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From the *dicey world* to the physical laws: dice toy models for bridging microscopic and macroscopic understanding of physical phenomena

Pasquale ONORATO, Luigi M GRATTON, Stefano OSS

Department of Physics, University of Trento - Povo (TN), Italy

Massimiliano MALGIERI

Department of Physics, University of Pavia - Pavia, Italy

Abstract. We discuss an educational approach to some different physical phenomena that can be explained by means of stochastic toy models, and explored by students with rolling dice. The discussion of the physical principles governing the phenomena, proceeds through a recurrent comparison between the outcomes obtained with the toy models and the results of real experiments.

The educational proposal allows students to compare experimental data they obtain to both analytical results and simulations.

The effectiveness of our approach was tested both with groups of undergraduate students and with a group of on service teachers.

1. Introduction

In this contribution to the conference we resume a series of papers sharing the common idea of describing microscopic phenomena by means of non-deterministic rules of game. Thus we discuss an educational approach to several different physical phenomena that can be explained with the help of stochastic toy models, and explored by students with rolling dice. The discussion of the physical principles governing the phenomena, proceeds through a recurrent comparison between the outcomes obtained with the toy models and the results of real experiments which students can made in a laboratory by themselves by using simple and inexpensive apparatuses.[1]

The educational proposal, grounded on a microscopic basis, goes on by considering the problem in statistical and probabilistic terms, and allows students to compare experimental data they obtain to both analytical results and simulations.

We analyze phenomena concerning different branches of physics from thermodynamics to optics, from material science to nuclear physics. We discuss some simple experiments easy to be carried on by undergraduates:

- the measure of the exponential decay and the lifetime of the photoluminescent compounds contained in the coating of fluorescent compact lamp;
- the measure of the Beer-Lambert's law which relates the attenuation of light to the properties of the material through which the light is travelling;
- the experiment on thermal contact of two masses of water at different temperature reaching the thermal equilibrium.

Each experiment can be explained by using an "ad hoc" stochastic toy model which can be realised using dice and directly explored by students as a real experiment.

In Figure 1 we summarize the approach.

- 1) The students' activity starts from a real experiment which students can made in a laboratory by themselves by using simple and inexpensive apparatuses. This experiment can be supported or



replaced by a simulated experiment. Data analysis allows the discovery of phenomenological laws able to explain the experiment.

- 2) A stochastic toy model, based on dice, is introduced and students explore it. The toy model highlights the microscopic basis of the measured phenomenon, it goes on by considering the problem in statistical and probabilistic terms, and allows students to compare experimental data they obtain to both analytical results and simulations.
- 3) The discussion of the physical principles governing the phenomena, proceeds through a recurrent comparison between the outcomes obtained with the toy models and the results of real experiments.

