficiently synthesize discrete arrays having arbitrary upper bounds and feeding layout.

Acknowledgments

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