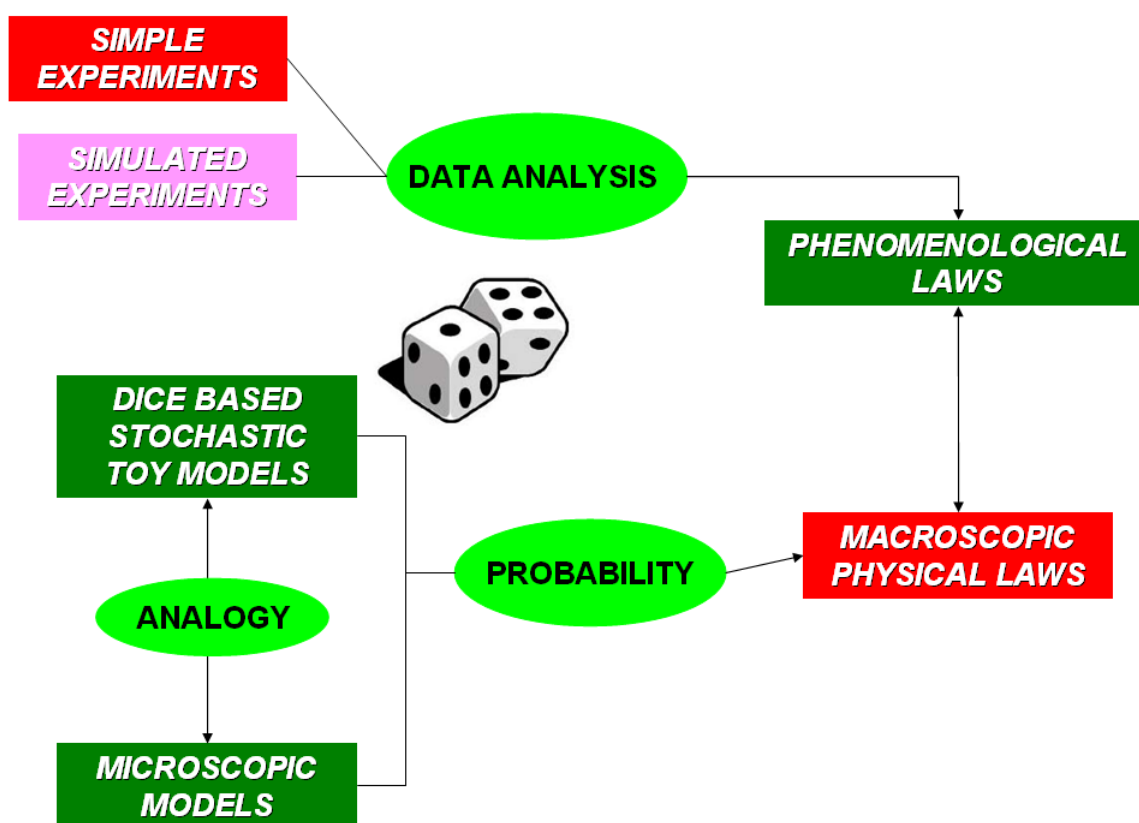


Figure 1. Schematic summary of the approach.

Some of the activities discussed in this work were tested both with a group of undergraduate students of the University of Trento and with a group of on service teachers.

2. The exponential decay

Dice rolling [2] is a famous and useful pedagogical tool to introduce students to the concepts of radioactivity. In fact phenomena such as radioactivity and fluorescence are typically discussed with students using the "radioactive dice" model and game [3,4,5], which efficiently allows to achieve the exponential decay describing phenomena ruled by "first-order kinetics".

With the aim of using the radioactive dice game to allow a comparison between the results of the toy model and the outcomes of a real experiment we propose an experimental activity on fluorescent decay. We choose the study of fluorescence because the use of a radioactive source is potentially hazardous and in general quite expensive. However, also without using a dangerous radioactive

source, experiments about exponential decay can be performed both by using simulated experiments [6,7] and by working in a Radioactivity Remote Laboratory.

The experiment about the exponential decay of photoluminescence is discussed in details in ref.[8] where we examined how some low cost apparatuses based on the use of sensors for didactic lab or commercial digital photo cameras can be employed to measure the exponential decay and the lifetime of the photoluminescent compounds contained in the coating of fluorescent compact lamp. The presence of different fluorescent materials, such Europium and Terbium, in the coating produces exponential decay with different lifetime and the intensity is described by a multi exponential law,

$$I(t) \approx I_{Tb} e^{-t/\tau_{Tb}} + I_{Eu} e^{-t/\tau_{Eu}} \quad (1)$$

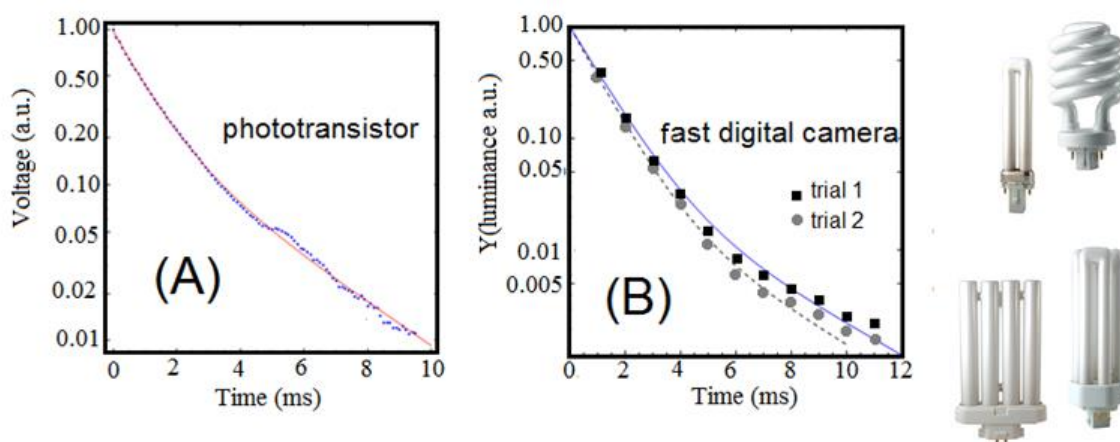


Figure 2. The multiple exponential decay from data acquired by a phototransistor (A) and by the Tracker analysis of a video (B) acquired with a fast digital camera[8].

In the experiment low cost apparatuses based on the use of sensors for didactic lab or commercial digital photo cameras are employed. In Figure 2.A we show the decay curve measured with phototransistor at 10 kHz in a semi-log scale and measurements are fitted using a double exponential model. The same results can be obtained by analyzing with Tracker[9] videos acquired by a fast digital camera at 1 kHz.

In the dice based stochastic toy model a large number of dice are thrown simultaneously. Those showing a particular number are deemed to have decayed like excited atoms. These dice are removed and the remaining ‘undecayed’ dice are counted. This number of ‘undecayed’ dice is recorded and represents the number of undecayed atoms remaining after a certain interval of time (obviously in a real experiment we measure the number of decayed nuclei or atoms). The ‘undecayed’ dice are then thrown and so on. This goes on for a number of throws, resulting in a reduction in the number of ‘undecayed’ dice as time goes by.

To reproduce the radiative decay of the fluorescent coating of a lamp we can use two different kinds of dice (e.g. as in Figure 3 20 32-side dice and 80 6-side dice) and plot the number of undecayed dice. At the end of the game students can compare results of experiments and outcomes of the game.

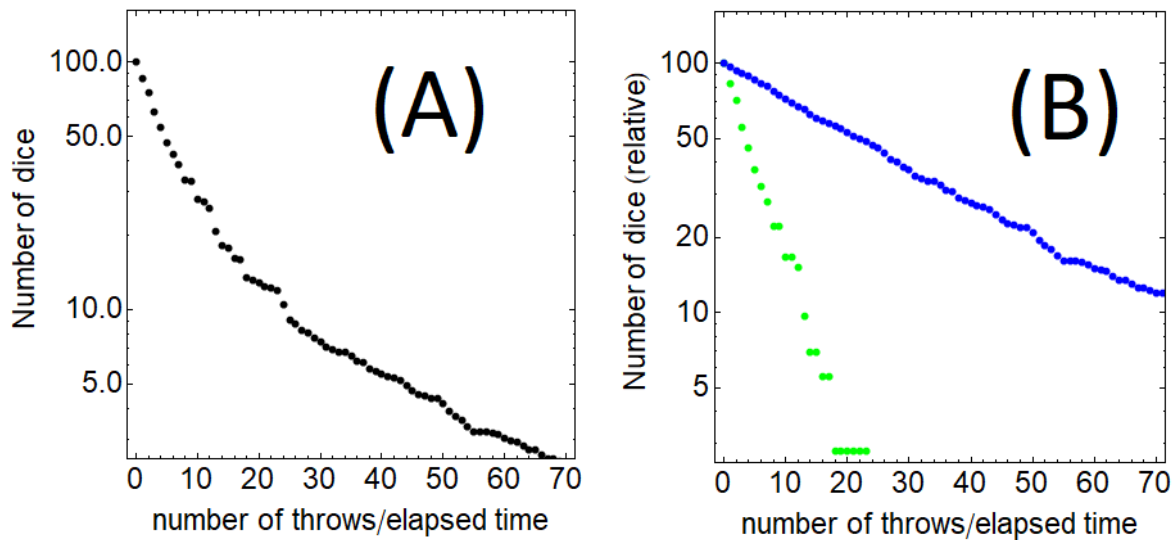


Figure 3 (A) Multiexponential decay of an ensemble of 20 32-side dice and 80 6-side dice. (B) the exponential decay of each family of dice. Results obtained as an average of 12 trials.

After the game students use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of exponential decay, as shown in Figure 4. Thanks to the laws of probability in the large number limit they deduce from the discrete stochastic toy model the continuous macroscopic physical law.

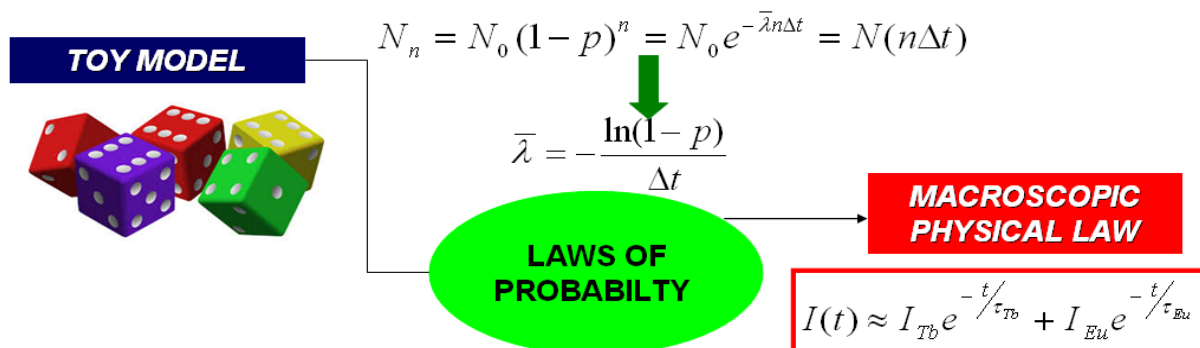


Figure 4 We show how students can use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of multiple exponential decay starting from the radioactive dice model.

3. The Beer-Lambert's law

In the second experiment students use a smartphone based apparatus as a tool for investigating the optical absorption of a material and to obtain the exponential decay predicted by the Beer's law.[610] In Figure 5 A and B we show the experimental set up: the light from a LED goes through a liquid soap in a glass put on a black cardboard with a window. The ambient light sensor of a smartphone measures the intensity while the thickness of the soap is increased with a graduate syringe. The Intensity versus thickness measured for different LEDs shows the typical exponential decay with a characteristic length strongly dependent on wavelength (Figure 5 E).

The same phenomenological law could be obtained by students by performing the simulated experiment[11] (Figure 5 D).

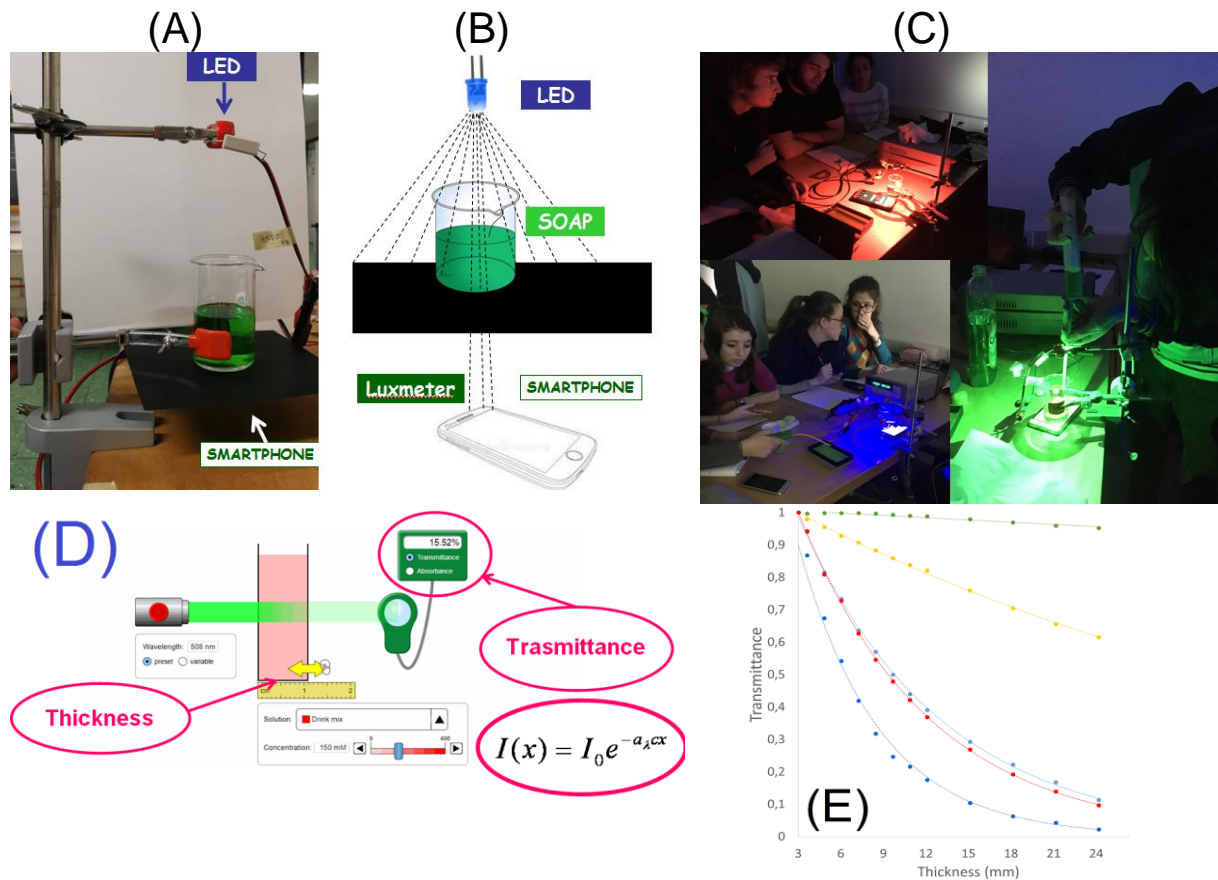


Figure 5 (A) and (B) Experimental set up. Light emitted by the LED goes through the liquid absorber in the glass (green soap). The transmitted light enter the smartphone sensor which has a very small area. The thickness of the liquid is increased about one millimeter at a time by using a graduate syringe. (C) Students added liquid in the glass and measured the intensity of light as a function of the thickness (D) The simulation of the experiment[11]. (E) Measurements obtained by students with different LEDs using green soap as absorber.

The toy model is based on the use of a game board, where incident photons are represented by rows of squares, in each square of the table can be placed an X i.e. microscopic scatterers are placed random according the roll of a dice (Figure 6). During the activity, students rolled the die many times, each launch corresponding to a column, and inserted an X in the box corresponding to the line corresponding to the extracted number. In this way they distributed the scatterers in a stochastic manner one by one. Finally they can infer from the simulation the exponential decay of transmitted light and the mean free path allowed to the simulated photons. Thus a corpuscular derivation of Beer's law is given emphasizing the stochastic laws which rule the microscopic world. The discussion of the law in these terms is advantageous because it provides more physical insight than the common approaches.[12]

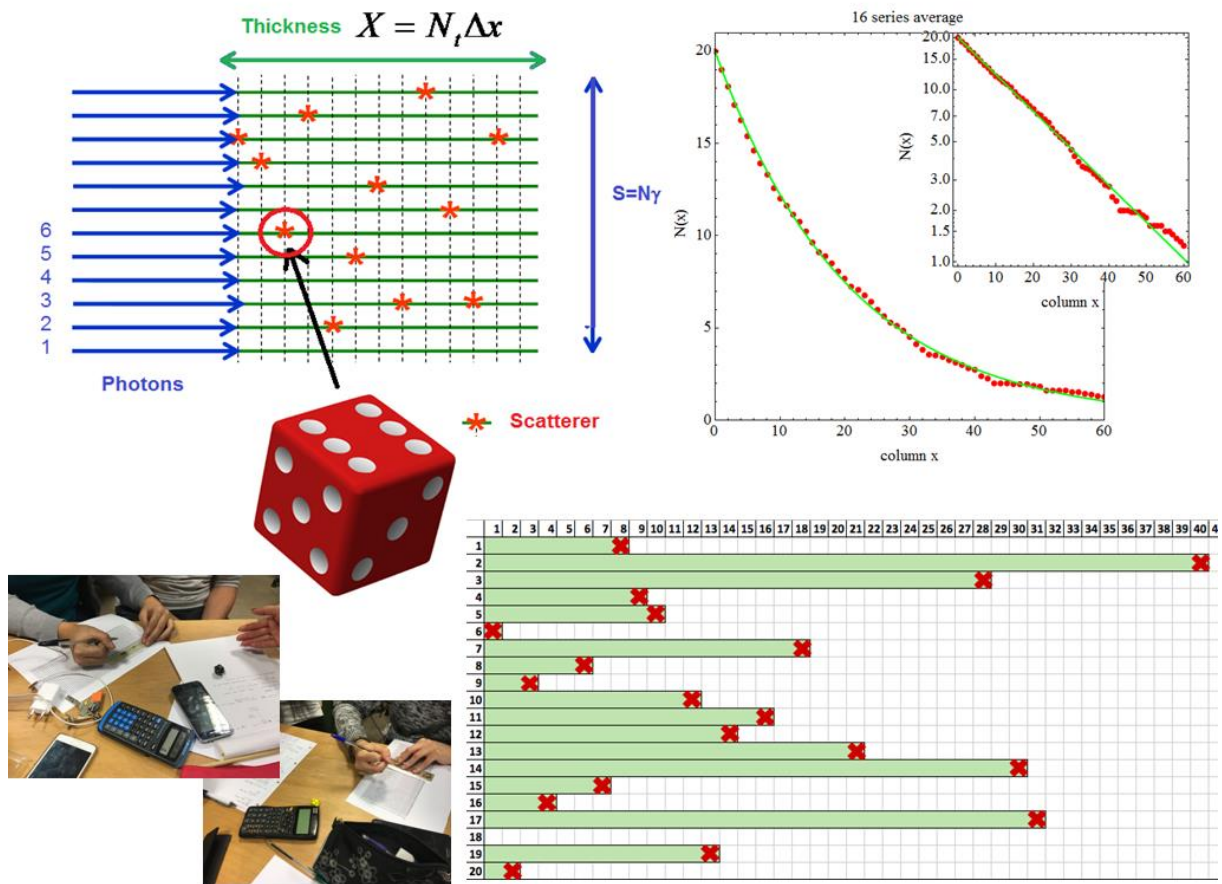


Figure 6 Model of the board with a grid where students introduce stochastically the scatterers (red Xs) and then compute the Intensity decay and the mean free path. Results obtained by students: number of filled boxes per column. The data are the average of the results obtained from 8 groups that have repeated each experiment twice. In the inset data reported on a logarithmic scale.

After the game students use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of the attenuation Law as shown in Figure 7.

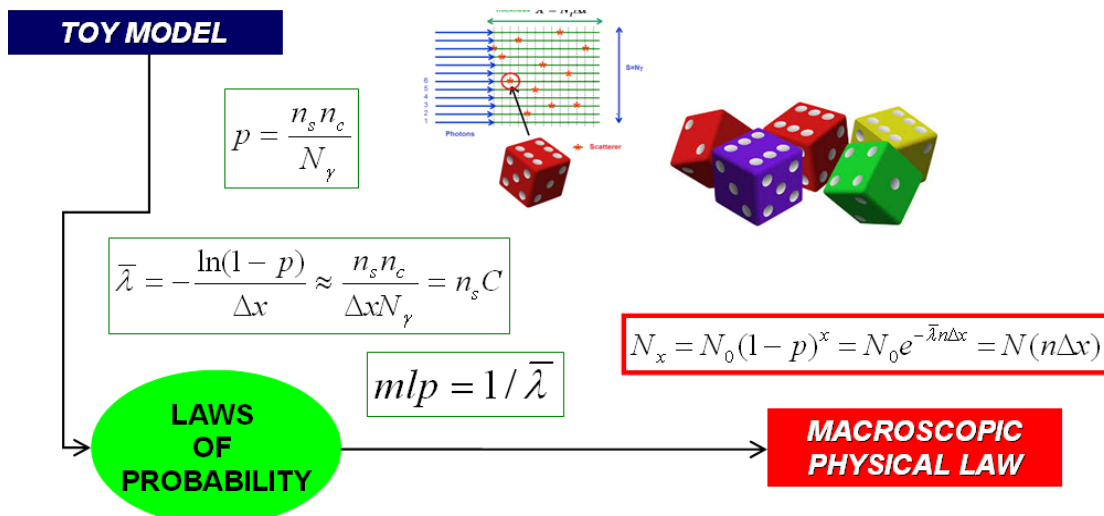


Figure 7 We show how students can use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of the exponential attenuation of transmitted light.

4. The thermal equilibrium.

In the third experiment students, using temperature sensors, measured the time evolution of two bodies in thermal contact. The experiment was carried out using a thermos filled with water at room temperature, and a small metal cylinder, immersed in the thermos water, in which students poured a small quantity of water at much higher high temperature.[1] (see Figure 7).

The experiment can be simulated thanks to a software as Energy2D[13] , as we show in Figure 7.

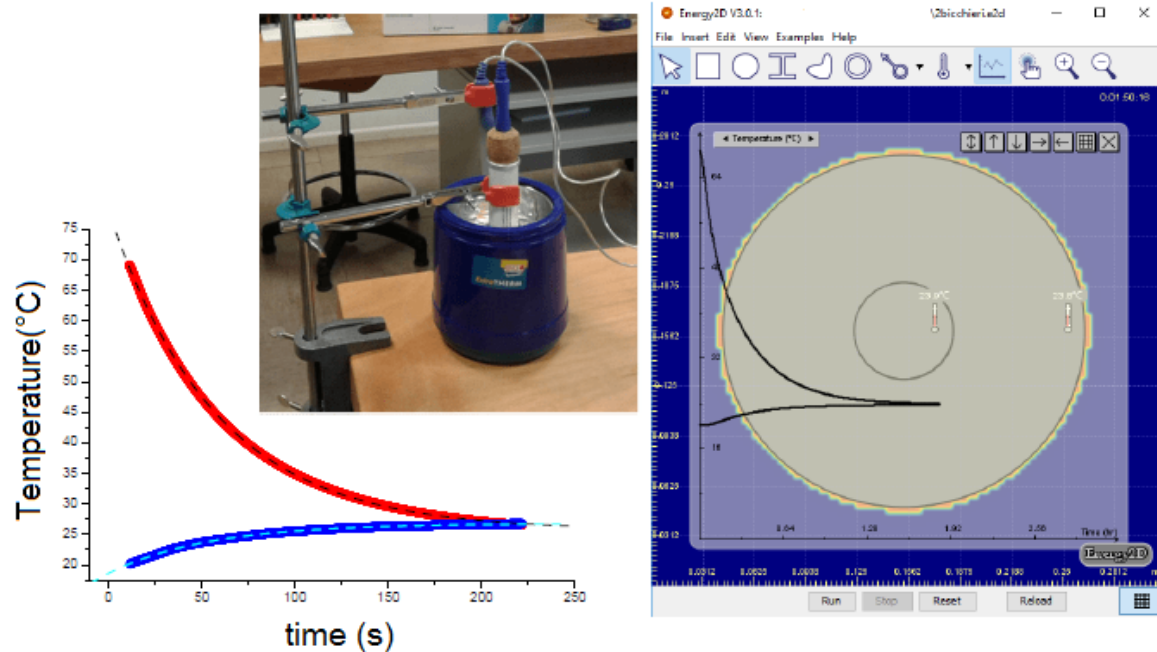


Figure 8 (Left. Inset) Experimental setup: a thermos filled with water at room temperature and a small metal cylinder which is filled with a smaller amount of water at a higher temperature. The metal cylinder is immersed in the thermos water. (Left) Typical time evolution of temperature in the two bodies of water measured by the students for the first 250 s. (Right) The same system simulated with Energy 2D[13].

The corresponding toy model is based on a board with two rows of different length of numbered squares; on each square of the row one coin (at most) can be placed. Bodies in thermal contact are represented by rows of squares on a cardboard table, which exchange coins placed on the squares based on the roll of two dice. Students can deduce from the model the exponential approach to equilibrium, the determination of the equilibrium temperature, the interpretation of the equilibrium state as the most probable macrostate.

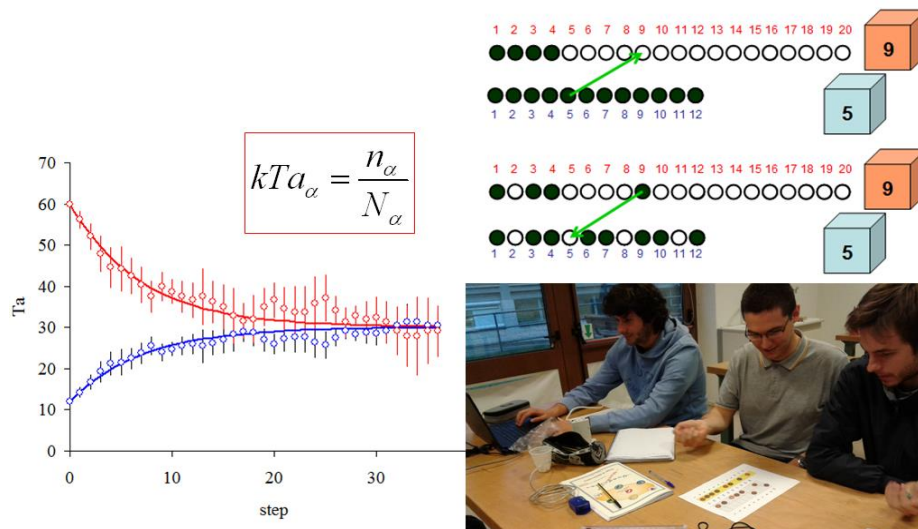


Figure 9 (Left) Data and exponential fit for the toy model experiment obtained by the students. Data are averaged over 12 repetitions of the dice rolling, (three for each of four student groups). (Right) Students during the activity of dice rolling. In the activity, each group has a cardboard with two rows of numbered squares, two dice and a fixed number of coins to place.

After the game students use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of the heat transfer and equilibrium law.

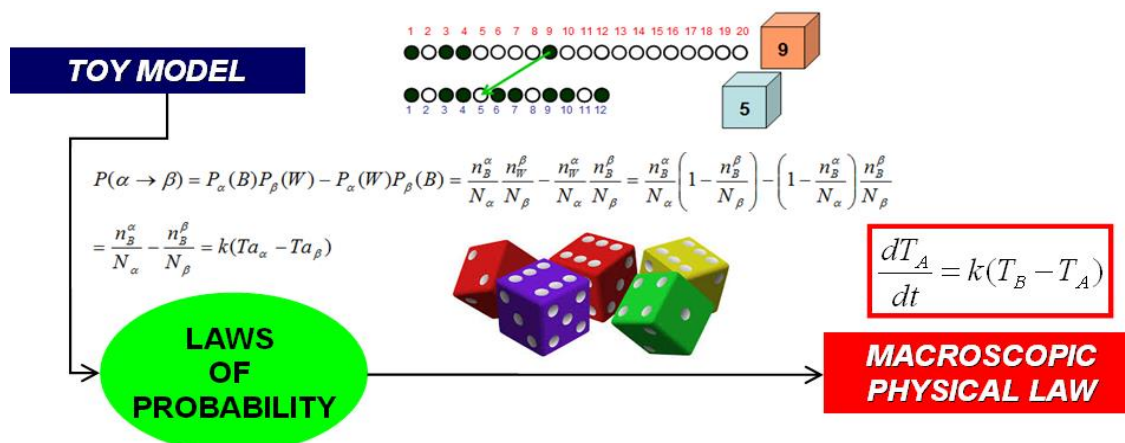


Figure 10 We show how students can use the probabilistic and statistic laws for bridging microscopic and macroscopic understanding of the law of heat transfer.

5. Conclusions

In this work we summarised a series of papers sharing the common idea of describing microscopic phenomena by means of non-deterministic rules of game. The approach, which we propose, based on corpuscular stochastic models is a simple and easily understandable tool that allows a derivation of many macroscopic physical laws. This derivation is mathematically much simpler than many others that have been proposed and is thought to be more pedagogically valid.

We tested the efficacy of our approach both with groups of undergraduate students and with a group of on service teachers.

We asked students what they think about the didactic use of stochastic models to describe the microscopic mechanism underlying macroscopic processes. Only few students (less than 10%) respondents said they felt confused and uncomfortable while others found the use of the dice "a fun and yet useful and intuitive way" to present the phenomena emphasizing the "simplicity" or the utility to "understand complex topics "and" what lies behind the phenomena ".

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